Theory of the Nuclear Binding Energy

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Abstract: Here, within the Scale-Symmetric Theory (SST), we present the mathematically very simple theory of nuclear binding energy. We start from the Newtonian dynamics. It leads to binding energy proportional to coupling constant (or running coupling) of interaction and inversely proportional to distance between interacting objects. Applying the new formula, as some examples, we calculated binding energy of electron in ground state in hydrogen atom (13.6 eV), mean binding energy per nucleon in the alpha particle (7.07 MeV), mean binding energy per nucleon in the nucleus of iron atom (8.79 MeV), in nucleus of nobelium atom (7.21 MeV), for nucleus of oxygen atom (7.91 MeV) and for nuclei containing more than 56 nucleons. We present also the theory of deuteron. Obtained results are consistent with experimental data.

1. Introduction

Here, within the Scale-Symmetric Theory (SST), we present the mathematically very simple theory of nuclear binding energy and theory of deuteron.

The General Relativity leads to the non-gravitating Higgs field composed of tachyons [1A]. On the other hand, the Scale-Symmetric Theory (SST) shows that the succeeding phase transitions of such Higgs field lead to the different scales of sizes/energies [1A]. Due to the saturation of interactions via the Higgs field and due to the law of conservation of the half-integral spin that is obligatory for all scales, there consequently appear the superluminal binary systems of closed strings (entanglons) responsible for the quantum entanglement (it is the quantum-entanglement scale), stable neutrinos and luminal neutrino-antineutrino pairs which are the components of the luminal Einstein spacetime (it is the Planck scale), cores of baryons (it is the electric-charges scale), and the cosmic structures (protoworlds; it is the cosmological scale) that evolution leads to the dark matter, dark energy and expanding universes (the "soft" big bangs) [1A], [1B]. The non-gravitating tachyons have infinitesimal spin so all listed structures have internal helicity (helicities) which distinguishes particles from their antiparticles [1A]. SST shows that a fundamental theory should start from infinite nothingness and pieces of space [1A]. Sizes of pieces of space depend on their velocities [1A]. The inflation field started as the liquid-like field composed of non-gravitating pieces of space [1A]. Cosmoses composed of universes are created because of collisions of big pieces

of space [1A], [1B]. During the inflation, the liquid-like inflation field (the non-gravitating superluminal Higgs field) transformed partially into the luminal Einstein spacetime (the big bang) [1A], [1B]. In our Cosmos, the two-component spacetime is surrounded by timeless wall – it causes that the fundamental constants are invariant [1A], [1B].

SST shows that to obtain results consistent with experimental data, the big piece of space that transformed into the inflation field had before the collision a rotational energy very low in comparison with kinetic energy [1A]. It leads to conclusion that there was low anisotropy of the inflation field i.e. of the expanding superluminal non-gravitating Higgs field. It means that to such field we can apply the Kasner metric, [2], that is a solution to the vacuum Einstein equations so the Ricci tensor always vanishes. The Kasner metric is for an anisotropic cosmos without matter so it is a vacuum solution for the Higgs field. The one of the two semi-symmetrical Kasner solution, (2/3, 2/3 −1/3), we interpret as virtual Higgs cyclones with toroidal and poloidal motions. Such tori appear in the succeeding phase transitions of the Higgs field [1A].

Due to the symmetrical decays of bosons on the equator of the core of baryons, there appears the atom-like structure of baryons described by the Titius-Bode orbits for the nuclear strong interactions: $R_d = A + dB$, where $A = 0.6974425$ fm, $A/B = 1.3898$ and $d = 0$, 1, 2, 4 [1A].

Applying 7 parameters only and a few new symmetries we calculated a thousand of basic physical (and mathematical) quantities (there are derived the physical and mathematical constants as well) consistent or very close to experimental data and observational facts (http://vixra.org/author/sylwester_kornowski). In SST there do not appear approximations, mathematical tricks, and free parameters which are characteristic for the mainstream particle physics and mainstream cosmology.

2. The four-shell model of atomic nucleus
Within the Scale-Symmetric Theory (SST) we can formulate the four-shell model of atomic nuclei.

2.1 Volumetric binding energy of a nucleus per nucleon
The sum of the masses of the relativistic charged and neutral pions W_d in the $d = 1$ state is 424.403 MeV [1A]. The nucleons that an alpha particle is composed of, occupies the vertices of the square with the diagonal of the square equal to $A + 4B$. The exchanged pions are most frequently located in the centre of this square. As $A / R_d = v^2 / c^2$, $m_{W(+-o),d} = m_{pion(+-o)} / (1 − (v^2 / c^2))^{1/2}$, and the nucleon-pion distance is $(A + 4B) / 2$, the sum of the masses of the charged and neutral W_d pions is 394.499 MeV [1A]. The distance between the mass of the unbound and bound states is 29.904 MeV per two nucleons. We can see that when side of the square is

$$Side = (A + 4B) / 2^{1/2} \tag{1}$$

then the volumetric binding energy per nucleon is 14.952 MeV.

2.2 The magic numbers and radii of nuclei
Each nucleon occupies a cube which has a side equal to $a_c = (A + 4B) / 2^{1/2} = 1.91258 \cdot 10^{-15}$ m. We can assume that the nucleons inside a nucleus are placed on the concentric spheres where the distances between them equal a_c. This means that the radius of

the first sphere is equal to $a_c/2$. This, therefore, leads to the following formula for the radii of the spheres (they are not the radii of the nuclei because the spheres have a thickness)

$$r_{sn} = (n - 0.5)\, a_c ,\qquad(2)$$

where $n = 1, 2, 3, 4$.

The maximum number of nucleons placed on a sphere is (one nucleon occupies square a_c^2)

$$A_n = 4\,\pi\, r_{sn}^2 / a_c^2 = 4\,\pi\,(n - 0.5)^2 \qquad(3)$$

i.e. $A_1 = 3.14$, $A_2 = 28.27$, $A_3 = 78.54$ and $A_4 = 153.94$.

If we round these numbers to the nearest even number (nuclei containing an even number of nucleons are more stable), we obtain the following series: 4, 28, 78, and 154. This means that on the first four wholly filled spheres there are 264 nucleons. As we see by the first two numbers, the sum of the first and third and the result of subtracting the third and second, and the fourth and second numbers, we can see that the result is the well-known magic numbers of 4, 28, 82, 50, 126. This cannot be a coincidence which confirms that we are on the right path in order to build the correct theory of an atomic nucleus. When the number of neutrons becomes equal to one of the magic numbers then transitions of the protons between lower and higher spheres occurs. This increases the binding energy of a nucleus.

To calculate the electric radius of a nucleus (i.e. the radius of a nucleus obtained in experiments based on the bombardment of a nucleus by electrons) we have to add the electric radius of the nucleon to the radius of the last sphere. The charged pions in the nucleons are placed in the $d = 1$ state so due to the thermal motions of nucleons, we must add to radius of last sphere the radius of the $d = 1$ state i.e. $A + B = 1.19927{\cdot}10^{-15}$ m. Furthermore, the electric radius of the nucleus $A_n = 110$ is

$$r_{je}(A_n{=}110) = 2.5a_c + (A + B) = 5.98{\cdot}10^{-15}\ \text{m.}\qquad(4)$$

If we define the electric radius by using the formula

$$r_{je} = r_{oe} A_n^{1/3},\qquad(5)$$

then for a nucleus containing $A_n = 110$ nucleons we obtain $r_{oe} = 1.25$ fm. The value r_{oe} changes from 1.28 fm for $A_n = 32$ to 1.23 fm for $A_n = 264$.

The range of the nuclear strong interactions of a nucleon is $A + 4B$ so the radius of a nucleus for strong interactions (i.e. the radius of a nucleus obtained during experiments based on the bombardment of a nucleus by nucleons having energy of approximately 20 MeV) is greater than the electric radius

$$r_{jj}(A_n{=}110) = 2.5a_c + (A + 4B) = 7.49{\cdot}10^{-15}\ \text{m.}\qquad(6)$$

If we define such a radius by using the formula

$$r_{jj} = r_{oj} A_n^{1/3},\qquad(7)$$

then for a nucleus containing $A_n = 110$ nucleons we obtain $r_{oj} = 1.56$ fm. The value r_{oj} changes from 1.76 fm for $A_n = 32$ to 1.47 fm for $A_n = 264$.

2.3 Model of dynamic supersymmetry for nuclei
From [3] results that the nucleons in a nuclei are grouped in following way

   a = 2 protons and 2 neutrons,
   b = 3 protons and 5 neutrons,
   c = 3 protons and 4 neutrons,
   d = 1 proton and 1 neutron.

The new theory explains the above as follows

 a) A proton exists in two states with the probabilities [1A]:
   $y = 0.50838$ and $1 - y = 0.49162$.

If we multiply these probabilities by two (for a deuteron) or by four (for an alpha particle), we obtain the integers (approximately) because the probabilities are that y and $1 - y$ have almost the same values.

 b) A neutron exists in two states with the probabilities [1A]:
   $x = 0.62554$ and $1 - x = 0.37446$.

If we multiply these probabilities by eight, we obtain in the integers approximately 5 (5.004) and 3 (2.996). The 8 is the smallest integer which leads to integers (in approximation).

 c) For a system containing 50% of **a**) and 50% of **b**), we obtain the following probabilities
   $(x + y) / 2 = 0.56696$ and $(1 - x + 1 - y) / 2 = 0.43304$.

This factor is equal to 7 – then we obtain 3.969 i.e. approximately 4, and 3.031 i.e. approximately 3.

A nucleus chooses a mixture of the states **a**, **b**, **c** and **d** in such a manner the binding energy was the greatest. The *2p2n* groups appear when the interactions of protons dominate whereas the *3p5n* groups appear when the interactions of neutrons dominate.

2.4 Theory of the deuteron that follows from SST [1A]
The magnetic moment of a deuteron is only slightly lower than the sum of the magnetic moments of a proton and a neutron. This suggests that the *pn* binary system is bound for short times in a region having a high negative pressure. We can assume that negative pressure appears due to the exchanges of the "free" neutral pions. The "free" neutral pions appear due to the weak interactions because then pions can run out from the nuclear strong field. Since in neutron is the resting neutral pion in the $H^o Z^o \pi^o$ state (probability is $1 - x = 0.3744629$) then emissions and absorptions of neutral pions do not change magnetic moment of neutron. Calculate probability of emission of the neutral pion by a proton. Due to the $W^o \rightarrow Z^o \pi^o$ transitions, the emission of neutral pion by proton changes its magnetic moment – it follows from the fact that such transition changes the probabilities y and $y - 1$ calculated for free protons. In such transition, the angular momentum of the relativistic W^o cannot change. This condition causes that during the emission of the pion π^o, the electromagnetic loop Z^o (spin speed of this loop is equal to the speed c) is in the $d = 4$ tunnel, i.e. in the last tunnel for strong interactions, because then the angular momentum of $W^o_{d=1}$ is close to the angular momentum of Z^o (the ratio of these two angular momentums is $u = 0.9575329 \approx 1$).

Probability of emission of the "free" neutral pion (it is the binary system of photon loops) is equal to the ratio of the fine-structure constant (it concerns the photon loops; ($\alpha_{EM} = $

1/137.036) and the coupling constant for the nuclear strong interactions of the neutral pion ($\alpha_{s(pion)} = 1$) i.e. $z = \alpha_{EM} / \alpha_{s(pion)} = 0.00729735$. SST shows that probability of the H^+W^o state is $y = 0.5083856$. Probability of the H^+W^o and $H^+Z^o\pi^o$ states of proton in the neutron-proton bound state is $w = y + z = 0.515683$ whereas of the H^oW^+ state is $1 - w = 0.484317$.

Applying the formula for magnetic moment derived within SST, we obtain that the changed proton magnetic moment in the bound state in deuteron in the nuclear magneton is

$$\mu_{proton}* / \mu_o = m_{proton}\, w / m_{H(+)} + m_{proton}\, (1 - w) / m_{W(+),d=1} = +2.77128. \qquad (8)$$

The neutron magnetic moment in the nuclear magneton calculated within SST is

$$\mu_{neutron} / \mu_o = m_{proton}\, x / m_{H(+)} - m_{proton}\, x / m_{W(-),d=1} = -1.91343. \qquad (9)$$

We can see that the sum, i.e. the deuteron magnetic moment in the nuclear magneton, is

$$\mu_{deuteron} / \mu_o = \mu_{proton}* / \mu_o + \mu_{neutron} / \mu_o = +0.85785. \qquad (10)$$

Calculate the matter radius of deuteron. We can assume that it is a cylinder. The radius of the base of the cylinder is $R_{CB} = A + 4B \approx 2.7048$ fm. SST shows that the distance between the nucleons in deuteron is $2\pi A/3$ and it is the height of the cylinder $R_{CH} = 2\pi A/3 \approx 1.4607$ fm. To calculate the mean radius, we assume that volume of an abstract sphere with radius $R_{Sph,matter}$ is the same as the cylinder

$$R_{Sph,matter} = (3\, R_{CB}^2\, R_{CH} / 4)^{1/3} \approx 2.001 \text{ fm.} \qquad (11)$$

To obtain the binding energy for a deuteron we must take into account the electric interactions.

The W_d^- and W_d^+ interact from distance equal to $D = 2\pi A/3 = 1.46072$ fm for a period equal to $1 - w$.

The H^+_{proton} and W^- interact from L for a period equal to $x - (1 - y)$, where

$$L = [(2\,\pi\,A\,/\,3)^2 + (A + B)^2]^{1/2} = 1.70266 \text{ fm.} \qquad (12)$$

The H^+_{proton} and $H^+_{neutron}$ interact from D for a period equal to $x - (1 - y)$.
The $H^+_{neutron}$ and W^+_{proton} interact from L for a period equal to $1 - w$.
This leads to the proton-neutron electric attraction in a deuteron equal to

$$\Delta E_{em} / c^2 = e^2 (x + y + w - 2) (1/L - 1/D) / (10^7 f) = 0.0490819 \text{ MeV,} \qquad (13)$$

where $f = 1.7826617 \cdot 10^{-30}$ kg/MeV.

Therefore, the binding energy of deuteron emitting two free neutral pions and bound due to the volumetric binding energy equal to $\Delta E_{volumetric} = 29.903738$ MeV and electromagnetic energy $\Delta E_{em} / c^2 = 0.0490819$ MeV is

$$\Delta E_{np} = (2m_{pion(o)} + \Delta E_{volumetric})\, z + \Delta E_{em} = 2.22372 \text{ MeV.} \qquad (14)$$

2.5 The path of stability

Applying the model of dynamic supersymmetry for nuclei (see Paragraph **2.3**), in Table 1 we showed the abundances of the structures **a**, **b**, **c** and **d** in most stable nuclei.

Table 1 *Main path of stability of nuclei*

ZXA	a	b	c	d	ZXA	a	b	c	d	ZXA	a	b	c	d
1H1					36Kr84	9	6			71Lu175	10	16	1	
2He4**m**	1				37Rb85	9	5	1	1	72Hf180	9	18		
3Li7			1		38Sr88**m**	10	6			73Ta181	9	17	1	1
4Be9			1	1	39Y89	10	5	1	1	74W184	10	18		
5B11	1		1		40Zr90**m**	12	5		1	75Re187	9	18	1	
6C12	3				41Nb93	11	5	1	1	76Os192	8	20		
7N14	3			1	42Mo98	10	7		1	77Ir193	8	19	1	1
8O16**m**	4				43Tc97	12	5	1	1	78Pt194**?**	10	19		1
9F19	3		1		44Ru102	11	7		1	79Au197	9	19	1	1
10Ne20	5				45Rh103	12	6	1		80Hg202	8	21		1
11Na23	4		1		46Pd106	12	7		1	81Tl205	7	21	1	1
12Mg24	6				47Ag107	13	6	1		82Pb208**m**	8	22		
13Al27	5		1		48Cd114	10	9		1	83Bi209	8	21	1	1
14Si28	7				49In115	11	8	1		84Po209	10	20	1	1
15P31	6		1		50Sn120**m**	10	10			85At210	12	20		1
16S32	8				51Sb121	10	9	1	1	86Rn222	5	25		1
17Cl35	7		1		52Te130	6	13		1	87Fr223	6	24	1	
18Ar40	6	2			53I127	10	10	1		88Ra226	6	25		1
19K39	8		1		54Xe132	9	12			89Ac227	7	24	1	
20Ca40**m**	10				55Cs133	9	11	1	1	90$^{\text{Th}}$232	6	26		
21Sc45	7	1	1	1	56Ba138	8	13		1	91Pa231	8	24	1	
22Ti48	8	2			57La139	9	12	1		92U238	5	27		1
23V51**m**	7	2	1		58Ce140	11	12			93Np237	7	25	1	1
24Cr52**m**	9	2			59Pr141	11	11	1	1	94Pu244	5	28		
25Mn55	8	2	1		60Nd142	13	11		1	95Am243	7	26	1	
26Fe56	10	2			61Pm147	11	12	1		96Cm247	6	27	1	
27Co59	9	2	1		62Sm152	10	14			97Bk247	8	26	1	
28Ni58**m**	12	1		1	63Eu153	10	13	1	1	98Cf251	7	27	1	
29Cu63	10	2	1		64Gd158	9	15		1	99Es254	7	28		1
30Zn64	10	2	1	1	65Tb159	10	14	1		100Fm253	9	26	1	1
31Ga69	9	3	1	1	66Dy164	9	16			101Md258	8	28		1
32Ge74	8	5		1	67Ho165	9	15	1	1	102No256	12	26		
33As75	9	4	1		68Er166	11	15		1	103Lr256	14	25		
34Se80	8	6			69Tm169	10	15	1	1	104Ku260	13	26		
35Br79	10	4	1		70Yb174	9	17		1					

ZXA – denotes the atomic-number/symbol-of-element/mass-number
a = 2p + 2n = 2He4;   **b** = 3p + 5n;   **c** = 3p + 4n = 3Li7;   **d** = p + n = 1D2
? - denotes the discrepancy with the results in the periodic table of elements
m – denotes magic-number nucleus

The consistency with the experimental data is very high – only one result is inconsistent with experimental data. SST shows that the abundance of the 78Pt194 should be slightly higher than the 78Pt195.

3. Coupling constants

According to SST, the spacetime as a whole is flat. According to Newton's second law, in the regular 3-dimensional Euclidean space is

$$F_i = d\, p_i\, /\, d\, t. \tag{15}$$

According to SST, constants of interactions G_i are directly proportional to the inertial mass densities of fields carrying the interactions [1A]. The following formula defines the coupling constants (or running couplings), α_i, of all interactions [1A] (notice that m_i can be both mass or massless energy responsible for interactions)

$$\alpha_i = G_i\, M_i\, m_i\, /\, (c\, \hbar), \tag{16}$$

where M_i defines the sum of the mass of the sources of interaction plus the mass of the component of the field, whereas m_i defines the mass/energy of the carrier of interactions.

The strong coupling constant for pions exchanging the large loop (its mass is a little higher than a half of the mass of neutral pion, 67.5444 MeV) is $\alpha_S^{\pi\pi\gamma} = 1$ [1A]. Coupling constant for strongly interacting protons, at low energies (as it is in the atomic nuclei), is $\alpha_S^{pp\pi} = 14.4038$ whereas for strongly interacting neutrons is $\alpha_S^{nn\pi} = 14.4230$ [1A]. To the alpha particle, we can apply the mean value $\alpha_S^{NN\pi} = 14.4134$. When we accelerate a baryon, then there decreases the spin speed of the large loop so mass of it decreases as well – it leads to the running coupling for the nuclear strong interactions [1A], [4].

Assume that a carrier of interactions interacts simultaneously, for example, strongly and electromagnetically. Then, strong mass is $\alpha_S\, m$ whereas electromagnetic mass of the strong mass is $\alpha_{em}\, \alpha_S\, m$. It leads to conclusion that resultant coupling constant, α, is the product, Π, of coupling constants involved in the interactions

$$\alpha = \Pi\, \alpha_i. \tag{17}$$

When a carrier is a binary system then there appears the factor 2 i.e. $\alpha = 2\, \Pi\, \alpha_i$.

4. Calculations
4.1 Derivation of the main formula

Due to the radial emissions of carriers of interactions or radial polarization of virtual pairs, there is the inverse square law

$$F_i = G_i\, M_i\, m_i\, /\, r^2. \tag{18}$$

Applying formulae (15) – (18), we obtain

$$\int dp_i = \Pi\, \alpha_i\, c\, \hbar \int (1\, /\, r^2)\, dt. \tag{19a}$$

$$p\, v = \Pi\, \alpha_i\, c\, \hbar \int (1\, /\, r^2)\, dr. \tag{19b}$$

The radial kinetic energy, E_{kin}, transforms into radiation energy, $E_{radiation}$, so into binding energy, $E_{binding}$, as well i.e. $E_{kin} = p\ v\ /\ 2 = E_{radiation} = -\ E_{binding}$. We can rewrite formula (19b) as follows

$$E_{binding} = -\ \Pi\ \alpha_i\ c\ \hbar\ /\ (2\ r). \tag{20}$$

When we express this energy in MeV then there appears the factor f

$$E_{binding}\ [\text{MeV}] = m_{binding}\ c^2 = -\ \Pi\ \alpha_i\ c\ \hbar\ /\ (2\ r\ f), \tag{21}$$

where $f = 1.7826617 \cdot 10^{-30}$ kg/MeV.

Introduce symbol k

$$k = \hbar\ /\ (2\ c\ f) = 9.86635 \cdot 10^{-14}\ [\text{MeV m}]. \tag{22}$$

Formulae (21) and (22) lead to

$$m_{binding}\ [\text{MeV}] = -\ k\ \Pi\ \alpha_i\ /\ r. \tag{23}$$

It is the main formula.

4.2 Hydrogen atom

Calculate binding energy of electron in the ground state in hydrogen atom i.e. $\Pi\ \alpha_i = \alpha_{em} = 1/137.036$ and $r = 0.529177 \cdot 10^{-10}$ m. Applying formula (23), we obtain

$$m_{binding}\ [\text{MeV}] = -\ 13.6 \cdot 10^{-6}\ \text{MeV}. \tag{24}$$

This value is consistent with experimental data.

4.3 Alpha particle

Calculate the mean binding energy per nucleon in the alpha particle.

According to the SST, the two protons and two neutrons are placed in vertices of square which diagonal is $D = A + 4B$, where $A = 0.6974425$ fm is the radius of the core of baryons, whereas $B = 0.5018395$ fm [1A]. The $D = 2.7048$ fm defines radius of the last shell in baryons for strongly interacting pions [1A]. There are 6 directions of strong interactions i.e. the 4 sides of the square and its two diagonal directions. It leads to conclusion that mean distance of strong interactions is

$$R = [2\ D + 4\ D\ /\ sqrt(2)]\ /\ 6 = 2.176655\ \text{fm}. \tag{25}$$

The strong interactions of the four nucleons follow from the exchanges of the charged pions. It means that they interact strongly, $\alpha_S^{NN\pi} = 14.4134$, and electromagnetically $\alpha_{em} = 1/137.036$ i.e.

$$\Pi\ \alpha_i = \alpha_{em}\ \alpha_S^{NN\pi} = 0.105180. \tag{26}$$

From formulae (23), (25) and (26) we obtain the total strong binding energy for the alpha particle

$$m_{binding,S,total} \text{ [MeV]} = -6 \, k \, \alpha_{em} \, \alpha_S \, / \, R = -28.606 \text{ MeV}. \tag{27}$$

From the obtained absolute value 28.606 MeV, we must subtract the energy E_{em} which follows from the electrostatic repulsion of the protons. They can occupy two arbitrary vertices of the square so we obtain (there are the six different directions but simultaneously is only one direction of electrostatic repulsion contrary to the previous six simultaneous directions of strong attraction)

$$E_{em} = k \, \alpha_{em} \, / \, R = 0.33077 \text{ MeV}. \tag{28}$$

The mean binding energy per nucleon, ΔE, in the alpha particle is

$$\Delta E = (m_{binding,S,total} + E_{em}) \, / \, 4 = -7.069 \text{ MeV} \approx -7.07 \text{ MeV}. \tag{29}$$

From experimental data follows that mass of alpha particle is $3727.379240(82)$ MeV [5]. Mass of two protons and two neutrons is $3755.67485(84)$ MeV [6]. It leads to the mean binding energy per nucleon about -7.074 MeV. We can see that the theoretical result, -7.069 MeV, is very close to experimental data.

4.4 Nucleus of iron atom

According to the extension of supersymmetry in nuclear structure, [3], Paragraph 2.3, in atomic nuclei there are groups of nucleons containing following number of protons, p, and neutrons, n: $A*(2p2n)$ – it is the alpha particle, $B*(3p5n)$, $C*(3p4n)$ – it is the nucleus of lithium, $D*(1p1n)$ – it is the deuteron.

Here, within SST we proved that there dominate the groups $A*(2p2n)$ and $B*(3p5n)$.

Nucleus of iron contains 10 $A*(2p2n)$ alpha-particles/squares and 2 Type $B*(3p5n)$ groups/rectangular-prisms. As the bases of the rectangular-prisms are the alpha particles (the Type $2A* = 4p + 4n$ prism) or one alpha particle and one group containing one proton and three neutrons (Type $B*$ prism). The strong interactions between the bases should not destroy the bases so the strong fields of the bases should be tangent only i.e. the centre-to-centre distance between the bases should be $D = A + 4B$. It leads to conclusion that there appear only four additional directions of strong interactions with mean distance equal to D.

In the nucleus of iron atom there are 7 rectangular prisms. In the vertices of each rectangular prism are placed 8 nucleons. Calculate the total strong binding energy of such nuclear structure.

There are $6 + 6 + 4 = 16$ directions of strong interactions i.e. the 8 interactions on distance $D/sqrt(2)$ and 8 on distance D. The mean range of strong interactions is

$$R_{RP} = [8 \, D + 8 \, D \, / \, sqrt(2)] \, / \, 16 = 2.308691 \text{ fm}. \tag{30}$$

The strong interactions of the 8 nucleons follow from the exchanges of the charged pions. It means that they interact strongly, $\alpha_{S,2A*}{}^{NN\pi} = 14.4134$, $\alpha_{S,B*}{}^{NN\pi} = (3\alpha_S{}^{pp\pi} + 5\alpha_S{}^{nn\pi})/8 = 14.4158$, and electromagnetically $\alpha_{em} = 1/137.036$ i.e.

$$(\Pi\, \alpha_i\,)_{2A*} = \alpha_{em}\, \alpha_{S,2A*}{}^{NN\pi} = 0.105180, \qquad (31a)$$

$$(\Pi\, \alpha_i\,)_{B*} = \alpha_{em}\, \alpha_{S,B*}{}^{NN\pi} = 0.105197. \qquad (31b)$$

The total strong binding energy for the Type $2A*$ and $B*$ rectangular prisms are

$$m_{binding,S,2A*}\,[\text{MeV}] = -\,16\,k\,\alpha_{em}\,\alpha_{S,2A*}\,/\,R_{RP} = -\,71.919\text{ MeV}, \qquad (32a)$$

$$m_{binding,S,B*}\,[\text{MeV}] = -\,16\,k\,\alpha_{em}\,\alpha_{S,B*}\,/\,R_{RP} = -\,71.931\text{ MeV}. \qquad (32b)$$

The total strong binding energy of nucleus of iron atom is $m_{binding,S,Fe} = -503.456$ MeV (i.e. $5(-71.919) + 2(-71.931)$ MeV).

From the obtained absolute value $+503.456$ MeV, we must subtract the energy E_{em} which follows from the electrostatic repulsion of the protons. They can occupy arbitrary vertices so we obtain (there are 6 different directions of electrostatic repulsion in $2A*$ and 3 in $B*$)

$$E_{em,2A*} = 6\,k\,\alpha_{em}\,/\,R_{RP} = 1.8711\text{ MeV}, \qquad (33a)$$

$$E_{em,B*} = 3\,k\,\alpha_{em}\,/\,R_{RP} = 0.9356\text{ MeV}. \qquad (33b)$$

In nucleus of iron atom, the heights of the prisms are perfectly parallel and their bases lie on the same two parallel planes. It causes that there is high screening of the electric charges (protons) by neutrons placed between them. It leads to conclusion that practically we can neglect in nucleus of iron atom and nuclei with low number of prisms, the electrostatic repulsion between the prisms. When number of the prisms increases then there appears higher and higher chaos in their orientation. We can assume that in nuclei containing about 256 nucleons there is no electrostatic screening.

The strong interactions between the prisms are in the cost of their inner binding energy.

The mean binding energy per nucleon, ΔE_{Fe}, in the nucleus of iron atom is

$$\Delta E_{Fe} = (m_{binding,S,Fe} + 5\,E_{em,2A*} + 2\,E_{em,B*})\,/\,56 = $$
$$= -\,8.790\text{ MeV} = -\,8.79\text{ MeV}. \qquad (34)$$

4.5 Nucleus of nobelium atom

Here, applying the same method, we calculate the mean binding energy per nucleon in the nucleus of the nobelium atom.

According to the SST, nucleus of nobelium atom (main path of stability) consists of 12 alpha-particles/squares and 26 the Type $B*(3p5n)$ groups/rectangular-prisms i.e. there are 6 Type $2A*$ and 26 Type $B*$ rectangular prisms. It leads to conclusion that the total strong binding energy of nucleus of nobelium atom is $m_{binding,S,No} = -2301.720$ MeV (i.e. we have $6(-71.919) + 26(-71.931)$ MeV).

Number of rectangular prisms in nobelium is about $32 / 7 \approx 4.57$ times greater than in iron so we should not neglect the higher orders concerning the electrostatic repulsion of the rectangular prisms. Assume that mean distance between the 32 rectangular prisms is D so the strong interactions between them cannot destroy them. There are $N = 496$ (i.e. $31 + 30 + 29 + \ldots + 3 + 2 + 1 = 496$) directions between the rectangular prisms associated with their electrostatic repulsion. Mean relative electric charge of a rectangular prism is $Q/e = (6{\cdot}4 + 26{\cdot}3) / 32 = 3.1875$ (notice that the charged pions carry elementary electric charge). The second order concerning the electrostatic repulsion leads to

$$E_{em,second} = N \, (Q / e) \, k \, \alpha_{em} / D = 420.842 \text{ MeV}. \tag{35}$$

The mean binding energy per nucleon, ΔE_{No}, in the nucleus of nobelium atom is

$$\Delta E_{No} = (m_{binding,S,No} + 6 \, E_{em,2A*} + 26 \, E_{em,B*} + E_{em,second}) / 256 = $$
$$= -7.208 \text{ MeV} \approx -7.21 \text{ MeV}. \tag{36}$$

This result is consistent with experimental data.

4.6 Nuclei containing more than 56 nucleons

In the nuclei containing $1 + 6 = 7$ prisms (one prism in centre and six satellite prisms – it is the nucleus of iron; it looks as two parallel **H** letters in distance D) or $2 + 6 = 8$ prisms (two prisms in centre and six satellite prisms) or $2 + (2 + 6) = 10$ prisms which are some analog to electrons in $n = 1$ and $n = 2$ states in atoms, the prisms should be parallel. It leads to conclusion that we can neglect the electrostatic repulsion of prisms for atomic nuclei composed of about 56 to 80 nucleons. Average nuclear binding energy per nucleon for such nuclei should be close to 8.8 MeV.

For nuclei with atomic number, A_N, greater than 80, the electrostatic repulsive energy between the prisms should increase from zero for $A_N = 80$ to $E_{em,second} = 420.842$ MeV for $A_N = 256$ (formula (35)). It leads to following formula

$$E_{em,second}(A_N \geq 80) = X \, (A_N - 80) / 8 \text{ [MeV]}, \tag{37}$$

where $X = 19.1292$ MeV (notice that similar value, 19.367 MeV, appears in the theory of hyperons [1A].

The mean binding energy per nucleon for nuclei $A_N \geq 80$ is

$$\Delta E(A_N \geq 80) = (m_{binding,S} + x \, E_{em,2A*} + y \, E_{em,B*} + E_{em,second}(A_N \geq 80)) / A_N. \tag{38}$$

4.7 Nucleus of oxygen atom

In nucleus of oxygen atom there should be two Type $2A*$ prisms. But there is one Type $2A*$ prism and two satellite alpha particles and both alpha particles lie on one of the two planes associated with the two bases of the Type $2A*$ prism. The nucleus of oxygen looks as letter **T**. For such structure, the screening of electrostatic repulsions between the Type $2A*$ prism and the two satellite alpha particles are more effective than for structure composed of two Type $2A*$ prisms.

Applying formulae (27), (28), (32a) and (33a), for average binding energy per nucleon, ΔE_O, in the nucleus of oxygen, we obtain

$$\Delta E_O = (2m_{binding,S,total} + 2E_{em} + m_{binding,S,2A*} + E_{em,2A*}) / 16 =$$
$$= -7.912 \text{ MeV} \approx -7.91 \text{ MeV}. \tag{39}$$

Generally, in the nuclei for which $A_N < 50$, there is less the prisms and more the alpha particles.

5. Summary

Here, within the Scale-Symmetric Theory (SST), we present the mathematically very simple theory of nuclear binding energy. We start from the Newtonian dynamics. It leads to binding energy proportional to coupling constant (or running coupling) of interaction and inversely proportional to distance between interacting objects. Applying the new formula, as some examples, we calculated binding energy of electron in ground state in hydrogen atom (13.6 eV), mean binding energy per nucleon in the alpha particle (7.07 MeV), mean binding energy per nucleon in the nucleus of iron atom (8.79 MeV), in nucleus of nobelium atom (7.21 MeV), for nucleus of oxygen atom (7.91 MeV) and for nuclei containing more than 56 nucleons.

We present also the theory of deuteron.

Obtained results are consistent with experimental data.

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