Theory of the Nuclear Binding Energy

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Abstract: Here, within the Scale-Symmetric Everlasting Theory (S-SET), I present the mathematically very simple theory of nuclear binding energy. We start from the Newtonian dynamics. It leads to binding energy proportional to coupling constant (or running coupling) of interaction and inversely proportional to distance between interacting objects. Applying the new formula, as some examples, I calculated binding energy of electron in ground state in hydrogen atom (13.6 eV), mean binding energy per nucleon in the alpha particle (7.07 MeV) mean binding energy per nucleon in the nucleus of iron atom (8.79 MeV) and in nucleus of nobelium atom (7.21 MeV). Obtained results are consistent with experimental data.

1. Introduction

According to the Scale-Symmetric Everlasting Theory (S-SET), [1], the spacetime as a whole is flat. According to Newton's second law, in the regular 3-dimensional Euclidean space is

$$F_i = d\, p_i\, /\, d\, t. \tag{1}$$

According to S-SET, constants of interactions G_i are directly proportional to the inertial mass densities of fields carrying the interactions ([1]: formulae (12) and (11)). The following formula defines the coupling constants (or running couplings), α_i, of all interactions ([1]: formula (76)); notice that m_i can be both mass or massless energy responsible for interactions)

$$\alpha_i = G_i\, M_i\, m_i\, /\, (c\, \hbar), \tag{2}$$

where M_i defines the sum of the mass of the sources of interaction plus the mass of the component of the field, whereas m_i defines the mass/energy of the carrier of interactions.

The strong coupling constant for pions exchanging the large loop ([1]: its mass is a little higher than a half of the mass of neutral pion, 67.5444 MeV) is $\alpha_S^{\pi\pi\gamma} = 1$ ([1]: formula (77)).

Coupling constant for strongly interacting protons, at low energies (as it is in the atomic nuclei), is $\alpha_S^{pp\pi} = 14.4038$ ([1]: formula (78)) whereas for strongly interacting neutrons is $\alpha_S^{nn\pi} = 14.4230$. To the alpha particle we can apply the mean value $\alpha_S^{NN\pi} = 14.4134$. When we accelerate a baryon, then there decreases the spin speed of the large loop so mass of it decreases as well – it leads to the running coupling for strong interactions [1], [2].

Assume that a carrier of interactions interacts simultaneously, for example, strongly and electromagnetically. Then, strong mass is $\alpha_S m$ whereas electromagnetic mass of the strong mass is $\alpha_{em}\alpha_S m$. It leads to conclusion that resultant coupling constant, α, is the product, Π, of coupling constants involved in the interactions

$$\alpha = \Pi\alpha_i. \tag{3}$$

When a carrier is a binary system then there appear the factor 2 i.e. $\alpha = 2\ \Pi\alpha_i$.

Here, within the S-SET, I showed that binding energy is proportional to coupling constant (or running coupling) of interaction and inversely proportional to distance between interacting objects. Applying the new formula, as some examples, I calculated binding energy of electron in ground state in hydrogen atom (13.6 eV), mean binding energy per nucleon in the alpha particle (7.07 MeV) mean binding energy per nucleon in the nucleus of iron atom (8.79 MeV) and in nucleus of nobelium atom (7.21 MeV). Obtained results are consistent with experimental data.

2. Calculations
2.1 Derivation of the main formula

Due to the radial emissions of carriers of interactions or radial polarization of virtual pairs, there is the inverse square law

$$F_i = G_i\ M_i\ m_i\ /\ r^2. \tag{4}$$

Applying formulae (1) – (4), we obtain

$$\int dp_i = \Pi\alpha_i\ c\ \hbar \int (1\ /\ r^2)\ dt. \tag{5a}$$

$$p\ v = \Pi\alpha_i\ c\ \hbar \int (1\ /\ r^2)\ dr. \tag{5b}$$

The radial kinetic energy, E_{kin}, transforms into radiation energy, $E_{radiation}$, so into binding energy, $E_{binding}$, as well i.e. $E_{kin} = p\ v\ /\ 2 = E_{radiation} = -\ E_{binding}$. We can rewrite formula (5b) as follows

$$E_{binding} = -\ \Pi\alpha_i\ c\ \hbar\ /\ (2\ r). \tag{6}$$

$$E_{binding}\ [MeV] = m_{binding}\ c^2 = -\ \Pi\alpha_i\ c\ \hbar\ /\ (2\ r\ f), \tag{7}$$

where $f = 1.78266168\cdot10^{-30}$ kg/MeV.

Introduce symbol k

$$k = \hbar\ /\ (2\ c\ f) = 9.86635\cdot10^{-14}\ [MeV\ m]. \tag{8}$$

Formulae (7) and (8) lead to

$$m_{binding} \, [MeV] = - \, k \, \Pi\alpha_i \, / \, r. \tag{9}$$

It is the main formula.

2.2 Hydrogen atom

Calculate binding energy of electron in the ground state in hydrogen atom i.e. $\Pi\alpha_i = \alpha_{em} = 1/137.036$ and $r = 0.529177 \cdot 10^{-10}$ m. Applying formula (9), we obtain

$$m_{binding} \, [MeV] = - \, 13.6 \cdot 10^{-6} \, MeV. \tag{10}$$

This value is consistent with experimental data.

2.3 Alpha particle

Calculate the mean binding energy per nucleon in the alpha particle.

According to the S-SET, the two protons and two neutrons are placed in vertices of square which diagonal is $D = A + 4B$, where $A = 0.6974425$ fm is the radius of the core of baryons, whereas $B = 0.5018395$ fm [1]. The $D = 2.7048$ fm defines radius of the last shell in baryons for strongly interacting pions [1]. There are 6 directions of strong interactions i.e. the 4 sides of the square and its two diagonal directions. It leads to conclusion that mean distance of strong interactions is

$$R = [2 \, D + 4 \, D \, / \, sqrt(2)] \, / \, 6 = 2.176655 \, fm. \tag{11}$$

The strong interactions of the four nucleons follow from the exchanges of the charged pions. It means that they interact strongly, $\alpha_S^{NN\pi} = 14.4134$, and electromagnetically $\alpha_{em} = 1/137.036$ i.e.

$$\Pi\alpha_i = \alpha_{em}\alpha_S^{NN\pi} = 0.105180. \tag{12}$$

From formulae (9), (11) and (12) we obtain the total strong binding energy for the alpha particle

$$m_{binding,S,total} \, [MeV] = - \, 6 \, k \, \alpha_{em}\alpha_S \, / \, R = - \, 28.606 \, MeV. \tag{13}$$

From the obtained absolute value 28.606 MeV, we must subtract the energy E_{em} which follows from the electrostatic repulsion of the protons. They can occupy two arbitrary vertices of the square so we obtain (there are the six different directions but simultaneously is only one direction of electrostatic repulsion contrary to the previous six simultaneous directions of strong attraction)

$$E_{em} = k \, \alpha_{em} \, / \, R = 0.33077 \, MeV. \tag{14}$$

The mean binding energy per nucleon, ΔE, in the alpha particle is

$$\Delta E = (m_{binding,S,total} + E_{em}) \, / \, 4 = - \, 7.069 \, MeV \approx 7.07 \, MeV. \tag{15}$$

From experimental data follows that mass of alpha particle is 3727.379240(82) MeV [3]. Mass of two protons and two neutrons is 3755.67485(84) [4]. It leads to the mean binding energy per nucleon about –7.074 MeV. We can see that the theoretical result, –7.069 MeV, is very close to experimental data.

2.4 Nucleus of iron atom

According to the extension of supersymmetry in nuclear structure, [5], in atomic nuclei there are groups of nucleons containing following number of protons, p, and neutrons, n: A*(2p2n) – it is the alpha particie, B*(3p5n), C*(3p4n), D*(1p1n).

Within S-SET is described the origin of such supersymmetry in nuclear structure ([1]: Chapter "Four-shell Model of an Atomic Nucleus"). We proved that there dominate the groups A*(2p2n) and B*(3p5n) ([1]: Table 17 'Main path of stability of nuclei').

Nucleus of iron contains 10 A*(2p2n) alpha-particles/squares and 2 Type B*(3p5n) groups/rectangular-prisms [1]. As the bases of the rectangular-prisms are the alpha particles or one alpha particle and one group containing one proton and three neutrons. The strong interactions between the bases should not destroy the bases so the strong fields of the bases should be tangent only i.e. the centre-to-centre distance between the bases should be $D = A + 4B$. It leads to conclusion that there appear only four additional directions of strong interactions with mean distance equal to D.

In the nucleus of iron atom there are 7 rectangular prisms. In the vertices of each rectangular prism are placed 8 nucleons. Calculate the total strong binding energy of such nuclear structure.

There are $6 + 6 + 4 = 16$ directions of strong interactions i.e. the 8 interactions on distance D/sqrt(2) and 8 on distance D. The mean range of strong interactions is

$$R_{RP} = [8\ D + 8\ D\ /\ sqrt(2)]\ /\ 16 = 2.308691\ fm. \tag{16}$$

The strong interactions of the 8 nucleons follow from the exchanges of the charged pions. It means that they interact strongly, $\alpha_{S,A*}^{NN\pi} = 14.4134$, $\alpha_{S,B*}^{NN\pi} = (3\alpha_S^{pp\pi} + 5\alpha_S^{nn\pi})/8 = 14.4158$, and electromagnetically $\alpha_{em} = 1/137.036$ i.e.

$$(\ \Pi\alpha_i\ )_{A*} = \alpha_{em}\alpha_{S,A*}^{NN\pi} = 0.105180, \tag{17a}$$

$$(\ \Pi\alpha_i\ )_{B*} = \alpha_{em}\alpha_{S,B*}^{NN\pi} = 0.105197. \tag{17b}$$

The total strong binding energy for the Type A* and B* rectangular prisms are

$$m_{binding,S,A*}\ [MeV] = -\ 16\ k\ \alpha_{em}\alpha_{S,A*}\ /\ R_{RP} = -\ 71.919\ MeV, \tag{18a}$$

$$m_{binding,S,B*}\ [MeV] = -\ 16\ k\ \alpha_{em}\alpha_{S,B*}\ /\ R_{RP} = -\ 71.931\ MeV. \tag{18b}$$

The total strong binding energy of nucleus of iron atom is $m_{binding,S,Fe} = -503.456$ MeV (i.e. 5(-71.919) + 2(-71.931) MeV).

From the obtained absolute value +503.456 MeV, we must subtract the energy E_{em} which follows from the electrostatic repulsion of the protons. They can occupy arbitrary vertices so we obtain (there are 6 different directions of electrostatic repulsion in A* and 3 in B*)

$$E_{em,A*} = 6 \, k \, \alpha_{em} \, / \, R_{RP} = 1.8711 \text{ MeV}, \tag{19a}$$

$$E_{em,B*} = 3 \, k \, \alpha_{em} \, / \, R_{RP} = 0.9356 \text{ MeV}. \tag{19b}$$

The mean binding energy per nucleon, ΔE_{Fe}, in the nucleus of iron atom is

$$\Delta E_{Fe} = (m_{binding,S,Fe} + 5 \, E_{em,A*} + 2 \, E_{em,B*}) \, / \, 56 =$$
$$= -\,8.790 \text{ MeV} = -\,8.79 \text{ MeV}. \tag{20}$$

2.5 Nucleus of nobelium atom

Here, applying the same method, we calculate the mean binding energy per nucleon in the nucleus of the nobelium atom.

According to the S-SET, nucleus of nobelium atom (main path of stability) consists of 12 alpha-particles/squares and 26 the Type B*(3p5n) groups/rectangular-prisms i.e. there are 6 Type A* and 26 Type B* rectangular prisms. It leads to conclusion that the total strong binding energy of nucleus of nobelium atom is $m_{binding,S,No} = -\,2301.720$ MeV (6(-71.919) + 26(-71.931) MeV).

Number of rectangular prisms in nobelium is about $32/7 \approx 4.57$ times greater than in iron so we should not neglect the higher orders concerning the electrostatic repulsion of the rectangular prisms. Assume that mean distance between the 32 rectangular prisms is D so the strong interactions between them cannot destroy them. There are N = 496 (31 + 30 + 29 + … + 3 + 2 + 1 = 496) directions between the rectangular prisms associated with their electrostatic repulsion. Mean relative electric charge of an rectangular prism is $Q/e = (6\cdot4 + 26\cdot3)/32 = 3.1875$ (notice that the charged pions carry elementary electric charge). The second order concerning the electrostatic repulsion leads to

$$E_{em,second} = N \, (Q \, / \, e) \, k \, \alpha_{em} \, / \, D = 420.842 \text{ MeV}. \tag{21}$$

The mean binding energy per nucleon, ΔE_{No}, in the nucleus of nobelium atom is

$$\Delta E_{No} = (m_{binding,S,No} + 6 \, E_{em,A*} + 26 \, E_{em,B*} + E_{em,second}) \, / \, 256 =$$
$$= -\,7.208 \text{ MeV} \approx -\,7.21 \text{ MeV}. \tag{22}$$

3. Summary

Here, within the Scale-Symmetric Everlasting Theory (S-SET), I present the mathematically very simple theory of nuclear binding energy. We start from the Newtonian dynamics. It leads to binding energy proportional to coupling constant (or running coupling) of interaction and inversely proportional to distance between interacting objects. Applying the new formula, as some examples, I calculated binding energy of electron in ground state in hydrogen atom (13.6 eV), mean binding energy per nucleon in the alpha particle (7.07 MeV) mean binding energy per nucleon in the nucleus of iron atom (8.79 MeV) and in nucleus of nobelium atom (7.21 MeV). Obtained results are consistent with experimental data.

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