Foundations of a mathematical model of physical reality

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Last modified: 14 maart 2015

Abstract

This paper starts from the idea that physical reality implements a network of a small number of mathematical structures. Only in that way can be explained that observations of physical reality fit so well with mathematical methods.

The mathematical structures do not contain mechanisms that ensure coherence. Thus apart from the network of mathematical structures a model of physical reality must contain mechanisms that manage coherence such that dynamical chaos is prevented.

Reducing complexity appears to be the general strategy. The structures appear in chains that start with a foundation. The strategy asks that especially in the lower levels, the subsequent members of the chain emerge with inescapable self-evidence from the previous member. The chains are interrelated and in this way they enforce mutual restrictions.

As a consequence the lowest levels of a corresponding mathematical model of physical reality are rather simple and can be comprehended by skilled mathematicians.

In order to explain the claimed setup of physical reality, the paper selects a special foundation for the major chain. That foundation is a skeleton relational structure and it was already discovered and introduced in 1936.

The paper does not touch more than the first development levels. The base model that is reached in this way puts already very strong restrictions to more extensive models.

Some of the features of the base model are investigated and compared with results of contemporary physics.

If the model introduces new science, then it has fulfilled its purpose.

Contents

1 Introduction

Physical reality is that what physicists try to model in their theories. It appears that observations of features and phenomena of physical reality can often be explained by mathematical structures and mathematical methods.

This leads to the unorthodox idea that physical reality itself mimics a small set of mathematical structures. In that case physical reality will show the features and phenomena of these structures.

In humanly developed mathematics, mathematical structures appear in chains that start from a foundation and subsequent members of the chain emerge with inescapable self-evidence from the previous member. The chains are often interrelated and impose then mutual restrictions. It is obvious to expect a similar setup for the structures that are maintained by physical reality.

Physical reality is known to show coherence. Its behavior is far from chaotic. The mimicked mathematical structures do not contain mechanisms that ensure coherence. Thus apart from the network of mathematical structures a model of physical reality must contain mechanisms that manage coherence such that dynamical chaos is prevented. In physical reality, reducing complexity appears to be the general strategy.

One chain is expected to play a major role and its foundation can be viewed as the major foundation of the investigated model of physical reality. The discovery of this foundation is essential for explaining how the network of mimicked mathematical structures is configured.

2 The major chain

2.1 The foundation

This paper uses the skeleton relational structure that in 1936 was discovered by Garret Birkhoff and John von Neumann as the major foundation of the model. Birkhoff and von Neumann named it “quantum logic”[1].

The ~25 axioms that define an orthocomplemented weakly modular lattice form the first principles on which the model of physical reality is supposed to be built [2]. Another name for this lattice is ***orthomodular lattice***. Quantum logic has this lattice structure. Classical logic has a slightly different lattice structure. It is an orthocomplemented modular lattice. Due to this resemblance, the discoverers of the orthomodular lattice gave quantum logic its name. The treacherous name “quantum logic” has invited many scientists to deliberate in vain about the significance of the elements of the orthomodular lattice as logical propositions. For our purpose it is better to interpret the elements of the orthomodular lattice as ***construction elements*** rather than as ***logic propositions***.

The selected foundation can be considered as part of a recipe for modular construction. What is missing are the binding mechanism and a way to hide part of the relations that exist inside the modules from the outside of the modules. That functionality is realized in higher levels of the model.

2.2 Extending the major chain

The next level of the major chain of mathematical structures ***emerges with inescapable self-evidence*** from the selected foundation. Not only quantum logic forms an orthomodular lattice, but also the set of closed subspaces of an infinite dimensional separable Hilbert space forms an orthomodular lattice [1].

Where the orthomodular lattice was discovered in the thirties, the Hilbert space was introduced shortly before that time [3].

The Hilbert space adds extra functionality to this orthomodular lattice. This extra functionality concerns the superposition principle and the possibility to store numeric data in eigenspaces of normal operators. In the form of Hilbert vectors the Hilbert space features a finer structure than the orthomodular lattice has.

Numbers do not exist in the realm of a pure orthomodular lattice. Via the Hilbert space number systems emerge into the model. Number systems do not find their foundation in the major chain. Instead they belong to another chain of mathematical structures. The foundation of that chain concerns mathematical sets.

The Hilbert space can only handle members of a division ring for specifying superposition coefficients, for the eigenvalues of its operators and for the values of its inner products. Only three suitable division rings exist: the real numbers, the complex numbers and the quaternions. These facts were known in the thirties but became a thorough mathematical prove in the sixties [4].

Separable Hilbert spaces act as structured storage media for discrete data that can be stored in real numbers, complex numbers or quaternions. Quaternions enable the storage of 1+3D data that have an Euclidean geometric structure.

The confinement to division rings puts strong restrictions onto the model. These restrictions reduce the complexity of the whole model.

Thus, selecting a skeleton relational structure that is an orthomodular lattice as the foundation of the model already puts significant restrictions to the model. On the other hand, as can be shown, this choice promotes modular construction. In this way it eases system configuration and the choice significantly reduces the relational complexity of the final model.

3 Consequences of the currently obtained model

The orthomodular lattice can be interpreted as a part of a recipe for modular construction. What is missing are means to bind modules and means to hide relations that stay inside the module. This functionality must be supplied by extensions of the model. It is partly supplied by the superposition principle, which is introduced via the separable Hilbert space.

The current model does not yet support coherent dynamics. The selected foundation and its extension to a separable Hilbert space can be interpreted in the following ways:

- Each discrete construct in this model is supposed to expose the skeleton relational structure that is defined as an orthomodular lattice.
- Each discrete construct in this model is either a module or a modular system.
- Every discrete construct in this model can be represented by a closed subspace of a single infinite dimensional separable quaternionic Hilbert space.
- Every module and every modular system in this model can be represented by a closed subspace of a single infinite dimensional separable quaternionic Hilbert space.

The modular construction recipe is certainly the most influential rule that exists in the generation of physical reality. Even without intelligent design it achieved the construction of intelligent species.

4 Supporting continuums

The separable Hilbert space can only handle discrete numeric data. Physical reality also supports continuums. The eigenspaces of the operators of the separable Hilbert space are countable. Continuums are not countable.

Soon after the introduction of the Hilbert space scientists tried to extend the separable Hilbert space to a non-separable version that supports operators, which feature continuums as eigenspaces. With his bra-ket notation for Hilbert vectors and operators and by introducing generic functions, such as the Dirac delta function Paul Dirac introduced ways to handle continuums [5]. This approach became proper mathematical support in the sixties when the Gelfand triple was introduced [6].

Every infinite dimensional separable Hilbert space owns a Gelfand triple. In fact the separable Hilbert space can be seen as embedded inside this Gelfand triple. How this embedding occurs in mathematical terms is still obscure. It appears that the embedding process allows a certain amount of freedom that is exploited by the mechanisms, which are contained in physical reality and that have the task to ensure coherence.

In the separable Hilbert space the closed subspaces have a well-defined numeric dimension. In contrast, in the non-separable companion the dimension of closed subspaces is in general not defined. The embedding of subspaces of the separable Hilbert space in a subspace of the non-separable Hilbert space that represents an encapsulating composite will at least partly hide the embedded constituents. This hiding is required for constituents of modular systems.

4.1.1 Representing continuums and continuous functions

Paul Dirac introduced the bra-ket notation that eases the formulation of Hilbert space habits [5]. By using bra-ket notation, operators that reside in the separable Hilbert space and correspond to continuous functions, can easily be defined starting from an orthogonal base of vectors. This works both in separable Hilbert spaces as well as in non-separable Hilbert spaces.

Let $\{q_i\}$ be the set of rational quaternions and $\{|q_i\rangle\}$ be the set of corresponding base vectors. They are eigenvectors of a normal operator $|q_i\rangle q_i\langle q_i|$. Here we enumerate the base vectors with index i.

$|q_i\rangle q_i\langle q_i|$ is the configuration parameter space operator.

Let $f(q)$ be a quaternionic function.

$|q_i\rangle f(q_i)\langle q_i|$ defines a new operator that is based on function $f(q)$.

In a non-separable Hilbert space, such as the Gelfand triple, the continuous function $f(q)$ can be used to define an operator, which features a continuum eigenspace that acts as target space of the function and uses the eigenspace of the reference operator $|q\rangle q\langle q|$. The eigenspace reference operator $|q\rangle q\langle q|$ acts as a flat parameter space that is spanned by a quaternionic number system.

$|q\rangle f(q)\langle q|$ defines a curved continuum.

Here we no longer enumerate the base vectors with index i. We just use the name of the parameter.

In general the dimension of a subspace loses its significance in the non-separable Hilbert space.

The continuums that appear as eigenspaces in the non-separable Hilbert space can be considered as quaternionic functions that also have a representation in the corresponding infinite dimensional separable Hilbert space. Both representations use a flat parameter space that is spanned by quaternions.

5 The orthomodular base model

Now we have achieved a level in which the major chain of mathematical structures does no longer offer an inescapable self-evident extension. The model uses separable and non-separable Hilbert spaces in order to store numeric data that can describe a series of discrete objects that are embedded in a continuum. The real parts of the parameters can be used to order the parameters and the target values of functions. If properly ordered these descriptions can represent a sequence of static status quos. However, this model contains no means to control the coherence between the subsequent members of the sequence.

We will call this stage of the model development "***The orthomodular base model***". Any further development of the model involves the insertion of mechanisms that ensure the coherence between the subsequent members of the sequence of static status quos.

The orthomodular base model describes the relational structure of modular systems. Via the management mechanisms it can add characteristics to the modules. These characteristics are based on eigenvalues of normal operators that reside in the separable Hilbert space and have eigenvectors in the closed subspace that represents the module.

The numeric data that occur in the orthonormal model must be taken from division rings. The most elaborate choice for these data are quaternions. The peculiarities of these quaternions influence the features and the behavior of the discrete objects and the fields that occur in the orthonormal model.

Many of these peculiarities are hardly known by scientists. As far as they apply to this paper these subjects are treated in the Appendix.

6 Embedding

The orthomodular base model consist of two related Hilbert spaces.

- A separable Hilbert space $\mathfrak{H}$ that acts as a descriptor of the properties of all discrete objects.
- A non-separable Hilbert space $\mathcal{H}$ that acts as a descriptor of the properties of all continuums.

The Hilbert space $\mathfrak{H}$ embeds into Hilbert space $\mathcal{H}$.

A closed subspace in $\mathfrak{H}$ maps in a subspace in $\mathcal{H}$.

Due to the four dimensions of quaternions, quaternionic number systems exist in 16 versions $\{q^x\}$ that differ only in their discrete symmetry set. The quaternionic number systems $\{q^x\}$ correspond to 16 versions $\{q_i^x\}$ of rational quaternions.

The index x can be ⓪,①,②,③,④,⑤,⑥,⑦,⑧,⑨,⑩,⑪,⑫,⑬,⑭, or ⑮.

A reference operator $\mathcal{R}^x = |q_i^x\rangle q_i^x \langle q_i^x|$ in $\mathfrak{H}$ maps into a reference operator $\mathfrak{R}^x = |q^x\rangle q^x \langle q^x|$ in $\mathcal{H}$.

In $\mathcal{H}$ the operator $\mathfrak{C} = |q^{⓪}\rangle \mathfrak{C}(q^{⓪}) \langle q^{⓪}|$ represents an embedding continuum.

In the form of eigenvalues of reference operator $\mathcal{R}^x$ the set $\{a_i^x\}$ correspond to sets of eigenvectors $\{|a_i^x\rangle\}$ that span a corresponding closed subspace. This restricts operator $\mathcal{R}^x$ to operator $\wp^x = |a_j^x\rangle a_j^x\langle a_j^x|$.

The embedder $\wp^x$ maps subsets$\{a_i^x\}$ of $\{q_i^x\}$ onto the continuum $\mathfrak{C}$ defined by function $\mathfrak{C}(q)$.

Its action can be split into three steps.

The two first steps form a map from a subspace of the eigenspace of $\mathcal{R}^x$ to the corresponding eigenspace of $\mathfrak{R}^{⓪}$.

The first step converts $\mathcal{R}^x$ into $\mathcal{R}^{⓪}$. It only switches the symmetry flavor of the reference operator.

The second step embeds $\mathfrak{H}$ into $\mathcal{H}$ by mapping $\mathcal{R}^{⓪}$ to $\mathfrak{R}^{⓪}$.It is a map between quaternions with rational valued components and a continuum consisting of quaternions that have real valued components. The discrete set and the continuum have the same symmetry flavor, which is the reference symmetry flavor.

The third step is performed completely inside $\mathcal{H}$ by operator $\mathfrak{C}$.

The symmetry flavor switch occurs in $\mathfrak{H}$ and the curvature of the continuum occurs in $\mathcal{H}$.

6.1 Coherence

Closed subspaces of a separable Hilbert space are characterized by a countable set of eigenvalues of a normal operator. Dedicated mechanisms ensure the coherence of the set of eigenvalues.

Coherence is quite obvious for continuums and continuous quaternionic functions. However, due to the four dimensions of quaternions, quaternionic number systems exist in 16 versions that only differ in their discrete symmetry set. For example right handed quaternions exist and left handed quaternions exist.

A ***coherent set*** of discrete quaternions is defined by two criteria:

1. All members of the set belong to the same symmetry flavor.
2. The set can be described by a continuous density distribution.

The second requirement involves a ***map*** $\wp^x(\{a_i^x\})$ onto a continuum that ***embeds*** the elements $\{a_i^x\}$ of the coherent set. The continuum is defined by the ***quaternionic function*** $\mathfrak{C}(q^{⓪})$, which has a flat parameter space that is spanned by a quaternionic number system $\{q^{⓪}\}$. The real valued continuous location density distribution $\rho_0(q^{⓪})$ describes the density distribution of set $\{a_j^x\}$ within set $\{q_i^x\}$.

An ***ordered coherent set*** is ordered with respect to the real parts of its members.

In a ***well-ordered coherent set*** all members have different real parts.

A well-ordered coherent set contains a well-defined hopping path. Also the hops form a discrete distribution. The landing locations form a well ordered swarm and the hops are also well-ordered. However, the subsequent hops have quite stochastic directions and sizes. Still the continuous location density distribution $\rho_0(q^{⓪})$ that describes the set of locations also characterizes the density distribution of the hops. Both are functions of the progression that is stored in the real parts of the eigenvalues.

The hops are eigenvalues of a hop operator. The hop operator and the landing location operator share the corresponding eigenvectors.

It is possible to define an imaginary function $\boldsymbol{\rho}(q^{⓪})$ that defines the ***average local displacement***. Together with the location density distribution $\rho_0(q^{⓪})$ it forms a quaternionic function:

$$\rho(q^{⓪}) = \rho_0(q^{⓪}) + \boldsymbol{\rho}(q^{⓪}).$$

We will call this function a ***density function***. The well-ordered coherent set $\{a_j^x\}$, which can be described by a dynamic continuous density distribution $\rho(q^{⓪})$ may also have a Fourier transform $\tilde{\rho}(p)$. In that case we call the set a ***coherent swarm***. The coherent swarm owns a ***displacement generator***. This means that at first approximation the swarm $\{a_j^x\}$ ***moves as one unit***. Having a Fourier transform is a higher level coherence requirement.

Defined in this way, the density function has lost its relation with the symmetry flavor of the discrete set $\{a_j^x\}$. However, it is possible to restore that relation by defining:

$$\rho^x(q^{⓪}) = \rho_0(q^{⓪}) + \boldsymbol{\rho}^x(q^{⓪})$$

The directions of the hops are stochastically distributed. This would mean that $\boldsymbol{\rho}^x(q^{⓪}) = \mathbf{0}$. However, the embedding causes an extra curvature of the continuum. This means that the curvature of the embedding continuum $\mathfrak{C}$ may change and that a corresponding flow is generated in this continuum. This produces a relative flow of the map of density distribution ρ^x with respect to $\mathfrak{C}$.

6.2 Embedding set elements

Embedding a single element a_j^x of the subset $\{a_j^x\}$ of the eigenspace of $\mathcal{R}^x$ in continuum $\mathfrak{C}$ involves first the conversion to the reference symmetry flavor. Next this element is mapped from the eigenspace of $\mathcal{R}^{⓪}$ in $\mathfrak{H}$ into to the eigenspace of $\mathfrak{R}^{⓪}$ in $\mathcal{H}$. Finally this discrete quaternion is embedded in the continuum $\mathfrak{C}$.

Locally the curved continuum $\mathfrak{C}$ is represented by ψ, which is nearly flat. For that reason for ψ we can use the quaternionic nabla ∇.

$$\nabla = \left\{\frac{\partial}{\partial\tau}, \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right\}$$

ψ is considered to cover the images of all elements of $\{a_j^x\}$. This makes ψ a normalizable function.

The duration of the embedding is very short and is quickly released. The continuum is touched and as a reaction it gets curved. The embedded particle will vanish, but traces in the continuum stay and represent the curvature. However, also these traces fade away. What happens can be described by the wave equation.

$$\nabla\nabla^*\psi = \rho_j$$

Before and after the embedding ρ_j equals zero. During the embedding ρ_j represents the embedded discrete quaternion. The embedding results in the emission of a spherical wave front, which is a solution of the homogeneous wave equation

Solutions of the wave equation can be found via the continuity equations:

$$\nabla\psi = \phi \; ; \; \nabla^*\phi = \rho_j;$$

and

$$\nabla^*\psi = \varphi\,;\nabla\,\varphi = \rho_j$$

Solutions of the homogeneous wave equation that cover an odd number of dimensions are known to represent wave fronts or combinations of wave fronts. These wave fronts proceed with fixed speed c. However, due to their diminishing amplitude, the spherical wave fronts fade away.

Embedding a single element of $\{a_j^x\}$ may cause the emission of a single spherical wave front. The amplitude of spherical wave fronts diminishes as 1/r with distance r from the source. This is also the form of the Green's function of the spatial part of the inhomogeneous wave equation for the three dimensional isotropic case. This fact forms the origin of the curvature of the embedding continuum ψ.

Embedding a single hop may cause the emission of a single one dimensional wave front. The amplitude of one dimensional wave fronts keeps constant. The direction of the one dimensional wave front relates to the direction of the hop. This phenomenon may represent quanta that leave or enter the object that is represented by the swarm $\{a_j^x\}$.

6.3 Embedding the full set

If embedding of the full set $\{a_j^x\}$ is considered, then ρ represents the density distribution of the full set. In that case the continuity equations: $\nabla\,\varphi = \rho$ and $\nabla^*\phi = \rho$ determine what happens to the embedding continuum ψ, which locally represents $\mathfrak{C}$. As already indicated, due to the extra curvature the map of ρ may flow relative to ψ.

The set $\{a_j^x\}$ is well-ordered. It means that each of its elements exists during a small interval. Before that interval the element did not exist. It is ***generated*** by a stochastic mechanism. After the embedding this element of $\{a_j^x\}$ vanishes into history. Only its value is stored in an eigenvalue of operator $\mathcal{O}^x = |a_j^x\rangle a_j^x \langle a_j^x|$ that maps the subspace spanned by $\{|a_j^x\}$ onto itself. The operator $\mathcal{O}^x$ and the corresponding subspace have a dynamic definition. That definition covers a certain period, which represents a progression window.

In the embedding continuum $\mathfrak{C}$, the traces of what happened are the emitted wave fronts that independent of the progression window keep proceeding. The spherical wave fronts do not vanish, but they fade away. With them the curvature also fades away. However, the recurrent embedding process keeps this curvature alive in a dynamical fashion. It drags the curvature with the subspace that represents the corresponding module.

The Green's functions indicate the averaged effects of the recurrent embedding on the curvature of ψ.

6.4 Subspace dimension

In $\mathfrak{H}$ the dimension of the subspace that represents the set $\{a_j^x\}$ has a clear significance. In order to comprehend what this dimension and the spread of the set do to the function ψ we use the Green's function. The Green's function represents the influence of the embedding of a single point-like artifact into ψ. That artifact can be a landing point or a hop. If we do this for the three dimensional case, then the shape of the Green's function is $\mathcal{g}_j = 1/r$.

We replace ρ_j by ρ/N, multiply by the Green's function $\mathcal{g}_j$ and integrate over the space covered by ψ. Here N represents the number of elements in the set. ρ_j represents the effect of the single element a_j^x. For example, in case of an isotropic Gaussian distribution ρ/N the contributions to the integral will equal $\mathfrak{G}(r) = \mathrm{ERF}(r)/r$. In total N of those contributions [7] will be added. $N\ \mathfrak{G}(r)$ represents the gravitation potential.

This indicates that N directly relates to mass, which determines the strength of curvature of ψ.

If $\|\rho\| = N$, then $\nabla\, \varphi = \rho$ means $\|\nabla\, \varphi\| = N$.

This is a version of the coupling equation, which holds for all quaternionic normalizable functions φ and ρ, where φ is differentiable. If there are N landing locations, then there are also N hops.

7 Attaching characteristics to a module

7.1 Module subspace

We take one closed subspace as an example.

In free translation, the spectral theorem for normal operators that reside in a separable Hilbert space states: “If a normal operator maps a closed subspace onto itself, then the subspace is spanned by an orthonormal base consisting of eigenvectors of the operator.”

The corresponding eigenvalues characterize this closed subspace.

The normal operator $\mathscr{a}^x = |a_i^x\rangle a_i^x \langle a_i^x|$ that maps the closed subspace onto itself ***may*** correspond to a ***companion operator*** $|\wp^x(a_i^x)\rangle \wp^x(a_i^x) \langle \wp^x(a_i^x)|$ that resides in the non-separable companion of the Hilbert space. $\wp^x$ represents the map. Its target is a curved continuum that is characterized by the reference symmetry flavor. The index ^{x} indicates the symmetry flavor of the set $\{a_i^x\}$ of eigenvalues of operator $\mathscr{a}^x$.

The Hilbert spaces are structured storage places and in that way they can describe things. They possess no means that enable them to control what happens. That is the task of management mechanisms. However, the mechanism is restricted by the properties of the Hilbert spaces.

Here we take the position that the eigenvalues of operator $\mathscr{a}^x = |a_i^x\rangle a_i^x \langle a_i^x|$ are generated by a mechanism that implements a stochastic process. This process does not reside in the Hilbert spaces, but part of its behavior can be described by a series of operators. Some of these operators reside in the separable Hilbert space $\mathfrak{H}$. Other participating operators reside in the non-separable Hilbert space $\mathcal{H}$.

$\{a_i^x\}$ forms a well-ordered coherent set. All elements belong to different progression values. They belong to the same symmetry flavor and with respect to the quaternionic number system $\{q^x\}$ they own a continuous density distribution $\rho_0(q^x)$.

The stochastic process can be considered as a combination of a stochastic selector, such as a Poisson process and a binomial process, which is implemented by a 3D spread function $\mathcal{S}$. This ***stochastic spread function*** produces a distribution of discrete locations that can be described by a density distribution ρ.

The involved operators and mechanisms are:

- In the separable Hilbert space a ***reference operator*** $\mathcal{R}^x = |q_i^x\rangle q_i^x \langle q_i^x|$ provides the parameter space of involved functions. The set of eigenvalues $\{q_i^x\}$ of this operator represent all rational members of a quaternionic number system $\{q^x\}$ that features a symmetry flavor, which is indicated with index ^{x}.
- In the non-separable Hilbert space a ***reference operator*** $\mathfrak{R}^x = |q^x\rangle q^x \langle q^x|$ provides the parameter space of involved functions. The set of eigenvalues $\{q^x\}$ of this operator represent

all members of a quaternionic number system $\{q^x\}$ that features a symmetry flavor, which is indicated with index ^{x}.

- The ***density operator*** $|a_j^x\rangle\rho(a_j^x)\langle a_j^x|$, resides in separable Hilbert space $\mathfrak{H}$ and represents the density $\rho(q_i^x)$ of the discrete distribution $\{a_j^x\}$ that is generated by the stochastic spread function $\mathcal{S}$ during a period of progression that covers the progression values of the set $\{a_j^x\}$.
- The ***stochastic selection mechanism*** selects parameter values a_j^x according to the density operator $|a_j^x\rangle\rho(a_j^x)\langle a_j^x|$ that represents the density $\rho(q_i^x)$ of the discrete distribution $\{a_j^x\}$ within the set $\{q_i^x\}$ that is generated by the stochastic spread function $\mathcal{S}$.
- The eigenvectors$\{|a_j^x\rangle\}$ that belong to the eigenvalues $\{a_j^x\}$ of operator $\mathscr{o}^x = |a_j^x\rangle a_j^x\langle a_j^x|$ span the considered closed subspace and characterize the module that is represented by this subspace.
- The ***target space operator*** $|q^{⓪}\rangle\mathfrak{C}^{⓪}(q^{⓪})\langle q^{⓪}|$ resides in the non-separable Hilbert space $\mathcal{H}$ and is implemented by a continuous mapping function $\mathfrak{C}^{⓪}(q^{⓪})$.
- The ***density operator*** $|q^x\rangle\wp^x((\rho(q^x)))\langle q^x|$ resides in the non-separable Hilbert space $\mathcal{H}$ and represents the density $\wp^x((\rho(q^x)))$ of the discrete distribution $\{\wp^x(a_j^x)\}$ that is generated by the stochastic spread function $\mathcal{S}$ via the convolution $\mathcal{P} = \wp \circ \mathcal{S}$ of the map $\wp$ and the spread function $\mathcal{S}$.

Thus the selection mechanism and the combination of the operators that reside in the separable Hilbert space produce a sequence of eigenvalues $\{a_j^x\}$ of operator $\mathscr{o}^x = |a_i^x\rangle a_i^x\langle a_i^x|$ that map onto the closed target set in the continuum that is formed by the density operator $|q^x\rangle\wp^x((\rho(q^x)))\langle q^x|$ that represents the convolution $\mathcal{P} = \wp \circ \mathcal{S}$.

$\{a_j^x\}$ is a coherent subset of $\{q_i^x\}$, which form the eigenvalues of $\mathcal{R}^x = |q_i^x\rangle q_i^x\langle q_i^x|$.

$\wp^{⓪}(q^{⓪})$ represents the continuum eigenspace of the target space operator $|q^{⓪}\rangle\wp^{⓪}(q^{⓪})\langle q^{⓪}|$.

Since $\mathcal{P}(q)$ is a continuous function, $\{\mathcal{P}(a_j^x)\}$ is a discrete coherent subset of the continuous target space $\{\wp^{⓪}(q^{⓪})\}$.

The target subset $\{\mathcal{P}(a_j^x)\}$ represents the freedom that is left by the embedding of the separable Hilbert space into the non-separable Hilbert space. This imaging process is described by the convolution:

$$\mathcal{P} = \wp \circ \mathcal{S}_j \tag{1}$$

$\mathcal{S}_j$ is a stochastic spatial spread function and varies with each subsequent progression step.

$\wp$ produces an exact map.

The exact target location $\mathcal{P}(a_j^x)$ is not known beforehand, but after selection of the source eigenvalue a_j^x the image $\wp^x(a_j^x)$ is exactly known and is stored in the eigenspaces of the respective operators.

Averaged over all selections, $\mathcal{P}$ produces a blurred image.

The average $\boldsymbol{a}^x$ of the imaginary parts of all $\{a_j^x\}$ is the center location of the set. The combination of all involved operators and the selection mechanism produces a blurred image of $\boldsymbol{a}^x$.

The blur only concerns the imaginary part of the quaternion(s).

7.2 History

In the orthomodular base model, the eigenvalues of the reference operators are not touched by management mechanisms or by the embedding process.

In the orthomodular base model ***history is an artificial concept***. History is defined with respect to the current real value of the eigenvalues of the reference operators.

The eigenspaces of operators other than reference operators exactly describe the history. The history is fixed. Thus also the historic eigenvalues are not touched by management mechanisms or by the embedding process. However, these operators do not yet describe the future. The future is constructed by the management mechanisms and the embedding process.

The subspace that represents a module covers a sliding part of the last history. The dimension N of the subspace determines the number of covered progression instances.

The progression window covers a recycling period in which the statistical properties of the set $\{a_j^x\}_N$ stabilize. This period is a property of the stochastic generation mechanism.

7.3 Map of well-ordered coherent set

Since the source eigenvalues $\{a_j^x\}$ are all quaternions, they can be ordered with respect to their real value. All source eigenvalues have different real parts. That real value contains the sequence number. The set of source eigenvalues forms a ***well-ordered coherent set***. As a consequence, the image of the map of the source eigenvalues onto the continuum eigenspace can be described by a dynamic continuous location density distribution in which the sequence number acts as the progression parameter. This also means that $\{a_j^x\}$ describes a ***hopping path***.

7.4 Coherent swarm

The well-ordered coherent set $\{a_j^x\}$, which can be described by a dynamic continuous location density distribution $\rho(q^x)$ may also have a Fourier transform. In that case we call the set a ***coherent swarm***. The coherent swarm owns a displacement generator. This means that at first approximation the swarm $\{a_j^x\}$ ***moves as one unit***. Having a Fourier transform is a higher level coherence requirement.

Having a Fourier transform means that the swarm can be represented by a wave package. On movement, wave packages tend to disperse. Since the dynamic continuous location density distribution only describes the swarm, it is continuously regenerated. As a consequence, movement does not disperse the swarm. Thus due to recurrent regeneration, no danger of dispersion exists.

On the other hand the representation by a wave package indicates that the swarm $\{a_j^x\}$ may take the form of an interference pattern. That interference pattern is still a location swarm. It is not constructed by interfering waves!

7.5 The coherent map

Thus in the ***special case*** that a companion operator$|\wp^x(a_i^x)\rangle\wp^x(a_i^x)\langle\wp^x(a_i^x)|$ of the normal operator $|a_i^x\rangle a_i^x\langle a_i^x|$ that maps the subspace onto itself exists and the source eigenvalues $\{a_i^x\}$ form a well ordered coherent set, then the embedding of the module can be described by a progression dependent continuous mapping function $\wp$, which produces a blurred image $\mathcal{P}(\boldsymbol{a})$ of the average of the source eigenvalues. $\wp$ uses a flat parameter space that is spanned by a quaternionic number system. The coherent set of source eigenvalues can be considered to be generated by a mechanism that can be characterized by a source location spread function $\mathcal{S}$. This function has fixed statistical characteristics, uses quaternions as its target values and progression as its parameter value. The progression parameter is taken from the parameter space of $\wp$. Now the blurred image $\mathcal{P}$ is the convolution of the mapping function $\wp$ and the source location spread function $\mathcal{S}$.

$$\mathcal{P} = \wp \circ \mathcal{S} \tag{1}$$

The coherent set of source eigenvalues can be described by a discrete source location density distribution $\{a_i^x\}$. If these eigenvalues are generated in a sequence, then for each member of this sequence the represented object can be considered to occupy a single source location. In this way the object can be considered to hop between the elements of the coherent swarm of eigenvalues. Each landing location corresponds with a hop. The sequence number can act as the progression parameter. The progression parameter is stored in the real part of the landing location eigenvalue. It was already there before we decided to order the sequence with respect to that parameter.

We will call this special case “the coherent map”.

7.6 Generation cycle

The generation by the stochastic spatial spread function $\mathcal{S}$ is done before the map $\wp$. This means that it occurs in the realm of the separable Hilbert space and this generation process is not (yet) affected by the embedding in the non-separable Hilbert space.

The stochastic generation process determines the short term cyclic part of the dynamical behavior of the object. The corresponding cycle period lasts until the spatial statistical characteristics of the generation result stabilize. Thus, the stochastic generation process is characterized by spatial statistical characteristics that are obtained after averaging over complete cycles of the generation process. These characteristics are the statistical characteristics of the coherent swarm.

The collection $\{\mathcal{P}(a_i^x)\}$ taken over the full generation cycle represents a spatial map of the cyclic dynamic behavior of the object.

7.7 Model wide progression steps and cycles

Each closed subspace that represents a coherent swarm is governed by a mechanism that ensures dynamic and spatial coherence. In fact many different types of such mechanisms exist. They correspond to elementary particle types. If these modules combine into composites, then the generation cycles must synchronize. This asks for a model wide progression step that is shorter than any cycle. A RTOS-like management mechanism must schedule the generation of composites from completed modules.

7.8 Swarm behavior

The coherent swarm moves as one unit. This means that the represented object features two kinds of kinetics. The first kind stays internal to the swarm. The second kind concerns the swarm as a whole.

Inside the swarm, the represented object hops from swarm element to swarm element. The hopping path is folded and if the swarm is at rest, then the hopping path is closed. Adding extra hops causes movement of the swarm. Adding a closed string of hops in a cyclic fashion causes an oscillation of the swarm. From observations it follows that in composites, such as atoms only certain oscillation modes are tolerated. Adding an arbitrary open string of hops opens the hopping path. In that case the sum of all hops is no longer zero. As a consequence the swarm will move. This motion gets its origin in the separable Hilbert space. And is mapped onto the continuum.

A dynamic local change of the mapping function $\wp$ may move the swarm relative to other swarms. Such changes may occur when discrete objects curve the embedding continuum. This kind of movement gets its origin in the non-separable Hilbert space.

7.9 Swarm characteristics

The swarm has a central location, which in separable Hilbert space is defined as the average a of the coherent set of source eigenvalues $\{a_i^x\}$ and in the non-separable Hilbert space it is defined by the image $\wp(\boldsymbol{a})$. This target value corresponds to an object source location $\boldsymbol{a}$ in the flat parameter space of $\wp$. The source location may move as a function of progression.

In the continuum the image of the swarm cannot move faster than the speed with which information can be transported.

The speed of transfer of information is set by the speed of information carriers. These information carriers are one-dimensional wave fronts. The quaternionic wave equation describes the way in which these wave fronts proceed.

The statistical characteristics of the swarm and the symmetry flavor of the swarm are sources for the properties that characterize the types of the objects that are represented by a coherent swarm.

7.10 Swarm diversity

The mechanism that generates the swarm determines the characteristics of the swarm. Apart from the number of elements of the swarm, the properties of the swarm appear to depend on its symmetry flavor. Due to the four dimensions of quaternions will quaternionic number systems, coherent swarms, quaternionic continuums and continuous quaternionic functions exist In 16 versions that only differ in their symmetry flavor.

Here we use the diversity that is represented by the standard model of contemporary physics as reference for naming elementary object types.

Elementary particle types have different masses. In the orthomodular base model this means that the corresponding closed subspaces have different dimensions and that correspondingly the swarms have different numbers of elements.

7.10.1 Fermions

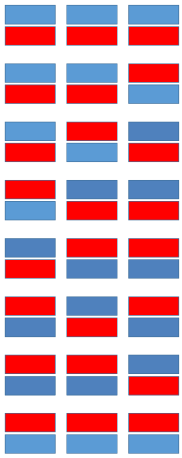
Embedding couples coherent swarms that possess symmetry flavor ψ^x to an embedding continuum that has symmetry flavor $\varphi^{⓪}$. If this symmetry flavor of the embedding continuum is fixed, then varying the symmetry flavor of the coherent swarm creates sixteen different elementary object types. Half of these types concern anti-particles. Again half of these sub-types concern left-handed quaternions and the other half are right-handed. Isotropic types represent another category. Anisotropic types occur in three versions that are deviated by the dimension in which the anisotropy occurs.

The difference in the symmetry flavors between the members of the pair $\{\psi^x, \varphi^y\}$ can be related to the electric charge, the color charge and the spin of the corresponding elementary particle.

Fermions are known to have half integer spin. In contemporary physics, their “color” structure becomes noticeable when composites are formed.

- Symmetry flavors are marked by special indices, for example $\boldsymbol{\psi}^{④}$
- They are also marked by colors $N, R, G, B, \bar{B}, \bar{G}, \bar{R}, \bar{N}$
- Half of them is right handed, **R**
- The other half is left handed, **L**
- $\boldsymbol{\psi}^{⓪}$is the reference symmetry flavor
- The colored rectangles reflect the directions of the axes

Result of coupling ψ^x to $\varphi^{⓪}$



$\psi^{⓪}$	*neutrino*	0	**R**
$\psi^{①}$	*R upquark*	⅔	**L**
$\psi^{②}$	*G upquark*	⅔	**L**
$\psi^{③}$	*B upquark*	⅔	**L**
$\psi^{④}$	$\overline{B}$ *downquark*	-⅓	**R**
$\psi^{⑤}$	$\overline{G}$ *downquark*	-⅓	**R**
$\psi^{⑥}$	$\overline{R}$ *downquark*	-⅓	**R**
$\psi^{⑦}$	*electron*	-1	**L**

Electric charge relates to the number of dimensions in which symmetry flavors differ. The sign of the electric charge relates to the direction in which the difference occurs.

Color charge appears to relate to the index of the dimension in which the difference occurs. Isotropic differences correspond to “neutral” colors.

Quarks have “partial” electric charge. Up-quarks have electric charge + ⅔e. Down-quarks have electric charge - ⅓ e.

7.10.2 Bosons

Massive bosons couple to an embedding continuum in a similar way as fermions do. Fermions and bosons appear to contribute to a common gravitation potential. This means that bosons embed in the same field as fermions do. Boson swarms feature color-neutral symmetry flavors. Bosons are known to feature integer spin.

Massive bosons are observable as W_+, W_- and Z particles. Their “color” structure cannot be observed. Until now, quark-like bosons are not observed.

7.10.3 Spin axis

Fermion swarms and boson swarms contain a hopping path that can be walked into two directions. That hopping path may implement spin.

If the swarm is at rest (does not move), then the hopping path is closed.

For bosons the spin axis may be coupled to the polar axis. The polar angle runs from 0 through 2π. For fermions the spin axis may be coupled to the azimuth axis. The azimuth angle runs from 0 through π.

Nothing is said yet about the fact and the corresponding influence that the number of hops can be even or odd. And nothing is said yet about whether the opening hop and the closing hop are coupled in a symmetric or asymmetric sense.

7.11 Mass and energy

7.11.1 Having mass

Having mass can be interpreted as the capability to curve the continuum that embeds the concerned object. More mass corresponds to more curvature.

The dimension of the closed subspace, which represents a discrete object has a physical significance. Any eigenvector that contributes to spanning the closed subspace increases the dimension of the subspace. If all elements of the swarm contribute separately to the curvature of the embedding continuum, then the total curvature is proportional to the dimension of the subspace. In that case, this dimension relates to the mass of the object that corresponds to the swarm. If extra hops are added that cause movements or oscillations, then this adds to the mass in the form of kinetic energy. The extra hops may enter or leave in strings. Inside the swarm the hops that cause oscillation are stored as closed strings. Outside of the swarm the strings are open and appear as information messengers.

The fact that fermions and massive bosons contribute to a common gravitation potential means that they curve the same embedding continuum.

7.11.2 Information messengers

Information messengers represent open strings of hops. At the same time they are solutions of the wave equation. This means that they can be viewed as strings of one dimensional wave fronts. One dimensional wave fronts do not diminish their amplitude as function of the distance to their emission point. In an otherwise flat continuum the one dimensional wave fronts and thus the information messengers proceed with the speed of information transfer. The energy carried by information messengers is proportional to the number of one-dimensional wave fronts that they contain. As a consequence, the apparent frequency of information messengers is proportional to their energy.

In contemporary physics the information messengers are known as ***photons***. From experiments we know that the energy of photons is proportional to their frequency. Thus if photons are information messengers then this suggests that the emission, the absorption and the passage of information messengers takes a fixed number of progression cycles.

7.11.3 Mass energy equivalence

Creation and annihilation of elementary particles shows the equivalence of mass and energy.

7.11.3.1 Suggested creation process

Creation of elementary particles starts with the combination of two photons that came from opposite directions into an intermediate object. The intermediate object is a very short lived massive object that consists of as many paired elements as wave fronts are contained in the constituting photons. The wave fronts will convert into hops. The long chain of paired hops will then rip apart into two folded hopping strings that each form a coherent location swarm. Next the two swarms will split and move in opposite directions.

7.11.3.2 Suggested annihilation process

Annihilation of elementary particles starts with the combination of an elementary particle and its anti-particle that come from opposite directions in an intermediate object. The intermediate object is a very short lived massive object that consists of as many paired elements as elements are contained in the constituting coherent location swarms. The hops will convert into wave fronts. The long chain of paired wave fronts will then rip apart into two separate chains of wave fronts. Next these photons leave in opposite directions.

Relation to the wave function

The concept of wave function is used by contemporary physics in order to represent the state of a quantum physical object. The wave function is a complex amplitude probability distribution. Its squared modulus is a normalized density distribution of locations where the owner of the wave function can be detected. The value of this continuous distribution equals the probability of finding the owner at the location that is defined by the value of the parameter of the distribution.

If the detection is actually performed, then the object will be converted into something else. By the adherents of the Copenhagen interpretation, this fact is known as "the collapse of the wave function".

The normalized density distribution of locations where the owner of the wave function can be detected corresponds to the map of a coherent swarm on a flat continuum eigenspace of the companion operator in the orthomodular base model.

Thus the concept of the coherent map of a well-ordered coherent set on a flat continuum eigenspace of the companion operator in the orthonormal base model leads directly to an equivalent of the concept of the wave function in contemporary physics. Both concepts cannot be verified by experiments. The equivalence indicates that the suggested coherent map extension of the orthomodular base model runs in a sensible direction.

8 Traces of embedding

The embedding of a discrete eigenvalue in the continuum does not last longer than a single progression step. For each object, the embedding occurs only once at every used progression step. The source eigenvalue a_j is stored in the eigenspace of the location operator that resides in the separable Hilbert space. Immediately afterwards the embedding is released and is replaced by another embedding at a slightly different location a_{j+1} in the target continuum. This recurrent embedding process generates the map of the well-ordered coherent set of source eigenvalues $\{a_j\}$.

In the non-separable Hilbert space the map $\{\wp(a_j)\}$ affects the target subspace of the continuum eigenspace. This is done in a special way. The effect is determined by the ***wave equation***. The homogeneous wave equation controls the situation just before and after the actual embedding action. The inhomogeneous wave equation determines the situation during the actual embedding action. The embedding results in the emission of a 3D ***wave front***. That wave front folds and thus ***curves*** the target subspace of the continuum. After release of the embedding, the wave front keeps proceeding, but then it will quickly diminish its amplitude as function of the distance to the emission location. The effects of the 3D wave fronts of all elements of the swarm combine and form a ***potential***.

8.1 Embedding potentials

In this model, embedding potentials form the averages over a small period of progression and over a region of space of the effects of wave fronts that are emitted during the embedding of particles. Mathematically these potentials are described by Green's functions or by weighted averages of these Green's functions. The shape of the Green's function of a single embedding corresponds with the shape of the amplitude of the wave front that is emitted at the embedding instant.

The wave fronts that are emitted during the embedding of the members of the location swarm are isotropic 3D wave fronts. Their spreading is controlled by the 3D version of the Huygens principle. This means that their amplitude decreases with the distance r from the source as $1/r$.

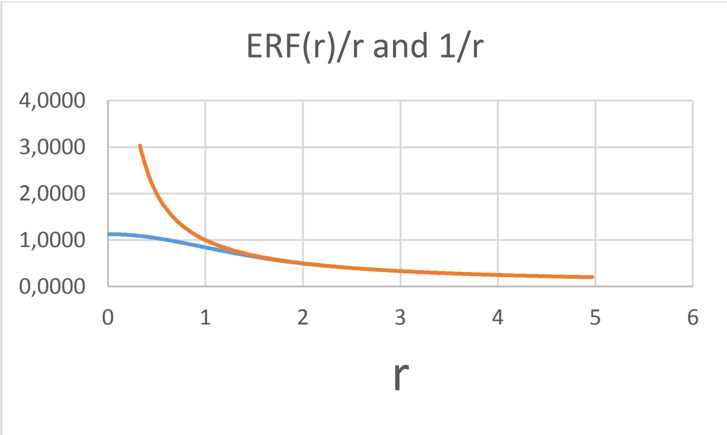
Here we consider a simplified situation. With an isotropic density distribution $\rho_0(r)$ in the swarm the scalar potential $\varphi_0(R)$ can be estimated as:

$$\varphi_0(R) = \int_0^R \rho_0(r)dr \tag{1}$$

R is the distance to the center of the swarm.

If the density distribution approaches a 3D Gaussian distribution, then this integral equals [10]:

$$\varphi_0(R) = \mathrm{ERF}(R)/R \tag{2}$$



We suppose that this distribution is a good estimate for the structure of the swarm of a free electron. It is remarkable that this potential (the blue curve) has no singularity at $R = 0$. At the same time, already at a short distance of the center the function very closely approaches $1/R$ (the orange curve).

The term $\mathrm{ERF}(R)$ indicates the influence of the spread of the embedding locations. This view can be used to determine the spatially averaged effect of the single embeddings. The set $\{a_j^x\}_N$ corresponds to N instances of such spatially averaged contributions. This approach shows that curvature and thus mass is directly related to the size of the set and to dimension of the subspace that represents the module.

In contemporary physics the embedding potential $\varphi_0(R)$ is known as the gravitation potential. It describes the curvature of the embedding continuum.

8.2 Symmetry related potential

All elements of the coherent swarm have the same symmetry flavor. The effects of symmetry flavor coupling work over the whole reach of the coherent swarm. The source of this influence is located at the target value of the mapping function $\wp(a)$. The charge at this location depends on the difference between the symmetry flavor of the coherent swarm and the symmetry flavor of the embedding continuum.

Also here the quaternionic wave equation describes what happens, but the charge stays at its center location. If the swarm stays at rest, then the charge stays static as well and the governing equation is:

$$\nabla^*\nabla\varphi = \nabla_0\nabla_0\varphi + \langle\boldsymbol{\nabla},\boldsymbol{\nabla}\rangle\varphi = \rho$$

Here φ represents the quaternionic electric potential and ρ represents the distribution of electric charges.

For the electrostatic potential this reduces to

$$\langle\boldsymbol{\nabla},\boldsymbol{\nabla}\varphi_0\rangle = \rho$$

8.2.1 Difference with gravitation potential

The electrostatic potential deviates in many aspects from the gravitation potential. Where every element of the swarm contributes separately to the gravitation potential, will the electrostatic potential only depend on the symmetry flavor of the swarm. It is generated by the complete swarm and not by the separate elements. The virtual location of the electrostatic charge coincides with the location of the center of mass of the swarm. For elementary particles, the strength of the symmetry related potential does not depend on the number of involved swarm elements.

9 Composites

Closed subspaces can combine into wider subspaces. If in the disjunction no eigenvectors of the location operator are shared between the constituents, then the constituents stay independent and keep their characteristics. Still superposition coefficients may rule the relative contribution of these properties. The properties are added per property type and these sums are not affected by the superposition.

9.1 Closed strings

Elementary particles are represented by coherent location swarms that also implement a folded hopping path. At rest this hopping path is closed. Adding extra hops may open the hopping path. This means that the sum of all hops may no longer equal zero. As a consequence the swarm moves. If a closed string of hops is added, then on average the swarm still stays at the same location, but at the same time the swarm oscillates. Such oscillations occur inside atoms.

The added hops act for the whole swarm as displacement generators. The corresponding quaternions act as superposition coefficients.

Quaternionic superposition coefficients may act as rotators. Special rotators can switch the color charge of quarks. They do not affect color-neutral swarms.

9.2 Open strings

The closed strings of superposition coefficients enter and leave the composite as open strings.

Messengers are open strings that relate to particular swarm oscillations. They are known as ***photons***.

Messengers are also represented by strings of one-dimensional wave fronts.

Gluons are open strings that relate to swarm rotations. They can switch the color charge of quarks

Color confinement stimulates that in composites the combined color charge is neutralized.

9.3 Binding

The potentials are a means to bind constituents of composites.

9.3.1 Orthomodular model

The orthomodular base model suggests that at every progression step in every participating elementary particle only one swarm element is influenced by the currently existing potentials.

9.3.2 Gravitation

In the orthomodular base model, this is obvious for the gravitation potential which describes the curvature of the embedding continuum that is caused by these constituents. All embedding events contribute separately to the curvature of the embedding continuum. The constituents produce pitches into the embedding continuum and when they oscillate these pitches transform into ditches. The strength of the gravitation potential depends on the number of involved swarm elements.

9.3.3 Symmetry related potential

The origin of the symmetry related potential can also take a role in the binding of constituents, but this is questionable. The source of the symmetry related potential is probably located at the center of mass of the composite and is not located at the centers of mass of the constituents. If the sources of this potential would be located on the centers of mass of the constituents, then in case of oscillating constituents, this would result in ongoing emission of electromagnetic radiation.

9.4 Contemporary physics

Here we compare with results of contemporary physics.

9.4.1 Atoms

For stable composites, such as atoms, an ongoing emission of electromagnetic radiation is obviously not the case. Still the behavior of atoms with respect to absorption and emission of photons indicate that the electrons oscillate in concordance with the patterns of spherical harmonics.

For atoms and its composites, the strength of the symmetry related potential does not depend on the number of involved swarm elements.

9.4.2 Hadrons

In hadrons the situation is different. There the binding is regulated by gluons. Gluons are capable of rotating quarks such that their color charge switches to another value. Gluons can join in strings. As rotators they act in pairs. Gluons do not affect isotropic swarms.

9.4.3 Standard model

In the standard model of contemporary physics the symmetry related potential that governs the biding of electrons in atoms is considered to be the electromagnetic potential.

The standard model suggests the existence of other potentials that implement weak and strong forces. Gluons play a role in the strong force. Massive bosons play a role in the weak force. Introducing strong and weak forces suggests that the potentials act on the full swarm and not on the individual swarm elements. At least the forces suggest that the corresponding potentials act in an equal way on each of the swarm elements.

10 Restricting the orthomodular base model

Not all closed subspaces of the separable Hilbert space will represent modules that act as construction elements. Only closed subspaces for which a location generating mechanism governs, will act as modular construction elements. The management mechanisms that ensure spatial coherence will enforce this rule. The mechanisms appear to work in a step-wise fashion. This introduces a model-wide notion of progression in the model. Progression steps in the separable Hilbert space and it flows in the non-separable Hilbert space. The restriction converts the static model into a dynamic model in which special mechanisms ensure spatial and dynamical coherence. These are coupled due to the fact that the well-ordered coherent set of source eigenvalues represents a spatial map of the dynamic behavior of the source eigenvalue. At the same time the continuity of the mapping function $\wp$ ensures that the coherence is preserved in the image $\mathcal{P}$ of the set.

11 Role of the incoherent subspaces

Incoherent subspaces correspond to closed subspaces that do not correspond to a well-ordered coherent set of eigenvalues. If the subspace still is spanned by eigenvectors of the reference operator, then these eigenvalues may still produce an image in the continuum eigenspace of the companion location operator in the non-separable Hilbert space. Those images may produce spurious traces of embedding.

12 Conclusion

It appears sensible to suggest that physical reality mimics a network of mathematical structures that is controlled by a set of coherence ensuring management mechanisms. This setup aims at reducing relational complexity and it prevents dynamical chaos. The network consists of chains of structures that each start with a rather simple foundation. The major chain starts with an orthomodular lattice.

In this way an orthomodular base model emerges with inescapable evidence. This model treats all discrete objects as modules or modular systems that are embedded in continuums. This is supported by an infinite dimensional separable Hilbert space and a companion non-separable Hilbert space. Both Hilbert spaces act as structured storage media. The management mechanisms ensure the dynamic and spatial coherence. This leads to a model in which progression steps in the discrete part and flows in the continuous part of the model.

The habits and diversity of quaternions play an essential role in the extension of the orthomodular base model. These habits cause a large variety of module types that differ in their properties and in their behavior. The generation of the modules is controlled both by these habits and by stochastic management mechanisms. The behavior of the modules and of the continuums is restricted by the embedding process.

The paper shows that leading physicists did not always provide the most sensible choice. The models of contemporary physics are more complicated than is necessary and do not reach as deep as is possible.

Appendix

1 Quaternionic calculus

Quaternions have features and capabilities that are hardly known [8]. Some of them are treated here.

Quaternions are hyper-complex numbers that consist of a real scalar and a three dimensional real vector [8]. The vector plays the role of the imaginary part. Quaternions keep these parts in one compact unit. This has the advantage that it is immediately clear that these parts belong together.

It is not necessary to treat quaternions as one unit. Contemporary physics has selected for the option to treat the real part and the imaginary part separately. This has generated unhappy far reaching consequences.

1.1 Quaternions

We indicate the real part of quaternion a by the suffix a_0.

We indicate the imaginary part of quaternion a by bold face $\boldsymbol{a}$.

$$a = a_0 + \boldsymbol{a} \tag{1}$$

The product of two quaternions does not commute and exists in two versions:

$$f = f_0 + \boldsymbol{f} = d\, e$$

$$f_0 = d_0 e_0 - \langle \boldsymbol{d}, \boldsymbol{e} \rangle \tag{2}$$

$$\boldsymbol{f} = d_0 \boldsymbol{e} + e_0 \boldsymbol{d} \pm \boldsymbol{d} \times \boldsymbol{e} \tag{3}$$

The $\pm$ sign indicates the influence of right or left handedness of the number system.

$\langle \boldsymbol{d}, \boldsymbol{e} \rangle$ is the inner product of $\boldsymbol{d}$ and $\boldsymbol{e}$.

$\boldsymbol{d} \times \boldsymbol{e}$ is the outer product of $\boldsymbol{d}$ and $\boldsymbol{e}$.

1.2 Symmetry flavors

Due to their four dimensions, quaternionic number systems exist in 16 versions that differ in their discrete symmetry sets. Half of these versions are right handed and the other half are left handed.

Quaternions can be mapped to Cartesian coordinates along the orthonormal base vectors 1, $\boldsymbol{i}$, $\boldsymbol{j}$ and $\boldsymbol{k}$; with $\boldsymbol{ij} = \boldsymbol{k}$

- If the real part is ignored, then still ***8*** symmetry flavors result
- Symmetry flavors are marked by special indices, for example $\boldsymbol{a}^{\textcircled{4}}$
- They are also marked by colors $N, R, G, B, \bar{B}, \bar{G}, \bar{R}, \bar{N}$
- Half of them is right handed, **R**
- The other half is left handed, **L**
- The colored rectangles reflect the directions of the coordinate axes

Symmetry flavors of members of coherent sets:

 $\boldsymbol{a}^{\textcircled{0}} N\ \boldsymbol{R}$

 $\boldsymbol{a}^{\textcircled{1}} R\ \boldsymbol{L}$

 $\boldsymbol{a}^{\textcircled{2}} G\ \boldsymbol{L}$

 $\boldsymbol{a}^{\textcircled{3}} B\ \boldsymbol{L}$

 $\boldsymbol{a}^{\textcircled{4}} \bar{B}\ \boldsymbol{R}$

 $\boldsymbol{a}^{\textcircled{5}} \bar{G}\ \boldsymbol{R}$

 $\boldsymbol{a}^{\textcircled{6}} \bar{R}\ \boldsymbol{R}$

 $\boldsymbol{a}^{\textcircled{7}} W\ \boldsymbol{L}$

Members of coherent sets $\{a_i\}$ of quaternions all feature the same symmetry flavor.

Continuous quaternionic functions $\psi(q)$ do not switch to other symmetry flavors.

The reference symmetry flavor of function $\psi(q)$ is the symmetry flavor of its parameter space .

Also continuous functions and continuums feature a symmetry flavor. The reference symmetry flavor of a continuous function $\psi(q)$ is the symmetry flavor of the parameter space $\{q\}$.

If the parameter space is a flat continuum, then it is a coherent set.

If the continuous quaternionic function describes the density distribution of a set $\{a_i\}$ of discrete objects a_i, then this set must be attributed with the same symmetry flavor.

1.3 Symmetry flavor conversion tools

1.3.1 Conjugation

Quaternionic conjugation

$$(\psi^x)^* = \psi^{(7-x)}; x = ⓪, ①, ②, ③, ④, ⑤, ⑥, ⑦ \tag{1}$$

1.3.2 Rotation

Quaternions are often used to represent rotations.

$$c = ab/a \tag{1}$$

rotates the imaginary part of b that is perpendicular to the imaginary part of a over an angle 2θ, where $a = |a|\ exp(2\pi \boldsymbol{i} \theta)$.

Via quaternionic rotation, the following normalized quaternions ϱ^x can shift the indices of symmetry flavors of coordinate mapped quaternions and for quaternionic functions:

$$\varrho^{①} = \frac{1+\boldsymbol{i}}{\sqrt{2}};\ \varrho^{②} = \frac{1+\boldsymbol{j}}{\sqrt{2}}; \varrho^{③} = \frac{1+\boldsymbol{k}}{\sqrt{2}};\ \varrho^{④} = \frac{1-\boldsymbol{k}}{\sqrt{2}};\ \varrho^{⑤} = \frac{1-\boldsymbol{j}}{\sqrt{2}};\ \varrho^{⑥} = \frac{1-\boldsymbol{i}}{\sqrt{2}} \tag{2}$$

$$\boldsymbol{ij} = \boldsymbol{k};\ \ \boldsymbol{jk} = \boldsymbol{i};\ \ \boldsymbol{ki} = \boldsymbol{j} \tag{2}$$

$$\varrho^{⑥} = \left(\varrho^{①}\right)^* \tag{3}$$

For example

$$\psi^{③} = \varrho^{①}\psi^{②}/\varrho^{①} \tag{4}$$

$$\psi^{③}\varrho^{①} = \varrho^{①}\psi^{②} \tag{5}$$

$$\psi^{⓪} = \varrho^x\psi^{⓪}/\varrho^x;\ \psi^{⑦} = \varrho^x\psi^{⑦}/\varrho^x \tag{6}$$

Also strings of symmetry flavor convertors may change the index of symmetry flavor of the multiplied quaternion or quaternionic function. The convertors can act on each other.

For example:

$$\varrho^{①}\varrho^{②} = \varrho^{②}\varrho^{③} = \varrho^{③}\varrho^{①} = \frac{1 + \boldsymbol{i} + \boldsymbol{j} + \boldsymbol{k}}{2} \tag{7}$$

The result is an isotropic quaternion. This means:

$$\varrho^{①}\psi^{②}/\varrho^{x} = \varrho^{②}\psi^{③}/\varrho^{x} = \psi^{(x+1)} \tag{8}$$

Here $(x+1)$ means $\boldsymbol{i} \to \boldsymbol{j} \to \boldsymbol{k} \to \boldsymbol{i} \to \boldsymbol{j} \to \boldsymbol{k}$, or ①→②→③→①→②→③ and so on.

1.4 Differential calculus

In a flat continuum we can use the quaternionic nabla

$$\nabla = \left\{\frac{\partial}{\partial\tau}, \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right\} = \frac{\partial}{\partial\tau} + \boldsymbol{i}\frac{\partial}{\partial x} + \boldsymbol{j}\frac{\partial}{\partial y} + \boldsymbol{k}\frac{\partial}{\partial z} = \nabla_0 + \boldsymbol{\nabla} \tag{1}$$

$$\Phi = \Phi_0 + \boldsymbol{\Phi} = \nabla\psi \tag{2}$$

$$\Phi_0 = \nabla_0\psi_0 - \langle\boldsymbol{\nabla}, \boldsymbol{\psi}\rangle \tag{3}$$

$$\boldsymbol{\Phi} = \nabla_0\boldsymbol{\psi} + \boldsymbol{\nabla}\psi_0 \pm \boldsymbol{\nabla} \times \boldsymbol{\psi} \tag{4}$$

In Maxwell equations the equivalent terms have been given separate names. Maxwell equations use coordinate time t rather than proper time τ. See section on space-progression models.

1.4.1 The coupling equation

The coupling equation represents a peculiar property of the differential equation.

We start with two normalized functions ψ and φ and a normalizable function $\Phi = m\,\varphi$.

$$\|\psi\| = \|\varphi\| = 1 \tag{1}$$

These normalized functions are supposed to be related by:

$$\Phi = \nabla\psi = m\,\varphi \tag{2}$$

$\Phi = \nabla\psi$ defines the ***differential equation***. (3)

$\nabla\psi = \Phi$ formulates a differential ***continuity equation***. (4)

$\nabla\psi = m\,\varphi$ formulates the ***coupling equation***. (5)

1.4.1.1 Special forms of the coupling equation

The existence of symmetry flavors of quaternionic functions gives rise to special forms of the coupling equation for symmetry flavors $\{\psi^x, \psi^y\}$ of the shared base function $\psi^{⓪}$.

$$\nabla\psi^x = m_{xy}\,\psi^y \tag{1}$$

For example the Dirac equation for the free electron in quaternionic format runs:

$$\nabla\psi = m_e\,\psi^* \tag{2}$$

ψ^* and ψ are symmetry flavors of the same base function.

The Dirac equation for the free positron runs:

$$\nabla^*\psi^* = m_e\,\psi \tag{3}$$

Thus

$$\nabla^*\nabla\psi = m_e\nabla^*\,\psi^* = m_e^2\,\psi \tag{4}$$

Thus, for electrons ψ represents its own normalized object density distribution.

This analysis suggests that for elementary particles the equivalent of the coupling equation runs:

$$\nabla\psi^x = m_{xy}\,\psi^y \tag{5}$$

where ψ^x and ψ^y are symmetry flavors of the same base function $\psi^{⓪}$.

1.4.2 Transformations

The value of ϕ in

$$\phi = \nabla\psi \tag{1}$$

does not change after the transformation

$$\psi \to \psi + \xi = \psi + \nabla^* \chi \tag{2}$$

where

$$\nabla \xi = \nabla \nabla^* \chi = 0 \tag{3}$$

1.4.3 The wave equation

Locally, the wave function is considered to act in a flat continuum χ.

The quaternionic wave equation exists in a homogeneous ($\rho = 0$) and in in-homogeneous ($\rho \neq 0$) form.

$$\nabla^* \nabla \chi = \nabla_0 \nabla_0 \chi + \langle \boldsymbol{\nabla}, \boldsymbol{\nabla} \rangle \chi = \rho \tag{1}$$

The function ρ represents the temporary presence of one or more discrepant discrete objects.

Near the embedding location the homogeneous wave equation applies between two embedding occurrences and the in-homogeneous wave equation applies during the embedding.

$$\nabla^* \nabla \chi_0 = 0 \tag{2}$$

Equation (3) has 3D isotropic wave fronts as its solution. χ_0 is a scalar function. By changing to polar coordinates it can be deduced that a general solution is given by:

$$\chi_0(r, \tau) = \frac{f_0(\boldsymbol{i} r - c\tau)}{r} \tag{3}$$

Where $c = \pm 1$ and $\boldsymbol{i}$ represents a base vector in radial direction. In fact the parameter $\boldsymbol{i} r - c\tau$ of f_0 can be considered as a complex number valued function.

$$\nabla^* \nabla \boldsymbol{\chi} = 0 \tag{4}$$

Here $\boldsymbol{\chi}$ is a vector function.

Equation (4) has one dimensional wave fronts as solutions:

$$\boldsymbol{\chi}(z, \tau) = \boldsymbol{f}(\boldsymbol{i} z - c\tau) \tag{5}$$

Again the parameter $\boldsymbol{i} z - c\tau$ of $\boldsymbol{f}$ can be interpreted as a complex number based function.

The imaginary $\boldsymbol{i}$ represents the base vector in the x, y plane. Its orientation θ may be a function of z.

That orientation determines the polarization of the one dimensional wave front.

1.5 Poisson equation

The Poisson equation is a special condition of the wave equation in which some terms are zero or have a special value.

$$\nabla^*\nabla\chi = \nabla_0\nabla_0\chi + \langle\boldsymbol{\nabla},\boldsymbol{\nabla}\rangle\chi = \rho; \tag{1}$$

$$\nabla_0\nabla_0\chi = -\lambda^2\,\chi \tag{2}$$

$$\langle\boldsymbol{\nabla},\boldsymbol{\nabla}\rangle\chi - \lambda^2\chi = \rho \tag{3}$$

The 3D solution of this equation is determined by the screened Green's function $G(r)$.

Green functions represent solutions for point sources.

$$G(r) = \frac{\exp(-\lambda\, r)}{r} \tag{4}$$

$$\chi = \iiint G(\boldsymbol{r} - \boldsymbol{r}')\,\rho(\boldsymbol{r}')\,d^3\boldsymbol{r}' \tag{5}$$

G(r) has the shape of the Yukawa potential [14]

In case of $\lambda = 0$ it is the Coulomb or gravitation potential of a point source.

1.6 Space-progression models

The orthomodular base model applies the quaternionic wave equation for establishing the model's speed of information transfer.

In his introduction of special relativity in 1905, Einstein used the Maxwell based wave equation [10] in order to derive the speed of information transfer in his models. This resulted in a spacetime model that features a Minkowski signature.

The Maxwell based wave equation uses coordinate time t. The quaternionic wave equation uses progression τ. Comparing these two parameters becomes difficult when space is curved, but for infinitesimal steps space can be considered to be flat and the progression step becomes a proper time step. In that situation holds:

$$\text{Coordinate time step vector} = \text{proper time step vector} + \text{spatial step vector} \tag{1}$$

Or in Pythagoras format:

$$(\Delta t)^2 = (\Delta\tau)^2 + (\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2 \quad (2)$$

The formula indicates that the coordinate time step corresponds to the step of a full quaternion, which is a superposition of a proper time step and a spatial step.

An infinitesimal spacetime step Δs is usually presented as an infinitesimal proper time step $\Delta\tau$.

$$(\Delta s)^2 = (\Delta t)^2 - (\Delta x)^2 - (\Delta y)^2 - (\Delta z)^2, \text{ with signature } +{-}{-}{-}. \quad (3)$$

The Lorentz transform uses a parameter that is compared with the maximum speed of information transfer. Einstein and contemporary physics models use coordinate time for this purpose.

The orthomodular base model will use progression for that purpose. As a consequence it supports a space-progression model that features an Euclidean signature.

1.6.1 The Maxwell-Huygens wave equation

In Maxell format the wave equation uses coordinate time t. It runs as:

$$\partial^2\psi/\partial t^2 - \partial^2\psi/\partial x^2 - \partial^2\psi/\partial y^2 - \partial^2\psi/\partial z^2 = 0 \quad (1)$$

Papers on Huygens principle work with this formula or it uses the version with polar coordinates.

For isotropic 3D the general solution runs:

$$\psi = f(r - ct)/r, \text{ where } c = \pm 1; f \text{ is real} \quad (2)$$

For 1D the general solution runs:

$$\psi = f(x - ct), \text{ where } c = \pm 1; f \text{ is real} \quad (3)$$

2 Related historic discoveries

[1] Quantum logic was introduced by Garret Birkhoff and John von Neumann in their 1936 paper. G. Birkhoff and J. von Neumann, *The Logic of Quantum Mechanics*, Annals of Mathematics, Vol. 37, pp. 823–843

[2] The lattices of quantum logic and classical logic are treated in detail in: http://vixra.org/abs/1411.0175 .

[3] The Hilbert space was discovered in the first decades of the 20-th century by David Hilbert and others. http://en.wikipedia.org/wiki/Hilbert_space.

[4] In the sixties Constantin Piron and Maria Pia Solèr proved that the number systems that a separable Hilbert space can use must be division rings. See: “Division algebras and quantum theory” by John Baez. http://arxiv.org/abs/1101.5690

[5] Paul Dirac introduced the bra-ket notation, which popularized the usage of Hilbert spaces. Dirac also introduced its delta function, which is a generalized function. Spaces of generalized functions offered continuums before the Gelfand triple arrived.

[6] In the sixties Israel Gelfand and Georgyi Shilov introduced a way to model continuums via an extension of the separable Hilbert space into a so called Gelfand triple. The Gelfand triple often gets the name rigged Hilbert space, which is confusing, because this construct is not a separable Hilbert space. http://www.encyclopediaofmath.org/index.php?title=Rigged_Hilbert_space .

[7] Potential of a Gaussian charge density:
http://en.wikipedia.org/wiki/Poisson%27s_equation#Potential_of_a_Gaussian_charge_density .

[8] Quaternionic function theory and quaternionic Hilbert spaces are treated in:
http://vixra.org/abs/1411.0178 .

[9] In 1843 quaternions were discovered by Rowan Hamilton.
http://en.wikipedia.org/wiki/History_of_quaternions

[10] http://en.wikipedia.org/wiki/Wave_equation#Derivation_of_the_wave_equation

[11] http://en.wikipedia.org/wiki/Yukawa_potential

[12] These discoveries are also used as foundations by the author's e-book "The Hilbert Book Model Game". http://vixra.org/abs/1405.0340 .