Florentin Smarandache

Neutrosophic Theory and its Applications

Collected Papers, Vol. I

Brussels, 2014

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EuropaNova

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INTRODUCTION TO NEUTROSOPHIC THEORY

(Preface)

Neutrosophic Theory means Neutrosophy applied in many fields in order to solve problems related to indeterminacy.

Neutrosophy is a new branch of philosophy that studies the origin, nature, and scope of neutralities, as well as their interactions with different ideational spectra.

This theory considers every entity <A> together with its opposite or negation <antiA> and with their spectrum of neutralities <neutA> in between them (i.e. entities supporting neither <A> nor <antiA>). The <neutA> and <antiA> ideas together are referred to as <nonA>.

Neutrosophy is a generalization of Hegel's dialectics (the last one is based on <A> and <antiA> only). According to this theory every entity <A> tends to be neutralized and balanced by <antiA> and <nonA> entities - as a state of equilibrium. In a classical way <A>, <neutA>, <antiA> are disjoint two by two. But, since in many cases the borders between notions are vague, imprecise, Sorites, it is possible that <A>, <neutA>, <antiA> (and <nonA> of course) have common parts two by two, or even all three of them as well.

Hence, in one hand, the Neutrosophic Theory is based on the triad <A>, <neutA>, and <antiA>. In the other hand, Neutrosophic Theory studies the indeterminacy, labelled as I, with $I^n = I$ for $n \geq 1$, and $mI + nI = (m+n)I$, in neutrosophic structures developed in algebra, geometry, topology etc.

The most developed fields of the *Neutrosophic Theory* are Neutrosophic Set, Neutrosophic Logic, Neutrosophic Probability, and Neutrosophic Statistics - that started in 1995, and recently Neutrosophic Precalculus and Neutrosophic Calculus, together with their applications in practice.

Neutrosophic Set and *Neutrosophic Logic* are generalizations of the fuzzy set and respectively fuzzy logic (especially of intuitionistic fuzzy set and respectively intuitionistic fuzzy logic). In neutrosophic logic a proposition has a degree of truth (T), a degree of indeterminacy (I), and a degree of falsity (F), where T, I, F are standard or non-standard subsets of $]^-0, 1^+[$.

Neutrosophic Probability is a generalization of the classical probability and imprecise probability.

Neutrosophic Statistics is a generalization of the classical statistics.

What distinguishes the neutrosophics from other fields is the <neutA>, which means neither <A> nor <antiA>. And <neutA>, which of course depends on <A>, can be indeterminacy, neutrality, tie (game), unknown, contradiction, vagueness, ignorance, incompleteness, imprecision, etc.

This volume contains 45 papers, written by the author alone or in collaboration with the following co-authors: Mumtaz Ali, Said Broumi, Sukanto Bhattacharya, Mamoni Dhar, Irfan Deli, Mincong Deng, Alexandru Gal, Valeri Kroumov, Pabitra Kumar Maji, Maikel Leyva-Vazquez, Feng Liu, Pinaki Majumdar, Munazza Naz, Karina Perez-Teruel, Rıdvan Sahin, A. A. Salama, Muhammad Shabir, Rajshekhar Sunderraman, Luige Vladareanu, Magdalena Vladila, Stefan Vladutescu, Haibin Wang, Hongnian Yu, Yan-Qing Zhang, about discounting of a neutrosophic mass in terms of reliability and respectively the importance of the source, evolution of sets from fuzzy set to neutrosophic set, classes of neutrosophic norm (n-norm) and neutrosophic conorm (n-conorm), applications of neutrosophic logic to physics, connections between extension logic and refined neutrosophic logic, approaches of neutrosophic logic to RABOT real time control, some applications of the neutrosophic logic to robotics, correlation coefficients of interval valued neutrosophic set, cosine similarity between interval valued neutrosophic sets, distance and similarity measures of interval neutrosophic soft sets, generalized interval neutrosophic soft sets and their operations, G-neutrosophic space, neutrosophic orbit, neutrosophic stabilizer, intuitionistic neutrosophic sets, intuitionistic neutrosophic soft sets, neutrosophic multi relation (NMR) defined on neutrosophic multisets, neutrosophic loops and biloops, neutrosophic N-loops and soft neutrosophic N-loops, operations on intuitionistic fuzzy soft sets, fuzzy soft matrix and new operations, such as fuzzy soft complement matrix and trace of fuzzy soft matrix based on reference function related properties, neutrosophic parameterized (NP) soft sets, NP-aggregation operator, and many more.

References:

Information about the Neutrosophics you get from the UNM website: http://fs.gallup.unm.edu/neutrosophy .

An international journal called *Neutrosophic Sets and Systems* is at http://fs.gallup.unm.edu/NSS .

A variety of scientific books in many languages can be downloaded freely from the *Digital Library of Science*: http://fs.gallup.unm.edu/eBooks-otherformats.htm

Florentin Smarandache

Reliability and Importance Discounting of Neutrosophic Masses

Florentin Smarandache

Abstract. In this paper, we introduce for the first time the discounting of a neutrosophic mass in terms of reliability and respectively the importance of the source.

We show that reliability and importance discounts commute when dealing with classical masses.

1. **Introduction.** Let $\Phi = \{\Phi_1, \Phi_2, \dots, \Phi_n\}$ be the frame of discernment, where $n \geq 2$, and the set of **focal elements**:

$$F = \{A_1, A_2, \dots, A_m\}, \text{for } m \geq 1, F \subset G^{\Phi}. \text{ (1)}$$

Let $G^{\Phi} = (\Phi, \cup, \cap, \mathcal{C})$ be the **fusion space**.

A **neutrosophic mass** is defined as follows:

$$m_n: G \rightarrow [0,1]^3$$

for any $x \in G$, $m_n(x) = (t(x), i(x), f(x))$, (2)

where $t(x) =$ believe that x will occur (truth);

$i(x) =$ indeterminacy about occurence;

and $f(x) =$ believe that x will not occur (falsity).

Simply, we say in neutrosophic logic:

$t(x) =$ believe in x;

$i(x) =$ believe in neut(x)

[the neutral of x, i.e. neither x nor anti(x)];

and $f(x) =$ believe in anti(x) [the opposite of x].

Of course, $t(x), \ i(x), \ f(x) \in [0,1]$, and

$$\sum_{x\in G}[t(x) + i(x) + f(x)] = 1, (3)$$

while

$$m_n(\phi) = (0,0,0). \ (4)$$

It is possible that according to some parameters (or data) a source is able to predict the believe in a hypothesis x to occur, while according to other parameters (or other data) the same source may be able to find the believe in x not occuring, and upon a third category of parameters (or data) the source may find some indeterminacy (ambiguity) about hypothesis occurence.

An element $x \in G$ is called **focal** if

$$n_m(x) \neq (0,0,0), (5)$$

i.e. $t(x) > 0$ or $i(x) > 0$ or $f(x) > 0$.

Any **classical mass**:

$$m : G^{\phi} \to [0,1] \ (6)$$

can be simply written as a neutrosophic mass as:

$$m(A) = (m(A), 0, 0). \ (7)$$

2. Discounting a Neutrosophic Mass due to Reliability of the Source.

Let $\alpha = (\alpha_1, \alpha_2, \alpha_3)$ be the reliability coefficient of the source, $\alpha \in [0,1]^3$.

Then, for any $x \in G^\theta \setminus \{\theta, I_t\}$,

where $\theta =$ the empty set

and $I_t =$ total ignorance,

$$m_n(x)_\alpha = \big(\alpha_1 t(x), \alpha_2 i(x), \alpha_3 f(x)\big), \ (8)$$

and

$$m_n(I_t)_\alpha = \left(t(I_t) + (1-\alpha_1) \sum_{x \in G^\theta \setminus \{\phi, I_t\}} t(x)\,, \right.$$
$$\left. i(I_t) + (1-\alpha_2) \sum_{x \in G^\theta \setminus \{\phi, I_t\}} i(x), f(I_t) + (1-\alpha_3) \sum_{x \in G^\theta \setminus \{\phi, I_t\}} f(x) \right)$$

(9),

and, of course,

$$m_n(\phi)_\alpha = (0, 0, 0).$$

The missing mass of each element x, for $x \neq \phi, x \neq I_t$, is transferred to the mass of the total ignorance in the following way:

$t(x) - \alpha_1 t(x) = (1 - \alpha_1) \cdot t(x)$ is transferred to $t(I_t)$, (10)

$i(x) - \alpha_2 i(x) = (1 - \alpha_2) \cdot i(x)$ is transferred to $i(I_t)$, (11) and $f(x) - \alpha_3 f(x) = (1 - \alpha_3) \cdot f(x)$ is transferred to $f(I_t)$. (12)

3. Discounting a Neutrosophic Mass due to the Importance of the Source.

Let $\beta \in [0, 1]$ be the importance coefficient of the source. This discounting can be done in several ways.

a. For any $x \in G^\theta \setminus \{\phi\}$,

$$m_n(x)_{\beta_1} = \big(\beta \cdot t(x), i(x), f(x) + (1 - \beta) \cdot t(x)\big), (13)$$

which means that $t(x)$, the believe in x, is diminished to $\beta \cdot t(x)$, and the missing mass, $t(x) - \beta \cdot t(x) = (1 - \beta) \cdot t(x)$, is transferred to the believe in $anti(x)$.

b. Another way:

For any $x \in G^\theta \setminus \{\phi\}$,

$$m_n(x)_{\beta_2} = \big(\beta \cdot t(x), i(x) + (1 - \beta) \cdot t(x), f(x)\big), (14)$$

which means that $t(x)$, the believe in x, is similarly diminished to $\beta \cdot t(x)$, and the missing mass $(1 - \beta) \cdot t(x)$ is now transferred to the believe in $neut(x)$.

c. The third way is the most general, putting together the first and second ways.

For any $x \in G^\theta \setminus \{\phi\}$,

$$m_n(x)_{\beta_3} = \big(\beta \cdot t(x), i(x) + (1 - \beta) \cdot t(x) \cdot \gamma, f(x) + (1 - \beta) \cdot t(x) \cdot (1 - \gamma)\big), (15)$$

where $\gamma \in [0,1]$ is a parameter that splits the missing mass $(1-\beta)\cdot t(x)$ a part to $i(x)$ and the other part to $f(x)$.

For $\gamma = 0$, one gets the first way of distribution, and when $\gamma = 1$, one gets the second way of distribution.

4. Discounting of Reliability and Importance of Sources in General Do Not Commute.

a. Reliability first, Importance second.

For any $x \in G^\theta \setminus \{\phi, I_t\}$, one has after reliability α discounting, where

$$\alpha = (\alpha_1, \alpha_2, \alpha_3):$$

$$m_n(x)_\alpha = \big(\alpha_1 \cdot t(x), \alpha_2 \cdot t(x), \alpha_3 \cdot f(x)\big), (16)$$

and

$$m_n(I_t)_\alpha = \left(t(I_t) + (1-\alpha_1) \cdot \sum_{x \in G^\theta \setminus \{\phi, I_t\}} t(x)\, , i(I_t) + (1-\alpha_2) \cdot \sum_{x \in G^\theta \setminus \{\phi, I_t\}} i(x)\, , f(I_t) + (1-\alpha_3) \cdot \sum_{x \in G^\theta \setminus \{\phi, I_t\}} f(x) \right) \stackrel{\text{def}}{=} \left(T_{I_t}, I_{I_t}, F_{I_t}\right). \quad (17)$$

Now we do the importance β discounting method, the third importance discounting way which is the most general:

$$m_n(x)_{\alpha\beta_3} = \big(\beta\alpha_1 t(x), \alpha_2 i(x) + (1-\beta)\alpha_1 t(x)\gamma, \alpha_3 f(x) + (1-\beta)\alpha_1 t(x)(1-\gamma)\big)$$

(18)

and

$$m_n(I_t)_{\alpha\beta_3} = \left(\beta \cdot T_{I_t}, I_{I_t} + (1-\beta)T_{I_t} \cdot \gamma, F_{I_t} + (1-\beta)T_{I_t}(1-\gamma)\right). \quad (19)$$

b. Importance first, Reliability second.

For any $x \in G^\theta \setminus \{\phi, I_t\}$, one has after importance β discounting (third way):

$$m_n(x)_{\beta_3} = \big(\beta \cdot t(x), i(x) + (1-\beta)t(x)\gamma, f(x) + (1-\beta)t(x)(1-\gamma)\big) \quad (20)$$

and

$$m_n(I_t)_{\beta_3} = \left(\beta \cdot t\left(I_{I_t}\right), i(I_{I_t}) + (1-\beta)t(I_t)\gamma,\ f(I_t) + (1-\beta)t(I_t)(1-\gamma)\right).$$

(21)

Now we do the reliability $\alpha = (\alpha_1, \alpha_2, \alpha_3)$ discounting, and one gets:

$$m_n(x)_{\beta_3\alpha} = \big(\alpha_1 \cdot \beta \cdot t(x), \alpha_2 \cdot i(x) + \alpha_2(1-\beta)t(x)\gamma, \alpha_3 \cdot f(x) + \alpha_3 \cdot (1-\beta)t(x)(1-\gamma)\big) \quad (22)$$

and

$$m_n(I_t)_{\beta_3\alpha} = \big(\alpha_1 \cdot \beta \cdot t(I_t), \alpha_2 \cdot i(I_t) + \alpha_2(1-\beta)t(I_t)\gamma, \alpha_3 \cdot f(I_t) + \alpha_3(1-\beta)t(I_t)(1-\gamma)\big). \quad (23)$$

Remark.

We see that (a) and (b) are in general different, so reliability of sources does not commute with the importance of sources.

5. Particular Case when Reliability and Importance Discounting of Masses Commute.

Let's consider a classical mass

$$m: G^\theta \to [0, 1] \text{ (24)}$$

and the focal set $F \subset G^\theta$,

$$F = \{A_1, A_2, \dots, A_m\}, m \geq 1, \text{ (25)}$$

and of course $m(A_i) > 0$, for $1 \leq i \leq m$.

Suppose $m(A_i) = a_i \in (0,1]$. (26)

a. Reliability first, Importance second.

Let $\alpha \in [0, 1]$ be the reliability coefficient of $m\ (\cdot)$.

For $x \in G^\theta \setminus \{\phi, I_t\}$, one has

$$m(x)_\alpha = \alpha \cdot m(x), \text{ (27)}$$

$$\text{and } m(I_t) = \alpha \cdot m(I_t) + 1 - \alpha. \text{ (28)}$$

Let $\beta \in [0, 1]$ be the importance coefficient of $m\ (\cdot)$.

Then, for $x \in G^\theta \setminus \{\phi, I_t\}$,

$$m(x)_{\alpha\beta} = \big(\beta\alpha m(x), \alpha m(x) - \beta\alpha m(x)\big) = \alpha \cdot m(x) \cdot (\beta, 1 - \beta), \text{ (29)}$$

considering only two components: believe that x occurs and, respectively, believe that x does not occur.

Further on,

$$m(I_t)_{\alpha\beta} = (\beta\alpha m(I_t) + \beta - \beta\alpha, \alpha m(I_t) + 1 - \alpha - \beta\alpha m(I_t) - \beta + \beta\alpha) =$$
$$[\alpha m(I_t) + 1 - \alpha] \cdot (\beta, 1 - \beta). \text{ (30)}$$

b. Importance first, Reliability second.

For $x \in G^\theta \setminus \{\phi, I_t\}$, one has

$$m(x)_\beta = \big(\beta \cdot m(x), m(x) - \beta \cdot m(x)\big) = m(x) \cdot (\beta, 1 - \beta), \text{ (31)}$$

and $m(I_t)_\beta = \big(\beta m(I_t), m(I_t) - \beta m(I_t)\big) = m(I_t) \cdot (\beta, 1 - \beta)$. (32)

Then, for the reliability discounting scaler α one has:

$$m(x)_{\beta\alpha} = \alpha m(x)(\beta, 1 - \beta) = \big(\alpha m(x)\beta, \alpha m(x) - \alpha\beta m(m)\big) \text{ (33)}$$

and $m(I_t)_{\beta\alpha} = \alpha \cdot m(I_t)(\beta, 1 - \beta) + (1 - \alpha)(\beta, 1 - \beta) = [\alpha m(I_t) + 1 - \alpha] \cdot (\beta, 1 - \beta) = (\alpha m(I_t)\beta, \alpha m(I_t) - \alpha m(I_t)\beta) + (\beta - \alpha\beta, 1 - \alpha - \beta + \alpha\beta) = (\alpha\beta m(I_t) + \beta - \alpha\beta, \alpha m(I_t) - \alpha\beta m(I_t) + 1 - \alpha - \beta - \alpha\beta)$. (34)

Hence (a) and (b) are equal in this case.

6. Examples.

1. Classical mass.

The following classical is given on $\theta = \{A, B\}$:

	A	B	AUB
m	0.4	0.5	0.1

(35)

Let $\alpha = 0.8$ be the reliability coefficient and $\beta = 0.7$ be the importance coefficient.

a. Reliability first, Importance second.

	A	B	AUB
m_α	0.32	0.40	0.28
$m_{\alpha\beta}$	(0.224, 0.096)	(0.280, 0.120)	(0.196, 0.084)

(36)

We have computed in the following way:

$$m_\alpha(A) = 0.8m(A) = 0.8(0.4) = 0.32, \quad (37)$$

$$m_\alpha(B) = 0.8m(B) = 0.8(0.5) = 0.40, \quad (38)$$

$$m_\alpha(AUB) = 0.8(\text{AUB}) + 1 - 0.8 = 0.8(0.1) + 0.2 = 0.28, \quad (39)$$

and

$$m_{\alpha\beta}(B) = \big(0.7m_\alpha(A), m_\alpha(A) - 0.7m_\alpha(A)\big) = \big(0.7(0.32), 0.32 - 0.7(0.32)\big) = (0.224, 0.096), \quad (40)$$

$$m_{\alpha\beta}(B) = \left(0.7m_\alpha(B), m_\alpha(B) - 0.7m_\alpha(B)\right) = \left(0.7(0.40), 0.40 - 0.7(0.40)\right) = (0.280, 0.120), \quad (41)$$

$$m_{\alpha\beta}(AUB) = \left(0.7m_\alpha(AUB), m_\alpha(AUB) - 0.7m_\alpha(AUB)\right) = \left(0.7(0.28), 0.28 - 0.7(0.28)\right) = (0.196, 0.084). \quad (42)$$

b. Importance first, Reliability second.

	A	B	AUB
m	0.4	0.5	0.1
m_β	(0.28, 0.12)	(0.35, 0.15)	(0.07, 0.03)
$m_{\beta\alpha}$	(0.224, 0.096	(0.280, 0.120)	(0.196, 0.084)

(43)

We computed in the following way:

$$m_\beta(A) = \left(\beta m(A), (1-\beta)m(A)\right) = \left(0.7(0.4), (1-0.7)(0.4)\right) = (0.280, 0.120), \quad (44)$$

$$m_\beta(B) = \left(\beta m(B), (1-\beta)m(B)\right) = \left(0.7(0.5), (1-0.7)(0.5)\right) = (0.35, 0.15), \quad (45)$$

$$m_\beta(AUB) = \left(\beta m(AUB), (1-\beta)m(AUB)\right) = \left(0.7(0.1), (1-0.1)(0.1)\right) = (0.07, 0.03), \quad (46)$$

and $m_{\beta\alpha}(A) = \alpha m_\beta(A) = 0.8(0.28, 0.12) = \left(0.8(0.28), 0.8(0.12)\right) = (0.224, 0.096)$, (47)

$$m_{\beta\alpha}(B) = \alpha m_\beta(B) = 0.8(0.35, 0.15) = \left(0.8(0.35), 0.8(0.15)\right) = (0.280, 0.120), \quad (48)$$

$$m_{\beta\alpha}(AUB) = \alpha m(AUB)(\beta, 1-\beta) + (1-\alpha)(\beta, 1-\beta) = 0.8(0.1)(0.7, 1-0.7) + (1-0.8)(0.7, 1-0.7) = 0.08(0.7, 0.3) + 0.2(0.7, 0.3) = (0.056, 0.024) + (0.140, 0.060) = (0.056 + 0.140, 0.024 + 0.060) = (0.196, 0.084). \quad (49)$$

Therefore reliability discount commutes with importance discount of sources when one has classical masses.

The result is interpreted this way: believe in A is 0.224 and believe in $nonA$ is 0.096, believe in B is 0.280 and believe in $nonB$ is 0.120, and believe in total ignorance AUB is 0.196, and believe in non-ignorance is 0.084.

7. Same Example with Different Redistribution of Masses Related to Importance of Sources.

Let's consider the third way of redistribution of masses related to importance coefficient of sources. $\beta = 0.7$, but $\gamma = 0.4$, which means that 40% of β is redistributed to $i(x)$ and 60% of β is redistributed to $f(x)$ for each $x \in G^\theta \setminus \{\phi\}$; and $\alpha = 0.8$.

a. Reliability first, Importance second.

	A	B	AUB
m	0.4	0.5	0.1
m_α	0.32	0.40	0.28
$m_{\alpha\beta}$	(0.2240, 0.0384, 0.0576)	(0.2800, 0.0480, 0.0720)	(0.1960, 0.0336, 0.0504).

(50)

We computed m_α in the same way.

But:

$$m_{\alpha\beta}(A) = \big(\beta \cdot m_\alpha(A), i_\alpha(A) + (1-\beta)m_\alpha(A) \cdot \gamma, f_\alpha(A) + (1-\beta)m_\alpha(A)(1-\gamma)\big) = \big(0.7(0.32), 0 + (1-0.7)(0.32)(0.4), 0 + (1-0.7)(0.32)(1-0.4)\big) = (0.2240, 0.0384, 0.0576). \quad (51)$$

Similarly for $m_{\alpha\beta}(B)$ and $m_{\alpha\beta}(AUB)$.

b. Importance first, Reliability second.

	A	B	AUB
m	0.4	0.5	0.1
m_β	(0.280, 0.048, 0.072)	(0.350, 0.060, 0.090)	(0.070, 0.012, 0.018)
$m_\beta\alpha$	(0.2240, 0.0384, 0.0576)	(0.2800, 0.0480, 0.0720)	(0.1960, 0.0336, 0.0504).

(52)

We computed $m_\beta(\cdot)$ in the following way:

$$m_\beta(A) = \big(\beta \cdot t(A), i(A) + (1-\beta)t(A) \cdot \gamma, f(A) + (1-\beta)t(A)(1-\gamma)\big) = \big(0.7(0.4), 0 + (1-0.7)(0.4)(0.4), 0 + (1-0.7)0.4(1-0.4)\big) = (0.280, 0.048, 0.072). \quad (53)$$

Similarly for $m_\beta(B)$ and $m_\beta(AUB)$.

To compute $m_{\beta\alpha}(\cdot)$, we take $\alpha_1 = \alpha_2 = \alpha_3 = 0.8$, (54)

in formulas (8) and (9).

$$m_{\beta\alpha}(A) = \alpha \cdot m_\beta(A) = 0.8(0.280, 0.048, 0.072)$$
$$= \big(0.8(0.280), 0.8(0.048), 0.8(0.072)\big)$$
$$= (0.2240, 0.0384, 0.0576). (55)$$

Similarly $m_{\beta\alpha}(B) = 0.8(0.350, 0.060, 0.090) = (0.2800, 0.0480, 0.0720)$. (56)

For $m_{\beta\alpha}(AUB)$ we use formula (9):

$$m_{\beta\alpha}(AUB) = \big(t_\beta(AUB) + (1-\alpha)\big[t_\beta(A) + t_\beta(B)\big],\ i_\beta(AUB) + (1-\alpha)\big[i_\beta(A) + i_\beta(B)\big], f_\beta(AUB) + (1-\alpha)\big[f_\beta(A) + f_\beta(B)\big]\big)$$
$$= (0.070 + (1-0.8)[0.280 + 0.350], 0.012 + (1-0.8)[0.048 + 0.060], 0.018 + (1-0.8)[0.072 + 0.090])$$
$$= (0.1960, 0.0336, 0.0504).$$

Again, the reliability discount and importance discount commute.

8. Conclusion.

In this paper we have defined a new way of discounting a classical and neutrosophic mass with respect to its importance. We have also defined the discounting of a neutrosophic source with respect to its reliability.

In general, the reliability discount and importance discount do not commute. But if one uses classical masses, they commute (as in Examples 1 and 2).

Acknowledgement.

The author would like to thank Dr. Jean Dezert for his opinions about this paper.

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A Geometric Interpretation of the Neutrosophic Set, A Generalization of the Intuitionistic Fuzzy Set

Florentin Smarandache

***Abstract*:**
In this paper we give a geometric interpretation of the Neutrosophic Set using the Neutrosophic Cube. Distinctions between the neutrosophic set and intuitionistic fuzzy set are also presented.

***Keywords and Phrases*:**
Intuitionistic Fuzzy Set, Paraconsistent Set, Intuitionistic Set, Neutrosophic Set, Neutrosophic Cube, Non-standard Analysis, Dialectics

1. Introduction:

One first presents the evolution of sets from fuzzy set to neutrosophic set. Then one introduces the neutrosophic components T, I, F which represent the membership, indeterminacy, and non-membership values respectively, where $]^-0, 1^+[$ is the non-standard unit interval, and thus one defines the neutrosophic set.

2. Short History:

The *fuzzy set* (FS) was introduced by L. Zadeh in 1965, where each element had a degree of membership.

The *intuitionistic fuzzy set* (IFS) on a universe X was introduced by K. Atanassov in 1983 as a generalization of FS, where besides the degree of membership $\mu_A(x) \in [0,1]$ of each element $x \in X$ to a set A there was considered a degree of non-membership $\nu_A(x) \in [0,1]$, but such that

$$\forall x \in X, \mu_A(x)+ \nu_A(x) \leq 1. \quad (2.1)$$

According to Deschrijver & Kerre (2003) the *vague set* defined by Gau and Buehrer (1993) was proven by Bustine & Burillo (1996) to be the same as IFS.

Goguen (1967) defined the *L-fuzzy Set* in X as a mapping X→L such that $(L^*, \leq_L^*)$ is a complete lattice, where $L^*=\{(x_1,x_2) \in [0,1]^2, x_1+x_2\leq 1\}$ and $(x_1,x_2) \leq L^* (y_1,y_2) \Leftrightarrow x_1\leq y_1$ and $x_2\geq y_2$. The *interval-valued fuzzy set* (IVFS) apparently first studied by Sambuc (1975), which were called by Deng (1989) *grey sets*, and IFS are specific kinds of L-fuzzy sets.

According to Cornelis et al. (2003), Gehrke et al. (1996) stated that “Many people believe that assigning an exact number to an expert’s opinion is too restrictive, and the assignment of an interval of values is more realistic”, which is somehow similar with the imprecise probability theory where instead of a crisp probability one has an interval (upper and lower) probabilities as in Walley (1991).

Atanassov (1999) defined the *interval-valued intuitionistic fuzzy set* (IVIFS) on a universe X as an object A such that:

$$A= \{(x, M_A(X), N_A(x)), x\in X\}, \quad (2.2)$$

with $M_A:X\to Int([0,1])$ and $N_A:X\to Int([0,1])$ (2.3)

and $\forall x \in X, \sup M_A(x)+ \sup N_A(x) \leq 1$. (2.4)

Belnap (1977) defined a four-valued logic, with truth (T), false (F), unknown (U), and contradiction (C). He used a billatice where the four components were inter-related.

In 1995, starting from philosophy (when I fretted to distinguish between *absolute truth* and *relative truth* or between *absolute falsehood* and *relative falsehood* in logics, and respectively between *absolute membership* and *relative membership* or *absolute non-membership* and *relative non-membership* in set theory) I began to use the non-standard analysis. Also, inspired from the sport games (winning, defeating, or tie scores), from votes (pro, contra, null/black votes), from positive/negative/zero numbers, from yes/no/NA, from decision making and control theory (making a decision, not making, or hesitating), from accepted/rejected/pending, etc. and guided by the fact that the law of excluded middle did not work any longer in the modern logics, I combined the non-standard analysis with a tri-component logic/set/probability theory and with philosophy (I was excited by paradoxism in science and arts and letters, as well as by paraconsistency and incompleteness in knowledge). How to deal with all of them at once, is it possible to unity them?

I proposed the term "neutrosophic" because "neutrosophic" etymologically comes from "neutrosophy" [French *neutre* < Latin *neuter*, neutral, and Greek *sophia*, skill/wisdom] which means knowledge of neutral thought, and this third/neutral represents the main distinction between "fuzzy" and "intuitionistic fuzzy" logic/set, i.e. the *included middle* component (Lupasco-Nicolescu's logic in philosophy), i.e. the neutral/indeterminate/unknown part (besides the "truth"/"membership" and "falsehood"/"non-membership" components that both appear in fuzzy logic/set). See the Proceedings of the First International Conference on Neutrosophic Logic, The University of New Mexico, Gallup Campus, 1-3 December 2001, at http://www.gallup.unm.edu/~smarandache/FirstNeutConf.htm.

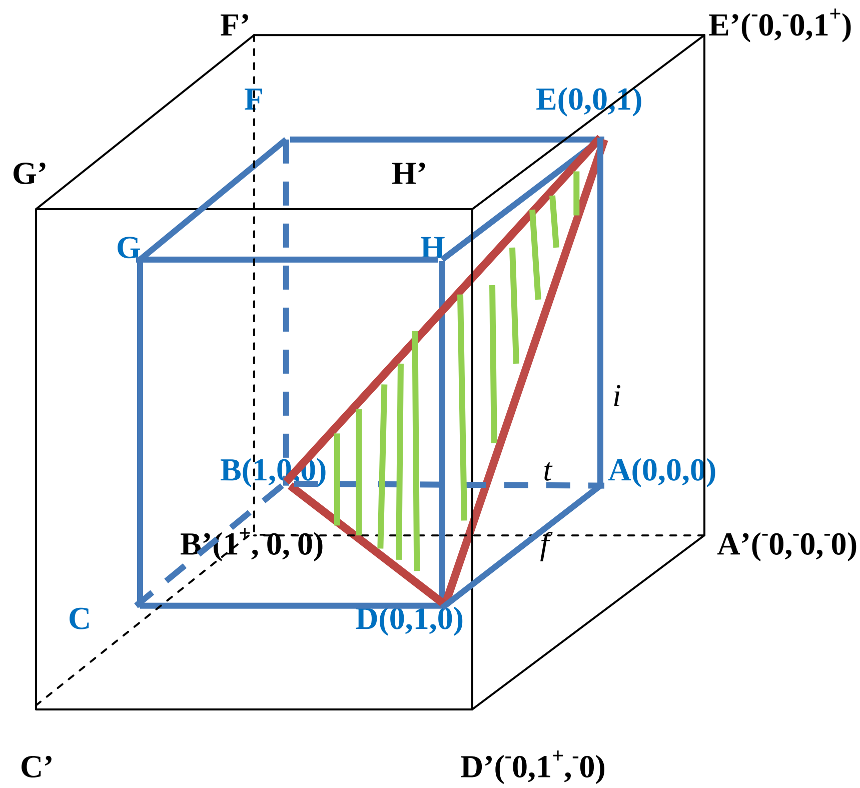
3. Definition of Neutrosophic Set:

Let T, I, F be real standard or non-standard subsets of $]^-0, 1^+[$,
with sup T = t_sup, inf T = t_inf,
sup I = i_sup, inf I = i_inf,
sup F = f_sup, inf F = f_inf,
and n_sup = t_sup+i_sup+f_sup,
n_inf = t_inf+i_inf+f_inf.
T, I, F are called *neutrosophic components*.
Let U be a universe of discourse, and M a set included in U. An element x from U is noted with respect to the set M as x(T, I, F) and belongs to M in the following way:
it is t% true in the set, i% indeterminate (unknown if it is) in the set, and f% false, where t varies in T, i varies in I, f varies in F.

4. Neutrosophic Cube as Geometric Interpretation of the Neutrosophic Set:

The most important distinction between IFS and NS is showed in the below **Neutrosophic Cube** A'B'C'D'E'F'G'H' introduced by J. Dezert in 2002.

Because in technical applications only the classical interval $[0,1]$ is used as range for the neutrosophic parameters t, i, f, we call the cube $ABCDEDGH$ the **technical neutrosophic cube** and its extension $A'B'C'D'E'D'G'H'$ the **neutrosophic cube** (or **absolute neutrosophic cube**), used in the fields where we need to differentiate between *absolute* and *relative* (as in philosophy) notions.



Let’s consider a 3D Cartesian system of coordinates, where t is the truth axis with value range in $\left]^{-}0,1^{+}\right[$, f is the false axis with value range in $\left]^{-}0,1^{+}\right[$, and similarly i is the indeterminate axis with value range in $\left]^{-}0,1^{+}\right[$.

We now divide the technical neutrosophic cube $ABCDEDGH$ into three disjoint regions:

1) The equilateral triangle BDE, whose sides are equal to $\sqrt{2}$, which represents the geometrical locus of the points whose sum of the coordinates is 1.

If a point Q is situated on the sides of the triangle BDE or inside of it, then $t_Q + i_Q + f_Q = 1$ as in Atanassov-intuitionistic fuzzy set $(A-IFS)$.

2) The pyramid $EABD$ {situated in the right side of the ΔEBD, including its faces ΔABD (base), ΔEBA, and ΔEDA (lateral faces), but excluding its face ΔBDE } is the locus of the points whose sum of coordinates is less than 1.

If $P \in EABD$ then $t_P + i_P + f_P < 1$ as in intuitionistic set (with incomplete information).

3) In the left side of ΔBDE in the cube there is the solid $EFGCDEBD$ (excluding ΔBDE) which is the locus of points whose sum of their coordinates is greater than 1 as in the paraconsistent set.

If a point $R \in EFGCDEBD$, then $t_R + i_R + f_R > 1$.

It is possible to get the **sum of coordinates strictly less than 1 or strictly greater than 1**. For example:

- We have a source which is capable to find only the degree of membership of an element; but it is unable to find the degree of non-membership;
- Another source which is capable to find only the degree of non-membership of an element;
- Or a source which only computes the indeterminacy.

Thus, when we put the results together of these sources, it is possible that their sum is not 1, but smaller or greater.

Also, in information fusion, when dealing with indeterminate models (i.e. elements of the fusion space which are indeterminate/unknown, such as intersections we don't know if they are empty or not since we don't have enough information, similarly for complements of indeterminate elements, etc.): if we compute the believe in that element (truth), the disbelieve in that element (falsehood), and the indeterminacy part of that element, then the sum of these three components is strictly less than 1 (the difference to 1 is the missing information).

5. More distinctions between the Neutrosophic Set and Intuitionistic Fuzzy Set

a) Neutrosophic Set can distinguish between *absolute membership* (i.e. membership in all possible worlds; we have extended Leibniz's absolute truth to absolute membership) and *relative membership* (membership in at least one world but not in all), because NS(absolute membership element)=1^+ while NS(relative membership element)=1. This has application in philosophy (see the neutrosophy). That's why the unitary standard interval [0, 1] used in IFS has been extended to the unitary non-standard interval $]^-0, 1^+[$ in NS.
Similar distinctions for *absolute or relative non-membership*, and *absolute or relative indeterminant appurtenance* are allowed in NS.

b) In NS there is no restriction on T, I, F other than they are subsets of $]^-0, 1^+[$,
thus: $^-0 \leq \inf T + \inf I + \inf F \leq \sup T + \sup I + \sup F \leq 3^+$.
The inequalities (2.1) and (2.4) of IFS are relaxed in NS.
This non-restriction allows paraconsistent, dialetheist, and incomplete information to be characterized in NS – as in above Neutrosophic Cube - {i.e. the sum of all three components if they are defined as points, or sum of superior limits of all three components if they are defined as subsets can be >1 (for paraconsistent information coming from different sources), or < 1 for incomplete information}, while that information cannot be described in IFS because in IFS the components T (membership), I (indeterminacy), F (non-membership) are restricted either to t+i+f=1 or to $t^2 + f^2 \leq 1$, if T, I, F are all reduced to the points t, i, f respectively, or to sup T + sup I + sup F = 1 if T, I, F are subsets of [0, 1].
Of course, there are cases when paraconsistent and incomplete informations can be normalized to 1, but this procedure is not always suitable.

c) Relation (2.3) from interval-valued intuitionistic fuzzy set is relaxed in NS, i.e. the intervals do not necessarily belong to Int[0,1] but to [0,1], even more general to $]^-0, 1^+[$.

d) In NS the components T, I, F can also be ***non-standard subsets*** included in the unitary non-standard interval $]^-0, 1^+[$, not only *standard* subsets included in the unitary standard interval [0, 1] as in IFS.

e) NS, like dialetheism, can describe **paradoxist elements**, NS(paradoxist element) = (1, I, 1), while IFL cannot describe a paradox because the sum of components should be 1 in IFS.

f) The connectors in IFS are defined with respect to T and F, i.e. membership and nonmembership only (hence the Indeterminacy is what's left from 1), while in NS they can be defined with respect to any of them (no restriction).

g) Component "I", indeterminacy, can be split into more subcomponents in order to better catch the vague information we work with, and such, for example, one can get more accurate answers to the *Question-Answering Systems* initiated by Zadeh (2003).

{In Belnap's four-valued logic (1977) indeterminacy is split into Uncertainty (U) and Contradiction (C), but they were interrelated.}

Even more, one can split "I" into Contradiction, Uncertainty, and Unknown, and we get a five-valued logic.

In a general **Refined Neutrosophic Set**, "T" can be split into subcomponents $T_1, T_2, ..., T_m$, and "I" into $I_1, I_2, ..., I_n$, and "F" into $F_1, F_2, ..., F_p$.

h) NS has a better and clear terminology (name) as "neutrosophic" (which means the neutral part: i.e. neither true/membership nor false/nonmembership), while IFS's name "intuitionistic" produces confusion with Intuitionistic Logic, which is something different (see the article by Didier Dubois et al., 2005).

i) The **Neutrosophic Numbers** have been introduced by W.B. Vasantha Kandasamy and F. Smarandache, which are numbers of the form N = a+bI, where a, b are real or complex numbers, while "I" is the indeterminacy part of the neutrosophic number N, such that $I^2 = I$ and $\alpha I+\beta I = (\alpha+\beta)I$.
Of course, indeterminacy "I" is different from the imaginary $i = \sqrt{-1}$.
In general one has $I^n = I$ if $n > 0$, and is undefined if $n \leq 0$.
The algebraic structures using neutrosophic numbers gave birth to the **neutrosophic algebraic structures** [see for example "neutrosophic groups", "neutrosophic rings", "neutrosophic vector space", "neutrosophic matrices, bimatrices, …, n-matrices", etc.], introduced by W.B. Vasantha Kandasamy, F. Smarandache et al.

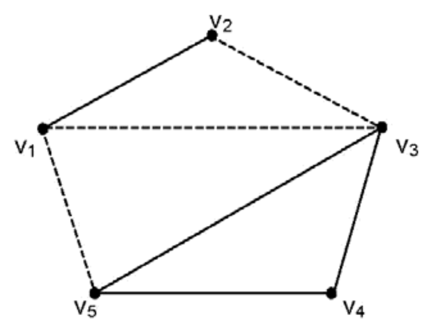
Example of Neutrosophic Matrix: $\begin{bmatrix} 1 & 2+I & -5 \\ 0 & 1/3 & I \\ -1+4I & 6 & 5I \end{bmatrix}$.

Example of Neutrosophic Ring: ({a+bI, with a, b ϵ R}, +, ·), where of course (a+bI)+(c+dI) = (a+c)+(b+d)I, and (a+bI) · (c+dI) = (ac) + (ad+bc+bd)I.

j) Also, "I" led to the definition of the **neutrosophic graphs** (graphs which have at least either one indeterminate edge or one indeterminate node), and **neutrosophic trees** (trees which have at least either one indeterminate edge or one indeterminate node), which have many applications in social sciences.
As a consequence, the neutrosophic cognitive maps and neutrosophic relational maps are generalizations of fuzzy cognitive maps and respectively fuzzy relational maps (W.B. Vasantha Kandasamy, F. Smarandache et al.).
A Neutrosophic Cognitive Map is a neutrosophic directed graph with concepts like policies, events etc. as nodes and causalities or indeterminates as edges. It represents the causal relationship between concepts.

An edge is said indeterminate if we don't know if it is any relationship between the nodes it connects, or for a directed graph we don't know if it is a directly or inversely proportional relationship.
A node is indeterminate if we don't know what kind of node it is since we have incomplete information.

Example of Neutrosophic Graph (edges V_1V_3, V_1V_5, V_2V_3 are indeterminate and they are drawn as dotted):

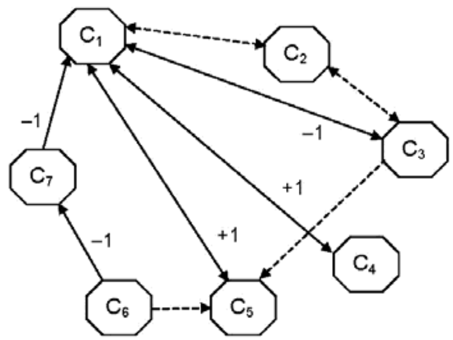


and its neutrosophic adjacency matrix is:

$$\begin{bmatrix} 0 & 1 & I & 0 & I \\ 1 & 0 & I & 0 & 0 \\ I & I & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ I & 0 & 1 & 1 & 0 \end{bmatrix}$$

The edges mean: 0 = no connection between nodes, 1 = connection between nodes, I = indeterminate connection (not known if it is or if it is not).
Such notions are not used in the fuzzy theory.

Example of Neutrosophic Cognitive Map (NCM), which is a generalization of the Fuzzy Cognitive Maps.
Let's have the following nodes:
C_1 - Child Labor
C_2 - Political Leaders
C_3 - Good Teachers
C_4 - Poverty
C_5 - Industrialists
C_6 - Public practicing/encouraging Child Labor
C_7 - Good Non-Governmental Organizations (NGOs)



The corresponding neutrosophic adjacency matrix related to this neutrosophic cognitive map is:

$$\begin{bmatrix} 0 & I & -1 & 1 & 1 & 0 & 0 \\ I & 0 & I & 0 & 0 & 0 & 0 \\ -1 & I & 0 & 0 & I & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & I & 0 & -1 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The edges mean: 0 = no connection between nodes, 1 = directly proportional connection, -1 = inversely proportionally connection, and I = indeterminate connection (not knowing what kind of relationship is between the nodes the edge connects).

k) The neutrosophics introduced (in 1995) the **Neutrosophic Probability** (NP), which is a generalization of the classical and imprecise probabilities. NP of an event $\mathcal{E}$ is the chance that event $\mathcal{E}$ occurs, the chance that event $\mathcal{E}$ doesn't occur, and the chance of indeterminacy (not knowing if the event $\mathcal{E}$ occurs or not).

In classical probability $n_{sup} \leq 1$, while in neutrosophic probability $n_{sup} \leq 3^+$.

In imprecise probability: the probability of an event is a subset T in [0, 1], not a number p in [0, 1], what's left is supposed to be the opposite, subset F (also from the unit interval [0, 1]); there is no indeterminate subset I in imprecise probability.

And consequently the **Neutrosophic Statistics**, which is the analysis of the neutrosophic events. Neutrosophic statistics deals with neutrosophic numbers, neutrosophic probability distribution, neutrosophic estimation, neutrosophic regression.

The function that models the neutrosophic probability of a random variable x is called *neutrosophic distribution*: NP(x) = (T(x), I(x), F(x)), where T(x) represents the probability that value x occurs, F(x) represents the probability that value x does not occur, and I(x) represents the indeterminate / unknown probability of value x.

l) Neutrosophy opened a new field in philosophy.

Neutrosophy is a new branch of philosophy that studies the origin, nature, and scope of neutralities, as well as their interactions with different ideational spectra.

This theory considers every notion or idea <A> together with its opposite or negation <Anti-A> and the spectrum of "neutralities" <Neut-A> (i.e. notions or ideas located between the two extremes, supporting neither <A> nor <Anti-A>). The <Neut-A> and <Anti-A> ideas together are referred to as <Non-A>.

According to this theory every idea <A> tends to be neutralized and balanced by <Anti-A> and <Non-A> ideas - as a state of equilibrium.

In a classical way <A>, <Neut-A>, <Anti-A> are disjoint two by two.

But, since in many cases the borders between notions are vague, imprecise, Sorites, it is possible that <A>, <Neut-A>, <Anti-A> (and <Non-A> of course) have common parts two by two as well.

Neutrosophy is the base of neutrosophic logic, neutrosophic set, neutrosophic probability and statistics used in engineering applications (especially for software and information fusion), medicine, military, cybernetics, physics.

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Presented at 2011 IEEE International Conference on Granular Computing, edited by Tzung-Pei Hong, Yasuo Kudo, Mineichi Kudo, Tsau-Young Lin, Been-Chian Chien, Shyue-Liang Wang, Masahiro Inuiguchi, GuiLong Liu, IEEE Computer Society, National University of Kaohsiung, Taiwan, 602-606, 8-10 November 2011.

n-Valued Refined Neutrosophic Logic and Its Applications to Physics

Florentin Smarandache

Abstract.

In this paper we present a short history of logics: from particular cases of *2*-symbol or numerical valued logic to the general case of *n*-symbol or numerical valued logic. We show generalizations of *2*-valued Boolean logic to fuzzy logic, also from the Kleene's and Lukasiewicz' *3*-symbol valued logics or Belnap's *4*-symbol valued logic to the most general *n-symbol or numerical valued refined neutrosophic logic*. Two classes of neutrosophic norm (*n-norm*) and neutrosophic conorm (*n-conorm*) are defined. Examples of applications of neutrosophic logic to physics are listed in the last section.

Similar generalizations can be done for *n-Valued Refined Neutrosophic Set*, and respectively *n-Valued Refined Neutrosopjhic Probability*.

1. Two-Valued Logic

a) The Two Symbol-Valued Logic.

It is the Chinese philosophy: *Yin* and *Yang* (or Femininity and Masculinity) as contraries:

Fig 1. Ying and Yang

It is also the Classical or *Boolean Logic*, which has two symbol-values: truth *T* and falsity *F*.

b) The Two Numerical-Valued Logic.

It is also the Classical or *Boolean Logic*, which has two numerical-values: truth *1* and falsity *0*.

More general it is the *Fuzzy Logic*, where the truth (T) and the falsity (F) can be any numbers in $[0,1]$ such that $T + F = 1$.

Even more general, T and F can be subsets of $[0, 1]$.

2. Three-Valued Logic

a) The Three Symbol-Valued Logics:

i) *Łukasiewicz 's Logic*: True, False, and Possible.
ii) *Kleene's Logic*: True, False, Unknown (or Undefined).
iii) Chinese philosophy extended to: *Yin, Yang,* and *Neuter* (or Femininity, Masculinity, and Neutrality) – as in Neutrosophy.

Neutrosophy philosophy was born from neutrality between various philosophies. Connected with Extenics (Prof. Cai Wen, 1983), and Paradoxism (F. Smarandache, 1980).
2. Three-Valued Logic
Neutrosophy is a new branch of philosophy that studies the origin, nature, and scope of neutralities, as well as their interactions with different ideational spectra.
This theory considers every notion or idea <A> together with its opposite or negation <antiA> and with their spectrum of neutralities <neutA> in between them (i.e. notions or ideas supporting neither <A> nor <antiA>).
The <neutA> and <antiA> ideas together are referred to as <nonA>.
Neutrosophy is a generalization of Hegel's dialectics (the last one is based on <A> and <antiA> only).
According to this theory every idea <A> tends to be neutralized and balanced by <antiA> and <nonA> ideas - as a state of equilibrium.
In a classical way <A>, <neutA>, <antiA> are disjoint two by two. But, since in many cases the borders between notions are vague, imprecise, Sorites, it is possible that <A>, <neutA>, <antiA> (and <nonA> of course) have common parts two by two, or even all three of them as well. Such contradictions involves Extenics. Neutrosophy is the base of all neutrosophics and it is used in engineering applications (especially for software and information fusion), medicine, military, airspace, cybernetics, physics.

b) The Three Numerical-Valued Logic:

i) Kleene's Logic: True (*1*), False (*0*), Unknown (or Undefined) (*1/2*), and uses "min" for $\wedge$, "max" for $\vee$, and "1-" for negation.
ii) More general is the *Neutrosophic Logic* [Smarandache, 1995], where the truth (T) and the falsity (F) and the indeterminacy (I) can be any numbers in $[0, 1]$, then $0 \le T + I + F \le 3$.
More general: Truth (T), Falsity (F), and Indeterminacy (I) are standard or nonstandard subsets of the nonstandard interval $]^-0, 1^+[$.

3. Four-Valued Logic

a) The Four Symbol-Valued Logic

i) It is *Belnap's Logic:* True (T), False (F), Unknown (U), and Contradiction (C), where T, F, U, C are symbols, not numbers.
Below is the Belnap's conjunction operator table:

∩	F	U	C	T
F	F	F	F	F
U	F	U	F	U
C	F	F	C	C
T	F	U	C	T

Table 1.

Restricted to *T, F, U,* and to *T, F, C,* the Belnap connectives coincide with the connectives in Kleene's logic.
ii) Let *G* = Ignorance. We can also propose the following two 4-Symbol Valued Logics:
(T, F, U, G), and *(T, F, C, G).*
iii) *Absolute-Relative 2-, 3-, 4-, 5-, or 6-Symbol Valued Logics* [Smarandache, 1995].
Let T_A be truth in all possible worlds (according to Leibniz's definition);
T_R be truth in at last one world but not in all worlds;
and similarly let I_A be indeterminacy in all possible worlds;
I_R be indeterminacy in at last one world but not in all worlds;
also let F_A be falsity in all possible worlds;
F_R be falsity in at last one world but not in all worlds;
Then we can form several Absolute-Relative 2-, 3-, 4-, 5-, or 6-Symbol Valued Logics just taking combinations of the symbols T_A, T_R, I_A, I_R, F_A, and F_R.

As particular cases, very interesting would be to study the Absolute-Relative 4-Symbol Valued Logic (T_A, T_R, F_A, F_R), as well as the Absolute-Relative 6-Symbol Valued Logic $(T_A, T_R, I_A, I_R, F_A, F_R)$.

b) *Four Numerical-Valued Neutrosophic Logic:* Indeterminacy I is refined (split) as U = Unknown, and C = contradiction.
T, F, U, C are subsets of [0, 1], instead of symbols;
This logic generalizes Belnap's logic since one gets a degree of truth, a degree of falsity, a degree of unknown, and a degree of contradiction.
Since C = T/\F, this logic involves the Extenics.

4. Five-Valued Logic

a) Five Symbol-Valued Neutrosophic Logic [Smarandache, 1995]:
Indeterminacy *I* is refined (split) as *U* = Unknown, *C* = contradiction, and *G* = ignorance; where the symbols represent:
T = truth;
F = falsity;
U = neither *T* nor *F* (undefined);
C = *T/\F,* which involves the Extenics;
G = *T\/F.*

b) If *T, F, U, C, G* are subsets of *[0, 1]* then we get: a *Five Numerical-Valued Neutrosophic Logic*.

5. Seven-Valued Logic

a) Seven Symbol-Valued Neutrosophic Logic [Smarandache, 1995]:
I is refined (split) as *U, C, G,* but *T* also is refined as T_A = absolute truth and T_R = relative truth, and *F* is refined as F_A = absolute falsity and F_R = relative falsity. Where:
U = neither (T_A *or* T_R) nor (F_A *or* F_R) (i.e. undefined);

$C = (T_A \text{ or } T_R) \wedge (F_A \text{ or } F_R)$ (i.e. Contradiction), which involves the Extenics;
$G = (T_A \text{ or } T_R) \vee (F_A \text{ or } F_R)$ (i.e. Ignorance).
All are symbols.

b) But if $T_A, T_R, F_A, F_R, U, C, G$ are subsets of $[0, 1]$, then we get a *Seven Numerical-Valued Neutrosophic Logic*.

6. n-Valued Logic

a) *The n-Symbol-Valued Refined Neutrosophic Logic* [Smarandache, 1995].
In general:
T can be split into many types of truths: $T_1, T_2, ..., T_p$, and I into many types of indeterminacies: $I_1, I_2, ..., I_r$, and F into many types of falsities: $F_1, F_2, ..., F_s$, where all $p, r, s \geq 1$ are integers, and $p + r + s = n$. Even more: T, I, and/or F (or any of their subcomponents T_j, I_k, and/or F_l) can be countable or uncountable infinite sets.
All subcomponents T_j, I_k, F_l are symbols for $j \in \{1,2,\ldots,p\}$, $k \in \{1,2,\ldots,r\}$, and $l \in \{1,2,\ldots,s\}$.
If at least one $I_k = T_j \wedge F_l$ = contradiction, we get again the Extenics.

b) *The n-Numerical-Valued Refined Neutrosophic Logic.*
In the same way, but all subcomponents T_j, I_k, F_l are not symbols, but subsets of $[0,1]$, for all

$j \in \{1,2,\ldots,p\}$, all $k \in \{1,2,\ldots,r\}$, and all $l \in \{1,2,\ldots,s\}$.
If all sources of information that separately provide neutrosophic values for a specific subcomponent are independent sources, then in the general case we consider that each of the subcomponents T_j, I_k, F_l is independent with respect to the others and it is in the non-standard set $]^-0, 1^+[$. Therefore per total we have for crisp neutrosophic value subcomponents T_j, I_k, F_l that:

$$^-0 \leq \sum_{j=1}^{p} T_j + \sum_{k=1}^{r} I_k + \sum_{l=1}^{s} F_l \leq n^+ \quad (1)$$

where of course $n = p + r + s$ as above.

If there are some dependent sources (or respectively some dependent subcomponents), we can treat those dependent subcomponents together. For example, if T_2 and I_3 are dependent, we put them together as $^-0 \leq T_2 + I_3 \leq 1^+$.
The non-standard unit interval $]^-0, 1^+[$, used to make a distinction between *absolute* and *relative* truth/indeterminacy/falsehood in philosophical applications, is replace for simplicity with the standard (classical) unit interval $[0, 1]$ for technical applications.

For at least one $I_k = T_j \wedge F_l$ = contradiction, we get again the Extenics.

7. n-Valued Neutrosophic Logic Connectors

a) n-Norm and n-Conorm defined on combinations of t-Norm and t-Conorm

The n-norm is actually the neutrosophic conjunction operator, NEUTROSOPHIC AND ($\wedge_n$); while the n-conorm is the neutrosophic disjunction operator, NEUTROSOPHIC OR ($\vee_n$).

One can use the t-norm and t-conorm operators from the fuzzy logic in order to define the **n-norm** and respectively **n-conorm** in neutrosophic logic:

$$n\text{-}norm((T_j)_{j=\{1,2,\ldots,p\}}, (I_k)_{k=\{1,2,\ldots,r\}}, (F_l)_{l=\{1,2,\ldots,s\}}) = \qquad (2)$$

$$([t\text{-}norm(T_j)]_{j=\{1,2,\ldots,p\}}, [t\text{-}conorm(I_k)]_{k=\{1,2,\ldots,r\}}, [t\text{-}conorm(F_l)]_{l=\{1,2,\ldots,s\}})$$

and

$$n\text{-}conorm((T_j)_{j=\{1,2,\ldots,p\}}, (I_k)_{k=\{1,2,\ldots,r\}}, (F_l)_{l=\{1,2,\ldots,s\}}) = \qquad (3)$$

$$([t\text{-}conorm(T_j)]_{j=\{1,2,\ldots,p\}}, [t\text{-}norm(I_k)]_{k=\{1,2,\ldots,r\}}, [t\text{-}norm(F_l)]_{l=\{1,2,\ldots,s\}})$$

and then one normalizes if needed.

Since the n-norms/n-conorms, alike t-norms/t-conorms, can only approximate the inter-connectivity between two n-Valued Neutrosophic Propositions, there are many versions of these approximations.

For example, for the n-norm:

the indeterminate (sub)components I_k alone can be combined with the t-conorm in a pessimistic way [i.e. lower bound], or with the t-norm in an optimistic way [upper bound];

while for the n-conorm:

the indeterminate (sub)components I_k alone can be combined with the t-norm in a pessimistic way [i.e. lower bound], or with the t-conorm in an optimistic way [upper bound].

In general, if one uses in defining an n-norm/n-conorm for example the t-norm $min\{x, y\}$ then it is indicated that the corresponding t-conorm used be $max\{x, y\}$; or if the t-norm used is the product $x\cdot y$ then the corresponding t-conorm should be $x+y-x\cdot y$; and similarly if the t-norm used is $max\{0, x+y-1\}$ then the corresponding t-conorm should be $min\{x+y, 1\}$; and so on.

Yet, it is still possible to define the n-norm and n-conorm using different types of t-norms and t-conorms.

b) N-norm and n-conorm based on priorities.

For the n-norm we can consider the priority: T < I < F, where the subcomponents are supposed to conform with similar priorities, i.e.

$$T_1 < T_2 <\ldots < T_p < I_1 < I_2 < \ldots <I_r < F_1 < F_2 < \ldots< F_s. \qquad (4)$$

While for the n-conorm one has the opposite priorities: T > I > F, or for the refined case:

$$T_1 > T_2 >\ldots > T_p > I_1 > I_2 > \ldots >I_r > F_1 > F_2 > \ldots> F_s. \qquad (5)$$

By definition A < B means that all products between A and B go to B (the bigger).

Let's say, one has two neutrosophic values in simple (non-refined case):

(T_x, I_x, F_x) (6)

and

(T_y, I_y, F_y). (7)

Applying the n-norm to both of them, with priorities T < I < F, we get:

$(T_x, I_x, F_x) \wedge_n (T_y, I_y, F_y) = (T_xT_y, T_xI_y + T_yI_x + I_xI_y, T_xF_y + T_yF_x + I_xF_y + I_yF_x + F_xF_y)$ (8)

Applying the n-conorm to both of them, with priorities T > I > F, we get:

$(T_x, I_x, F_x) \vee_n (T_y, I_y, F_y) = (T_xT_y + T_xI_y + T_yI_x + T_xF_y + T_yF_x, I_xI_y + I_xF_y + I_yF_x, F_xF_y)$. (9)

In a lower bound (pessimistic) n-norm one considers the priorities T < I < F, while in an upper bound (optimistic) n-norm one considers the priorities I < T < F.

Whereas, in an upper bound (optimistic) n-conorm one considers T > I > F, while in a lower bound (pessimistic) n-conorm one considers the priorities T > F > I.

Various priorities can be employed by other researchers depending on each particular application.

8. Particular Cases

If in *6 a)* and *b)* one has all $I_k = 0$, $k = \{1,2,\ldots,r\}$, we get the **n-Valued Refined Fuzzy Logic**.

If in *6 a)* and *b)* one has only one type of indeterminacy, i.e. $k = 1$, hence $I_1 = I > 0$, we get the **n-Valued Refined Intuitionistic Fuzzy Logic**.

9. Distinction between Neutrosophic Physics and Paradoxist Physics

Firstly, we make a distinction between Neutrosophic Physics and Paradoxist Physics.

a) Neutrosophic Physics.

Let <A> be a physical entity (i.e. concept, notion, object, space, field, idea, law, property, state, attribute, theorem, theory, etc.), <antiA> be the opposite of <A>, and <neutA> be their neutral (i.e. neither <A> nor <antiA>, but in between).

Neutrosophic Physics is a mixture of two or three of these entities <A>, <antiA>, and <neutA> that hold together.

Therefore, we can have neutrosophic fields, and neutrosophic objects, neutrosophic states, etc.

b) Paradoxist Physics.

Neutrosophic Physics is an extension of Paradoxist Physics, since Paradoxist Physics is a combination of physical contradictories *<A>* and *<antiA>* only that hold together, without referring to their neutrality *<neutA>*. Paradoxist Physics describes collections of objects or states that are individually characterized by contradictory properties, or are characterized neither by a property nor by the opposite of that property, or are composed of contradictory sub-elements. Such objects or states are called *paradoxist entities.*

These domains of research were set up in the *1995* within the frame of neutrosophy, neutrosophic logic/set/probability/statistics.

10. n-Valued Refined Neutrosophic Logic Applied to Physics

There are many cases in the scientific (and also in humanistic) fields that two or three of these items <*A*>*,* <*antiA*>*,* and <*neutA*> simultaneously coexist.

Several **Examples** of paradoxist and neutrosophic entities:

- anions in two spatial dimensions are arbitrary spin particles that are neither bosons (integer spin) nor fermions (half integer spin);

- among possible *Dark Matter* candidates there may be exotic particles that are neither *Dirac* nor *Majorana fermions;*

- mercury (Hg) is a state that is neither liquid nor solid under normal conditions at room temperature;

- non-magnetic materials are neither ferromagnetic nor anti-ferromagnetic;

- quark gluon plasma (QGP) is a phase formed by quasi-free quarks and gluons that behaves neither like a conventional plasma nor as an ordinary liquid;

- *unmatter,* which is formed by matter and antimatter that bind together (F. Smarandache, 2004);

- neutral kaon, which is a pion & anti-pion composite (R. M. Santilli, 1978) and thus a form of unmatter;

- neutrosophic methods in General Relativity (D. Rabounski, F. Smarandache, L. Borissova, 2005);

- neutrosophic cosmological model (D. Rabounski, L. Borissova, 2011);

- neutrosophic gravitation (D. Rabounski);

- qubit and generally quantum superposition of states;

- semiconductors are neither conductors nor isolators;

- semi-transparent optical components are neither opaque nor perfectly transparent to light;

- quantum states are metastable (neither perfectly stable, nor unstable);

- neutrino-photon doublet (E. Goldfain);

- the “multiplet” of elementary particles is a kind of ‘neutrosophic field’ with two or more values (E. Goldfain, 2011);

- A "neutrosophic field" can be generalized to that of operators whose action is selective. The effect of the neutrosophic field is somehow equivalent with the “tunneling” from the solid physics, or with the “spontaneous symmetry breaking" (SSB) where there is an internal symmetry which is broken by a particular selection of the vacuum state (E. Goldfain).

Conclusion

Many types of logics have been presented above. For the most general logic, the n-valued refined neutrosophic logic, we presented two classes of neutrosophic operators to be used in combinations of neutrosophic valued propositions in physics.

Similar generalizations are done for **n-Valued Refined Neutrosophic Set**, and respectively **n-Valued Refined Neutrosopjhic Probability**.

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Published in *Bulletin of the American Physical Society* 2013 Annual Fall Meeting of the APS Ohio-Region Section Volume 58, Number 9. Friday–Saturday, October 4–5, 2013; Cincinnati, Ohio, http://meetings.aps.org/Meeting/OSF13/Event/205641.

Replacing the Conjunctive Rule and Disjunctive Rule with T-norms and T-conorms respectively (Tchamova-Smarandache)

Florentin Smarandache

A **T-norm** is a function T_n: $[0, 1]^2 \rightarrow [0, 1]$, defined in fuzzy/neutrosophic set theory and fuzzy/neutrosophic logic to represent the "intersection" of two fuzzy/neutrosophic sets and the fuzzy/neutrosophic logical operator "and" respectively. Extended to the fusion theory the T-norm will be a substitute for the conjunctive rule.
The T-norm satisfies the conditions:
a) Boundary Conditions: $T_n(0, 0) = 0$, $T_n(x, 1) = x$.
b) Commutativity: $T_n(x, y) = T_n(y, x)$.
c) Monotonicity: If $x \leq u$ and $y \leq v$, then $T_n(x, y) \leq T_n(u, v)$.
d) Associativity: $T_n(T_n(x, y), z) = T_n(x, T_n(y, z))$.
There are many functions which satisfy the T-norm conditions. We present below the most known ones:
The Algebraic Product T-norm:
$T_{n\text{-algebraic}}(x, y) = x\cdot y$
The Bounded T-norm:
$T_{n\text{-bounded}}(x, y) = \max\{0, x+y-1\}$
The Default (min) T-norm (introduced by Zadeh):
$T_{n\text{-min}}(x, y) = \min\{x, y\}$.

Min rule can be interpreted as an optimistic lower bound for combination of bba and the below Max rule as a prudent/pessimistic upper bound. (Jean Dezert)

A **T-conorm** is a function T_c: $[0, 1]^2 \rightarrow [0, 1]$, defined in fuzzy/neutrosophic set theory and fuzzy/neutrosophic logic to represent the "union" of two fuzzy/neutrosophic sets and the fuzzy/neutrosophic logical operator "or" respectively. Extended to the fusion theory the T-conorm will be a substitute for the disjunctive rule.
The T-conorm satisfies the conditions:
a) Boundary Conditions: $T_c(1, 1) = 1$, $T_c(x, 0) = x$.
b) Commutativity: $T_c(x, y) = T_c(y, x)$.
c) Monotonicity: if $x \leq u$ and $y \leq v$, then $T_c(x, y) \leq T_c(u, v)$.
d) Associativity: $T_c(T_c(x, y), z) = T_c(x, T_c(y, z))$.

There are many functions which satisfy the T-conorm conditions. We present below the most known ones:
The Algebraic Product T-conorm:

$T_{c\text{-algebraic}}(x, y) = x+y-x \cdot y$
The Bounded T-conorm:
$T_{c\text{-bounded}}(x, y) = \min\{1, x+y\}$
The Default (max) T-conorm (introduced by Zadeh):
$T_{c\text{-max}}(x, y) = \max\{x, y\}$.

Then, the T-norm Fusion rule is defined as follows:

$$m_{\cap 12}(A) = \sum_{\substack{X,Y\in\Theta \\ X\cap Y=A}} Tn(m1(X), m2(Y))$$

and the T-conorm Fusion rule is defined as follows:

$$m_{\cup 12}(A) = \sum_{\substack{X,Y\in\Theta \\ X\cup Y=A}} Tc(m1(X), m2(Y))$$

The T-norms/conorms are commutative, associative, isotone, and have a neutral element.

Connections between Extension Logic and Refined Neutrosophic Logic

Florentin Smarandache

Abstract.

The aim of this presentation is to connect Extension Logic with new fields of research, i.e. fuzzy logic and neutrosophic logic.

We show herein:

- How Extension Logic is connected to the 3-Valued Neutrosophic Logic,
- How Extension Logic is connected to the 4-Valued Neutrosophic Logic,
- How Extension Logic is connected to the n-Valued Neutrosophic Logic,

when contradictions occurs. As extension transformation one uses the normalization of the neutrosophic logic components.

Introduction.

In this paper we present a short history of logics: from particular cases of *2*-symbol or numerical valued logic to the general case of *n*-symbol or numerical valued logic, and the way they are connected to Prof. Cai Wen's Extension Logic Theory (1983). We show generalizations of *2*-valued Boolean logic to fuzzy logic, also from the Kleene's and Lukasiewicz' *3*-symbol valued logics or Belnap's *4*-symbol valued logic to the most general *n-symbol or numerical valued refined neutrosophic logic*. Two classes of neutrosophic norm (*n-norm*) and neutrosophic conorm (*n-conorm*) are defined. Examples of applications of neutrosophic logic to physics are listed in the last section.
Similar generalizations can be done for *n-Valued Refined Neutrosophic Set*, and respectively *n-Valued Refined Neutrosopjhic Probability* in connections with Extension Logic.

The essential difference between extension logic and neutrosophic logic is that the sum of the components in the extension logic is greater than 1. And the relationship between extension logic and refined neutrosophic logic is that both of them can be normalized (by dividing each logical component by the sum of all components), thus using an extension transformation.

1. Two-Valued Logic

a) The Two Symbol-Valued Logic.

It is the Chinese philosophy: *Yin* and *Yang* (or Femininity and Masculinity) as contraries:

Fig 1. Ying and Yang

It is also the Classical or *Boolean Logic*, which has two symbol-values: truth *T* and falsity *F*.

b) The Two Numerical-Valued Logic.
It is also the Classical or *Boolean Logic*, which has two numerical-values: truth *1* and falsity *0*.
More general it is the *Fuzzy Logic*, where the truth (*T*) and the falsity (*F*) can be any numbers in $[0,1]$ such that $T + F = 1$.
Even more general, *T* and *F* can be subsets of $[0, 1]$.

2. Three-Valued Logic

a) The Three Symbol-Valued Logics:
i) *Łukasiewicz 's Logic*: True, False, and Possible.
ii) *Kleene's Logic*: True, False, Unknown (or Undefined).
iii) Chinese philosophy extended to: *Yin, Yang,* and *Neuter* (or Femininity, Masculinity, and Neutrality) – as in Neutrosophy.
Neutrosophy philosophy was born from neutrality between various philosophies. Connected with Extension Logic (Prof. Cai Wen, 1983), and Paradoxism (F. Smarandache, 1980).
Neutrosophy is a new branch of philosophy that studies the origin, nature, and scope of neutralities, as well as their interactions with different ideational spectra.
This theory considers every notion or idea <A> together with its opposite or negation <antiA> and with their spectrum of neutralities <neutA> in between them (i.e. notions or ideas supporting neither <A> nor <antiA>).
The <neutA> and <antiA> ideas together are referred to as <nonA>.
Neutrosophy is a generalization of Hegel's dialectics (the last one is based on <A> and <antiA> only).
According to this theory every idea <A> tends to be neutralized and balanced by <antiA> and <nonA> ideas - as a state of equilibrium.
In a classical way <A>, <neutA>, <antiA> are disjoint two by two. But, since in many cases the borders between notions are vague, imprecise, Sorites, it is possible that <A>, <neutA>, <antiA> (and <nonA> of course) have common parts two by two, or even all three of them as well. Such contradictions involves Extension Logic.
Neutrosophy is the base of all neutrosophics and it is used in engineering applications (especially for software and information fusion), medicine, military, airspace, cybernetics, physics.

b) The Three Numerical-Valued Logic:
i) Kleene's Logic: True (*1*), False (*0*), Unknown (or Undefined) (*1/2*),
and uses "min" for $\wedge$, "max" for $\vee$, and "1-" for negation.
ii) More general is the *Neutrosophic Logic* [Smarandache, 1995], where the truth (T) and the falsity (F) and the indeterminacy (I) can be any numbers in $[0, 1]$, then $0 \leq T + I + F \leq 3$.
More general: Truth (T), Falsity (F), and Indeterminacy (I) are standard or nonstandard subsets of the nonstandard interval $]^-0, 1^+[$.

When t + f > 1 we have conflict, hence Extension Logic.

3. Four-Valued Logic

a) The Four Symbol-Valued Logic

i) It is *Belnap's Logic:* True (T), False (F), Unknown (U), and Contradiction (C), where T, F, U, C are symbols, not numbers.

Now we have Extension Logic, thanks to C = contradiction.
Below is the Belnap's conjunction operator table:

∩	F	U	C	T
F	F	F	F	F
U	F	U	F	U
C	F	F	C	C
T	F	U	C	T

Table 1.

Restricted to T, F, U, and to T, F, C, the Belnap connectives coincide with the connectives in Kleene's logic.
ii) Let G = Ignorance. We can also propose the following two 4-Symbol Valued Logics: (T, F, U, G), and (T, F, C, G).
iii) *Absolute-Relative 2-, 3-, 4-, 5-, or 6-Symbol Valued Logics* [Smarandache, 1995].
Let T_A be truth in all possible worlds (according to Leibniz's definition);
T_R be truth in at last one world but not in all worlds;
and similarly let I_A be indeterminacy in all possible worlds;
I_R be indeterminacy in at last one world but not in all worlds;
also let F_A be falsity in all possible worlds;
F_R be falsity in at last one world but not in all worlds;

Then we can form several Absolute-Relative 2-, 3-, 4-, 5-, or 6-Symbol Valued Logics just taking combinations of the symbols T_A, T_R, I_A, I_R, F_A, and F_R.

As particular cases, very interesting would be to study the Absolute-Relative 4-Symbol Valued Logic (T_A, T_R, F_A, F_R), as well as the Absolute-Relative 6-Symbol Valued Logic $(T_A, T_R, I_A, I_R, F_A, F_R)$.

b) *Four Numerical-Valued Neutrosophic Logic:* Indeterminacy I is refined (split) as U =

Unknown, and C = contradiction.
T, F, U, C are subsets of [0, 1], instead of symbols;
This logic generalizes Belnap's logic since one gets a degree of truth, a degree of falsity, a degree of unknown, and a degree of contradiction.
Since C = T/\F, this logic involves the Extension Logic.

4. Five-Valued Logic

a) *Five Symbol-Valued Neutrosophic Logic* [Smarandache, 1995]:
Indeterminacy *I* is refined (split) as *U* = Unknown, *C* = contradiction, and *G* = ignorance; where the symbols represent:
T = truth;
F = falsity;
U = neither *T* nor *F* (undefined);
C = *T/\F,* which involves the Extension Logic;
G = *T\/F.*

b) If *T, F, U, C, G* are subsets of *[0, 1]* then we get: a *Five Numerical-Valued Neutrosophic Logic*.

5. Seven-Valued Logic

a) *Seven Symbol-Valued Neutrosophic Logic* [Smarandache, 1995]:
I is refined (split) as *U, C, G,* but *T* also is refined as T_A = absolute truth and T_R = relative truth, and *F* is refined as F_A = absolute falsity and F_R = relative falsity. Where:
U = neither (T_A *or* T_R) nor (F_A *or* F_R) (i.e. undefined);
C = (T_A *or* T_R) /\ (F_A *or* F_R) (i.e. Contradiction), which involves the Extension Logic;
G = (T_A *or* T_R) \/ (F_A *or* F_R) (i.e. Ignorance).
All are symbols.

b) But if T_A, T_R, F_A, F_R, *U, C, G* are subsets of *[0, 1]*, then we get a *Seven Numerical-Valued Neutrosophic Logic*.

6. n-Valued Logic

a) *The n-Symbol-Valued Refined Neutrosophic Logic* [Smarandache, 1995].
In general:
T can be split into many types of truths: $T_1, T_2, ..., T_p$, and *I* into many types of indeterminacies: $I_1, I_2, ..., I_r$, and *F* into many types of falsities: $F_1, F_2, ..., F_s$, where all $p, r, s \geq 1$ are integers, and $p + r + s = n$.

All subcomponents T_j, I_k, F_l are symbols for $j \in \{1,2,\ldots,p\}$, $k \in \{1,2,\ldots,r\}$, and $l \in \{1,2,\ldots,s\}$.
If at least one $I_k = T_j \wedge F_l$ = contradiction, we get again the Extension Logic.

b) *The n-Numerical-Valued Refined Neutrosophic Logic.*

In the same way, but all subcomponents T_j, I_k, F_l are not symbols, but subsets of $[0,1]$, for all

$j \in \{1,2,\ldots,p\}$, all $k \in \{1,2,\ldots,r\}$, and all $l \in \{1,2,\ldots,s\}$.

If all sources of information that separately provide neutrosophic values for a specific subcomponent are independent sources, then in the general case we consider that each of the subcomponents T_j, I_k, F_l is independent with respect to the others and it is in the non-standard set $]^-0, 1^+[$. Therefore per total we have for crisp neutrosophic value subcomponents T_j, I_k, F_l that:

$$^-0 \leq \sum_{j=1}^{p} T_j + \sum_{k=1}^{r} I_k + \sum_{l=1}^{s} F_l \leq n^+ \tag{1}$$

where of course $n = p + r + s$ as above.

If there are some dependent sources (or respectively some dependent subcomponents), we can treat those dependent subcomponents together. For example, if T_2 and I_3 are dependent, we put them together as $^-0 \leq T_2 + I_3 \leq 1^+$.

The non-standard unit interval $]^-0, 1^+[$, used to make a distinction between *absolute* and *relative* truth/indeterminacy/falsehood in philosophical applications, is replace for simplicity with the standard (classical) unit interval $[0, 1]$ for technical applications.

For at least one $I_k = T_j \wedge F_l$ = contradiction, we get again the Extension Logic.

7. Neutrosophic Cube and its Extension Logic Part

The most important distinction between IFS and NS is showed in the below **Neutrosophic Cube** A'B'C'D'E'F'G'H' introduced by J. Dezert in 2002.

Because in technical applications only the classical interval is used as range for the neutrosophic parameters , we call the cube the **technical neutrosophic cube** and its extension the **neutrosophic cube** (or **absolute neutrosophic cube**), used in the fields where we need to differentiate between *absolute* and *relative* (as in philosophy) notions.

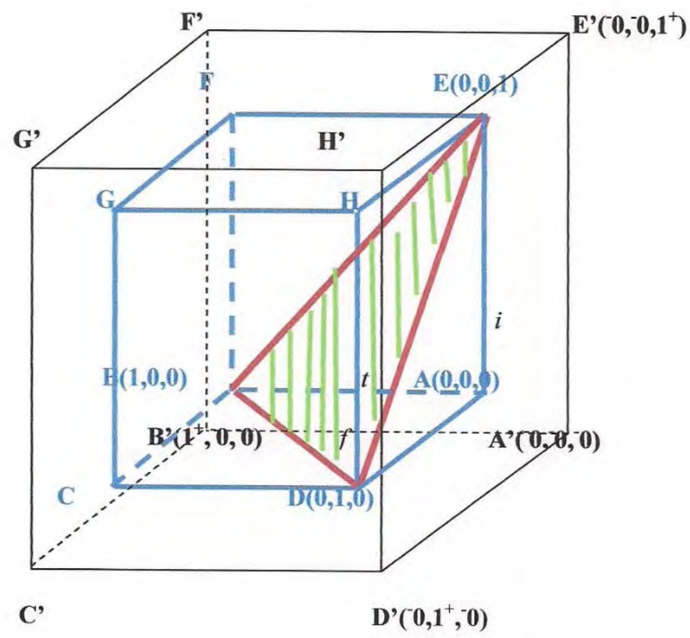

Fig. 2. Neutrosophic Cube

Let's consider a 3D-Cartesian system of coordinates, where t is the truth axis with value range in $]^-0,1^+[$, i is the false axis with value range in $]^-0,1^+[$, and similarly f is the indeterminate axis with value range in $]^-0,1^+[$.

We now divide the technical neutrosophic cube ABCDEFGH into three disjoint regions:

1) The equilateral triangle BDE, whose sides are equal to √(2) which represents the geometrical locus of the points whose sum of the coordinates is 1.

   If a point Q is situated on the sides of the triangle BDE or inside of it, then tQ+iQ+fQ=1 as in Atanassov-intuitionistic fuzzy set (A-IFS).
2) The pyramid EABD {situated in the right side of the triangle EBD, including its faces triangle ABD(base), triangle EBA, and triangle EDA (lateral faces), but excluding its face: triangle BDE } is the locus of the points whose sum of coordinates is less than 1 (Incomplete Logic).

3) In the left side of triangle BDE in the cube there is the solid EFGCDEBD (excluding triangle BDE) which is the locus of points whose sum of their coordinates is greater than 1 as in the paraconsistent logic. This is the **Extension Logic** part.

It is possible to get the **sum of coordinates strictly less than 1** (in Incomplete information), or **strictly greater than 1 (in contradictory** Extension Logic**)**. For example:

We have a source which is capable to find only the degree of membership of an element; but it is unable to find the degree of non-membership;

Another source which is capable to find only the degree of non-membership of an element;

Or a source which only computes the indeterminacy.

Thus, when we put the results together of these sources, it is possible that their sum is not 1, but smaller (Incomplete) or greater (Extension Logic).

8. Example of Extension Logic in 3-Valued Neutrosophic Logic

About a proposition P, the first source of information provides the truth-value T=0.8.

Second source of information provides the false-value F=0.7.

Third source of information provides the indeterminacy-value I=0.2.

Hence NL3(P) = (0.8, 0.2, 0.7).

Got Extension Logic, since Contradiction: $T + F = 0.8 + 0.7 > 1$.

Can remove Contradiction by normalization:

NL(P) = (0.47, 0.12, 0.41); now $T+F \leq 1$.

9. Example of Extension Logic in 4-Valued Neutrosophic Logic

About a proposition Q, the first source of information provides the truth-value T=0.4, second source provides the false-value F=0.3, third source provides the undefined-value U=0.1, fourth source provides the contradiction-value C=0.2.

Hence NL4(Q) = (0.4, 0.1, 0.2, 0.3).

Got Extension Logic, since Contradiction $C = 0.2 > 0$.

Since $C = T \wedge F$, we can remove it by transferring its value 0.2 to T and F (since T and F were involved in the conflict) proportionally w.r.t. their values 0. 4,0.3:

$xT/0.4 = yF/0.3 = 0.2/(0.4+0.3)$, whence $xT=0.11$, $yF=0.09$.

Thus $T=0.4+0.11=0.51$, $F=0.3+0.09=0.39$, $U=0.1$, $C=0$.

Conclusion

Many types of logics have been presented above related with Extension Logic. Examples of Neutrosophic Cube and its Extension Logic part, and Extension Logic in 3-Valued and 4-Valued Neutrosophic Logics are given.

Similar generalizations are done for **n-Valued Refined Neutrosophic Set**, and respectively **n-Valued Refined Neutrosopjhic Probability** in connections with Extension Logic.

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Neutrosophic Logic Approaches Applied to ”RABOT” Real Time Control

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***Abstract*— In this paper we present a way of deciding which control law should operate at a time for a mobile walking robot. The proposed deciding method is based on the new research field, called Neutrosophic Logic. The results are presented as a simulated system for which the output is related to the inputs according to the Neutrosophic Logic.**

***Keywords*— Neutrosophic Logic, Hybrid Control, Walking Robots**

I. Introduction

The mobile robot control represents a real interest due to its industry applications, but also due to its ideas of using robots in households. Because of its complexity, one can say there are three major types of robot control[9]. The first one is formed out of the PID (proportional – integrative – derivative) control or PD (proportional – derivative) control[10 - 13], in which the tracking errors along with their integrative and derivative part are amplified with certain gain values and then given as input values to the actuation system. The second type of robot control laws is formed by the adaptive control [14-20], in which the control law modifies its parameters according to the robot and environment dynamics and also to compensate the outside perturbations. The thirst control law type is represented by the iterative control laws in which the motors torque is computed by summing in a certain way the previous torques [21 - 23]. Other methods of control include Sliding Motion Control, Switching Control, Robust Control, etc.

All these types of control mentioned, are very good for certain applications. This is why, if we can’t fit an application to a certain category for which, there are efficient control laws already made, then we need to design another control law for the robot. Another way is to use several control laws, each specialized for a certain task. But this is not possible unless you use a switching mechanism between the robot control laws. This is why, we need that the switching law used in selecting a different control law specialized for a certain task, and according to the wish of the designer/engineer and also according to different environmental factors given by sensors and transducers.

In this paper, we presented a new method for deciding how to switch between several control laws, and in particular between a kinematic control law (a PID controller) and a dynamic control law (a Sliding Motion Control Law). These control laws that were used, were thought to be used for controlling a mobile walking robot, laws that have the objective of following as good as possible a given trajectory for the robot foot.

This new switching method, is based on the new scientific area called Neutrosophy[7] and more precise on its derivate Neutrosophic logic. The neutrosophic logic was applied by using the classic Dezert-Smarandache[8] theory, but also the research of Smarandache and Vladareanu[6]. By making a

Acknowledgment

This work was supported in part by the Romanian Academy and the FP7 IRSES RABOT project no. 318902/2012-2016.

simulation of the conditions encountered by a walking robot foot, in Matlab Simulink, we could observe how the switching technique behaves, compared to a classic fuzzy switching method.

II. Neutrosophic logic and DSMT

The neutrosophic logic is a generalization of fuzzy logic. In neutrosophic logic a statement is t% true, f% false and i% indeterminate, and t, f, i are real values taken from the sets T, F, I. These three sets can be of any form and the sum = t+f+i has no restrictions. Neutrosophic logic is related to other logics through the true and false parameters but it introduces the percentage of indeterminacy which expresses the percentage of unknown parameters or states [7].

If we choose U to be a universe of discourse, and M a set included in U, then an element x from U is noted with respect to the set M as x(T,I,F) and belongs to M in the following way:

- x is t% true that it is in the set M
- x is i% indeterminate that it is in the set M (the value of unknown)
- x is f% false that it is in the set M

where the value of t varies in T, the value of i varies in I and the value of f varies in F[8].

A distinctive part of DSmT (Dezert Smarandache Theory) is the notion of hyper-power set. Let $\Theta=\{\theta_1,...,\theta_n\}$ be a finite set of "n" exhaustive elements. Then the DSmT hyper-power set D^Θ is defined as the set of all composite propositions built from elements of Θ with the operators $\cup$ and $\cap$ such that [8]:

1. $\phi, \theta_1, ..., \theta_n \in D^\Theta$
2. If $A,B\in D^\Theta$, then $A\cap B\in D^\Theta$ and $A\cup B\in D^\Theta$

Within the same set Θ and with $m(\times): D^\Theta \to [0,1]$ we have:

$$m(\varnothing): 0 \ and \sum_{A\in D^\Theta} m(A) = 1 \quad (1)$$

where m(A) is called the generalized basic belief assignment or mass (gbba) of A[8].

By using the belief function

$$Bel(A) = \sum_{\substack{B\subseteq A \\ B\in D^\Theta}} m(B) \quad (2)$$

associated with two sources (observers) $m_1(.)$ and $m_2(.)$ we can define the classic DSm rule of combination:

$$\forall C\in D^\Theta, m_{M^f(\Theta)}(C) \equiv m(C) = \sum_{\substack{A,B\subseteq D^\Theta \\ A\cap B=C}} m_1(A)\times m_2(B) \quad (3)$$

Since D^Θ is closed under the set operators $\cup$ and $\cap$ this Dezert-Smarandache rule of combination guarantees that m(.) is a proper belief mass. Meaning that $m(\times): D^\Theta \to [0,1]$. The rule of combination described is commutative and associative. Also one can extend the rule for as many sources as required.

III. Appling the neutrosophic logic to a walking robot leg control

For the walking robot kinematic structure, one can imagine any kind of biped or hexapod structure, for it doesn't affect the neutrosophic decision making. Bearing this in mind, we have simulated the approach of the robot foot to the support surface through a well thought sine signal. By knowing where the support surface is at, we could say if the robot foot is near the surface, or is in contact with it. According to this distance we could compute the contact force between the robot foot and the contact surface / ground.

Having simulated these two sensors, we have chosen these two as our two observers for the Neutrosophic computations. Knowing this, we defined in figure 1, the basic diagram of how the neutrosophic logic is applied. Also we need to specify that the decision will be made between two control techniques for the walking robot leg control. These two control laws were chosen to be based on motion control for the foot trajectory. One will be based on a dynamic control law and the other will be based on a kinematic control law. Also, the two control laws were not implemented, but were only used in presenting the neutrosophic decision.

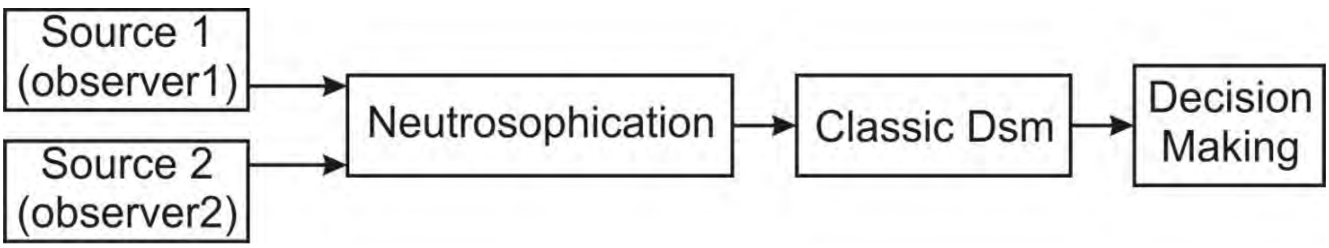

Fig. 1 Neutrosophic logic applied for two observers

As one can see, the first part of the neutrosophic diagram is formed from the two observers which we have chosen as the Proximity and Force sensors. After that, there is a stage of Neutrosophication in which the sensors values are converted as in fuzzy logic, into values from the interval [0,1].

For the neutrosophication stage, we have to bear in mind that the neutrosophic logic has functions that work with values of Truth, Indeterminacy and Falsity. Because of this, we'll have similar to a fuzzification graph, three signals of Low, Medium and High areas, which are attributed to the percentages of Truth, Indeterminacy and Falsity according to a specific statement for each sensor.

For the proximity sensor, we have the member function in figure 2, in which one can see the three Low, Medium and High values. These three values correspond to the percentage values of truth, indeterminacy/unknown and falsity for the dynamic and kinematic control in the following manner (table 1).

TABLE I. NEUTROSPHICATION CORRESPONDENCE OF FUZZY VALUES FOR THE PROXIMITY SENSOR

Control type	Low	Medium	High
Dynamic Control	Truth percentage	Indeterminacy/unknown percentage	Falsity percentage
Kinematic Control	Falsity percentage	Indeterminacy/unknown percentage	Truth percentage

For the force sensor diagram, we'll have a slightly different correspondence (table 2):

TABLE II. NEUTROSPHICATION CORRESPONDENCE OF FUZZY VALUES FOR THE FORCE SENSOR

Control type	Low	Medium	High
Dynamic Control	Falsity percentage	Indeterminacy/unknown percentage	Truth percentage
Kinematic Control	Truth percentage	Indeterminacy/unknown percentage	Falsity percentage

For these two member functions, one can see that in figures 2 and 3 we have a threshold for the sensor values according to which, the values of the neutrosophication are directly influenced. This threshold is chosen according to the application in which the neutrosophication logic is used and is also adjusted by trial and error after seeing the experimental data.

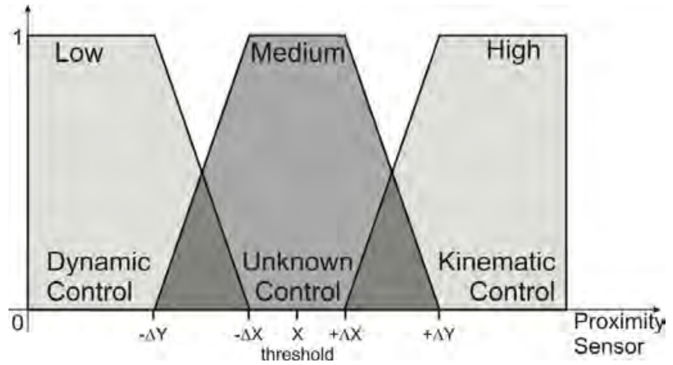

Fig.2 Neutrosophication for the Proximity sensor data

Therefore, when the robot foot is in contact with a support surface, it means that the proximity sensor will provide a 0 value or one very close to it, and it also means that the force transducer should provide a value higher than the set threshold.

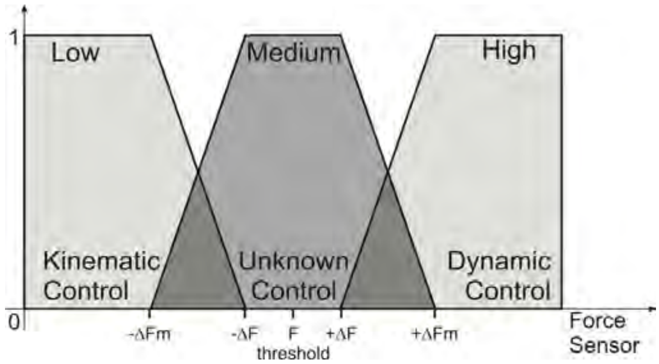

Fig. 3 Neutrosophication for the Force sensor data

Knowing these facts we developed the neutrosophic switching block control based on the theory presented in this paper, and its results are discussed in the next chapter. Also, we used a classic fuzzy control so we can compare the results obtained to a very common and known switching design.

IV. Results and Conclusion

To prove the validity of our proposed switching technique we developed a simulated system in Matlab Simulink, in which we built two loops one for the Neutrosophic logic and one for the Fuzzy logic so we can compare the results. Thus, figure 4 presents the switching system.

In the presented diagram of figure 4, one can identify the block that defines the reference values, made out of the robot vertical position, its foot position according to the distance between the robot platform and foot, and the third reference signal is the one that defines the ground position. The second diagram bloc, named Sensors computes the reference data and provides to the decision making block the values of force and proximity which in a real system would be provided by two real sensors.

By using the sensor data, we have defined two switching blocks. The first one is called Neutrosophic Decision Control and was made using the data presented in this paper, and the second one, is called Fuzzy Decision Control and was made using a simple fuzzy rule which was not presented because is not this paper objective, but was used to compare the final results. The output data was plotted to observe how the switching system behaves.

Figure 5, presents two of the reference signals. The first one defines the sine signal for the foot vertical position and the second the ground position which was made to look like a descending stair. The third signal that defines the robot position was not presented due to the fact that it was taken of value 0. Thus, one can observe that the foot reference position does not stop at the ground level, so that we can compute the force parameter due to the negative value of the proximity computed sensor. This was done only for the reason to present different cases that the robot can encounter.

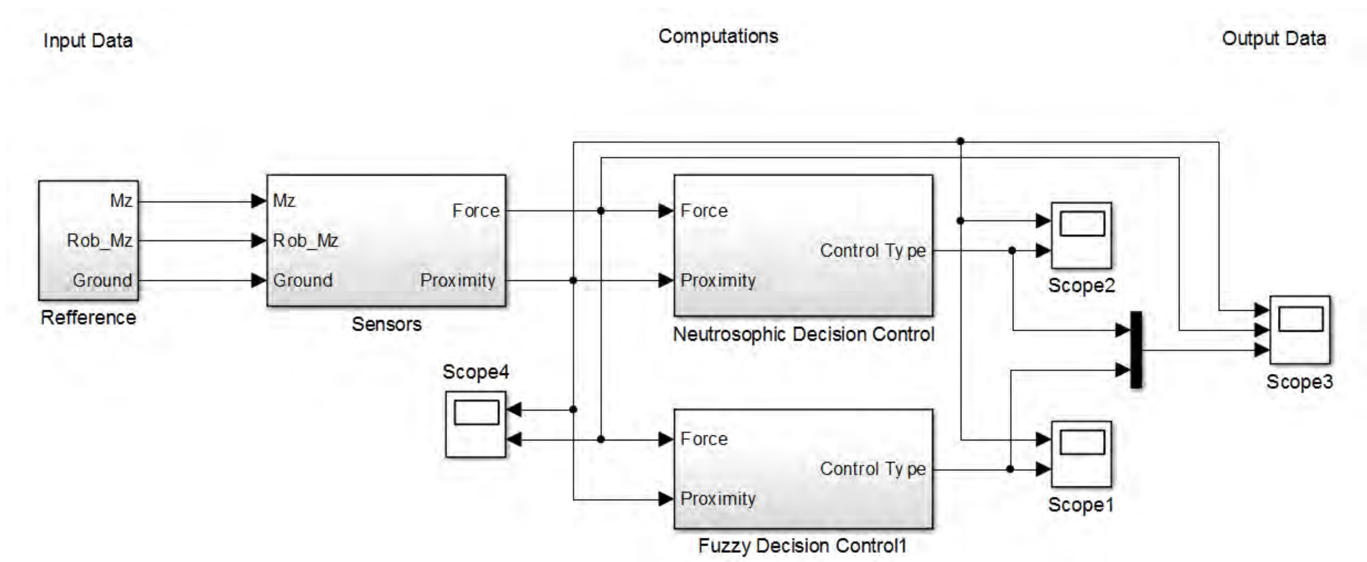

Fig. 4 The simulated switching system

After the simulation was done, the output data provided by the simulated sensors is shown in figure 6, the top two diagrams. These signals are for proximity data and the computed force. The third diagram of figure 6 presents the switching data provided by the neutrosophic and fuzzy decision blocks. The full line represents the neutrosophic decision and the dashed line the fuzzy decision. Also, the decision to choose the kinematic control law is when the output value of the switching law is equal to 10 and for the dynamic control law we have chosen the 0 value. Before the neutrosophic decision is made, we had to compute the four parameters on which the neutrosophic switching is based. These parameters are presented in figure 7.

One can observe that the value of the indeterminacy parameter is always 0 because the values provided by the sensors do not make our system to be in an unknown state.

One can see how the value of truthiness, indeterminacy, falsity and contradiction varies according to the values of proximity and force sensors. Also, we have to point out that these values correspond to the level of truthiness, indeterminacy and falsity for choosing the dynamic control law, and the kinematic control law is chosen when the dynamic one fails to be selected.

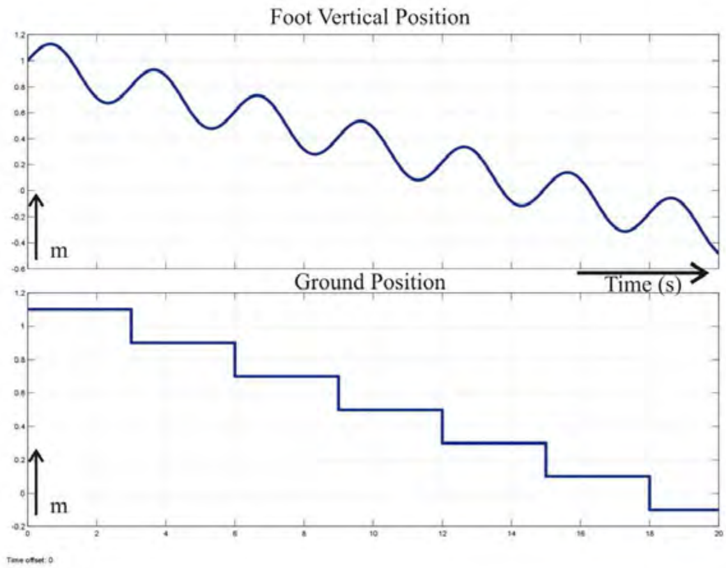

Fig. 5 The reference signals for the robot foot and ground

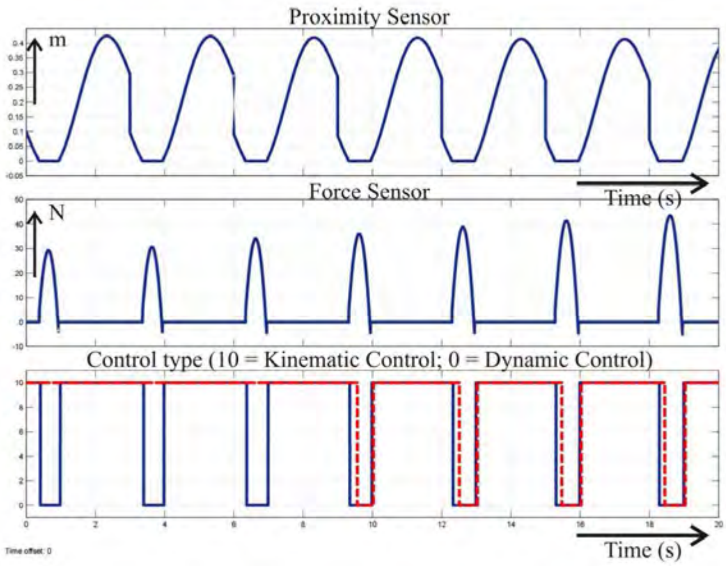

Fig. 6 The output data from sensors and the switching decisions

After the neutrosophication phase, in which we computed the truthiness, indeterminacy, falsity and contradiction parameters, we have applied the classic Neutrosophic decision, described in this paper. After that, we have chosen the control law, by simply comparing the results of the truthiness, indeterminacy, falsity and contradiction parameters to each other, and obtained the first diagram from figure 8.

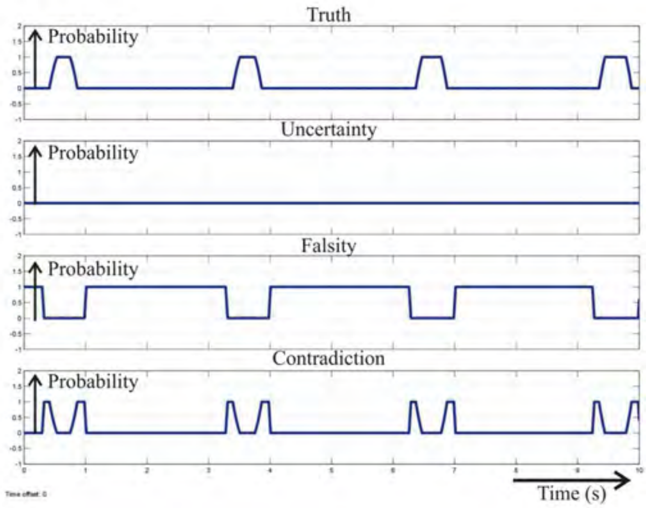

Fig. 7 Parameters after neutrosophication (Truthiness, Indeterminacy, Falsity and Contradiction)

The second diagram of figure 8, shows the output of the fuzzy switching block in which the decision was made with the help of a threshold value of 0,5for the fuzzification values.

As one can see from figure 8, the neutrosophic based switching law has commuted from the kinematic control law to the dynamic control law when the robot foot was near and then in contact with the support surface.

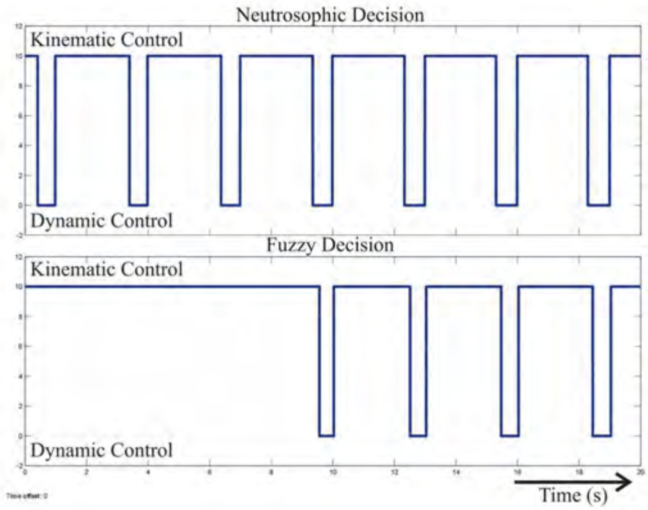

Fig. 8 Output data of the two switching techniques

After the contact has ended, the control law has been switched back from the dynamic control law to the kinematic control law. This was done at every step of the stairs. But, in comparison, the fuzzy based switching law did not behave like we wanted because it failed to switch to a dynamic control law for the first 3 steps, and after that, at the last 4 steps the robot has taken, it commuted the control laws too late to be efficient.

The main conclusion that can be drawn is that the neutrosophic technique behaves really well in different conditions of uncertainties, that can occur during the robot motion, due to the errors form the sensors or uneven ground surface in the case of the force sensor.

Further work will focus on implementing this switching technique on a simulation of a walking robot in which one will be able to see how the switching in influencing the motion of the walking robot. And after that, the second step will be to implement it on a real robot.

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Applications of Neutrosophic Logic to Robotics

An Introduction

Florentin Smarandache Luige Vladareanu

Abstract— In this paper we present the N-norms/N-conorms in neutrosophic logic and set as extensions of T-norms/T-conorms in fuzzy logic and set.
Then we show some applications of the neutrosophic logic to robotics.

Keywords: N-norm, N-conorm, N-pseudonorm, N-pseudoconorm, Neutrosophic set, Neutrosophic logic, Robotics

I. DEFINITION OF NEUTROSOPHIC SET

Let T, I, F be real standard or non-standard subsets of $]^-0, 1^+[$,
with sup T = t_sup, inf T = t_inf,
sup I = i_sup, inf I = i_inf,
sup F = f_sup, inf F = f_inf,
and n_sup = t_sup+i_sup+f_sup,
n_inf = t_inf+i_inf+f_inf.

Let U be a universe of discourse, and M a set included in U. An element x from U is noted with respect to the set M as x(T, I, F) and belongs to M in the following way: it is t% true in the set, i% indeterminate (unknown if it is or not) in the set, and f% false, where t varies in T, i varies in I, f varies in F ([1], [3]).

Statically T, I, F are subsets, but dynamically T, I, F are functions/operators depending on many known or unknown parameters.

II. DEFINITION OF NEUTROSOPHIC LOGIC

In a similar way we define the **Neutrosophic Logic**:
A logic in which each proposition x is T% true, I% indeterminate, and F% false, and we write it x(T,I,F), where T, I, F are defined above.

III. PARTIAL ORDER

We define a *partial order relationship* on the neutrosophic set/logic in the following way:

$x(T_1, I_1, F_1) \leq y(T_2, I_2, F_2)$ iff (if and only if)
$T_1 \leq T_2, I_1 \geq I_2, F_1 \geq F_2$ for crisp components.

And, in general, for subunitary set components:

$x(T_1, I_1, F_1) \leq y(T_2, I_2, F_2)$ iff
$\inf T_1 \leq \inf T_2, \sup T_1 \leq \sup T_2$,
$\inf I_1 \geq \inf I_2, \sup I_1 \geq \sup I_2$,
$\inf F_1 \geq \inf F_2, \sup F_1 \geq \sup F_2$.

If we have mixed - crisp and subunitary - components, or only crisp components, we can transform any crisp component, say "a" with a Ï [0,1] or a Ï $]^-0, 1^+[$, into a subunitary set [a, a]. So, the definitions for subunitary set components should work in any case.

IV. N-NORM AND N-CONORM

As a generalization of T-norm and T-conorm from the Fuzzy Logic and Set, we now introduce the **N-norms and N-conorms** for the Neutrosophic Logic and Set.

A. N-norm

N_n: $( ]^-0,1^+[\times]^-0,1^+[\times]^-0,1^+[ )^2 \to]^-0,1^+[\times]^-0,1^+[\times]^-0,1^+[$
$N_n (x(T_1,I_1,F_1), y(T_2,I_2,F_2)) = (N_nT(x,y), N_nI(x,y), N_nF(x,y))$, where $N_nT(.,.)$, $N_nI(.,.)$, $N_nF(.,.)$ are the truth/membership, indeterminacy, and respectively falsehood/nonmembership components.

N_n have to satisfy, for any x, y, z in the neutrosophic logic/set M of the universe of discourse U, the following axioms:
a) Boundary Conditions: $N_n(x, \mathbf{0}) = \mathbf{0}$, $N_n(x, \mathbf{1}) = x$.
b) Commutativity: $N_n(x, y) = N_n(y, x)$.
c) Monotonicity: If $x \leq y$, then $N_n(x, z) \leq N_n(y, z)$.
d) Associativity: $N_n(N_n (x, y), z) = N_n(x, N_n(y, z))$.

There are cases when not all these axioms are satisfied, for example the associativity when dealing with the neutrosophic normalization after each neutrosophic operation. But, since we work with approximations, we can call these **N-pseudo-norms**, which still give good results in practice.

N_n represent the *and* operator in neutrosophic logic, and respectively the *intersection* operator in neutrosophic set theory.

Let $J \in \{T, I, F\}$ be a component.
Most known N-norms, as in fuzzy logic and set the T-norms, are:
• The Algebraic Product N-norm: $N_{n-algebraic}J(x, y) = x \cdot y$
• The Bounded N-Norm: $N_{n-bounded}J(x, y) = \max\{0, x + y - 1\}$
• The Default (min) N-norm: $N_{n-min}J(x, y) = \min\{x, y\}$.

A general example of N-norm would be this.
Let $x(T_1, I_1, F_1)$ and $y(T_2, I_2, F_2)$ be in the neutrosophic set/logic M. Then:

$$\mathbf{N_n(x, y) = (T_1 \wedge T_2, I_1 \vee I_2, F_1 \vee F_2)}$$

where the "$\wedge$" operator, acting on two (standard or non-standard) subunitary sets, is a N-norm (verifying the above N-norms axioms); while the "$\vee$" operator, also acting on two (standard or non-standard) subunitary sets, is a N-conorm (verifying the below N-conorms axioms).

For example, $\wedge$ can be the Algebraic Product T-norm/N-norm, so $T_1 \wedge T_2 = T_1 \cdot T_2$ (herein we have a product of two subunitary sets – using simplified notation); and $\vee$ can be the Algebraic Product T-conorm/N-conorm, so $T_1 \vee T_2 = T_1+T_2-T_1 \cdot T_2$ (herein we have a sum, then a product, and afterwards a subtraction of two subunitary sets).

Or $\wedge$ can be any T-norm/N-norm, and $\vee$ any T-conorm/N-conorm from the above and below; for example the easiest way would be to consider the *min* for crisp components (or *inf* for subset components) and respectively *max* for crisp components (or *sup* for subset components).

If we have crisp numbers, we can at the end neutrosophically normalize.

B. N-conorm

N_c: $( ]^-0,1^+[\times]^-0,1^+[\times]^-0,1^+[ )^2 \to]^-0,1^+[\times]^-0,1^+[\times]^-0,1^+[$
$N_c (x(T_1,I_1,F_1), y(T_2,I_2,F_2)) = (N_cT(x,y), N_cI(x,y), N_cF(x,y))$, where $N_nT(.,.)$, $N_nI(.,.)$, $N_nF(.,.)$ are the truth/membership, indeterminacy, and respectively falsehood/nonmembership components.

N_c have to satisfy, for any x, y, z in the neutrosophic logic/set M of universe of discourse U, the following axioms:

a) Boundary Conditions: $N_c(x, \mathbf{1}) = \mathbf{1}$, $N_c(x, \mathbf{0}) = x$.
b) Commutativity: $N_c (x, y) = N_c(y, x)$.
c) Monotonicity: if $x \le y$, then $N_c(x, z) \le N_c(y, z)$.
d) Associativity: $N_c (N_c(x, y), z) = N_c(x, N_c(y, z))$.

There are cases when not all these axioms are satisfied, for example the associativity when dealing with the neutrosophic normalization after each neutrosophic operation. But, since we work with approximations, we can call these **N-pseudo-conorms**, which still give good results in practice.

N_c represent the *or* operator in neutrosophic logic, and respectively the *union* operator in neutrosophic set theory.

Let $J \in \{T, I, F\}$ be a component.
Most known N-conorms, as in fuzzy logic and set the T-conorms, are:

- The Algebraic Product N-conorm: $N_{c-algebraic}J(x, y) = x + y - x \cdot y$
- The Bounded N-conorm: $N_{c-bounded}J(x, y) = \min\{1, x + y\}$
- The Default (max) N-conorm: $N_{c-max}J(x, y) = \max\{x, y\}$.

A general example of N-conorm would be this.
Let $x(T_1, I_1, F_1)$ and $y(T_2, I_2, F_2)$ be in the neutrosophic set/logic M. Then:

$$\mathbf{N_n(x, y) = (T_1 \vee T_2, I_1 \wedge I_2, F_1 \wedge F_2)}$$

Where – as above - the "$\wedge$" operator, acting on two (standard or non-standard) subunitary sets, is a N-norm (verifying the above N-norms axioms); while the "$\vee$" operator, also acting on two (standard or non-standard) subunitary sets, is a N-conorm (verifying the above N-conorms axioms).

For example, $\wedge$ can be the Algebraic Product T-norm/N-norm, so $T_1 \wedge T_2 = T_1 \cdot T_2$ (herein we have a product of two subunitary sets); and $\vee$ can be the Algebraic Product T-conorm/N-conorm, so $T_1 \vee T_2 = T_1+T_2-T_1 \cdot T_2$ (herein we have a sum, then a product, and afterwards a subtraction of two subunitary sets).

Or $\wedge$ can be any T-norm/N-norm, and $\vee$ any T-conorm/N-conorm from the above; for example the easiest way would be to consider the *min* for crisp components (or *inf* for subset components) and respectively *max* for crisp components (or *sup* for subset components).

If we have crisp numbers, we can at the end neutrosophically normalize.

Since the min/max (or inf/sup) operators work the best for subunitary set components, let's present their definitions below. They are extensions from subunitary intervals {defined in [3]} to any subunitary sets. Analogously we can do for all neutrosophic operators defined in [3].

Let $x(T_1, I_1, F_1)$ and $y(T_2, I_2, F_2)$ be in the neutrosophic set/logic M.

C. More Neutrosophic Operators

Neutrosophic Conjunction/Intersection:

$x \wedge y=(T_\wedge,I_\wedge,F_\wedge)$,
where $\inf T_\wedge = \min\{\inf T_1, \inf T_2\}$
$\sup T_\wedge = \min\{\sup T_1, \sup T_2\}$
$\inf I_\wedge = \max\{\inf I_1, \inf I_2\}$
$\sup I_\wedge = \max\{\sup I_1, \sup I_2\}$
$\inf F_\wedge = \max\{\inf F_1, \inf F_2\}$
$\sup F_\wedge = \max\{\sup F_1, \sup F_2\}$

Neutrosophic Disjunction/Union:

$x \vee y=(T_\vee,I_\vee,F_\vee)$,
where $\inf T_\vee = \max\{\inf T_1, \inf T_2\}$
$\sup T_\vee = \max\{\sup T_1, \sup T_2\}$
$\inf I_\vee = \min\{\inf I_1, \inf I_2\}$
$\sup I_\vee = \min\{\sup I_1, \sup I_2\}$
$\inf F_\vee = \min\{\inf F_1, \inf F_2\}$
$\sup F_\vee = \min\{\sup F_1, \sup F_2\}$

Neutrosophic Negation/Complement:

$C(x) = (T_C,I_C,F_C)$,
where $T_C = F_1$
$\inf I_C = 1-\sup I_1$

$$\sup I_C = 1\text{-}\inf I_1$$
$$F_C = T_1$$

Upon the above Neutrosophic Conjunction/Intersection, we can define the

Neutrosophic Containment:
We say that the neutrosophic set A is included in the neutrosophic set B of the universe of discourse U, iff for any $x(T_A, I_A, F_A) \in A$ with $x(T_B, I_B, F_B) \in B$ we have:
$\inf T_A \le \inf T_B$; $\sup T_A \le \sup T_B$;
$\inf I_A \ge \inf I_B$; $\sup I_A \ge \sup I_B$;
$\inf F_A \ge \inf F_B$; $\sup F_A \ge \sup F_B$.

D. Remarks

a) The non-standard unit interval $]^-0, 1^+[$ is merely used for philosophical applications, especially when we want to make a distinction between relative truth (truth in at least one world) and absolute truth (truth in all possible worlds), and similarly for distinction between relative or absolute falsehood, and between relative or absolute indeterminacy.

But, for technical applications of neutrosophic logic and set, the domain of definition and range of the N-norm and N-conorm can be restrained to the normal standard real unit interval [0, 1], which is easier to use, therefore:

$$N_n: ([0,1] \times [0,1] \times [0,1])^2 \rightarrow [0,1] \times [0,1] \times [0,1]$$
and
$$N_c: ([0,1] \times [0,1] \times [0,1])^2 \rightarrow [0,1] \times [0,1] \times [0,1].$$

b) Since in NL and NS the sum of the components (in the case when T, I, F are crisp numbers, not sets) is not necessary equal to 1 (so the normalization is not required), we can keep the final result un-normalized.
But, if the normalization is needed for special applications, we can normalize at the end by dividing each component by the sum all components.
If we work with intuitionistic logic/set (when the information is incomplete, i.e. the sum of the crisp components is less than 1, i.e. *sub-normalized*), or with paraconsistent logic/set (when the information overlaps and it is contradictory, i.e. the sum of crisp components is greater than 1, i.e. *over-normalized*), we need to define the neutrosophic measure of a proposition/set.
If x(T,I,F) is a NL/NS, and T,I,F are crisp numbers in [0,1], then the **neutrosophic vector norm** of variable/set x is the sum of its components:
$$N_{vector\text{-}norm}(x) = T+I+F.$$

Now, if we apply the N_n and N_c to two propositions/sets which maybe intuitionistic or paraconsistent or normalized (i.e. the sum of components less than 1, bigger than 1, or equal to 1), x and y, what should be the neutrosophic measure of the results $N_n(x,y)$ and $N_c(x,y)$?
Herein again we have more possibilities:

- either the product of neutrosophic measures of x and y:
$N_{vector\text{-}norm}(N_n(x,y)) = N_{vector\text{-}norm}(x) \cdot N_{vector\text{-}norm}(y)$,
- or their average:
$N_{vector\text{-}norm}(N_n(x,y)) = (N_{vector\text{-}norm}(x) + N_{vector\text{-}norm}(y))/2$,
- or other function of the initial neutrosophic measures:

$N_{vector\text{-}norm}(N_n(x,y)) = f(N_{vector\text{-}norm}(x), N_{vector\text{-}norm}(y))$, where f(.,.) is a function to be determined according to each application.

Similarly for $N_{vector\text{-}norm}(N_c(x,y))$.
Depending on the adopted neutrosophic vector norm, after applying each neutrosophic operator the result is neutrosophically normalized. We'd like to mention that "**neutrosophically normalizing**" doesn't mean that the sum of the resulting crisp components should be 1 as in fuzzy logic/set or intuitionistic fuzzy logic/set, but the sum of the components should be as above: either equal to the product of neutrosophic vector norms of the initial propositions/sets, or equal to the neutrosophic average of the initial propositions/sets vector norms, etc.
In conclusion, we neutrosophically normalize the resulting crisp components T\`,I\`,F\` by multiplying each neutrosophic component T\`,I\`,F\` with S/(T\`+I\`+F\`), where
$S= N_{vector\text{-}norm}(N_n(x,y))$ for a N-norm or $S= N_{vector\text{-}norm}(N_c(x,y))$ for a N-conorm - as defined above.

c) If T, I, F are subsets of [0, 1] the problem of neutrosophic normalization is more difficult.
   i) If sup(T)+sup(I)+sup(F) < 1, we have an *intuitionistic proposition/set*.
   ii) If inf(T)+inf(I)+inf(F) > 1, we have a *paraconsistent proposition/set*.
   iii) If there exist the crisp numbers $t \in T$, $i \in I$, and $f \in F$ such that t+i+f =1, then we can say that we have a *plausible normalized proposition/set*.
   But in many such cases, besides the normalized particular case showed herein, we also have crisp numbers, say $t_1 \in T$, $i_1 \in I$, and $f_1 \in F$ such that $t_1+i_1+f_1 < 1$ (incomplete

information) and $t_2 \in T$, $i_2 \in I$, and $f_2 \in F$ such that $t_2+i_2+f_2 > 1$ (paraconsistent information).

E. *Examples of Neutrosophic Operators which are N-norms or N-pseudonorms or, respectively N-conorms or N-pseudoconorms*

We define a binary **neutrosophic conjunction (intersection)** operator, which is a particular case of a N-norm (neutrosophic norm, a generalization of the fuzzy T-norm):

$$c_N^{TIF} : \left([0,1]\times[0,1]\times[0,1]\right)^2 \to [0,1]\times[0,1]\times[0,1]$$

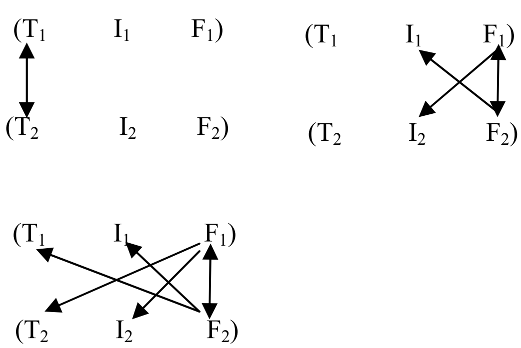
$$c_N^{TIF}(x,y) = \left(T_1T_2, I_1I_2 + I_1T_2 + T_1I_2, F_1F_2 + F_1I_2 + F_1T_2 + F_2T_1 + F_2I_1\right)$$

.

The neutrosophic conjunction (intersection) operator $x \wedge_N y$ component truth, indeterminacy, and falsehood values result from the multiplication

$$\left(T_1 + I_1 + F_1\right)\cdot\left(T_2 + I_2 + F_2\right)$$

since we consider in a prudent way $T \prec I \prec F$, where "$\prec$" is a **neutrosophic relationship** and means "weaker", i.e. the products T_iI_j will go to I, T_iF_j will go to F, and I_iF_j will go to F for all i, j $\in \{1,2\}$, i $\neq$ j, while of course the product T_1T_2 will go to T, I_1I_2 will go to I, and F_1F_2 will go to F (or reciprocally we can say that F prevails in front of I which prevails in front of T, and this neutrosophic relationship is transitive):



So, the truth value is T_1T_2, the indeterminacy value is $I_1I_2 + I_1T_2 + T_1I_2$ and the false value is $F_1F_2 + F_1I_2 + F_1T_2 + F_2T_1 + F_2I_1$. The norm of $x \wedge_N y$ is $(T_1 + I_1 + F_1)\cdot(T_2 + I_2 + F_2)$. Thus, if x and y are normalized, then $x \wedge_N y$ is also normalized. Of course, the reader can redefine the neutrosophic conjunction operator, depending on application, in a different way, for example in a more optimistic way, i.e. $I \prec T \prec F$ or T prevails with respect to I, then we get:

$$c_N^{ITF}(x,y) = \left(T_1T_2 + T_1I_2 + T_2I_1, I_1I_2, F_1F_2 + F_1I_2 + F_1T_2 + F_2T_1 + F_2I_1\right)$$

Or, the reader can consider the order $T \prec F \prec I$, etc.

V. ROBOT POSITION CONTROL BASED ON KINEMATICS EQUATIONS

A robot can be considered as a mathematical relation of actuated joints which ensures coordinate transformation from one axis to the other connected as a serial link manipulator where the links sequence exists. Considering the case of revolute-geometry robot all joints are rotational around the freedom ax [4, 5]. In general having a six degrees of freedom the manipulator mathematical analysis becomes very complicated. There are two dominant coordinate systems: Cartesian coordinates and joints coordinates. Joint coordinates represent angles between links and link extensions. They form the coordinates where the robot links are moving with direct control by the actuators.

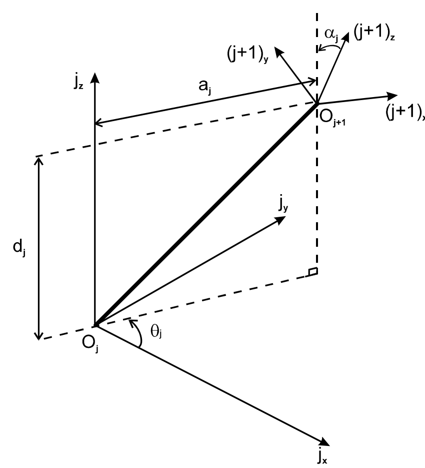


Fig.1. The robot control through DH transformation.

The position and orientation of each segment of the linkage structure can be described using Denavit-Hartenberg [DH] transformation [6]. To determine the D-H transformation matrix (Fig. 1) it is assumed that the *Z*-axis (which is the system's axis in relation to the motion surface) is the axis of rotation in each frame, with the following notations: θ_j - joint angled is the joint angle positive in the right hand sense about j_Z; a_j - link length is the length of the common normal, positive in the direction of $(j+1)_X$; α_j - twist angled is the angle between j_Z and $(j+1)_Z$, positive in the right hand sense about the common normal; d_j - offset distance is the value of j_Z at which the common normal intersects j_Z; as well if j_X and $(j+1)_X$ are parallel and in the same direction, then $\theta_j = 0$; $(j+1)_X$ - is chosen to be collinear with the common normal between j_Z and $(j+1)_Z$ [7, 8] . Figure 1 illustrates a robot position control based on the Denavit-Hartenberg transformation. The robot joint angles, θ_c, are transformed in X_c - Cartesian coordinates with D-H transformation. Considering that a point in j, respectively j+1 is given by:

$$\begin{vmatrix} X \\ Y \\ Z \\ 1 \end{vmatrix}_j = {}^jP \quad \text{and} \quad \begin{vmatrix} X \\ Y \\ Z \\ 1 \end{vmatrix}_{j+1} = {}^{j+1}P' \qquad (1)$$

then ${}^jP$ can be determined in relation to ${}^{j+1}P$ through the equation :

$${}^jP = {}^jA_{j+1} \cdot {}^{j+1}P, \qquad (2)$$

where the transformation matrix ${}^jA_{j+1}$ is:

$${}^jA_{j+1} = \begin{bmatrix} \cos\theta_j & -\sin\theta_j \cdot \cos\alpha_j & +\sin\theta_j \cdot \sin\alpha_j & a_j \cdot \cos\alpha_j \\ \sin\theta_j & -\cos\theta_j \cdot \cos\alpha_j & -\cos\theta_j \cdot \sin\alpha_j & a_j \cdot \sin\alpha_j \\ 0 & \sin\theta_j & \cos\theta_j & d_j \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

Control through forward kinematics consists of the transformation of robot coordinates at any given moment, resulting directly from the measurement transducers of each axis, to Cartesian coordinates and comparing to the desired target's Cartesian coordinates (reference point). The resulting error is the difference of position, represented in Cartesian coordinates, which requires changing. Using the inverted Jacobean matrix ensures the transformation into robot coordinates of the position error from Cartesian coordinates, which allows the generating of angle errors for the direct control of the actuator on each axis.

The control using forward kinematics consists of transforming the actual joint coordinates, resulting from transducers, to Cartesian coordinates and comparing them with the desired Cartesian coordinates. The resulted error is a required position change, which must be obtained on every axis. Using the Jacobean matrix inverting it will manage to transform the change in joint coordinates that will generate angle errors for the motor axis control.

Figure 2 illustrates a robot position control system based on the Denavit-Hartenberg transformation. The robot joint angles, θ_c, are transformed in X_c - Cartesian coordinates with D-H transformation, where a matrix results from (1) and (2) with θ_j -joint angle, d_j -offset distance, a_j - link length, α_j - twist.

Position and orientation of the end effector with respect to the base coordinate frame is given by X_C :

$$X_C = A_1 \cdot A_2 \cdot A_3 \cdot \ldots\ldots\ldots \cdot A_6 \qquad (3)$$

Position error ΔX is obtained as a difference between desired and current position. There is difficulty in controlling robot trajectory, if the desired conditions are specified using position difference ΔX with continuously measurement of current position $\theta_{1,2,\ldots..6}$.

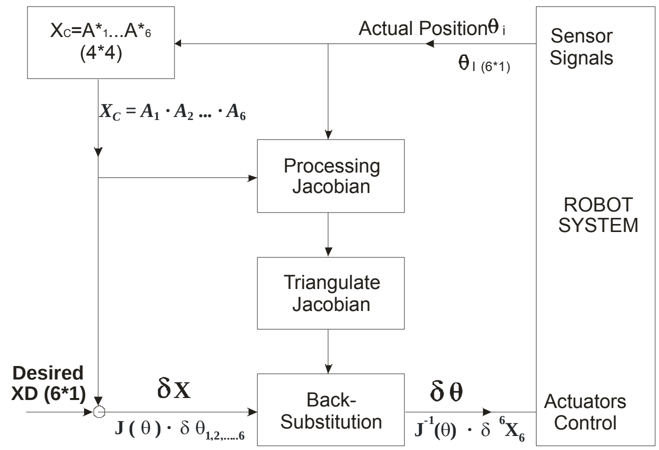


Fig. 2. Robot position control system based on the Denavit-Hartenberg transformation

The relation, between given by end-effector's position and orientation considered in Cartesian coordinates and the robot joint angles $\theta_{1,2,\ldots..6}$, it is :

$$x_i = f_i(\theta) \qquad (4)$$

where θ is vector representing the degrees of freedom of robot. By differentiating we will have: $\delta\, {}^6X_6 = J(\theta) \cdot \delta\theta_{1,2,\ldots..6}$, where $\delta\, {}^6X_6$ represents differential linear and angular changes in the end effector at the currently values of X_6 and $\delta\theta_{1,2,\ldots..6}$ represents the differential change of the set of joint angles. $J(\theta)$ is the Jacobean matrix in which the elements a_{ij} satisfy the relation: $a_{ij} = \delta f_{i-1} / \delta\theta_{j-1}$, (x.6) where i, j are corresponding to the dimensions of x respectively θ. The inverse Jacobian transforms the Cartesian position δ^6X_6 respectively ΔX in joint angle error ($\Delta\theta$): $\delta\theta_{1,2,\ldots6} = J^{-1}(\theta) \cdot \delta\, {}^6X_6$.

VI. HYBRID POSITION AND FORCE CONTROL OF ROBOTS

Hybrid position and force control of industrial robots equipped with compliant joints must take into consideration the passive compliance of the system. The generalized area where a robot works can be defined in a constraint space with six degrees of freedom (DOF), with position constrains along the normal force of this area and force constrains along the tangents. On the basis of these two constrains there is described the general scheme of hybrid position and force control in figure 3. Variables X_C and F_C represent the Cartesian position and the Cartesian force exerted onto the environment. Considering X_C and F_C expressed in specific frame of coordinates, its can be determinate selection matrices S_x and S_f, which are diagonal matrices with 0 and 1 diagonal elements, and which satisfy relation: $S_x + S_f = I_d$, where S_x and S_f are methodically deduced from kinematics constrains imposed by the working environment [9, 10].

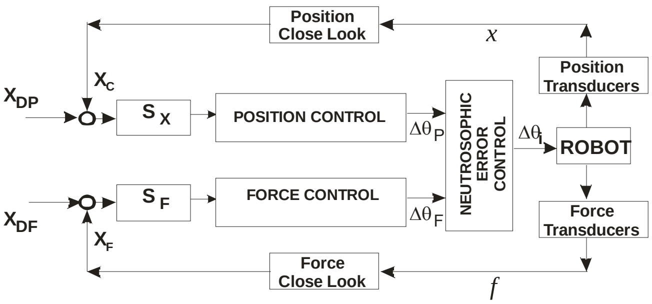


Fig. 3. General structure of hybrid control.

Mathematical equations for the hybrid position-force control. A system of hybrid position–force control normally achieves the simultaneous position–force control. In order to determine the control relations in this situation, ΔX_P – the measured deviation of Cartesian coordinate command system is split in two sets: ΔX^F corresponds to force controlled component and ΔX^P corresponds to position control with axis actuating in accordance with the selected matrixes S_f and S_x. If there is considered only positional control on the directions established by the selection matrix S_x there can be determined the desired end - effector differential motions that correspond to position control in the relation: $\Delta X_P = K_P \Delta X^P$, where K_P is the gain matrix, respectively desired motion joint on position controlled axis: $\Delta\theta_P = J^{-1}(\theta) \cdot \Delta X_P$ [11, 12].

Now taking into consideration the force control on the other directions left, the relation between the desired joint motion of end-effector and the force error ΔX_F is given by the relation: $\Delta\theta_F = J^{-1}(\theta) \cdot \Delta X_F$, where the position error due to force ΔX_F is the motion difference between ΔX^F– current position deviation measured by the control system that generates position deviation for force controlled axis and ΔX_D – position deviation because of desired residual force. Noting the given desired residual force as F_D and the physical rigidity K_W there is obtained the relation: $\Delta X_D = K_W^{-1} \cdot F_D$.

Thus, ΔX_F can be calculated from the relation: $\Delta X_F = K_F (\Delta X^F - \Delta X_D)$, where K_F is the dimensionless ratio of the stiffness matrix. Finally, the motion variation on the robot axis matched to the motion variation of the end-effectors is obtained through the relation: $\Delta\theta = J^{-1}(\theta)\, \Delta X_F + J^{-1}(\theta)\, \Delta X_P$. Starting from this representation the architecture of the hybrid position – force control system was developed with the corresponding coordinate transformations applicable to systems with open architecture and a distributed and decentralized structure.

For the fusion of information received from various sensors, information that can be conflicting in a certain degree, the robot uses the fuzzy and neutrosophic logic or set [3]. In a real time it is used a neutrosophic dynamic fusion, so an autonomous robot can take a decision at any moment.

CONCLUSION

In this paper we have provided in the first part an introduction to the neutrosophic logic and set operators and in the second part a short description of mathematical dynamics of a robot and then a way of applying neutrosophic science to robotics. Further study would be done in this direction in order to develop a **robot neutrosophic control**.

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Presented at 2011 IEEE International Conference on Granular Computing, National University of Kaohsiung, Taiwan, 8-10 November 2011.

Correlation Coefficient of Interval Neutrosophic Set

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Keywords: Neutrosophic Set, Correlation Coefficient of Interval Neutrosophic Set, Weighted Correlation Coefficient of Interval Neutrosophic Set.

Abstract. In this paper we introduce for the first time the concept of correlation coefficients of interval valued neutrosophic set (INS for short). Respective numerical examples are presented.

1. Introduction

Neutrosophy was pioneered by Smarandache [1]. It is a branch of philosophy which studies the origin, nature, and scope of neutralities, as well as their interactions with different ideational spectra [2]. Neutrosophic set theory is a powerful formal framework which generalizes the concept of the classic set, fuzzy set [3], interval-valued fuzzy set [4], intuitionistic fuzzy set [5], interval-valued intuitionistic fuzzy set [6], and so on. Neutrosophy introduces a new concept called <NeutA> which represents indeterminacy with respect to <A>. It can solve certain problems that cannot be solved by fuzzy logic. For example, a paper is sent to two reviewers, one says it is 90% acceptable and another says it is 90% unacceptable. But the two reviewers may have different backgrounds. One is an expert, and another is a new comer in this field. The impacts on the final decision of the paper by the two reviewers should be different, even though they give the same grade level of the acceptance. There are many similar problems, such as weather forecasting, stock price prediction, and political elections containing indeterminate conditions that fuzzy set theory does not handle well. This theory deals with imprecise and vague situations where exact analysis is either difficult or impossible.
After the pioneering work of Smarandache. In 2005, Wang et al. [7] introduced the notion of interval neutrosophic set (INS) which is a particular case of the neutrosophic set (NS) that can be described by a membership interval, a non-membership interval, and an indeterminate interval, thus the NS is flexible and practical, and the NS provides a more reasonable mathematical framework to deal with indeterminate and inconsistent information.
The theories of both neutrosophic set and interval neutrosophic set have achieved great success in various areas such as medical diagnosis [8], database [9,10], topology[11], image processing [12,13,14], and decision making problem [15].
Although several distance measures, similarity measures, and correlation measure of neutrosophic sets have been recently presented in [16, 17], there is a rare investigation on correlation of interval neutrosophic sets.
It is very common in statistical analysis of data to finding the correlation between variables or attributes, where the correlation coefficient is defined on ordinary crisp sets, fuzzy sets [18], intuitionistic fuzzy sets [19,20,21], and neutrosophic set [16,17] respectively. In this paper we first discuss and derive a formula for the correlation coefficient defined on the domain of interval neutrosophic sets. The paper unfolds as follows. The next section briefly introduces some definitions related to the method. Section III presents the correlation and weighted correlation coefficient of the interval neutrosophic set. Conclusions appear in the last section.

2. Preliminaries

In this section, we mainly recall some notions related to neutrosophic sets, and interval neutrosophic sets relevant to the present work. See especially [1, 7, 17] for further details and background.

2.1 Definition ([1]). Let U be an universe of discourse; then the neutrosophic set A is an object having the form A = {< x: $T_A(x)$,$I_A(x)$,$F_A(x)$>,x ∈ U}, where the functions T,I,F : U→]$^-$0,1$^+$[define respectively the degree of membership, the degree of indeterminacy, and the degree of non-membership of the element x ∈ U to the set A with the condition:

$$^-0 \leq T_A(x)+ I_A(x)+ F_A(x) \leq 3^+. \quad (1)$$

From philosophical point of view, the neutrosophic set takes the value from real standard or non-standard subsets of]$^-$0,1$^+$[.So instead of]$^-$0,1$^+$[we need to take the interval [0,1] for technical applications, because]$^-$0,1$^+$[will be difficult to apply in the real applications such as in scientific and engineering problems.

2.2 Definition ([7]). Let X be a space of points (objects) with generic elements in X denoted by x. An interval neutrosophic set A in X is characterized by truth-membership function $T_A(x)$, indeteminacy-membership function $I_A(x)$, and falsity-membership function $F_A(x)$. For each point x in X, we have that $T_A(x)$, $I_A(x)$, $F_A(x) \subset [0,1]$.

Remark 1. An INS is clearly a NS.

2.3 Definition ([7]).

- An INS A is empty if $inf T_A(x) = sup T_A(x)$= 0, $inf I_A(x) = sup I_A(x)$= 1, $inf F_A(x) = sup F_A(x)$= 0, for all x in A.
- Let $\underline{0}$ = <0, 1 ,1> and $\underline{1}$ = <1, 0 ,0>

2.4 Correlation Coefficient of Neutrosophic Set ([17]).

Let A and B be two neutrosophic sets in the universe of discourse X = {x_1, x_2, …,x_n}.

The *correlation coefficient* of A and B is given by

$$R(A,B)=\frac{C(A,B)}{(E(A)\cdot E(B))^{\frac{1}{2}}} \quad (2)$$

where the *correlation* of two NSs A and B is given by

$$C(A,B) =\sum_{1=0}^{n}(T_A(x_i)T_B(x_i) + I_A(x_i)I_B(x_i) + F_A(x_i)F_B(x_i)) \quad (3)$$

And the *informational energy* of two NSs A and B are given by

$$E(A) = \sum_{1=0}^{n}(T_A^2(x_i) + I_A^2(x_i) + F_A^2(x_i)) \quad (4)$$

$$E(B) = \sum_{1=0}^{n}(T_B^2(x_i) + I_B^2(x_i) + F_B^2(x_i)) \quad (5)$$

Respectively, the correlation coefficient of two neutrosophic sets A and B satisfies the following properties:

(1) $0 \leq R(A,B) \leq 1$ (6)
(2) R(A,B)= R(B,A) (7)
(3) R(A,B)= 1 if A=B (8)

3. Correlation of Two Interval Neutrosophic Sets

In this section, following the correlation between two neutrosophic sets defined by A. A. Salama in [17], we extend this definition to interval neutrosophic sets. If we have a random non-crisp set, with a triple membership form, for each of two interval neutrosophic sets, we get the interest in comparing the degree of their relationship. We check if there is any linear relationship between the two interval neutrosophic sets; thus we need a formula for the sample correlation coefficient of two interval neutrosophic sets in order to find the relationship between them.

3.1.Definition

Assume that two interval neutrosophic sets A and B in the universe of discourse $X = \{x_1, x_2, x_3, \ldots, x_n\}$ are denoted by

$$A=\frac{\sum_1^n<[\inf T_A(x_i)\ \sup T_A(x_i)],[\inf I_A(x_i)\ \sup I_A(x_i)],[\inf T_A(x_i)\ \sup F_A(x_i)],>}{x_i},\ x_i \in X, \text{ and} \quad (9)$$

$$B=\frac{\sum_1^n<[\inf T_B(x_i)\ \sup T_B(x_i)],[\inf I_B(x_i)\ \sup I_B(x_i)],[\inf T_B(x_i)\ \sup F_B(x_i)],>}{x_i},\ x_i \in X, \text{ where} \quad (10)$$

$\inf T_A(x_i) \le \sup T_A(x_i)$,$\inf F_A(x_i) \le \sup F_A(x_i)$, $\inf I_A(x_i) \le \sup I_A(x_i)$, $\inf T_B(x_i) \le \sup T_B(x_i)$, $\inf F_B(x_i) \le \sup F_B(x_i)$, $\inf I_B(x_i) \le \sup I_B(x_i)$, and they all belong to [0, 1];

then we define the correlation of the interval neutrosophic sets A and B in X by the formula

$$C_{INS}(A,B)= \sum_1^n\{\inf T_A(x_i)\cdot \inf T_B(x_i) + \sup T_A(x_i)\cdot \sup T_B(x_i) + \inf I_A(x_i)\cdot \inf I_B(x_i) + \sup I_A(x_i)\cdot \sup I_B(x_i) + \inf F_A(x_i)\cdot \inf F_B(x_i) + \sup F_A(x_i)\cdot \sup F_B(x_i)\} \quad (11)$$

Let us notice that this formula coincides with that given by A. A Salama [17] when $\inf T_A(x_i) = \sup T_A(x_i)$, $\inf F_A(x_i) = \sup F_A(x_i)$, $\inf I_A(x_i) = \sup I_A(x_i)$ and $\inf T_B(x_i) = \sup T_B(x_i)$, $\inf F_B(x_i) = \sup F_B(x_i)$, $\inf I_B(x_i) = \sup I_B(x_i)$ and the correlation coefficient of the interval neutrosophic sets A and B given by

$$K_{INS}(A,B) = \frac{C_{INS}(A,B)}{(E(A)\cdot E(B))^{1/2}} \in [0, 1^+[\quad (12)$$

where

$$E(A)=\sum_1^n[T_{AL}{}^2(x_i) + T_{AU}{}^2(x_i) + I_{AL}{}^2(x_i) + I_{AU}{}^2(x_i) + F_{AL}{}^2(x_i) + F_{AU}{}^2(x_i)] \quad (13)$$

$$E(B)=\sum_1^n[T_{BL}{}^2(x_i) + T_{BU}{}^2(x_i) + I_{BL}{}^2(x_i) + I_{BU}{}^2(x_i) + F_{BL}{}^2(x_i) + F_{BU}{}^2(x_i)] \quad (14)$$

express the so-called informational energy of the interval neutrosophic sets A and B respectively.

Remark 2: For the sake of simplicity we shall use the symbols:

$\inf T_A(x_i) = T_{AL}$, $\sup T_A(x_i) = T_{AU}$, (15)
$\inf T_B(x_i) = T_{BL}$, $\sup T_B(x_i) = T_{BU}$, (16)
$\inf I_A(x_i) = I_{AL}$,$\sup I_A(x_i) = I_{AU}$, (17)
$\inf I_B(x_i) = I_{BL}$, $\sup I_B(x_i) = I_{BU}$, (18)
$\inf F_A(x_i) = F_{AL}$, $\sup F_A(x_i) = F_{AU}$, (19)
$\inf F_B(x_i) = F_{BL}$, $\sup F_B(x_i) = F_{BU}$, (20)

For the correlation of interval neutrosophic set, the following proposition is immediate from the definition.

3.2. Proposition

For A, B $\in$ $INSs$ in the universe of discourse X={$x_1,x_2,x_3,\ldots,x_n$} the correlation of interval neutrosophic set have the following properties:

$$(1)\, C_{INS}(A,A) = E(A) \tag{21}$$

$$(2)\, C_{INS}(A,B) = C_{INS}(B,A) \tag{22}$$

3.3. Theorem. For all INSs A, B the correlation coefficient satisfies the following properties:

(3) If $A = B$ *, then* $K_{INS}(A,B) = 1$. (23)

(4) $K_{INS}(A,B) = K_{INS}(B,A)$. (24)

(5) $0 \le K_{INS}(A,B) \le 1$. (25)

Proof. Conditions (1) and (2) are evident; we shall prove condition (3). $K_{INS}(A,B) \ge 0$ is evident. We will prove that $K_{INS}(A,B) \le 1$. From the Schwartz inequality, we obtain

$$K_{INS}(A,B) = (\sum_1^n [T_{AL}(x_i)T_{BL}(x_i) + T_{AU}(x_i)T_{BU}(x_i) + I_{AL}(x_i)I_{BL}(x_i) + I_{AU}(x_i)I_{BU}(x_i) + F_{AL}(x_i)F_{BL}(x_i) + F_{AU}(x_i)F_{BU}(x_i)])$$

$$\left(\sum_1^n T_{AL}{}^2(x_i) + T_{AU}{}^2(x_i) + I_{AL}{}^2(x_i) + I_{AU}{}^2(x_i) + F_{AL}{}^2(x_i) + F_{AU}{}^2(x_i)\right)^{\frac{-1}{2}} \cdot \left(\sum_1^n T_{BL}{}^2(x_i) + T_{BU}{}^2(x_i) + I_{BL}{}^2(x_i) + I_{BU}{}^2(x_i) + F_{BL}{}^2(x_i) + F_{BU}{}^2(x_i)\right)^{\frac{-1}{2}} \le \Big\{(\sum_{i=1}^n T_{AL}{}^2(x_i)\sum_{i=1}^n T_{BL}{}^2(x_i))^{\frac{1}{2}} + (\sum_{i=1}^n T_{AU}{}^2(x_i)\sum_{i=1}^n T_{BU}{}^2(x_i))^{\frac{1}{2}} + (\sum_{i=1}^n I_{AL}{}^2(x_i)\sum_{i=1}^n I_{BL}{}^2(x_i))^{\frac{1}{2}} + (\sum_{i=1}^n I_{AU}{}^2(x_i)\sum_{i=1}^n I_{BU}{}^2(x_i))^{\frac{1}{2}} + (\sum_{i=1}^n F_{AL}{}^2(x_i)\sum_{i=1}^n F_{BL}{}^2(x_i))^{\frac{1}{2}} + (\sum_{i=1}^n F_{AU}{}^2(x_i)\sum_{i=1}^n F_{BU}{}^2(x_i))^{\frac{1}{2}}\Big\} \cdot \{(\sum_{i=1}^n \boldsymbol{T_{AL}}{}^2(\boldsymbol{x_i}) + \sum_{i=1}^n \boldsymbol{T_{AU}}{}^2(\boldsymbol{x_i}) + \sum_{i=1}^n \boldsymbol{I_{AL}}{}^2(\boldsymbol{x_i}) + \sum_{i=1}^n \boldsymbol{I_{AU}}{}^2(\boldsymbol{x_i}) + \sum_{i=1}^n \boldsymbol{F_{AL}}{}^2(\boldsymbol{x_i}) + \sum_{i=1}^n \boldsymbol{F_{AU}}{}^2(\boldsymbol{x_i})) \cdot (\sum_{i=1}^n \boldsymbol{T_{BL}}{}^2(\boldsymbol{x_i}) +$$

$$\times \quad \sum_{i=1}^n \boldsymbol{T_{BU}}{}^2(\boldsymbol{x_i}) + \sum_{i=1}^n \boldsymbol{I_{BL}}{}^2(\boldsymbol{x_i}) + \sum_{i=1}^n \boldsymbol{I_{BU}}{}^2(\boldsymbol{x_i}) + \sum_{i=1}^n \boldsymbol{F_{BL}}{}^2(\boldsymbol{x_i}) + \sum_{i=1}^n \boldsymbol{F_{BU}}{}^2(\boldsymbol{x_i}))\}^{\frac{-1}{2}} \tag{26}$$

Let us adopt the following notations:

$$\sum_{i=1}^n T_{AL}{}^2(x_i) = a \quad \sum_{i=1}^n T_{BL}{}^2(x_i) = b \quad \sum_{i=1}^n T_{AU}{}^2(x_i) = c \quad \sum_{i=1}^n T_{BU}{}^2(x_i) = d \tag{27}$$

$$\sum_{i=1}^n I_{AL}{}^2(x_i) = e \quad \sum_{i=1}^n I_{BL}{}^2(x_i) = f \quad \sum_{i=1}^n I_{AU}{}^2(x_i) = g \quad \sum_{i=1}^n I_{BU}{}^2(x_i) = h \tag{28}$$

$$\sum_{i=1}^n F_{AL}{}^2(x_i) = i \quad \sum_{i=1}^n F_{BL}{}^2(x_i) = j \quad \sum_{i=1}^n F_{AU}{}^2(x_i) = k \quad \sum_{i=1}^n F_{BU}{}^2(x_i) = l \tag{29}$$

The above inequality is equivalent to

$$K_{INS}(A,B) \le \frac{\sqrt{ab}+\sqrt{cd}+\sqrt{ef}+\sqrt{gh}+\sqrt{ij}+\sqrt{kl}}{\sqrt{(a+c+e+g+i+k)}\sqrt{b+d+f+h+j+l}} \tag{30}$$

Then, since $K_{INS}(A,B) \ge 0$ we have

$$K_{INS}^2(A,B) \le \frac{\left(\sqrt{ab}+\sqrt{cd}+\sqrt{ef}+\sqrt{gh}+\sqrt{ij}+\sqrt{kl}\right)^2}{(a+c+e+g+i+k)(b+d+f+h+j+l)}$$

=1-

$$\Big\{\left(\sqrt{ad}-\sqrt{bc}\right)^2+\left(\sqrt{af}-\sqrt{be}\right)^2+\left(\sqrt{ah}-\sqrt{bg}\right)^2+\left(\sqrt{aj}-\left(\sqrt{bi}\right)^2+\sqrt{al}-\sqrt{bk}\right)^2+$$
$$\left(\sqrt{cf}-\sqrt{de}\right)^2+\left(\sqrt{ch}-\sqrt{dg}\right)^2+\left(\sqrt{cj}-\sqrt{di}\right)^2+\left(\sqrt{cl}-\left(\sqrt{dk}\right)^2+\sqrt{eh}-\sqrt{fg}\right)^2+$$
$$\left(\sqrt{ej}-\sqrt{fi}\right)^2+\left(\sqrt{el}-\sqrt{fk}\right)^2+\left(\sqrt{gi}-\sqrt{fk}\right)^2+\left(\sqrt{gl}-\left(\sqrt{hk}\right)^2+\sqrt{il}-\sqrt{jk}\right)^2\Big\}\times$$
$$\{(a+c+e+g+i+k)(b+d+h+j+l)\}^{-1}\leq$$
1. (31)

And thus we have $0 \leq K_{INS}(A,B) \leq 1.$ (32)

Remark 3: From the following counter-example, we can easily check that

$K_{INS}(A,B)$ = 1 but A $\neq B$. (33)

Remark 4:

Let A and B be two interval neutrosophic set defined on the universe X = {x_1}

A={ x_1: < [0.5, 0.5] [0.5, 0.5] [0.5, 0.5]>}

B={ x_1: < [0.25, 0.25] [0.25, 0.25] [0.25, 0.25]>}

$K_{INS}(A,B)$ = 1 but A$\neq B$.

3.4. Weighted Correlation Coefficient of Interval Neutrosophic Sets

In order to investigate the difference of importance considered in the elements in the universe of discourse, we need to take the weights of the elements x_i(i = 1, 2, 3, …, n). In the following we develop a weighted correlation coefficient between the interval neutrosophic sets as follows:

$$W_{INS}(A,B)=\frac{\sum_1^n w_i\{T_{AL}(x_i)\cdot T_{BL}(x_i)+T_{AU}(x_i)\cdot T_{BU}(x_i)+I_{AL}(x_i)\cdot I_{BL}(x_i)+I_{AU}(x_i)\cdot I_{BU}(x_i)+F_{AL}(x_i)\cdot F_{BL}(x_i)+F_{AU}(x_i)\cdot F_{BU}(x_i)\}}{\left(\sum_1^n w_i\left(T_{AL}{}^2(x_i)+T_{AU}{}^2(x_i)+I_{AL}{}^2(x_i)+I_{AU}{}^2(x_i)+F_{AL}{}^2(x_i)+F_{AU}{}^2(x_i)\right)\cdot\sum_1^n w_i\left(T_{BL}{}^2(x_i)+T_{BU}{}^2(x_i)+I_{BL}{}^2(x_i)+I_{BU}{}^2(x_i)+F_{BL}{}^2(x_i)+F_{BU}{}^2(x_i)\right)\right)^{1/2}}\in$$
$[0, 1^+[$ (34)

If w = $\{\frac{1}{n},\frac{1}{n},\dots,\frac{1}{n}\}$, equation (34) is reduced to the correlation coefficient (12); it is easy to check that the weighted correlation coefficient $w_{INS}(A,B)$ between INSs A and B also satisfies the properties:

(1) $0 \leq w_{INS}(A,B) \leq 1$ (35)

(2) $w_{INS}(A,B) = w_{INS}(B,A)$ (36)

(3) $w_{INS}(A,B) = 1$ if A = B (37)

3.5. Numerical Illustration.

In this section we present, an example to depict the method defined above, where the data is represented by an interval neutrosophic sets.

Example. For a finite universal set X = {x_1, x_2}, if two interval neutrosophic sets are written, respectively

A = {x_1:<[0.2, 0.3] [0.4, 0.5] [0.1, 0.2]>; x_2:<[0.3, 0.5] [0.1, 0.2] [0.4, 0.5]>}

B = {x_1:<[0.1, 0.2] [0.3, 0.4] [0.1, 0.3]>; x_2:<[0.4, 0.5] [0.2, 0.3] [0.1, 0.2]>}

Therefore, we have

$$K_{INS}(A,B) = \frac{1{,}06}{(1{,}39)^{\frac{1}{2}}(0{,}99)^{1/2}} \in [0, 1^+[$$

E(A) = 1,39

E(B) = 0,99

$$K_{INS}(A,B) = 0.90$$

It shows that the interval neutrosophic sets A and B have a good positively correlation.

Conclusion:
In this paper we introduced a method to calculate the correlation coefficient of two interval neutrosophic sets.

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Cosine Similarity Measure of Interval Valued Neutrosophic Sets

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Abstract. In this paper, we define a new cosine similarity between two interval valued neutrosophic sets based on Bhattacharya's distance [19]. The notions of interval valued neutrosophic sets (IVNS, for short) will be used as vector representations in 3D-vector space. Based on the comparative analysis of the existing similarity measures for IVNS, we find that our proposed similarity measure is better and more robust. An illustrative example of the pattern recognition shows that the proposed method is simple and effective.

Keywords: Cosine Similarity Measure; Interval Valued Neutrosophic Sets .

I. INTRODUCTION

The neutrsophic sets (**NS**), pioneered by F. Smarandache [1], has been studied and applied in different fields, including decision making problems [2, 3, 4 , 5, 23], databases [6-7], medical diagnosis problems [8] , topology [9], control theory [10], Image processing [11,12,13] and so on. The character of NSs is that the values of its membership function, non-membership function and indeterminacy function are subsets. The concept of neutrosophic sets generalizes the following concepts: the classic set, fuzzy set, interval valued fuzzy set, Intuitionistic fuzzy set, and interval valued intuitionistic fuzzy set and so on, from a philosophical point of view. Therefore, Wang et al [14] introduced an instance of neutrosophic sets known as single valued neutrosophic sets (SVNS), which were motivated from the practical point of view and that can be used in real scientific and engineering application, and provide the set theoretic operators and various properties of SVNSs. However, in many applications, due to lack of knowledge or data about the problem domains, the decision information may be provided with intervals, instead of real numbers. Thus, interval valued neutrosophic sets (IVNS), as a useful generation of NS, was introduced by Wang et al [15], which is characterized by a membership function, non-membership function and an indeterminacy function, whose values are intervals rather than real numbers. Also, the interval valued neutrosophic set can represent uncertain, imprecise, incomplete and inconsistent information which exist in the real world. As an important extension of NS, IVNS has many applications in real life [16, 17].

Many methods have been proposed for measuring the degree of similarity between neutrosophic set, S.Broumi and F. Smarandache [22] proposed several definitions of similarity measure between NS. P.Majumdar and S.K.Samanta [21] suggested some new methods for measuring the similarity between neutrosophic set. However, there is a little investigation on the similarity measure of IVNS, although some method on measure of similarity between intervals valued neutrosophic sets have been presented in [5] recently.

Pattern recognition has been one of the fastest growing areas during the last two decades because of its usefulness and fascination. In pattern recognition, on the basis of the knowledge of known pattern, our aim is to classify the unknown pattern. Because of the complex and uncertain nature of the problems. The problem pattern recognition is given in the form of interval valued neutrosophic sets.

In this paper, motivated by the cosine similarity measure based on Bhattacharya's distance [19], we propose a new method called "cosine similarity measure for interval valued neutrosophic sets. Also the proposed and existing similarity measures are compared to show that the proposed similarity measure is more reasonable than some similarity measures. The proposed similarity measure is applied to pattern recognition

This paper is organized as follow: In section 2 some basic definitions of neutrosophic set, single valued neutrosophic set, interval valued neutrosophic set and cosine similarity measure are presented briefly. In section 3, cosine similarity measure of interval valued neutrosophic sets and their proofs are introduced. In section 4, results of the proposed similarity measure and existing similarity measures are compared .In section 5, the proposed similarity measure is applied to deal with the problem related to medical diagnosis. Finally we conclude the paper.

II. PRELIMINARIE

This section gives a brief overview of the concepts of neutrosophic set, single valued neutrosophic set, interval valued neutrosophic set and cosine similarity measure.

A. *Neutrosophic Sets*

1) *Definition [1]*

Let U be an universe of discourse then the neutrosophic set A is an object having the form

A = {< x: $T_A(x)$, $I_A(x)$, $F_A(x)$>, x ∈ U}, where the functions T, I, F : U→]−0, 1+[define respectively the degree of membership (or Truth) , the degree of indeterminacy, and the degree of non-membership (or Falsehood) of the element x ∈ U to the set A with the condition.

$$^{-}0 \leq T_A(x)+I_A(x)+F_A(x) \leq 3^{+}. \qquad \textbf{(1)}$$

From philosophical point of view, the neutrosophic set takes the value from real standard or non-standard subsets of $]^{-}0, 1^{+}[$. So instead of] $-0, 1^{+}[$ we need to take the interval [0, 1] for technical applications, because $]^{-}0, 1^{+}[$will be difficult to apply in the real applications such as in scientific and engineering problems.

For two NS, A_{NS} = {<x,$T_A(x)$, $I_A(x)$, $F_A(x)$> | x ∈ X}

And B_{NS} = {<x, $T_B(x)$, $I_B(x)$, $F_B(x)$> | x ∈ X} the two relations are defined as follows:

(1) $A_{NS} \subseteq B_{NS}$ If and only if $T_A(x) \leq T_B(x)$, $I_A(x) \geq I_B(x)$, $F_A(x) \geq F_B(x)$

(2) $A_{NS} = B_{NS}$ if and only if, $T_A(x)$=$T_B(x)$, $I_A(x)$ =$I_B(x)$, $F_A(x)$ =$F_B(x)$

B. *Single Valued Neutrosophic Sets*

1) Definition [14]

Let X be a space of points (objects) with generic elements in X denoted by x. An SVNS A in X is characterized by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity-membership function $F_A(x)$, for each point x in X, $T_A(x)$, $I_A(x)$, $F_A(x)$ ∈ [0, 1].

When X is continuous, an SVNS A can be written as

$$A=\int_X \frac{<T_A(x), I_A(x), F_A(x),>}{x}, x \in X. \qquad \textbf{(2)}$$

When X is discrete, an SVNS A can be written as

$$A=\sum_1^n \frac{<T_A(x_i), I_A(x_i), F_A(x_i),>}{x_i}, x_i \in X \qquad \textbf{(3)}$$

For two SVNS, A_{SVNS} = {<x,$T_A(x)$, $I_A(x)$, $F_A(x)$> | x ∈ X}

And B_{SVNS} ={ <x, $T_B(x)$, $I_B(x)$, $F_B(x)$> | x ∈ X} the two relations are defined as follows:

(1) $A_{SVNS} \subseteq B_{SVNS}$ if and only if $T_A(x) \leq T_B(x)$, $I_A(x) \geq I_B(x)$, $F_A(x) \geq F_B(x)$

(2) $A_{SVNS} = B_{SVNS}$ if and only if , $T_A(x)$ =$T_B(x)$, $I_A(x)$ =$I_B(x)$, $F_A(x)$ =$F_B(x)$ for any x ∈ X.

C. *Interval Valued Neutrosophic Sets*

1) Definition [15]

Let X be a space of points (objects) with generic elements in X denoted by x. An interval valued neutrosophic set (for short IVNS) A in X is characterized by truth-membership function $T_A(x)$, indeteminacy-membership function $I_A(x)$ and falsity-membership function $F_A(x)$. For each point x in X, we have that$T_A(x)$, $I_A(x)$, $F_A(x)$ [0, 1] .

For two IVNS, $_{IVNS}$ ={<x, [$T_A^L(x)$, $T_A^U(x)$] , [$F_A^L(x)$ $F_A^U(x)$] [$I_A^L(x)$, $I_A^U(x)$] > | x ∈ X}

And B_{IVNS}= {<x, [$T_B^L(x)$,$T_B^U(x)$], [$F_B^L(x)$, $F_B^U(x)$], [$I_B^L(x)$, $I_B^U(x)$]>| x ∈ X} the two relations are defined as follows:

(1) $A_{IVNS} \subseteq B_{IVNS}$ if and only if $T_A^L(x) \leq T_B^L(x)$,$T_A^U(x) \leq T_B^U(x)$, $I_A^L(x) \geq I_B^L(x)$, $I_A^U(x) \geq I_B^U(x)$, $F_A^L(x) \geq F_B^L(x)$, $F_A^U(x) \geq F_B^U(x)$.

(2) $A_{IVNS} = B_{IVNS}$ if and only if , $T_A^L(x)$=$T_B^L(x)$, $T_A^U(x)$ =$T_B^U(x)$, $I_A^L(x)$=$I_B^L(x)$, $I_A^U(x)$ =$I_B^U(x)$, $F_A^L(x)$=$F_B^L(x)$, $F_A^U(x)$ =$F_B^U(x)$ for any x ∈ X.

D. *Cosine Similarity*

1) Definition

Cosine similarity is a fundamental angle-based measure of similarity between two vectors of n dimensions using the cosine of the angle between them Candan and Sapino [20]. It measures the similarity between two vectors based only on the direction, ignoring the impact of the distance between them. Given two vectors of attributes, X = (x_1, x_2, … , x_n) and Y= (y_1, y_2, … , y_n), the cosine similarity, cosθ, is represented using a dot product and magnitude as

$$\mathbf{Cos\theta} = \frac{\sum_{i=1}^{n} x_i y_i}{\sqrt{\sum_{i=1}^{n} x_i^2}\sqrt{\sum_{i=1}^{n} y_i^2}} \qquad \textbf{(4)}$$

In vector space, a cosine similarity measure based on Bhattacharya's distance [19] between two fuzzy set $\mu_A(x_i)$ and $\mu_B(x_i)$ defined as follows:

$$C_F(A,B) = \frac{\sum_{i=1}^{n} \mu_A(x_i)\ \mu_B(x_i)}{\sqrt{\sum_{i=1}^{n} \mu_A(x_i)^2}\sqrt{\sum_{i=1}^{n} \mu_B(x_i)^2}} \qquad \textbf{(5)}$$

The cosine of the angle between the vectors is within the values between 0 and 1.

In 2-D vector space, J. Ye [18] defines cosine similarity measure between IFS as follows:

$$C_{IFS}(A,B) = \frac{\sum_{i=1}^{n} \mu_A(x_i)\ \mu_B(x_i)+\nu_A(x_i)\nu_B(x_i)}{\sqrt{\sum_{i=1}^{n} \mu_A(x_i)^2+\nu_A(x_i)^2}\sqrt{\sum_{i=1}^{n} \mu_B(x_i)^2+\nu_B(x_i)^2}} \qquad \textbf{(6)}$$

III . COSINE SIMILARITY MEASURE FOR INTERVAL VALUED NEUTROSOPHIC SETS.

The existing cosine similarity measure is defined as the inner product of these two vectors divided by the product of their lengths. The cosine similarity measure is the cosine of the angle between the vector representations of the two fuzzy sets. The cosine similarity measure is a classic measure used in information retrieval and is the most widely reported measures of vector similarity [19]. However, to the best of our Knowledge, the existing cosine similarity measures does not deal with interval valued neutrosophic sets. Therefore, to overcome this limitation in this section, a new cosine similarity measure between interval valued neutrosophic sets is proposed in 3-D vector space.

Let A be an interval valued neutrosophic sets in a universe of discourse X ={x}, the interval valued neutrosophic sets is characterized by the interval of membership [$T_A^L(x)$, $T_A^U(x)$] ,the interval degree of non-membership [$F_A^L(x)$, $F_A^U(x)$] and the interval degree of indeterminacy [$I_A^L(x)$, $I_A^U(x)$] which can be considered as a vector representation with the three elements. Therefore, a cosine similarity measure for interval neutrosophic sets is proposed in an analogous manner to the cosine similarity measure proposed by J. Ye [18].

E. Definition

Assume that there are two interval neutrosophic sets A and B in X $=\{x_1, x_2, \ldots, x_n\}$ Based on the extension measure for fuzzy sets, a cosine similarity measure between interval valued neutrosophic sets A and B is proposed as follows

$$C_N(A,B)=\frac{1}{n}\sum_{i=1}^{n}\frac{(T_A^L(x_i)+T_A^U(x_i))(T_B^L(x_i)+T_B^U(x_i))+(I_A^L(x_i)+I_A^U(x_i))(I_B^L(x_i)+I_B^U(x_i))+(F_A^L(x_i)+F_A^U(x_i))(F_B^L(x_i)+F_B^U(x_i))}{\sqrt{(T_A^L(x_i)+T_A^U(x_i))^2+(I_A^L(x_i)+I_A^U(x_i))^2+(F_A^L(x_i)+F_A^U(x_i))^2}\sqrt{(T_B^L(x_i)+T_B^U(x_i))^2+(I_B^L(x_i)+I_B^U(x_i))^2+(F_B^L(x_i)+F_B^U(x_i))^2}} \quad \textbf{(7)}$$

F. Proposition

Let A and B be interval valued neutrosophic sets then

- **i.** $\mathbf{0 \le} C_N(A,B) \mathbf{\le 1}$
- ii. $C_N(A,B) = C_N(B,A)$
- iii. $C_N(A,B) = 1$ if A= B i.e

$T_A^L(x_i) = T_B^L(x_i)$, $T_A^U(x_i) = T_B^U(x_i)$,
$I_A^L(x_i) = I_B^L(x_i)$, $I_A^U(x_i) = I_B^U(x_i)$ and
$F_A^L(x_i) = F_B^L(x_i)$, $F_A^U(x_i) = F_B^U(x_i)$ for i=1,2,…., n

Proof : (i) it is obvious that the proposition is true according to the cosine valued

(ii) it is obvious that the proposition is true.

(iii) when A =B, there are
$T_A^L(x_i) = T_B^L(x_i)$, $T_A^U(x_i) = T_B^U(x_i)$,
$I_A^L(x_i) = I_B^L(x_i)$, $I_A^U(x_i) = I_B^U(x_i)$ and
$F_A^L(x_i) = F_B^L(x_i)$ $F_A^U(x_i) = F_B^U(x_i)$ for i=1,2,…, n , So there is $C_N(A,B) = 1$

If we consider the weights of each element x_i, a weighted cosine similarity measure between IVNSs A and B is given as follows:

$$C_{WN}(A,B)=\frac{1}{n}\sum_{i=1}^{n}w_i\frac{(T_A^L(x_i)+T_A^U(x_i))(T_B^L(x_i)+T_B^U(x_i))+(I_A^L(x_i)+I_A^U(x_i))(I_B^L(x_i)+I_B^U(x_i))+(F_A^L(x_i)+F_A^U(x_i))(F_B^L(x_i)+F_B^U(x_i))}{\sqrt{(T_A^L(x_i)+T_A^U(x_i))^2+(I_A^L(x_i)+I_A^U(x_i))^2+(F_A^L(x_i)+F_A^U(x_i))^2}\sqrt{(T_B^L(x_i)+T_B^U(x_i))^2+(I_B^L(x_i)+I_B^U(x_i))^2+(F_B^L(x_i)+F_B^U(x_i))^2}} \quad \textbf{(8)}$$

Where $w_i \in [0.1]$,i =1,2,…,n ,and $\sum_{i=1}^{n} w_i = 1$.

If we take $w_i = \frac{1}{n}$, i =1,2,…,n , then there is $C_{WN}(A,B) = C_N(A,B)$.

The weighted cosine similarity measure between two IVNSs A and B also satisfies the following properties:

- **i.** $\mathbf{0 \le} C_{WN}(A,B) \mathbf{\le 1}$
- ii. $C_{WN}(A,B) = C_{WN}(B,A)$
- iii. $C_{WN}(A,B) = 1$ if A= B i.e

$T_A^L(x_i) = T_B^L(x_i)$, $T_A^U(x_i) = T_B^U(x_i)$, $I_A^L(x_i) = I_B^L(x_i)$, $I_A^U(x_i) = I_B^U(x_i)$ and $F_A^L(x_i) = F_B^L(x_i)$, $F_A^U(x_i) = F_B^U(x_i)$ for i=1,2,…, n

G. Proposition

Let the distance measure of the angle as **d(A,B)= arcos ($\boldsymbol{C_N(A,B)}$)** then it satisfies the following properties.

- **i.** d(A, B) ≥ 0, if $\mathbf{0 \le} C_s(A,B) \mathbf{\le 1}$
- **ii.** d(A, B) = **arcos(1) = 0, if** $\boldsymbol{C_N(A,B) = 1}$
- **iii.** d(A, B) = d(B, A) if $C_N(A,B) = C_N(B,A)$
- **iv.** d(A, C) ≤ d(A, B) + d(B, C) if A ⊆ B ⊆ C for any interval valued neutrosophic sets C.

Proof : obviously, d(A,B) satisfies the (i) – (iii). In the following , d(A,B) will be proved to satisfy the (iv).

For any C = { x_i}, A ⊆ B ⊆ C since Eq (7) is the sum of terms. Let us consider the distance measure of the angle between vectors:

d_i(A(x_i), B(x_i)) = **arcos($\boldsymbol{C_N(A(x_i), B(x_i))}$),**
d_i(B(x_i), C(x_i)) = **arcos($\boldsymbol{C_N(B(x_i), C(x_i))}$) and**
d_i(A(x_i), C(x_i)) = **arcos($\boldsymbol{C_N(A(x_i), C(x_i))}$),** for i=1, 2, .., n, where

$$C_N(A,B)=\frac{1}{n}\sum_{i=1}^{n}\frac{(T_A^L(x_i)+T_A^U(x_i))(T_B^L(x_i)+T_B^U(x_i))+(I_A^L(x_i)+I_A^U(x_i))(I_B^L(x_i)+I_B^U(x_i))+(F_A^L(x_i)+F_A^U(x_i))(F_B^L(x_i)+F_B^U(x_i))}{\sqrt{(T_A^L(x_i)+T_A^U(x_i))^2+(I_A^L(x_i)+I_A^U(x_i))^2+(F_A^L(x_i)+F_A^U(x_i))^2}\sqrt{(T_B^L(x_i)+T_B^U(x_i))^2+(I_B^L(x_i)+I_B^U(x_i))^2+(F_B^L(x_i)+F_B^U(x_i))^2}} \quad \textbf{(9)}$$

$$C_N(B,C)=\frac{1}{n}\sum_{i=1}^{n}\frac{(T_B^L(x_i)+T_B^U(x_i))(T_C^L(x_i)+T_C^U(x_i))+(I_B^L(x_i)+I_B^U(x_i))(I_C^L(x_i)+I_C^U(x_i))+(F_B^L(x_i)+F_B^U(x_i))(F_C^L(x_i)+F_C^U(x_i))}{\sqrt{(T_B^L(x_i)+T_B^U(x_i))^2+(I_B^L(x_i)+I_B^U(x_i))^2+(F_B^L(x_i)+F_B^U(x_i))^2}\sqrt{(T_C^L(x_i)+T_C^U(x_i))^2+(I_C^L(x_i)+I_C^U(x_i))^2+(F_C^L(x_i)+F_C^U(x_i))^2}} \quad \textbf{(10)}$$

$$C_N(A,C)=\frac{1}{n}\sum_{i=1}^{n}\frac{(T_A^L(x_i)+T_A^U(x_i))(T_C^L(x_i)+T_C^U(x_i))+(I_A^L(x_i)+I_A^U(x_i))(I_C^L(x_i)+I_C^U(x_i))+(F_A^L(x_i)+F_A^U(x_i))(F_C^L(x_i)+F_C^U(x_i))}{\sqrt{(T_A^L(x_i)+T_A^U(x_i))^2+(I_A^L(x_i)+I_A^U(x_i))^2+(F_A^L(x_i)+F_A^U(x_i))^2}\sqrt{(T_C^L(x_i)+T_C^U(x_i))^2+(I_C^L(x_i)+I_C^U(x_i))^2+(F_C^L(x_i)+F_C^U(x_i))^2}} \quad \textbf{(11)}$$

For three vectors

A(x_i) = <x_i, [$T_A^L(x_i)$, $T_A^U(x_i)$], [$I_A^L(x_i)$, $I_A^U(x_i)$], [$F_A^L(x_i)$, $F_A^U(x_i)$] >

B(x_i) = < x_i, [$T_B^L(x_i)$, $T_B^U(x_i)$],[$I_B^L(x_i)$, $I_B^U(x_i)$], [$F_B^L(x_i)$, $F_B^U(x_i)$] >

C(x_i) = <x_i, [$T_C^L(x_i)$, $T_C^U(x_i)$], [$I_C^L(x_i)$, $I_C^U(x_i)$] , [$F_C^L(x_i)$, $F_C^U(x_i)$] >, in a plane , If A $(x_i) \subseteq$ B $(x_i) \subseteq$ C (x_i) (I =1, 2,…, n), then it is obvious that d(A(x_i), C(x_i)) ≤ d(A(x_i), B(x_i)) + d(B(x_i), C(x_i)), According to the triangle inequality. Combining the inequality with Eq (7), we can obtain d(A, C) ≤ d(A, B) + d(B, C)

Thus, d(A,B) satisfies the property (iv). So we have finished the proof.

IV. Comparison of New Similarity Measure with the Existing Measures.

Let A and B be two interval neutrosophic set in the universe of discourse X={x_1, x_2,.,x_n}. For the cosine similarity and the existing similarity measures of interval valued neutrosophic sets introduced in [5, 21], they are listed as follows:

Pinaki's similarity I [21]

$$S_{PI}=\frac{\sum_{i=1}^{n}\{\min\{T_A(x_i),T_B(x_i)\}+\min\{I_A(x_i),I_B(x_i)\}+\min\{F_A(x_i),F_B(x_i)\}\}}{\sum_{i=1}^{n}\{\max\{T_A(x_i),T_B(x_i)\}+\max\{I_A(x_i),I_B(x_i)\}+\max\{F_A(x_i),F_B(x_i)\}\}} \quad \textbf{(12)}$$

Also ,P. Majumdar [21] proposed weighted similarity measure for neutrosophic set as follows:

$$S_{PII}=\frac{\sum_{i=1}^{n} w_i\,(T_A(x_i)\cdot T_B(x_i)+(I_A(x_i)\cdot I_B(x_i)+(F_A(x_i)\cdot F_B(x_i))}{\max(w_i\sqrt{T_A(x_i)^2+I_A(x_i)^2+F_A(x_i)^2}\,,\,w_i\sqrt{T_B(x_i)^2+I_B(x_i)^2+F_B(x_i)^2}))} \quad \textbf{(13)}$$

Where, S_{PI}, S_{PII} denotes Pinaki's similarity I and Pinaki's similarity II

Ye's similarity [5] is defined as the following:

$$S_{ye}(A,B)=1-\frac{1}{6}\sum_{i=1}^{n} w_i\,[|\inf T_A(x_i)-\inf T_B(x_i)|+|\sup T_A(x_i)-\sup T_B(x_i)|+|\inf I_A(x_i)-\inf I_B(x_i)|+|\sup I_A(x_i)-\sup I_B(x_i)|+|\inf F_A(x_i)-\inf F_B(x_i)|+|\sup F_A(x_i)-\sup F_B(x_i)|] \quad (14)$$

Example 1:

Let A = {<x, (0.2, 0.2 0.3)>} and B= {<x, (0.5, 0.2 0.5)>}

Pinaki similarity I = 0.58

Pinaki similarity II (with w_i =1) = 0.29

Ye similarity (**w**ith w_i =1) = 0.83

Cosine similarity $\mathbf{C_N(A,B)}$ = 0.95

Example 2:

Let A= {<x, ([0.2, 0.3], [0.5, 0.6] ,[0.3, 0.5])>} and B{<x, ([0.5, 0.6], [0.3, 0.6] ,[0.5, 0.6])>}

Pinaki similarty I = NA

Pinaki similarty II(With w_i =1) = NA

Ye similarity (With w_i =1) =0.81

Cosine similarity $\mathbf{C_N(A,B)}$ = 0.92

On the basis of computational study. J.Ye [5] have shown that their measure is more effective and reasonable .A similar kind of study with the help of the proposed new measure based on the cosine similarity, has been done and it is found that the obtained results are more refined and accurate. It may be observed from the example 1 and 2 that the values of similarity measures are more closer to 1 with $\mathbf{C_N(A,B)}$,the proposed similarity measure. This implies that we may be more deterministic for correct diagnosis and proper treatment.

V. Application of Cosine Similarity Measure for Interval Valued Neutrosophic Numbers to Pattern Recognition

In order to demonstrate the application of the proposed cosine similarity measure for interval valued neutrosophic numbers to pattern recognition, we discuss the medical diagnosis problem as follows:

For example the patient reported temperature claiming that the patient has temperature between 0.5 and 0.7 severity /certainty, some how it is between 0.2 and 0.4 indeterminable if temperature is cause or the effect of his current disease. And it between 0.1 and 0.2 sure that temperature has no relation with his main disease. This piece of information about one patient and one symptom may be written as:

(patient , Temperature) = <[0.5, 0.7], [0.2 ,0.4], [0.1, 0.2]>
(patient , Headache) = < [0.2, 0.3], [0.3 ,0.5], [0.3, 0.6]>
(patient , Cough) = <[0.4, 0.5], [0.6 ,0.7], [0.3, 0.4]>

Then, P= {< x_1, [0.5, 0.7], [0.2 ,0.4], [0.1, 0.2] >, < x_2, [0.2, 0.3], [0.3, 0.5], [0.3, 0.6] > ,< x_3, [0.4, 0.5], [0.6 ,0.7], [0.3, 0.4]>}

And each diagnosis A_i (i=1, 2, 3) can also be represented by interval valued neutrosophic numbers with respect to all the symptoms as follows:

A_1= {< x_1, [0.5, 0.6], [0.2 ,0.3], [0.4, 0.5] >, < x_2 , [0.2 , 0.6], [0.3 ,0.4], [0.6 , 0.7]>,< x_3, [0.1, 0.2], [0.3 ,0.6], [0.7, 0.8]>}

A_2= {< x_1, [0.4, 0.5], [0.3, 0.4], [0.5, 0.6] >, < x_2 , [0.3, 0.5], [0.4 ,0.6], [0.2, 0.4]> ,< x_3 , [0.3, 0.6], [0.1, 0.2], [0.5, 0.6]>}

A_3= {< x_1, [0.6, 0.8], [0.4 ,0.5], [0.3, 0.4]>, <x_2 , [0.3, 0.7], [0.2, 0.3], [0.4, 0.7]> ,< x_3, [0.3, 0.5], [0.4, 0.7], [0.2, 0.6]>}

Our aim is to classify the pattern P in one of the classes A_1, A_2, $_3$ .According to the recognition principle of maximum degree of similarity measure between interval valued neutrosophic numbers, the process of diagnosis A_k to patient P is derived according to

$$k = \arg\max\{\, C_N(A_i, \boldsymbol{P})\}$$

from the previous formula (7) , we can compute the cosine similarity between A_i (i=1, 2, 3) and P as follows;

$\boldsymbol{C_N}(A_1, \boldsymbol{P})$**=0.8988,** $\boldsymbol{C_N}(A_2, \boldsymbol{P})$**=0.8560,** $\boldsymbol{C_N}(A_3, \boldsymbol{P})$ **=0.9654**

Then, we can assign the patient to diagnosis A_3 (Typoid) according to recognition of principal.

VI. Conclusions.

In this paper a cosine similarity measure between two and weighted interval valued neutrosophic sets is proposed. The results of the proposed similarity measure and existing similarity measure are compared. Finally, the proposed cosine similarity measure is applied to pattern recognition.

ACKNOWLEDGMENT

The authors are very grateful to the anonymous referees for their insightful and constructive comments and suggestions, which have been very helpful in improving the paper.

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Published in *Neutrosophic Sets and Systems*, Vol. 5, 15-20, 2014.

Distance and Similarity Measures of Interval Neutrosophic Soft Sets

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Abstract: In this paper several distance and similarity measures of interval neutrosophic soft sets are introduced. The measures are examined based on the geometric model, the set theoretic approach and the matching function. Finally, we have successfully shown an application of this similarity measure of interval neutrosophic soft sets.

Keywords: Distance, Similarity Measure, Neutrosophic set, Interval Neutrosohic sets, Interval Neutrosohic Soft sets.

1. Introduction

In 1965, fuzzy set theory was firstly given by Zadeh [2] which is applied in many real applications to handle uncertainty.Then,interval-valued fuzzy set [3],intuitionisticfuzzy set theory[4] and interval valued intuitionistic fuzzy sets[5] was introduced by Türkşen, Atanassov and Atanassov and Gargov, respectively. This theories can only handle incomplete information not the indeterminate information and inconsistent information which exists commonly in belief systems. So, Neutrsophic sets, founded by F.Smarandache [1], has capapility to deal with uncertainty, imprecise, incomplete and inconsistent information which exist in real world from philosophical point of view. The theory is a powerful tool formal framework which generalizes the concept of the classic set, fuzzy set [2], interval-valued fuzzy set [3], intuitionistic fuzzy set [4] interval-valued intuitionistic fuzzy set [5], and so on.

In the actual applications, sometimes, it is not easy to express the truth-membership, indeterminacy-membership and falsity-membership by crisp value, and they may be easier to expressed by interval numbers. The neutrosophic set and their operators need to be specified from scientific or engineering point of view. So, after the pioneering work of Smarandache, in 2005, Wang [6] proposed the notion of interval neutrosophic set (INS for short) which is another extension of neutrosophic sets. INS can be described by a membership interval, a non-membership interval and indeterminate interval, thus the interval value (INS) has the virtue of complementing NS, which is more flexible and practical than neutrosophic set. The sets provides a more reasonable mathematical framework to deal with indeterminate and inconsistent information.A lot of works about neutrosophic set theory have been studied by several researches [7,11,13,14,15,16,17,18,19,20].

In 1999, soft theory was introduced byMolodtsov[45] as a completely new mathematical toolfor modeling uncertainties. After Molodsov, based on the several operations on softsets introduced in [33,34,35,36,46],some more properties and algebra may be found in [32,34]. We can found some new conceptc ombined with fuzzy set in [28,29,37,39,42], interval-valued fuzzy set in [38], intuitionistic fuzzy set in [50], rough set in [43,47], interval-valued intuitionistic fuzzy set in [45], neutrosophic set in [8,9,27], interval neutrosophic set [31].

Also in some problems it is often needed to compare two sets such as fuzzy, soft, neutrosophic etc. Therefore, some researchers has studied of similarity measurement between fuzzysets in [24,48], interval valued fuzzy in [48], neutrosophic set in [23,26], interval neutrosophic set in [10,12].Recently similarity measure of softsets [40,49], intuitionistic fuzzy soft sets [30]was studied. Similarity measure between two sets such as fuzzy, soft has been defined by many authors which are based on both distances and matching function. The significant differences between similarity measure based on matching function and similarity measure based on distance is that if intersection of the two sets equals empty, then between similarity measure based on matching function the two sets is zero in but similarity measure based on distance may not be equal to zero. Distance-based measures are also popular because it is easier to calculate the intermediate distance between two fuzzy sets or soft sets. It's mentioned in [40]. In this paper several distance and similarity measures of interval neutrosophic soft sets are introduced. The measures are examined based on the geometric model, the set-theoretic approach and the matching function. Finally, we give an application for similarity measures of interval neutrosophic soft sets

2. Prelimiairies

This section gives a brief overview of concepts of neutrosophic set [1], and interval valued neutrosophic set [6], soft set [41],neutrosophic soft set [27] and interval valued neutrosophic soft set [31]. More detailed explanations related to this subsection may be found in [8,9,27,31,36].

Definition 2.1[1]Neutrosophic Sets

Let X be an universe of discourse, with a generic element in X denoted by x, the neutrosophic (NS) set is an object having the form

$A = \{< x: T_A(x), I_A(x), F_A(x)>, x \in X\}$, where the functions T, I, F : $X \to]^-0, 1^+[$ define respectively the degree of membership (or Truth) , the degree of indeterminacy, and the degree of non-membership (or Falsehood) of the element $x \in X$ to the set A with the condition.

$$^-0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3^+. \quad (1)$$

From philosophical point of view, the neutrosophic set takes the value from real standard or non-standard subsets of $]^-0, 1^+[$. So instead of $]-0, 1^+[$ we need to take the interval [0, 1] for technical applications, because $]^-0, 1^+[$ will be difficult to apply in the real applications such as in scientific and engineering problems.

For two NS ,A_{NS} ={ <x , $T_A(x)$, $I_A(x)$, $F_A(x)$> | x ∈ X } (2)

And B_{NS}={<x , $T_B(x)$, $I_B(x)$, $F_B(x)$> | x ∈ X } the two relations are defined as follows:

(1)$A_{NS} \subseteq B_{NS}$ if and only if $T_A(x) \le T_B(x)$,$I_A(x) \ge I_B(x)$,$F_A(x) \ge F_B(x)$

(2)$A_{NS} = B_{NS}$ if and only if , $T_A(x) = T_B(x)$,$I_A(x) = I_B(x)$,$F_A(x) = F_B(x)$

Definition 2.2 [6] Interval Valued Neutrosophic Sets

Let X be a universe of discourse, with generic element in X denoted by x. An interval valued neutrosophic set (for short IVNS) A in X is characterized by truth-membership function $T_A(x)$, indeteminacy-membership function $I_A(x)$ and falsity-membership function $F_A(x)$. For each point x in X, we have that $T_A(x), I_A(x), F_A(x) \in [0,1]$.

For two IVNS , A_{IVNS} ={ <x , $[T_A^L(x), T_A^U(x)]$, $[I_A^L(x), I_A^U(x)]$, $[F_A^L(x), F_A^U(x)]$> | x ∈ X } (3)

And B_{IVNS} ={<x , $[T_B^L(x), T_B^U(x)]$, $[I_B^L(x), I_B^U(x)]$, $[F_B^L(x), F_B^U(x)]$ > | x ∈ X } the two relations are defined as follows:

(1)$A_{\mathrm{IVNS}} \subseteq B_{\mathrm{IVNS}}$ if and only if $T_A^L(x) \le T_B^L(x)$,$T_A^U(x) \le T_B^U(x)$,$I_A^L(x) \ge I_B^L(x)$,$I_A^U(x) \ge I_B^U(x)$, $F_A^L(x) \ge F_B^L(x)$,$F_A^U(x) \ge F_B^U(x)$

(2)$A_{\mathrm{IVNS}} = B_{\mathrm{IVNS}}$ if and only if , $T_A^L(x_i) = T_B^L(x_i)$,$T_A^U(x_i) = T_B^U(x_i)$,$I_A^L(x_i) = I_B^L(x_i)$, $I_A^U(x_i) = I_B^U(x_i)$,$F_A^L(x_i) = F_B^L(x_i)$ and $F_A^U(x_i) = F_B^U(x_i)$ for any x ∈ X.

The complement of A_{IVNS} is denoted by A_{IVNS}^o and is defined by

A_{IVNS}^o={<x , $[F_A^L(x), F_A^U(x)]$> , $[1 - I_A^U(x), 1 - I_A^L(x)]$, $[T_A^L(x), T_A^U(x)]$| x ∈ X }

Definition 2.3 [41] Soft Sets

Let U be an initial universe set and E be a set of parameters. Let P(U) denotes the power set of U. Consider a nonempty set A, A ⊂ E. A pair (F, A) is called a soft set over U, where F is a mapping given by F: A → P (U).

It can be written a set of ordered pairs(F, A) = {(x, F (x)): x∈A}.

As an illustration, let us consider the following example.

Example 1 Suppose that U is the set of houses under consideration, say U = {h_1, h_2, . . ., h_5}. Let E be the set of some attributes of such houses, say E = {e_1, e_2, . . ., e_6}, where e_1, e_2, . . ., e_6 stand for the attributes “expensive”, “beautiful”, “wooden”, “cheap”, “modern”, and “in bad repair”, respectively.

In this case, to define a soft set means to point out expensive houses, beautiful houses, and so on. For example, the soft set (F,A) that describes the “attractiveness of the houses” in the opinion of a buyer, say Thomas, may be defined like this:

A={e_1,e_2,e_3,e_4,e_5};

F(e_1) = {h_2, h_3, h_5}, F(e_2) = {h_2, h_4}, F(e_3) = {h_1}, F(e_4) = U, F(e_5) = {h_3, h_5}.

Definition 2.4 Neutrosophic soft Sets [27]

Let U be an initial universe set and E be a set of parameters. Consider A⊆E. LetN(U) denotes the set of all neutrosophic sets of U. Thecollection (F,A) is termed to be the soft neutrosophic set over Udenoted by N, where F is amappinggivenbyF : A→P(U).

It can be written a set of ordered pairs N= {(x, F (x)): x∈A}.

Definition 2.5 Interval Valued Neutrosophic Soft Sets [31]

Let U be an universe set, IVN(U) denotes the set of all interval valued neutrosophic sets of U and E be a set of parameters that are describe the elements of U. Thecollection (K, E) is termed to be the interval valued neutrosophic soft sets (ivn-soft sets)over U denoted byϒ, where K is a mapping given by K : E→IVN(U).

It can be written a set of ordered pairs

ϒ= {(x, K(x)): x∈E}

Here, K which is interval valued neutrosophic sets, is called approximate function of the ivn-soft sets ϒ and K(x) is called x-approximate value of x ∈E.

Generally, K, L, M,... will be used as an approximate functions of ϒ, Ψ , Ω… respectively.

Note that the sets of all ivn-soft sets over U will be denoted by IVNS(U).

Then a relation form of ϒ is defined by R_K= { (r_K(e,u)/(e, u)) : u∈U, e∈E}
where
r_K: ExU→IV NS(U) and r_K (e_i,u_j)=$a_{ij}$for all e_i ∈ E and u_j ∈U.

Here,
1. ϒ is an ivn-soft subset of Ψ, denoted by ϒ ⋐ Ψ, if K(e) ⊆L(e) for alle∈E.
2. ϒ is an ivn-soft equals toΨ, denoted by ϒ = Ψ, if K(e)=L(e) for all e∈E.
3. The complement of ϒ is denoted byϒ^c, and is defined by ϒ^c = {(x, K^o (x)): x∈E}

As an illustration for ivn-soft, let us consider the following example.

Example 2. Suppose that U is the set of houses under consideration, say U = {h_1, h_2, h_3}. Let E be the set of some attributes of such houses, say E = {e_1, e_2, e_3, e_4}, where e_1, e_2, . . ., e_6 stand for the attributes "expensive", "beautiful", "wooden", "cheap", "modern", and "in bad repair", respectively.
In this case we give an ivn-soft set as;

ϒ= { (e_1,{<h_1,[0.5, 0.6], [0.6 ,0.7],[0.3,0.4]> ,<h_2, [0.5, 0.6], [0.6 ,0.7],[0.3,0.4]> ,

<h_3, [0.5, 0.6], [0.6,0.7],[0.3,0.4]> }),(e_2,{<h_1,[0.2, 0.3], [0.5 ,0.6],[0.3,0.6]> ,

<h_2, [0.4, 0.6], [0.2 ,0.3],[0.2,0.3]> , <h_3, [0.5, 0.6], [0.6 ,0.7],[0.3,0.4]> }),

(e_3,{<h_1,[0.3, 0.4], [0.1 ,0.5],[0.2,0.4]>,<h_2, [0.2, 0.5], [0.3,0.4],[0.4,0.5]>,

<h_3, [0.5, 0.6], [0.6,0.7],[0.3,0.4]> }),(e_4,{<h_1,[0.4, 0.6], [0.3 ,0.5],[0.3,0.4]>,

<h_2, [0.4, 0.6], [0.2 ,0.3],[0.2,0.3]>) ,<h_3, [0.3, 0.4], [0.2,0.7],[0.1,0.4]>}) }

Definition 2.6 (*Distance axioms)*

Let E be a set of parameters. Suppose that ϒ = <K,E>, Ψ = <L,E> and Ω = <M,E>; are three ivn-soft sets in universe U. Assume d is a mapping,

d :IVNS(U) x IVNS(U) ⟶ [0, 1].If d satisfies the following properties ((1)-(4)) :

(1) d (ϒ, Ψ) ≥ 0;

(2) d (ϒ, Ψ) = d (Ψ, ϒ) ;

(3) d (ϒ, Ψ) = 0 iff Ψ = ϒ;

(4) d (ϒ,Ψ) + d (Ψ, Ω) ≥ d (ϒ,Ω) .

Hence d(ϒ,Ψ) is called a distance measure between ivn-soft sets ϒ and Ψ.

Definition 2.7(*similarity axioms)*

A real function S: INS(U) x INS(U) ⟶ [0, 1] is named a similarity measure between two ivn-soft set ϒ=(K,E) and Ψ =(M,E) if S satisfies all the following properties:

(1) S (ϒ, Ψ) ∈ [0, 1];

(2) S(ϒ, ϒ)=S(Ψ, Ψ) = 1;

(3) S(ϒ, Ψ) = S(Ψ, ϒ);

(4) S (ϒ, Ω) ≤ S (ϒ,Ψ) and S (ϒ, Ω) ≤ S (Ψ,Ω) if ϒ ⊆ Ψ ⊆ Ω

Hence S(ϒ,Ψ) is called a similarity measure between ivn-soft setsϒ and Ψ.

For more details on the algebra and operations on interval neutrosophic set and soft set and interval neutrosophic soft set, the reader may refer to [5,6,8,9,12, 31,45,52].

3. Distance Measure between Interval Valued Neutrosophic Soft Sets

In this section, we present the definitions of the Hamming and Euclidean distances between ivn-soft sets and the similarity measures between ivn-soft sets based on the distances, which can be used in real scientific and engineering applications.

Based on Hamming distance between two interval neutrosophic set proposed by Ye[12] as follow:

D (A,B)=$\frac{1}{6}\sum_{i=1}^{n}\left[\left|T_A^L(x_i)-T_B^L(x_i)\right|+\left|T_A^U(x_i)-T_B^U(x_i)\right|+\left|I_A^L(x_i)-I_B^L(x_i)\right|+\left|I_A^U(x_i)-I_B^U(x_i)\right|+\left|F_A^L(x_i)-F_B^L\right|+\left|F_A^L(x_i)-F_B^U(x_i)\right|\right]$

We extended it to the case of ivn-soft sets as follows:

Definition 3.1 Let ϒ = (K,E) =$[a_{ij}]_{mxn}$ and Ψ = (M,E)=$[b_{ij}]_{mxn}$ be two ivn-soft sets.

K(e) = {<x, $[T_{K(e)}^L(x),T_{K(e)}^U(x)]$, $[I_{K(e)}^L(x),I_{K(e)}^U(x)]$, $[F_{K(e)}^L(x),T_{K(e)}^U(x)]$>: x ∈ X}

M(e) = {<x, $[T_{M(e)}^L(x),T_{M(e)}^U(x)]$, $[I_{M(e)}^L(x),I_{M(e)}^U(x)]$, $[F_{M(e)}^L(x),F_{M(e)}^U(x)]$>: x ∈ X}

Then we define the following distances for ϒ and Ψ

(1) The Hamming distance d_{IVNSS}^H(ϒ ,Ψ),

$$d_{IVNSS}^H(ϒ,Ψ)=\sum_{j=1}^{n}\sum_{i=1}^{m}\frac{\left[\left|\Delta_{ij}^L T\right|+\left|\Delta_{ij}^U T\right|+\left|\Delta_{ij}^L I\right|+\left|\Delta_{ij}^U I\right|+\left|\Delta_{ij}^L F\right|+\left|\Delta_{ij}^U F\right|\right]}{6}$$

Where $\Delta_{ij}^L T= T_{K(e)}^L(x_i)-T_{M(e)}^L(x_i)$,$\Delta_{ij}^U T= T_{K(e)}^U(x_i)-T_{M(e)}^U(x_i)$,$\Delta_{ij}^L I= I_{K(e)}^L(x_i)-I_{M(e)}^L(x_i)$,$\Delta_{ij}^U I= I_{K(e)}^U(x_i)-I_{M(e)}^U(x_i)$,$\Delta_{ij}^L F= F_{K(e)}^L(x_i)-F_{M(e)}^L(x_i)$ and $\Delta_{ij}^U F= F_{K(e)}^U(x_i)-F_{M(e)}^U(x_i)$

(2) The normalized Hamming distance d_{IVNSS}^{nH}(ϒ ,Ψ),

$$d_{IVNSS}^{nH}(ϒ,Ψ)=\frac{d_{IVNSS}^H(ϒ,Ψ)}{mn}$$

(3)The Euclidean distanced_{IVNSS}^E(ϒ ,Ψ),

$$d_{IVNSS}^E(ϒ,Ψ)=\sqrt{\sum_{j=1}^{n}\sum_{i=1}^{m}\frac{(\Delta_{ij}^L T)^2+(\Delta_{ij}^U T)^2+(\Delta_{ij}^L I)^2+(\Delta_{ij}^U I)^2+(\Delta_{ij}^L F)^2+(\Delta_{ij}^U F)^2}{6}}$$

Where $\Delta_{ij}^L T= T_{K(e)}^L(x_i)-T_{M(e)}^L(x_i)$,$\Delta_{ij}^U T= T_{K(e)}^U(x_i)-T_{M(e)}^U(x_i)$,$\Delta_{ij}^L I= I_{K(e)}^L(x_i)-I_{M(e)}^L(x_i)$,$\Delta_{ij}^U I= I_{K(e)}^U(x_i)-I_{M(e)}^U(x_i)$,$\Delta_{ij}^L F= F_{K(e)}^L(x_i)-F_{M(e)}^L(x_i)$ and $\Delta_{ij}^U F= F_{K(e)}^U(x_i)-F_{M(e)}^U(x_i)$

(4)The normalized Euclidean distance d_{IVNSS}^{nE}(ϒ ,Ψ),

$$d_{IVNSS}^{nE}(ϒ,Ψ)=\frac{d_{IVNSS}^E(ϒ,Ψ)}{\sqrt{mn}}$$

Here, it is clear that the following properties hold:

(1) $0 \le d_{IVNSS}^H(ϒ,Ψ) \le$ m n and $0 \le d_{IVNSS}^{nH}(ϒ,Ψ) \le 1$;
(2) $0 \le d_{IVNSS}^E(ϒ,Ψ) \le \sqrt{mn}$ and $0 \le d_{IVNSS}^{nE}(ϒ,Ψ) \le 1$;

Example 3. Assume that two interval neutrosophic soft sets ϒ and Ψ are defined as follows

K (e1) =(<x_1,[0.5, 0.6],[0.6 ,0.7],[0.3,0.4]>,<x_2, [0.5, 0.6], [0.6 ,0.7],[0.3,0.4]>), K (e2) =(<x_1,[0.2, 0.3], [0.5 ,0.6],[0.3,0.6]> ,<x_2, [0.4, 0.6], [0.2 ,0.3],[0.2,0.3]>),

M (e1) =(<x_1,[0.3, 0.4], [0.1 ,0.5],[0.2,0.4]> ,<x_2, [0.2, 0.5], [0.3 ,0.4],[0.4,0.5]>),

M (e2) =(<x_1,[0.4, 0.6], [0.3 ,0.5],[0.3,0.4]> ,<x_2, [0.3, 0.4], [0.2,0.7],[0.1,0.4]>),

$$d^H_{IVNSS}(\Upsilon ,\Psi) = \sum_{j=1}^{2}\sum_{i=1}^{2}\frac{\left[\left|\Delta^L_{ij}T\right|+\left|\Delta^U_{ij}T\right|+\left|\Delta^L_{ij}I\right|+\left|\Delta^U_{ij}I\right|+\left|\Delta^L_{ij}F\right|+\left|\Delta^U_{ij}F\right|\right]}{6}$$

$$=\frac{|0.5-0.3|+|0.6-0.4|+|0.6-0.1|+|0.7-0.5|+|0.3-0.2|+|0.4-0.4|}{6}$$
$$+\frac{|0.5-0.2|+|0.6-0.5|+|0.6-0.3|+|0.7-0.4|+|0.3-0.4|+|0.4-0.5|}{6}+$$
$$\frac{|0.2-0.4|+|0.3-0.6|+|0.5-0.3|+|0.6-0.5|+|0.3-0.3|+|0.6-0.4|}{6}+$$
$$\frac{|0.4-0.3|+|0.6-0.4|+|0.2-0.2|+|0.3-0.7|+|0.2-0.1|+|0.3-0.4|}{6}$$

$d^H_{IVNSS}(\Upsilon ,\Psi)$ =0.71

Theorem 3.2 The functions $d^H_{IVNSS}(\Upsilon ,\Psi)$, $d^{nH}_{IVNSS}(\Upsilon ,\Psi)$, $d^E_{IVNSS}(\Upsilon ,\Psi)$, $d^{nE}_{IVNSS}(\Upsilon ,\Psi)$: *IVNS*(*U*) → R^+given by Definition 3.1 respectively are metrics, where R^+ is the set of all non-negative real numbers.

Proof. The proof is straightforward.

4. Generalized weighted distance measure between two interval valued nutrosophic soft sets.

Let A and B be two interval neutrosophic sets, then S.Broumi and F.Smarandache[11] proposed a generalized interval valued neutrosophic weighted distance measure between A and B as follows:

$$d_\lambda(A ,B) = \left\{\frac{1}{6}\sum_{j=1}^{m} \sum_{i=1}^{n} w_i\left[|T^L_A(x_i)-T^L_B(x_i)|^\lambda+|T^U_A(x_i)-T^U_B(x_i)|^\lambda+|I^L_A(x_i)-I^L_B(x_i)|^\lambda+|I^U_A(x_i)-I^U_B(x_i)|^\lambda+|F^L_A(x_i)-F^L_B(x_i)|^\lambda+|F^U_A(x_i)-F^U_B(x_i)|^\lambda\right]\right\}^{\frac{1}{\lambda}} \quad (4)$$

where

$\lambda>0$and $T^L_A(x_i)$,$T^U_A(x_i)$,$I^L_A(x_i)$,$I^U_A(x_i)$, ,$F^L_A(x_i)$,$F^U_A(x_i)$, $T^L_B(x_i)$,$T^U_B(x_i)$, $I^L_B(x_i)$,$I^U_B(x_i)$, $F^L_B(x_i)$,$F^U_B(x_i)$, ∈ [0, 1]

,we extended the above equation (4) distance to the case of interval valued neutrosophic soft set between ϒ and Ψ as follow:

$$d_\lambda(\Upsilon ,\Psi) = \left\{\frac{1}{6}\sum_{j=1}^{m} \sum_{i=1}^{n} w_i\left[\left|\Delta^L_{ij}T\right|^\lambda+\left|\Delta^U_{ij}T\right|^\lambda+\left|\Delta^L_{ij}I\right|^\lambda+\left|\Delta^U_{ij}I\right|^\lambda+\left|\Delta^L_{ij}F\right|^\lambda+\left|\Delta^U_{ij}F\right|^\lambda\right]\right\}^{\frac{1}{\lambda}} \quad (5)$$

Where $\Delta^L_{ij}T= T^L_{K(e)}(x_i)-T^L_{M(e)}(x_i)$,$\Delta^U_{ij}T= T^U_{K(e)}(x_i)-T^U_{M(e)}(x_i)$,$\Delta^L_{ij}I= I^L_{K(e)}(x_i)-I^L_{M(e)}(x_i)$,$\Delta^U_{ij}I= I^U_{K(e)}(x_i)-I^U_{M(e)}(x_i)$,$\Delta^L_{ij}F= F^L_{K(e)}(x_i)-F^L_{M(e)}(x_i)$ and $\Delta^U_{ij}F= F^U_{K(e)}(x_i)-F^U_{M(e)}(x_i)$.

Normalized generalized interval neutrosophic distance is

$$d_{\lambda}^{n}(\Upsilon,\Psi)=\left\{\frac{1}{6n}\sum_{j=1}^{m}\sum_{i=1}^{n}w_i\left[\left|\Delta_{ij}^{L}T\right|^{\lambda}+\left|\Delta_{ij}^{U}T\right|^{\lambda}+\left|\Delta_{ij}^{L}I\right|^{\lambda}+\left|\Delta_{ij}^{U}I\right|^{\lambda}+\left|\Delta_{ij}^{L}F\right|^{\lambda}+\left|\Delta_{ij}^{U}F\right|^{\lambda}\right]\right\}^{\frac{1}{\lambda}} \quad (6)$$

If w=$\{\frac{1}{n},\frac{1}{n},\ldots,\frac{1}{n}\}$,the Eq. (6) is reduced to the following distances:

$$d_{\lambda}(\Upsilon,\Psi)=\left\{\frac{1}{6}\sum_{j=1}^{m}\sum_{i=1}^{n}\left[\left|\Delta_{ij}^{L}T\right|^{\lambda}+\left|\Delta_{ij}^{U}T\right|^{\lambda}+\left|\Delta_{ij}^{L}I\right|^{\lambda}+\left|\Delta_{ij}^{U}I\right|^{\lambda}+\left|\Delta_{ij}^{L}F\right|^{\lambda}+\left|\Delta_{ij}^{U}F\right|^{\lambda}\right]\right\}^{\frac{1}{\lambda}} \quad (7)$$

$$d_{\lambda}(\Upsilon,\Psi)=\left\{\frac{1}{6n}\sum_{j=1}^{m}\sum_{i=1}^{n}\left[\left|\Delta_{ij}^{L}T\right|^{\lambda}+\left|\Delta_{ij}^{U}T\right|^{\lambda}+\left|\Delta_{ij}^{L}I\right|^{\lambda}+\left|\Delta_{ij}^{U}I\right|^{\lambda}+\left|\Delta_{ij}^{L}F\right|^{\lambda}+\left|\Delta_{ij}^{U}F\right|^{\lambda}\right]\right\}^{\frac{1}{\lambda}} \quad (8)$$

Particular case

(i) if λ =1 then the equation (7), (8) is reduced to the following hamming distance and normalized hamming distance between interval valued neutrosophic soft set

$$d_{IVNSS}^{H}(\Upsilon,\Psi)=\sum_{j=1}^{n}\sum_{i=1}^{m}\frac{\left[\left|\Delta_{ij}^{L}T\right|+\left|\Delta_{ij}^{U}T\right|+\left|\Delta_{ij}^{L}I\right|+\left|\Delta_{ij}^{U}I\right|+\left|\Delta_{ij}^{L}F\right|+\left|\Delta_{ij}^{U}F\right|\right]}{6} \quad (9)$$

$$d_{IVNSS}^{nH}(\Upsilon,\Psi)=\frac{d_{IVNSS}^{H}(\Upsilon,\Psi)}{mn} \quad (10)$$

(ii) If λ =2 then the equation (7) , (8) is reduced to the following Euclidean distance and normalized Euclidean distance between interval valued neutrosophic soft set

$$d_{IVNSS}^{E}(\Upsilon,\Psi)=\sqrt{\sum_{j=1}^{n}\sum_{i=1}^{m}\frac{(\Delta_{ij}^{L}T)^2+(\Delta_{ij}^{U}T)^2+(\Delta_{ij}^{L}I)^2+(\Delta_{ij}^{U}I)^2+(\Delta_{ij}^{L}F)^2+(\Delta_{ij}^{U}F)^2}{6}} \quad (11)$$

$$d_{IVNSS}^{NE}(\Upsilon,\Psi)=\frac{d_{IVNSS}^{E}(\Upsilon,\Psi)}{\sqrt{mn}} \quad (12)$$

5. Similarity Measures between Interval Valued Neutrosophic Soft Sets

This section proposes several similarity measures of interval neutrosophic soft sets.

It is well known that similarity measures can be generated from distance measures. Therefore, we may use the proposed distance measures to define similarity measures. Based on the relationship of similarity measures and distance measures, we can define some similarity measures between IVNSSs Υ = (K,E) and Ψ = (M,E) as follows:

5.1. Similarity measure based on the geometric distance model

Now for each e_i ∈E , K(e_i) and M(e_i) are interval neutrosophic set. To find similarity between Υ and Ψ. We first find the similarity between K(e_i) and M(e_i).

Based on the distance measures defined above the similarity as follows:

$S_{IVNSS}^{H}(\Upsilon,\Psi)=\frac{1}{1+d_{IVNSS}^{H}(\Upsilon,\Psi)}$ and $S_{IVNSS}^{E}(\Upsilon,\Psi)=\frac{1}{1+d_{IVNSS}^{E}(\Upsilon,\Psi)}$

$S^{nH}_{IVNSS}(\Upsilon,\Psi)=\frac{1}{1+d^{nH}_{IVNSS}(\Upsilon,\Psi)}$ and $S^{nE}_{IVNSS}(\Upsilon,\Psi)=\frac{1}{1+d^{nE}_{IVNSS}(\Upsilon,\Psi)}$

Example 4 : Based on example 3,then

$S^{H}_{IVNSS}(\Upsilon,\Psi)=\frac{1}{1+0.71}=\frac{1}{1.71}=0.58$

Based on (4), we define the similarity measure between the interval valued neutrosophic soft sets $\Upsilon$and Ψ as follows:

$$S_{DM}(\Upsilon,\Psi)= 1-\left\{\frac{1}{6n}\sum_{i=1}^{n}\left[\left|T^L_{K(e)}(x_i)-T^L_{M(e)}(x_i)\right|^\lambda+\left|T^U_{K(e)}(x_i)-T^U_{M(e)}(x_i)\right|^\lambda+\left|I^L_{K(e)}(x_i)-I^L_{M(e)}(x_i)\right|^\lambda+\left|I^U_{K(e)}(x_i)-I^U_{M(e)}(x_i)\right|^\lambda+\left|F^L_{K(e)}(x_i)-F^L_{M(e)}(x_i)\right|^\lambda+\left|F^U_{K(e)}(x_i)-F^U_{M(e)}(x_i)\right|^\lambda\right]\right\}^{\frac{1}{\lambda}} \quad (13)$$

Where $\lambda>0$ *and* $S_{DM}(\Upsilon,\Psi)$ is the degree of similarity of A and B .

If we take the weight of each element $x_i\in X$ into account, then

$$S^w_{DM}(\Upsilon,\Psi)= 1-\left\{\frac{1}{6}\sum_{i=1}^{n}w_i\left[\left[\left|T^L_{K(e)}(x_i)-T^L_{M(e)}(x_i)\right|^\lambda+\left|T^U_{K(e)}(x_i)-T^U_{M(e)}(x_i)\right|^\lambda+\left|I^L_{K(e)}(x_i)-I^L_{M(e)}(x_i)\right|^\lambda+\left|I^U_{K(e)}(x_i)-I^U_{M(e)}(x_i)\right|^\lambda+\left|F^L_{K(e)}(x_i)-F^L_{M(e)}(x_i)\right|^\lambda+\left|F^U_{K(e)}(x_i)-F^U_{M(e)}(x_i)\right|^\lambda\right]\right]\right\}^{\frac{1}{\lambda}} \quad (14)$$

If each elements has the same importance ,i.e w $=\{\frac{1}{n},\frac{1}{n},\ldots,\frac{1}{n}\}$, then (14) reduces to (13)

By definition 2.7 it can easily be known that $S_{DM}(\Upsilon,\Psi)$ satisfies all the properties of definition..

$$\left[\left|\Delta^L_{ij}T\right|+\left|\Delta^U_{ij}T\right|+\left|\Delta^L_{ij}I\right|+\left|\Delta^U_{ij}I\right|+\left|\Delta^L_{ij}F\right|+\left|\Delta^U_{ij}F\right|\right]$$

Similarly , we define another similarity measure of Υ and Ψ as:

$$S(\Upsilon,\Psi)=1-\left[\frac{\sum_{i=1}^{n}\left(\left|\Delta^L_{ij}T\right|^\lambda+\left|\Delta^U_{ij}T\right|^\lambda+\left|\Delta^L_{ij}I\right|^\lambda+\left|\Delta^U_{ij}I\right|^\lambda+\left|\Delta^L_{ij}F\right|^\lambda+\left|\Delta^U_{ij}F\right|^\lambda\right)}{\sum_{i=1}^{n}\left(\left|T^L_K(x_i)+T^L_M(x_i)\right|^\lambda+\left|T^U_K(x_i)+T^U_M(x_i)\right|^\lambda+\left|I^L_K(x_i)+I^L_M(x_i)\right|^\lambda+\left|I^U_K(x_i)+I^U_M(x_i)\right|^\lambda+\left|F^L_K(x_i)+F^L_M(x_i)\right|^\lambda+\left|F^U_K(x_i)+F^U_M(x_i)\right|^\lambda\right)}\right]^{\frac{1}{\lambda}} \quad (15)$$

If we take the weight of each element $x_i\in X$ into account, then

$$S(\Upsilon,\Psi) = 1 - \left[\frac{\sum_{i=1}^{n} w_i\left(\left|\Delta_{ij}^{L}T\right|^{\lambda}+\left|\Delta_{ij}^{U}T\right|^{\lambda}+\left|\Delta_{ij}^{L}I\right|^{\lambda}+\left|\Delta_{ij}^{U}I\right|^{\lambda}+\left|\Delta_{ij}^{L}F\right|^{\lambda}+\left|\Delta_{ij}^{U}F\right|^{\lambda}\right)}{\sum_{i=1}^{n} w_i\left(\left|T_K^L(x_i)+T_M^L(x_i)\right|^{\lambda}+\left|T_K^U(x_i)+T_M^U(x_i)\right|^{\lambda}+\left|I_K^L(x_i)+I_M^L(x_i)\right|^{\lambda}+\left|I_K^U(x_i)+I_M^U(x_i)\right|^{\lambda}+\left|F_K^L(x_i)+F_M^L(x_i)\right|^{\lambda}+\left|F_K^U(x_i)+F_M^U(x_i)\right|^{\lambda}\right)}\right]^{\frac{1}{\lambda}} \quad (16)$$

This also has been proved that all the properties of definition are satisfied, If each elements has the same importance, and then (16) reduces to (15)

5.2.Similarity measure based on the interval valuedneutrosophic theoretic approach:

In this section, following the similarity measure between two interval neutrosophic sets defined by S.Broumi and F.Samarandache in [11], we extend this definition to interval valued neutrosophic soft sets.

Let $S_i(\Upsilon,\Psi)$ indicates the similarity between the interval neutrosophic soft sets Υ and Ψ .To find the similarity between Υ and Ψ first we have to find the similarity between their e - approximations. Let $S_i(\Upsilon,\Psi)$ denote the similarity between the two e_i- approximations K(e_i) and M(e_i).

Let Υ and Ψ be two interval valued neutrosophic soft **sets**, then we define a similarity measure between K(e_i) and M(e_i) as follows:

$$S_i(\Upsilon,\Psi)= \frac{\sum_{i=1}^{n}\{\min\{T^L_{K(e_i)}(x_j),T^L_{M(e_i)}(x_j)\}+\min\{T^U_{K(e_i)}(x_j),T^U_{M(e_i)}(x_j)\}+\min\{I^L_{K(e_i)}(x_j),I^L_{M(e_i)}(x_j)\}+\min\{I^U_{K(e_i)}(x_j),I^U_{M(e_i)}(x_j)\}+\min\{F^L_{K(e_i)}(x_j),F^L_{M(e_i)}(x_j)\}+\min\{F^U_{K(e_i)}(x_j),F^U_{M(e_i)}(x_j)\}}{\sum_{i=1}^{n}\{\max\{T^L_{K(e_i)}(x_j),T^L_{M(e_i)}(x_j)\}+\max\{T^U_{K(e_i)}(x_j),T^U_{M(e_i)}(x_j)\}+\max\{I^L_{K(e_i)}(x_j),I^L_{M(e_i)}(x_j)\}+\max\{I^U_{K(e_i)}(x_j),I^U_{M(e_i)}(x_j)\}+\max\{F^L_{K(e_i)}(x_j),F^L_{M(e_i)}(x_j)\}+\max\{F^U_{K(e_i)}(x_j),F^U_{M(e_i)}(x_j)\}} \quad (17))$$

Then $S(\Upsilon,\Psi) = \max_i S_i(\Upsilon,\Psi)$

The similarity measure has the following proposition

Proposition 4.2

Let Υ and Ψ be interval valued neutrosophic soft sets then

i. $\mathbf{0 \leq S(\Upsilon,\Psi) \leq 1}$
ii. $S(\Upsilon,\Psi) = S(\Psi,\Upsilon)$
iii. $S(\Upsilon,\Psi) = 1$ if $\Upsilon = \Psi$

$T^L_{K(e)}(x_i) = T^L_{M(e)}(x_i)$, $T^U_{K(e)}(x_i) = T^U_{M(e)}(x_i)$, $I^L_{K(e)}(x_i) = I^L_{M(e)}(x_i)$, $I^U_{K(e)}(x_i) = I^U_{M(e)}(x_i)$ and$F^L_{K(e)}(x_i) = F^L_{M(e)}(x_i)$, $F^U_{K(e)}(x_i) = F^U_{M(e)}(x_i)$for i=1,2,…., n

Iv .$\Upsilon \subseteq \Psi \subseteq \Omega \Rightarrow S(\Upsilon,\Psi) \leq \min(S(\Upsilon,\Psi), S(\Psi,\Omega))$

Proof. Properties (i) and(ii) follows from definition

(iii) it is clearly that if $\Upsilon = \Psi \Rightarrow S(\Upsilon, \Psi) = 1$

$\Rightarrow \sum_{i=1}^{n}\{\min\{T^{L}_{K(e)}(x_i), T^{L}_{M(e)}(x_i)\} + \min\{T^{U}_{K(e)}(x_i), T^{U}_{M(e)}(x_i)\} + \min\{I^{L}_{K(e)}(x_i), I^{L}_{M(e)}(x_i)\} + \min\{I^{U}_{K(e)}(x_i), I^{U}_{M(e)}(x_i)\} + \min\{F^{L}_{K(e)}(x_i), F^{L}_{M(e)}(x_i)\} + \min\{F^{U}_{K(e)}(x_i), F^{U}_{M(e)}(x_i)\}$
$=\sum_{i=1}^{n}\{\max\{T^{L}_{K(e)}(x_i), T^{L}_{M(e)}(x_i)\} + \max\{T^{U}_{K(e)}(x_i), T^{U}_{M(e)}(x_i)\} + \max\{I^{L}_{K(e)}(x_i), I^{L}_{M(e)}(x_i)\} + \max\{I^{U}_{K(e)}(x_i), I^{U}_{M(e)}(x_i)\} + \max\{F^{L}_{K(e)}(x_i), F^{L}_{M(e)}(x_i)\} + \max\{F^{U}_{K(e)}(x_i), F^{U}_{M(e)}(x_i)\}$

$\Rightarrow \sum_{i=1}^{n}\{[\min\{T^{L}_{K(e)}(x_i), T^{L}_{M(e)}(x_i)\} - \max\{T^{L}_{K(e)}(x_i), T^{L}_{M(e)}(x_i)\}] + [\min\{T^{U}_{K(e)}(x_i), T^{U}_{M(e)}(x_i)\} - \max\{T^{U}_{K(e)}(x_i), T^{U}_{M(e)}(x_i)\}] + [\min\{I^{L}_{K(e)}(x_i), I^{L}_{M(e)}(x_i)\} - \max\{I^{L}_{K(e)}(x_i), I^{L}_{M(e)}(x_i)\}] + [\min\{I^{U}_{K(e)}(x_i), I^{U}_{M(e)}(x_i)\} - \max\{I^{U}_{K(e)}(x_i), I^{U}_{M(e)}(x_i)\}] + [\min\{F^{L}_{K(e)}(x_i), F^{L}_{M(e)}(x_i)\} - \max\{F^{L}_{K(e)}(x_i), F^{L}_{M(e)}(x_i)\}] + [\min\{F^{U}_{K(e)}(x_i), F^{U}_{M(e)}(x_i)\} - \max\{F^{U}_{K(e)}(x_i), F^{U}_{M(e)}(x_i)]\} = 0$

Thus for each x,

$[\min\{T^{L}_{K(e)}(x_i), T^{L}_{M(e)}(x_i)\} - \max\{T^{L}_{K(e)}(x_i), T^{L}_{M(e)}(x_i)\}] = 0$

$[\min\{T^{U}_{K(e)}(x_i), T^{U}_{M(e)}(x_i)\} - \max\{T^{U}_{K(e)}(x_i), T^{U}_{M(e)}(x_i)\}] = 0$

$[\min\{I^{L}_{K(e)}(x_i), I^{L}_{M(e)}(x_i)\} - \max\{I^{L}_{K(e)}(x_i), I^{L}_{M(e)}(x_i)\}] = 0$

$[\min\{I^{U}_{K(e)}(x_i), I^{U}_{M(e)}(x_i)\} - \max\{I^{U}_{K(e)}(x_i), I^{U}_{M(e)}(x_i)\}] = 0$

$[\min\{F^{L}_{K(e)}(x_i), F^{L}_{M(e)}(x_i)\} - \max\{F^{L}_{K(e)}(x_i), F^{L}_{M(e)}(x_i)\}] = 0$

$[\min\{F^{U}_{K(e)}(x_i), F^{U}_{M(e)}(x_i)\} - \max\{F^{U}_{K(e)}(x_i), F^{U}_{M(e)}(x_i)]\} = 0$ holds

Thus $T^{L}_{K(e)}(x_i) = T^{L}_{M(e)}(x_i)$, $T^{U}_{K(e)}(x_i) = T^{U}_{M(e)}(x_i)$, $I^{L}_{K(e)}(x_i) = I^{L}_{M(e)}(x_i)$, $I^{U}_{K(e)}(x_i) = I^{U}_{M(e)}(x_i)$, $F^{L}_{A}(x_i) = F^{L}_{B}(x_i)$ and $F^{U}_{K(e)}(x_i) = F^{U}_{M(e)}(x_i) \Rightarrow \Upsilon = \Psi$

(iv) now we prove the last result.

Let $\Upsilon \subseteq \Psi \subseteq \Omega$, then we have

$T^{L}_{K(e)}(x_i) \le T^{L}_{M(e)}(x_i) \le T^{L}_{C}(x)$, $T^{U}_{K(e)}(x_i) \le T^{U}_{M(e)}(x_i) \le T^{L}_{C}(x)$, $I^{L}_{K(e)}(x) \ge I^{L}_{M(e)}(x) \ge I^{L}_{C}(x)$, $I^{U}_{A}(x) \ge I^{U}_{M(e)}(x) \ge I^{U}_{C}(x)$, $F^{L}_{K(e)}(x) \ge F^{L}_{M(e)}(x) \ge F^{L}_{C}(x)$, $F^{U}_{K(e)}(x) \ge F^{U}_{M(e)}(x) \ge F^{U}_{C}(x)$ for all $x \in X$ .Now

$T^{L}_{K}(x) + T^{U}_{K}(x) + I^{L}_{K}(x) + I^{U}_{K}(x) + F^{L}_{M}(x) + F^{U}_{M}(x) \ge T^{L}_{K}(x) + T^{U}_{K}(x) + I^{L}_{K}(x) + I^{U}_{K}(x) + F^{L}_{C}(x) + F^{U}_{C}(x)$

And

$T^{L}_{M}(x) + T^{U}_{M}(x) + I^{L}_{M}(x) + I^{U}_{M}(x) + F^{L}_{K}(x) + F^{U}_{K}(x) \ge T^{L}_{C}(x) + T^{U}_{C}(x) + I^{L}_{C}(x) + I^{U}_{C}(x) + F^{L}_{K}(x) + F^{U}_{K}(x)$

$$S(\Upsilon,\Psi) = \frac{T_K^L(x) + T_K^U(x) + I_K^L(x) + I_K^U(x) + F_M^L(x) + F_M^U(x)}{T_M^L(x) + T_M^U(x) + I_M^L(x) + I_M^U(x) + F_K^L(x) + F_K^U(x)} \geq \frac{T_K^L(x) + T_K^U(x) + I_K^L(x) + I_K^U(x) + F_C^L(x) + F_C^U(x)}{T_C^L(x) + T_C^U(x) + I_C^L(x) + I_C^U(x) + F_K^L(x) + F_K^U(x)} = S(\Upsilon,\Omega)$$

Again similarly we have

$$T_M^L(x) + T_M^U(x) + I_M^L(x) + I_M^U(x) + F_C^L(x) + F_C^U(x) \geq T_K^L(x) + T_K^U(x) + I_K^L(x) + I_K^U(x) + F_C^L(x) + F_C^U(x)$$

$$T_C^L(x) + T_C^U(x) + I_C^L(x) + I_C^U(x) + F_K^L(x) + F_K^U(x) \geq T_C^L(x) + T_C^U(x) + I_C^L(x) + I_C^U(x) + F_M^L(x) + F_M^U(x)$$

$$S(\Psi,\Omega) = \frac{T_M^L(x) + T_M^U(x) + I_M^L(x) + I_M^U(x) + F_C^L(x) + F_C^U(x)}{T_C^L(x) + T_C^U(x) + I_C^L(x) + I_C^U(x) + F_M^L(x) + F_M^U(x)} \geq \frac{T_K^L(x) + T_K^U(x) + I_K^L(x) + I_K^U(x) + F_C^L(x) + F_C^U(x)}{T_C^L(x) + T_C^U(x) + I_C^L(x) + I_C^U(x) + F_K^L(x) + F_K^U(x)} = S(\Upsilon,\Omega)$$

$\Rightarrow S(\Upsilon,\Omega) \leq \min(S(\Upsilon,\Psi), S(\Psi,\Omega))$

Hence the proof of this proposition

If we take the weight of each element $x_i \in$ X into account, then

$$S(\Upsilon,\Psi) = \frac{\sum_{i=1}^{n} w_i\{\min\{T_A^L(x_i),T_B^L(x_i)\}+\min\{T_A^U(x_i),T_B^U(x_i)\}+\min\{I_A^L(x_i),I_B^L(x_i)\}+\min\{I_A^U(x_i),I_B^U(x_i)\}+\min\{F_A^L(x_i),F_B^L(x_i)\}+\min\{F_A^U(x_i),F_B^U(x_i)\}}{\sum_{i=1}^{n} w_i\{\max\{T_A^L(x_i),T_B^L(x_i)\}+\max\{T_A^U(x_i),T_B^U(x_i)\}+\max\{I_A^L(x_i),I_B^L(x_i)\}+\max\{I_A^U(x_i),I_B^U(x_i)\}+\max\{F_A^L(x_i),F_B^L(x_i)\}+\max\{F_A^U(x_i),F_B^U(x_i)\}} \quad (18)$$

Particularly ,if each element has the same importance, then (18) is reduced to (17) ,clearly this also satisfies all the properties of definition .

Theorem Υ = <K,E>, Ψ = <L,E> and Ω = <M,E>; are three ivn-soft sets in universe U such that $\Upsilon$is a ivn-soft subset of Ψ and $\Psi$is a soft subset of Ω then, $S(\Upsilon, \Omega) \leq S(\Psi, \Omega)$.

Proof. The proof is straightforward.

5.3. Similarity measure based for matching function by using interval neutrosophic sets:

Chen [24] and Chen et al. [25]) introduced a matching function to calculate the degree of similarity between fuzzy sets. In the following, we extend the matching function to deal with the similarity measure of interval valued neutrosophic soft sets.

Let Υ = A and Ψ=B be two interval valued neutrosophic soft sets, then we define a similarity measure between Υ and Ψ as follows:

$$S_{MF}(\Upsilon,\Psi) = \frac{\sum_{i=1}^{n}\Big(\big(T_A^L(x_i)\cdot T_B^L(x_i)\big)+\big(T_A^U(x_i)\cdot T_B^U(x_i)\big)+\big(I_A^L(x_i)\cdot I_B^L(x_i)\big)+\big(I_A^U(x_i)\cdot I_B^U(x_i)\big)+\big(F_A^L(x_i)\cdot F_B^L(x_i)\big)+\big(F_A^U(x_i)\cdot F_B^U(x_i)\big)\Big)}{\max\big(\sum_{i=}^{n}(T_A^L(x_i)^2+T_A^U(x_i)^2+I_A^L(x_i)^2+I_A^U(x_i)^2+F_A^L(x_i)^2+F_A^U(x_i)^2),\ \sum_{i=}^{n}(T_B^L(x_i)^2+T_B^U(x_i)^2+I_B^L(x_i)^2+I_B^U(x_i)^2+F_B^L(x_i)^2+F_B^U(x_i)^2)\big)}$$

$T_{K(e)}^L(x_i) = T_{M(e)}^L(x_i)$, $T_{K(e)}^U(x_i) = T_{M(e)}^U(x_i)$, $I_{K(e)}^L(x_i) = I_{M(e)}^L(x_i)$, $I_{K(e)}^U(x_i) = I_{M(e)}^U(x_i)$ and $F_{K(e)}^L(x_i) = F_{M(e)}^L(x_i)$, $F_{K(e)}^U(x_i) = F_{M(e)}^U(x_i)$
(19)

Proof.

i. $\mathbf{0} \le S_{MF}(\Upsilon, \Psi) \le \mathbf{1}$

The inequality $S_{MF}(\Upsilon, \Psi) \ge 0$ is obvious. Thus, we only prove the inequality $S(\Upsilon, \Psi) \le 1$.

$$S_{MF}(\Upsilon,\Psi)=\sum_{\mathbf{i=1}}^{\mathbf{n}}\Big(\big(\mathrm{T}_{\mathrm{K(e)}}^{\mathrm{L}}(\mathrm{x_i}) \cdot T_{M(e)}^{L}(\mathrm{x_i})\big) + \big(\mathrm{T}_{\mathrm{K(e)}}^{\mathrm{U}}(\mathrm{x_i}) \cdot T_{M(e)}^{U}(\mathrm{x_i})\big) + \big(I_{K(e)}^{L}(x_i) \cdot I_{M(e)}^{L}(\mathrm{x_i})\big) + \big(I_{K(e)}^{U}(x_i) \cdot I_{M(e)}^{U}(\mathrm{x_i})\big) + \big(F_{K(e)}^{L}(x_i) \cdot F_{M(e)}^{L}(\mathrm{x_i})\big) + \big(F_{K(e)}^{U}(x_i) \cdot F_{M(e)}^{U}(\mathrm{x_i})\big)\Big)$$

$$=T_{K(e)}^{L}(x_1) \cdot T_{M(e)}^{L}(x_1)+T_{K(e)}^{L}(x_2) \cdot T_{M(e)}^{L}(x_2)+\ldots+T_{K(e)}^{L}(x_n) \cdot T_{M(e)}^{L}(x_n)+T_{K(e)}^{U}(x_1) \cdot T_{M(e)}^{U}(x_1)+T_{K(e)}^{U}(x_2) \cdot T_{M(e)}^{U}(x_2)+\ldots+T_{K(e)}^{U}(x_n) \cdot T_{M(e)}^{U}(x_n)+$$

$$I_{K(e)}^{L}(x_1) \cdot I_{M(e)}^{L}(x_1)+I_{K(e)}^{L}(x_2) \cdot I_{M(e)}^{L}(x_2)+\ldots+I_{K(e)}^{L}(x_n) \cdot I_{M(e)}^{L}(x_n)+I_{K(e)}^{U}(x_1) \cdot I_{M(e)}^{U}(x_1)+I_{K(e)}^{U}(x_2) \cdot I_{M(e)}^{U}(x_2)+\ldots+I_{K(e)}^{U}(x_n) \cdot I_{M(e)}^{U}(x_n)+$$

$$F_{K(e)}^{L}(x_1) \cdot F_{M(e)}^{L}(x_1)+F_{K(e)}^{L}(x_2) \cdot F_{M(e)}^{L}(x_2)+\ldots+F_{K(e)}^{L}(x_n) \cdot F_{M(e)}^{L}(x_n)+F_{K(e)}^{U}(x_1) \cdot F_{M(e)}^{U}(x_1)+F_{K(e)}^{U}(x_2) \cdot F_{M(e)}^{U}(x_2)+\ldots+F_{K(e)}^{U}(x_n) \cdot F_{M(e)}^{U}(x_n)+$$

According to the Cauchy–Schwarz inequality:

$$(x_1 \cdot y_1 + x_2 \cdot y_2 + \cdots + x_n \cdot y_n)^2 \le ({x_1}^2 + {x_2}^2 + \cdots + {x_n}^2) \cdot ({y_1}^2 + {y_2}^2 + \cdots + {y_n}^2)$$

where $(x_1, x_2, \ldots, x_n) \in R^n$ *and* $(y_1, y_2, \ldots, y_n) \in R^n$ *we can obtain*

$$[S_{MF}(\Upsilon, \Psi)]^2 \le \sum_{\boldsymbol{i=1}}^{\boldsymbol{n}}\big(T_{K(e)}^{L}(x_i)^2 + T_{K(e)}^{U}(x_i)^2 + I_{K(e)}^{L}(x_i)^2 + I_{K(e)}^{U}(x_i)^2 + F_{K(e)}^{L}(x_i)^2 + F_{K(e)}^{U}(x_i)^2\big) \cdot$$

$$\sum_{\boldsymbol{i=1}}^{\boldsymbol{n}}\big(T_{M(e)}^{L}(x_i)^2 + T_{M(e)}^{U}(x_i)^2 + I_{M(e)}^{L}(x_i)^2 + I_{M(e)}^{U}(x_i)^2 + F_{M(e)}^{L}(x_i)^2 + F_{M(e)}^{U}(x_i)^2\big)=\mathrm{S}(\Upsilon, \Upsilon) \cdot \mathrm{S}(\Psi, \Psi)$$

Thus $S_{MF}(\Upsilon,\Psi) \le [S(\Upsilon, \Upsilon)]^{\frac{1}{2}} \cdot [S(\Psi, \Psi)]^{\frac{1}{2}}$

Then $S_{MF}(\Upsilon,\Psi) \le \max\{\mathrm{S}(\Upsilon, \Upsilon), \mathrm{S}(\Psi, \Psi)]$

Therefore, $S_{MF}(\Upsilon,\Psi) \le 1$.

If we take the weight of each element $x_i \in$ X into account, then

$$S_{MF}^{w}(\Upsilon,\Psi)= \frac{\sum_{\boldsymbol{i=1}}^{\boldsymbol{n}} w_i\Big(\big(T_A^L(x_i) \cdot T_B^L(x_i)\big) + \big(T_A^U(x_i) \cdot T_B^U(x_i)\big) + \big(I_A^L(x_i) \cdot I_B^L(x_i)\big) + \big(I_A^U(x_i) \cdot I_B^U(x_i)\big) + \big(F_A^L(x_i) \cdot F_B^L(x_i)\big) + \big(F_A^U(x_i) \cdot F_B^U(x_i)\big)\Big)}{\max\big(\sum_{i=}^{n} w_i\,(\mathrm{T_A^L(x_i)^2 + T_A^U(x_i)^2 + I_A^L(x_i)^2 + I_A^U(x_i)^2 + F_A^L(x_i)^2 + F_A^U(x_i)^2}),\ \sum_{i=}^{n} w_i\,(\mathrm{T_B^L(x_i)^2 + T_B^U(x_i)^2 + I_B^L(x_i)^2 + I_B^U(x_i)^2 + F_B^L(x_i)^2 + F_B^U(x_i)^2})\big)} \tag{20}$$

Particularly ,if each element has the same importance, then (20) is reduced to (19) ,clearly this also satisfies all the properties of definition .

The larger the value of S(ϒ,Ψ) ,the more the similarity between ϒ and Ψ.

Majumdar and Samanta [40] compared the properties of the two measures of soft sets and proposed α-similar of two soft sets. In the following, we extend to interval valued neutrosophic soft sets as;

Let $X_{\Upsilon,\Psi}$ denote the similarity measure between two ivn-soft sets ϒ and Ψ Table compares the properties of the two measures of similarity of ivn-soft soft sets discussed here. It can be seen that most of the properties are common to both.and few differences between them do exist.

Property	S (**geometric**)	S (**theoretic**)	S (**matching)**
$S(\Upsilon,\Psi) = S(\Psi,\Upsilon)$	Yes	Yes	Yes
$0 \leq S(\Upsilon,\Psi) \leq 1$	Yes	Yes	Yes
$\Upsilon = \Psi \Rightarrow S(\Upsilon,\Psi) = 1$	Yes	No	?
$S(\Upsilon,\Psi) = 1 \Rightarrow \Upsilon = \Psi$	Yes	Yes	?
$\Upsilon \cap \Psi = \emptyset \Rightarrow S(\Upsilon,\Psi) = 0$	No	No	?
$S(\Upsilon,\Upsilon^c) = 0$	No	No	?

Definition A relation α≈ on IVNS(U), called α-similar, as follows:two inv-soft sets ϒ and Ψ are said to be α-similar, denoted as ϒα≈Ψ iff $S(\Upsilon,\Psi) \geq \alpha$ for α ∈(0, 1).

Here, we call the two ivn-soft sets significantly similar if $S(\Upsilon,\Psi) > 0.5$

Lemma [40] α≈is reflexive and symmetric, but not transitive.

Majumdar and Samanta [40] introduced a technique of similarity measure of two soft sets which can be applied to detect whether an ill person is suffering from a certain disease or not. In a example, they was tried to estimate the possibility that an ill person having certain visible symptoms is suffering from pneumonia.Therefore, they were given an example by using similarity measure of two soft sets. In the following application, similarly we will try for ivn-soft sets in same example. Some of it is quoted from [40] .

6. An Application

This technique of similarity measure of two inv-soft sets can be applied to detect whether an ill person is suffering from a certain disease or not. In the followinge xample, we will try to estimate the possibility that an ill person having certain visible symptoms is suffering from pneumonia. For this, we first construct a model inv-soft set for pneumonia and the inv-soft set for the ill person. Next we find the similarity measure of these two sets. If they are significantly similar, then we conclude that the person is possibly suffering from pneumonia.

Let our universal set contain only two elements yes and no, i.e. $U = \{yes=h_1, no=h_2\}$. Here the set of parameters E is the set of certain visible symptoms. Let E = {e1, e2, e3, e4, e5, e6}, where *e*1 = high body temperature, *e*2 = cough with chest congestion, *e*3 = body ache, *e*4 = headache, *e*5 = loose motion, and *e*6 = breathing trouble. Our model inv-soft for pneumoniaϒis given below and this can be prepared with the help of a medical person:

ϒ= { (e_1,{<h_1,[0.5, 0.6], [0.6 ,0.7],[0.3,0.4]> ,<h_2, [0.5, 0.6], [0.6 ,0.7],[0.3,0.4]>}),

(e_2,{< h_1, [0.5, 0.6], [0.6,0.7],[0.3,0.4]>,<h_2,[0.2, 0.3], [0.5 ,0.6],[0.3,0.6]>}),

(e_3,{<h_1, [0.4, 0.6], [0.2 ,0.3],[0.2,0.3]> , <h_2,[0.5, 0.6], [0.6 ,0.7],[0.3,0.4]> }),

(e_4,{<h_1,[0.3, 0.4], [0.1 ,0.5],[0.2,0.4]>,<h_2, [0.2, 0.5],[0.3,0.4],[0.4,0.5]> }),

(e_5,{<h_1,[0.5, 0.6],[0 .6,0.7],[0.3,0.4]>,<h_2,[0.4, 0.6], [0.3 ,0.5],[0.3,0.4]>}),

(e_6,{<h_1, [0.4, 0.6], [0.2 ,0.3],[0.2,0.3]>) , <h_2, [0.3, 0.4], [0.2,0.7],[0.1,0.4]>}) }

Now the ill person is having fever, cough and headache. After talking to him, we can construct his ivn-soft Ψ as follows:

Ψ = { (e_1,{<h_1,[0.1, 0.2], [0.1 ,0.2],[0.8,0.9]> ,<h_2, [0.1, 0.2], [0.0 ,0.1],[0.8,0.9]>}),

(e_2,{< h_1, [0.8, 0.9], [0.1,0.2],[0.2,0.9]> ,<h_2,[0.8, 0.9], [0.2 ,0.9],[0.8,0.9]>}),

(e_3,{<h_1,[0.1, 0.9], [0.7 ,0.8],[0.6,0.9]> , <h_2, [0.1, 0.8], [0.6 ,0.7],[0.8,0.7]> }),

(e_4,{<h_1,[0.8, 0.8], [0.1 ,0.9],[0.3,0.3]>,<h_2, [0.6, 0.9],[0.5,0.9],[0.8,0.9]> }),

(e_5,{<h_1, [0.3, 0.4],[0 .1,0.2],[0.8,0.8]> ,<h_2,[0.5, 0.9], [0.8 ,0.9],[0.1,0.2]>}),

(e_6,{<h_1, [0.1, 0.2], [0.8 ,0.9],[0.7,0.7]>) , <h_2, [0.7, 0.8], [0.8,0.9],[0.0,0.4]>}) }

Then we find the similarity measure of these two ivn-soft sets as:

$S^H_{IVNSS}(\Upsilon ,\Psi)= \frac{1}{1+d^H_{IVNSS}(\Upsilon ,\Psi)} =0.17$

Hence the two ivn-softsets, i.e. two symptoms ϒ and Ψare not significantly similar. Therefore, we conclude that the person is not possibly suffering from pneumonia. A person suffering from the following symptoms whose corresponding ivn-soft set Ω is given below:

Ω= { (e_1,{<h_1,[0.5, 0.7], [0.5 ,0.7],[0.3,0.5]> ,<h_2, [0.6, 0.6], [0.6 ,0.8],[0.3,0.5]>}),

(e_2,{< h_1, [0.5, 0.7], [0.5,0.7],[0.3,0.4]> ,<h_2,[0.2, 0.4], [0.6 ,0.7],[0.2,0.7]>}),

(e_3,{<h_1, [0.4, 0.7], [0.2 ,0.2],[0.1,0.3]> , <h_2, [0.4, 0.8], [0.2 ,0.8],[0.2,0.8]> }),

(e_4,{<h_1,[0.3, 0.4], [0.1 ,0.5],[0.2,0.6]>,<h_2, [0.2, 0.5],[0.3,0.4],[0.4,0.5]> }),

(e_5,{<h_1, [0.5, 0.6],[0 .6,0.7],[0.3,0.4]> ,<h_2,[0.4, 0.6], [0.3 ,0.5],[0.1,0.8]>}),

(e_6,{<h_1, [0.4, 0.7], [0.3 ,0.7],[0.2,0.8]>) , <h_2, [0.5, 0.2], [0.3,0.5],[0.2,0.5]>}) }

Then,

$S^H_{IVNSS}(\Upsilon ,\Omega)= \frac{1}{1+d^H_{IVNSS}(\Upsilon ,\Psi)} = 0.512$

Here the two ivn-soft sets, i.e. two symptoms ϒ and Ω are significantly similar. Therefore, we conclude that the person is possibly suffering from pneumonia. This is only a simple example to show the possibility of using this method for diagnosis of diseases which could be improved by incorporating clinical results and other competing diagnosis.

Conclusions

In this paper we have defined, for the first time, the notion of distance and similarity measures between two interval neutrosophic soft sets. We have studied few properties of distance and similarity measures. The similarity measures have natural applications in the field of pattern recognition, feature extraction, region extraction, image processing, coding theory etc. The results of the proposed similarity measure and existing similarity measure are compared. We also give an application for similarity measures of interval neutrosophic soft sets.

Acknowledgements

The authors are very grateful to the anonymous referees for their insightful and constructive comments and suggestions, which have been very helpful in improving the paper.

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Published in *Proceedings of the 17th International Conference on Information Fusion,* Salamanca, Spain, 7-10 July 2014.

Generalized Interval Neutrosophic Soft Set and its Decision Making Problem

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Abstract – In this work, we introduce the concept of generalized interval neutrosophic soft set and study their operations. Finally, we present an application of generalized interval neutrosophic soft set in decision making problem.

Keywords – *Soft set, neutrosophic set, neutrosophic soft set, decision making*

1. Introduction

Neutrosophic sets, founded by Smarandache [8] has capability to deal with uncertainty, imprecise, incomplete and inconsistent information which exist in real world. Neutrosophic set theory is a powerful tool which generalizes the concept of the classic set, fuzzy set [16], interval-valued fuzzy set [10], intuitionistic fuzzy set [13] interval-valued intuitionistic fuzzy set [14], and so on.

After the pioneering work of Smarandache, Wang [9] introduced the notion of interval neutrosophic set (INS) which is another extension of neutrosophic set. INS can be described by a membership interval, a non-membership interval and indeterminate interval, thus the interval value (INS) has the virtue of complementing NS, which is more flexible and practical than neutrosophic set, and interval neutrosophic set provides a morereasonable mathematical framework to deal with indeterminate and inconsistent information.The theory of neutrosophic sets and their hybrid structures has proven useful in many different fields such as control theory [25], databases [17,18], medical diagnosis problem [3,11], decision making problem [1,2,15,19,23,24,27,28,29,30,31,32,34], physics[7], and etc.
In 1999, a Russian researcher [5] firstly gave the soft set theory as a general mathematical tool for dealing with uncertainty and vagueness. Soft set theory is free from the parameterization inadequacy syndrome of fuzzy set theory, rough set theory, probability theory. Recently, some authors have introduced new mathematical tools by generalizing and extending Molodtsov's classical soft set theory;

fuzzy soft set [22], vague soft set [35], intuitionistic fuzzy soft set [20], interval valued intuitionistic fuzzy set [36].

Similarity, combining neutrosophic set models with other mathematical models has attracted the attention of many researchers: neutrosophic soft set [21], intuitionistic neutrosophic soft set [26], generalized neutrosophic soft set [23], interval neutrosophic soft set [12].

Broumi et al. [33] presented the concept of rough neutrosophic set which is based on a combination of the neutrosophic set and rough set models. Recently, Şahin and Küçük [23] generalized the concept of neutrosophic soft set with a degree of which is attached with the parameterization of fuzzy sets while defining a neutrosophic soft set, and investigated some basic properties of the generalized neutrosophic soft sets.

In this paper our main objective is to extend the concept of generalized neutrosophic soft set introduced by Şahin and Küçük [23] to the case of interval neutrosophic soft set [12].

The paper is structured as follows. In Section 2, we first recall the necessary background on neutrosophic sets,soft set and generalized neutrosophic soft set. The concept of generalized interval neutrosophic soft sets and some of their properties are presented in Section 3.In Section 4, we present an application of generalized interval neutrosophic soft sets in decision making. Finally we conclude the paper.

2. Preliminaries

In this section, we will briefly recall the basic concepts of neutrosophic set,soft sets and generalized neutrosophic soft sets. Let U be an initial universe set of objects and E the set of parameters in relation to objects in U. Parameters are often attributes, characteristics or properties of objects. Let $P(U)$ denote the power set of U and $A \subseteq E$.

2.1 Neutrosophic Sets

Definition 2.1 [8]. Let U be an universe of discourse.The neutrosophic set A is an object having the form $A = \{< x: u_A(x), w_A(x), v_A(x) > : x \in U\}$,where the functions $u, w, v : U \to]0^-, 1^+[$define respectively the degree of membership, the degree of indeterminacy, and the degree of non-membership of the element $x \in U$ to the set A with the condition.

$$0^- \leq u_A(x) + w_A(x) + v_A(x)) \leq 3^+$$

From philosophical point of view, the neutrosophic set takes the value from real standard or non-standard subsets of $]^-0,1^+[$. So instead of $]^-0,1^+[$ we need to take the interval [0,1] for technical applications, because $]^-0,1^+[$will be difficult to apply in the real applicationssuch as in scientific and engineering problems.

Definition 2.2 [8] A neutrosophicset $A$is contained in the other neutrosophic set B, $A \subseteq B$ iff $\inf u_A(x) \leq \inf u_B(x)$, $\sup u_A(x) \leq \sup u_B(x)$, $\inf w_A(x) \geq \inf w_B(x)$, $\sup w_A(x) \geq \sup w_B(x)$and $\inf v_A(x) \geq \inf v_B(x)$, $\sup v_A(x) \geq \sup v_B(x)$ for all $x \in U$.

An INS is an instance of a neutrosophic set, which can be used in real scientific and engineering applications. In the following, we introduce the definition of an INS.

2.2 Interval Neutrosophic Sets

Definition 2.3 [9] Let U be a space of points (objects) and Int[0,1] be the set of all closed subsets of [0,1]. An INS A in U is defined with the form

$$A = \{\langle x, u_A(x), w_A(x), v_A(x)\rangle : x \in U\}$$

where $u_A(x): U \to \text{int}[0,1]$, $w_A(x): U \to \text{int}[0,1]$ and $v_A(x): U \to \text{int}[0,1]$ with $0 \le \sup u_A(x) + \sup w_A(x) + \sup v_A(x) \le 3$ for all $x \in U$. The intervals $u_A(x), w_A(x)$ and $v_A(x)$ denote the truth-membership degree, the indeterminacy-membership degree and the falsity membership degree of $x$to A, respectively.

For convenience,

if let $u_A(x) = [u_A^-(x), u_A^+(x)]$, $w_A(x) = [w_A^-(x), w_A^+(x)]$ and $v(x) = [v_A^-(x), v_A^+(x)]$, then

$$A = \{\langle x, [u_A^-(x), u_A^+(x)], [w_A^-(x), w_A^+(x)], [v_A^-(x), v_A^+(x)]\rangle : x \in U\}$$

with the condition, $0 \le \sup u_A^+(x) + \sup w_A^+(x) + \sup v_A^+(x) \le 3$ for all $x \in U$. Here, we only consider the sub-unitary interval of [0,1]. Therefore, an INS is clearly a neutrosophic set.

Definition 2.4 [9] Let A and B be two interval neutrosophic sets,

$$A = \{\langle x, [u_A^-(x), u_A^+(x)], [w_A^-(x), w_A^+(x)], [v_A^-(x), v_A^+(x)]\rangle : x \in U\}$$
$$B = \{\langle x, [u_B^-(x), u_B^+(x)], [w_B^-(x), w_B^+(x)], [v_B^-(x), v_B^+(x)]\rangle : x \in U\}.$$

Then some operations can be defined as follows:

(1) $A \subseteq B$ iff $u_A^-(x) \le u_B^-(x), u_A^+(x) \le u_B^+(x), w_A^-(x) \ge w_B^-(x), w_A^+(x) \ge w_B^+(x) v_A^-(x) \ge v_B^-(x), v_A^+(x) \ge v_B^+(x)$ for each $x \in U$.
(2) $A = B$ iff $A \subseteq B$ and $B \subseteq A$.
(3) $A^c = \{\langle x, [v_A^-(x), v_A^+(x)], [1 - w_A^+(x), 1 - w_A^-(x)], [u_A^-(x), u_A^+(x)]\rangle : x \in U\}$

2.3 Soft Sets

Defnition2.5 [5] A pair (F, A) is called a soft set over, where F is a mapping given by $F : A \to P(U)$. In other words, a soft set over U is a mapping from parameters to the power set of U, and it is not a kind of set in ordinary sense, but a parameterized family of subsets of U. For any parameter$e \in A$, $F(e)$ may be considered as the set of e −approximate elements of the soft set (F, A).

Example 2.6 Suppose that U is the set of houses under consideration, say $U = \{h_1, h_2, \dots, h_5\}$. Let E be the set of some attributes of such houses, say $E = \{e_1, e_2, e_3, e_4\}$, where e_1, e_2, e_3, e_4 stand for the attributes “beautiful”, “costly”, “in the green surroundings” and “moderate”, respectively.
In this case, to define a soft set means to point out expensive houses, beautiful houses, and so on. For example, the soft set (F, A) that describes the “attractiveness of the houses” in the opinion of a buyer, say Thomas, may be defined like this:

$$F(e_1) = \{h_2, h_3, h_5\}, F(e_2) = \{h_2, h_4\}, F(e_4) = \{h_3, h_5\} \text{ for } A = \{e_1, e_2, e_4\}.$$

2.4 Neutrosophic Soft Sets

Definition 2.7 [21] Let $\boldsymbol{U}$ be an initial universe set and $\boldsymbol{A} \subset \boldsymbol{E}$ be a set of parameters. Let NS(U) denotes the set of all neutrosophic subsets of $\boldsymbol{U}$. The collection $(\boldsymbol{F}, \boldsymbol{A})$ is termed to be the neutrosophic soft set over $\boldsymbol{U}$, where **F** is a mapping given by $\boldsymbol{F}: \boldsymbol{A} \rightarrow \boldsymbol{NS}(\boldsymbol{U})$.

Example 2.8 [21] Let U be the set of houses under consideration and E is the set of parameters. Each parameter is a neutrosophic word or sentence involving neutrosophic words. Consider E ={beautiful, wooden, costly, very costly, moderate, green surroundings, in good repair, in bad repair, cheap, expensive}. In this case, to define a neutrosophic soft set means to point out beautiful houses, wooden houses, houses in the green surroundings and so on. Suppose that, there are five houses in the universe U given by $U = \{h_1, h_2, \ldots, h_5\}$ and the set of parameters

$A = \{e_1, e_2, e_3, e_4\}$, where e_1 stands for the parameter `beautiful', $e_2$ stands for the parameter `wooden', e_3 stands for the parameter `costly' and the parameter $e_4$ stands for `moderate'. Then the neutrosophic set (F, A) is defined as follows:

$$(F,A) = \begin{Bmatrix} \left(e_1\left\{\frac{h_1}{(0.5,0.6,0.3)},\frac{h_2}{(0.4,0.7,0.6)},\frac{h_3}{(0.6,0.2,0.3)},\frac{h_4}{(0.7,0.3,0.2)},\frac{h_5}{(0.8,0.2,0.3)}\right\}\right) \\ \left(e_2\left\{\frac{h_1}{(0.6,0.3,0.5)},\frac{h_2}{(0.7,0.4,0.3)},\frac{h_3}{(0.8,0.1,0.2)},\frac{h_4}{(0.7,0.1,0.3)},\frac{h_5}{(0.8,0.3,0.6)}\right\}\right) \\ \left(e_3\left\{\frac{h_1}{(0.7,0.4,0.3)},\frac{h_2}{(0.6,0.7,0.2)},\frac{h_3}{(0.7,0.2,0.5)},\frac{h_4}{(0.5,0.2,0.6)},\frac{h_5}{(0.7,0.3,0.4)}\right\}\right) \\ \left(e_4\left\{\frac{h_1}{(0.8,0.6,0.4)},\frac{h_2}{(0.7,0.9,0.6)},\frac{h_3}{(0.7,0.6,0.4)},\frac{h_4}{(0.7,0.8,0.6)},\frac{h_5}{(0.9,0.5,0.7)}\right\}\right) \end{Bmatrix}$$

2.5 Interval Neutrosophic Soft Sets

Definition 2.9 [12] Let $\boldsymbol{U}$ be an initial universe set and $\boldsymbol{A} \subset \boldsymbol{E}$ be a set of parameters. Let INS(U) denotes the set of all interval neutrosophic subsets of $\boldsymbol{U}$. The collection $(\boldsymbol{F}, \boldsymbol{A})$ is termed to be the interval neutrosophic soft set over $\boldsymbol{U}$, where **F** is a mapping given by $\boldsymbol{F}: \boldsymbol{A} \rightarrow \boldsymbol{INS}(\boldsymbol{U})$.

Example 2.10 [12] Let $\boldsymbol{U} = \{\boldsymbol{x_1}, \boldsymbol{x_2}\}$ be set of houses under consideration and **E** is a set of parameters which is a neutrosophic word. Let **E** be the set of some attributes of such houses, say $\boldsymbol{E} = \{\boldsymbol{e_1}, \boldsymbol{e_2}, \boldsymbol{e_3}, \boldsymbol{e_4}\}$, where $\mathbf{e_1}, \mathbf{e_2}, \mathbf{e_3}, \mathbf{e_4}$ stand for the attributes $\mathbf{e_1}$ = cheap, $\mathbf{e_2}$ = beautiful, $\mathbf{e_3}$ = in the green surroundings, $\mathbf{e_4}$ = costly and $\mathbf{e_5}$ = large, respectively. Then we define the interval neutrosophic soft set **A** as follows:

$$(\boldsymbol{F},\boldsymbol{A}) = \begin{Bmatrix} \left(\boldsymbol{e_1}\left\{\frac{\boldsymbol{x_1}}{[\mathbf{0.5},\mathbf{0.8}],[\mathbf{0.5},\mathbf{0.9}],[\mathbf{0.2},\mathbf{0.5}]},\frac{\boldsymbol{x_2}}{[\mathbf{0.4},\mathbf{0.8}],[\mathbf{0.2},\mathbf{0.5}],[\mathbf{0.5},\mathbf{0.6}]}\right\}\right) \\ \left(\boldsymbol{e_2}\left\{\frac{\boldsymbol{x_1}}{[\mathbf{0.5},\mathbf{0.8}],[\mathbf{0.2},\mathbf{0.8}],[\mathbf{0.3},\mathbf{0.7}]},\frac{\boldsymbol{x_2}}{[\mathbf{0.1},\mathbf{0.9}],[\mathbf{0.6},\mathbf{0.7}],[\mathbf{0.2},\mathbf{0.3}]}\right\}\right) \\ \left(\boldsymbol{e_3}\left\{\frac{\boldsymbol{x_1}}{[\mathbf{0.2},\mathbf{0.7}],[\mathbf{0.1},\mathbf{0.5}],[\mathbf{0.5},\mathbf{0.8}]},\frac{\boldsymbol{x_2}}{[\mathbf{0.5},\mathbf{0.7}],[\mathbf{0.1},\mathbf{0.4}],[\mathbf{0.6},\mathbf{0.7}]}\right\}\right) \\ \left(\boldsymbol{e_4}\left\{\frac{\boldsymbol{x_1}}{[\mathbf{0.4},\mathbf{0.5}],[\mathbf{0.4},\mathbf{0.9}],[\mathbf{0.4},\mathbf{0.9}]},\frac{\boldsymbol{x_2}}{[\mathbf{0.3},\mathbf{0.4}],[\mathbf{0.6},\mathbf{0.7}],[\mathbf{0.1},\mathbf{0.5}]}\right\}\right) \\ \left(\boldsymbol{e_5}\left\{\frac{\boldsymbol{x_1}}{[\mathbf{0.1},\mathbf{0.7}],[\mathbf{0.5},\mathbf{0.6}],[\mathbf{0.1},\mathbf{0.5}]},\frac{\boldsymbol{x_2}}{[\mathbf{0.6},\mathbf{0.7}],[\mathbf{0.2},\mathbf{0.4}],[\mathbf{0.3},\mathbf{0.7}]}\right\}\right) \end{Bmatrix}$$

2.6 Generalized Neutrosophic Soft Sets

The concept of generalized neutrosophic soft is defined by Şahin and Küçük [23] as follows:

Definition 2.11 [23] Let U be an intial universe and E be a set of parameters. Let $NS(U)$ be the set of all neutrosophic sets of U. A generalized neutrosophic soft set F^{μ} over U is defined by the set of ordered pairs

$$F^{\mu} = \{(F(e), \mu(e)): e \in E, F(e) \in N(U), \mu(e) \in [0,1]\},$$

where F isa mapping given by $F: E \to NS(U) \times I$ and μ is a fuzzy set such that $\mu: E \to I = [0,1]$. Here, F^{μ} is a mapping defined by $F^{\mu}: E \to NS(U) \times I$.

For any parameter $e \in E$, $F(e)$ is referred as the neutrosophic value set of parameter e, i.e,

$$F(e) = \{\langle x, u_{F(e)}(x), w_{F(e)}(x), v_{F(e)}(x)\rangle : x \in U\}$$

where u, w, v : U→ [0 ,1] are the memberships functions of truth, indeterminacy and falsity respectively of the element $x \in U$. For any $x \in U$ and $e \in E$,

$$0 \le u_{F(e)}(x) + w_{F(e)}(x) + v_{F(e)}(x) \le 3.$$

In fact, F^{μ} is a parameterized family of neutrosophic sets over U, which has the degree of possibility of the approximate value set which is represented by $\mu(e)$ for each parameter e, so F^{μ} can be expressed as follows:

$$F^{\mu}(e) = \left\{\left(\frac{x_1}{F(e)(x_1)}, \frac{x_2}{F(e)(x_2)}, \ldots\ldots, \frac{x_n}{F(e)(x_n)}\right), \mu(\mathrm{e})\right\}.$$

Definition 2.12 [4] A binary operation $\otimes: [0,1] \times [0,1] \longrightarrow [0,1]$ is continuous $t-$norm if $\otimes$ satisfies the following conditions:

(1) $\otimes$ is commutative and associative,
(2) $\otimes$ is continuous,
(3) $a \otimes 1 = a, \forall a \in [0,1]$,
(4) $a \otimes b \le c \otimes d$ whenever $a \le c, b \le d$ and $a, b, c, d \in [0,1]$.

Definition 2.13 [4] A binary operation $\oplus: [0,1] \times [0,1] \longrightarrow [0,1]$ is continuous $t-$conorm if $\oplus$ satisfies the following conditions:

(1) $\oplus$ is commutative and associative,
(2) $\oplus$ is continuous,
(3) $a \oplus 0 = a, \forall a \in [0,1]$,
(4) $a \oplus b \le c \oplus d$ whenever $a \le c, b \le d$ and $a, b, c, d \in [0,1]$.

3. Generalized Interval Neutrosophic Soft Set

In this section, we define the generalized interval neutrosophic soft sets and investigate some basic properties.

Definition 3.1. Let U be an initial universe and E be a set of parameters.Suppose that$INS(U)$is the set of all interval neutrosophic sets over$U$ and $\text{int}[0,1]$is the set of all closed subsets of $[0,1]$. A generalized interval neutrosophic soft set F^{μ} over U is defined by the set of ordered pairs

$$F^{\mu} = \{(F(e), \mu\,(e)): e \in E\,, F(e) \in INS(U), \mu(e) \in [0,1]\},$$

where F is a mapping given by$F: E \to INS(U) \times I$ and μ is a fuzzy set such that $\mu: E \to I = [0,1]$. Here,$F^{\mu}$is a mapping defined by$F^{\mu}: E \to INS(U) \times I$.

For any parameter $e \in E$,$F(e)$ is referred as the interval neutrosophic value set of parameter e, i.e,

$$F(e) = \{\langle x, u_{F(e)}(x), w_{F(e)}(x), v_{F(e)}(x)\rangle: x \in U\}$$

where $u_{F(e)}, w_{F(e)}, v_{F(e)}: U \to \text{int}[0\,,1]$with the condition

$$0 \leq \sup u_{F(e)}(x) + \sup w_{F(e)}(x) + \sup v_{F(e)}\,(x) \leq 3$$

for all $x \in U$.

The intervals $u_{F(e)}(x), w_{F(e)}(x)$ and $v_{F(e)}(x)$are the interval memberships functions of truth, interval indeterminacy and interval falsity of the element $x \in U$, respectively.

For convenience, if let

$$u_{F(e)}(x) = [u^{L}_{F(e)}(x), u^{U}_{F(e)}(x)]$$
$$w_{F(e)}(x) = [w^{L}_{F(e)}(x), w^{U}_{F(e)}(x)]$$
$$v_{F(e)}(x) = [v^{L}_{F(e)}(x), v^{U}_{F(e)}(x)]$$

then

$$F(e) = \{\langle x, [u^{L}_{F(e)}(x), u^{U}_{F(e)}(x)], [w^{L}_{F(e)}(x), w^{U}_{F(e)}(x)], [v^{L}_{F(e)}(x), v^{U}_{F(e)}(x)]\rangle: x \in U\}$$

In fact, $F^{\mu}$is a parameterized family of interval neutrosophic sets on U, which has the degree of possibility of the approximate value set which is represented by $\mu\,(e)$ for each parameter e, so F^{μ}can be expressed as follows:

$$F^{\mu}(e) = \left\{\left(\frac{x_1}{F(e)(x_1)}, \frac{x_2}{F(e)(x_2)}, \dots, \frac{x_n}{F(e)(x_n)}\right), \mu\,(\mathrm{e})\right\}$$

Example 3.2. Consider two generalized interval neutrosophic soft set $F^{\mu}$and G^{θ}. Suppose that $U = \{h_1, h_2, h_3\}$ is the set of house and $E = \{e_1, e_2, e_3\}$ is the set of parameters where e_1 =cheap,e_2 =moderate,e_3 =comfortable. Suppose that F^{μ} and $G^{\theta}$are given as follows, respectively:

$$F^{\mu}(e_1) = \left(\frac{h_1}{([0.2, 0.3], [0.3, 0.5], [0.2, 0.3])}, \frac{h_2}{([0.3, 0.4], [0.3, 0.4], [0.5, 0.6])}, \frac{h_3}{([0.5, 0.6], [0.2, 0.4], [0.5, 0.7])}\right), (0.2)$$
$$F^{\mu}(e_2) = \left(\frac{h_1}{([0.1, 0.4], [0.5, 0.6], [0.3, 0.4])}, \frac{h_2}{([0.6, 0.7], [0.4, 0.5], [0.5, 0.8])}, \frac{h_3}{([0.2, 0.4], [0.3, 0.6], [0.6, 0.9])}\right), (0.5)$$
$$F^{\mu}(e_3) = \left(\frac{h_1}{([0.2, 0.6], [0.2, 0.5], [0.1, 0.5])}, \frac{h_2}{([0.3, 0.5], [0.3, 0.6], [0.4, 0.5])}, \frac{h_3}{([0.6, 0.8], [0.3, 0.4], [0.2, 0.3])}\right), (0.6)$$

and

$$G^{\theta}(e_1) = \left(\frac{h_1}{([0.1, 0.2], [0.1, 0.2], [0.1, 0.2])}, \frac{h_2}{([0.4, 0.5], [0.2, 0.3], [0.3, 0.5])}, \frac{h_3}{([0.6, 0.7], [0.1, 0.3], [0.2, 0.3])}\right), (0.4)$$
$$G^{\theta}(e_2) = \left(\frac{h_1}{([0.2, 0.5], [0.3, 0.4], [0.2, 0.3])}, \frac{h_2}{([0.7, 0.8], [0.3, 0.4], [0.4, 0.6])}, \frac{h_3}{([0.3, 0.6], [0.2, 0.5], [0.4, 0.6])}\right), (0.7)$$
$$G^{\theta}(e_3) = \left(\frac{h_1}{([0.3, 0.5], [0.1, 0.3], [0.1, 0.3])}, \frac{h_2}{([0.4, 0.5], [0.1, 0.5], [0.2, 0.3])}, \frac{h_3}{([0.7, 0.9], [0.2, 0.3], [0.1, 0.2])}\right), (0.8)$$

For the purpose of storing a generalized interval neutrosophic soft sets in a computer, we can present it in matrix form. For example, the matrix form of F^{μ} can be expressed as follows;

$$\begin{pmatrix} ([0.2, 0.3], [0.3, 0.5], [0.2, 0.3]) & ([0.3, 0.4], [0.3, 0.4], [0.5, 0.6]) & ([0.5, 0.6], [0.2, 0.4], [0.5, 0.7]), (0.2) \\ ([0.1, 0.4], [0.5, 0.6], [0.3, 0.4]) & ([0.6, 0.7], [0.4, 0.5], [0.5, 0.8]) & ([0.2, 0.4], [0.3, 0.6], [0.6, 0.9]), (0.5) \\ ([0.2, 0.6], [0.2, 0.5], [0.1, 0.5]) & ([0.3, 0.5], [0.3, 0.6], [0.4\ 0.5]) & ([0.6, 0.8], [0.3, 0.4], [0.2, 0.3]), (0.6) \end{pmatrix}$$

Definition 3.3. A generalized interval neutrosophic soft set F^{μ} over U is said to be generalized null interval neutrosophic soft set ,denoted by $\emptyset^{\mu}$, if $\emptyset^{\mu}: E \to$ IN(U) ×I such that

$\emptyset^{\mu}(e) = \{(F(e), \mu\,(e)\}$, where $F(e) = \{< x, ([0, 0], [1,1], [1, 1]) >\}$ and $\mu\,(e) = 0$ for each $e \in E$ and $x \in U$.

Definition 3.4. A generalized interval neutrosophic soft set F^{μ} over U is said to be generalized absolute interval neutrosophic soft set, denoted by U^{μ}, if $U^{\mu}: E \to IN(U) \times I$ such that $U^{\mu}(e) = \{(F(e), \mu\,(e)\}$,where $F(e) = \{< x, ([1,1], [0\ ,0], [0, 0]) >\}$ and $\mu\,(e) = 1$ for each $e \in E$ and $x \in U$.

Definition 3.5. Let F^{μ} be a generalized interval neutrosophic soft set over U, where

$$F^{\mu}\ (\mathrm{e}) = \{(F(e), \mu\,(e)\}$$

and

$$F(e) = \left\{\langle x, \left[u^L_{F(e)}(x), u^U_{F(e)}(x)\right], \left[w^L_{F(e)}(x), w^U_{F(e)}(x)\right], [v^L_{F(e)}(x), v^U_{F(e)}(x)]\rangle : x \in U\right\}$$

for all $e \in E$. Then, for $e_m \in E$ and $x_n \in U$;

(1) $F^{\star} = [\mathrm{F}^{\star}_L, \mathrm{F}^{\star}_U]$ is said to be interval truth membership part of F^{μ} where $F^{\star} = \{(\mathrm{F}^{\star}{}_{mn}\,(e_m)\,, \mu\,(e_m))\}$ and $F^{\star}{}_{mn}(e_m) = \left\{\langle x_n, \left[u^L_{F(e_m)}(x_n), u^U_{F(e_m)}(x_n)\right]\rangle\right\}$,
(2) $\mathrm{F}^{\wr} = [\mathrm{F}^{\wr}_L, \mathrm{F}^{\wr}_U]$ is said to be interval indeterminacy membership part of F^{μ} where $F^{\wr} = \{F^{\wr}{}_{mn}\,(e_m)\,, \mu\,(e_m)\}$ and $F^{\wr}{}_{mn}(e_m) = \left\{\langle x_n, \left[w^L_{F(e_m)}(x_n), w^U_{F(e_m)}(x_n)\right]\rangle\right\}$,
(3) $\mathrm{F}^{\triangle} = [\mathrm{F}^{\triangle}_L, \mathrm{F}^{\triangle}_U]$ is said to be interval falsity membership part of F^{μ} where $\mathrm{F}^{\triangle} = \{\mathrm{F}^{\triangle}{}_{mn}\,(e_m)\,, \mu\,(e_m)\}$ and $\mathrm{F}^{\triangle}{}_{mn}(e_m) = \left\{\langle x_n, \left[v^L_{F(e_m)}(x_n), v^U_{F(e_m)}(x_n)\right]\rangle\right\}$.

We say that every part of F^{μ} is a component of itself and is denote by $F^{\mu} = (\mathrm{F}^{\star}, \mathrm{F}^{\wr}, \mathrm{F}^{\triangle})$. Then matrix forms of components of F^{μ} in example 3.2 can be expressed as follows:

$$F^{\star}=\begin{pmatrix} ([0.2,0.3],[0.3,0.6],[0.4,0.5]) & (0.1) \\ ([0.2,0.5],[0.3,0.5],[0.4,0.7]) & (0.4) \\ ([0.3,0.4],[0.1,0.3],[0.1,0.4]) & (0.6) \end{pmatrix}$$

$$F^{\wr}=\begin{pmatrix} ([0.2,0.3],[0.3,0.5],[0.2,0.5]) & (0.1) \\ ([0.2,0.5],[0.4,0.8],[0.3,0.8]) & (0.4) \\ ([0.3,0.4],[0.2,0.5],[0.2,0.3]) & (0.6) \end{pmatrix}$$

$$F^{\Delta}=\begin{pmatrix} ([0.2,0.3],[0.2,0.4],[0.2,0.6]) & (0.1) \\ ([0.2,0.5],[0.8,0.9],[0.3,0.4]) & (0.4) \\ ([0.7,0.9],[0.3,0.7],[0.5,0.7]) & (0.6) \end{pmatrix}$$

where

$$F^{\star}{}_{mn}(e_m) = \left\{\left\langle x_n, \left[u^L_{F(e_m)}(x_n), u^U_{F(e_m)}(x_n)\right]\right\rangle\right\}$$
$$F^{\wr}{}_{mn}(e_m) = \left\{\left\langle x_n, \left[w^L_{F(e_m)}(x_n), w^U_{F(e_m)}(x_n)\right]\right\rangle\right\}$$
$$F^{\Delta}{}_{mn}(e_m) = \left\{\left\langle x_n, \left[v^L_{F(e_m)}(x_n), v^U_{F(e_m)}(x_n)\right]\right\rangle\right\}$$

are defined as the interval truth, interval indeterminacy and interval falsity values of n −th element the according to m −th parameter, respectively.

Remark 3.6. Suppose that F^{μ} is a generalizedinterval neutrosophic soft set over U.Then we say that each components ofF^{μ}can be seen as the generalizedinterval valued vague soft set [15]. Also if it is taken $\mu\,(e)\;=1$ for all $e \in$ E,the our generalized interval neutrosophic soft set concides with the interval neutrosophic soft set [12].

Definition 3.7. Let U be an universe and E be a of parameters, F^{μ} and G^{θ} be two generalized interval neutrosophic soft sets, we say that F^{μ} is a generalized interval neutrosophic soft subset G^{θ} if

(1) μ is a fuzzy subset of θ,
(2) For $e \in E$, $F(e)$ is an interval neutrosophic subset of$G(e)$, i.e, for all $e_m \in E$ and $m, n \in \wedge$,

$F^{\star}{}_{mn}(e_m) \le G^{\star}{}_{mn}(e_m)$, $F^{\wr}{}_{mn}(e_m) \ge G^{\wr}{}_{mn}(e_m)$ and $F^{\Delta}{}_{mn}(e_m) \ge G^{\Delta}{}_{mn}(e_m)$ where,

$$u^L_{F(e_m)}(x_n) \le\; u^L_{G(e_m)}(x_n), u^U_{F(e_m)}(x_n) \le\; u^U_{G(e_m)}(x_n)$$
$$w^L_{F(e_m)}(x_n) \ge w^L_{G(e_m)}(x_n), w^U_{F(e_m)}(x_n) \ge w^U_{G(e_m)}(x_n)$$
$$v^L_{F(e_m)}(x_n) \ge\; v^L_{G(e_m)}(x_n), v^U_{F(e_m)}(x_n) \ge\; v^U_{G(e_m)}(x_n)$$

For $x_n \in U$.

We denote this relationship by$F^{\mu} \sqsubseteq G^{\theta}$. Moreover if$G^{\theta}$ is generalized interval neutrosophic soft subset of F^{μ}, thenF^{μ} is called a generalized interval neutrosophic soft superset of G^{θ} this relation is denoted by $F^{\mu} \sqsupseteq G^{\theta}$.

Example 3.8. Consider two generalized interval neutrosophic soft set F^{μ} and G^{θ}.suppose that U= { h_1 , h_2 , h_3] is the set of houses and E $=\{e_1, e_2, e_3\}$ is the set of parameters where e_1=cheap,e_2 =moderate,e_3 =comfortable. Suppose that F^{μ} and $G^{\theta}$are given as follows respectively:

$$\begin{cases} F^{\mu}(e_1) = \left(\frac{h_1}{([0.1,0.2],[0.3,0.5],[0.2,0.3])}, \frac{h_2}{([0.3,0.4],[0.3,0.4],[0.5,0.6])}, \frac{h_3}{([0.5,0.6],[0.2,0.4],[0.5,0.7])}\right),(0.2) \\ F^{\mu}(e_2) = \left(\frac{h_1}{([0.1,0.4],[0.5,0.6],[0.3,0.4])}, \frac{h_2}{([0.6,0.7],[0.4,0.5],[0.5,0.8])}, \frac{h_3}{([0.2,0.4],[0.3,0.6],[0.6,0.9])}\right),(0.5) \\ F^{\mu}(e_3) = \left(\frac{h_1}{([0.2,0.6],[0.2,0.5],[0.1,0.5])}, \frac{h_2}{([0.3,0.5],[0.3,0.6],[0.4,0.5])}, \frac{h_3}{([0.6,0.8],[0.3,0.4],[0.2,0.3])}\right),(0.6) \end{cases}$$

and

$$\begin{cases} G^{\theta}(e_1) = \left(\frac{h_1}{([0.2,0.3],[0.1,0.2],[0.1,0.2])}, \frac{h_2}{([0.4,0.5],[0.2,0.3],[0.3,0.5])}, \frac{h_3}{([0.6,0.7],[0.1,0.3],[0.2,0.3])}\right),(0.4) \\ G^{\theta}(e_2) = \left(\frac{h_1}{([0.2,0.5],[0.3,0.4],[0.2,0.3])}, \frac{h_2}{([0.7,0.8],[0.3,0.4],[0.4,0.6])}, \frac{h_3}{([0.3,0.6],[0.2,0.5],[0.4,0.6])}\right),(0.7) \\ G^{\theta}(e_3) = \left(\frac{h_1}{([0.3,0.7],[0.1,0.3],[0.1,0.3])}, \frac{h_2}{([0.4,0.5],[0.1,0.5],[0.2,0.3])}, \frac{h_3}{([0.7,0.9],[0.2,0.3],[0.1,0.2])}\right),(0.8) \end{cases}$$

Then $F^{\mu}$is a generalized interval neutrosophic soft subset ofG^{θ}, that is$F^{\mu} \sqsubseteq G^{\theta}$.

Definition3.9. The union of two generalized interval neutrosophic soft setsF^{μ}andG^{θ}over U, denoted by $\mathrm{H}^{\lambda} = F^{\mu} \sqcup G^{\theta}$ is a generalized interval neutrosophic soft setH^{λ}defined by

$$\mathrm{H}^{\lambda} = ([\, \mathrm{H}_L^{\star}, \mathrm{H}_U^{\star}], [\, \mathrm{H}_L^{\wr}, \mathrm{H}_U^{\wr}], [\, \mathrm{H}_L^{\triangle}, \mathrm{H}_U^{\triangle}])$$

where$\lambda\ (e_m) = \mu\ (e_m) \oplus \theta\ (e_m)$,

$$\begin{aligned} \mathrm{H}_{L_{mn}}^{\star} &= \mathrm{F}_{L_{mn}}^{\star}(e_m) \oplus \mathrm{G}_{L_{mn}}^{\star}(e_m) \\ \mathrm{H}_{L_{mn}}^{\wr} &= \mathrm{F}_{L_{mn}}^{\wr}(e_m) \otimes \mathrm{G}_{L_{mn}}^{\wr}(e_m) \\ \mathrm{H}_{L_{mn}}^{\triangle} &= \mathrm{F}_{L_{mn}}^{\triangle}\ (e_m) \otimes \mathrm{G}_{L_{mn}}^{\triangle}(e_m) \end{aligned}$$

and

$$\begin{aligned} \mathrm{H}_{U_{mn}}^{\star} &= \mathrm{F}_{U_{mn}}^{\star}(e_m) \oplus \mathrm{G}_{U_{mn}}^{\star}(e_m) \\ \mathrm{H}_{U_{mn}}^{\wr} &= \mathrm{F}_{U_{mn}}^{\wr}(e_m) \otimes \mathrm{G}_{U_{mn}}^{\wr}(e_m) \\ \mathrm{H}_{U_{mn}}^{\triangle} &= \mathrm{F}_{U_{mn}}^{\triangle}\ (e_m) \otimes \mathrm{G}_{U_{mn}}^{\triangle}(e_m) \end{aligned}$$

for all $e_m \in$ E and $m, n \in \Lambda$.

Definition 3.10. The intersection of two generalized interval neutrosophic soft sets$F^{\mu}\ and\ G^{\theta}$over U, denoted by $\mathrm{K}^{\varepsilon} = F^{\mu} \sqcap G^{\theta}$isa generalized interval neutrosophic soft setK^{ε}defined by

$$\mathrm{K}^{\varepsilon} = ([\, \mathrm{K}_L^{\star}, \mathrm{K}_U^{\star}], [\, \mathrm{K}_L^{\wr}, \mathrm{K}_U^{\wr}], [\, \mathrm{K}_L^{\triangle}, \mathrm{K}_U^{\triangle}])$$

where$\varepsilon\ (e_m) = \mu\ (e_m) \otimes\ \theta\ (e_m)$,

$$\begin{aligned} \mathrm{K}_{L_{mn}}^{\star} &= \mathrm{F}_{L_{mn}}^{\star}(e_m) \otimes \mathrm{G}_{L_{mn}}^{\star}(e_m) \\ \mathrm{K}_{L_{mn}}^{\wr} &= \mathrm{F}_{L_{mn}}^{\wr}(e_m) \oplus \mathrm{G}_{L_{mn}}^{\wr}(e_m) \\ \mathrm{K}_{L_{mn}}^{\triangle} &= \mathrm{F}_{L_{mn}}^{\triangle}\ (e_m) \oplus \mathrm{G}_{L_{mn}}^{\triangle}(e_m) \end{aligned}$$

and

$$\begin{aligned} \mathrm{K}_{U_{mn}}^{\star} &= \mathrm{F}_{U_{mn}}^{\star}(e_m) \otimes \mathrm{G}_{U_{mn}}^{\star}(e_m) \\ \mathrm{K}_{U_{mn}}^{\wr} &= \mathrm{F}_{U_{mn}}^{\wr}(e_m) \oplus \mathrm{G}_{U_{mn}}^{\wr}(e_m) \\ \mathrm{K}_{U_{mn}}^{\triangle} &= \mathrm{F}_{U_{mn}}^{\triangle}\ (e_m) \oplus \mathrm{G}_{U_{mn}}^{\triangle}(e_m) \end{aligned}$$

for all $e_m \in$ E and $m, n \in \Lambda$.

Example 3.11. Let us consider the generalized interval neutrosophic soft sets $F^{\mu} and\ G^{\theta}$ defined in Example 3.2. Suppose that the t-conorm is defined by $\oplus(a,b) = \max\{a,b\}$ and the $t-$norm by $\otimes(a,b) = \min\{a,b\}$ for $a,b \in [0,1]$. Then $\mathrm{H}^{\lambda} = F^{\mu} \sqcup G^{\theta}$ is defined as follows:

$$\left\{\begin{array}{l} H(e_1) = \left(\frac{h_1}{([0.2,0.3],[0.1,0.2],[0.1,0.2])}, \frac{h_2}{([0.4,0.5],[0.2,0.3],[0.3,0.5])}, \frac{h_3}{([0.6,0.7],[0.1,0.3],[0.2,0.3])}\right),(0.4) \\ H(e_2) = \left(\frac{h_1}{([0.2,0.5],[0.3,0.4],[0.2,0.3])}, \frac{h_2}{([0.7,0.8],[0.3,0.4],[0.4,0.6])}, \frac{h_3}{([0.3,0.6],[0.2,0.5],[0.4,0.6])}\right),(0.7) \\ H(e_3) = \left(\frac{h_1}{([0.3,0.6],[0.1,0.3],[0.1,0.3])}, \frac{h_2}{([0.4,0.5],[0.1,0.5],[0.2,0.3])}, \frac{h_3}{([0.7,0.9],[0.2,0.3],[0.1,0.2])}\right),(0.8) \end{array}\right\}$$

Example 3.12. Let us consider the generalized interval neutrosophic soft sets $F^{\mu} and\ G^{\theta}$ defined in Example 3.2. Suppose that the $t-$conorm is defined by $\oplus(\mathrm{a},\mathrm{b}) = \max\{\mathrm{a},\mathrm{b}\}$ and the $t-$norm by $\otimes(a,b) = \min\{\mathrm{a},\mathrm{b}\}$ for $a,b \in [0,1]$. Then $\mathrm{K}^{\varepsilon} = F^{\mu} \sqcap G^{\theta}$ is defined as follows:

$$\left\{\begin{array}{l} K(e_1) = \left(\frac{h_1}{([0.1,0.2],[0.3,0.5],[0.2,0.3])}, \frac{h_2}{([0.3,0.4],[0.3,0.4],[0.5,0.6])}, \frac{h_3}{([0.5,0.6],[0.2,0.4],[0.5,0.7])}\right),(0.2) \\ K(e_2) = \left(\frac{h_1}{([0.1,0.4],[0.5,0.6],[0.3,0.4])}, \frac{h_2}{([0.6,0.7],[0.4,0.5],[0.5,0.8])}, \frac{h_3}{([0.2,0.4],[0.3,0.6],[0.6,0.9])}\right),(0.5) \\ K(e_3) = \left(\frac{h_1}{([0.2,0.5],[0.2,0.5],[0.1,0.5])}, \frac{h_2}{([0.3,0.5],[0.3,0.6],[0.4,0.5])}, \frac{h_3}{([0.6,0.8],[0.3,0.4],[0.2,0.3])}\right),(0.6) \end{array}\right\}$$

Proposition 3.13. Let F^{μ}, G^{θ} and H^{λ} be three generalized interval neutrosophic soft sets over U. Then

(1) $F^{\mu} \sqcup G^{\theta} = G^{\theta} \sqcup F^{\mu}$,
(2) $F^{\mu} \sqcap G^{\theta} = G^{\theta} \sqcap F^{\mu}$,
(3) $(F^{\mu} \sqcup G^{\theta}) \sqcup H^{\lambda} = F^{\mu} \sqcup (G^{\theta} \sqcup H^{\lambda})$,
(4) $(F^{\mu} \sqcap G^{\theta}) \sqcap H^{\lambda} = F^{\mu} \sqcap (G^{\theta} \sqcap H^{\lambda})$.

Proof. The proofs are trivial.

Proposition 3.14. Let F^{μ}, G^{θ} and H^{λ} be three generalized interval neutrosophic soft sets over U. If we consider the $t-$conorm defined by $\oplus(a,b) = max\{a,b\}$ and the $t-$norm defined by $\otimes(a,b) = min\{a,b\}$ for $a,b \in [0,1]$, then the following relations holds:

(1) $H^{\lambda} \sqcap (F^{\mu} \sqcup G^{\theta}) = (H^{\lambda} \sqcap F^{\mu}) \sqcup (H^{\lambda} \sqcap G^{\theta})$,
(2) $H^{\lambda} \sqcup (F^{\mu} \sqcap G^{\theta}) = (H^{\lambda} \sqcup F^{\mu}) \sqcap (H^{\lambda} \sqcup G^{\theta})$.

Remark 3.15. The relations in above proposition does not hold in general.

Definition 3.16. The complement of a generalized interval neutrosophic soft sets F^{μ} over U, denoted by $F^{\mu(c)}$ is defined by $F^{\mu(c)} = ([\mathrm{F}_L^{\star(c)}, \mathrm{F}_U^{\star(c)}], [\mathrm{F}_L^{\wr(c)}, \mathrm{F}_U^{\wr(c)}], [\mathrm{F}_L^{\Delta(c)}, \mathrm{F}_U^{\Delta(c)}])$ where

$$\mu^{(c)}(e_m) = 1 - \mu(e_m)$$

and

$$\mathrm{F}_{L_{mn}}^{\star(c)} = \mathrm{F}_{L_{mn}}^{\Delta};\ \mathrm{F}_{L_{mn}}^{\wr(c)} = 1 - \mathrm{F}_{U_{mn}}^{\wr};\ \mathrm{F}_{L_{mn}}^{\Delta(c)} = \mathrm{F}_{L_{mn}}^{\star}$$

$$\mathrm{F}_{U_{mn}}^{\star(c)} = \mathrm{F}_{U_{mn}}^{\triangle} \; ; \; \mathrm{F}_{U_{mn}}^{\wr(c)} = 1 - \mathrm{F}_{L_{mn}}^{\wr} ; \; \mathrm{F}_{U_{mn}}^{\triangle(c)} = \mathrm{F}_{U_{mn}}^{\star}$$

Example 3.17. Consider Example 3.2. Complement of the generalized interval neutrosophic soft set F^{μ} denoted by $F^{\mu(c)}$ is given as follows:

$$\begin{cases} F^{\mu(c)}(e_1) = \left(\frac{h_1}{([0.2,0.3],[0.5,0.7],[0.2,0.3])}, \frac{h_2}{([0.5,0.6],[0.6,0.7],[0.3,0.4])}, \frac{h_3}{([0.5,0.7],[0.6,0.8],[0.5,0.6])}\right), (0.8) \\ F^{\mu(c)}(e_2) = \left(\frac{h_1}{([0.3,0.4],[0.4,0.5],[0.1,0.4])}, \frac{h_2}{([0.5,0.8],[0.5,0.6],[0.6,0.7])}, \frac{h_3}{([0.6,0.9],[0.4,0.7],[0.2,0.4])}\right), (0.5) \\ F^{\mu(c)}(e_3) = \left(\frac{h_1}{([0.1,0.5],[80.5,0.5],[0.2,0.6])}, \frac{h_2}{([0.4,0.5],[0.4,0.7],[0.3,0.5])}, \frac{h_3}{([0.2,0.3],[0.6,0.7],[0.6,0.8])}\right), (0.4) \end{cases}$$

Proposition 3.18. LetF^{μ} and G^{θ} be two generalized interval neutrosophic soft sets over U. Then,

(1) F^{μ} is a generalized interval neutrosophic soft subset of$F^{\mu} \sqcup F^{\mu(c)}$
(2) $F^{\mu} \sqcap F^{\mu(c)}$is a generalized interval neutrosophic soft subset ofF^{μ}.

Proof: It is clear.

Definition 3.19. "And" operation on two generalized interval neutrosophic soft sets F^{μ}andG^{θ} over U,denoted by$\mathrm{H}^{\lambda} = F^{\mu} \wedge G^{\theta}$ is the mapping$\mathrm{H}^{\lambda}: C \to \mathrm{IN(U)} \times \mathrm{I}$ defined by

$$\mathrm{H}^{\lambda} = ([\,\mathrm{H}_L^{\star}, \mathrm{H}_U^{\star}], [\,\mathrm{H}_L^{\wr}, \mathrm{H}_U^{\wr}], [\,\mathrm{H}_L^{\triangle}, \mathrm{H}_U^{\triangle}])$$

where$\lambda\,(e_m) = \min(\mu\,(e_k), \theta\,(e_h)$ and

$$\begin{gathered} \mathrm{H}_L^{\star}(e_m) = \min\{\mathrm{F}_L^{\star}(e_{kn}), \mathrm{G}_L^{\star}(e_{hn})\} \\ \mathrm{H}_L^{\wr}(e_m) = \max\{\mathrm{F}_L^{\wr}(e_{kn}), \mathrm{G}_L^{\wr}(e_{hn}) \\ \mathrm{H}_L^{\triangle}(e_m) = \max\{\mathrm{F}_L^{\triangle}(e_{kn}), \mathrm{G}_L^{\triangle}(e_{hn})\} \end{gathered}$$

and

$$\begin{gathered} \mathrm{H}_{\mathrm{U}}^{\star}(\mathrm{e_m}) = \min\{\mathrm{F}_{\mathrm{U}}^{\star}(\mathrm{e_{kn}}), \mathrm{G}_{\mathrm{U}}^{\star}(\mathrm{e_{hn}})\} \\ \mathrm{H}_U^{\wr}(e_m) = \max\{\mathrm{F}_U^{\wr}(e_{kn}), \mathrm{G}_U^{\wr}(e_{hn})\} \\ \mathrm{H}_U^{\triangle}(e_m) = \max\{\mathrm{F}_U^{\triangle}(e_{kn}), \mathrm{G}_U^{\triangle}(e_{hn})\} \end{gathered}$$

for all$e_m = (e_k, e_h) \in C \subseteq E \times E$ and $m, n, k, h \in \Lambda$.

Definition 3.20. "OR" operation on two generalized interval neutrosophic soft sets F^{μ}andG^{θ} over U,denoted by$\mathrm{K}^{\lambda} = F^{\mu} \vee G^{\theta}$ is the mapping$\mathrm{K}^{\varepsilon}: C \to \mathrm{IN(U)} \times$ Idefined by

$$\mathrm{K}^{\varepsilon} = ([\mathrm{K}_L^{\star}, \mathrm{K}_U^{\star}], [\mathrm{k}_L^{\wr}, \mathrm{K}_U^{\wr}], [\,\mathrm{K}_L^{\triangle}, \mathrm{K}_U^{\triangle}])$$

where $\varepsilon\,(e_m)$= max($\mu\,(e_k), \theta\,(e_h)$ and

$$\begin{gathered} \mathrm{K}_L^{\star}(e_m) = \max\{\mathrm{F}_L^{\star}(e_{kn}), \mathrm{G}_L^{\star}(e_{hn})\} \\ \mathrm{K}_L^{\wr}(e_m) = \min\{F_L^{\wr}(e_{kn}), G_L^{\wr}(e_{hn})\} \\ \mathrm{K}_L^{\triangle}(e_m) = \min\{F_L^{\triangle}(e_{kn}), G_L^{\triangle}(e_{hn})\} \end{gathered}$$

and

$$\begin{gathered} \mathrm{K}_{\mathrm{U}}^{\star}(\mathrm{e_m}) = \max\{F_U^{\star}(e_{kn}), G_U^{\star}(e_{hn})\} \\ \mathrm{K}_U^{\wr}(e_m) = \min\{\mathrm{F}_U^{\wr}(e_{kn}), \mathrm{G}_U^{\wr}(e_{hn})\} \\ \mathrm{K}_U^{\triangle}(e_m) = \min\{\mathrm{F}_U^{\triangle}(e_{kn}), \mathrm{G}_U^{\triangle}(e_{hn})\} \end{gathered}$$

for all $e_m = (e_k, e_h) \in C \subseteq E \times E$ and $m, n, k, h \in \Lambda$.

Definition 3.21. LetF^{μ}andG^{θ} be two generalizedinterval neutrosophic soft sets over UandC ⊆ E × E , a function $R: C \rightarrow$IN(U) ×Idefined by R= $F^{\mu} \wedge G^{\theta}$and $R(e_m, e_h) = F^{\mu}(e_m) \wedge G^{\theta}(e_h)$is said to be a interval neutrosophic relation from F^{μ} to $G^{\theta}$for all $(e_m, e_h) \in C$.

4. Application of Generalized Interval Neutrosophic Soft Set

Now, we illustrate an application of generalized interval neutrosophic soft set in decision making problem.

Example 4.1. Supposethat the universe consists of three machines, that is$U = \{x_1, x_2, x_3\}$ and consider the set of parameters $E = \{e_1, e_2, e_3\}$ which describe their performances according to certain specific task. Assumethat a firm wants to buy one such machine depending on any two of the parameters only. Let there be two observations F^{μ} and $G^{\theta}$by two experts A and B respectively, defined as follows:

$$\begin{cases} F^{\mu}(e_1) = \left(\frac{h_1}{([0.2, 0.3], [0.2, 0.3], [0.2, 0.3])}, \frac{h_2}{([0.3, 0.6], [0.3, 0.5], [0.2, 0.4])}, \frac{h_3}{([0.4, 0.5], [0.2, 0.5], [0.2, 0.6])}\right), (0.2) \\ F^{\mu}(e_2) = \left(\frac{h_1}{([0.2, 0.5], [0.2, 0.5], [0.2, 0.5])}, \frac{h_2}{([0.3, 0.5], [0.4, 0.8], [0.8, 0.9])}, \frac{h_3}{([0.4, 0.7], [0.3, 0.8], [0.3, 0.4])}\right), (0.5) \\ F^{\mu}(e_3) = \left(\frac{h_1}{([0.3, 0.4], [0.3, 0.4], [0.7, 0.9])}, \frac{h_2}{([0.1, 0.3], [0.2, 0.5], [0.3, 0.7])}, \frac{h_3}{([0.1, 0.4], [0.2, 0.3], [0.5, 0.7])}\right), (0.6) \end{cases}$$

$$\begin{cases} G^{\theta}(e_1) = \left(\frac{h_1}{([0.2, 0.3], [0.3, 0.5], [0.2, 0.3])}, \frac{h_2}{([0.3, 0.4], [0.3, 0.4], [0.5, 0.6])}, \frac{h_3}{([0.5, 0.6], [0.2, 0.4], [0.5, 0.7])}\right), (0.3) \\ G^{\theta}(e_2) = \left(\frac{h_1}{([0.1, 0.4], [0.5, 0.6], [0.3, 0.4])}, \frac{h_2}{([0.6, 0.7], [0.4, 0.5], [0.5, 0.8])}, \frac{h_3}{([0.2, 0.4], [0.3, 0.6], [0.6, 0.9])}\right), (0.6) \\ G^{\theta}(e_3) = \left(\frac{h_1}{([0.2, 0.6], [0.2, 0.5], [0.1, 0.5])}, \frac{h_2}{([0.3, 0.5], [0.3, 0.6], [0.4, 0.5])}, \frac{h_3}{([0.6, 0.8], [0.3, 0.4], [0.2, 0.3])}\right), (0.4) \end{cases}$$

To find the "AND" between the two GINSSs, we have $F^{\mu}$and G^{θ},$R = F^{\mu} \wedge G^{\theta}$ where

$$(F^{\mu})^{\star} = \begin{pmatrix} e_1 & ([0.2, 0.3], [0.3, 0.6], [0.4, 0.5]) & (0.2) \\ e_2 & ([0.2, 0.5], [0.3, 0.5], [0.4, 0.7]) & (0.5) \\ e_3 & ([0.3, 0.4], [0.1, 0.3], [0.1, 0.4]) & (0.6) \end{pmatrix}$$

$$(F^{\mu})^{\wr} = \begin{pmatrix} e_1 & ([0.2, 0.3], [0.3, 0.5], [0.2, 0.5]) & (0.2) \\ e_2 & ([0.2, 0.5], [0.4, 0.8], [0.3, 0.8]) & (0.5) \\ e_3 & ([0.3, 0.4], [0.2, 0.5], [0.2, 0.3]) & (0.6) \end{pmatrix}$$

$$(F^{\mu})^{\Delta} = \begin{pmatrix} e_1 & ([0.2, 0.3], [0.2, 0.4], [0.2, 0.6]) & (0.2) \\ e_2 & ([0.2, 0.5], [0.8, 0.9], [0.3, 0.4]) & (0.5) \\ e_3 & ([0.7, 0.9], [0.3, 0.7], [0.5, 0.7]) & (0.6) \end{pmatrix}$$

$$(G^{\theta})^{\star} = \begin{pmatrix} e_1 & ([0.2, 0.3], [0.3, 0.4], [0.5, 0.6]) & (0.3) \\ e_2 & ([0.1, 0.4], [0.6, 0.7], [0.2, 0.4]) & (0.6) \\ e_3 & ([0.2, 0.6], [0.3, 0.5], [0.6, 0.8]) & (0.4) \end{pmatrix}$$

$$(G^{\theta})^{\wr} = \begin{pmatrix} e_1 & ([0.3, 0.5], [0.3, 0.4], [0.2, 0.4]) & (0.3) \\ e_2 & ([0.5, 0.6], [0.4, 0.5], [0.3, 0.6]) & (0.6) \\ e_2 & ([0.2, 0.5], [0.3, 0.6], [0.3, 0.4]) & (0.4) \end{pmatrix}$$

$$(G^{\theta})^{\Delta}=\begin{pmatrix} e_1 & ([0.2,0.3],[0.5,0.6],[0.5,0.7]) & (0.3) \\ e_2 & ([0.3,0.4],[0.5,0.8],[0.6,0.9]) & (0.6) \\ e_3 & ([0.1,0.5],[0.4,0.5],[0.2,0.3]) & (0.4) \end{pmatrix}$$

We present the table of three basic component of R, which are interval truth –membership, Interval indeterminacy membership and interval falsity-membership part.To choose the best candidate, we firstly propose the induced interval neutrosophic membership functions by taking the arithmetic average of the end point of the range, and mark the highest numerical grade (underline) in each row of each table. But here, since the last column is the grade of such belongingness of a candidate for each pair of parameters, its not taken into account while making. Then we calculate the score of each component of R by taking the sum of products of these numerical grades with the corresponding values of μ. Next, we calculate the final score by subtracting the score of falsity-membership part of R from the sum of scores of truth-membership part and of indeterminacy membership part of R.The machine with the highestscore is the desired machine by company.

For the interval truth membership function components we have:

$$(F^{\mu})^{\star}=\begin{pmatrix} e_1 & ([0.2,0.3],[0.3,0.6],[0.4,0.5]) & (0.2) \\ e_2 & ([0.2,0.5],[0.3,0.5],[0.4,0.7]) & (0.5) \\ e_3 & ([0.3,0.4],[0.1,0.3],[0.1,0.4]) & (0.6) \end{pmatrix}$$

$$(G^{\theta})^{\star}=\begin{pmatrix} e_1 & ([0.2,0.3],[0.3,0.4],[0.5,0.6]) & (0.3) \\ e_2 & ([0.1,0.4],[0.6,0.7],[0.2,0.4]) & (0.6) \\ e_3 & ([0.2,0.6],[0.3,0.5],[0.6,0.8]) & (0.4) \end{pmatrix}$$

$$(R)^{\star}=$$

$$(R)^{\star}(e_1,e_1)=\left\{\left(\frac{x_1}{[0.2,0.3]},\frac{x_2}{[0.3,0.4]},\frac{x_3}{[0.4,0.5]}\right),0.2\right\}$$

$$(R)^{\star}(e_1,e_2)=\left\{\left(\frac{x_1}{[0.1,0.3]},\frac{x_2}{[0.3,0.6]},\frac{x_3}{[0.2,0.5]}\right),0.2\right\}$$

$$(R)^{\star}(e_1,e_3)=\left\{\left(\frac{x_1}{[0.2,0.3]},\frac{x_2}{[0.3,0.5]},\frac{x_3}{[0.2,0.4]}\right),0.2\right\}$$

$$(R)^{\star}(e_2,e_1)=\left\{\left(\frac{x_1}{[0.2,0.3]},\frac{x_2}{[0.3,0.4]},\frac{x_3}{[0.4,0.6]}\right),0.3\right\}$$

$$(R)^{\star}(e_2,e_2)=\left\{\left(\frac{x_1}{[0.1,0.4]},\frac{x_2}{[0.3,0.5]},\frac{x_3}{[0.2,0.4]}\right),0.5\right\}$$

$$(R)^{\star}(e_2,e_3)=\left\{\left(\frac{x_1}{[0.2,0.5]},\frac{x_2}{[0.3,0.5]},\frac{x_3}{[0.4,0.7]}\right),0.4\right\}$$

$$(R)^{\star}(e_3,e_1)=\left\{\left(\frac{x_1}{[0.2,0.3]},\frac{x_2}{[0.1,0.3]},\frac{x_3}{[0.1,0.4]}\right),0.3\right\}$$

$$(R)^{\star}(e_3,e_2)=\left\{\left(\frac{x_1}{[0.1,0.4]},\frac{x_2}{[0.1,0.3]},\frac{x_3}{[0.1,0.4]}\right),0.6\right\}$$

$$(R)^{\star}(e_3,e_3)=\left\{\left(\frac{x_1}{[0.2,0.4]},\frac{x_2}{[0.1,0.3]},\frac{x_3}{[0.1,0.4]}\right),0.4\right\}$$

	x_1	x_2	x_3	μ
(e_1, e_1)	$[0.2, 0.3]$	$[0.3, 0.4]$	$[0.4, 0.5]$	0.2
(e_1, e_2)	$[0.1, 0.3]$	$[0.3, 0.6]$	$[0.2, 0.5]$	0.2
(e_1, e_3)	$[0.2, 0.3]$	$[0.3, 0.5]$	$[0.2, 0.4]$	0.2
(e_2, e_1)	$[0.2, 0.3]$	$[0.3, 0.4]$	$[0.4, 0.6]$	0.3
(e_2, e_2)	$[0.1, 0.4]$	$[0.3, 0.5]$	$[0.2, 0.4]$	0.5
(e_2, e_3)	$[0.2, 0.5]$	$[0.3, 0.5]$	$[0.4, 0.7]$	0.4
(e_3, e_1)	$[0.2, 0.3]$	$[0.1, 0.3]$	$[0.1, 0.4]$	0.3
(e_3, e_1)	$[0.1, 0.4]$	$[0.1, 0.3]$	$[0.1, 0.4]$	0.6
(e_3, e_2)	$[0.2, 0.4]$	$[0.1, 0.3]$	$[0.1, 0.4]$	0.4

Table 1: Interval truth membership function.

	x_1	x_2	x_3	μ
(e_1, e_1)	0.25	0.35	<u>0.45</u>	0.2
(e_1, e_2)	0.2	<u>0.45</u>	0.35	0.2
(e_1, e_3)	0.25	<u>0.4</u>	0.3	0.2
(e_2, e_1)	0.25	0.35	<u>0.5</u>	0.3
(e_2, e_2)	0.25	<u>0.4</u>	0.3	0.5
(e_2, e_3)	0.35	0.4	<u>0.55</u>	0.4
(e_3, e_1)	<u>0.25</u>	0.2	<u>0.25</u>	0.3
(e_3, e_1)	<u>0.25</u>	0.2	<u>0.25</u>	0.6
(e_3, e_2)	<u>0.3</u>	0.2	0.25	0.4

Table 2: Induced interval truth membership function.

The value of representation interval truth membership function $[a, b]$ are obtained using mean value.Then, the scores of interval truth membership function of x_1,x_2 and$x_3$are:

$$S_{(R)^\star}(x_1) = (0.25 \times 0.3) + (0.25 \times 0.6) + (0.3 \times 0.4) = \mathbf{0.325}$$
$$S_{(R)^\star}(x_2) = (0.45 \times 0.2) + (0.4 \times 0.2) + (0.4 \times 0.5)) = \mathbf{0.37}$$
$$S_{(R)^\star}(x_3) = (0.45 \times 0.2) + (0.5 \times 0.3) + (0.55 \times 0.4)) + (0.25 \times 0.3) + (0.25 \times 0.6) = \mathbf{0.685}.$$

For the interval indeterminacy membership function components we have:

$$(F^{\mu})^{\wr} = \begin{pmatrix} ([0.2, 0.3], [0.3, 0.5], [0.2, 0.5]) & (0.2) \\ ([0.2, 0.5], [0.4, 0.8], [0.3, 0.8]) & (0.5) \\ ([0.3, 0.4], [0.2, 0.5], [0.2, 0.3]) & (0.6) \end{pmatrix}$$

$$(G^{\theta})^{\wr} = \begin{pmatrix} ([0.3, 0.5], [0.3, 0.4], [0.2, 0.4]) & (0.3) \\ ([0.5, 0.6], [0.4, 0.5], [0.3, 0.6]) & (0.6) \\ ([0.2, 0.5], [0.3, 0.6], [0.3, 0.4]) & (0.4) \end{pmatrix}$$

$$(R)^{\wr} =$$
$$(R)^{\wr}(e_1, e_1) = \left\{\left(\frac{x_1}{[0.3, 0.5]}, \frac{x_2}{[0.3, 0.5]}, \frac{x_3}{[0.2, 0.5]}\right), 0.3\right\}$$

$$(R)^{\wr}(e_1\,, e_2) = \left\{\left(\frac{x_1}{[0.5, 0.6]}, \frac{x_2}{[0.4, 0.5]}\,, \frac{x_3}{[0.3, 0.6]}\right), 0.6\right\}$$

$$(R)^{\wr}(e_1\,, e_3) = \left\{\left(\frac{x_1}{[0.2, 0.5]}, \frac{x_2}{[0.3, 0.6]}\,, \frac{x_3}{[0.3, 0.5]}\right), 0.4\right\}$$

$$(R)^{\wr}(e_2\,, e_1) = \left\{\left(\frac{x_1}{[0.3, 0.5]}, \frac{x_2}{[0.4, 0.8]}\,, \frac{x_3}{[0.3, 0.8]}\right), 0.5\right\}$$

$$(R)^{\wr}(e_2\,, e_2) = \left\{\left(\frac{x_1}{[0.5, 0.6]}, \frac{x_2}{[0.4, 0.8]}\,, \frac{x_3}{[0.3, 0.8]}\right), 0.6\right\}$$

$$(R)^{\wr}(e_2\,, e_3) = \left\{\left(\frac{x_1}{[0.2, 0.5]}, \frac{x_2}{[0.4, 0.8]}\,, \frac{x_3}{[0.3, 0.8]}\right), 0.5\right\}$$

$$(R)^{\wr}(e_3\,, e_1) = \left\{\left(\frac{x_1}{[0.3, 05]}, \frac{x_2}{[0.3, 0.5]}\,, \frac{x_3}{[0.2, 0.4]}\right), 0.6\right\}$$

$$(R)^{\wr}(e_3\,, e_2) = \left\{\left(\frac{x_1}{[0.5, 0.6]}, \frac{x_2}{[0.4, 0.5]}\,, \frac{x_3}{[0.3, 0.6]}\right), 0.6\right\}$$

$$(R)^{\wr}(e_3\,, e_3) = \left\{\left(\frac{x_1}{[03, 0.5]}, \frac{x_2}{[0.3, 0.6]}\,, \frac{x_3}{[0.3, 0.4]}\right), 0.6\right\}$$

	x_1	x_2	x_3	μ
$(e_1\,, e_1\,)$	[0.3, 0.5]	[0.3, 0.5]	[0.2, 0.5]	0.3
$(e_1\,, e_2\,)$	[0.5, 0.6]	[0.4, 0.5]	[0.3, 0.6]	0.6
$(e_1\,, e_3\,)$	[0.2, 0.5]	[0.3, 0.6]	[0.3, 0.5]	0.4
$(e_2\,, e_1\,)$	[0.3, 0.5]	[0.4, 0.8]	[0.3, 0.8]	0.5
$(e_2\,, e_2\,)$	[0.5, 0.6]	[0.4, 0.8]	[0.3, 0.8]	0.6
$(e_2\,, e_3\,)$	[0.2, 0.5]	[0.4, 0.8]	[0.3, 0.8]	0.5
$(e_3\,, e_1\,)$	[0.3, 05]	[0.3, 0.5]	[0.2, 0.4]	0.6
$(e_3\,, e_1\,)$	[0.5, 0.6]	[0.4, 0.5]	[0.3, 0.6]	0.6
$(e_3\,, e_2\,)$	[0.3, 0.5]	[0.3, 0.6]	[0.3, 0.4]	0.6

Table 3: Interval indeterminacy membership function

	x_1	x_2	x_3	μ
$(e_1\,, e_1\,)$	$\underline{0.4}$	$\underline{0.4}$	0.35	0.3
$(e_1\,, e_2\,)$	$\underline{0.55}$	0.45	0.45	0.6
$(e_1\,, e_3\,)$	0.35	$\underline{0.45}$	0.4	0.4
$(e_2\,, e_1\,)$	0.4	$\underline{0.6}$	0.55	0.5
$(e_2\,, e_2\,)$	0.55	$\underline{0.6}$	0.55	0.6
$(e_2\,, e_3\,)$	0.35	$\underline{0.6}$	0.55	0.5
$(e_3\,, e_1\,)$	$\underline{0.4}$	$\underline{0.4}$	0.3	0.6
$(e_3\,, e_1\,)$	$\underline{0.55}$	0.45	0.45	0.6
$(e_3\,, e_2\,)$	0.4	$\underline{0.45}$	0.35	0.6

Table 4: Induced interval indeterminacy membership function

The value of representation interval indeterminacy membership function$[a, b]$ are obtained using mean value. Then, the scores of interval indeterminacy membership function of x_1, x_2 and$x_3$are:

$$S_{(R)^{\wr}}(x_1) = (0.4 \times 0.3) + (0.55 \times 0.6) + (0.4 \times 0.6) + (0.55 \times 0.6) = \mathbf{1.02}$$

$$S_{(R)^{\wr}}(x_2) = (0.4 \times 0.3) + (0.45 \times 0.4) + (0.6 \times 0.5) + (0.6 \times 0.6) + (0.6 \times 0.5) + (0.4 \times 0.60) + (0.45 \times 0.6) + = \mathbf{1.77}$$

$$S_{I(R)^{\wr}}(x_2) = \mathbf{0}.$$

For the interval indeterminacy membership function components we have:

$$(F^{\mu})^{\Delta} = \begin{pmatrix} ([0.2, 0.3], [0.2, 0.4], [0.2, 0.6]) & (0.2) \\ ([0.2, 0.5], [0.8, 0.9], [0.3, 0.4]) & (0.5) \\ ([0.7, 0.9], [0.3, 0.7], [0.5, 0.7]) & (0.6) \end{pmatrix}$$

$$(G^{\theta})^{\Delta} = \begin{pmatrix} ([0.2, 0.3], [0.5, 0.6], [0.5, 0.7]) & (0.3) \\ ([0.3, 0.4], [0.5, 0.8], [0.6, 0.9]) & (0.6) \\ ([0.1, 0.5], [0.4, 0.5], [0.2, 0.3]) & (0.4) \end{pmatrix}$$

$$(R)^{\Delta} =$$

$$(R)^{\Delta}(e_1, e_1) = \left\{\left(\frac{x_1}{[0.2, 0.3]}, \frac{x_2}{[0.5, 0.6]}, \frac{x_3}{[0.5, 0.7]}\right), 0.3\right\}$$

$$(R)^{\Delta}(e_1, e_2) = \left\{\left(\frac{x_1}{[0.3, 0.4]}, \frac{x_2}{[0.5, 0.8]}, \frac{x_3}{[0.6, 0.9]}\right), 0.6\right\}$$

$$(R)^{\Delta}(e_1, e_3) = \left\{\left(\frac{x_1}{[0.2, 0.5]}, \frac{x_2}{[0.4, 0.5]}, \frac{x_3}{[0.2, 0.6]}\right), 0.4\right\}$$

$$(R)^{\Delta}(e_2, e_1) = \left\{\left(\frac{x_1}{[0.2, 0.5]}, \frac{x_2}{[0.8, 0.9]}, \frac{x_3}{[0.5, 0.7]}\right), 0.5\right\}$$

$$(R)^{\Delta}(e_2, e_2) = \left\{\left(\frac{x_1}{[0.3, 0.5]}, \frac{x_2}{[0.8, 0.9]}, \frac{x_3}{[0.6, 0.9]}\right), 0.6\right\}$$

$$(R)^{\Delta}(e_2, e_3) = \left\{\left(\frac{x_1}{[0.2, 0.5]}, \frac{x_2}{[0.8, 0.9]}, \frac{x_3}{[0.3, 0.4]}\right), 0.5\right\}$$

$$(R)^{\Delta}(e_3, e_1) = \left\{\left(\frac{x_1}{[0.7, 0.9]}, \frac{x_2}{[0.5, 0.7]}, \frac{x_3}{[0.5, 0.7]}\right), 0.6\right\}$$

$$(R)^{\Delta}(e_3, e_2) = \left\{\left(\frac{x_1}{[0.7, 0.9]}, \frac{x_2}{[0.5, 0.8]}, \frac{x_3}{[0.6, 0.9]}\right), 0.6\right\}$$

$$(R)^{\Delta}(e_3, e_3) = \left\{\left(\frac{x_1}{[0.7, 0.9]}, \frac{x_2}{[0.4, 0.7]}, \frac{x_3}{[0.5, 0.7]}\right), 0.6\right\}$$

	x_1	x_2	x_3	μ
(e_1, e_1)	$[0.2, 0.3]$	$[0.5, 0.6]$	$[0.5, 0.7]$	0.3
(e_1, e_2)	$[0.3, 0.4]$	$[0.5, 0.8]$	$[0.6, 0.9]$	0.6
(e_1, e_3)	$[0.2, 0.5]$	$[0.4, 0.5]$	$[0.2, 0.6]$	0.4
(e_2, e_1)	$[0.2, 0.5]$	$[0.8, 0.9]$	$[0.5, 0.7]$	0.5
(e_2, e_2)	$[0.3, 0.5]$	$[0.8, 0.9]$	$[0.6, 0.9]$	0.6
(e_2, e_3)	$[0.2, 0.5]$	$[0.8, 0.9]$	$[0.3, 0.4]$	0.5
(e_3, e_1)	$[0.7, 0.9]$	$[0.5, 0.7]$	$[0.5, 0.7]$	0.6
(e_3, e_1)	$[0.7, 0.9]$	$[0.5, 0.8]$	$[0.6, 0.9]$	0.6
(e_3, e_2)	$[0.7, 0.9]$	$[0.4, 0.7]$	$[0.5, 0.7]$	0.6

Table 5: Interval falsity membership function.

	x_1	x_2	x_3	μ
(e_1, e_1)	0.25	0.55	<u>0.6</u>	0.3
(e_1, e_2)	0.35	0.43	<u>0.75</u>	0.6
(e_1, e_3)	0.35	<u>0.45</u>	0.4	0.4
(e_2, e_1)	0.35	<u>0.85</u>	0.6	0.5
(e_2, e_2)	0.4	<u>0.85</u>	0.75	0.6
(e_2, e_3)	0.35	<u>0.85</u>	0.35	0.5
(e_3, e_1)	<u>0.8</u>	0.6	0.6	0.6
(e_3, e_1)	<u>0.8</u>	0.43	0.75	0.6
(e_3, e_2)	<u>0.8</u>	0.55	0.6	0.6

Table 6: Induced interval falsity membership function.

The value of representation interval falsity membership function $[a, b]$ are obtained using mean value. Then, the scores of interval falsity membership function of x_1, x_2 and $x_3$are:

$$S_{(R)^{\vartriangle}}(x_1) = (0.8 \times 0.6) + (0.8 \times 0.6) + (0.8 \times 0.6) = \mathbf{1.44}$$
$$S_{(R)^{\vartriangle}}(x_2) = (0.45 \times 0.4) + (0.85 \times 0.5) + (0.85 \times 0.6) + (0.85 \times 0.5) = \mathbf{1.54}$$
$$S_{(R)^{\vartriangle}}(x_3) = (0.6 \times 0.3) + (0.75 \times 0.6) = \mathbf{0.63}.$$

Thus, we conclude the problem by calculating final score, using the following formula:

$$\mathrm{S}(x_\mathrm{i}) = \mathrm{S}_{(\mathrm{R})^{\star}}(x_\mathrm{i}) + \mathrm{S}_{(\mathrm{R})^{\wr}}(x_\mathrm{i}) - \mathrm{S}_{(\mathrm{R})^{\vartriangle}}(x_\mathrm{i})$$

so,

$$\mathrm{S}(x_1) = 0.325 + 1.02 - 1.44 = -0.095$$

$$\mathrm{S}(x_2) = 0.37 + 1.77 - 1.54 = 0.6$$

$$\mathrm{S}(x_3) = 0.685 + 0 - 0.63 = 0.055.$$

Then the optimal selection for Mr.X is the x_2.

Table 1, Table 3 and Table 5 present the truth–membership function, indeterminacy-membership function and falsity-membership function in generalized interval neutrosophic soft set respectively.

5. Conclusions

This paper can be viewed as a continuation of the study of Sahin and Küçük [23]. We extended the generalized neutrosophic soft set to the case of interval valued neutrosophic soft set and also gave the application of GINSS in dealing with some decision making problems. In future work, will study another type of generalized interval neutrosophic soft set where the degree of possibility are interval.

Acknowledgements

The authors are very grateful to the anonymous referees for their insightful and constructive comments and suggestions, which have been very helpful in improving the paper.

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Published in *Journal of New Research in Science*, No. 7 (2014), pp. 29-47, 19 p.,
pages 97 – 115.

G-Neutrosophic Space

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In this article we give an extension of group action theory to neutrosophic theory and develop G-neutrosophic spaces by certain valuable techniques. Every G-neutrosophic space always contains a G-space. A G-neutrosophic space has neutrosophic orbits as well as strong neutrosophic orbits. Then we give an important theorem for orbits which tells us that how many orbits of a G-neutrosophic space. We also introduce new notions called pseudo neutrosophic space and ideal space and then give the important result that the transitive property implies to ideal property

.

Keywords: Group action, G-space, orbit, stabilizer, G-neutrosophic space, neutrosophic orbit, neutrosophic stabilizer.

1. Introduction

The Concept of a G-space came into being as a consequence of Group action on an ordinary set. Over the history of Mathematics and Algebra, theory of group action has emerged and proven to be an applicable and effective framework for the study of different kinds of structures to make connection among them. The applications of group action in different areas of science such as physics, chemistry, biology, computer science, game theory, cryptography etc has been worked out very well. The abstraction provided by group actions is a powerful one, because it allows geometrical ideas to be applied to more abstract objects. Many objects in Mathematics have natural group actions defined on them. In particular, groups can act on other groups, or even on themselves. Despite this generality, the theory of group actions contains wide-reaching theorems, such as the orbit stabilizer theorem, which can be used to prove deep results in several fields. Neutrosophy is a branch of neutrosophic philosophy which handles the origin and stages of neutralities. Neutrosophic science is a newly emerging science which has been firstly introduced by Florentin Smarandache in 1995. This is quite a general phenomenon which can be found almost everywhere in the nature. Neutrosophic approach provides a generosity to absorbing almost all the corresponding algebraic structures open heartedly. This tradition is also maintained in our work here. The combination of neutrosophy and group action gives some extra ordinary excitement while forming this new structure called G-neutrosophic space. This is a generalization of all the work of the past and some new notions are also raised due to this approach. Some new types of spaces and their core properties have been discovered here for the first time. Examples and counter examples have been illustrated wherever required. In this paper we have also coined a new term called pseudo neutrosophic spaces and a new property called ideal property. The link of transitivity with ideal property and the corresponding results are established.

2. Basic Concepts

Group Action

Definition 1: Let Ω be a non empty set and G be a group. Let $\mu:\Omega\times G\to\Omega$ be a mapping. Then μ is called an action of G on Ω such that for all $\omega\in\Omega$ and $g,h\in G$.

1) $\mu(\mu(\omega,g),h)=\mu(\omega,gh)$

2) $\mu(\omega,1)=\omega$, where 1 is the identity element in G.

Usually we write ω^g instead of $\mu(\omega,g)$. Therefore (1) and (2) becomes as

1) $(\omega^g)^h=(\omega)^{gh}$. For all $\omega\in\Omega$ and $g,h\in G$.

2) $\omega^1=\omega$.

Definition 2: Let Ω be a G-space. Let $\Omega_1\neq\phi$ be a subset of Ω. Then Ω_1 is called G-subspace of Ω if $\omega^g\in\Omega_1$ for all $\omega\in\Omega_1$ and $g\in G$.

Definition 3: We say that Ω is transitive G-space if for any $\alpha,\beta\in G$, there exist $g\in G$ such that $\alpha^g=\beta$.

Definition 4: Let $\alpha\in\Omega$, then α^G or αG is called G-orbit and is defined as $\alpha^G=\{\alpha^g:g\in G\}$.

A transitive G-subspace is also called an orbit.

Remark 1: A transitive G space has only one orbit.

Definition 5: Let G be a group acting on Ω and if $\alpha\in\Omega$, we denote stabilizer of α by G_α and is define as $G_\alpha=stab_G(\alpha)=\{g\in G:\alpha^g=\alpha\}$.

Lemma 1: Let Ω be a G-space and $\alpha\in\Omega$. Then

1) $G_\alpha\leq G$ and

2) There is one-one correspondence between the right cosets of G_α and the G-orbit α^G in G.

Corollary 1: If G is finite, then $|G|=|G_\alpha|.|\alpha^G|$

Definition 6: Let Ω be a G-space and $g \in G$. Then

$$fix_{\Omega}\ g\ =\ \ \alpha \in \Omega : \alpha^{g} = \alpha\ \ .$$

Theorem 1: Let Ω and G be finite. Then

$$\left| Orb_{\Omega}\ G\ \right| = \frac{1}{|G|} \sum_{g \in G} \left| fix_{\Omega}\ g\ \right|,$$

where $\left| Orb_{\Omega}\ G\ \right|$ is the number of orbits of G in Ω.

3. Neutrosophic Spaces

Definition 10: Let Ω be a G-space. Then $N\ \Omega$ is called G-neutrosophic space if $N\ \Omega\ = \langle \Omega \cup I \rangle$ which is generated by Ω and I.

Example 1: Let $\Omega = \ e, x, x^{2}, y, xy, x^{2}y\ $ =S_3 and $G = \ e, y\ $. Let $\mu : \Omega \times G \to \Omega$ be an action of G on Ω defined by $\mu\ \omega, g\ = g\ \omega\ $, for all $\omega \in \Omega$ and $g \in G$. Then Ω be a G-space under this action. Let $N\ \Omega$ be the corresponding G-neutrosophic space, where

$$N\ \Omega\ = \langle \Omega \cup I \rangle = \ e, x, x^{2}, y, xy, x^{2}y, I, Ix, Ix^{2}, Iy, Ixy, Ix^{2}y$$

Theorem 3: $N\ \Omega$ always contains Ω.

Definition 11: Let $N\ \Omega$ be a neutrosophic space and $N\ \Omega_{1}$ be a subset of $N\ \Omega$. Then $N\ \Omega_{1}$ is called neutrosophic subspace of $N\ \Omega$ if $x^{g} \in N\ \Omega_{1}$ for all $x \in N\ \Omega_{1}$ and $g \in G$.

Example 2: In the above example 1 . Let $N\ \Omega_{1}\ =\ x, xy$ and $N\ \Omega_{2}\ =\ Ix^{2}, Ix^{2}y$ are subsets of $N\ \Omega$. Then clearly $N\ \Omega_{1}$ and $N\ \Omega_{2}$ are neutrosophic subspaces of $N\ \Omega$.

Theorem 4: Let $N\ \Omega$ be a G-neutrosophic space and Ω be a G-space. Then Ω is always a neutrosophic subspace of $N\ \Omega$.

Proof: The proof is straightforward.

Definition 12: A neutrosophic subspace $N\ \Omega_1$ is called strong neutrosophic subspace or pure neutrosophic subspace if all the elements of $N\ \Omega_1$ are neutrosophic elements.

Example 3: In example 1 , the neutrosophic subspace $N\ \Omega_2 = Ix^2, Ix^2 y$ is a strong neutrosophic subspace or pure neutrosophic subspace of $N\ \Omega$.

Remark 2: Every strong neutrosophic subspace or pure neutrosophic subspace is trivially neutrosophic subspace.

The converse of the above remark is not true.

Example 4: In previous example $N\ \Omega_1 = x, xy$ is a neutrosophic subspace but it is not strong neutrosophic subspace or pure neutrosophic subspace of $N\ \Omega$.

Definition 13: Let $N\ \Omega$ be a G-neutrosophic space. Then $N\ \Omega$ is said to be transitive G-neutrosophic space if for any $x, y \in N\ \Omega$, there exists $g \in G$ such that $x^g = y$.

Example 5: Let $\Omega = G = Z_4, +$, where Z_4 is a group under addition modulo 4 . Let $\mu : \Omega \times G \to \Omega$ be an action of G on itself defined by $\mu\ \omega, g = \omega + g$, for all $\omega \in \Omega$ and $g \in G$. Then Ω is a G-space and $N\ \Omega$ be the corresponding G-neutrosophic space , where

$$N\ \Omega = 0,1,2,3,I,2I,3I,4I,1+I,2+I,3+I,1+2I,2+2I,2+3I,3+2I,3+3I$$

Then $N\ \Omega$ is not transitive neutrosophic space.

Theorem 5: All the G-neutrosophic spaces are intransitive G-neutrosophic spaces.

Definition 14: Let $n \in N \;\; \Omega$, the neutrosophic orbit of n is denoted by NO_n and is defined as $NO_n = \;\; n^g : g \in G \;\;$.

Equivalently neutrosophic orbit is a transitive neutrosophic subspace.

Example 6: In example 1 , the neutrosophic space $N \;\; \Omega$ has 6 neutrosophic orbits which are given below

$$NO_e = \;\; e, y \;\; , NO_x = \;\; x, xy \;\; ,$$
$$NO_{x^2} = \;\; x^2, x^2 y \;\; , NO_I = \;\; I, Iy \;\; ,$$
$$NO_{Ix} = \;\; Ix, Ixy \;\; , NO_{Ix^2} = \;\; Ix^2, Ix^2 y \;\; .$$

Definition 15: A neutrosophic orbit NO_n is called strong neutrosophic orbit or pure neutrosophic orbit if it has only neutrosophic elements.

Example 7: In example 1 ,

$$NO_I = \;\; I, Iy \;\; ,$$
$$NO_{Ix} = \;\; Ix, Ixy \;\; ,$$
$$NO_{Ix^2} = \;\; Ix^2, Ix^2 y \;\; .$$

are strong neutrosophic orbits or pure neutrosophic orbits of $N \;\; \Omega$.

Theorem 7: All strong neutrosophic orbits or pure neutrosophic orbits are neutrosophic orbits.

Proof: Straightforward

To show that the converse is not true, let us check the following example.

Example 8: In example 1

$$NO_e = \;\; e, y \;\; ,$$
$$NO_x = \;\; x, xy \;\; ,$$
$$NO_{x^2} = \;\; x^2, x^2 y \;\; .$$

are neutrosophic orbits of $N \ \ \Omega$ but they are not strong or pure neutrosophic orbits.

Definition 16: Let G be a group acting on Ω and $x \in N \ \ \Omega$. The neutrosophic stabilizer of x is defined as $G_x = stab_G \ \ x \ \ = \ \ g \in G : x^g = x$.

Example 9: Let $\Omega = \ e, x, x^2, y, xy, x^2y$ and $G = \ e, x, x^2$. Let $\mu : \Omega \times G \to \Omega$ be an action of G on Ω defined by $\mu \ \ \omega, g \ \ = g \ \ \omega$, for all $\omega \in \Omega$ and $g \in G$. Then Ω is a G-space under this action. Now $N \ \ \Omega$ be the G-neutrosophic space, where

$$N \ \ \Omega \ \ = \ \ e, x, x^2, y, xy, x^2y, I, Ix, Ix^2, Iy, Ixy, Ix^2y$$

Let $x \in N \ \ \Omega$, then the neutrosophic stabilizer of x is $G_x = \ \ e$ and also let $I \in N \ \ \Omega$, so the neutrosophic stabilizer of I is $G_I = \ \ e$.

Lemma 2: Let $N \ \ \Omega$ be a neutrosophic space and $x \in N \ \ \Omega$, then

1) $G_x \leq G$.
2) There is also one-one correspondence between the right cosets of G_x and the neutrosophic orbit NO_x.

Corollary 2: Let G is finite and $x \in N \ \ \Omega$, then $|G| = |G_x|.|NO_x|$.

Definition 17: Let $x \in N \ \ \Omega$, then the neutrosophic stabilize of x is called strong neutrosophic stabilizer or pure neutrosophic stabilizer if and only if x is a neutrosophic element of $N \ \ \Omega$.

Example 10: In above example (9), $G_I = \ \ e$ is a strong neutrosophic or pure neutrosophic stabilizer of neutrosophic element I , where $I \in N \ \ \Omega$.

Remark 3: Every strong neutrosophic stabilizer or pure neutrosophic stabilizer is always a neutrosophic stabilizer

but the converse is not true.

Example 11: Let $x \in N(\Omega)$, where

$$N(\Omega) = \{e, x, x^2, y, xy, x^2y, I, Ix, Ix^2, Iy, Ixy, Ix^2y\}$$

Then $G_x = \{e\}$ is the neutrosophic stabilize of x but it is not strong neutrosophic stabilizer or pure neutrosophic stabilizer as x is not a neutrosophic element of $N(\Omega)$.

Definition 18: Le $N(\Omega)$ be a neutrosophic space and G be a finite group acting on Ω. For $g \in G$, $fix_{N(\Omega)}(g) = \{x \in N(\Omega) : x^g = x\}$

Example 12: In example 11,

$fix_{N(\Omega)}(e) = \{e, x, x^2, y, xy, x^2y, I, Ix, Ix^2, Iy, Ixy, Ix^2y\}$

$fix_{N(\Omega)}(g) = \phi$, where $g \neq e$.

Theorem 8: Let $N(\Omega)$ be a finite neutrosophic space, then

$$\left|NO_{N(\Omega)}(G)\right| = \frac{1}{|G|}\sum_{g \in G}\left|fix_{N(\Omega)}(g)\right|.$$

Proof: The proof is same as in group action.

Example 13: Consider example (1), only identity element of G fixes all the elements of $N(\Omega)$. Hence $fix_{N(\Omega)}(e) = \{e, x, x^2, y, xy, x^2y, I, Ix, Ix^2, Iy, Ixy, Ix^2y\}$ and hence $\left|fix_{N(\Omega)}(e)\right| = 12$.

The number of neutrosophic orbits of $N(\Omega)$ are given by above theorem

$$\left|NO_{N(\Omega)}(G)\right| = \frac{1}{2}(12) = 6$$

Hence $N(\Omega)$ has 6 neutrosophic orbits.

4. Pseudo Neutrosophic Space

Definition 19: A neutrosophic space $N(\Omega)$ is called pseudo neutrosophic space which does not contain a proper set which is a G-space.

Example 14: Let $\Omega = G = Z_2$ where Z_2 is a group under addition modulo 2. Let $\mu : \Omega \times G \to \Omega$ be an action of G on Ω defined by $\mu(\omega, g) = \omega + g$, for all

$\omega \in \Omega$ and $g \in G$. Then Ω be a G-space under this action and $N(\Omega)$ be the G-neutrosophic space, where $N(\Omega) = \{0,1,I,1+I\}$.

Then clearly $N(\Omega)$ is a pseudo neutrosophic space.

Theorem 9: Every pseudo neutrosophic space is a neutrosophic space but the converse is not true in general.

Example 15: In example (1), $N(\Omega)$ is a neutrosophic space but it is not pseudo neutrosophic space because $\{e,y\}$, $\{x,xy\}$ and $\{x^2,x^2y\}$ are proper subsets which are G-spaces.

Definition 20: Let $N(\Omega)$ be a neutrosophic space and $N(\Omega_1)$ be a neutrosophic subspace of $N(\Omega)$. Then $N(\Omega_1)$ is called pseudo neutrosophic subspace of $N(\Omega)$ if $N(\Omega_1)$ does not contain a proper subset of Ω which is a G-subspace of Ω.

Example 16: In example (1), $\{e,y\}$, $\{Ix,Ixy\}$ etc are pseudo neutrosophic subspaces of $N(\Omega)$ but $\{e,y,Ix,Ixy\}$ is not pseudo neutrosophic subspace of $N(\Omega)$ as $\{e,y\}$ is a proper G-subspace of Ω.

Theorem 10: All pseudo neutrosophic subspaces are neutrosophic subspaces but the converse is not true in general.

Example 17: In example (1), $\{e,y,Ix,Ixy\}$ is a neutrosophic subspace of $N(\Omega)$ but it is not pseudo neutrosophic subspace of $N(\Omega)$.

Theorem 11: A neutrosophic space $N(\Omega)$ has neutrosophic subspaces as well as pseudo neutrosophic subspaces.

Proof : The proof is obvious.

Theorem 12: A transitive neutrosophic subspace is always a pseudo neutrosophic subspace but the converse is not true in general.

Proof: A transitive neutrosophic subspace is a neutrosophic orbit and hence neutrosophic orbit does not contain any other subspace and so pseudo neutrosophic subspace.

The converse of the above theorem does not holds in general. For instance let see the following example.

Example 18: In example 1 , I, Iy, Ix, Ixy is a pseudo neutrosophic subspace of $N \ \Omega$ but it is not transitive.

Theorem 13: All transitive pseudo neutrosophic subspaces are always neutrosophic orbits.

Proof: The proof is followed from by definition.

Definition 21: The pseudo property in a pseudo neutrosophic subspace is called ideal property.

Theorem 14: The transitive property implies ideal property but the converse is not true.

Proof: Let us suppose that $N \ \Omega_1$ be a transitive neutrosophic subspace of $N \ \Omega$. Then by following above theorem, $N \ \Omega_1$ is pseudo neutrosophic subspace of $N \ \Omega$ and hence transitivity implies ideal property.
The converse of the above theorem is not holds.

Example 19: In example 1 , I, Iy, Ix, Ixy is a pseudo neutrosophic subspace of $N \ \Omega$ but it is not transitive.

Theorem 15: The ideal property and transitivity both implies to each other in neutrosophic orbits.

Proof: The proof is straightforward.

Definition 22: A neutrosophic space $N \ \Omega$ is called ideal space or simply if all of its proper neutrosophic subspaces are pseudo neutrosophic subspaces.

Example 20: In example (14), the neutrosophic space $N(\Omega)$ is an ideal space because $\{0,1\}$, $\{I,1+I\}$ are only proper neutrosophic subspaces which are pseudo neutrosophic subspaces of $N(\Omega)$.

Theorem 16: Every ideal space is trivially a neutrosophic space but the converse is not true.

For converse, we take the following example

Example 21: In example (1), $N(\Omega)$ is a neutrosophic space but it is not an ideal space.

Theorem 17: A neutrosophic space $N(\Omega)$ is an ideal space If Ω is transitive G-space.

Theorem 18: Let $N(\Omega)$ be a neutrosophic space, then $N(\Omega)$ is pseudo neutrosophic space if and only if $N(\Omega)$ is an ideal space.

Proof: Suppose that $N(\Omega)$ is a pseudo neutrosophic space and hence by definition all proper neutrosophic subspaces are also pseudo neutrosophic subspaces. Thus $N(\Omega)$ is an ideal space.
Conversely suppose that $N(\Omega)$ is an ideal space and so all the proper neutrosophic subspaces are pseudo neutrosophic subspaces and hence $N(\Omega)$ does not contain any proper set which is G-subspace and consequently $N(\Omega)$ is a pseudo neutrosophic space.

Theorem 19: If the neutrosophic orbits are only the neutrosophic proper subspaces of $N(\Omega)$, then $N(\Omega)$ is an ideal space.

Proof: The proof is obvious.

Theorem 20: A neutrosophic space $N(\Omega)$ is an ideal space if $\left|NO_{N(\Omega)}(G)\right| = 2$

Theorem 21: A neutrosophic space $N(\Omega)$ is ideal space if all of its proper neutrosophic subspaces are neutrosophic orbits.

6. Conclusions

The main theme of this paper is the extension of neutrosophy to group action and G-spaces to form G-neutrosophic spaces. Our aim is to see the newly generated structures and finding their links to the old versions in a logical manner. Fortunately enough, we have found some new type of algebraic structures here, such as ideal space, Pseudo spaces. Pure parts of neutrosophy and their corresponding properties and theorems are discussed in detail with a sufficient supply of examples.

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Published in „U.P.B. Sci. Bull.", 11 p., pages 116 - 126.

Interval Neutrosophic Rough Set

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Abstract –This paper combines interval-valued neutrosophic sets and rough sets. It studies rougheness in interval-valued neutrosophic sets and some of its properties. Finally we propose a Hamming distance between lower an upper approximations of interval neutrosophic sets.

Keywords -

Interval Neutrosophic, Rough Set, Interval Neutrosophic Rough Set.

1.Introduction

Neutrosophic set (NS for short), a part of neutrosophy introduced by Smarandache [1] as a new branch of philosophy, is a mathematical tool dealing with problems involving imprecise, indeterminacy and inconsistent knowledge. Contrary to fuzzy sets and intuitionistic fuzzy sets, a neutrosophic set consists of three basic membership functions independently of each other, which are truth, indeterminacy and falsity. This theory has been well developed in both theories and applications. After the pioneering work of Smarandache, In 2005, Wang [2] introduced the notion of interval neutrosophic sets (INS for short) which is another extension of neutrosophic sets. INS can be described by a membership interval, a non-membership interval and indeterminate interval, thus the interval neutrosophic (INS) has the virtue of complementing NS, which is more flexible and practical than neutrosophic set, and Interval Neutrosophic Set (INS) provides a more reasonable mathematical framework to deal with indeterminate and inconsistent information. The interval neutrosophic set generalize, the classical set ,fuzzy set [3] , the interval valued fuzzy set [4], intuitionistic fuzzy set [5] , interval valued intuitionstic fuzzy set [6] and so on. Many scholars have performed studies on neutrosophic sets , interval neutrosophic sets and their properties [7,8,9,10,11,12,13]. Interval neutrosophic sets have also been widely applied to many fields [14,15,16,17,18,19].

The rough set theory was introduced by Pawlak [20] in 1982, which is a technique for managing the uncertainty and imperfection, can analyze incomplete information effectively. Therefore, many models have been built upon different aspect, i.e, univers, relations, object, operators by many scholars [21,22,23,24,25,26]such as rough fuzzy sets, fuzzy rough sets, generalized fuzzy rough, rough intuitionistic fuzzy set. intuitionistic fuzzy rough sets[27]. It has been successfully applied in many fields such as attribute reduction [28,29,30,31], feature selection [32,33,34], rule extraction [35,36,37,38] and so on. The rough sets theory approximates any subset of objects of the universe by two sets, called the lower and upper approximations. It focuses on the ambiguity caused by the limited discernibility of objects in the universe of discourse.
More recently, S.Broumi et al [39] combined neutrosophic sets with rough sets in a new hybrid mathematical structure called "rough neutrosophic sets" handling incomplete and indeterminate information . The concept of rough neutrosophic sets generalizes fuzzy rough sets and intuitionistic fuzzy rough sets. Based on the equivalence relation on the universe of discourse, A.Mukherjee et al [40] introduced lower and upper approximation of interval valued intuitionistic fuzzy set in Pawlak's approximation space . Motivated by this ,we extend the interval intuitionistic fuzzy lower and upper approximations to the case of interval valued neutrosophic set. The concept of interval valued neutrosophic rough set is introduced by coupling both interval neutrosophic sets and rough sets.

The organization of this paper is as follow : In section 2, we briefly present some basic definitions and preliminary results are given which will be used in the rest of the paper. In section 3 , basic concept of rough approximation of an interval valued neutrosophic sets and their properties are presented. In section 4, Hamming distance between lower approximation and upper approximation of interval neutrosophic set is introduced, Finally, we concludes the paper

2.Preliminaries

Throughout this paper, We now recall some basic notions of neutrosophic set , interval neutrosophic set , rough set theory and intuitionistic fuzzy rough set. More can found in ref [1, 2,20,27].

Definition 1 [1]

Let U be an universe of discourse then the neutrosophic set A is an object having the form A= $\{< x: \mu_{A(x)}, \nu_{A(x)}, \omega_{A(x)} >, x \in U\}$, where the functions $\mu, \nu, \omega : U \rightarrow]^-0,1^+[$ define respectively the degree of membership , the degree of indeterminacy, and the degree of non-membership of the element x ∈ X to the set A with the condition.

$$^-0 \leq \mu_{A(x)} + \nu_{A(x)} + \omega_{A(x)} \leq 3^+. \qquad (1)$$

From philosophical point of view, the neutrosophic set takes the value from real standard or non-standard subsets of $]^-0,1^+[$.so instead of $]^-0,1^+[$ we need to take the interval [0,1] for technical applications, because $]^-0,1^+[$will be difficult to apply in the real applications such as in scientific and engineering problems.

Definition 2 [2]

Let X be a space of points (objects) with generic elements in X denoted by x. An interval valued neutrosophic set (for short IVNS) A in X is characterized by truth-membership function $\mu_A(x)$, indeteminacy-membership function $\nu_A(x)$ and falsity-membership function $\omega_A(x)$. For each point x in X, we have that $\mu_A(x), \nu_A(x), \omega_A(x) \in [0,1]$.

For two IVNS, A= $\{<x, [\mu_A^L(x), \mu_A^U(x)], [\nu_A^L(x), \nu_A^U(x)], [\omega_A^L(x), \omega_A^U(x)] > | x \in X\}$ (2)

And B= $\{<x, [\mu_B^L(x), \mu_B^U(x)], [\nu_B^L(x), \nu_B^U(x)], [\omega_B^L(x), \omega_B^U(x)]> | x \in X\}$ the two relations are defined as follows:

(1) $A \subseteq B$ if and only if $\mu_A^L(x) \leq \mu_B^L(x), \mu_A^U(x) \leq \mu_B^U(x), \nu_A^L(x) \geq \nu_B^L(x), \omega_A^U(x) \geq \omega_B^U(x)$, $\omega_A^L(x) \geq \omega_B^L(x), \omega_A^U(x) \geq \omega_B^U(x)$

(2) $A = B$ if and only if, $\mu_A(x) = \mu_B(x), \nu_A(x) = \nu_B(x), \omega_A(x) = \omega_B(x)$ for any $x \in X$

The complement of A_{IVNS} is denoted by A_{IVNS}^o and is defined by

$A^o = \{<x, [\omega_A^L(x), \omega_A^U(x)]>, [1 - \nu_A^U(x), 1 - \nu_A^L(x)], [\mu_A^L(x), \mu_A^U(x)] | x \in X\}$

$A \cap B = \{<x, [\min(\mu_A^L(x), \mu_B^L(x)), \min(\mu_A^U(x), \mu_B^U(x))], [\max(\nu_A^L(x), \nu_B^L(x)), \max(\nu_A^U(x), \nu_B^U(x)], [\max(\omega_A^L(x), \omega_B^L(x)), \max(\omega_A^U(x), \omega_B^U(x))] >: x \in X\}$

$A \cup B = \{<x, [\max(\mu_A^L(x), \mu_B^L(x)), \max(\mu_A^U(x), \mu_B^U(x))], [\min(\nu_A^L(x), \nu_B^L(x)), \min(\nu_A^U(x), \nu_B^U(x)], [\min(\omega_A^L(x), \omega_B^L(x)), \min(\omega_A^U(x), \omega_B^U(x))] >: x \in X\}$

$0_N = \{<x, [0, 0], [1, 1], [1, 1] >| x \in X\}$, denote the neutrosophic empty set ϕ

$1_N = \{<x, [0, 0], [0, 0], [1, 1] >| x \in X\}$, denote the neutrosophic universe set U

As an illustration, let us consider the following example.

Example 1.Assume that the universe of discourse U={x1, x2, x3}, where x1characterizes the capability, x2characterizes the trustworthiness and x3 indicates the prices of the objects. It may be further assumed that the values of x1, x2 and x3 are in [0, 1] and they are obtained from some questionnaires of some experts. The experts may impose their opinion in three components viz. the degree of goodness, the degree of indeterminacy and that of poorness to explain the characteristics of the objects. Suppose A is an interval neutrosophic set (INS) of U, such that,

A = {< x1,[0.3 0.4],[0.5 0.6],[0.4 0.5] >,< x2, ,[0.1 0.2],[0.3 0.4],[0.6 0.7]>,< x3, [0.2 0.4],[0.4 0.5],[0.4 0.6] >}, where the degree of goodness of capability is 0.3, degree of indeterminacy of capability is 0.5 and degree of falsity of capability is 0.4 etc.

Definition 3 [20]

Let R be an equivalence relation on the universal set U. Then the pair (U, R) is called a Pawlak approximation space. An equivalence class of R containing x will be denoted by $[x]_R$. Now

for $X \subseteq U$, the lower and upper approximation of X with respect to (U, R) are denoted by respectively R^*X and $R_* X$ and are defined by
$R_* X = \{x \in U: [x]_R \subseteq X\}$,
$R^*X = \{x \in U: [x]_R \cap X \neq \emptyset\}$.
Now if $R^*X = R_* X$, then X is called definable; otherwise X is called a rough set.
Definition 4 [27]
Let U be a universe and X , a rough set in U. An IF rough set A in U is characterized by a membership function $\mu_A : U \to [0, 1]$ and non-membership function $\nu_A : U \to [0, 1]$ such that
$\mu_A(\underline{R} X) = 1$, $\nu_A(\underline{R} X) = 0$
Or $[\mu_A(x), \nu_A(x)] = [1, 0]$ if $x \in (\underline{R} X)$ and $\mu_A(U - \overline{R} X) = 0$, $\nu_A(U - \overline{R} X) = 1$

Or $[\mu_A(x), \nu_A(x)] = [0, 1]$ if $x \in U - \overline{R} X$,
$0 \leq \mu_A(\overline{R} X - \underline{R} X) + \nu_A(\overline{R} X - \underline{R} X) \leq 1$

Example 2: Example of IF Rough Sets
Let U= {Child, Pre-Teen, Teen, Youth, Teenager, Young-Adult, Adult, Senior, Elderly} be a universe.
Let the equivalence relation R be defined as follows:
R*= {[Child, Pre-Teen], [Teen, Youth, Teenager], [Young-Adult, Adult],[Senior, Elderly]}.
Let X = {Child, Pre-Teen, Youth, Young-Adult} be a subset of univers U.
We can define X in terms of its lower and upper approximations:
$\underline{R} X$ = {Child, Pre-Teen}, and $\overline{R} X$ = {Child, Pre-Teen, Teen, Youth, Teenager, Young-Adult, Adult}.
The membership and non-membership functions
$\mu_A : U \to]1, 0[$ and $\nu_A : U \to]1, 0[$ on a set A are defined as follows:
μ_AChild) = 1, μ_A (Pre-Teen) = 1 and μ_A (Child) = 0, μ_A(Pre-Teen) = 0
μ_A (Young-Adult) = 0, μ_A (Adult) = 0, μ_A(Senior) = 0, μ_A (Elderly) = 0

3.Basic Concept of Rough Approximations of an Interval Valued Neutrosophic Set and their Properties.

In this section we define the notion of interval valued neutrosophic rough sets (in brief ivn-rough set) by combining both rough sets and interval neutrosophic sets. IVN- rough sets are the generalizations of interval valued intuitionistic fuzzy rough sets, that give more information about uncertain or boundary region.

Definition 5 : Let (U,R) be a pawlak approximation space ,for an interval valued neutrosophic set

$A = \{<x, [\mu_A^L(x), \mu_A^U(x)], [\nu_A^L(x), \nu_A^U(x)], [\omega_A^L(x), \omega_A^U(x)]> | x \in U\}$ neutrosophic set of.
The lower approximation $\underline{A}_R$ and $\overline{A}_R$ upper approximations of A in the pawlak approwimation space (U,R) are defined as:

$\underline{A}_R = \{<x, [\wedge_{y \in [x]_R}\{\mu_A^L(y)\}, \wedge_{y \in [x]_R}\{\mu_A^U(y)\}], [\vee_{y \in [x]_R}\{\nu_A^L(y)\}, \vee_{y \in [x]_R}\{\nu_A^U(y)\}],$
$[\vee_{y \in [x]_R}\{\omega_A^L(y)\}, \vee_{y \in [x]_R}\{\omega_A^U(y)\}]>: x \in U\}$.

$\overline{A}_R$**={<x,** [$\vee_{y\in[x]_R}\{\mu_A^L(y)\}$, $\vee_{y\in[x]_R}\{\mu_A^U(y)\}$], [$\wedge_{y\in[x]_R}\{\nu_A^L(y)\}$, $\wedge_{y\in[x]_R}\{\nu_A^U(y)\}$],
[$\wedge_{y\in[x]_R}\{\omega_A^L(y)\}$, $\wedge_{y\in[x]_R}\{\omega_A^U(y)\}$]:x ∈ U}.

Where " ∧ " means " min" and " ∨ " means " max", R denote an equivalence relation for interval valued neutrosophic set A.

Here $[x]_R$ is the equivalence class of the element x.

It is easy to see that

[$\wedge_{y\in[x]_R}\{\mu_A^L(y)\}$, $\wedge_{y\in[x]_R}\{\mu_A^U(y)\}$] ⊂ [0 ,1]

[$\vee_{y\in[x]_R}\{\nu_A^L(y)\}$, $\vee_{y\in[x]_R}\{\nu_A^U(y)\}$] ⊂ [0 ,1]

[$\vee_{y\in[x]_R}\{\omega_A^L(y)\}$, $\vee_{y\in[x]_R}\{\omega_A^U(y)\}$] ⊂ [0 ,1]

And

$\mathbf{0} \leq \wedge_{y\in[x]_R}\{\mu_A^U(y)\} + \vee_{y\in[x]_R}\{\nu_A^U(y)\} + \vee_{y\in[x]_R}\{\omega_A^U(y)\} \leq \mathbf{3}$

Then, $\underline{A}_R$ is an interval neutrosophic set

Similarly , we have

[$\vee_{y\in[x]_R}\{\mu_A^L(y)\}$, $\vee_{y\in[x]_R}\{\mu_A^U(y)\}$] ⊂ [0 ,1]

[$\wedge_{y\in[x]_R}\{\nu_A^L(y)\}$, $\wedge_{y\in[x]_R}\{\nu_A^U(y)\}$] ⊂ [0 ,1]

[$\wedge_{y\in[x]_R}\{\omega_A^L(y)\}$, $\wedge_{y\in[x]_R}\{\omega_A^U(y)\}$] ⊂ [0 ,1]

And

$\mathbf{0} \leq \vee_{y\in[x]_R}\{\mu_A^U(y)\} + \wedge_{y\in[x]_R}\{\nu_A^U(y)\} + \wedge_{y\in[x]_R}\{\omega_A^U(y)\} \leq \mathbf{3}$

Then, $\overline{A}_R$ is an interval neutrosophic set

If $\underline{A}_R = \overline{A}_R$,then A is a definable set, otherwise A is an interval valued neutrosophic rough set, $\underline{A}_R$ and $\overline{A}_R$ are called the lower and upper approximations of interval valued neutrosophic set with respect to approximation space (U, R), respectively. $\underline{A}_R$ and $\overline{A}_R$ are simply denoted by $\underline{A}$ and $\overline{A}$.

In the following , we employ an example to illustrate the above concepts

Example:

Theorem 1. Let A, B be interval neutrosophic sets and $\underline{A}$ and $\overline{A}$ the lower and upper approximation of interval –valued neutrosophic set A with respect to approximation space

(U,R) ,respectively. $\underline{B}$ and $\overline{B}$ the lower and upper approximation of interval –valued neutrosophic set B with respect to approximation space (U,R) ,respectively.Then we have

i. $\underline{A} \subseteq A \subseteq \overline{A}$
ii. $\overline{A \cup B} = \overline{A} \cup \overline{B}$, $\underline{A \cap B} = \underline{A} \cap \underline{B}$
iii. $\underline{A} \cup \underline{B} = \underline{A \cup B}$, $\overline{A \cap B} = \overline{A} \cap \overline{B}$
iv. $\overline{(\overline{A})} = \underline{(\overline{A})} = \overline{A}$, $\underline{(\underline{A})} = \overline{(\underline{A})} = \underline{A}$
v. $\underline{U} = U$; $\overline{\phi} = \phi$
vi. If $A \subseteq B$,then $\underline{A} \subseteq \underline{B}$ and $\overline{A} \subseteq \overline{B}$
vii. $\underline{A^c} = (\overline{A})^c$, $\overline{A^c} = (\underline{A})^c$

Proof:we prove only i,ii,iii, the others are trivial

(i)

Let $A = \{<x , [\mu_A^L(x), \mu_A^U(x)] , [\nu_A^L(x), \nu_A^U(x)] , [\omega_A^L(x), \omega_A^U(x)] > | x \in X \}$ be interval neutrosophic set

From definition of $\underline{A}_R$ and $\overline{A}_R$, we have

Which implies that

$\mu_{\underline{A}}^L(x) \leq \mu_A^L(x) \leq \mu_{\overline{A}}^L(x)$; $\mu_{\underline{A}}^U(x) \leq \mu_A^U(x) \leq \mu_{\overline{A}}^U(x)$ for all $x \in X$

$\nu_{\underline{A}}^L(x) \geq \nu_A^L(x) \geq \nu_{\overline{A}}^L(x)$; $\nu_{\underline{A}}^U(x) \geq \nu_A^U(x) \geq \nu_{\overline{A}}^U(x)$ for all $x \in X$

$\omega_{\underline{A}}^L(x) \geq \omega_A^L(x) \geq \omega_{\overline{A}}^L(x)$; $\omega_{\underline{A}}^U(x) \geq \omega_A^U(x) \geq \omega_{\overline{A}}^U(x)$ for all $x \in X$

$([\mu_{\underline{A}}^L , \mu_{\underline{A}}^U], [\nu_{\underline{A}}^L , \nu_{\underline{A}}^U], [\omega_{\underline{A}}^L , \omega_{\underline{A}}^U]) \subseteq ([\mu_A^L , \mu_A^U], [\nu_A^L , \nu_A^U], [\omega_A^L , \omega_A^U]) \subseteq ([\mu_{\overline{A}}^L , \mu_{\overline{A}}^U], [\nu_{\overline{A}}^L , \nu_{\overline{A}}^U], [\omega_{\overline{A}}^L , \omega_{\overline{A}}^U])$ .Hence $\underline{A}_R \subseteq A \subseteq \overline{A}_R$

(ii) Let $A = \{<x , [\mu_A^L(x), \mu_A^U(x)] , [\nu_A^L(x), \nu_A^U(x)] , [\omega_A^L(x), \omega_A^U(x)] > | x \in X \}$ and

$B = \{<x , [\mu_B^L(x), \mu_B^U(x)] , [\nu_B^L(x), \nu_B^U(x)] , [\omega_B^L(x), \omega_B^U(x)] > | x \in X \}$ are two interval neutrosophic set and

$\overline{A \cup B} = \{<x , [\mu_{\overline{A \cup B}}^L(x), \mu_{\overline{A \cup B}}^U(x)] , [\nu_{\overline{A \cup B}}^L(x), \nu_{\overline{A \cup B}}^U(x)] , [\omega_{\overline{A \cup B}}^L(x), \omega_{\overline{A \cup B}}^U(x)] > | x \in X \}$

$\overline{A} \cup \overline{B} = \{x, [\max(\mu_{\overline{A}}^L(x) , \mu_{\overline{B}}^L(x)) ,\max(\mu_{\overline{A}}^U(x) , \mu_{\overline{B}}^U(x))],[\min(\nu_{\overline{A}}^L(x) , \nu_{\overline{B}}^L(x)) ,\min(\nu_{\overline{A}}^U(x) , \nu_{\overline{B}}^U(x))],[\min(\omega_{\overline{A}}^L(x) , \omega_{\overline{B}}^L(x)) ,\min(\omega_{\overline{A}}^U(x) , \omega_{\overline{B}}^U(x))]$

for all $x \in X$

$$\mu_{\overline{A \cup B}}^L(x) = \vee\{ \mu_{A \cup B}^L(y) \mid y \in [x]_R\}$$
$$= \vee \{\mu_A^L(y) \vee \mu_B^L(y) \mid y \in [x]_R\}$$
$$= (\vee \mu_A^L(y) \mid y \in [x]_R) \vee (\vee \mu_A^L(y) \mid y \in [x]_R)$$

$=(\mu_{\overline{A}}^{L} \vee \mu_{\overline{B}}^{L})(x)$

$\mu_{\overline{A \cup B}}^{U}(x) = \vee\{\mu_{A \cup B}^{u}(y) \mid y \in [x]_R\}$

$= \vee \{\mu_{A}^{U}(y) \vee \mu_{B}^{U}(y) \mid y \in [x]_R\}$

$= (\vee \mu_{A}^{u}(y) \mid y \in [x]_R) \vee (\vee \mu_{A}^{U}(y) \mid y \in [x]_R)$

$=(\mu_{\overline{A}}^{U} \vee \mu_{\overline{B}}^{U})(x)$

$\nu_{\overline{A \cup B}}^{L}(x) = \wedge\{\nu_{A \cup B}^{L}(y) \mid y \in [x]_R\}$

$= \wedge \{\nu_{A}^{L}(y) \wedge \nu_{B}^{L}(y) \mid y \in [x]_R\}$

$= (\wedge \nu_{A}^{L}(y) \mid y \in [x]_R) \wedge (\wedge \nu_{B}^{L}(y) \mid y \in [x]_R)$

$=(\nu_{\overline{A}}^{L} \wedge \nu_{\overline{B}}^{L})(x)$

$\nu_{\overline{A \cup B}}^{U}(x) = \wedge\{\nu_{A \cup B}^{U}(y) \mid y \in [x]_R\}$

$= \wedge \{\nu_{A}^{U}(y) \wedge \nu_{B}^{U}(y) \mid y \in [x]_R\}$

$= (\wedge \nu_{A}^{U}(y) \mid y \in [x]_R) \wedge (\wedge \nu_{B}^{U}(y) \mid y \in [x]_R)$

$=(\nu_{\overline{A}}^{U}(y) \wedge \nu_{\overline{B}}^{U}(y))(x)$

$\omega_{\overline{A \cup B}}^{L}(x) = \wedge\{\omega_{A \cup B}^{L}(y) \mid y \in [x]_R\}$

$= \wedge \{\omega_{A}^{L}(y) \wedge \omega_{B}^{L}(y) \mid y \in [x]_R\}$

$= (\wedge \omega_{A}^{L}(y) \mid y \in [x]_R) \wedge (\wedge \omega_{B}^{L}(y) \mid y \in [x]_R)$

$=(\omega_{\overline{A}}^{L} \wedge \omega_{\overline{B}}^{L})(x)$

$\omega_{\overline{A \cup B}}^{U}(x) = \wedge\{\omega_{A \cup B}^{U}(y) \mid y \in [x]_R\}$

$= \wedge \{\omega_{A}^{U}(y) \wedge \nu_{B}^{U}(y) \mid y \in [x]_R\}$

$= (\wedge \omega_{A}^{U}(y) \mid y \in [x]_R) \wedge (\wedge \omega_{B}^{U}(y) \mid y \in [x]_R)$

$=(\omega_{\overline{A}}^{U} \wedge \omega_{\overline{B}}^{U})(x)$

Hence, $\overline{A \cup B} = \overline{A} \cup \overline{B}$

Also for $\underline{A \cap B} = \underline{A} \cap \underline{B}$ for all x ∈ A

$\mu_{\underline{A \cap B}}^{L}(x) = \wedge\{\mu_{A \cap B}^{L}(y) \mid y \in [x]_R\}$

$= \wedge \{\mu_{A}^{L}(y) \wedge \mu_{B}^{L}(y) \mid y \in [x]_R\}$

$= \wedge\ (\mu_A^L(y) \mid y\ \in [x]_R) \wedge\ (\vee \mu_B^L(y) \mid y\ \in [x]_R)$

$= \mu_{\underline{A}}^L(x) \wedge \mu_{\underline{B}}^L(x)$

$= (\mu_{\underline{A}}^L \wedge \mu_{\underline{B}}^L)(x)$

Also

$\mu_{\underline{A \cap B}}^U(x) = \wedge\{\mu_{A\cap B}^U(y) \mid y\ \in [x]_R\}$

$= \wedge\ \{\mu_A^U(y)\ \wedge\ \mu_B^U(y) \mid y\ \in [x]_R\}$

$= \wedge\ (\mu_A^U(y) \mid y\ \in [x]_R) \wedge\ (\vee \mu_B^U(y) \mid y\ \in [x]_R)$

$= \mu_{\underline{A}}^U(x) \wedge \mu_{\underline{B}}^U(x)$

$= (\mu_{\underline{A}}^U \wedge \mu_{\underline{B}}^U)(x)$

$\nu_{\underline{A \cap B}}^L(x) = \vee\{\nu_{A\cap B}^L(y) \mid y\ \in [x]_R\}$

$= \vee\ \{\nu_A^L(y)\ \vee\ \nu_B^L(y) \mid y\ \in [x]_R\}$

$= \vee\ (\nu_A^L(y) \mid y\ \in [x]_R) \vee\ (\vee \nu_B^L(y) \mid y\ \in [x]_R)$

$= \nu_{\underline{A}}^L(x) \vee \nu_{\underline{B}}^L(x)$

$= (\nu_{\underline{A}}^L \vee \nu_{\underline{B}}^L)(x)$

$\nu_{\underline{A \cap B}}^U(x) = \vee\{\nu_{A\cap B}^U(y) \mid y\ \in [x]_R\}$

$= \vee\ \{\nu_A^U(y)\ \vee\ \nu_B^U(y) \mid y\ \in [x]_R\}$

$= \vee\ (\nu_A^U(y) \mid y\ \in [x]_R) \vee\ (\vee \nu_B^U(y) \mid y\ \in [x]_R)$

$= \nu_{\underline{A}}^U(x) \vee \nu_{\underline{B}}^U(x)$

$= (\nu_{\underline{A}}^U \vee \nu_{\underline{B}}^U)(x)$

$\omega_{\underline{A \cap B}}^L(x) = \vee\{\omega_{A\cap B}^L(y) \mid y\ \in [x]_R\}$

$= \vee\ \{\omega_A^L(y)\ \vee\ \omega_B^L(y) \mid y\ \in [x]_R\}$

$= \vee\ (\omega_A^L(y) \mid y\ \in [x]_R) \vee\ (\vee \omega_B^L(y) \mid y\ \in [x]_R)$

$= \omega_{\underline{A}}^L(x) \vee \nu\omega_{\underline{B}}^L(x)$

$= (\omega_{\underline{A}}^L \vee \omega_{\underline{B}}^L)(x)$

$\omega^{\mathrm{U}}_{\underline{A\cap B}}(\mathrm{x}) = \vee\{\, \omega^{\mathrm{U}}_{A\cap B}(\mathrm{y}) \mid y \in [\mathrm{x}]_R\}$

$= \vee\ \{\omega^{\mathrm{U}}_{\mathrm{A}}(\mathrm{y}) \ \vee\ \omega^{\mathrm{U}}_{\mathrm{B}}(\mathrm{y}) \mid y \in [\mathrm{x}]_R\}$

$= \vee\ (\omega^{\mathrm{U}}_{\mathrm{A}}(\mathrm{y}) \mid y \in [\mathrm{x}]_R) \vee\ (\vee\, \omega^{\mathrm{U}}_{\mathrm{B}}(\mathrm{y}) \mid y \in [\mathrm{x}]_R)$

$=\omega^{\mathrm{U}}_{\underline{A}}(\mathrm{x}) \vee \omega^{\mathrm{U}}_{\underline{B}}(\mathrm{x})$

$=(\omega^{\mathrm{U}}_{\underline{A}} \vee \omega^{\mathrm{U}}_{\underline{B}})(\mathrm{x})$

(iii)

$\mu^{\mathrm{U}}_{\overline{A\cap B}}(\mathrm{x}) = \vee\{\, \mu^{\mathrm{U}}_{A\cap B}(\mathrm{y}) \mid y \in [\mathrm{x}]_R\}$

$= \vee\ \{\mu^{\mathrm{U}}_{\mathrm{A}}(\mathrm{y}) \ \wedge\ \mu^{\mathrm{U}}_{\mathrm{B}}(\mathrm{y}) \mid y \in [\mathrm{x}]_R\}$

$=(\vee\ (\mu^{\mathrm{U}}_{\mathrm{A}}(\mathrm{y}) \mid y \in [\mathrm{x}]_R)) \wedge (\vee\ (\mu^{\mathrm{U}}_{\mathrm{A}}(\mathrm{y}) \mid y \in [\mathrm{x}]_R))$

$= \mu^{\mathrm{U}}_{\overline{A}}(\mathrm{x}) \vee \mu^{\mathrm{U}}_{\overline{B}}(\mathrm{x})$

$=(\mu^{\mathrm{U}}_{\overline{A}} \vee \mu^{\mathrm{U}}_{\overline{B}})(\mathrm{x})$

$\nu^{\mathrm{U}}_{\overline{A\cap B}}(\mathrm{x}) = \wedge\{\, \nu^{\mathrm{U}}_{A\cap B}(\mathrm{y}) \mid y \in [\mathrm{x}]_R\}$

$= \wedge\ \{\nu^{\mathrm{U}}_{\mathrm{A}}(\mathrm{y}) \ \wedge\ \nu^{\mathrm{U}}_{\mathrm{B}}(\mathrm{y}) \mid y \in [\mathrm{x}]_R\}$

$=(\wedge\ (\nu^{\mathrm{U}}_{\mathrm{A}}(\mathrm{y}) \mid y \in [\mathrm{x}]_R)) \vee (\wedge\ (\nu^{\mathrm{U}}_{\mathrm{A}}(\mathrm{y}) \mid y \in [\mathrm{x}]_R))$

$= \nu^{\mathrm{U}}_{\overline{A}}(\mathrm{x}) \vee \nu^{\mathrm{U}}_{\overline{B}}(\mathrm{x})$

$=(\nu^{\mathrm{U}}_{\overline{A}} \vee \nu^{\mathrm{U}}_{\overline{B}})(\mathrm{x})$

$\omega^{\mathrm{U}}_{\overline{A\cap B}}(\mathrm{x}) = \wedge\{\, \omega^{\mathrm{U}}_{A\cap B}(\mathrm{y}) \mid y \in [\mathrm{x}]_R\}$

$= \wedge\ \{\omega^{\mathrm{U}}_{\mathrm{A}}(\mathrm{y}) \ \wedge\ \omega\nu^{\mathrm{U}}_{\mathrm{B}}(\mathrm{y}) \mid y \in [\mathrm{x}]_R\}$

$=(\wedge\ (\omega^{\mathrm{U}}_{\mathrm{A}}(\mathrm{y}) \mid y \in [\mathrm{x}]_R)) \vee (\wedge\ (\omega^{\mathrm{U}}_{\mathrm{A}}(\mathrm{y}) \mid y \in [\mathrm{x}]_R))$

$= \omega^{\mathrm{U}}_{\overline{A}}(\mathrm{x}) \vee \omega^{\mathrm{U}}_{\overline{B}}(\mathrm{x})$

$=(\omega^{\mathrm{U}}_{\overline{A}} \vee \omega^{\mathrm{U}}_{\overline{B}})(\mathrm{x})$

Hence follow that $\overline{A\cap B} = \overline{A} \cap \overline{B}$ .we get $\underline{A} \cup \underline{B} = \underline{A\cup B}$ by following the same procedure as above.

Definition 6:

Let (U,R) be a pawlak approximation space ,and A and B two interval valued neutrosophic sets over U.

If $\underline{A}$ =$\underline{B}$,then A and B are called interval valued neutrosophic lower rough equal.

If $\overline{A}$=$\overline{B}$, then A and B are called interval valued neutrosophic upper rough equal.

If $\underline{A}$ =$\underline{B}$, $\overline{A}$=$\overline{B}$, then A and B are called interval valued neutrosophic rough equal.

Theorem 2 .

Let (U,R) be a pawlak approximation space ,and A and B two interval valued neutrosophic sets over U. then

1. $\underline{A} = \underline{B} \Leftrightarrow \underline{A \cap B} = \underline{A}$, $\underline{A \cap B} = \underline{B}$
2. $\overline{A} = \overline{B} \Leftrightarrow \overline{A \cup B} = \overline{A}$, $\overline{A \cup B} = \overline{B}$
3. If $\overline{A} = \overline{A'}$ and $\overline{B} = \overline{B'}$,then $\overline{A \cup B} = \overline{A' \cup B'}$
4. If $\underline{A} = \underline{A'}$ and $\underline{B} = \underline{B'}$,Then
5. If $A \subseteq B$ and $\underline{B} = \underline{\phi}$,then $\underline{A} = \underline{\phi}$
6. If $A \subseteq B$ and $\underline{B} = \underline{U}$,then $\underline{A} = \underline{U}$
7. If $\underline{A} = \underline{\phi}$ or $\underline{B} = \underline{\phi}$ or then $\underline{A \cap B} = \underline{\phi}$
8. If $\overline{A} = \overline{U}$ or $\overline{B} = \overline{U}$,then $\overline{A \cup B} = \overline{U}$
9. $\overline{A} = \overline{U} \Leftrightarrow A = U$
10. $\overline{A} = \overline{\phi} \Leftrightarrow A = \phi$

Proof: the proof is trial

4.Hamming distance between Lower Approximation and Upper Approximation of IVNS

In this section , we will compute the Hamming distance between lower and upper approximations of interval neutrosophic sets based on Hamming distance introduced by **Ye [41]** of interval neutrosophic sets.

Based on Hamming distance between two interval neutrosophic set A and B as follow:

$$d(A,B)=\frac{1}{6}\sum_{i=1}^{n}\left[\left|\mu_A^L(x_i)-\mu_B^L(x_i)\right|+\left|\mu_A^U(x_i)-\mu_B^U(x_i)\right|+\left|\nu_A^L(x_i)-\nu_B^L(x_i)\right|+\left|\nu_A^U(x_i)-\nu_B^U(x_i)\right|+\left|\omega_A^L(x_i)-\omega_B^L(x_i)\right|+\left|\omega_A^L(x_i)-\nu_B^U(x_i)\right|\right]$$

we can obtain the standard hamming distance of $\underline{A}$ and $\overline{A}$ from

$$d_H(\underline{A},\overline{A})=\frac{1}{6}\sum_{i=1}^{n}\left[\left|\mu_{\underline{A}}^L(x_j)-\mu_{\overline{A}}^L(x_j)\right|+\left|\mu_{\underline{A}}^U(x_j)-\mu_{\overline{A}}^U(x_j)\right|+\left|\nu_{\underline{A}}^L(x_j)-\nu_{\overline{A}}^L(x_j)\right|+\left|\nu_{\underline{A}}^U(x_j)-\nu_{\overline{A}}^U(x_j)\right|+\left|\omega_{\underline{A}}^L(x_j)-\omega_{\overline{A}}^L(x_j)\right|+\left|\omega_{\underline{A}}^U(x_j)-\omega_{\overline{A}}^U(x_j)\right|\right]$$

Where

$\underline{A}_R$={<x, $[\wedge_{y \in [x]_R}\{\mu_A^L(y)\}, \wedge_{y \in [x]_R}\{\mu_A^U(y)\}], [\vee_{y \in [x]_R}\{\nu_A^L(y)\}, \vee_{y \in [x]_R}\{\nu_A^U(y)\}], [\vee_{y \in [x]_R}\{\omega_A^L(y)\}, \vee_{y \in [x]_R}\{\omega_A^U(y)\}]$>:x ∈ U}.

$\overline{A}_R$={<x, $[\vee_{y \in [x]_R}\{\mu_A^L(y)\}, \vee_{y \in [x]_R}\{\mu_A^U(y)\}], [\wedge_{y \in [x]_R}\{\nu_A^L(y)\}, \wedge_{y \in [x]_R}\{\nu_A^U(y)], [\wedge_{y \in [x]_R}\{\omega_A^L(y)\}, \wedge_{y \in [x]_R}\{\omega_A^U(y)\}]$:x ∈ U}.

$\mu_{\underline{A}}^L(x_j) = \wedge_{y \in [x]_R}\{\mu_A^L(y)\}$; $\mu_{\underline{A}}^U(x_j) = \wedge_{y \in [x]_R}\{\mu_A^U(y)\}$

$\nu_{\underline{A}}^L(x_j) = \vee_{y \in [x]_R}\{\nu_A^L(y)\}$; $\nu_{\underline{A}}^U(x_j) = \vee_{y \in [x]_R}\{\nu_A^U(y)\}$

$\omega_{\underline{A}}^L(x_j) = \vee_{y \in [x]_R}\{\omega_A^L(y)\}$; $\omega_{\underline{A}}^U(x_j) = \vee_{y \in [x]_R}\{\omega_A^U(y)\}$

$\mu_{\overline{A}}^L(x_j) = \vee_{y \in [x]_R}\{\mu_A^L(y)\}$; $\mu_{\overline{A}}^U(x_j) = \vee_{y \in [x]_R}\{\mu_A^U(y)\}$

$\mu_{\overline{A}}^L(x_j) = \wedge_{y \in [x]_R}\{\nu_A^L(y)\}$; $\mu_{\overline{A}}^U(x_j) = \wedge_{y \in [x]_R}\{\nu_A^U(y)\}$

$\omega_{\overline{A}}^L(x_j) = \wedge_{y \in [x]_R}\{\omega_A^L(y)\}$, ; $\omega_{\overline{A}}^U(x_j) = \wedge_{y \in [x]_R}\{\omega_A^U(y)\}$

Theorem 3. Let (U, R) be approximation space, A be an interval valued neutrosophic set over U . Then

(1) If d $(\underline{A}, \overline{A}) = 0$, then A is a definable set.

(2) If $0 < d(\underline{A}, \overline{A}) < 1$, then A is an interval-valued neutrosophic rough set.

Theorem 4. Let (U, R) be a Pawlak approximation space, and A and B two interval-valued neutrosophic sets over U . Then

1. $d(\underline{A}, \overline{A}) \geq d(\underline{A}, A)$ and $d(\underline{A}, \overline{A}) \geq d(A, \overline{A})$;
2. $d(\overline{A \cup B}, \overline{A} \cup \overline{B}) = 0$, $d(\underline{A \cap B}, \underline{A} \cap \underline{B}) = 0$.
3. $d(\underline{A} \cup \underline{B}, A \cup B) \geq d(\underline{A} \cup \underline{B}, \underline{A \cup B})$
   and $d(\underline{A} \cup \underline{B}, A \cup B) \geq d(\underline{A \cup B}, A \cup B)$;
   and $d(A \cap B, \overline{A} \cap \overline{B}) \geq d(A \cap B, \overline{A \cap B})$
   and $d(A \cap B, \overline{A} \cap \overline{B}) \geq d(\overline{A \cap B}, \overline{A} \cap \overline{B})$
4. $d(\overline{(\overline{A})}, \underline{(\overline{A})}) = 0$, $d(\overline{(\overline{A})}, \overline{A}) = 0$, $d(\underline{(\overline{A})}, \overline{A}) = 0$;
   $d(\underline{(\underline{A})}, \underline{(\overline{A})}) = 0$, $d(\underline{(\underline{A})}, , \overline{A}) = 0$, $d(\underline{(\overline{A})}, \underline{A}) = 0$,
5. $d(\underline{U}, U) = 0$, $d(\overline{\phi}, \phi) = 0$
6. if A B ,then $d(\overline{A}, B) \geq d(\underline{A}, \underline{B})$ and $d(\underline{A}, B) \geq d(\underline{B}, B)$
   $d(A, \overline{B}) \geq d(A, \overline{A})$ and $d(A, \overline{B}) = \geq d(\overline{A}, \overline{B})$
7. $d(\underline{A^c}, (\overline{A})^c) = 0$, $d(\overline{A^c}, (\underline{A})^c) = 0$

5-Conclusion

In this paper we have defined the notion of interval valued neutrosophic rough sets. We have also studied some properties on them and proved some propositions. The concept combines two different theories which are rough sets theory and interval valued neutrosophic set theory. Further, we have introduced the Hamming distance between two interval neutrosophic rough sets. We hope that our results can also be extended to other algebraic system.

Acknowledgements

The authors would like to thank the anonymous reviewer for their careful reading of this research paper and for their helpful comments.

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Published in *Journal of New Research in Science*, 14 p., pages 127 – 140.

Interval Neutrosophic Logic

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Abstract

In this paper, we present a novel interval neutrosophic logic that generalizes the interval valued fuzzy logic, the intuitionistic fuzzy logic and paraconsistent logics which only consider truth-degree or falsity-degree of a proposition. In the interval neutrosophic logic, we consider not only truth-degree and falsity-degree but also indeterminacy-degree which can reliably capture more information under uncertainty. We introduce mathematical definitions of an interval neutrosophic propositional calculus and an interval neutrosophic predicate calculus. We propose a general method to design an interval neutrosophic logic system which consists of neutrosophication, neutrosophic inference, a neutrosophic rule base, neutrosophic type reduction and deneutrosophication. A neutrosophic rule contains input neutrosophic linguistic variables and output neutrosophic linguistic variables. A neutrosophic linguistic variable has neutrosophic linguistic values which defined by interval neutrosophic sets characterized by three membership functions: truth-membership, falsity-membership and indeterminacy-membership. The interval neutrosophic logic can be applied to many potential real applications where information is imprecise, uncertain, incomplete and inconsistent such as Web intelligence, medical informatics, bioinformatics, decision making, etc.

Index Terms

Interval neutrosophic sets, interval neutrosophic logic, interval valued fuzzy logic, intuitionistic fuzzy logic, paraconsistent logics, interval neutrosophic logic system.

I. Introduction

The concept of fuzzy sets was introduced by Zadeh in 1965 [1]. Since then fuzzy sets and fuzzy logic have been applied to many real applications to handle uncertainty. The traditional fuzzy set uses one real value $\mu_A(x) \in [0,1]$ to represent the grade of membership of fuzzy set A defined on universe X. The corresponding fuzzy logic associates each proposition p with a real value $\mu(p) \in [0,1]$ which represents the degree of truth. Sometimes $\mu_A(x)$ itself is uncertain and hard to be defined by a crisp value. So the concept of interval valued fuzzy sets was proposed [2] to capture the uncertainty of grade of membership. The interval valued fuzzy set uses an interval value $[\mu_A^L(x), \mu_A^U(x)]$ with $0 \leq \mu_A^L(x) \leq \mu_A^U(x) \leq 1$ to represent the grade of membership of fuzzy set. The traditional fuzzy logic can be easily extended to the interval valued fuzzy logic. There are related works such as type-2 fuzzy sets and type-2 fuzzy logic [3], [4], [5]. The family of fuzzy sets and fuzzy logic can only handle "complete" information that is if a grade of truth-memhership is $\mu_A(x)$ then a grade of false-membership is $1 - \mu_A(x)$ by default. In some applications such as expert systems, decision making systems and information fusion systems, the information is both uncertain and incomplete. That is beyond the scope of traditional fuzzy sets and fuzzy logic. In 1986, Atanassov introduced the intuitionistic fuzzy set [6] which is a generalization of a fuzzy set and provably equivalent to an interval valued fuzzy set. The intuitionistic fuzzy sets consider both truth-membership and false-membership. The corresponding intuitionistic fuzzy logic [7], [8], [9] associates each proposition p with two real values $\mu(p)$-truth degree and $\nu(p)$-falsity degree, respectively, where $\mu(p), \nu(p) \in [0,1], \mu(p) + \nu(p) \leq 1$. So intuitionistic fuzzy sets and intuitionistic fuzzy logic can handle uncertain and incomplete information.

However, inconsistent information exists in a lot of real situations such as those mentioned above. It is obvious that the intuitionistic fuzzy logic cannot reason with inconsistency because it requires $\mu(p) + \nu(p) \leq 1$. Generally, two basic approaches are used to solve the inconsistency problem in knowledge bases: the belief revision and paraconsistent logics. The goal of the first approach is to make an inconsistent theory consistent, either by revising it or by representing it by a consistent semantics. On the other hand, the paraconsistent approach allows reasoning in the presence of inconsistency as contradictory information can be derived or introduced without trivialization [10]. de Costa's C_w logic [11] and Belnap's four-valued logic [12] are two well-known paraconsistent logics.

Neutrosophy was introduced by Smarandache in 1995. "Neutrosophy is a branch of philosophy which studies the origin, nature and scope of neutralities, as well as their interactions with different ideational spectra" [13]. Neutrosophy includes neutrosophic probability, neutrosophic sets and neutrosophic logic. In a neutrosophic set (neutrosophic logic), indeterminacy is quantified explicitly and truth-membership (truth-degree), indeterminacy-membership (indeterminacy-degree) and false-membership (falsity-degree) are independent. The independence assumption is very important in a lot of applications such as information fusion when we try to combine different data from different sensors. A neutrosophic set (neutrosophic logic) is different from an intuitionistic fuzzy set (intuitionistic fuzzy logic) where indeterminacy membership (indeterminacy-degree) is $1 - \mu_A(x) - \nu_A(x)$ $(1 - \mu(p) - \nu(p))$ by default.

The neutrosophic set generalizes the above mentioned sets from a philosophical point of view. From a scientific or engineering point of view, the neutrosophic set and set-theoretic operators need to be specified meaningfully. Otherwise, it will be difficult to apply to the real applications. In [14] we discussed a special neutrosophic set called an interval neutrosophic set and defined a set of set-theoretic operators. It is natural to define the interval neutrosophic logic based on interval neutrosophic sets. In this paper, we give mathematical definitions of an interval neutrosophic propositional calculus and a first order interval neutrosophic predicate calculus.

The rest of paper is organized as follows. Section II gives a brief review of interval neutrosophic sets. Section III gives the mathematical definition of the interval neutrosophic propositional calculus. Section IV gives the mathematical definition of the first order interval neutrosophic predicate calculus. Section V provides one application example of interval neutrosophic logic as the foundation for the design of interval neutrosophic logic system. In section VI we conclude the paper and discuss the future research directions.

II. Interval Neutrosophic Sets

This section gives a brief overview of concepts of interval neutrosophic sets defined in [14]. An interval neutrosophic set is an instance of the neutrosophic set introduced in [15] which can be used in real scientific and engineering applications.

Definition 1 (Interval Neutrosophic Set): Let X be a space of points (objects), with a generic element in X denoted by x. An interval neutrosophic set (INS) A in X is characterized by truth-membership function T_A, indeterminacy-membership function I_A and false-membership function F_A. For each point x in X, $T_A(x), I_A(x), F_A(x) \subseteq [0,1]$. □

When X is continuous, an INS A can be written as

$$A = \int_X \langle T(x), I(x), F(x)\rangle / x, x \in X$$

When X is discrete, an INS A can be written as

$$A = \sum_{i=1}^{n} \langle T(x_i), I(x_i), F(x_i)\rangle / x_i, x_i \in X$$

Example 1: Consider parameters such as capability, trustworthiness, and price of semantic Web services. These parameters are commonly used to define quality of service of semantic Web services [16]. Assume that $X = [x_1, x_2, x_3]$. x_1 is capability, x_2 is trustworthiness and x_3 is price. The values of x_1, x_2 and x_3 are a subset of $[0,1]$. They are obtained from the questionnaire of some domain experts, their option could be a degree of "good service", a degree of indeterminacy and a degree of "poor service". A is an interval neutrosophic set of X defined by $A = \langle [0.2, 0.4], [0.3, 0.5], [0.3, 0.5]\rangle / x_1 + \langle [0.5, 0.7], [0, 0.2], [0.2, 0.3]\rangle / x_2 + \langle [0.6, 0.8], [0.2, 0.3], [0.2, 0.3]\rangle / x_3$. □

Definition 2: An interval neutrosophic set A is empty if and only if its $\inf T_A(x) = \sup T_A(x) = 0, \inf I_A(x) = \sup I_A(x) = 1$ and $\inf F_A(x) = \sup F_A(x) = 0$, for all x in X. □

Let A be an interval neutrosophic set on X, then $A(x) = \langle T_A(x), I_A(x), F_A(x)\rangle$. Let $\underline{0} = \langle 0, 0, 1\rangle$ and $\underline{1} = \langle 1, 1, 0\rangle$.

Definition 3: Let A and B be two interval neutrosophic sets defined on X. $A(x) \leq B(x)$ if and only if

$$\inf T_A(x) \leq \inf T_B(x) \quad , \quad \sup T_A(x) \leq \sup T_B(x), \tag{1}$$

$$\inf I_A(x) \leq \inf I_B(x) \quad , \quad \sup I_A(x) \leq \sup I_B(x), \tag{2}$$

$$\inf F_A(x) \geq \inf F_B(x) \quad , \quad \sup F_A(x) \geq \sup F_B(x). \tag{3}$$

□

Definition 4 (Containment): An interval neutrosophic set A is contained in the other interval neutrosophic set B, $A \subseteq B$, if and only if $A(x) \leq B(x)$, for all x in X. □

Definition 5: Two interval neutrosophic sets A and B are equal, written as $A = B$, if and only if $A \subseteq B$ and $B \subseteq A$. □

Let $N = \langle [0,1] \times [0,1], [0,1] \times [0,1], [0,1] \times [0,1]\rangle$.

Definition 6 (Complement): Let C_N denote a neutrosophic *complement* of A. Then C_N is a function

$$C_N : N \to N$$

and C_N must satisfy at least the following three axiomatic requirements:

1) $C_N(\underline{0}) = \underline{1}$ and $C_N(\underline{1}) = \underline{0}$ (boundary conditions).
2) Let A and B be two interval neutrosophic sets defined on X, if $A(x) \leq B(x)$, then $C_N(A(x)) \geq C_N(B(x))$, for all x in X. (monotonicity).
3) Let A be an interval neutrosophic set defined on X, then $C_N(C_N(A(x))) = A(x)$, for all x in X. (involutivity).

□

There are many functions which satisfy the requirement to be the complement operator of interval neutrosophic sets. Here we give one example.

Definition 7 (Complement C_{N_1}): The complement of an interval neutrosophic set A is denoted by $\bar{A}$ and is defined by

$$T_{\bar{A}}(x) = F_A(x), \tag{4}$$

$$\inf I_{\bar{A}}(x) = 1 - \sup I_A(x), \tag{5}$$

$$\sup I_{\bar{A}}(x) = 1 - \inf I_A(x), \tag{6}$$

$$F_{\bar{A}}(x) = T_A(x), \tag{7}$$

for all x in X. □

Definition 8 (N-norm): Let I_N denote a neutrosophic *intersection* of two interval neutrosophic sets A and B. Then I_N is a function

$$I_N : N \times N \to N$$

and I_N must satisfy at least the following four axiomatic requirements:

1) $I_N(A(x), \underline{1}) = A(x)$, for all x in X. (boundary condition).
2) $B(x) \leq C(x)$ implies $I_N(A(x), B(x)) \leq I_N(A(x), C(x))$, for all x in X. (monotonicity).
3) $I_N(A(x), B(x)) = I_N(B(x), A(x))$, for all x in X. (commutativity).
4) $I_N(A(x), I_N(B(x), C(x))) = I_N(I_N(A(x), B(x)), C(x))$, for all x in X. (associativity).

□

Here we give one example of intersection of two interval neutrosophic sets which satisfies above N-norm axiomatic requirements. Other different definitions can be given for different applications.

Definition 9 (Intersection I_{N_1}): The intersection of two interval neutrosophic sets A and B is an interval neutrosophic set C, written as $C = A \cap B$, whose truth-membership, indeterminacy-membership, and false-membership are related to those of A and B by

$$\begin{aligned}
\inf T_C(x) &= \min(\inf T_A(x), \inf T_B(x)), &(8)\\
\sup T_C(x) &= \min(\sup T_A(x), \sup T_B(x)), &(9)\\
\inf I_C(x) &= \min(\inf I_A(x), \inf I_B(x)), &(10)\\
\sup I_C(x) &= \min(\sup I_A(x), \sup I_B(x)), &(11)\\
\inf F_C(x) &= \max(\inf F_A(x), \inf F_B(x)), &(12)\\
\sup F_C(x) &= \max(\sup F_A(x), \sup F_B(x)), &(13)
\end{aligned}$$

for all x in X. □

Definition 10 (N-conorm): Let U_N denote a neutrosophic *union* of two interval neutrosophic sets A and B. Then U_N is a function

$$U_N : N \times N \to N$$

and U_N must satisfy at least the following four axiomatic requirements:

1) $U_N(A(x), \underline{0}) = A(x)$, for all x in X. (boundary condition).
2) $B(x) \leq C(x)$ implies $U_N(A(x), B(x)) \leq U_N(A(x), C(x))$, for all x in X. (monotonic ity).
3) $U_N(A(x), B(x)) = U_N(B(x), A(x))$, for all x in X. (commutativity).
4) $U_N(A(x), U_N(B(x), C(x))) = U_N(U_N(A(x), B(x)), C(x))$, for all x in X. (associativity).

□

Here we give one example of union of two interval neutrosophic sets which satisfies above N-conorm axiomatic requirements. Other different definitions can be given for different applications.

Definition 11 (Union U_{N_1}): The union of two interval neutrosophic sets A and B is an interval neutrosophic set C, written as $C = A \cup B$, whose truth-membership, indeterminacy-membership, and false-membership are related to those of A and B by

$$\begin{aligned}
\inf T_C(x) &= \max(\inf T_A(x), \inf T_B(x)), &(14)\\
\sup T_C(x) &= \max(\sup T_A(x), \sup T_B(x)), &(15)\\
\inf I_C(x) &= \max(\inf I_A(x), \inf I_B(x)), &(16)\\
\sup I_C(x) &= \max(\sup I_A(x), \sup I_B(x)), &(17)\\
\inf F_C(x) &= \min(\inf F_A(x), \inf F_B(x)), &(18)\\
\sup F_C(x) &= \min(\sup F_A(x), \sup F_B(x)), &(19)
\end{aligned}$$

for all x in X. □

Theorem 1: Let P be the power set of all interval neutrosophic sets defined in the universe X. Then $\langle P; I_{N_1}, U_{N_1} \rangle$ is a distributive lattice.

Proof: Let A, B, C be the arbitrary interval neutrosophic sets defined on X. It is easy to verify that $A \cap A = A, A \cup A = A$ (idempotency), $A \cap B = B \cap A, A \cup B = B \cup A$ (commutativity), $(A \cap B) \cap C = A \cap (B \cap C), (A \cup B) \cup C = A \cup (B \cup C)$ (associativity), and $A \cap (B \cup C) = (A \cap B) \cup (A \cap C), A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ (distributivity). ■

Definition 12 (Interval neutrosophic relation): Let X and Y be two non-empty crisp sets. An interval neutrosophic relation $R(X, Y)$ is a subset of product space $X \times Y$, and is characterized by the truth membership function $T_R(x, y)$, the indeterminacy membership function $I_R(x, y)$, and the falsity membership function $F_R(x, y)$, where $x \in X$ and $y \in Y$ and $T_R(x, y), I_R(x, y), F_R(x, y) \subseteq [0, 1]$.

Definition 13 (Interval Neutrosophic Composition Functions): The membership functions for the composition of interval neutrosophic relations $R(X, Y)$ and $S(Y, Z)$ are given by the *interval neutrosophic sup-star composition* of R and S

$$T_{R\circ S}(x,z) = \sup_{y\in Y} \min(T_R(x,y), T_S(y,z)), \quad (20)$$

$$I_{R\circ S}(x,z) = \sup_{y\in Y} \min(I_R(x,y), I_S(y,z)), \quad (21)$$

$$F_{R\circ S}(x,z) = \inf_{y\in Y} \max(F_R(x,y), F_S(y,z)). \quad (22)$$

If R is an interval neutrosophic set rather than an interval neutrosophic relation, then $Y = X$ and $\sup_{y\in Y} \min(T_R(x,y), T_S(y,z))$ becomes $\sup_{y\in Y} \min(T_R(x), T_S(y,z))$, which is only a function of the output variable z. It is similar for $\sup_{y\in Y} \min(I_R(x,y), I_S(y,z))$ and $\inf_{y\in Y} \max(F_R(x,y), F_S(y,z))$. Hence, the notation of $T_{R\circ S}(x,z)$ can be simplified to $T_{R\circ S}(z)$, so that in the case of R being just an interval neutrosophic set,

$$T_{R\circ S}(z) = \sup_{x\in X} \min(T_R(x), T_S(x,z)), \quad (23)$$

$$I_{R\circ S}(z) = \sup_{x\in X} \min(I_R(x), I_S(x,z)), \quad (24)$$

$$F_{R\circ S}(z) = \inf_{x\in X} \max(F_R(x), F_S(x,z)). \quad (25)$$

Definition 14 (Truth-favorite): The truth-favorite of an interval neutrosophic set A is an interval neutrosophic set B, written as $B = \triangle A$, whose truth-membership and false-membership are related to those of A by

$$\inf T_B(x) = \min(\inf T_A(x) + \inf I_A(x), 1), \quad (26)$$

$$\sup T_B(x) = \min(\sup T_A(x) + \sup I_A(x), 1), \quad (27)$$

$$\inf I_B(x) = 0, \quad (28)$$

$$\sup I_B(x) = 0, \quad (29)$$

$$\inf F_B(x) = \inf F_A(x), \quad (30)$$

$$\sup F_B(x) = \sup F_A(x), \quad (31)$$

for all x in X. □

Definition 15 (False-favorite): The false-favorite of an interval neutrosophic set A is an interval neutrosophic set

B, written as $B = \nabla A$, whose truth-membership and false-membership are related to those of A by

$$\inf T_B(x) = \inf T_A(x), \tag{32}$$

$$\sup T_B(x) = \sup T_A(x), \tag{33}$$

$$\inf I_B(x) = 0, \tag{34}$$

$$\sup I_B(x) = 0, \tag{35}$$

$$\inf F_B(x) = \min(\inf F_A(x) + \inf I_A(x), 1), \tag{36}$$

$$\sup F_B(x) = \min(\sup F_A(x) + \sup I_A(x), 1), \tag{37}$$

for all x in X. □

III. Interval Neutrosophic Propositional Calculus

In this section, we introduce the elements of an interval neutrosophic propositional calculus based on the definition of the interval neutrosophic sets by using the notations from the theory of classical propositional calculus [17].

A. Syntax of Interval Neutrosophic Propositional Calculus

Here we give the formalization of syntax of the interval neutrosophic propositional calculus.

Definition 16: An *alphabet* of the interval neutrosophic propositional calculus consists of three classes of symbols:

1) A set of *interval neutrosophic propositional variables*, denoted by lower-case letters, sometimes indexed;
2) Five *connectives* $\wedge, \vee, \neg, \rightarrow, \leftrightarrow$ which are called conjunction, disjunction, negation, implication, and biimplication symbols respectively;
3) The parentheses (and).

□

The alphabet of the interval neutrosophic propositional calculus has combinations obtained by assembling connectives and interval neutrosophic propositional variables in strings. The purpose of the construction rules is to have the specification of distinguished combinations, called formulas.

Definition 17: The set of formulas (well-formed formulas) of interval neutrosophic propositional calculus is defined as follows.

1) Every *interval neutrosophic propositional variable* is a formula;
2) If p is a formula, then so is $(\neg p)$;
3) If p and q are formulas, then so are
   a) $(p \wedge q)$,
   b) $(p \vee q)$,
   c) $(p \rightarrow q)$, and
   d) $(p \leftrightarrow q)$.

4) No sequence of symbols is a formula which is not required to be by 1, 2, and 3.

□

To avoid having formulas cluttered with parentheses, we adopt the following precedence hierarchy, with the highest precedence at the top:

$$\neg,$$
$$\wedge, \vee,$$
$$\rightarrow, \leftrightarrow.$$

Here is an example of the interval neutrosophic propositional calculus formula:

$$\neg p_1 \wedge p_2 \vee (p_1 \rightarrow p_3) \rightarrow p_2 \wedge \neg p_3$$

Definition 18: The *language of interval neutrosophic propositional calculus* given by an alphabet consists of the set of all formulas constructed from the symbols of the alphabet. □

B. Semantics of Interval Neutrosophic Propositional Calculus

The study of interval neutrosophic propositional calculus comprises, among others, a *syntax*, which has the distinction of well-formed formulas, and a *semantics*, the purpose of which is the assignment of a meaning to well-formed formulas.

To each interval neutrosophic proposition p, we associate it with an ordered triple components $\langle t(p), i(p), f(p)\rangle$, where $t(p), i(p), f(p) \subseteq [0,1]$. $t(p), i(p), f(p)$ is called truth-degree, indeterminacy-degree and falsity-degree of p, respectively. Let this assignment be provided by an *interpretation function* or *interpretation* INL defined over a set of propositions P in such a way that

$$INL(p) = \langle t(p), i(p), f(p)\rangle.$$

Hence, the function $INL : P \rightarrow N$ gives the truth, indeterminacy and falsity degrees of all propositions in P. We assume that the interpretation function INL assigns to the logical truth $T : INL(T) = \langle 1, 1, 0\rangle$, and to $F : INL(F) = \langle 0, 0, 1\rangle$.

An interpretation which makes a formula true is a *model* of the formula.

Let i, l be the subinterval of $[0,1]$. Then $i + l = [\inf i + \inf l, \sup i + \sup l]$, $i - l = [\inf i - \sup l, \sup i - \inf l]$, $\max(i,l) = [\max(\inf i, \inf l), \max(\sup i, \sup l)]$, $\min(i,l) = [\min(\inf i, \inf l), \min(\sup i, \sup l)]$.

The semantics of five interval neutrosophic propositional connectives is given in Table I. Note that $p \leftrightarrow q$ if and only if $p \rightarrow q$ and $q \rightarrow p$.

Example 2: $INL(p) = \langle 0.5, 0.4, 0.7\rangle$ and $INL(q) = \langle 1, 0.7, 0.2\rangle$. Then, $INL(\neg p) = \langle 0.7, 0.6, 0.5\rangle$, $INL(p \wedge \neg p) = \langle 0.5, 0.4, 0.7\rangle$, $INL(p \vee q) = \langle 1, 0.7, 0.2\rangle$, $INL(p \rightarrow q) = \langle 1, 1, 0\rangle$. □

TABLE I

SEMANTICS OF FIVE CONNECTIVES IN INTERVAL NEUTROSOPHIC PROPOSITIONAL LOGIC

Connectives	Semantics
$INL(\neg p)$	$\langle f(p), 1 - i(p), t(p) \rangle$
$INL(p \wedge q)$	$\langle \min(t(p), t(q)), \min(i(p), i(q)), \max(f(p), f(q)) \rangle$
$INL(p \vee q)$	$\langle \max(t(p), t(q)), \max(i(p), i(q)), \min(f(p), f(q)) \rangle$
$INL(p \rightarrow q)$	$\langle \min(1, 1 - t(p) + t(q)), \min(1, 1 - i(p) + i(q)), \max(0, f(q) - f(p)) \rangle$
$INL(p \leftrightarrow q)$	$\langle \min(1 - t(p) + t(q), 1 - t(q) + t(p)), \min(1 - i(p) + i(q), 1 - i(q) + i(p)), \max(f(p) - f(q), f(q) - f(p)) \rangle$

A given well-formed interval neutrosophic propositional formula will be called a tautology (valid) if $INL(A) = \langle 1, 1, 0 \rangle$, for all interpretation functions INL. It will be called a contradiction (inconsistent) if $INL(A) = \langle 0, 0, 1 \rangle$, for all interpretation functions INL.

Definition 19: Two formulas p and q are said to be *equivalent*, denoted $p = q$, if and only if the $INL(p) = INL(q)$ for every interpretation function INL. □

Theorem 2: Let F be the set of formulas and $\wedge$ be the meet and $\vee$ the join, then $\langle F; \wedge, \vee \rangle$ is a distributive lattice.

Proof: It is analogous to the proof of Theorem 1. ■

Theorem 3: If p and $p \rightarrow q$ are tautologies, then q is also a tautology.

Proof: Since p and $p \rightarrow q$ are tautologies then for every INL, $INL(p) = INL(p \rightarrow q) = \langle 1, 1, 0 \rangle$, that is $t(p) = i(p) = 1, f(p) = 0$, $t(p \rightarrow q) = \min(1, 1 - t(p) + t(q)) = 1$, $i(p \rightarrow q) = \min(1, 1 - i(p) + i(q)) = 1$, $f(p \rightarrow q) = \max(0, f(q) - f(p)) = 0$. Hence,
t(q) = i(q) = 1, f(q) = 0. So q is a tautology. ■

C. *Proof Theory of Interval Neutrosophic Propositional Calculus*

Here we give the proof theory for interval neutrosophic propositional logic to complement the semantics part.

Definition 20: The interval neutrosophic propositional logic is defined by the following axiom schema.

$$p \rightarrow (q \rightarrow p)$$

$$p_1 \wedge \ldots \wedge p_m \rightarrow q_1 \vee \ldots q_n \text{ provided some } p_i \text{ is some } q_j$$

$$p \rightarrow (q \rightarrow p \wedge q)$$

$$(p \rightarrow r) \rightarrow ((q \rightarrow r) \rightarrow (p \vee q \rightarrow r))$$

$$(p \vee q) \rightarrow r \text{ iff } p \rightarrow r \text{ and } q \rightarrow r$$

$$p \rightarrow q \text{ iff } \neg q \rightarrow \neg p$$

$$p \rightarrow q \text{ and } q \rightarrow r \text{ implies } p \rightarrow r$$

$$p \rightarrow q \text{ iff } p \leftrightarrow (p \wedge q) \text{ iff } q \rightarrow (p \vee q)$$

□

The concept of (formal) deduction of a formula from a set of formulas, that is, using the standard notation, $\Gamma \vdash p$, is defined as usual; in this case, we say that p is a syntactical consequence of the formulas in T.

Theorem 4: For interval neutrosophic propositional logic, we have

1) $\{p\} \vdash p$,
2) $\Gamma \vdash p$ entails $\Gamma \cup \Delta \vdash p$,
3) if $\Gamma \vdash p$ for any $p \in \Delta$ and $\Delta \vdash q$, then $\Gamma \vdash q$.

Proof: It is immediate from the standard definition of the syntactical consequence ($\vdash$). ■

Theorem 5: In interval neutrosophic propositional logic, we have:

1) $\neg\neg p \leftrightarrow p$
2) $\neg(p \wedge q) \leftrightarrow \neg p \vee \neg q$
3) $\neg(p \vee q) \leftrightarrow \neg p \wedge \neg q$

Proof: Proof is straight forward by following the semantics of interval neutrosophic propositional logic. ■

Theorem 6: In interval neutrosophic propositional logic, the following schemas do not hold:

1) $p \vee \neg p$
2) $\neg(p \wedge \neg p)$
3) $p \wedge \neg p \rightarrow q$
4) $p \wedge \neg p \rightarrow \neg q$
5) $\{p, p \rightarrow q\} \vdash q$
6) $\{p \rightarrow q, \neg q\} \vdash \neg p$
7) $\{p \vee q, \neg q\} \vdash p$
8) $\neg p \vee q \leftrightarrow p \rightarrow q$

Proof: Immediate from the semantics of interval neutrosophic propositional logic. ■

Example 3: To illustrate the use of the interval neutrosophic propositional consequence relation, let's consider the following example.

$$p \rightarrow (q \wedge r)$$

$$r \rightarrow s$$

$$q \rightarrow \neg s$$

$$a$$

From $p \rightarrow (q \wedge r)$, we get $p \rightarrow q$ and $p \rightarrow r$. From $p \rightarrow q$ and $q \rightarrow \neg s$, we get $p \rightarrow \neg s$. From $p \rightarrow r$ and $r \rightarrow s$, we get $p \rightarrow s$. Hence, p is equivalent to $p \wedge s$ and $p \wedge \neg s$. However, we cannot detach s from p nor $\neg s$ from p. This is in part due to interval neutrosophic propositional logic incorporating neither modus ponens nor and elimination. □

IV. Interval Neutrosophic Predicate Calculus

In this section, we will extend our consideration to the full language of first order interval neutrosophic predicate logic. First we give the formalization of syntax of first order interval neutrosophic predicate logic as in classical first-order predicate logic.

A. Syntax of Interval Neutrosophic Predicate Calculus

Definition 21: An *alphabet* of the first order interval neutrosophic predicate calculus consists of seven classes of symbols:

1) *variables*, denoted by lower-case letters, sometimes indexed;
2) *constants*, denoted by lower-case letters;
3) *function symbols*, denoted by lower-case letters, sometimes indexed;
4) *predicate symbols*, denoted by lower-case letters, sometimes indexed;
5) Five *connectives* $\wedge, \vee, \neg, \rightarrow, \leftrightarrow$ which are called the conjunction, disjunction, negation, implication, and biimplication symbols respectively;
6) Two *quantifiers*, the *universal quantifier* $\forall$ (for all) and the *existential quantifier* $\exists$ (there exists);
7) The parentheses (and).

□

To avoid having formulas cluttered with brackets, we adopt the following precedence hierarchy, with the highest precedence at the top:

$$\neg, \forall, \exists$$

$$\wedge, \vee$$

$$\rightarrow, \leftrightarrow$$

Next we turn to the definition of the first order interval neutrosophic language given by an alphabet.

Definition 22: A *term* is defined as follows:

1) A variable is a term.
2) A constant is a term.
3) If f is an n-ary function symbol and $t_1, \ldots, t_n$ are terms, then $f(t_1, \ldots, f_n)$ is a term.

□

Definition 23: A *(well-formed)formula* is defined inductively as follows:

1) If p is an n-ary predicate symbol and $t_1, \ldots, t_n$ are terms, then $p(t_1, \ldots, t_n)$ is a formula (called an *atomic formula* or, more simply, an *atom*).
2) If F and G are formulas, then so are $(\neg F), (F \wedge G), (F \vee G), (F \rightarrow G)$ and $(F \leftrightarrow G)$.

3) If F is a formula and x is a variable, then $(\forall x F)$ and $(\exists x F)$ are formulas.

□

Definition 24: The *first order interval neutrosophic language* given by an alphabet consists of the set of all formulas constructed from the symbols of the alphabet. □

Example 4: $\forall x \exists y (p(x,y) \rightarrow q(x)), \neg \exists x (p(x,a) \wedge q(x))$ are formulas. □

Definition 25: The *scope* of $\forall x$ (resp. $\exists x$) in $\forall x F$ (resp. $\exists x F$) is F. A *bound occurrence* of a variable in a formula is an occurrence immediately following a quantifier or an occurrence within the scope of a quantifier, which has the same variable immediately after the quantifier. Any other occurrence of a variable is *free*. □

Example 5: In the formula $\forall x p(x,y) \vee q(x)$, the first two occurrences of x are bound, while the third occurrence is free, since the scope of $\forall x$ is $p(x,y)$ and y is free. □

B. Semantics of Interval Neutrosophic Predicate Calculus

In this section, we study the semantics of interval neutrosophic predicate calculus, the purpose of which is the assignment of a meaning to well-formed formulas. In the interval neutrosophic propositional logic, an interpretation is an assignment of truth values (ordered triple component) to propositions. In the first order interval neutrosophic predicate logic, since there are variables involved, we have to do more than that. To define an interpretation for a well-formed formula in this logic, we have to specify two things, the domain and an assignment to constants and predicate symbols occurring in the formula. The following is the formal definition of an interpretation of a formula in the first order interval neutrosophic predicate logic.

Definition 26: An *interpretation function (or interpretation)* of a formula F in the first order interval neutrosophic predicate logic consists of a nonempty domain D, and an assignment of "values" to each constant and predicate symbol occurring in F as follows:

1) To each constant, we assign an element in D.
2) To each n-ary function symbol, we assign a mapping from D^n to D. (Note that $D^n = \{(x_1, \ldots, x_n) | x_1 \in D, \ldots, x_n \in D\}$).
3) Predicate symbols get their meaning through evaluation functions E which assign to each variable x an element $E(x) \in D$. To each n-ary predicate symbol p, there is a function $INP(p) : D^n \rightarrow N$. So $INP(p(x_1, \ldots, x_n)) = INP(p)(E(x_1), \ldots, E(x_n))$.

□

That is, $INP(p)(a_1, \ldots, a_n) = \langle t(p(a_1, \ldots, a_n)), i(p(a_1, \ldots, a_n)), f(p(a_1, \ldots, a_n)),$ where $t(p(a_1, \ldots, a_n)), i(p(a_1, \ldots, a_n)), f(p(a_1, \ldots, a_n)) \subseteq [0,1]$. They are called truth-degree, indeterminacy-degree and falsity-degree of $p(a_1, \ldots, a_n)$ respectively. We assume that the interpretation function INP assigns to the logical truth $T : INP(T) = \langle 1, 1, 0 \rangle$, and to $F : INP(F) = \langle 0, 0, 1 \rangle$.

The semantics of five interval neutrosophic predicate connectives and two quantifiers is given in Table II. For simplification of notation, we use p to denote $p(a_1, \ldots, a_i)$. Note that $p \leftrightarrow q$ if and only if $p \rightarrow q$ and $q \rightarrow p$.

TABLE II

SEMANTICS OF FIVE CONNECTIVES AND TWO QUANTIFIERS IN INTERVAL NEUTROSOPHIC PREDICATE LOGIC

Connectives	Semantics
$INP(\neg p)$	$\langle f(p), 1-i(p), t(p)\rangle$
$INP(p \wedge q)$	$\langle \min(t(p), t(q)), \min(i(p), i(q)), \max(f(p), f(q))\rangle$
$INP(p \vee q)$	$\langle \max(t(p), t(q)), \max(i(p), i(q)), \min(f(p), f(q))\rangle$
$INP(p \rightarrow q)$	$\langle \min(1, 1-t(p)+t(q)), \min(1, 1-i(p)+i(q)), \max(0, f(q)-f(p))\rangle$
$INP(p \leftrightarrow q)$	$\langle \min(1-t(p)+t(q), 1-t(q)+t(p)), \min(1-i(p)+i(q), 1-i(q)+i(p)), \max(f(p)-f(q), f(q)-f(p))\rangle$
$INP(\forall xF)$	$\langle \min t(F(E(x))), \min i(F(E(x))), \max f(F(E(x)))\rangle$, $E(x) \in D$
$INP(\exists xF)$	$\langle \max t(F(E(x))), \max i(F(E(x))), \min f(F(E(x)))\rangle$, $E(x) \in D$

Example 6: Let $D = 1,2,3$ and $p(1) = \langle 0.5, 1, 0.4\rangle, p(2) = \langle 1, 0.2, 0\rangle, p(3) = \langle 0.7, 0.4, 0.7\rangle$. Then $INP(\forall xp(x)) = \langle 0.5, 0.2, 0.7\rangle$, and $INP(\exists xp(x)) = \langle 1, 1, 0\rangle$. □

Definition 27: A formula F is *consistent (satisfiable)* if and only if there exists an interpretation I such that F is evaluated to $\langle 1,1,0\rangle$ in I. If a formula F is T in an interpretation I, we say that I is a *model* of F and I *satisfies* F. □

Definition 28: A formula F is *inconsistent (unsatisfiable)* if and only if there exists no interpretation that satisfies F. □

Definition 29: A formula F is *valid* if and only if every interpretation of F satisfies F. □

Definition 30: A formula F is a *logical consequence* of formulas $F_1, \ldots, F_n$ if and only if for every interpretation I, if $F_1 \wedge \ldots \wedge F_n$ is true in I, F is also true in I. □

Example 7: $(\forall x)(p(x) \rightarrow (\exists y)p(y))$ is valid, $(\forall x)p(x) \wedge (\exists y)\neg p(y)$ is consistent. □

Theorem 7: There is no inconsistent formula in the first order interval neutrosophic predicate logic.

Proof: It is direct from the definition of semantics of interval neutrosophic predicate logic. ■

Note that the first order interval neutrosophic predicate logic can be considered as an extension of the interval neutrosophic propositional logic. When a formula in the first order logic contains no variables and quantifiers, it can be treated just as a formula in the propositional logic.

C. Proof Theory of Interval Neutrosophic Predicate Calculus

In this part, we give the proof theory for first order interval neutrosophic predicate logic to complement the semantics part.

Definition 31: The first order interval neutrosophic predicate logic is defined by the following axiom schema.

$$(p \rightarrow q(x)) \rightarrow (p \rightarrow \forall xq(x))$$
$$\forall xp(x) \rightarrow p(a)$$
$$p(x) \rightarrow \exists xp(x)$$
$$(p(x) \rightarrow q) \rightarrow (\exists xp(x) \rightarrow q)$$

□

Theorem 8: In the first order interval neutrosophic predicate logic, we have:

1) $p(x) \vdash \forall x p(x)$
2) $p(a) \vdash \exists x p(x)$
3) $\forall x p(x) \vdash p(y)$
4) $\Gamma \cup \{p(x)\} \vdash q$, then $\Gamma \cup \{\exists x p(x)\} \vdash q$

Proof: Directly from the definition of the semantics of first order interval neutrosophic predicate logic. ■

Theorem 9: In the first order interval neutrosophic predicate logic, the following schemes are valid, where r is a formula in which x does not appear free:

1) $\forall x r \leftrightarrow r$
2) $\exists x r \leftrightarrow r$
3) $\forall x \forall y p(x,y) \leftrightarrow \forall y \forall x p(x,y)$
4) $\exists x \exists y p(x,y) \leftrightarrow \exists y \exists x p(x,y)$
5) $\forall x \forall y p(x,y) \rightarrow \forall x p(x,x)$
6) $\exists x p(x,x) \rightarrow \exists x \exists y p(x,y)$
7) $\forall x p(x) \rightarrow \exists x p(x)$
8) $\exists x \forall y p(x,y) \rightarrow \forall y \exists x p(x,y)$
9) $\forall x (p(x) \wedge q(x)) \leftrightarrow \forall x p(x) \wedge \forall x q(x)$
10) $\exists x (p(x) \vee q(x)) \leftrightarrow \exists x p(x) \vee \exists x q(x)$
11) $p \wedge \forall x q(x) \leftrightarrow \forall x (p \wedge q(x))$
12) $p \vee \forall x q(x) \leftrightarrow \forall x (p \vee q(x))$
13) $p \wedge \exists x q(x) \leftrightarrow \exists x (p \wedge q(x))$
14) $p \vee \exists x q(x) \leftrightarrow \exists x (p \vee q(x))$
15) $\forall x (p(x) \rightarrow q(x)) \rightarrow (\forall x p(x) \rightarrow \forall x q(x))$
16) $\forall x (p(x) \rightarrow q(x)) \rightarrow (\exists x p(x) \rightarrow \exists x q(x))$
17) $\exists x (p(x) \wedge q(x)) \rightarrow \exists x p(x) \wedge \exists x q(x)$
18) $\forall x p(x) \vee \forall x q(x) \rightarrow \forall x (p(x) \vee q(x))$
19) $\neg \exists x \neg p(x) \leftrightarrow \forall x p(x)$
20) $\neg \forall x \neg p(x) \leftrightarrow \exists p(x)$
21) $\neg \exists x p(x) \leftrightarrow \forall x \neg p(x)$
22) $\exists x \neg p(x) \leftrightarrow \neg \forall x p(x)$

Proof: It is straightforward from the definition of the semantics and axiomatic schema of first order interval neutrosophic predicate logic. ■

V. An Application of Interval Neutrosophic Logic

In this section we provide one practical application of the interval neutrosophic logic – Interval Neutrosophic Logic System (INLS). INLS can handle rule uncertainty as same as type-2 FLS [4], besides, it can handle rule inconsistency without the danger of trivialization. Like the classical FLS, INLS is also characterized by IF-THEN rules. INLS consists of neutrosophication, neutrosophic inference, a neutrosophic rule base, neutrosophic type reduction and deneutrosophication. Given an input vector $x = (x_1, \ldots, x_n)$, where $x_1, \ldots, x_n$ can be crisp inputs or neutrosophic sets, the INLS will generate a crisp output y. The general scheme of INLS is shown in Fig. 1.

Suppose the neutrosophic rule base consists of M rules in which each rule has n antecedents and one consequent. Let the kth rule be denoted by R^k such that IF x_1 is A_1^k, x_2 is A_2^k, ..., and x_n is A_n^k, THEN y is B^k. A_i^k is an interval neutrosophic set defined on universe X_i with truth-membership function $T_{A_i^k}(x_i)$, indeterminacy-membership function $I_{A_i^k}(x_i)$ and falsity-membership function $F_{A_i^k}(x_i)$, where $T_{A_i^k}(x_i), I_{A_i^k}(x_i), F_{A_i^k}(x_i) \subseteq [0,1], 1 \leq i \leq n$. B^k is an interval neutrosophic set defined on universe Y with truth-membership function $T_{B^k}(y)$, indeterminacy-membership functiou $I_{B^k}(y)$ and falsity-membership function $F_{B^k}(y)$, where $T_{B^k}(y), I_{B^k}(y), F_{B^k}(y) \subseteq [0,1]$. Given fact x_1 is $\tilde{A}_1^k, x_2$ is $\tilde{A}_2^k, \ldots,$ and x_n is $\tilde{A}_n^k$, then consequence y is $\tilde{B}^k$. $\tilde{A}_i^k$ is an interval neutrosophic set defined on universe X_i with truth-membership function $T_{\tilde{A}_i^k}(x_i)$, indeterminacy-membership function $I_{\tilde{A}_i^k}(x_i)$ and falsity-membership function $F_{\tilde{A}_i^k}(x_i)$, where $T_{\tilde{A}_i^k}(x_i), I_{\tilde{A}_i^k}(x_i), F_{\tilde{A}_i^k}(x_i) \subseteq [0,1], 1 \leq i \leq n$. $\tilde{B}^k$ is an interval neutrosophic set defined on universe Y with truth-membership function $T_{\tilde{B}^k}(y)$, indeterminacy-membership function $I_{\tilde{B}^k}(y)$ and falsity-membership function $F_{\tilde{B}^k}(y)$, where $T_{\tilde{B}^k}(y), I_{\tilde{B}^k}(y), F_{\tilde{B}^k}(y) \subseteq [0,1]$. In this paper, we consider $a_i \leq x_i \leq b_i$ and $\alpha \leq y \leq \beta$.

An unconditional neutrosophic proposition is expressed by the phrase: "Z is C", where Z is a variable that receives values z from a universal set U, and C is an interval neutrosophic set defined on U that represents a neutrosophic predicate. Each neutrosophic proposition p is associated with $\langle t(p), i(p), f(p)\rangle$ with $t(p), i(p), f(p) \subseteq [0,1]$. In general, for any value z of Z, $\langle t(p), i(p), f(p)\rangle = \langle T_C(z), I_C(z), F_C(z)\rangle$.

For implication $p \rightarrow q$, we define the semantics as:

$$\sup t_{p\rightarrow q} = \min(\sup t(p), \sup t(q)), \tag{38}$$

$$\inf t_{p\rightarrow q} = \min(\inf t(p), \inf t(q)), \tag{39}$$

$$\sup i_{p\rightarrow q} = \min(\sup i(p), \sup i(q)), \tag{40}$$

$$\inf i_{p\rightarrow q} = \min(\inf i(p), \inf i(q)), \tag{41}$$

$$\sup f_{p\rightarrow q} = \max(\sup f(p), \sup f(q)), \tag{42}$$

$$\inf f_{p\rightarrow q} = \max(\inf f(p), \inf f(q)), \tag{43}$$

where $t(p), i(p), f(p), t(q), i(q), f(q) \subseteq [0,1]$.

Let $X = X_1 \times \cdots \times X_n$. The truth-membership function, indeterminacy-membership function and falsity-membership function $T_{\tilde{B}^k}(y), I_{\tilde{B}^k}(y), F_{\tilde{B}^k}(y)$ of a fired kth rule can be represented using the definition of interval neutrosophic composition functions (23–25) and the semantics of conjunction and disjunction defined in Table II and equations (38–43) as:

$$\begin{aligned}
\sup T_{\tilde{B}^k}(y) &= \sup_{x \in X} \min(\sup T_{\tilde{A}_1^k}(x_1), \sup T_{A_1^k}(x_1), \ldots, \sup T_{\tilde{A}_n^k}(x_n), \sup T_{A_n^k}(x_n), \sup T_{B^k}(y)), && (44)\\
\inf T_{\tilde{B}^k}(y) &= \sup_{x \in X} \min(\inf T_{\tilde{A}_1^k}(x_1), \inf T_{A_1^k}(x_1), \ldots, \inf T_{\tilde{A}_n^k}(x_n), \inf T_{A_n^k}(x_n), \inf T_{B^k}(y)), && (45)\\
\sup I_{\tilde{B}^k}(y) &= \sup_{x \in X} \min(\sup I_{\tilde{A}_1^k}(x_1), \sup I_{A_1^k}(x_1), \ldots, \sup I_{\tilde{A}_n^k}(x_n), \sup I_{A_n^k}(x_n), \sup I_{B^k}(y)), && (46)\\
\inf I_{\tilde{B}^k}(y) &= \sup_{x \in X} \min(\inf I_{\tilde{A}_1^k}(x_1), \inf I_{A_1^k}(x_1), \ldots, \inf I_{\tilde{A}_n^k}(x_n), \inf I_{A_n^k}(x_n), \inf I_{B^k}(y)), && (47)\\
\sup F_{\tilde{B}^k}(y) &= \inf_{x \in X} \max(\sup F_{\tilde{A}_1^k}(x_1), \sup F_{A_1^k}(x_1), \ldots, \sup F_{\tilde{A}_n^k}(x_n), \sup F_{A_n^k}(x_n), \sup F_{B^k}(y)), && (48)\\
\inf F_{\tilde{B}^k}(y) &= \inf_{x \in X} \max(\inf F_{\tilde{A}_1^k}(x_1), \inf F_{A_1^k}(x_1), \ldots, \inf F_{\tilde{A}_n^k}(x_n), \inf F_{A_n^k}(x_n), \inf F_{B^k}(y)), && (49)
\end{aligned}$$

where $y \in Y$.

Now, we give the algorithmic description of INLS.

BEGIN

Step 1: Neutrosophication

The purpose of neutrosophication is to map inputs into interval neutrosophic input sets. Let ${G_i}^k$ be an interval neutrosophic input set to represent the result of neutrosophication of ith input variable of kth rule, then

$$\begin{aligned}
\sup T_{{G_i}^k}(x_i) &= \sup_{x_i \in X_i} \min(\sup T_{\tilde{A}_i^k}(x_i), \sup T_{A_i^k}(x_i)), && (50)\\
\inf T_{G_i^k}(x_i) &= \sup_{x_i \in X_i} \min(\inf T_{\tilde{A}_i^k}(x_i), \inf T_{A_i^k}(x_i)), && (51)\\
\sup I_{G_i^k}(x_i) &= \sup_{x_i \in X_i} \min(\sup I_{\tilde{A}_i^k}(x_i), \sup I_{A_i^k}(x_i)), && (52)\\
\inf I_{G_i^k}(x_i) &= \sup_{x_i \in X_i} \min(\inf I_{\tilde{A}_i^k}(x_i), \inf I_{A_i^k}(x_i)), && (53)\\
\sup F_{G_i^k}(x_i) &= \inf_{x_i \in X_i} \max(\sup F_{\tilde{A}_i^k}(x_i), \sup F_{A_i^k}(x_i)), && (54)\\
\inf F_{G_i^k}(x_i) &= \inf_{x_i \in X_i} \max(\inf F_{\tilde{A}_i^k}(x_i), \inf F_{A_i^k}(x_i)), && (55)
\end{aligned}$$

where $x_i \in X_i$.

If x_i are crisp inputs, then equations (50–55) are simplified to

$$\begin{aligned}
\sup T_{G_i^k}(x_i) &= \sup T_{A_i^k}(x_i), && (56)\\
\inf T_{G_i^k}(x_i) &= \inf T_{A_i^k}(x_i), && (57)\\
\sup I_{G_i^k}(x_i) &= \sup I_{A_i^k}(x_i), && (58)\\
\inf I_{G_i^k}(x_i) &= \inf I_{A_i^k}(x_i), && (59)\\
\sup F_{G_i^k}(x_i) &= \sup F_{A_i^k}(x_i), && (60)\\
\inf F_{G_i^k}(x_i) &= \inf F_{A_i^k}(x_i), && (61)
\end{aligned}$$

where $x_i \in X_i$.

Fig. 2 shows the conceptual diagram for neutrosophication of a crisp input x_1.

Step 2: Neutrosophic Inference

The core of INLS is the neutrosophic inference, the principle of which has already been explained above. Suppose the kth rule is fired. Let G^k be an interval neutrosophic set to represent the result of the input and antecedent operation for kth rule, then

$$\sup T_{G^k}(x) = \sup_{x\in X} \min(\sup T_{\tilde{A}_1^k}(x_1), \sup T_{A_1^k}(x_1), \ldots, \sup T_{\tilde{A}_n^k}(x_n), \sup T_{A_n^k}(x_n)), \tag{62}$$

$$\inf T_{G^k}(x) = \sup_{x\in X} \min(\inf T_{\tilde{A}_1^k}(x_1), \inf T_{A_1^k}(x_1), \ldots, \inf T_{\tilde{A}_n^k}(x_n), \inf T_{A_n^k}(x_n)), \tag{63}$$

$$\sup I_{G^k}(x) = \sup_{x\in X} \min(\sup I_{\tilde{A}_1^k}(x_1), \sup I_{A_1^k}(x_1), \ldots, \sup I_{\tilde{A}_n^k}(x_n), \sup I_{A_n^k}(x_n)), \tag{64}$$

$$\inf I_{G^k}(x) = \sup_{x\in X} \min(\inf I_{\tilde{A}_1^k}(x_1), \inf I_{A_1^k}(x_1), \ldots, \inf I_{\tilde{A}_n^k}(x_n), \inf I_{A_n^k}(x_n)), \tag{65}$$

$$\sup F_{G^k}(x) = \inf_{x\in X} \max(\sup F_{\tilde{A}_1^k}(x_1), \sup F_{A_1^k}(x_1), \ldots, \sup F_{\tilde{A}_n^k}(x_n), \sup F_{A_n^k}(x_n)), \tag{66}$$

$$\inf F_{G^k}(x) = \inf_{x\in X} \max(\inf F_{\tilde{A}_1^k}(x_1), \inf F_{A_1^k}(x_1), \ldots, \inf F_{\tilde{A}_n^k}(x_n), \inf F_{A_n^k}(x_n)), \tag{67}$$

where $x_i \in X_i$.

Here we restate the result of neutrosophic inference:

$$\sup T_{\tilde{B}^k}(y) = \min(\sup T_{G_1^k}(x), \ldots, \sup T_{B^k}(y)), \tag{68}$$

$$\inf T_{\tilde{B}^k}(y) = \min(\inf T_{G^k}(x), \inf T_{B^k}(y)), \tag{69}$$

$$\sup I_{\tilde{B}^k}(y) = \min(\sup I_{G^k}(x), \sup I_{B^k}(y)), \tag{70}$$

$$\inf I_{\tilde{B}^k}(y) = \min(\inf I_{G^k}(x), \inf I_{B^k}(y)), \tag{71}$$

$$\sup F_{\tilde{B}^k}(y) = \max(\sup F_{G^k}(x), \sup F_{B^k}(y)), \tag{72}$$

$$\inf F_{\tilde{B}^k}(y) = \max(\inf F_{G^k}(x), \inf F_{B^k}(y)), \tag{73}$$

where $x \in X, y \in Y$.

Suppose that N rules in the neutrosophic rule base are fired, where $N \leq M$, then, the output interval neutrosophic set $\tilde{B}$ is:

$$\sup T_{\tilde{B}}(y) = \max_{k=1}^{N} \sup T_{\tilde{B}^k}(y), \tag{74}$$

$$\inf T_{\tilde{B}}(y) = \max_{k=1}^{N} \inf T_{\tilde{B}^k}(y), \tag{75}$$

$$\sup I_{\tilde{B}}(y) = \max_{k=1}^{N} \sup I_{\tilde{B}^k}(y), \tag{76}$$

$$\inf I_{\tilde{B}}(y) = \max_{k=1}^{N} \inf I_{\tilde{B}^k}(y), \tag{77}$$

$$\sup F_{\tilde{B}}(y) = \min_{k=1}^{N} \sup T_{\tilde{B}^k}(y), \tag{78}$$

$$\inf T_{\tilde{B}}(y) = \min_{k=1}^{N} \inf T_{\tilde{B}^k}(y), \tag{79}$$

where $y \in Y$.

Step 3: Neutrosophic type reduction

After neutrosophic inference, we will get an interval neutrosophic set $\tilde{B}$ with $T_{\tilde{B}}(y), I_{\tilde{B}}(y), F_{\tilde{B}}(y) \subseteq [0,1]$. Then, we do the neutrosophic type reduction to transform each interval into one number. There are many ways to do it, here, we give one method:

$$T'_{\tilde{B}}(y) = (\inf T_{\tilde{B}}(y) + \sup T_{\tilde{B}}(y))/2, \tag{80}$$

$$I'_{\tilde{B}}(y) = (\inf I_{\tilde{B}}(y) + \sup I_{\tilde{B}}(y))/2, \tag{81}$$

$$F'_{\tilde{B}}(y) = (\inf F_{\tilde{B}}(y) + \sup F_{\tilde{B}}(y))/2, \tag{82}$$

where $y \in Y$.

So, after neutrosophic type reduction, we will get an ordinary neutrosophic set (a type-1 neutrosophic set) $\tilde{B}$. Then we need to do the deneutrosophication to get a crisp output.

Step 4: Deneutrosophication

The purpose of deneutrosophication is to convert an ordinary neutrosophic set (a type-1 neutrosophic set) obtained by neutrosophic type reduction to a single real number which represents the real output. Similar to defuzzification [18], there are many deneutrosophication methods according to different applications. Here we give one method. The deneutrosophication process consists of two steps.

Step 4.1: *Synthesization*: It is the process to transform an ordinary neutrosophic set (a type-1 neutrosophic set) $\tilde{B}$ into a fuzzy set $\bar{B}$. It can be expressed using the following function:

$$f(T'_{\tilde{B}}(y), I'_{\tilde{B}}(y), F'_{\tilde{B}}(y)) : [0,1] \times [0,1] \times [0,1] \to [0,1] \tag{83}$$

Here we give one definition of f:

$$T_{\bar{B}}(y) = a * T'_{\tilde{B}}(y) + b * (1 - F'_{\tilde{B}}(y)) + c * I'_{\tilde{B}}(y)/2 + d * (1 - I'_{\tilde{B}}(y)/2), \tag{84}$$

where $0 \leq a, b, c, d \leq 1, a + b + c + d = 1$.

The purpose of synthesization is to calculate the overall truth degree according to three components: truth-membership function, indeterminacy-membership function and falsity-membership function. The component–truth-membership function gives the direct information about the truth-degree, so we use it directly in the formula; The component–falsity-membership function gives the indirect information about the truth-degree, so we use $(1 - F)$ in the formula. To understand the meaning of indeterminacy-membership function I, we give an example: a statement is "The quality of service is good", now firstly a person has to select a decision among $\{T, I, F\}$, secondly he or she has to answer the degree of the decision in $[0,1]$. If he or she chooses $I = 1$, it means 100% "not sure" about the statement, i.e., 50% true and 50% false for the statement (100% balanced), in this sense, $I = 1$ contains the potential truth value 0.5. If he or she chooses $I = 0$, it means 100% "sure" about the statement, i.e., either 100% true or 100% false for the statement (0% balanced), in this sense, $I = 0$ is related to two extreme cases, but we do not know which one is in his or her mind. So we have to consider both at the same time: $I = 0$ contains the potential truth value that is either 0 or 1. If I decreases from 1 to 0, then the potential truth value changes from one value 0.5 to two different possible values gradually to the final possible ones 0 and 1 (i.e., from 100% balanced to 0% balanced), since he or she does not choose either T or F but I, we do not know his or her final truth value. Therefore, the formula has to consider two potential truth values implicitly represented by I with different weights (c and d) because of lack of his or her final decision information after he or she has chosen I. Generally, $a > b > c, d$; c and d could be decided subjectively or objectively as long as enough information is available. The parameters a, b, c and d can be tuned using learning algorithms such as neural networks and genetic algorithms in the development of application to improve the performance of the INLS.

Step 4.2: *Calculation of a typical neutrosophic value*: Here we introduce one method of calculation of center of area. The method is sometimes called the *center of gravity method* or *centroid method*, the deneutrosophicated value, $dn(T_{\bar{B}}(y))$ is calculated by the formula

$$dn(T_{\bar{B}}(y)) = \frac{\int_{\alpha}^{\beta} T_{\bar{B}}(y) y dy}{\int_{\alpha}^{\beta} T_{\bar{B}} dy}. \tag{85}$$

END.

VI. Conclusions

In this paper, we give the formal definitions of interval neutrosophic logic which are extension of many other classical logics such as fuzzy logic, intuitionistic fuzzy logic and paraconsistent logics, etc. Interval neutrosophic logic include interval neutrosophic propositional logic and first order interval neutrosophic predicate logic. We call them classical (standard) neutrosophic logic. In the future, we also will discuss and explore the non-classical (non-standard) neutrosophic logic such as modal interval neutrosophic logic, temporal interval neutrosophic logic, etc. Interval neutrosophic logic can not only handle imprecise, fuzzy and incomplete propositions but also inconsistent propositions without the danger of trivialization. The paper also give one application based on the semantic notion of interval neutrosophic logic – the Interval Neutrosophic Logic Systems (INLS) which is the generalization of classical FLS and interval valued fuzzy FLS. Interval neutrosophic logic will have a lot of potential applications in computational Web intelligence [19]. For example, current fuzzy Web intelligence techniques can be improved by using more reliable interval neutrosophic logic methods because T, I and F are all used in decision making. In large, such robust interval neutrosophic logic methods can also be used in other applications such as medical informatics, bioinformatics and human-oriented decision-making under uncertainty. In fact, interval neutrosophic sets and interval neutrosophic logic could be applied in the fields that fuzzy sets and fuzz logic are suitable for, also the fields that paraconsistent logics are suitable for.

Acknowledgment

The authors would like to thank Dr. Albena Tchamova for her valuable suggestions.

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Published in *Advances in Fuzzy Sets and Systems*, Vol. 1, No. 3, 187-218, 2006.

Intuitionistic Neutrosophic Soft Set

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Abstract. In this paper we study the concept of intuitionistic neutrosophic set of Bhowmik and Pal. We have introduced this concept in soft sets and defined intuitionistic neutrosophic soft set. Some definitions and operations have been introduced on intuitionistic neutrosophic soft set. Some properties of this concept have been established.

Keywords: Soft sets, Neutrosophic set,Intuitionistic neutrosophic set, Intuitionistic neutrosophic soft set.

1. Introduction

In wide varities of real problems like , engineering problems, social, economic, computer science, medical science…etc. The data associated are often uncertain or imprecise, all real data are not necessarily crisp, precise, and deterministic because of their fuzzy nature. Most of these problem were solved by different theories, firstly by fuzzy set theory provided by Lotfi , Zadeh [1] ,Later several researches present a number of results using different direction of fuzzy set such as : interval fuzzy set [13], intuitionistic fuzzy set by Atanassov[2], all these are successful to some extent in dealing with the problems arising due to the vagueness present in the real world ,but there are also cases where these theories failed to give satisfactory results, possibly due to indeterminate and inconsistent information which exist in belif system, then in 1995, Smarandache [3] intiated the theory of neutrosophic as new mathematical tool for handling problems involving imprecise, indeterminacy,and inconsistent data. Later on authors like Bhowmik and Pal [7] have further studied the intuitionistic neutrosophic set and presented various properties of it. In 1999 Molodtsov [4] introduced the concept of soft set which was completely a new approche for dealing with vagueness and uncertainties ,this concept can be seen free from the inadequacy of parameterization tool. After Molodtsovs'work, there have been many researches in combining fuzzy set with soft set, which incorporates the beneficial properties of both fuzzy set and soft set techniques (see [12] [6] [8]). Recently , by the concept of neutrosophic set and soft set, first time, Maji [11] introduced neutrosophic soft set, established its application in decision making, and thus opened a new direction, new path of thinking to engineers, mathematicians, computer scientists and many others in various tests. This paper is an attempt to combine the concepts: intuitionistic neutrosophic set and soft set together by introducing a new concept called intuitionistic neutrosophic sof set, thus we introduce its operations namely equal ,subset, union ,and intersection, We also present an application of intuitionistic neutrosophic soft set in decision making problem.

The organization of this paper is as follow : in section 2, we briefly present some basic definitions and preliminary results are given which will be used in the rest of the paper. In section 3, Intuitionistic neutrosophic soft set. In section 4 an application of intuitionistic neutrosophic soft set in a decision making problem. Conclusions are there in the concluding section 5.

2. Preliminaries

Throughout this paper, let U be a universal set and E be the set of all possible parameters under consideration with respect to U, usually, parameters are attributes , characteristics, or properties of objects in U. We now recall some basic notions of neutrosophic set , intuitionistic neutrosophic set and soft set .

Definition 2.1 (see[3]). Let U be an universe of discourse then the neutrosophic set A is an object having the form A = {< x: $T_{A(x)}$,$I_{A(x)}$,$F_{A(x)}$ >,x ∈ U}, where the functions T,I,F : U→]$^-$0,1$^+$[define respectively the degree of membership , the degree of indeterminacy, and the degree of non-membership of the element x ∈ X to the set A with the condition.

$$^-0 \le T_{A(x)} + I_{A(x)} + F_{A(x)} \le 3^+.$$

From philosophical point of view, the neutrosophic set takes the value from real standard or non-standard subsets of]$^-$0,1$^+$[.so instead of]$^-$0,1$^+$[we need to take the interval [0,1] for technical applications, because]$^-$0,1$^+$[will be difficult to apply in the real applications such as in scientific and engineering problems.

Definition 2.2 (see [3]). A neutrosophic set A is contained in another neutrosophic set B i.e. A ⊆ B if ∀x ∈ U, $T_A(x) \le T_B(x)$, $I_A(x) \le I_B(x)$, $F_A(x) \ge F_B(x)$.

A complete account of the operations and application of neutrsophic set can be seen in [3] [10].

Definition 2.3(see[7]). intuitionistic neutrosophic set

An element x of U is called significant with respect to neutrsophic set A of U if the degree of truth-membership or falsity-membership or indeterminancy-membership value, i.e.,$T_{A(x)}$ or $F_{A(x)}$or $I_{A(x)} \le 0.5$. Otherwise, we call it insignificant. Also, for neutrosophic set the truth-membership, indeterminacy-membership and falsity-membership all can not be significant. We define an intuitionistic neutrosophic set by A = {< x: $T_{A(x)}$,$I_{A(x)}$,$F_{A(x)}$ >,x ∈ U},where

min { $T_{A(x)}$, $F_{A(x)}$ } ≤ 0.5,

min { $T_{A(x)}$, $I_{A(x)}$ } ≤ 0.5,

min { $F_{A(x)}$, $I_{A(x)}$ } ≤ 0.5, for all x ∈ U,

with the condition $0 \le T_{A(x)} + I_{A(x)} + F_{A(x)} \le 2$.

As an illustration ,let us consider the following example.

Example2.4. Assume that the universe of discourse U={x_1,x_2,x_3},where x_1 characterizes the capability, x_2 characterizes the trustworthiness and x_3 indicates the prices of the objects. It may be further assumed that the values of x_1, x_2 and x_3 are in [0,1] and they are obtained from some questionnaires of some experts. The experts may impose their opinion in three components viz. the degree of goodness, the degree of indeterminacy and that of poorness to explain the characteristics of the objects. Suppose A is an intuitionistic neutrosophic set (IN S) of U, such that,

A = {< x_1,0.3,0.5,0.4 >,< x_2,0.4,0.2,0.6 >,< x_3,0.7,0.3,0.5 >}, where the degree of goodness of capability is 0.3, degree of indeterminacy of capability is 0.5 and degree of falsity of capability is 0.4 etc.

Definition 2.5 (see[4]). Let U be an initial universe set and E be a set of parameters. Let P(U) denotes the power set of U. Consider a nonempty set A, A ⊂ E. A pair (F, A) is called a soft set over U, where F is a mapping given by F : A → P(U).

As an illustration ,let us consider the following example.

Example 2.6. Suppose that U is the set of houses under consideration, say U = {h_1, h_2, . . ., h_5}. Let E be the set of some attributes of such houses, say E = {e_1, e_2, . . ., e_8}, where e_1, e_2, . . ., e_8 stand for the attributes "expensive", "beautiful", "wooden", "cheap", "modern", and "in bad repair", respectively.

In this case, to define a soft set means to point out expensive houses, beautiful houses, and so on. For example, the soft set (F, A) that describes the "attractiveness of the houses" in the opinion of a buyer, say Thomas, may be defined like this:

A={e_1,e_2,e_3,e_4,e_5};

F(e_1) = {h_2, h_3, h_5}, F(e_2) = {h_2, h_4}, F(e_3) = {h_1}, F(e_4) = U, F(e_5) = {h_3, h_5}.

For more details on the algebra and operations on intuitionistic neutrosophic set and soft set, the reader may refer to [5,6,8,9,12].

3. Intuitionistic Neutrosophic Soft Set

In this section ,we will initiate the study on hybrid structure involving both intuitionstic neutrosophic set and soft set theory.

Definition 3.1. Let U be an initial universe set and $A \subset E$ be a set of parameters. Let N(U) denotes the set of all intuitionistic neutrosophic sets of U. The collection (F,A) is termed to be the soft intuitionistic neutrosophic set over U, where F is a mapping given by $F : A \rightarrow N(U)$.
Remark 3.2. we will denote the intuitionistic neutrosophic soft set defined over an universe by INSS.
Let us consider the following example.
Example 3.3. Let U be the set of blouses under consideration and E is the set of parameters (or qualities). Each parameter is a intuitionistic neutrosophic word or sentence involving intuitionistic neutrosophic words. Consider E = { Bright, Cheap, Costly, very costly, Colorful, Cotton, Polystyrene, long sleeve , expensive }. In this case, to define a intuitionistic neutrosophic soft set means to point out Bright blouses, Cheap blouses, Blouses in Cotton and so on. Suppose that, there are five blouses in the universe U given by, $U = \{b_1,b_2,b_3,b_4,b_5\}$ and the set of parameters $A = \{e_1,e_2,e_3,e_4\}$, where each e_i is a specific criterion for blouses:

e_1 stands for 'Bright',
e_2 stands for 'Cheap',
e_3 stands for 'costly',
e_4 stands for 'Colorful',

Suppose that,

F(Bright)={<b_1,0.5,0.6,0.3>,<b_2,0.4,0.7,0.2>,<b_3,0.6,0.2,0.3>,<b_4,0.7,0.3,0.2>
,<b_5,0.8,0.2,0.3>}.
F(Cheap)={<b_1,0.6,0.3,0.5>,<b_2,0.7,0.4,0.3>,<b_3,0.8,0.1,0.2>,<b_4,0.7,0.1,0.3>
,<b_5,0.8,0.3,0.4}.
F(Costly)={<b_1,0.7,0.4,0.3>,<b_2,0.6,0.1,0.2>,<b_3,0.7,0.2,0.5>,< b_4,0.5,0.2,0.6 >
,< b_5,0.7,0.3,0.2 >}.
F(Colorful)={<b_1,0.8,0.1,0.4>,<b_2,0.4,0.2,0.6>,<b_3,0.3,0.6,0.4>,<b_4,0.4,0.8,0.5>
,< b_5,0.3,0.5,0.7 >}.

The intuitionistic neutrosophic soft set (INSS) (F, E) is a parameterized family $\{F(e_i),i = 1,\cdots,10\}$ of all intuitionistic neutrosophic sets of U and describes a collection of approximation of an object. The mapping F here is 'blouses (.)', where dot(.) is to be filled up by a parameter $e_i \in E$. Therefore, $F(e_1)$ means 'blouses (Bright)' whose functional-value is the intuitionistic neutrosophic set
{< b_1,0.5,0.6,0.3 >,< b_2,0.4,0.7,0.2 >, < b_3,0.6,0.2,0.3 >,< b_4,0.7,0.3,0.2 >,< b_5,0.8,0.2,0.3 >}.
Thus we can view the intuitionistic neutrosophic soft set (INSS) (F, A) as a collection of approximation as below:
(F, A) = { Bright blouses= {< b_1,0.5,0.6,0.3 >,< b_2,0.4,0.7,0.2 >, < b_3,0.6,0.2,0.3 >,< b_4,0.7,0.3,0.2 >,< b_5,0.8,0.2,0.3 >}, Cheap blouses= {< b_1,0.6,0.3,0.5 >,< b_2,0.7,0.4,0.3 >,< b_3,0.8,0.1,0.2 >, < b_4,0.7,0.1,0.3 >,< b_5,0.8,0.3,0.4 >}, costly blouses= {< b_1,0.7,0.4,0.3 > ,< b_2,0.6,0.1,0.2 >,< b_3,0.7,0.2,0.5 >,< b_4,0.5,0.2,0.6 >,< b_5,0.7,0.3,0.2 >}, Colorful blouses= {< b_1,0.8,0.1,0.4 >,< b_2,0.4,0.2,0.6 >,< b_3,0.3,0.6,0.4 >, < b_4,0.4,0.8,0.5>,< b_5,0.3,0.5,0.7 >}}.
where each approximation has two parts: (i) a predicate p, and (ii) an approximate value-set v (or simply to be called value-set v).
For example, for the approximation 'Bright blouses= {< b_1,0.5,0.6,0.3 >, < b_2,0.4,0.7,0.2 >,<b_3,0.6,0.2,0.3>,<b_4,0.7,0.3,0.2>,<b_5,0.8,0.2,0.3>}'. we have (i) the predicate name 'Bright blouses', and (ii) the approximate value-set is{<b_1,0.5,0.6,0.3>,<b_2,0.4,0.7,0.2>,<b_3,0.6,0.2,0.3>,<b_4,0.7,0.3,0.2> ,< b_5,0.8,0.2,0.3 >}. Thus, an intuitionistic neutrosophic soft set (F, E) can be viewed as a collection of approximation like (F, E) = $\{p_1 = v_1,p_2 = v_2,\cdots ,p_{10} = v_{10}\}$. In order to store an intuitionistic neutrosophic soft set in a computer, we could represent it in the form of a table as shown below (corresponding to the intuitionistic neutrosophic soft set in the above example). In this table, the entries are c_{ij} corresponding to the blouse b_i and the parameter e_j, where c_{ij} = (true-membership value of b_i, indeterminacy-membership value of b_i, falsity membership value of b_i) in $F(e_j)$. The table 1 represent the intuitionistic neutrosophic soft set (F, A) described above.

U	bright	cheap	costly	colorful
b_1	(0.5,0.6, 0.3)	(0.6,0.3, 0.5)	(0.7,0.4, 0.3)	(0.8,0.1, 0.4)
b_2	(0.4,0.7, 0.2)	(0.7,0.4, 0.3)	(0.6,0.1, 0.2)	(0.4,0.2, 0.6)
b_3	(0.6,0.2, 0.3)	(0.8,0.1, 0.2)	(0.7,0.2, 0.5)	(0.3,0.6, 0.4)
b_4	(0.7,0.3, 0.2)	(0.7,0.1, 0.3)	(0.5,0.2, 0.6)	(0.4,0.8, 0.5)
b_5	(0.8,0.2, 0.3)	(0.8,0.3, 0.4)	(0.7,0.3, 0.2)	(0.3,0.5, 0.7)

Table 1: Tabular form of the INSS (F, A).

Remark 3.4. An intuitionistic neutrosophic soft set is not an intuituionistic neutrosophic set but a parametrized family of an intuitionistic neutrosophic subsets.

Definition 3.5. Containment of two intuitionistic neutrosophic soft sets.
For two intuitionistic neutrosophic soft sets (F, A) and (G, B) over the common universe U. We say that (F, A) is an intuitionistic neutrosophic soft subset of (G, B) if and only if

(i) $A \subset B$.

(ii) F(e) is an intuitionistic neutrosophic subset of G(e).

Or $T_{F(e)}(x) \leq T_{G(e)}(x)$, $I_{F(e)}(x) \leq I_{G(e)}(x)$, $F_{F(e)}(x) \geq F_{G(e)}(x)$, $\forall e \in A$, $x \in U$.

We denote this relationship by (F, A) ⊆ (G, B).
(F, A) is said to be intuitionistic neutrosophic soft super set of (G, B) if (G, B) is an intuitionistic neutrosophic soft subset of (F, A). We denote it by (F, A) ⊇ (G, B).

Example 3.6. Let (F,A) and (G,B) be two INSSs over the same universe $U = \{o_1,o_2,o_3,o_4,o_5\}$. The INSS (F,A) describes the sizes of the objects whereas the INSS (G, B) describes its surface textures. Consider the tabular representation of the INSS (F, A) is as follows.

U	small	large	colorful
O_1	(0.4,0.3, 0.6)	(0.3,0.1, 0.7)	(0.4,0.1, 0.5)
O_2	(0.3,0.1, 0.4)	(0.4,0.2, 0.8)	(0.6,0.3, 0.4)
O_3	(0.6,0.2, 0.5)	(0.3,0.1, 0.6)	(0.4,0.3, 0.8)
O_4	(0.5,0.1, 0.6)	(0.1,0.5, 0.7)	(0.3,0.3, 0.8)
O_5	(0.3,0.2, 0.4)	(0.3,0.1, 0.6)	(0.5,0.2, 0.4)

Table 2: Tabular form of the INSS (F, A).

The tabular representation of the INSS (G, B) is given by table 3.

U	small	large	colorful	very smooth
O1	(0.6,0.4, 0.3)	(0.7,0.2, 0.5)	(0.5,0.7, 0.4)	(0.1,0.8, 0.4)
O2	(0.7,0.5, 0.2)	(0.4,0.7, 0.3)	(0.7,0.3, 0.2)	(0.5,0.7, 0.3)
O3	(0.6,0.3, 0.5)	(0.7,0.2, 0.4)	(0.6,0.4, 0.3)	(0.2,0.9, 0.4)
O4	(0.8,0.1, 0.4)	(0.3,0.6, 0.4)	(0.4,0.5, 0.7)	(0.4,0.4, 0.5)
O5	(0.5,0.4, 0.2)	(0.4,0.1, 0.5)	(0.6,0.4, 0.3)	(0.5,0.8, 0.3)

Table 3: Tabular form of the INSS (G, B).

Clearly, by definition 3.5 we have (F, A) ⊂ (G, B).

Definition 3.7. **Equality of two intuitionistic neutrosophic soft sets.**
Two INSSs (F, A) and (G, B) over the common universe U are said to be intuitionistic neutrosophic soft equal if (F, A) is an intuitionistic neutrosophic soft subset of (G, B) and (G, B) is an intuitionistic neutrosophic soft subset of (F, A) which can be denoted by (F, A)= (G, B).

Definition 3.8. **NOT set of a set of parameters**.

Let $E = \{e_1, e_2, \cdots, e_n\}$ be a set of parameters. The NOT set of E is denoted by]E is defined by $]E = \{ \rceil e_1, \rceil e_2, \cdots, \rceil e_n\}$, where $\rceil e_i$ = not $e_i, \forall i$ (it may be noted that] and $\rceil$ are different operators).

Example 3.9. Consider the example 3.3. Here]E = { not bright, not cheap, not costly, not colorful }.

Definition 3.10. **Complement of an intuitionistic neutrosophic soft set.**

The complement of an intuitionistic neutrosophic soft set (F, A) is denoted by $(F,A)^c$ and is defined by $(F,A)^c = (F^c,]A)$, where $F^c :]A \rightarrow N(U)$ is a mapping given by $F^c(\alpha)$ = intutionistic neutrosophic soft complement with $T_{F^c(x)} = F_{F(x)}, I_{F^c(x)} = I_{F(x)}$ and $F_{F^c(x)} = T_{F(x)}$.

Example 3.11. As an illustration consider the example presented in the example 3.2. the complement $(F,A)^c$ describes the 'not attractiveness of the blouses'. Is given below.

F(not bright) = {< b_1,0.3,0.6,0.5 >,< b_2,0.2,0.7,0.4 >,< b_3,0.3,0.2,0.6 >,
< b4,0.2,0.3,0.7 >< b_5,0.3,0.2,0.8 >}.

F(not cheap) = {< b_1,0.5,0.3,0.6 >,< b_2,0.3,0.4,0.7 >,< b_3,0.2,0.1,0.8 >,
< b_4,0.3,0.1,0.7 >,< b_5,0.4,0.3,0.8 >}.

F(not costly) = {< b_1,0.3,0.4,0.7 >,< b_2,0.2,0.1,0.6 >,< b_3,0.5,0.2,0.7 >,
< b_4,0.6,0.2,0.5 >,< b_5,0.2,0.3,0.7 >}.

F(not colorful) = {< b_1,0.4,0.1,0.8 >,< b_2,0.6,0.2,0.4 >,< b_3,0.4,0.6,0.3 >,
< b_4,0.5,0.8,0.4 >< b_5,0.7,0.5,0.3 >}.

Definition 3.12:**Empty or Null intuitionistic neutrosopphic soft set.**

An intuitionistic neutrosophic soft set (F,A) over U is said to be empty or null intuitionistic neutrosophic soft (with respect to the set of parameters) denoted by Φ_A or (Φ,A) if $T_{F(e)}(m) = 0, F_{F(e)}(m) = 0$ and $I_{F(e)}(m) = 0, \forall m \in U, \forall e \in A$.

Example 3.13. Let $U = \{b_1,b_2,b_3,b_4,b_5\}$, the set of five blouses be considered as the universal set and A = { Bright, Cheap, Colorful } be the set of parameters that characterizes the blouses. Consider the intuitionistic neutrosophic soft set (F, A) which describes the cost of the blouses and

F(bright)={< b_1,0,0,0 >,< b_2,0,0,0 >,< b_3,0,0,0 >,< b_4,0,0,0 >, < b_5,0,0,0 >},

F(cheap)={< b_1,0,0,0 >,< b_2,0,0,0 >,< b_3,0,0,0 >,< b_4,0,0,0 >, < b_5,0,0,0 >},

F(colorful)={< b_1,0,0,0 >,< b_2,0,0,0 >,< b_3,0,0,0 >, < b_4,0,0,0 >,< b_5,0,0,0 >}.

Here the NINSS (F, A) is the null intuitionistic neutrosophic soft set.

Definition 3.14. Union of two intuitionistic neutrosophic soft sets.

Let (F,A) and (G,B) be two INSSs over the same universe U.Then the union of (F,A) and (G,B) is denoted by '(F,A)∪(G,B)' and is defined by (F,A)∪(G,B)=(K,C), where C=A∪B and the truth-membership, indeterminacy-membership and falsity-membership of (K,C) are as follows:

$T_{K(e)}(m) = T_{F(e)}(m)$, if $e \in A - B$,
$= T_{G(e)}(m)$, if $e \in B - A$,
$= \max (T_{F(e)}(m), T_{G(e)}(m))$, if $e \in A \cap B$.

$I_{K(e)}(m) = I_{F(e)}(m)$, if $e \in A - B$,
$= I_{G(e)}(m)$, if $e \in B - A$,
$= \min (I_{F(e)}(m), I_{G(e)}(m))$, if $e \in A \cap B$.

$F_{K(e)}(m) = F_{F(e)}(m)$, if $e \in A - B$,
$= F_{G(e)}(m)$, if $e \in B - A$,
$= \min (F_{F(e)}(m), F_{G(e)}(m))$, if $e \in A \cap B$.

Example 3.15. Let (F, A) and (G, B) be two INSSs over the common universe U. Consider the tabular representation of the INSS (F, A) is as follow:

	Bright	Cheap	Colorful
b_1	(0.6,0.3, 0.5)	(0.7,0.3, 0.4)	(0.4,0.2, 0.6)
b_2	(0.5,0.1, 0.8)	(0.6,0.1, 0.3)	(0.6,0.4, 0.4)
b_3	(0.7,0.4, 0.3)	(0.8,0.3, 0.5)	(0.5,0.7, 0.2)
b_4	(0.8,0.4, 0.1)	(0.6,0.3, 0.2)	(0.8,0.2, 0.3

b_5	(0.6,0.3, 0.2)	(0.7,0.3, 0.5)	(0.3,0.6, 0.5

Table 4: Tabular form of the INSS (F, A).

The tabular representation of the INSS (G, B) is as follow:

U	Costly	Colorful
b_1	(0.6,0.2, 0.3)	(0.4,0.6, 0.2)
b_2	(0.2,0.7, 0.2)	(0.2,0.8, 0.3)
b_3	(0.3,0.6, 0.5)	(0.6,0.3, 0.4)
b_4	(0.8,0.4, 0.1)	(0.2,0.8, 0.3)
b_5	(0.7,0.1, 0.4)	(0.5,0.6, 0.4)

Table 5: Tabular form of the INSS (G, B).

Using definition 3.12 the union of two INSS (F, A) and (G, B) is (K, C) can be represented into the following Table.

U	Bright	Cheap	Colorful	Costly
b_1	(0.6,0.3, 0.5)	(0.7,0.3, 0.4)	(0.4,0.2, 0.2)	(0.6,0.2, 0.3)
b_2	(0.5,0.1, 0.8)	(0.6,0.1, 0.3)	(0.6,0.4, 0.3)	(0.2,0.7, 0.2)
b_3	(0.7,0.4, 0.3)	(0.8,0.3, 0.5)	(0.6,0.3, 0.2)	(0.3,0.6, 0.5)
b_4	(0.8,0.4, 0.1)	(0.6,0.3, 0.2)	(0.8,0.2, 0.3)	(0.8,0.4, 0.1)
b_5	(0.6,0.3, 0.2)	(0.7,0.3, 0.5)	(0.5,0.6, 0.4)	(0.7,0.1, 0.4)

Table 6: Tabular form of the INSS (K, C).

Definition 3.16. Intersection of two intuitionistic neutrosophic soft sets.
Let (F,A) and (G,B) be two INSSs over the same universe U such that A ∩ B≠0. Then the intersection of (F,A) and (G,B) is denoted by '(F,A) ∩ (G, B)' and is defined by (F, A) ∩(G, B) = (K,C),where C =A∩B and the truth-membership, indeterminacy membership and falsity-membership of (K, C) are related to those of (F,A) and (G,B) by:

$T_{K(e)}(m) = \min (T_{F(e)}(m),T_{G(e)}(m))$,
$I_{K(e)}(m) = \min (I_{F(e)}(m),I_{G(e)}(m))$,
$F_{K(e)}(m) = \max (F_{F(e)}(m),F_{G(e)}(m))$, for all $e \in C$.

Example 3.17. Consider the above example 3.15. The intersection of (F, A) and (G, B) can be represented into the following table :

U	Colorful
b_1	(0.4,0.2,0.6)
b_2	(0.2,0.4,0.4)
b_3	(0.6,0.3,0.4)
b_4	(0.8,0.2,0.3)
b_5	(0.3,0.6,0.5)

Table 7: Tabular form of the INSS (K, C).

Proposition 3.18. If (F, A) and (G, B) are two INSSs over U and on the basis of the operations defined above ,then:

(1) idempotency laws: (F,A) ∪ (F,A) = (F,A).
(F,A) ∩ (F,A) = (F,A).
(2) Commutative laws : (F,A) ∪ (G,B) = (G,B) ∪ (F,A).
(F,A) ∩ (G,B) = (G,B) ∩ (F,A).
(3) (F,A) ∪ Φ = (F,A).
(4) (F,A) ∩ Φ = Φ.
(5) $[(F,A)^c]^c$ = (F,A).

Proof. The proof of the propositions 1 to 5 are obvious.

Proposition 3.19 . If (F, A), (G, B) and (K, C) are three INSSs over U,then:
(1) (F,A) ∩ [(G,B) ∩ (K,C)] = [(F,A) ∩ (G,B)] ∩ (K,C).
(2) (F,A) ∪ [(G,B) ∪ (K,C)] = [(F,A) ∪ (G,B)] ∪ (K,C).
(3) Distributive laws: (F,A) ∪ [(G,B) ∩ (K,C)] = [(F,A) ∪ (G,B)] ∩ [(F,A) ∪ (K,C)].
(4) (F,A) ∩ [(G,B) ∪ (K,C)] = [(H,A) ∩ (G,B)] ∪ [(F,A) ∩ (K,C)].

Exemple 3.20. Let (F,A) ={⟨b_1 ,0.6,0.3,0. 1 ⟩ ,⟨ b_2,0.4,0.7,0. 5) ,(b_3,0.4,0.1,0.8)} , (G,B) ={(b_1,0.2,0.2,0.6), (b_2 0.7,0.2,0.4), (b_3,0.1,0.6,0.7) } and (K,C) ={(b_1, 0.3,0.8,0.2) ,(b_2, 0.4,0.1,0.6) ,⟨ b_3,0.9,0.1,0.2)} be three INSSs of U, Then:

(F,A) ∪ (G,B) = { ⟨b_1, 0.6,0.2,0.1 ⟩ , ⟨b_2, 0.7,0.2,0.4 ⟩ , ⟨ b_3,0.4,0.1,0.7 ⟩ }.
(F,A) ∪ (K,C) = { ⟨ b_1,0.6,0.3,0.1 ⟩ , ⟨b_2, 0.4,0.1,0.5 ⟩ , ⟨ b_3,0.9,0.1,0.2 ⟩ }.
(G,B) ∩ (K,C)] = { ⟨ b_1,0.2,0.2,0.6 ⟩ , ⟨ b_2,0.4,0.1,0.6 ⟩ , ⟨b_3, 0.1,0.1,0.7 ⟩ }.
(F,A) ∪ [(G,B) ∩ (K,C)] = { ⟨ b_1,0.6,0.2,0.1 ⟩ , ⟨ b_2,0.4,0.1,0.5 ⟩ , ⟨ b_3,0.4,0.1,0.7 ⟩ }.
[(F,A) ∪ (G,B)] ∩ [(F,A) ∪ (K,C)] = {⟨b_1,0.6,0.2,0.1⟩,⟨b_2,0.4,0.1,0.5⟩,⟨b_3,0.4,0.1,0.7⟩}.

Hence distributive (3) proposition verified.

Proof, can be easily proved from definition 3.14.and 3.16.

Definition 3.21. **AND operation on two intuitionistic neutrosophic soft sets**.

Let (F, A) and (G, B) be two INSSs over the same universe U. then (F, A) ''AND (G, B) denoted by '(F, A) ∧ (G, B)and is defined by (F, A) ∧ (G, B) = (K, A × B), where K(α, β)=F(α)∩ B(β) and the truth-membership, indeterminacy-membership and falsity-membership of (K, A×B) are as follows:

$$T_{K(\alpha,\beta)}(m) = \min(T_{F(\alpha)}(m),T_{G(\beta)}(m)),\ I_{K(\alpha,\beta)}(m) = \min(I_{F(\alpha)}(m),I_{G(\beta)}(m))$$
$$F_{K(\alpha,\beta)}(m) = \max(F_{F(\alpha)}(m),F_{G(\beta)}(m)),\ \forall\alpha\in A,\forall\beta\in B.$$

Example 3.22. Consider the same example 3.15 above. Then the tabular representation of (F,A) AND (G, B) is as follow:

u	(bright, costly)	(bright, Colorful)	(cheap, costly)
b_1	(0.6,0.2, 0.5)	(0.4,0.3, 0.5)	(0.6,0.2, 0.4)
b_2	(0.2,0.1, 0.8)	(0.2,0.1, 0.8)	(0.2,0.1, 0.3)
b_3	(0.3,0.4, 0.5)	(0.6,0.3, 0.4)	(0.3,0.3, 0.5)
b_4	(0.8,0.4, 0.1)	(0.2,0.4,0.3)	(0.6,0.3, 0.2)
b_5	(0.6,0.1, 0.4)	(0.5,0.3, 0.4)	(0.7,0.1, 0.5)
u	(cheap, colorful)	(colorful, costly)	(colorful, colorful)
b_1	(0.4,0.3, 0.4)	(0.4,0.2, 0.6)	(0.4,0.2, 0.6)
b_2	(0.2,0.1, 0.3)	(0.2,0.4, 0.4)	(0.2,0.4, 0.4)
b_3	(0.6,0.3, 0.5)	(0.3,0.6, 0.5)	(0.5,0.3, 0.4)
b_4	(0.2,0.3, 0.3)	(0.8,0.2, 0.3)	(0.2,0.2, 0.3)
b_5	(0.5,0.3, 0.5)	(0.3,0.1, 0.5)	(0.3,0.6, 0.5)

Table 8: Tabular representation of the INSS (K, A × B).

Definition 3.23. If (F,A) and (G,B) be two INSSs over the common universe U then '(F,A) OR(G,B)' denoted by (F,A) ∨ (G,B) is defined by (F, A) ∨ (G, B) = (O,A×B), where, the truth-membership, indeterminacy membership and falsity-membership of O(α, β) are given as follows:

$T_{O(\alpha,\beta)}(m) = \max(T_{F(\alpha)}(m), T_{G(\beta)}(m))$,
$I_{O(\alpha,\beta)}(m) = \min(I_{F(\alpha)}(m), I_{G(\beta)}(m))$,
$F_{O(\alpha,\beta)}(m) = \min(F_{F(\alpha)}(m), F_{G(\beta)}(m)), \forall \alpha \in A, \forall \beta \in B.$

Example 3.24 Consider the same example 3.14 above. Then the tabular representation of (F, A) OR (G, B) is as follow:

u	(bright, costly)	(bright, colorful)	(cheap, costly)
b_1	(0.6,0.2, 0.3)	(0.6,0.3, 0.2)	(0.7,0.2, 0.3)
b_2	(0.5,0.1, 0.2)	(0.5,0.1, 0.3)	(0.6,0.1, 0.2)
b_3	(0.7,0.4, 0.3)	(0.7,0.3, 0.3)	(0.8,0.3, 0.5)
b_4	(0.8,0.4, 0.1)	(0.8,0.4, 0.1)	(0.8,0.3, 0.1)
b_5	(0.7,0.1, 0.2)	(0.6,0.3, 0.4)	(0.7,0.1, 0.4)
u	(cheap, colorful)	(colorful, costly)	(colorful, colorful)
b_1	(0.7,0.3, 0.2)	(0.6,0.2, 0.3)	(0.4,0.2, 0.2)
b_2	(0.6,0.1, 0.3)	(0.6,0.4, 0.2)	(0.6,0.4, 0.3)
b_3	(0.8,0.3, 0.4)	(0.5,0.6, 0.2)	(0.5,0.7, 0.2)
b_4	(0.6,0.3, 0.2)	(0.8,0.2, 0.1)	(0.8,0.2, 0.3)
b_5	(0.7,0.3, 0.4)	(0.7,0.1, 0.4)	(0.5,0.6, 0.4)

Table 9: Tabular representation of the INSS (O, A × B).

Proposition 3.25. if (F, A) and (G, B) are two INSSs over U, then **:**

$$(1)\ [(F,A) \wedge (G,B)]^c = (F,A)^c \vee (G,B)^c$$
$$(2)\ [(F,A) \vee (G,B)]^c = (F,A)^c \wedge (G,B)^c$$

Proof1. Let $(F,A)=\{<b, T_{F(x)}(b), I_{F(x)}(b), F_{F(x)}(b)>|b \in U\}$
and
$$(G,B) = \{< b, T_{G(x)}(b), I_{G(x)}(b), F_{G(x)}(b) > |b \in U\}$$
be two INSSs over the common universe U. Also let $(K,A \times B) = (F,A) \wedge (G,B)$,
where, $K(\alpha, \beta) = F(\alpha) \cap G(\beta)$ for all $(\alpha, \beta) \in A \times B$ then
$K(\alpha, \beta) = \{< b, \min(T_{F(\alpha)}(b),T_{G(\beta)}(b)), \min(I_{F(\alpha)}(b),I_{G(\beta)}(b)), \max(F_{F(\alpha)}(b),F_{G(\beta)}(b)) >| b \in U\}$.
Therefore,
$[(F,A) \wedge (G,B)]^c = (K,A \times B)^c$
$= \{< b, \max(F_{F(\alpha)}(b),F_{G(\beta)}(b)), \min(I_{F(\alpha)}(b),I_{G(\beta)}(b)), \min(T_{F(\alpha)}(b),T_{G(\beta)}(b)) >|b \in U\}$.
Again
$(F,A)^c \vee (G,B)^c$
$= \{< b, \max(F_{F^c(\alpha)}(b)),F_{G^c(\beta)}(b)), \min(I_{F^c(\alpha)}(b),I_{G^c(\beta)}(b)), \min(T_{F^c(\alpha)}(b), T_{G^c(\beta)}(b)) >| b \in U\}$.
$= \{< b, \min(T_{F(\alpha)}(b),T_{G(\beta)}(b)), \min(I_{F(\alpha)}(b),I_{G(\beta)}(b)), \max(F_{F(\alpha)}(b),F_{G(\beta)}(b)) >| b \in U\}^c$.
$= \{< b, \max(F_{F(\alpha)}(b), F_{G(\beta)}(b)), \min(I_{F(\alpha)}(b),I_{G(\beta)}(b)), \min(T_{F(\alpha)}(b),T_{G(\beta)}(b)) >| b \in U\}$.

It follows that $[(F,A) \wedge (G,B)]^c = (F,A)^c \vee (G,B)^c$.

Proof 2.
Let $(F, A) = \{< b, T_{F(x)}(b), I_{F(x)}(b), F_{F(x)}(b) > |b \in U\}$ and
$(G,B) = \{< b, T_{G(x)}(b),I_{G(x)}(b),F_{G(x)}(b) > |b \in U\}$ be two INSSs over the common universe U. Also let $(O,A \times B) = (F,A) \vee (G,B)$, where, $O (\alpha,\beta) = F(\alpha) \cup G(\beta)$ for all $(\alpha,\beta) \in A \times B$. then
$O(\alpha,\beta) = \{< b, \max(T_{F(\alpha)}(b),T_{G(\beta)}(b)), \min(I_{F(\alpha)}(b),I_{G(\beta)}(b)), \min(F_{F(\alpha)}(b),F_{G(\beta)}(b)) > |b \in U\}$.
$[(F,A)\vee(G,B)]^c = (O,A\times B)^c =\{< b, \min(F_{F(\alpha)}(b),F_{G(\beta)}(b)), \min(I_{F(\alpha)}(b),I_{G(\beta)}(b)),$
$\max(T_{F(\alpha)}(b),T_{G(\beta)}(b)) > |b \in U\}$.
Again
$(H,A)^c \wedge (G,B)^c$
$= \{< b,\min(F_{F^c(\alpha)}(b),F_{G^c(\beta)}(b)),\min(I_{F^c(\alpha)}(b),I_{G^c(\beta)}(b)), \max(T_{F^c(\alpha)}(b),T_{G^c(\beta)}(b)),>| b \in U\}$.

$= \{< b, \max(T_{F(\alpha)}(b),T_{G(\beta)}(b)), \min(I_{F}{}^{c}{}_{(\alpha)}(b),I_{G}{}^{c}{}_{(\beta)}(b)), \min(F_{F(\alpha)}(b),F_{G(\beta)}(b))>| \, b \in U\}^{c}$.
$= \{< b, \min(F_{F(\alpha)}(b),F_{G(\beta)}(b)), \min(I_{F(\alpha)}(b),I_{G(\beta)}(b)), \max(T_{F(\alpha)}(b),T_{G(\beta)}(b)) >| \, b \in U\}$.
It follows that $[(F,A) \vee (G,B)]^{c} = (F,A)^{c} \wedge (G,B)^{c}$.

4. An application of intuitionistic neutrosophic soft set in a decision making problem

For a concrete example of the concept described above, we revisit the blouse purchase problem in Example 3.3. So let us consider the intuitionistic neutrosophic soft set S = (F,P) (see also Table 10 for its tabular representation), which describes the "attractiveness of the blouses" that Mrs. X is going to buy. on the basis of her m number of parameters $(e_1,e_2,\ldots,e_m)$ out of n number of blouses$(b_1,b_2,\ldots,b_n)$. We also assume that corresponding to the parameter $e_j(j =1,2,\cdots,m)$ the performance value of the blouse b_i $(i = 1,2,\cdots,n)$ is a tuple $p_{ij} = (T_{F(ej)}(b_i), I_{F(ej)}(b_i), T_{F(ej)}(b_i))$, such that for a fixed i that values p_{ij} $(j = 1,2,\cdots,m)$ represents an intuitionistic neutrosophic soft set of all the n objects. Thus the performance values could be arranged in the form of a matrix called the 'criteria matrix'. The more are the criteria values, the more preferability of the corresponding object is. Our problem is to select the most suitable object i.e. the object which dominates each of the objects of the spectrum of the parameters e_j. Since the data are not crisp but intuitionistic neutrosophic soft the selection is not straightforward. Our aim is to find out the most suitable blouse with the choice parameters for Mrs. X. The blouse which is suitable for Mrs. X need not be suitable for Mrs. Y or Mrs. Z, as the selection is dependent on the choice parameters of each buyer. We use the technique to calculate the score for the objects.

4.1. Definition: Comparison matrix

The Comparison matrix is a matrix whose rows are labelled by the object names of the universe such as $b_1,b_2,\cdots,b_n$ and the columns are labelled by the parameters $e_1,e_2,\cdots,e_m$. The entries are c_{ij}, where c_{ij}, is the number of parameters for which the value of b_i exceeds or is equal to the value b_j. The entries are calculated by $c_{ij} = a + d - c$, where 'a' is the integer calculated as 'how many times $T_{bi}(e_j)$ exceeds or equal to $T_{bk}(e_j)$', for $b_i \neq b_k$, $\forall\, b_k \in U$, 'd'is the integer calculated as 'how many times $I_{bi(ej)}$ exceeds or equal to $I_{bk(ej)}$', for $b_i \neq b_k$, $\forall\, b_k \in U$ and 'c' is the integer 'how many times $F_{bi(ej)}$ exceeds or equal to $F_{bk}(e_j)$', for $b_i \neq b_k$, $\forall\, b_k \in U$.

Definition 4.2. Score of an object. The score of an object b_i is S_i and is calculated as :

$$S_i = \sum_j c_{ij}$$

Now the algorithm for most appropriate selection of an object will be as follows.

Algorithm

(1) input the intuitionistic Neutrosophic Soft Set (F, A).
(2) input P, the choice parameters of Mrs. X which is a subset of A.
(3) consider the INSS (F, P) and write it in tabular form.
(4) compute the comparison matrix of the INSS (F, P).
(5) compute the score S_i of $b_i,\forall i$.
(6) find S_k = maxi S_i
(7) if k has more than one value then any one of b_i may be chosen.

To illustrate the basic idea of the algorithm, now we apply it to the intuitionistic neutrosophic soft set based decision making problem.

Suppose the wishing parameters for Mrs. X where P={Bright,Costly, Polystyreneing,Colorful,Cheap}.

Consider the INSS (F, P) presented into the following table.

U	Bright	costly	Polystyreneing	Colorful	Cheap
b_1	(0.6,0.3, 0.4)	(0.5,0.2, 0.6)	(0.5,0.3, 0.4)	(0.8,0.2, 0.3)	(0.6,0.3, 0.2)
b_2	(0.7,0.2, 0.5)	(0.6,0.3, 0.4)	(0.4,0.2, 0.6)	(0.4,0.8, 0.3)	(0.8,0.1, 0.2)
b_3	(0.8,0.3, 0.4)	(0.8,0.5, 0.1)	(0.3,0.5, 0.6)	(0.7,0.2, 0.1)	(0.7,0.2, 0.5)
b_4	(0.7,0.5, 0.2)	(0.4,0.8, 0.3)	(0.8,0.2, 0.4)	(0.8,0.3, 0.4)	(0.8,0.3, 0.4)

b_5	(0.3,0.8, 0.4)	(0.3,0.6, 0.1)	(0.7,0.3, 0.2)	(0.6,0.2, 0.4)	(0.6,0.4, 0,2)

Table 10: Tabular form of the INSS (F, P).

The comparison-matrix of the above INSS (F, P) is represented into the following table.

U	Bright	Costly	Polystyreneing	Colorful	Cheap
b_1	0	-2	3	0	2
b_2	-1	1	-2	2	2
b_3	3	5	0	4	-1
b_4	6	3	3	3	4
b_5	7	2	6	-1	3

Table 11: Comparison matrix of the INSS (F, P).

Next we compute the score for each b_i as shown below:

U	Score (S_i)
b_1	3
b_2	2
b_3	11
b_4	19
b_5	17

Clearly, the maximum score is the score 19, shown in the table above for the blouse b_4.
Hence the best decision for Mrs. X is to select b_4**,** followed by b_5 **.**

5. Conclusions

In this paper we study the notion of intuitionistic neutrosophic set initiated by Bhowmik and Pal. We use this concept in soft sets considering the fact that the parameters (which are words or sentences) are mostly intutionistic neutrosophic set; but both the concepts deal with imprecision, We have also defined some operations on INSS and prove some propositions. Finally, we present an application of INSS in a decision making problem.

Acknowledgements.

The authors are thankful to the anonymous referee for his valuable and constructive remarks that helped to improve the clarity and the completeness of this paper.

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Published in *Journal of Information and Computing Science,* Vol. 8, No. 2, 2013, pp. 130-140, 11 p., pages 161 - 171.

Intuitionistic Neutrosphic Soft Set over Rings

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Abstract . S.Broumi and F.Smarandache introduced the concept of intuitionistic neutrosophic soft set as an extension of the soft set theory. In this paper we have applied the concept of intuitionistic neutrosophic soft set to rings theory .The notion of intuitionistic neutrosophic soft set over ring (INSSOR for short) is introduced and their basic properties have been investigated.The definitions of intersection, union, AND, and OR operations over ring (INSSOR) have also been defined. Finally, we have defined the product of two intuitionistic neutrosophic soft set over ring.

Keywords Intuitionistic Neutrosphic Soft Set, Intuitionistic Neutrosphic Soft Set over Ring, Soft Set, Neutrosphic Soft Set

1. Introduction

The theory of neutrosophic set (NS), which is the generalization of the classical sets, conventional fuzzy set [1], intuitionistic fuzzy set [2] and interval valued fuzzy set [3],was introduced by Samarandache [4]. This concept has recently motivated new research in several directions such as databases [5,6], medical diagnosis problem [7],decision making problem [8],topology [9],control theory [10]and so on. We become handicapped to use fuzzy sets, intuitionistic fuzzy sets or interval valued fuzzy sets when the indeterministic part of uncertain data plays an important role to make a decision. In this context some works can be found in [11,12,13,14].

Another important concept that addresses uncertain information is the soft set theory originated by Molodtsov[15]. This concept is free from the parameterization inadequacy syndrome of fuzzy set theory, rough set theory, probability theory. Molodtsov has successfully applied the soft set theory in many different fields such as smoothness of functions, game theory, operations research, Riemann integration, Perron integration, and probability.

In recent years, soft set theory has been received much attention since its appearance. There are many papers devoted to fuzzify the concept of soft set theory which leads to a series of mathematical models such as fuzzy soft set [16,17,18,19,20], generalized fuzzy soft set [21,22], possibility fuzzy soft set [23] and so on. Thereafter ,P.K.Maji and his coworker[24]introduced the notion of intuitionistic fuzzy soft set which is based on a combination of the intuitionistic fuzzy sets and soft set models and studied the properties of intuitionistic fuzzy soft set. Later, a lot of extentions of intuitionistic fuzzy soft are appeared such as generalized intuitionistic fuzzy soft set [25], possibility Intuitionistic fuzzy soft set [26] and so on. Furthermore, few researchers have contributed a lot towards neutrosophication of soft set theory. In [27] P.K.Maji, first proposed a new mathematical model called “neutrosophic soft set” and investigate some properties regarding neutrosophic soft union, neutrosophic soft intersection, complement of a neutrosophic soft set ,De Morgan’s laws. In 2013, S.Broumi and F. Smarandache [28]combined the intuitionistic neutrosophic set and soft set which lead to a new mathematical model called” intutionistic neutrosophic soft sets”. They studied the notions of intuitionistic neutrosophic soft set union, intuitionistic neutrosophic soft set intersection, complement of intuitionistic neutrosophic soft set and several other properties of intuitionistic neutrosophic soft set along with examples and proofs of certain results. S.Broumi [29]presented the concept of“generalized neutrosophic soft set” by combining the generalized neutrosophic sets[13]and soft set models, studied some properties on it, and presented an application of generalized neutrosophic soft Set in decision making problem.

The algebraic structure of soft set theories has been explored in recent years. In [30], Aktas and Cagman gave a definition of soft groups and compared soft sets to the related concepts of fuzzy sets and rough sets. Sezgin and Atagün [33] defined the notion of normalistic soft groups and corrected some of the problematic cases in paper by Aktas and Cagman [30]. Aygunoglu and Aygun [31] introduced the notion of fuzzy soft groups based on Rosenfeld’s approach [32]and studied its properties. In 2010, Acar et al. [34] introduced the basic notion of soft rings which are actually a parametrized family of subrings. Ghosh, Binda and Samanta [35] introduced the notion of fuzzy soft rings and fuzzy soft ideals and studied some of its algebraic properties. Inan and Ozturk [36]concurrently studied the notion of fuzzy soft rings and fuzzy soft ideals but they dealt with these concepts in a more detailed manner compared to Ghosh et al.[35]. In 2012, B.P.Varol et al [37]introduced the notion of fuzzy soft ring in different way and studied several of their basic properties. J. Zhan et al[38]introduced soft rings related to fuzzy set theory. G. Selvachandran and A. R. Salleh[39]introduced vague soft rings and vague soft ideals and studied some of their basic properties. Z.Zhang [40] introduced intuitionistic fuzzy soft rings studied the algebraic properties of intuitionistic fuzzy soft ring. Studies of fuzzy soft rings are carried out by several researchers but the notion of neutrosophic soft rings is not studied. So, in this work we first study with the algebraic properties of intuitionistic neutrosophic soft set in ring theory. This paper is organized as follows. In section 2 we gives some known and useful preliminary definitions and notations on soft set theory, neutrosophic set, intuitionistic neutrosophic set, intuitionistic neutrosophic soft set and ring theory. In section 3 we discuss intuitionistic neutrosophic soft set over ring (INSSOR). In section 4 concludes the paper.

2. Preliminaries

In this section we recapitulate some relevant definitions viz, soft set, neutrosophic set, intuitionistic neutrosophic set, intuitionistic neutrosophic soft sets, fuzzy subring for better understanding of this article.

2.1. Definition [15]

Molodtsov defined the notion of a soft set in the following way: Let U be an initial universe and E be a set of parameters. Let ζ(U) denotes the power set of U and A be a non-empty subset of E. Then a pair (P, A) is called a soft set over U, where P is a mapping given by P : A $\rightarrow$ ζ(U). In other words, a soft set over U is a parameterized family of subsets of the universe U. For $\varepsilon \in$ A, P (ε) may be considered as the set of ε -approximate elements of the soft set (P, A).

2.2. Definition [4]

Let U be an universe of discourse then the neutrosophic set A is an object having the form A={<x: $T_A(x)$, $I_A(x)$, $F_A(\mathbf{x})$>,x $\in$U}, where the functions T, I, F : U$\rightarrow]^-0,1^+[$define respectively the degree of membership , the degree of indeterminacy, and the degree of non-membership of the element x $\in$ X to the set A with the condition.

$$^-0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3^+. \quad (1)$$

From philosophical point of view, the neutrosophic set takes the value from real standard or non-standard subsets of $]^-0,1^+[$.So instead of $]^-0,1^+[$we need to take the interval[0,1]for technical applications, because $]^-0,1^+[$will be difficult to apply in the real applications such as in scientific and engineering problems.

2.3. Definition [11]

An element x of U is called significant with respect to neutrosophic set A of U if the degree of truth-membership or falsity-membership or indeterminancy-membership value, i.e., $T_A(x)$ or $I_A(x)$ or $F_A(\mathbf{x}) \leq 0.5$. Otherwise, we call it insignificant. Also, for neutrosophic set the truth-membership, indeterminacy-membership and falsity-membership all can not be significant. We define an intuitionistic neutrosophic set by

A={<x: $T_A(x)$, $I_A(x)$, $F_A(\mathbf{x})$ >,x $\in$U},where

$$\min \{ T_A(x), F_A(\mathbf{x}) \} \leq 0.5,$$

$$\min \{ T_A(x), I_A(x) \} \leq 0.5,$$

$$\min \{ F_A(\mathbf{x}) , I_A(x)\} \leq 0.5, \text{ for all } x \in U,$$

with the condition $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 2$. (2)

As an illustration, let us consider the following example.

2.4. Example

Assume that the universe of discourse U={x_1,x_2,x_3}, where x_1 characterizes the capability, x_2 characterizes the trustworthiness and x_3 indicates the prices of the objects. Further, It may be assumed that the values of x_1, x_2 and x_3 are in[0,1]and they are obtained from some questionnaires of some experts. The experts may impose their opinion in three components viz. the degree of goodness, the degree of indeterminacy and that of poorness to explain the characteristics of the objects. Suppose A is an intuitionistic neutrosophic set (INS) of U, such that,

A = {< x_1,0.3, 0.5, 0.4 >,< x_2,0.4, 0.2, 0.6>, < x_3, 0.7, 0.3, 0.5 >}, where the degree of goodness of capability is 0.3, degree of indeterminacy of capability is 0.5 and degree of falsity of capability is 0.4 etc.

2.5. Definition [28]

Let U be an initial universe set and A $\subset$ E be a set of parameters. Let N(U) denotes the set of all intuitionistic neutrosophic sets of U. The collection (P, A) is termed to be the soft intuitionistic neutrosophic set over U, where P is a mapping given by P: A $\rightarrow$ N(U).

2.6. Remark

We will denote the intuitionistic neutrosophic soft set defined over a universe by INSS.

Let us consider the following example.

2.7. Example

Let U be the set of blouses under consideration and E is the set of parameters (or qualities). Each parameter is a intuitionistic neutrosophic word or sentence involving intuitionistic neutrosophic words. Consider E = { Bright, Cheap, Costly, very costly, Colorful, Cotton, Polystyrene, long sleeve , expensive }. In this case, to define a intuitionistic neutrosophic soft set means to point out Bright blouses, Cheap blouses, Blouses in Cotton and so on. Suppose that, there are five blouses in the universe U given by, U = {b_1, b_2, b_3, b_4, b_5} and the set of parameters A = {e_1, e_2, e_3, e_4}, where each e_i is a specific criterion for blouses:
e_1 stands for 'Bright',
e_2 stands for 'Cheap',
e_3 stands for 'costly',
e_4 stands for 'Colorful',
Suppose that,
P(Bright)={<b_1,0.5,0.6,0.3>,<b_2,0.4,0.7,0.2>,<b_3,0.6,0.2,0.3>,<b_4,0.7,0.3,0.2>,<b_5,0.8,0.2,0.3>}.
P(Cheap)={<b_1,0.6,0.3,0.5>,<b_2,0.7,0.4,0.3>,<b_3,0.8,0.1,0.2>,<b_4,0.7,0.1,0.3> ,<b_5,0.8,0.3,0.4}.
P(Costly)={<b_1,0.7,0.4,0.3>,<b_2,0.6,0.1,0.2>,<b_3,0.7,0.2,0.5>,< b_4,0.5,0.2,0.6 > ,< b_5,0.7,0.3,0.2 >}.
P(Colorful)={<b_1,0.8,0.1,0.4>,<b_2,0.4,0.2,0.6>,<b_3,0.3,0.6,0.4>,<b_4,0.4,0.8,0.5> ,< b_5,0.3,0.5,0.7 >}.

2.8. Definition [28]

For two intuitionistic neutrosophic soft sets (P,A) and (Q,B) over the common universe U. We say that (P,A) is an intuitionistic neutrosophic soft subset of (Q,B) if and only if
(i) A ⊂ B.
(ii)P(e) is an intuitionistic neutrosophic subset of Q(e). $T_{P(e)}(x)$,
Or $T_{P(e)}(x) \le T_{Q(e)}(x)$, $I_{P(e)}(x) \ge I_{Q(e)}(x)$, $F_{P(e)}(x) \ge F_{Q(e)}(x)$, $\forall e \in A$, $x \in U$.

We denote this relationship by (P,A) ⊆ (Q,B).

(P,A) is said to be intuitionistic neutrosophic soft super set of (Q,B) if (Q,B) is an intuitionistic neutrosophic soft subset of (P,A). We denote it by (P, A) ⊇ (Q,B).

2.9. Definition [28]

Two INSSs (P, A) and (Q, B) over the common universe U are said to be equal if (P,A) is an intuitionistic neutrosophic soft subset of (Q,B) and (Q,B) is an intuitionistic neutrosophic soft subset of (P,A) which can be denoted by (P,A) = (Q,B).

2.10. Definition [28]

Let (P,A) and (Q,B) be two INSSs over the same universe U. Then the union of (P, A) and (Q, B) is denoted by '(P,A) ∪ (Q , B)' and is defined by (P,A) ∪ (Q,B)=(K, C), where C=A ∪ B and the truth-membership, indeterminacy-membership and falsity-membership of (K, C) are as follows:

$$T_{K(\varepsilon)}(m)=\begin{cases} T_{P(\varepsilon)}(m), \text{if } \varepsilon \in A - B, \\ T_{Q(\varepsilon)}(m), \text{if } \varepsilon \in B - A \\ \max(T_{P(\varepsilon)}(m), T_{Q(\varepsilon)}(m)), \text{if } \varepsilon \in A \cap B \end{cases}$$

$$I_{K(\varepsilon)}(m)=\begin{cases} I_{P(\varepsilon)}(m), \text{if } \varepsilon \in A - B, \\ I_{Q(\varepsilon)}(m), \text{if } \varepsilon \in B - A \\ \min(I_{P(\varepsilon)}(m), I_{Q(\varepsilon)}(m)), \text{if } \varepsilon \in A \cap B \end{cases}$$

$$F_{K(\varepsilon)}(m)=\begin{cases} F_{P(\varepsilon)}(m), \text{if } \varepsilon \in A - B, \\ F_{Q(\varepsilon)}(m), \text{if } \varepsilon \in B - A \\ \min(F_{P(\varepsilon)}(m), F_{Q(\varepsilon)}(m)), \text{if } \varepsilon \in A \cap B \end{cases} \tag{3}$$

2.11. Definition[28]

Let (P,A) and (Q,B) be two INSSs over the same universe U such that A ∩ B≠0. Then the intersection of (P, A) and (Q, B) is denoted by '(P,A) ∩ (Q,B)' and is defined by (P,A) ∩ (Q,B) = (L,C),where C =A ∩ B and the truth-membership, indeterminacy membership and falsity-membership of (L,C) are related to those of (P,A) and (Q,B) by:

$$T_{L(\varepsilon)}(m)=\begin{cases} T_{P(\varepsilon)}(m), \text{if } \varepsilon \in A - B, \\ T_{Q(\varepsilon)}(m), \text{if } \varepsilon \in B - A \\ \min(T_{P(\varepsilon)}(m), T_{Q(\varepsilon)}(m)), \text{if } \varepsilon \in A \cap B \end{cases}$$

$$I_{L(\varepsilon)}(m)=\begin{cases} I_{P(\varepsilon)}(m), \text{if } \varepsilon \in A - B, \\ I_{Q(\varepsilon)}(m), \text{if } \varepsilon \in B - A \\ \min(I_{P(\varepsilon)}(m), I_{Q(\varepsilon)}(m)), \text{if } \varepsilon \in A \cap B \end{cases}$$

$$F_{L(\varepsilon)}(m)=\begin{cases} F_{P(\varepsilon)}(m), \text{if } \varepsilon \in A - B, \\ F_{Q(\varepsilon)}(m), \text{if } \varepsilon \in B - A \\ \max(F_{P(\varepsilon)}(m), F_{Q(\varepsilon)}(m)), \text{if } \varepsilon \in A \cap B \end{cases} \tag{4}$$

2.12. Definition [27]

Let (P, A) be a soft set. The set Supp (P,A) = {x ∈ A | P (x)≠ ∅} is called the support of the soft set (P,A). A soft set (P, A) is non-null if Supp (P, A) ≠∅.

2.13. Definition [41]

A fuzzy subset μ of a ring R is called a fuzzy subring of R (in Rosenfeld' sense), if for all x, y ∈ R the following requirements are met:

$$\mu\,(x-y) \ge \min(\mu(x), \mu(y)) \text{ and}$$
$$\mu\,(xy) \ge \min(\mu(x), \mu(y)) \tag{5}$$

3. Intuitionistic Neutrosophic Soft Set over Ring

In this section, we introduce the notions of intuitionistic neutrosophic soft set over ring and intuitionistic neutrosophic soft subring in Rosenfeld's sense and study some of their properties related to this notions.

Throughout this paper. Let (R, + , .) be a ring . E be a parameter set and let A $\subseteq$ E. For the sake of simplicity , we will denote the ring (R, +, .) simply as R.

From now on, R denotes a commutative ring and all intuitionistic neutrosophic soft sets are considered over R.

3.1. Definition

Let $(\tilde{P}$,A) be an intuitionistic neutrosophic soft set. The set Supp($\tilde{P}$,A) = { $\varepsilon \in$ A |$\tilde{P}(\varepsilon) \neq \emptyset$} is called the support of the intuitionistic neutrosophic soft set ($\tilde{P}$,A).An intuitionistic neutrosophic soft set ($\tilde{P}$,A) is non-null if Supp ($\tilde{P}$,A) $\neq \emptyset$.

3.2. Definition

A pair ($\tilde{P}$,A) is called an intuitionistic neutrosophic soft set over ring, where $\tilde{P}$ is a mapping given by

$\tilde{P}$:A $\rightarrow$ $([0,1] \times [0,1] \times [0,1])^R$, $\tilde{P}(\varepsilon)$: R $\rightarrow [0,1] \times [0,1] \times [0,1]$,

$\tilde{P}(\varepsilon)= \left\{(x, T_{\tilde{P}(\varepsilon)}(x), I_{\tilde{P}(\varepsilon)}(x), F_{\tilde{P}(\varepsilon)}(x)) : x \in R\right\}$ for all $\varepsilon \in$ A,

If for all x ,y $\in$ R the following condition hold:

(1) $T_{\tilde{P}(\varepsilon)}(x-y) \geq T_{\tilde{P}(\varepsilon)}(x) \wedge T_{\tilde{P}(\varepsilon)}(y)$,
$F_{\tilde{P}(\varepsilon)}(x-y) \leq F_{\tilde{P}(\varepsilon)}(x) \vee F_{\tilde{P}(\varepsilon)}(y)$ and
$I_{\tilde{P}(\varepsilon)}(x-y) \leq I_{\tilde{P}(\varepsilon)}(x) \vee I_{\tilde{P}(\varepsilon)}(y)$ **(6)**

(2) $T_{\tilde{P}(\varepsilon)}(xy) \geq T_{\tilde{P}(\varepsilon)}(x) \wedge T_{\tilde{P}(\varepsilon)}(y)$,
$F_{\tilde{P}(\varepsilon)}(xy) \leq F_{\tilde{P}(\varepsilon)}(x) \vee F_{\tilde{P}(\varepsilon)}(y)$ and
$I_{\tilde{P}(\varepsilon)}(xy) \leq I_{\tilde{P}(\varepsilon)}(x) \vee I_{\tilde{P}(\varepsilon)}(y)$ (7)

3.3. Definition

For two intuitionistic neutrosophic soft set over ring ($\tilde{P}$,A) and ($\tilde{Q}$,B), we say that ($\tilde{P}$,A) is an intuitionistic neutrosophic soft subring of ($\tilde{Q}$,B) and write ($\tilde{P}$,A) $\subseteq$ ($\tilde{Q}$,B) if

(i) A $\subseteq$ B
(ii) for each x $\in R$, $\varepsilon \in$ A, $T_{\tilde{P}(\varepsilon)}(x) \leq T_{\tilde{Q}(\varepsilon)}(x)$,
$I_{\tilde{P}(\varepsilon)}(x) \geq I_{\tilde{Q}(\varepsilon)}(x)$, $F_{\tilde{P}(\varepsilon)}(x) \geq F_{\tilde{Q}(\varepsilon)}(x)$. **(8)**

3.4. Definition

Two intuitionist neutrosophic soft set over ring ($\tilde{P}$,A) and ($\tilde{Q}$,B) are said to be equal if ($\tilde{P}$,A) $\subseteq$ ($\tilde{Q}$,B) and ($\tilde{Q}$,B) $\subseteq$ ($\tilde{P}$,A).

3.5. Theorem

Let ($\tilde{P}$,A) and ($\tilde{Q}$,B) be two intuitionistic neutrosophic soft over ring R. if $\tilde{P}(\varepsilon) \leq \tilde{Q}(\varepsilon)$ for all $\varepsilon \in$ A and A $\subset$ B, then ($\tilde{P}$,A) is an intuitionistic neutrosophic soft subring of ($\tilde{Q}$,B).

Proof The proof is straightforward

3.6. Definition

The union of two intuitionistic neutrosophic soft set over ring ($\tilde{P}$,A) and ($\tilde{Q}$,B) is denoted by ($\tilde{P}$,A) $\tilde{\cup}$ ($\tilde{Q}$,B) and is defined by a intuitionistic neutrosophic soft set over ring

$\tilde{H}$:A $\cup$ B $\rightarrow$ $([0,1] \times [0,1] \times [0,1])^R$ such that for each $\varepsilon \in$ A $\cup$ B.

$$\tilde{H}(\varepsilon)= \begin{cases} < x, T_{\tilde{P}(\varepsilon)}(x), I_{\tilde{P}(\varepsilon)}(x), F_{\tilde{P}(\varepsilon)}(x) > if\ \varepsilon \in A-B \\ < x, T_{\tilde{Q}(\varepsilon)}(x), I_{\tilde{Q}(\varepsilon)}(x), F_{\tilde{Q}(\varepsilon)}(x) > if\ \varepsilon \in B-A \\ < x, T_{\tilde{P}(\varepsilon)}(x) \vee T_{\tilde{Q}(\varepsilon)}(x), I_{\tilde{P}(\varepsilon)}(x) \wedge I_{\tilde{Q}(\varepsilon)}(x), F_{\tilde{P}(\varepsilon)}(x) \wedge F_{\tilde{Q}(\varepsilon)}(x) >, \\ \quad if\ \varepsilon \in A \cap B \end{cases}$$

(9)

This is denoted by $(\tilde{H}, C)$=($\tilde{P}$,A)$\tilde{\cup}$ ($\tilde{Q}$,B), where C= A$\cup$B.

3.7. Theorem

If ($\tilde{P}$,A) and ($\tilde{Q}$,B) are two intuitionistic neutrosophic soft set over ring R, then , so are ($\tilde{P}$,A) $\tilde{\cup}$ ($\tilde{Q}$,B) .

Proof. For any $\varepsilon \in$ A$\cup$B and x, y $\in$ R,we consider the following cases.

Case 1. Let $\varepsilon \in$ A $-$ B .Then,

$$\begin{aligned} T_{\tilde{H}(\varepsilon)}(x-y) &= T_{\tilde{P}(\varepsilon)}(x-y) \\ &\geq T_{\tilde{P}(\varepsilon)}(x) \wedge T_{\tilde{P}(\varepsilon)}(y) \\ &= T_{\tilde{H}(\varepsilon)}(x) \wedge T_{\tilde{H}(\varepsilon)}(y), \\ T_{\tilde{H}(\varepsilon)}(xy) &= T_{\tilde{P}(\varepsilon)}(xy) \\ &\geq T_{\tilde{P}(\varepsilon)}(x) \wedge T_{\tilde{P}(\varepsilon)}(y) \\ &= T_{\tilde{H}(\varepsilon)}(x) \wedge T_{\tilde{H}(\varepsilon)}(y), \\ I_{\tilde{H}(\varepsilon)}(x-y) &= I_{\tilde{P}(\varepsilon)}(x-y) \\ &\leq I_{\tilde{P}(\varepsilon)}(x) \vee I_{\tilde{P}(\varepsilon)}(y) \\ &= I_{\tilde{H}(\varepsilon)}(x) \vee I_{\tilde{H}(\varepsilon)}(y), \\ I_{\tilde{H}(\varepsilon)}(xy) &= I_{\tilde{P}(\varepsilon)}(xy) \\ &\leq I_{\tilde{P}(\varepsilon)}(x) \vee I_{\tilde{P}(\varepsilon)}(y) \\ &= I_{\tilde{H}(\varepsilon)}(x) \vee I_{\tilde{H}(\varepsilon)}(y), \\ F_{\tilde{H}(\varepsilon)}(x-y) &= F_{\tilde{P}(\varepsilon)}(x-y) \\ &\leq F_{\tilde{P}(\varepsilon)}(x) \vee F_{\tilde{P}(\varepsilon)}(y) \\ &= F_{\tilde{H}(\varepsilon)}(x) \vee F_{\tilde{H}(\varepsilon)}(y), \\ F_{\tilde{H}(\varepsilon)}(xy) &= F_{\tilde{P}(\varepsilon)}(xy) \\ &\leq F_{\tilde{P}(\varepsilon)}(x) \vee F_{\tilde{P}(\varepsilon)}(y) \\ &= F_{\tilde{H}(\varepsilon)}(x) \vee F_{\tilde{H}(\varepsilon)}(y), \end{aligned}$$

Case 2. Let $if\ \varepsilon \in$ B $-$ A .Then, analogous to the proof of case 1, we have

$$T_{\tilde{H}(\varepsilon)}(x-y) \geq T_{\tilde{H}(\varepsilon)}(x) \wedge T_{\tilde{H}(\varepsilon)}(y)$$

$$T_{\tilde{H}(\varepsilon)}(xy) \geq T_{\tilde{H}(\varepsilon)}(x) \wedge T_{\tilde{H}(\varepsilon)}(y)$$

$$I_{\tilde{H}(\varepsilon)}(x-y) \leq I_{\tilde{H}(\varepsilon)}(x) \vee I_{\tilde{H}(\varepsilon)}(y)$$

$$I_{\tilde{H}(\varepsilon)}(xy) \leq I_{\tilde{H}(\varepsilon)}(x) \vee I_{\tilde{H}(\varepsilon)}(y)$$

$F_{\tilde{H}(\varepsilon)}(x-y) \leq F_{\tilde{H}(\varepsilon)}(x) \vee F_{\tilde{H}(\varepsilon)}(y)$

$F_{\tilde{H}(\varepsilon)}(xy) \leq F_{\tilde{H}(\varepsilon)}(x) \vee F_{\tilde{H}(\varepsilon)}(y)$

Case 3. Let $\varepsilon \in A \cap B$. In this case the proof is straightforward. thus ,in any cases ,we have

$T_{\tilde{H}(\varepsilon)}(x-y) \geq T_{\tilde{H}(\varepsilon)}(x) \wedge T_{\tilde{H}(\varepsilon)}(y)$

$T_{\tilde{H}(\varepsilon)}(xy) \geq T_{\tilde{H}(\varepsilon)}(x) \wedge T_{\tilde{H}(\varepsilon)}(y)$

$I_{\tilde{H}(\varepsilon)}(x-y) \leq I_{\tilde{H}(\varepsilon)}(x) \vee I_{\tilde{H}(\varepsilon)}(y)$

$I_{\tilde{H}(\varepsilon)}(xy) \leq I_{\tilde{H}(\varepsilon)}(x) \vee I_{\tilde{H}(\varepsilon)}(y)$

$F_{\tilde{H}(\varepsilon)}(x-y) \leq F_{\tilde{H}(\varepsilon)}(x) \vee F_{\tilde{H}(\varepsilon)}(y)$

$F_{\tilde{H}(\varepsilon)}(xy) \leq F_{\tilde{H}(\varepsilon)}(x) \vee F_{\tilde{H}(\varepsilon)}(y)$

Therefore , $(\tilde{P}$, A)$\tilde{\cup}$($\tilde{Q}$, B) is an intuitionistic neutrosophic soft set over ring

3.8. Definition

The intersection of two intuitionistic neutrosophic soft set over ring $(\tilde{P}$,A) and $(\tilde{Q}$,B) is denoted by $(\tilde{P}$,A) $\tilde{\cap}$ $(\tilde{Q}$,B) and is defined by an intuitionistic neutrosophic soft set over ring.

$\tilde{M}$:A $\cup$ B $\rightarrow$ $([0,1] \times [0,1] \times [0,1])^R$ such that for each $\varepsilon \in$ A $\cup$ B.

$\tilde{M}(\varepsilon)=$

$$\begin{cases} < x, T_{\tilde{P}(\varepsilon)}(x), I_{\tilde{P}(\varepsilon)}(x), F_{\tilde{P}(\varepsilon)}(x) > if\ \varepsilon \in \mathrm{A-B} \\ < x, T_{\tilde{Q}(\varepsilon)}(x), I_{\tilde{Q}(\varepsilon)}(x), F_{\tilde{Q}(\varepsilon)}(x) > if\ \varepsilon \in \mathrm{B-A} \\ < x, T_{\tilde{P}(\varepsilon)}(x) \wedge T_{\tilde{Q}(\varepsilon)}(x), I_{\tilde{P}(\varepsilon)}(x) \wedge I_{\tilde{Q}(\varepsilon)}(x), F_{\tilde{P}(\varepsilon)}(x) \vee F_{\tilde{Q}(\varepsilon)}(x) >, \\ if\ \varepsilon \in \mathrm{A} \cap \mathrm{B} \end{cases} \quad \textbf{(10)}$$

This is denoted by $(\tilde{M}, C)=$ $(\tilde{P}$,A) $\tilde{\cap}$ $(\tilde{Q}$,B), where C = A$\cup$ B .

3.9. Theorem

If $(\tilde{P}$,A) and $(\tilde{Q}$,B) are two intuitionistic neutrosophic soft set over ring, then , so are $(\tilde{P}$,A) $\tilde{\cap}$ $(\tilde{Q}$,B).

Proof. The proof is similar to that of Theorem 3.8.

3.10. Definition

Let $(\tilde{P}$,A) and $(\tilde{Q}$,B) be two intuitionistic neutrosophic soft set over ring R. Then , “$(\tilde{P}$,A) AND $(\tilde{Q}$,B)” is denoted by $(\tilde{P}$,A) $\tilde{\wedge}$ $(\tilde{Q}$,B) and is defined by $(\tilde{P}$,A) $\tilde{\wedge}$ $(\tilde{Q}$,B)= $(\tilde{N}, C)$,where C= A×B and $\tilde{H}$:C $\rightarrow$ $([0,1]^3 \times [0,1]^3)^R$ is defined as

$$\tilde{N}(\alpha, \beta) = \tilde{P}(\alpha) \cap \tilde{Q}(\beta), \text{ for all } (\alpha, \beta) \in C.$$

3.11. Theorem

If $(\tilde{P}$,A) and $(\tilde{Q}$,B) are two intuitionistic neutrosophic soft set over ring R, then , so is $(\tilde{P}$,A)$\tilde{\wedge}$ $(\tilde{Q}$,B).

Proof. For all x, y $\in$ R and $(\alpha, \beta) \in$ A x B we have

$T_{\tilde{N}(\alpha,\beta)}(x-y) = (T_{\tilde{P}(\alpha)}(x-y) \wedge T_{\tilde{Q}(\beta)}(x-y))$

$\geq (T_{\tilde{P}(\alpha)}(x) \wedge T_{\tilde{P}(\alpha)}(y)) \wedge (T_{\tilde{Q}(\beta)}(x) \wedge T_{\tilde{Q}(\beta)}(y))$

$= (T_{\tilde{P}(\alpha)}(x) \wedge T_{\tilde{Q}(\beta)}(x)) \wedge (T_{\tilde{P}(\alpha)}(y) \wedge T_{\tilde{Q}(\beta)}(y))$

$= T_{\tilde{N}(\alpha,\beta)}(x) \wedge T_{\tilde{N}(\alpha,\beta)}(y)$

$T_{\tilde{N}(\alpha,\beta)}(xy) = (T_{\tilde{P}(\alpha)}(xy) \wedge T_{\tilde{Q}(\beta)}(xy))$

$\geq (T_{\tilde{P}(\alpha)}(x) \wedge T_{\tilde{P}(\alpha)}(y)) \wedge (T_{\tilde{Q}(\beta)}(x) \wedge T_{\tilde{Q}(\beta)}(y))$

$= (T_{\tilde{P}(\alpha)}(x) \wedge T_{\tilde{Q}(\beta)}(x)) \wedge (T_{\tilde{P}(\alpha)}(y) \wedge T_{\tilde{Q}(\beta)}(y))$

$= T_{\tilde{N}(\alpha,\beta)}(x) \wedge T_{\tilde{N}(\alpha,\beta)}(y)$

In a similar way ,we have

$I_{\tilde{N}(\alpha,\beta)}(x-y) \leq I_{\tilde{N}(\alpha,\beta)}(x) \vee I_{\tilde{N}(\alpha,\beta)}(y)$

$I_{\tilde{N}(\alpha,\beta)}(xy) \leq I_{\tilde{N}(\alpha,\beta)}(x) \vee I_{\tilde{N}(\alpha,\beta)}(y)$

$F_{\tilde{N}(\alpha,\beta)}(x-y) \leq F_{\tilde{N}(\alpha,\beta)}(x) \vee F_{\tilde{N}(\alpha,\beta)}(y)$

$F_{\tilde{N}(\alpha,\beta)}(xy) \leq F_{\tilde{N}(\alpha,\beta)}(x) \vee F_{\tilde{N}(\alpha,\beta)}(y)$

For all x, y $\in$ R and $(\alpha, \beta) \in$ C .It follows that $(\tilde{P}$,A)$\tilde{\wedge}$ $(\tilde{Q}$,B) is an intuitionistic neutrosophic soft set over ring R.

3.12. Definition

Let $(\tilde{P}$,A) and $(\tilde{Q}$,B) be two intuitionistic neutrosophic soft set over ring R. Then , “$(\tilde{P}$,A) OR $(\tilde{Q}$,B)” is denoted by $(\tilde{P}$,A) $\tilde{\vee}$ $(\tilde{Q}$,B) and is defined by $(\tilde{P}$,A) $\tilde{\vee}$ $(\tilde{Q}$,B)= $(\tilde{O}, C)$,where C= A × B and $\tilde{O}$: C $\rightarrow$ $([0,1]^3 \times [0,1]^3)^R$ is defined as

$$\tilde{O}(\alpha, \beta) = \tilde{P}(\alpha) \tilde{\cup} \tilde{Q}(\beta), \text{ for all } (\alpha, \beta) \in C.$$

3.13. Theorem

If ($\tilde{P}$, A) and ($\tilde{Q}$, B) are two intuitionist neutrosophic soft set over ring R, then , so are ($\tilde{P}$, A)$\tilde{\vee}$($\tilde{Q}$, B).

Proof. The proof is straightforward.

The following theorem is a generalization of previous results.

3.14. Theorem

Let ($\tilde{P}$, A) be an intuitionist neutrosophic soft set over ring R, and let $\{(\tilde{P}_i, A_i)\}_{i \in I}$ be a nonempty family of intuitionistic neutrosophic soft set over ring, where I is an index set .Then , one has the following:

(1) $\wedge_{i \in I}(\tilde{P}_i, A_i)$ is an intuitionistic neutrosophic soft set over ring R.

(2) if $A_i \cap A_j = 0$,for all i, j $\in$ I ,then $\vee_{i \in I}(\tilde{P}_i, A_i)$ is an intuitionistic neutrosophic soft set over ring R.

3.15. Definition

Let $(\tilde{P}$,A) and $(\tilde{Q}$,B) be two intuitionistic neutrosophic soft set over ring R. Then ,the product of $(\tilde{P}$,A) and $(\tilde{Q}$,B) is defined to be the intuitionistic neutrosophic soft set over ring $(\tilde{P} \circ \tilde{Q}$, C) ,where C= A∪ B and

$$T_{(\tilde{P} \circ \tilde{Q})(\varepsilon)}(x) = \begin{cases} T_{\tilde{P}(\varepsilon)}(x) \text{ if } \varepsilon \in A-B \\ T_{\tilde{Q}(\varepsilon)}(x) \text{ if } \varepsilon \in B-A \\ \vee_{x=ab}\{T_{\tilde{P}(\varepsilon)}(a) \wedge T_{\tilde{Q}(\varepsilon)}(b)\} \text{ if } \varepsilon \in A \cap B \end{cases} \quad (11)$$

$$I_{(\tilde{P} \circ \tilde{Q})(\varepsilon)}(x) = \begin{cases} I_{\tilde{P}(\varepsilon)}(x) \text{ if } \varepsilon \in A-B \\ I_{\tilde{Q}(\varepsilon)}(x) \text{ if } \varepsilon \in B-A \\ \wedge_{x=ab}\{I_{\tilde{P}(\varepsilon)}(a) \vee I_{\tilde{Q}(\varepsilon)}(b)\} \text{ if } \varepsilon \in A \cap B \end{cases}$$

$$F_{(\tilde{P} \circ \tilde{Q})(\varepsilon)}(x) = \begin{cases} F_{\tilde{P}(\varepsilon)}(x) \text{ if } \varepsilon \in A-B \\ F_{\tilde{Q}(\varepsilon)}(x) \text{ if } \varepsilon \in B-A \\ \wedge_{x=ab}\{F_{\tilde{P}(\varepsilon)}(a) \vee F_{\tilde{Q}(\varepsilon)}(b)\} \text{ if } \varepsilon \in A \cap B, a, b \in R \end{cases}$$

For all $\varepsilon \in$ C and a , b ∈ R .This is denoted by $(\tilde{P} \circ \tilde{Q}$, C) = $(\tilde{P}$,A)∘ $(\tilde{Q}$,B).

3.16. Theorem

If $(\tilde{P}$,A) and $(\tilde{Q}$,B) are two intuitionistic neutrosophic soft set over ring R. Then , so is $(\tilde{P}$,A) ∘ $(\tilde{Q}$,B) .

Proof. Let $(\tilde{P}$,A) and $(\tilde{Q}$,B) be two intuitionistic neutrosophic soft set over ring R. Then ,for any $\varepsilon \in$ A ∪ B ,and x ,y ∈ R, we consider the following cases.

Case 1. Let $\varepsilon \in$ A − B. Then,

$$\begin{aligned} T_{(\tilde{P} \circ \tilde{Q})(\varepsilon)}(x-y) &= T_{\tilde{P}(\varepsilon)}(x-y) \\ &\geq T_{\tilde{P}(\varepsilon)}(x) \wedge T_{\tilde{P}(\varepsilon)}(y) \\ &= T_{(\tilde{P} \circ \tilde{Q})(\varepsilon)}(x) \wedge T_{(\tilde{P} \circ \tilde{Q})(\varepsilon)}(y), \end{aligned}$$

$$\begin{aligned} T_{(\tilde{P} \circ \tilde{Q})(\varepsilon)}(xy) &= T_{\tilde{P}(\varepsilon)}(xy) \\ &\geq T_{\tilde{P}(\varepsilon)}(x) \wedge T_{\tilde{P}(\varepsilon)}(y) \\ &= T_{(\tilde{P} \circ \tilde{Q})(\varepsilon)}(x) \wedge T_{(\tilde{P} \circ \tilde{Q})(\varepsilon)}(y) \end{aligned}$$

$$\begin{aligned} I_{(\tilde{P} \circ \tilde{Q})(\varepsilon)}(x-y) &= I_{\tilde{P}(\varepsilon)}(x-y) \\ &\leq I_{\tilde{P}(\varepsilon)}(x) \vee I_{\tilde{P}(\varepsilon)}(y) \\ &= I_{(\tilde{P} \circ \tilde{Q})(\varepsilon)}(x) \vee I_{(\tilde{P} \circ \tilde{Q})(\varepsilon)}(y), \end{aligned}$$

$$\begin{aligned} I_{(\tilde{P} \circ \tilde{Q})(\varepsilon)}(xy) &= I_{\tilde{P}(\varepsilon)}(xy) \\ &\leq I_{\tilde{P}(\varepsilon)}(x) \vee I_{\tilde{P}(\varepsilon)}(y) \\ &= I_{(\tilde{P} \circ \tilde{Q})(\varepsilon)}(x) \vee I_{(\tilde{P} \circ \tilde{Q})(\varepsilon)}(y) \end{aligned}$$

$$\begin{aligned} F_{(\tilde{P} \circ \tilde{Q})(\varepsilon)}(x-y) &= F_{\tilde{P}(\varepsilon)}(x-y) \\ &\leq F_{\tilde{P}(\varepsilon)}(x) \vee F_{\tilde{P}(\varepsilon)}(y) \\ &= F_{(\tilde{P} \circ \tilde{Q})(\varepsilon)}(x) \vee F_{(\tilde{P} \circ \tilde{Q})(\varepsilon)}(y), \end{aligned}$$

$$\begin{aligned} F_{(\tilde{P} \circ \tilde{Q})(\varepsilon)}(xy) &= F_{\tilde{P}(\varepsilon)}(xy) \\ &\leq F_{\tilde{P}(\varepsilon)}(x) \vee F_{\tilde{P}(\varepsilon)}(y) \\ &= F_{(\tilde{P} \circ \tilde{Q})(\varepsilon)}(x) \vee F_{(\tilde{P} \circ \tilde{Q})(\varepsilon)}(y) \end{aligned}$$

Case 2. Let $\varepsilon \in$ B − A. Then, analogous to the proof of case 1,the proof is straightforward.

Case 3. Let $\varepsilon \in$ A ∩ B. Then,

$$\begin{aligned} T_{(\tilde{P} \circ \tilde{Q})(\varepsilon)}(x) &= \vee_{x=ab}(T_{\tilde{P}(\varepsilon)}(a) \wedge T_{\tilde{Q}(\varepsilon)}(b)) \\ &\geq \vee_{xy=aby}(T_{\tilde{P}(\varepsilon)}(a) \wedge T_{\tilde{Q}(\varepsilon)}(by)) \\ &\geq \vee_{xy=cd}(T_{\tilde{P}(\varepsilon)}(c) \wedge T_{\tilde{Q}(\varepsilon)}(d)) \\ &= T_{(\tilde{P} \circ \tilde{Q})(\varepsilon)}(xy) \end{aligned}$$

Similarly ,we have $T_{(\tilde{P} \circ \tilde{Q})(\varepsilon)}(xy) \geq T_{(\tilde{P} \circ \tilde{Q})(\varepsilon)}(y)$, and so

$T_{(\tilde{P} \circ \tilde{Q})(\varepsilon)}(xy) \geq T_{(\tilde{P} \circ \tilde{Q})(\varepsilon)}(x) \wedge T_{(\tilde{P} \circ \tilde{Q})(\varepsilon)}(y)$

In a similar way , we prove that

$I_{(\tilde{P} \circ \tilde{Q})(\varepsilon)}(xy) \leq I_{(\tilde{P} \circ \tilde{Q})(\varepsilon)}(x) \vee I_{(\tilde{P} \circ \tilde{Q})(\varepsilon)}(y)$

and $F_{(\tilde{P} \circ \tilde{Q})(\varepsilon)}(xy) \leq F_{(\tilde{P} \circ \tilde{Q})(\varepsilon)}(x) \vee F_{(\tilde{P} \circ \tilde{Q})(\varepsilon)}(y)$

Therefore $(\tilde{P}$,A)∘ $(\tilde{Q}$,B) is an intuitionistic neutrosophic soft set over ring R.

4. Conclusion

In this paper we have introduced the concept of intuitionistic neutrosophic soft set over ring (INSSOR for short). We also studied and discussed some properties related to this concept. The definitions of intersection, union, AND, and OR operations over ring (INSSOR) have also been defined. we have defined the product of two intuitionistic neutrosophic soft set over ring. Finally, it is hoped that this concept will be useful for the researchers to further promote and advance the research in neutrosophic soft set theory.

Acknowledgments

The authors are highly grateful to the referees for their valuable comments and suggestions for improving the paper.

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Published in *Mathematics and Statistics*, No. 2(3), 2014, pp. 120-126, DOI: 10.13189/ms.2014.020303, 7 p., pages 172 - 178

More on Intuitionistic Neutrosophic Soft Sets

Said Broumi, Florentin Smarandache

Abstract Intuitionistic Neutrosophic soft set theory proposed by S.Broumi and F.Samarandache [28], has been regarded as an effective mathematical tool to deal with uncertainties. In this paper new operations on intuitionistic neutrosophic soft sets have been introduced . Some results relating to the properties of these operations have been established. Moreover ,we illustrate their interconnections between each other.

Keywords Soft Set, Intuitionistic Fuzzy Soft , Intuitionistic Neutrosophic Soft Sets, Necessity and Possibility Operations

1. Introduction

The theory of neutrosophic set (NS), which is the generalization of the classical sets, conventional fuzzy set [1], intuitionistic fuzzy set [2]and interval valued fuzzy set [3],was introduced by Samarandache [4]. This concept has been applied in many fields such as Databases [5, 6], Medical diagnosis problem [7], Decision making problem [8],Topology [9],control theory [10] and so on. The concept of neutrosophic set handle indeterminate data whereas fuzzy set theory, and intuitionstic fuzzy set theory failed when the relation are indeterminate.

Later on, several researchers have extended the neutrosophic set theory, such as Bhowmik and M.Pal in [11, 12], in their paper, they defined "intuitionistic neutrosophic set".In [13], A.A.Salam, S.A.Alblowi introduced another concept called "Generalized neutrosophic set". In [14], Wang et al. proposed another extension of neutrosophic set which is" single valued neutrosophic". In 1998 a Russian researcher, Molodtsov proposed a new mathematical tool called" Soft set theory" [15],for dealing with uncertainty and how soft set theory is free from the parameterization inadequacy syndrome of fuzzy set theory, rough set theory, probability theory.

In recent time, researchers have contributed a lot towards fuzzification of soft set theory which leads to a series of mathematical models such as Fuzzy soft set [17, 18, 19, 20], generalized fuzzy soft set [21, 22], possibility fuzzy soft set [23] and so on, therafter, P.K.Maji and his coworker [24] introduced the notion of intuitionistic fuzzy soft set which is based on a combination of the intuitionistic fuzzy setsand soft set models and studied the properties of intuitionistic fuzzy soft set. Later a lot of extentions of intuitionistic fuzzy soft are appeared such as generalized intuitionistic fuzzy soft set [25], Possibility intuitionistic fuzzy soft set [26]and so on. Few studies are focused on neutrosophication of soft set theory. In [25] P.K.Maji, first proposed a new mathematical model called "Neutrosophic Soft Set" and investigate some properties regarding neutrosophic soft union, neutrosophic soft intersection ,complement of a neutrosophic soft set ,De Morgan law etc. Furthermore , in 2013, S.Broumi and F. Smarandache [26] combined the intuitionistic neutrosophic and soft set which lead to a new mathematical model called" intutionistic neutrosophic soft set". They studied the notions of intuitionistic neutrosophic soft set union, intuitionistic neutrosophic soft set intersection, complement of intuitionistic neutrosophic soft set and several other properties of intuitionistic neutrosophic soft set along with examples and proofs of certain results. Also ,in [27] S.Broumi presentedthe concept of "Generalized neutrosophic soft set" by combining the generalized neutrosophic sets [13] and soft set models ,studied some properties on it, and presented an application of generalized neutrosophic soft set in decision making problem.

In the present work, we have extended the intuitionistic neutrosophic soft sets defining new operations on it. Some properties of these operations have also been studied.

The rest of this paper is organized as follow: section II deals with some definitions related to soft set theory ,neutrosophic set, intuitionistic neutrosophic set, intuitionistic neutrosophic soft set theory. Section III deals with the necessity operation on intuitionistic neutrosophic soft set. Section IV deals with the possibility operation on intuitionistic neutrosophic soft set. Finally ,section V give the conclusion.

2. Preliminaries

In this section we represent definitions needful for next section,we denote by N(u) the set of all intuitionistic neutrosophic set.

2.1. Soft Sets (see [15]).

Let U be a universe set and E be a set of parameters. Let ζ (U) denotes the power set of U and A $\subset$ E.

2.1.1. Definition [15]

A pair (P, A) is called a soft set over U, where F is a mapping given by P : A $\rightarrow$ ζ (U). In other words, a soft set over U is a parameterized family of subsets of the universe U. For e $\in$ A, P (e) may be considered as the set of e-approximate elements of the soft set (P, A).

2.2 Intuitionistic Fuzzy Soft Set

Let U be an initial universe set and E be the set of parameters. Let IF^U denote the collection of all intuitionistic fuzzy subsets of U. Let . A $\subseteq$ E pair (P A) is called an intuitionistic fuzzy soft set over U where P is a mapping given by P: A$\rightarrow IF^U$.

2.2.1. Defintion

Let P: A$\rightarrow IF^U$ then F is a function defined as P (ε) ={ x, $\boldsymbol{\mu_{P(\varepsilon)}(x)}$, $\boldsymbol{\nu_{P(\varepsilon)}(x) : x \in U}$, $\varepsilon \in \boldsymbol{E}$} where μ , ν denote the degree of membership and degree of non-membership respectively and π = 1- μ- ν , denote the hesitancy degree.

2.3. Neutrosophic Sets (see [4]).

Let U be an universe of discourse then the neutrosophic set A is an object having the form
A = {< x: $T_{A(x)}$,$I_{A(x)}$,$F_{A(x)}$>,x ∈ U}, where the functions T, I, F : U$\rightarrow$ $]^-0, 1^+[$ define respectively the degree of membership (or Truth) , the degree of indeterminacy, and the degree of non-membership (or Falsehood) of the element x ∈ U to the set A with the condition.

$$^-0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3^+. \qquad (1)$$

From philosophical point of view, the neutrosophic set takes the value from real standard or non-standard subsets of $]^-0, 1^+[$. So instead of $]^-0, 1^+[$ we need to take the interval [0, 1] for technical applications, because $]^-0, 1^+[$will be difficult to apply in the real applications such as in scientific and engineering problems.

2.4. Single Valued Neutrosophic Set(see [14]).

2.4.1. Definition (see [14])

Let X be a space of points (objects) with generic elements in X denoted by x. An SVNS A in X is characterized by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity-membership function $F_A(x)$ for each point x in X, $T_A(x), I_A(x), F_A(x) \in [0, 1]$.
When X is continuous, an SVNS A can be written as

$$A=\int_X \frac{<T_A(x), I_A(x), F_A(x),>}{x}, x \in X. \qquad (2)$$

When X is discrete, an SVNS A can be written as

$$A= \sum_1^n \frac{<T_A(x_i), I_A(x_i),F_A(x_i),>}{x_i}, x_i \in X \qquad (3)$$

2.4.2. Definition (see [4,14])

A neutrosophic set or single valued neutrosophic set (SVNS) A is contained in another neutrosophic set B i.e. A ⊆ B if ∀x ∈ U, $T_A(x) \leq T_B(x)$, $I_A(x) \geq I_B(x)$, $F_A(x) \geq F_B(x)$.

2.4.3. Definition (see [2])

The complement of a neutrosophic set A is denoted by A^c and is defined as $T_{A^c(x)} = F_{A(x)}$, $I_{A^c(x)} = I_{A(x)}$ and $F_{A^c(x)} = T_{A(x)}$ for every x in X.
A complete study of the operations and application of neutrosophic set can be found in [4] .

2.5. Intuitionistic Neutrosophic Set

2.5.1. Definition (see[11])

An element x of U is called significant with respect to neutrsophic set A of U if the degree of truth-membership or falsity-membership or indeterminancy-membership value, i.e., $T_A(x)$ or $F_A(x)$ or $I_A(x) \leq 0.5$. Otherwise, we call it insignificant. Also, for neutrosophic set the truth-membership, indeterminacy-membership and falsity-membership all can not be significant. We define an intuitionistic neutrosophic set by A = {< x: $T_A(x)$, $I_A(x)$, $F_A(x)$ >,x $\in$U},where

$$\min \{ T_A(x), F_A(x)\} \leq 0.5,$$
$$\min \{ T_A(x), , I_A(x)\} \leq 0.5,$$
$$\min \{ F_A(x), I_A(x) \} \leq 0.5, \text{ for all } x \in U, \qquad (4)$$

with the condition

$$0 \leq T_A(x) + I_A(x) + F_A(x) \leq 2. \qquad (5)$$

As an illustration ,let us consider the following example.

2.5.2. Example

Assume that the universe of discourse U={x_1,x_2,x_3},where x_1 characterizes the capability, x_2 characterizes the trustworthiness and $x_3$indicates the prices of the objects. It may be further assumed that the values of x_1, x_2 and x_3 are in [0,1] and they are obtained from some questionnaires of some experts. The experts may impose their opinion in three components viz. the degree of goodness, the degree of indeterminacy and that of poorness to explain the characteristics of the objects. Suppose A is an intuitionistic neutrosophic set (IN S) of U, such that,

$$A = \{< x_1, 0.3, 0.5, 0.4 >, < x_2,, 0.4, 0.2, 0.6 >, < x_3, 0.7, 0.3, 0.5 >\},$$

where the degree of goodness of capability is 0.3, degree of indeterminacy of capability is 0.5 and degree of falsity of capability is 0.4 etc.

2.6. Intuitionistic Neutrosophic Soft Sets (see [28]).

2.6.1. Definition

Let U be an initial universe set and $A \subset E$ be a set of parameters. Let N(U) denotes the set of all intuitionistic neutrosophic sets of U. The collection (P,A) is termed to be the soft intuitionistic neutrosophic set over U, where F is a mapping given by $P : A \to N(U)$.

2.6.2. Example

Let U be the set of blouses under consideration and E is the set of parameters (or qualities). Each parameter is a intuitionistic neutrosophic word or sentence involving intuitionistic neutrosophic words. Consider E = { Bright, Cheap, Costly, very costly, Colorful, Cotton, Polystyrene, long sleeve , expensive }. In this case, to define a intuitionistic neutrosophic soft set means to point out Bright blouses, Cheap blouses, Blouses in Cotton and so on. Suppose that, there are five blouses in the universe U given by, $U = \{b_1, b_2, b_3, b_4, b_5\}$ and the set of parameters $A = \{e_1,e_2,e_3,e_4\}$, where each e_i is a specific criterion for blouses:
e_1 stands for 'Bright',
e_2 stands for 'Cheap',
e_3 stands for 'Costly',
e_4 stands for 'Colorful',
Suppose that,
$P(\text{Bright})=\{< b_1,0.5,0.6,0.3>,<b_2,0.4,0.7,0.2>,<b_3,0.6,0.2,0.3>,<b_4,0.7,0.3,0.2> ,< b_5,0.8,0.2,0.3>\}$.
$P(\text{Cheap})=\{< b_1,0.6,0.3,0.5>,<b_2,0.7,0.4,0.3>,<b_3,0.8,0.1,0.2>,<b_4,0.7,0.1,0.3> ,< b_5,0.8,0.3,0.4\}$.
$P(\text{Costly})=\{< b_1,0.7,0.4,0.3>,<b_2,0.6,0.1,0.2>,<b_3,0.7,0.2,0.5>,< b_4,0.5,0.2,0.6 >,< b_5,0.7,0.3,0.2 >\}$.
$P(\text{Colorful})=\{< b_1,0.8,0.1,0.4>,<b_2,0.4,0.2,0.6>,<b_3,0.3,0.6,0.4>,<b_4,0.4,0.8,0.5> ,< b_5,0.3,0.5,0.7 >\}$.

2.6.3.Definition([28]). Containment of two intuitionistic neutrosophic soft sets

For two intuitionistic neutrosophic soft sets (P, A) and (Q, B) over the common universe U. We say that (P, A) is an intuitionistic neutrosophic soft subset of (Q, B) if and only if
(i) $A \subset B$.
(ii)P(e) is an intuitionistic neutrosophic subset of Q(e).
Or $T_{P(e)}(x) \leq T_{Q(e)}(m)$, $I_{P(e)}(m) \geq I_{Q(e)}(m)$, $F_{P(e)}(m) \geq F_{Q(e)}(m)$, $\forall e \in A$, $x \in U$.
We denote this relationship by (P, A) ⊆ (Q, B).
(P, A) is said to be intuitionistic neutrosophic soft super set of (Q, B) if (Q, B) is an intuitionistic neutrosophic soft subset of (P, A). We denote it by (P, A) ⊇ (Q, B).

2.6.4 .Definition [28]. Equality of two intuitionistic neutrosophic soft sets

Two INSSs (P, A) and (Q, B) over the common universe U are said to be intuitionistic neutrosophic soft equal if (P, A) is an intuitionistic neutrosophic soft subset of (Q, B) and (Q, B) is an intuitionistic neutrosophic soft subset of (P, A) which can be denoted by (P, A)= (Q, B).

2.6.5. Definition [28]. Complement of an intuitionistic neutrosophic soft set

The complement of an intuitionistic neutrosophic soft set (P, A) is denoted by $(P,A)^c$ and is defined by $(P,A)^c= (P^c,\rceil A)$, where $P^c :\rceil A \to N(U)$ is a mapping given by $P^c(\alpha)$ = intutionistic neutrosophic soft complement with $T_{P^c(x)} = F_{P(x)}$, $I_{P^c(x)} = I_{P(x)}$ and $F_{P^c(x)} = T_{P(x)}$.

2.6.6. Definition [28] Union of two intuitionistic neutrosophic soft sets

Let (P, A) and (Q, B) be two INSSs over the same universe U.Then the union of (P, A) and (Q, B) is denoted by '(P, A)∪(Q, B)' and is defined by (P,A)∪(Q, B) =(K, C), where C=A∪B and the truth-membership, indeterminacy-membership and falsity-membership of (K,C) are as follows:

$$T_{K(e)}(m) = \begin{cases} T_{P(e)}(m) \ , \ if \ e \in A - B \\ T_{Q(e)}(m) \ , if \ e \in B - A \\ max\{T_{P(e)}(m), T_{Q(e)}(m)\}, if \ e \in A \cap B \end{cases}$$

$$I_{K(e)}(\mathrm{m}) = \begin{cases} I_{P(e)}(\mathrm{m}) \ , \ if \ e \in \ \mathrm{A-B} \\ I_{Q(e)}(\mathrm{m}) \ , if \ e \in \ \mathrm{B-A} \\ min \ \{I_{P(e)}(\mathrm{m}), I_{Q(e)}(\mathrm{m})\}, if \ \ e \in A \ \cap B \end{cases}$$

$$F_{K(e)}(\mathrm{m}) = \begin{cases} F_{P(e)}(\mathrm{m}) \ , \ if \ e \in \ \mathrm{A-B} \\ F_{Q(e)}(\mathrm{m}) \ , if \ e \in \ \mathrm{B-A} \\ min\{F_{P(e)}(\mathrm{m}) \ , F_{Q(e)}(\mathrm{m}) \ \}, if \ \ e \in A \ \cap B \end{cases} \quad (6)$$

2.6.7. Definition. Intersection of two intuitionistic neutrosophic soft sets [28]

Let (P,A) and (Q,B) be two INSSs over the same universe U such that A ∩ B≠0. Then the intersection of (P,A) and (Q, B) is denoted by '(P,A) ∩ (Q, B)' and is defined by (P, A) ∩(Q, B) = (K,C),where C =A∩B and the truth-membership, indeterminacy membership and falsity-membership of (K, C) are related to those of (P,A) and (Q,B) by:

$$T_{K(e)}(\mathrm{m}) = \begin{cases} T_{P(e)}(\mathrm{m}) \ , \ if \ e \in \ \mathrm{A-B} \\ T_{Q(e)}(\mathrm{m}) \ , if \ e \in \ \mathrm{B-A} \\ min\{T_{P(e)}(\mathrm{m}), T_{Q(e)}(\mathrm{m})\}, if \ \ e \in A \ \cap B \end{cases}$$

$$I_{K(e)}(\mathrm{m}) = \begin{cases} I_{P(e)}(\mathrm{m}) \ , \ if \ e \in \ \mathrm{A-B} \\ I_{Q(e)}(\mathrm{m}) \ , if \ e \in \ \mathrm{B-A} \\ min \ \{I_{P(e)}(\mathrm{m}) \ , I_{Q(e)}(\mathrm{m})\}, if \ \ e \in A \ \cap B \end{cases}$$

$$F_{K(e)}(\mathrm{m}) = \begin{cases} F_{P(e)}(\mathrm{m}) \ , \ if \ e \in \ \mathrm{A-B} \\ F_{Q(e)}(\mathrm{m}) \ , if \ e \in \ \mathrm{B-A} \\ max\{F_{P(e)}(\mathrm{m}), F_{Q(e)}(\mathrm{m})\}, if \ \ e \in A \ \cap B \end{cases} \quad (7)$$

In this paper we are concerned with intuitionistic neutrosophic sets whose T_A, I_A and F_A values are single points in [0, 1] instead of subintervals/subsets in [0, 1]

3. The Necessity Operation on Intuitionistic Neutrosophic Soft Set

In this section,we shall introduce the necessity operation on intuitionistic neutrosophic soft set

3.1. Remark

$$s_A = T_A + I_A + F_A, \ s_B = T_B + I_B + F_B \quad .\text{if } \ s_A = s_B \ \text{ we put S} = s_A = s_B$$

3.2. Definition

The necessity operation on an intuitionistic neutrosophic soft set (P, A) is denoted by (P, A) and is defined as

$$\boxdot \ (\mathrm{P, A}) = \{<\mathrm{m}, \ \mathrm{T}_{P(e)}(m), \ \mathrm{I}_{P(e)}(m), \ s_A - \mathrm{T}_{P(e)}(m)> |\mathrm{m} \in \mathrm{U} \text{ and } \mathrm{e} \in \mathrm{A}\},$$

where s_A=T+I+F.

Here $T_{P(e)}(m)$ is the neutrosophic membership degree that object m hold on parameter e , $I_{P(e)}(m)$ represent the indeterminacy function and P is a mapping P : A → N(U), N(U) is the set of aintuitionistic neutrosophic sets of U.

3.3. Example

Let there are five objects as the universal set where U = { m_1, m_2, m_3, m_4, m_5 }and the set of parameters as E = { beautiful, moderate, wooden, muddy, cheap, costly }and

Let A = {beautiful, moderate, wooden}. Let the attractiveness of the objects represented by the intuitionistic neutrosophic soft sets (P, A) is given as

P(beautiful)={ $m_{1/(.6,.2,.4)}$, $m_{2/(.7, .3, .2)}$, $m_{3/(.5, .4, .4)}$, $m_{4/(.6, .4, .3)}$, $m_{5/(.8, .4, .1)}$},

P(moderate)={$m_{1/(.7, .3, .2)}$, $m_{2/(.8,.1, .1)}$, $m_{3/(.7, .5, .2)}$, $m_{4/(.8, .5, .1)}$, $m_{5/(1, .2, 0)}$}

and P(wooden) ={ $m_{1/(.8, .5, .1)}$, $m_{2/(.6, .4,0)}$, $m_{3/(.6, .5, .2)}$, $m_{4/(.2, .3, .4)}$, $m_{5/(.3, .2, .5)}$}.

Then the intuitionistic neutrosophicsoft sets (P,A) becomes as

$$P(\text{beautiful}) =\{ m_{1/(.6,.2,.6)}, m_{2/(.7, .3, .5)}, m_{3/(.5, .4, .8)}, m_{4/(.6, .4, .7)}, m_{5/(.8, .4, .5)}\},$$

$$P(\text{moderate}) =\{ m_{1/(.7, .3, .5)}, m_{2/(.8,.1, .2)}, m_{3/(.7, .5, .7)}, m_{4/(.8, .5, .6)}, m_{5/(1, .2,.2}\}$$

And

$$P(\text{wooden}) =\{ m_{1/(.8, .5, .6)}, m_{2/(.6, .4,.4)}, m_{3/(.6, .5, .7)}, m_{4/(.2, .3, .7)}, m_{5/(.3, .2, .7)}\}.$$

Let (P, A) and (Q, B) be two intuitionistic neutrosophic soft sets over a universe U and A, B be two sets of parameters. Then we have the following propositions:

3.4. Proposition

i. $\boxdot\ [(P, A) \cup (Q, B)] = \boxdot\ (P, A) \cup \boxdot\ (Q, B)$. (8)

ii. $\boxdot\ [(P, A) \cap (Q, B)] = \boxdot\ (P, A) \cap \boxdot\ (G, B)$. (9)

iii. $\boxdot\ \boxdot\ (P, A) = \boxdot\ (P, A)$. (10)

iv. $\boxdot\ [(P, A)]^n = [\boxdot\ (P, A)]^n$ (11)

for any finite positive integer n.

v. $\boxdot\ [(P,A) \cup (Q,B)]^n = [\boxdot\ (P,A) \cup \boxdot\ (Q,B)]^n$. (12)

vi. $\boxdot\ [(P,A) \cap (Q,B)]^n = [\boxdot\ (P,A) \cap \boxdot\ (Q,B)]^n$. (13)

Proof

i. [(P, A) ∪ (Q, B)]

suppose (P ,A) ∪ (Q , B) =(H, C) ,where C= A∪ B and for all e ∈ C and

$s_A = T_{P(e)}+I_{P(e)}+F_{P(e)}$ and $s_B = T_{Q(e)}+I_{Q(e)}+F_{Q(e)}$, $s_A - \mathrm{T}_{P(e)}(m) = \mathrm{I}_{P(e)}(m) + \mathrm{F}_{P(e)}(m)$, $s_B - \mathrm{T}_{Q(e)}(m) = \mathrm{I}_{Q(e)}(m) + \mathrm{F}_{Q(e)}(m)$,

$$T_{H(e)}(\mathrm{m}) = \begin{cases} \mathrm{T}_{P(e)}(m)\,, if\ e \in \mathrm{A-B} \\ \mathrm{T}_{Q(e)}(m)\,, if\ e \in \mathrm{B-A} \\ max\{T_{P(e)(m)}, T_{Q(e)(m)}\}, if\ e \in A \cap B \end{cases}$$

$$I_{H(e)}(\mathrm{m}) = \begin{cases} \mathrm{I}_{P(e)}(m)\,, if\ e \in \mathrm{A-B} \\ \mathrm{I}_{Q(e)}(m)\,, if\ e \in \mathrm{B-A} \\ min\{\mathrm{I}_{P(e)}(m), \mathrm{I}_{Q(e)}(m)\}, if\ e \in A \cap B \end{cases}$$

$$F_{H(e)}(\mathrm{m}) = \begin{cases} \mathrm{F}_{P(e)}(m), if\ e \in \mathrm{A-B} \\ \mathrm{F}_{Q(e)}(m)\,, if\ e \in \mathrm{B-A} \\ min\{\mathrm{F}_{P(e)}(m), \mathrm{F}_{Q(e)}(m)\}, if\ e \in A \cap B \end{cases}$$

Since [(P, A) ∪ (Q, B)] = (H, C) and m ∈ U, by definition 3.2 we Have

$$T_{H(e)}(\mathrm{m}) = \begin{cases} T_{P(e)}(\mathrm{m})\,, if\ e \in \mathrm{A-B} \\ T_{Q(e)}(\mathrm{m})\,, if\ e \in \mathrm{B-A} \\ max\{T_{P(e)}(\mathrm{m}), T_{Q(e)}(\mathrm{m})\}, if\ e \in A \cap B \end{cases}$$

$$I_{H(e)}(\mathrm{m}) = \begin{cases} I_{P(e)}\ (\mathrm{m}), if\ e \in \mathrm{A-B} \\ I_{Q(e)}\ (\mathrm{m}), if\ e \in \mathrm{B-A} \\ min\{I_{P(e)}(\mathrm{m}), I_{Q(e)}(\mathrm{m})\}, if\ e \in A \cap B \end{cases}$$

$$F_{H(e)}(\mathrm{m}) = \begin{cases} s_A - T_{P(e)}(\mathrm{m})\,, if\ e \in \mathrm{A-B}\,, \\ s_B - T_{Q(e)}(\mathrm{m})\,, if\ e \in \mathrm{B-A} \\ S - max\{T_{P(e)}(\mathrm{m}), T_{Q(e)}(\mathrm{m})\}, if\ e \in A \cap B \end{cases}$$

For all e ∈ C =A ∪ B and m ∈ U. Assume that $\boxdot$ (P, A)={<m, $T_{P(e)}(m)$,$I_{P(e)}(m)$,s_A-$T_{P(e)}(m)$>,m ∈ U} and $\boxdot$ (Q, A)={< m, $T_{O(e)}(\mathrm{m})$,$I_{O(e)}(\mathrm{m})$,s_B-$T_{O(e)}(\mathrm{m})$,m ∈ U} .Suppose that (P,A) ∪ (Q,B) =(O,C), where C= A ∪ B,and for all e ∈ C and m ∈ U.

$$T_{O(e)}(\mathrm{m}) = \begin{cases} T_{P(e)}(\mathrm{m}), if\ e \in \mathrm{A}-\mathrm{B} \\ T_{Q(e)}(\mathrm{m}), if\ e \in \mathrm{B}-\mathrm{A} \\ max\{T_{P(e)}(\mathrm{m}), T_{Q(e)}(\mathrm{m})\}, if\ e \in A \cap B \end{cases}$$

$$I_{O(e)}(\mathrm{m}) = \begin{cases} I_{P(e)}(\mathrm{m}), if\ e \in \mathrm{A}-\mathrm{B} \\ I_{Q(e)}(\mathrm{m}), if\ e \in \mathrm{B}-\mathrm{A} \\ min\{I_{P(e)}(\mathrm{m}), I_{Q(e)}(\mathrm{m})\}, if\ e \in A \cap B \end{cases}$$

$$F_{O(e)}(\mathrm{m}) = \begin{cases} s_A - T_{P(e)}(\mathrm{m}), if\ e \in \mathrm{A}-\mathrm{B} \\ s_B - T_{Q(e)}(\mathrm{m}), if\ e \in \mathrm{B}-\mathrm{A} \\ min\{s_A - T_{P(e)}(\mathrm{m}), s_A - T_{Q(e)}(\mathrm{m})\}, if\ e \in A \cap B \end{cases}$$

$$= \begin{cases} s_A - T_{P(e)}(\mathrm{m}), if\ e \in \mathrm{A}-\mathrm{B} \\ s_B - T_{Q(e)}(\mathrm{m}), if\ e \in \mathrm{B}-\mathrm{A} \\ S - max\{T_{P(e)}(\mathrm{m}), T_{Q(e)}(\mathrm{m})\}, \\ if\ e \in A \cap B\ with\ S = s_A = s_B \end{cases}$$

Consequently, (H,C) and (O, C) are the same intuitionistic neutrosophic soft sets.Thus ,

$$\boxdot\ ((\mathrm{P,A}) \cup (\mathrm{Q,B})) = \boxdot\ (\mathrm{P,A}) \cup \boxdot\ (\mathrm{Q,B}).$$

Hence the result is proved.
(ii) and (iii) are proved analogously.
iii. Let

$$(\mathrm{P, A}) = \{<\mathrm{m},\ \mathrm{T}_{\mathrm{P(e)}}(\mathrm{m}),\ \mathrm{I}_{\mathrm{P(e)}}(\mathrm{m}),\ \mathrm{F}_{\mathrm{P(e)}}(\mathrm{m}), >|\mathrm{m} \in \mathrm{U} \text{ and } \mathrm{e} \in \mathrm{A}\}.$$

Then

$$\boxdot\ (\mathrm{P, A}) = \{<\mathrm{m},\ \mathrm{T}_{\mathrm{P(e)}}(\mathrm{m}),\ \mathrm{I}_{\mathrm{P(e)}}(\mathrm{m}),\ \mathrm{s}_{\mathrm{A}} - \mathrm{T}_{\mathrm{P(e)}}(\mathrm{m}) >|\mathrm{m} \in \mathrm{U} \text{ and } \mathrm{e} \in \mathrm{A}\}.$$

So

$$\boxdot\ \boxdot\ (\mathrm{P, A}) = \{<\mathrm{m},\ \mathrm{T}_{\mathrm{P(e)}}(\mathrm{m}),\ \mathrm{I}_{P(e)}(\mathrm{m}),\ s_A\text{-}\ T_{P(e)}(\mathrm{m}) >|\mathrm{m} \in \mathrm{U} \text{ and } \mathrm{e} \in \mathrm{A}\}.$$

Hence the result follows.
iv. Let the intuitionistic neutrosophic soft set

$$(\mathrm{P, A}) = \{<\mathrm{m},\ T_{P(e)}(\mathrm{m}),\ \mathrm{I}_{P(e)}(\mathrm{m}),\ \mathrm{F}_{P(e)}(\mathrm{m})>|\mathrm{m} \in \mathrm{U} \text{ and } \mathrm{e} \in \mathrm{A}\}.$$

Then for any finite positive integer n

$$(\mathrm{P, A})^n = \{<\mathrm{m},\ [\mathrm{T}_{\mathrm{P(e)}}(\mathrm{m})]^n,\ [\mathrm{I}_{\mathrm{P(e)}}(\mathrm{m})]^n,\ s_A\text{-}[s_A - \mathrm{F}_{\mathrm{P(e)}}(\mathrm{m})]^n>|\mathrm{m} \in \mathrm{U} \text{ and } \mathrm{e} \in \mathrm{A}\}$$

So,

$$\boxdot\ (\mathrm{P, A})^n = \{<\mathrm{m},\ [\mathrm{T}_{\mathrm{P(e)}}(\mathrm{m})]^n,\ [\mathrm{I}_{\mathrm{P(e)}}(\mathrm{m})]^n, s_A\text{-}\ [\mathrm{T}_{\mathrm{P(e)}}(\mathrm{m})]^n>|\mathrm{m} \in \mathrm{U} \text{ and } \mathrm{e} \in \mathrm{A}\}.$$

Again, $[\boxdot\ (\mathrm{P, A})]^n = \{<\mathrm{m},\ [\mathrm{T}_{\mathrm{P(e)}}(\mathrm{m})]^n,\ [\mathrm{I}_{\mathrm{P(e)}}(\mathrm{m})]^n,\ s_A\text{-}\ [\mathrm{T}_{\mathrm{P(e)}}(\mathrm{m})]^n>|\mathrm{m} \in \mathrm{U} \text{ and } \mathrm{e} \in \mathrm{A}\}$ as

$$\boxdot\ (\mathrm{P, A}) = \{<\mathrm{m},\ T_{P(e)}(\mathrm{m}),\ \mathrm{I}_{P(e)}(\mathrm{m}),\ s_A - T_{P(e)}(\mathrm{m}) >|\mathrm{m} \in \mathrm{U} \text{ and } \mathrm{e} \in \mathrm{A}\}.$$

Hence the result.

v. As $(\mathrm{P,A})^n \cup (\mathrm{Q,B})^n = [(\mathrm{P,A}) \cup (\mathrm{Q,B})]^n$

$\boxdot\ [(\mathrm{P,A}) \cup (\mathrm{Q,B})]^n = [\boxdot\ [(\mathrm{P,A}) \cup (\mathrm{Q,B})]]^n$ by the proposition 3.4.iv
$= [\boxdot\ (\mathrm{P,A}) \cup \boxdot\ (\mathrm{Q,B})]^n$ by the proposition 3.4.i

vi. As $(\mathrm{P,A})^n \cap (\mathrm{Q,B})^n = [(\mathrm{P,A}) \cap (\mathrm{Q,B})]^n$

So, $\boxdot\ [(\mathrm{P,A}) \cap (\mathrm{Q,B})]^n = [\boxdot\ [(\mathrm{P,A}) \cap (\mathrm{Q,B})]]^n$ by the proposition3.4.iv
$= [\boxdot\ (\mathrm{P,A}) \cap \boxdot\ (\mathrm{Q,B})]^n$ by the proposition 3.4.ii
The result is proved.

The concept of necessity operation on intuitionistic neutrosophic soft set can also be applied to measure the necessity operation on intuitionistic fuzzy soft set (IFSS) ,proposed by P.K .Maji [30] ,where the indeterminacy degree $\mathrm{I}_{\mathrm{P(e)}}(\mathrm{m})$

should be replaced by $I_{P(e)}(m)$ = 1-$T_{P(e)}(m)$- $F_{P(e)}(m)$ in case of IFSS. In this case, we conclude that the necessity operation on intuitionistic neutrosophic soft set is a generalization of the necessity operation on intuitionistic fuzzy soft set

4. The Possibility Operation on Intuitionistic Neutrosophic Soft Sets

In this section, we shall define another operation, the possibility operation on intuitionistic neutrosophic soft sets.

Let U be a universal set. E be a set of parameters and A be a subset of E. Let the intuitionistic neutrosophic soft set. (P, A) = {<m, $T_{P(e)}(m)$, $I_{P(e)}(m)$, $F_{P(e)}(m)$ >| m ∈ U and e ∈A}, where $T_{P(e)}(m)$, $I_{P(e)}(m)$, $F_{P(e)}(m)$be the membership, indeterminacyand non-membership functions respectively.

4.1. Definition

Let U be the universal set and E be the set of parameters. The possibility operation on the intuitionistic neutrosophic soft set (P, A) is denoted by ◊(P, A) and is defined as

$$\Diamond (P, A) = \{<m,\ s_A - F_{P(e)}(m),\quad I_{P(e)}(m),\quad F_{P(e)}(m)\ >|\ m \in U \text{ and } e \in A \},$$

where

$$s_A = T_{P(e)}(m) + I_{P(e)}(m) + F_{P(e)}(m) \text{ and } 0^- \le s_A \le 3^+$$

4.2. Example

Let there are five objects as the universal set where U = {m_1, m_2, m_3, m_4, m_5}. Also let the set of parameters as E = { beautiful, costly, cheap, moderate, wooden, muddy } and A = { costly, cheap, moderate}. The cost of the objects represented by the intuitionistic neutrosophic soft sets
(P, A) is given as

$$P(\text{costly})=\{ m_{1/(.7, .1, .2)}, m_{2/(.8, .3, 0)}, m_{3/(.8, .2, .1)}, m_{4/(.9, .4, 0)}, m_{5/(.6, .2, .2)}\},$$

$$P(\text{cheap})=\{ m_{1/(.5, .3, .2)}, m_{2/(.7, .5, .1)}, m_{3/(.4, .3, .2)}, m_{4/(.8, .5, .1)}, m_{5/(.4, .4, .2)}\}$$

and

$$P(\text{moderate}) =\{ m_{1/(.8, .4, .2)}, m_{2/(.6, .1, .3)}, m_{3/(.5, .5, .1)}, m_{4/(.9, .4, 0)}, m_{5/(.7, .3,.1)}\}.$$

Then the neutrosophic soft set ◊(P, A) is as

$$P(\text{costly}) =\{ m_{1/(.8, .1, .2)}, m_{2/(1.1, .3, 0)}, m_{3/(1, .2, .1)}, m_{4/(1.3, .4, 0)}, m_{5/(.8, .2, .2)}\},$$

$$P(\text{cheap}) =\{ m_{1/(.8, .3, .2)}, m_{2/(1.2, .5, .1)}, m_{3/(.7, .3, .2)}, m_{4/(1.3, .5, .1)}, m_{5/(.8, .4, .2}\}$$

and

$$P(\text{moderate}) =\{ m_{1/(1.2, .4, .2)}, m_{2/(.7, .1, .3)}, m_{3/(1, .5, .1)}, m_{4/(1.3, .4, 0)}, m_{5/(1, .3,.1)}\}.$$

The concept of possibilty operation on intuitionistic neutrosophic soft set can also be applied to measure the necessity operation on intuitionistic fuzzy soft set (IFSS) ,proposed by P.K .Maji [30] ,where the indeterminacy degree $I_{P(e)}(m)$

should be replaced by $I_{P(e)}(m)$ = 1-$T_{P(e)}(m)$- $F_{P(e)}(m)$ in case of IFSS. In this case, we conclude that the possibility operation on intuitionistic neutrosophic soft set is a generalization of the possibility operation on intuitionistic fuzzy soft set.

Let (P, A) and (Q, B) be two intuitionistic neutrosophic soft sets over the same universe U and A, B be two sets of parameters. Then we have the propositions

4.3. Proposition

$$\text{i.}\Diamond\ [(P, A) \cup (Q, B)] = \Diamond (P, A) \cup \Diamond (Q, B). \quad (14)$$

$$\text{ii.}\Diamond\ [(P, A) \cap (Q, B)] = \Diamond (P, A) \cap \Diamond (Q, B) \quad (15)$$

$$\text{iii. } \Diamond\Diamond (P, A) = \Diamond (P, A). \quad (16)$$

$$\text{iv. } \Diamond [(P,A)]^n = [\Diamond (P,A)]^n \quad (17)$$

for any finite positive integer n.

v.◊ $[(\mathrm{P,A}) \cup (\mathrm{Q,B})]^n = [◊ (\mathrm{P,A}) \cup ◊ (\mathrm{Q,B})]^n$. (18)

vi.◊ $[(\mathrm{P,A}) \cap (\mathrm{Q,B})]^n = [◊ (\mathrm{P,A}) \cap ◊ (\mathrm{Q,B})]^n$ (19)

Proof

i. ◊ [(P, A) ∪ (Q, B)]

suppose (P ,A) ∪ (Q , B) =(H, C) ,where C= A∪ B and for all e ∈ C and

$$s_A = \mathrm{T}_{P(e)}(m) + \mathrm{I}_{P(e)}(m) + \mathrm{F}_{P(e)}(m) \quad \text{and } s_B = \mathrm{T}_{Q(e)}(m) + \mathrm{I}_{Q(e)}(m) + \mathrm{F}_{Q(e)}(m)$$

$$s_A - F_{P(e)}(\mathrm{m}) = I_{P(e)}(\mathrm{m}) + T_{P(e)}(\mathrm{m}) \;,$$

$$s_B - F_{Q(e)}(\mathrm{m}) = I_{P(e)}(\mathrm{m}) + T_{Q(e)}(\mathrm{m})$$

$$T_{H(e)}(\mathrm{m}) = \begin{cases} T_{P(e)}(\mathrm{m}) \;,\; if\; e \in \mathrm{A-B} \\ T_{Q(e)}(\mathrm{m}) \,, if\; e \in \mathrm{B-A} \\ max\{T_{P(e)}(\mathrm{m}) \;, T_{Q(e)}(\mathrm{m})\}, if\;\; e \in A \cap B \end{cases}$$

$$I_{H(e)}(\mathrm{m}) = \begin{cases} I_{P(e)}(\mathrm{m}) \;,\; if\; e \in \mathrm{A-B} \\ I_{Q(e)}(\mathrm{m}) \,, if\; e \in \mathrm{B-A} \\ min\,\{I_{P(e)}(\mathrm{m}), I_{Q(e)}(\mathrm{m})\}, if\;\; e \in A \cap B \end{cases}$$

$$F_{H(e)}(\mathrm{m}) = \begin{cases} F_{P(e)}(\mathrm{m}) \;,\; if\; e \in \mathrm{A-B} \\ F_{Q(e)}(\mathrm{m}) \,, if\; e \in \mathrm{B-A} \\ min\{F_{P(e)(m)}, F_{Q(e)(m)}\}, if\;\; e \in A \cap B \end{cases}$$

Since ◊ [(P, A) ∪ (Q, B)] =◊ (H, C) and m ∈ U, by definition 4.1 we Have

$$T_{H(e)}(\mathrm{m}) = \begin{cases} s_A - F_{P(e)}(\mathrm{m}) \quad, if\; e \in \mathrm{A-B} \\ s_B - F_{Q(e)}(\mathrm{m}) \,, if\; e \in \mathrm{B-A} \\ S - min\{F_{P(e)}(\mathrm{m}), F_{Q(e)}(\mathrm{m})\}, \\ if\;\; e \in A \cap B,\; with\; S = s_A = s_B \end{cases}$$

$$I_{H(e)}(\mathrm{m}) = \begin{cases} I_{P(e)}(\mathrm{m}) \;,\; if\; e \in \mathrm{A-B} \\ I_{Q(e)}(\mathrm{m}) \,, if\; e \in \mathrm{B-A} \\ min\{I_{P(e)}(\mathrm{m}), I_{Q(e)}(\mathrm{m})\}, if\;\; e \in A \cap B \end{cases}$$

$$F_{H(e)}(\mathrm{m}) = \begin{cases} F_{P(e)}(\mathrm{m}) \;,\; if\; e \in \mathrm{A-B} \\ F_{Q(e)}(\mathrm{m}) \,, if\; e \in \mathrm{B-A} \\ min\,\{F_{P(e)}(\mathrm{m}), F_{Q(e)}(\mathrm{m})\}, if\;\; e \in A \cap B \end{cases}$$

For all e ∈ C =A ∪ B and m ∈ U. Assume that

$$◊\,(\mathrm{P, A}) = \{< \mathrm{m},\; s_A\text{-}F_{P(e)}(\mathrm{m}),\; I_{P(e)}(\mathrm{m}) \;,\; F_{P(e)}(\mathrm{m}) >, \mathrm{m} \in \mathrm{U}\}$$

and

$$◊\,(\mathrm{Q, B}) = \{< m,\; s_B\text{-}F_{Q(e)}(\mathrm{m}) \,, I_{Q(e)}(\mathrm{m}) \;,\; F_{Q(e)}(\mathrm{m}) >, \mathrm{m} \in \mathrm{U}\}\,.$$

Suppose that

$$◊\,(\mathrm{P, A}) \cup ◊\,(\mathrm{Q, B}) = (\mathrm{O, C})\,,$$

where C= A ∪ B, and for all e ∈ C and m ∈ U.

$$T_{O(e)}(\mathrm{m}) = \begin{cases} s_A - F_{P(e)}(\mathrm{m}), \;, if\; e \in \mathrm{A-B} \\ s_B - F_{Q(e)}(\mathrm{m}), \;, if\; e \in \mathrm{B-A} \\ max\{s_A - F_{P(e)}(\mathrm{m}), s_B - F_{Q(e)}(\mathrm{m})\}, if\;\; e \in A \cap B \end{cases}$$

$$= \begin{cases} s_A - F_{P(e)}(\mathrm{m}) \;,\; if\; e \in \mathrm{A-B} \\ s_B - F_{Q(e)}(\mathrm{m}) \,, if\; e \in \mathrm{B-A} \\ S - min\{F_{P(e)}(\mathrm{m}), F_{Q(e)}(\mathrm{m})\}, if\;\; e \in A \cap B, \\ with\; S = s_A = s_B \end{cases}$$

$$I_{O(e)}(\mathrm{m}) = \begin{cases} I_{P(e)}(\mathrm{m}) \;,\; if\; e \in \mathrm{A-B} \\ I_{Q(e)}(\mathrm{m}) \,, if\; e \in \mathrm{B-A} \\ min\,\{I_{P(e)}(\mathrm{m}) \,, I_{Q(e)}(\mathrm{m})\}, if\;\; e \in A \cap B \end{cases}$$

$$F_{O(e)}(\mathrm{m}) = \begin{cases} F_{P(e)}(\mathrm{m}) \;,\; if\; e \in \mathrm{A-B} \\ F_{Q(e)}(\mathrm{m}) \,, if\; e \in \mathrm{B-A} \\ min\{F_{P(e)}(\mathrm{m}), F_{Q(e)}(\mathrm{m})\}, if\;\; e \in A \cap B \end{cases}$$

Consequently, ◊ (H,C) and (O, C) are the same intuitionistic neutrosophic soft sets.Thus ,

◊ ((P, A) ∪ (Q, B))=◊ (P, A) ∪ ◊ (Q, B).

Hence the result is proved.
(ii) and (iii) are proved analogously.

iii. ◊(P, A) = {<m, s_A- $\mathrm{F}_{P(e)}(m)$, $\mathrm{I}_{P(e)}(m)$], $\mathrm{F}_{P(e)}(m)$]>|m ∈U and e∈A}.

So

◊◊(P, A) = {<m, s_A–$\mathrm{F}_{P(e)}(m)$, $\mathrm{I}_{P(e)}(m)$, $\mathrm{F}_{P(e)}(m)$] >| m ∈ U and e ∈A}.

Hence the result.
iv. For any positive finite integer n,

$(\mathrm{P,A})^n$ ={<m, $[\mathrm{T}_{P(e)}(m)]^n$, $[\mathrm{I}_{P(e)}(m)]^n$, s_A-$[s_A - \mathrm{F}_{P(e)}(m)]]^n$>|m∈U } ∀e∈A,

So,

◊$(\mathrm{P,A})^n$= {<m, s_A- $[s_A$-$[s_A - \mathrm{F}_{P(e)}(m)]^n]$, $[\mathrm{I}_{P(e)}(m)]^n$, s_A-$[s_A - \mathrm{F}_{P(e)}(m)]^n$>|m ∈U }
= {<m, $[s_A - \mathrm{F}_{P(e)}(m)]^n$, $[\mathrm{I}_{P(e)}(m)]^n$, s_A-$[s_A - \mathrm{F}_{P(e)}(m)]^n$>|m∈U } ∀ e ∈A.

Again

$[$◊ $(\mathrm{P,A})]^n$ = {<m, $[s_A - \mathrm{F}_{P(e)}(m)]^n$, $[\mathrm{I}_{P(e)}(m)]^n$, s_A-$[s_A - \mathrm{F}_{P(e)}(m)]^n$ >| m∈U } ∀ e ∈A.

Hence the result follows.

v. As $[(\mathrm{P,A}) \cup (\mathrm{Q,B})]^n = (\mathrm{P,A})^n \cup (\mathrm{Q,B})^n$,

◊$[(\mathrm{P,A}) \cup (\mathrm{Q,B})]^n$= = ◊$(\mathrm{P,A})^n$ ∪ ◊ $(\mathrm{Q,B})^n$.

the result is proved

vi.As $[(\mathrm{P,A}) \cap (\mathrm{Q,B})]^n = (\mathrm{P,A})^n \cap (\mathrm{Q,\ B})^n$,

◊$[(\mathrm{P,A}) \cap (\mathrm{Q,B})]^n$= ◊ $(\mathrm{P,\ A})^n$∩◊ $(\mathrm{Q,B})^n$.

Hence the result follows.
For any intuitionistic neutrosophic soft set (P, A) we have the following propositions.

4.4. Proposition

i. ◊ ⊡ (P, A) = ⊡ (P, A) (20)

ii. ⊡ ◊ (P, A) = ◊ (P, A) (21)

Proof
i.Let (P, A) be a intuitionistic neutrosophic soft set over the universe U.
Then (P, A) = { <m, $\mathrm{T}_{P(e)}(m)$, $\mathrm{I}_{P(e)}(m)$, $\mathrm{F}_{P(e)}(m)$> |m ∈ U} where e ∈ A.
So, ⊡ (P, A) = { <m, $\mathrm{T}_{P(e)}(m)$, $\mathrm{I}_{P(e)}(m)$, s_A- $\mathrm{T}_{P(e)}(m)$> | m ∈ U}, and
◊ (P, A) = { <m, s_A- $\mathrm{F}_{P(e)}(m)$, $\mathrm{I}_{P(e)}(m)$, $\mathrm{F}_{P(e)}(m)$>| m ∈ U}.
So ◊⊡ (P, A) = { <m, s_A- (s_A- $\mathrm{T}_{P(e)}(m)$), $\mathrm{I}_{P(e)}(m)$, s_A- $\mathrm{T}_{P(e)}(m)$>| m ∈ U}.
= { <m, $\mathrm{T}_{P(e)}(m)$, $\mathrm{I}_{P(e)}(m)$, s_A- $\mathrm{T}_{P(e)}(m)$> | m ∈ U}.
= ⊡ (P, A)
ii.The proof is similar to the proof of the proposition 3.4.i.

Let (P, A) and (Q, B) be two intuitionistic neutrosophic soft sets over the common universe U, then we have the following propositions:

4.5. Proposition

$$\text{i. } \boxdot\ [(P, A) \wedge (Q, B)] = \boxdot\ (P, A) \wedge \boxdot\ (Q, B). \tag{22}$$

$$\text{ii. } \boxdot\ [(P, A) \vee (Q, B)] = \boxdot\ (P, A) \vee \boxdot\ (Q, B). \tag{23}$$

$$\text{iii.} \Diamond\ [(P, A) \wedge (Q, B)] = \Diamond\ (P, A) \wedge \Diamond\ (Q, B). \tag{24}$$

$$\text{iv.} \Diamond\ [(P, A) \vee (Q, B)] = \Diamond\ (P, A) \vee \Diamond\ (Q, B). \tag{25}$$

Proof

i. Let (H, A $\times$B) = (P, A) $\wedge$ (Q, B).

Hence,

$$(H, A \times B) = \{<m, T_{H(\alpha,\beta)}(m), I_{H(\alpha,\beta)}(m), F_{H(\alpha,\beta)}(m)(m)>|m \in U \},$$

where

$$T_{H(\alpha,\beta)}(m) = \min \{ T_{P(\alpha)}(m), T_{Q(\beta)}(m)\} ,\ F_{H(\alpha,\beta)}(m) = \max \{F_{P(\alpha)}(m), F_{Q(\beta)}(m) \}$$

and

$$I_{H(\alpha,\beta)}(m) = \max \{I_{P(\alpha)}(m), I_{Q(\beta)}(m) \}.$$

So,

$$\boxdot\ (H, A \times B) = \{ <m, T_{H(\alpha,\beta)}(m), I_{H(\alpha,\beta)}(m), S - T_{H(\alpha,\beta)}(m)>|m \in U \}, (\alpha, \beta) \in A \times B$$

$= \{ < m, \min (T_{P(\alpha)}(m), T_{Q(\beta)}(m)), \max (I_{P(\alpha)}(m), I_{Q(\beta)}(m)), S - \min (T_{P(\alpha)}(m), T_{Q(\beta)}(m)) > |m \in U \}$

$= \{ < m, \min (T_{P(\alpha)}(m), T_{Q(\beta)}(m)), \max (I_{P(\alpha)}(m), I_{Q(\beta)}(m)), \max (S - T_{P(\alpha)}(m), S- T_{Q(\beta)}(m)) > |m \in U \}$

$= \{ < m, T_{P(\alpha)}(m), I_{P(\alpha)}(m), S- T_{P(\alpha)}(m)>|m \in U\}$ AND $\{<m, T_{Q(\beta)}(m), I_{Q(\beta)}(m), S- T_{Q(\beta)}(m)>|m \in U\}$

$= \boxdot\ (P, A) \wedge \boxdot\ (Q, B)$.

Hence the result is proved

ii. Let (L, A $\times$ B) = (P, A) $\vee$ (Q, B).

Hence ,

$$(L, A \times B) = \{ <m, T_{L(\alpha,\beta)}(m), I_{L(\alpha,\beta)}(m), F_{L(\alpha,\beta)}(m)>|m \in U \},$$

where

$$T_{L(\alpha,\beta)}(m) = \max \{ T_{P(\alpha)}(m), T_{Q(\beta)}(m) \} , I_{L(\alpha,\beta)}(m) = \min \{I_{P(\alpha)}(m), I_{Q(\beta)}(m) \}$$

And $F_{L(\alpha,\beta)}(m) = \min\{ F_{P(\beta)}(m), F_{Q(\beta)}(m)\}$.

So,

$$\boxdot\ (L, A \times B) = \{ <m, T_{L(\alpha,\beta)}(m), I_{L(\alpha,\beta)}(m) , S - T_{L(\alpha,\beta)}(m)>|m \in U \}, \text{ for } (\alpha, \beta) \in A \times B$$

$= \{ < m, \max (T_{P(\alpha)}(m), T_{Q(\beta)}(m)), \min (I_{P(\alpha)}(m), I_{Q(\beta)}(m)), S - \max (T_{P(\alpha)}(m), T_{Q(\beta)}(m)) > |m \in U \}$

$= \{ < m, \max (T_{P(\alpha)}(m), T_{Q(\beta)}(m)), \min (I_{P(\alpha)}(m), I_{Q(\beta)}(m)), \min (S - T_{P(\alpha)}(m), S- T_{Q(\beta)}(m)) > |m \in U \}$

$= \{ < m, T_{P(\alpha)}(m), I_{P(\alpha)}(m), S- T_{P(\alpha)}(m)> |m \in U\}$ OR $\{<m, T_{Q(\beta)}(m), I_{Q(\beta)}(m), S- T_{Q(\beta)}(m)> |m \in U\}$

$= \boxdot\ (P, A) \vee \boxdot\ (Q, B)$.

Hence the result is proved

iii. Let (H, A $\times$ B) =(P, A) $\wedge$ (Q, B).

Hence,

$$(H, A \times B) = \{<m, T_{H(\alpha,\beta)}(m), I_{H(\alpha,\beta)}(m), F_{H(\alpha,\beta)}(m)>|m \in U \},$$

where

$$T_{H(\alpha,\beta)}(m) = \min \{T_{P(\alpha)}(m), T_{Q(\beta)}(m)\}, I_{H(\alpha,\beta)}(m) = \max \{I_{P(\alpha)}(m), I_{Q(\beta)}(m)\}.$$

and

$$\mathrm{F}_{H(\alpha,\beta)}(\mathrm{m}) = \max\{\mathrm{F}_{P(\alpha)}(\mathrm{m}), \mathrm{F}_{Q(\beta)}(\mathrm{m})\}.$$

So,

$$\Diamond(\mathrm{H}, \mathrm{A} \times \mathrm{B}) = \{ <\mathrm{m}, \mathrm{S} - \mathrm{F}_{H(\alpha,\beta)}(\mathrm{m}), \mathrm{I}_{H(\alpha,\beta)}(\mathrm{m}), \mathrm{F}_{H(\alpha,\beta)}(\mathrm{m})> | \mathrm{m} \in \mathrm{U} \}, \text{ for } (\alpha, \beta) \in \mathrm{A} \times \mathrm{B}$$

$$= \{ < \mathrm{m}, \mathrm{S} - \max(\mathrm{F}_{P(\alpha)}(\mathrm{m}), \mathrm{F}_{Q(\beta)}(\mathrm{m})), \max(\mathrm{I}_{P(\alpha)}(\mathrm{m}), \mathrm{I}_{Q(\beta)}(\mathrm{m})), \max(\mathrm{F}_{P(\alpha)}(\mathrm{m}), \mathrm{F}_{Q(\beta)}(\mathrm{m})) > | \mathrm{m} \in \mathrm{U} \}$$

$$= \{ < \mathrm{m}, \min(\mathrm{S}- \mathrm{F}_{P(\alpha)}(\mathrm{m}), \mathrm{S}- \mathrm{F}_{Q(\beta)}(\mathrm{m})), \max(\mathrm{I}_{P(\alpha)}(\mathrm{m}), \mathrm{I}_{Q(\beta)}(\mathrm{m})), \max(\mathrm{F}_{P(\alpha)}(\mathrm{m}), \mathrm{F}_{Q(\beta)}(\mathrm{m})) > | \mathrm{m} \in \mathrm{U} \}$$

$$= \{< \mathrm{m}, \mathrm{S}- \mathrm{F}_{P(\alpha)}(\mathrm{m}), \mathrm{I}_{P(\alpha)}(\mathrm{m}), \mathrm{F}_{P(\alpha)}(\mathrm{m})> | \mathrm{m} \in \mathrm{U}\} \text{ AND } \{<\mathrm{m}, \mathrm{S}- \mathrm{F}_{Q(\beta)}(\mathrm{m}), \mathrm{I}_{Q(\beta)}(\mathrm{m}), \mathrm{F}_{Q(\beta)}(\mathrm{m})> | \mathrm{m} \in \mathrm{U}\}$$

$= \Diamond(\mathrm{P}, \mathrm{A}) \wedge \Diamond(\mathrm{Q}, \mathrm{B})$. Hence the result is proved

iv. The proof is similar to the proof of the proposition 3.5.iii.

5. Conclusion

In the present work ,We have continued to study the properties of intuitionistic neutrosophic soft set. New operations such as necessity and possibility on the intuitionistic neutrosophic soft set are introduced. Some properties of these operations and their interconnection between each other are also presented and discussed. We conclude that necessity and possibility operations on the intuitionistic neutrosophic soft set are generalization of necessity and possibility operations on the intuitionistic fuzzy soft set. The new operations can be applied also on neutrosophic soft set [27] and generalized neutrosophic soft set [29]. We hope that the findings, in this paper will help researcher enhance the study on the intuitionistic neutrosophic soft set theory.

Acknowledgements

The authors would like to thank the anonymous reviewer for their careful reading of this article and for their helpful comments.

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Lower and Upper Soft Interval Valued Neutrosophic Rough Approximations of An IVNSS-Relation

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Abstract: In this paper, we extend the lower and upper soft interval valued intuitionstic fuzzy rough approximations of IVIFS –relations proposed by Anjan et al. to the case of interval valued neutrosophic soft set relation(IVNSS-relation for short)

Keywords: Interval valued neutrosophic soft , Interval valued neutrosophic soft set relation

0. Introduction

This paper is an attempt to extend the concept of interval valued intuitionistic fuzzy soft relation (IVIFSS-relations) introduced by A. Mukherjee et al [45]to IVNSS relation .

The organization of this paper is as follow: In section 2, we briefly present some basic definitions and preliminary results are given which will be used in the rest of the paper. In section 3, relation interval neutrosophic soft relation is presented. In section 4 various type of interval valued neutrosophic soft relations. In section 5, we concludes the paper

1. Preliminaries

Throughout this paper, let U be a universal set and E be the set of all possible parameters under consideration with respect to U, usually, parameters are attributes, characteristics, or properties of objects in U. We now recall some basic notions of neutrosophic set, interval neutrosophic set, soft set, neutrosophic soft set and interval neutrosophic soft set.

Definition 2.1.

Let U be an universe of discourse then the neutrosophic set A is an object having the form A= {< x: $\mu_{A(x)}$, $\nu_{A(x)}$, $\omega_{A(x)}$ >,x ∈ U}, where the functions $\mu, \nu, \omega : U \to]^-0,1^+[$ define respectively the degree of membership , the degree of indeterminacy, and the degree of non-membership of the element x ∈ X to the set A with the condition.

$$^-0 \leq \mu_{A(x)} + \nu_{A(x)} + \omega_{A(x)} \leq 3^+.$$

From philosophical point of view, the neutrosophic set takes the value from real standard or non-standard subsets of $]^{-}0,1^{+}[$.so instead of $]^{-}0,1^{+}[$ we need to take the interval [0,1] for technical applications, because $]^{-}0,1^{+}[$will be difficult to apply in the real applications such as in scientific and engineering problems.

Definition 2.2. A neutrosophic set A is contained in another neutrosophic set B i.e. $A \subseteq B$ if $\forall x \in U$, $\mu_A(x) \le \mu_B(x)$, $\nu_A(x) \ge \nu_B(x)$, $\omega_A(x) \ge \omega_B(x)$.

Definition 2.3. Let X be a space of points (objects) with generic elements in X denoted by x. An interval valued neutrosophic set (for short IVNS) A in X is characterized by truth-membership function $\boldsymbol{\mu_A(x)}$, indeteminacy-membership function $\boldsymbol{\nu_A(x)}$ and falsity-membership function $\boldsymbol{\omega_A(x)}$. For each point **x in X, we have that** $\boldsymbol{\mu_A(x), \nu_A(x), \omega_A(x) \in [0,1]}$ **.**
For two IVNS , $A_{\text{IVNS}} = \{ <x , [\mu_A^L(x), \mu_A^U(x)] , [\nu_A^L(x), \nu_A^U(x)] , [\omega_A^L(x), \omega_A^U(x)] > | x \in X \}$
And $B_{\text{IVNS}} = \{ <x , [\mu_B^L(x), \mu_B^U(x)] , [\nu_B^L(x), \nu_B^U(x)] , [\omega_B^L(x), \omega_B^U(x)]> | x \in X \}$ the two relations are defined as follows:
(1) $A_{\text{IVNS}} \subseteq B_{\text{IVNS}}$ if and only if $\mu_A^L(x) \le \mu_B^L(x)$,$\mu_A^U(x) \le \mu_B^U(x)$,$\nu_A^L(x) \ge \nu_B^L(x)$, $\omega_A^U(x) \ge \omega_B^U(x)$, $\omega_A^L(x) \ge \omega_B^L(x)$, $\omega_A^U(x) \ge \omega_B^U(x)$
(2) $A_{\text{IVNS}} = B_{\text{IVNS}}$ if and only if , $\mu_A(x) = \mu_B(x)$, $\nu_A(x) = \nu_B(x)$, $\omega_A(x) = \omega_B(x)$ for any $x \in X$
As an illustration ,let us consider the following example.
Example 2.4. Assume that the universe of discourse $U=\{x_1,x_2,x_3\}$,where x_1 characterizes the capability, x2 characterizes the trustworthiness and x3 indicates the prices of the objects. It may be further assumed that the values of x_1, x_2 and x_3 are in [0,1] and they are obtained from some questionnaires of some experts. The experts may impose their opinion in three components viz. the degree of goodness,
the degree of indeterminacy and that of poorness to explain the characteristics of the objects. Suppose A is an interval neutrosophic set (INS) of U, such that,
$A = \{< x_1,[0.3\ 0.4],[0.5\ 0.6],[0.4\ 0.5] >,< x_2, ,[0.1\ 0.2],[0.3\ 0.4],[0.6\ 0.7]>,< x_3, [0.2\ 0.4],[0.4\ 0.5],[0.4\ 0.6] >\}$, where the degree of goodness of capability is 0.3, degree of indeterminacy of capability is 0.5 and degree of falsity of capability is 0.4 etc.

Definition 2.5.
Let U be an initial universe set and E be a set of parameters. Let P(U) denotes the power set of U. Consider a nonempty set A, $A \subset E$. A pair (K, A) is called a soft set over U, where K is a mapping given by $K : A \to P(U)$.
As an illustration, let us consider the following example.
Example 2.6 .
Suppose that U is the set of houses under consideration, say $U = \{h_1, h_2, \ldots, h_5\}$. Let E be the set of some attributes of such houses, say $E = \{e_1, e_2, \ldots, e_8\}$, where $e_1, e_2, \ldots, e_8$ stand for the attributes "beautiful", "costly", "in the green surroundings'", "moderate", respectively.
In this case, to define a soft set means to point out expensive houses, beautiful houses, and so on. For example, the soft set (K,A) that describes the "attractiveness of the houses" in the opinion of a buyer, say Thomas, may be defined like this:
$A=\{e_1,e_2,e_3,e_4,e_5\}$;
$K(e_1) = \{h_2, h_3, h_5\}$, $K(e_2) = \{h_2, h_4\}$, $K(e_3) = \{h_1\}$, $K(e_4) = U$, $K(e_5) = \{h_3, h_5\}$.

Definition 2.7 .
Let U be an initial universe set and $A \subset E$ be a set of parameters. Let IVNS(U) denotes the

set of all interval neutrosophic subsets of U. The collection (K,A) is termed to be the soft interval neutrosophic set over U, where F is a mapping given by K : A → IVNS(U).
The interval neutrosophic soft set defined over an universe is denoted by INSS.
To illustrate let us consider the following example:
Let U be the set of houses under consideration and E is the set of parameters (or qualities). Each parameter is a interval neutrosophic word or sentence involving interval neutrosophic words. Consider E = { beautiful, costly, in the green surroundings, moderate, expensive }. In this case, to define a interval neutrosophic soft set means to point out beautiful houses, costly houses, and so on. Suppose that, there are five houses in the universe U given by, U = $\{h_1,h_2,h_3,h_4,h_5\}$ and the set of parameters A = $\{e_1,e_2,e_3,e_4\}$, where each e_i is a specific criterion for houses:
e_1 stands for 'beautiful',
e_2 stands for 'costly',
e_3 stands for 'in the green surroundings',
e_4 stands for 'moderate',
Suppose that,

K(beautiful)={< h_1,[0.5, 0.6], [0.6, 0.7], [0.3, 0.4]>,< h_2,[0.4, 0.5], [0.7 ,0.8], [0.2, 0.3] >, < h_3,[0.6, 0.7],[0.2 ,0.3],[0.3, 0.5] >,< h_4,[0.7 ,0.8],[0.3, 0.4],[0.2, 0.4] >,< h_5,[0.8, 0.4] ,[0.2 ,0.6],[0.3, 0.4] >}.K(costly)={< b_1,[0.5, 0.6], [0.6, 0.7], [0.3, 0.4]>,< h_2,[0.4, 0.5], [0.7 ,0.8], [0.2, 0.3] >, < h_3,[0.6, 0.7],[0.2 ,0.3],[0.3, 0.5] >,< h_4,[0.7 ,0.8],[0.3, 0.4],[0.2, 0.4] >,< h_5,[0.8, 0.4] ,[0.2 ,0.6],[0.3, 0.4] >}.
K(in the green surroundings)= {< h_1,[0.5, 0.6], [0.6, 0.7], [0.3, 0.4]>,< b_2,[0.4, 0.5], [0.7 ,0.8], [0.2, 0.3] >, < h_3,[0.6, 0.7],[0.2 ,0.3],[0.3, 0.5] >,< h_4,[0.7 ,0.8],[0.3, 0.4],[0.2, 0.4] >,< h_5,[0.8, 0.4] ,[0.2 ,0.6],[0.3, 0.4] >}.K(moderate)={< h_1,[0.5, 0.6], [0.6, 0.7], [0.3, 0.4]>,< h_2,[0.4, 0.5], [0.7 ,0.8], [0.2, 0.3] >, < h_3,[0.6, 0.7],[0.2 ,0.3],[0.3, 0.5] >,< h_4,[0.7 ,0.8],[0.3, 0.4],[0.2, 0.4] >,< h_5,[0.8, 0.4] ,[0.2 ,0.6],[0.3, 0.4] >}.

Definition 2.8.
Let U be an initial universe and (F,A) and (G,B) be two interval valued neutrosophic soft set . Then a relation between them is defined as a pair (H, AxB), where H is mapping given by H: AxB→IVNS(U). This is called an interval valued neutrosophic soft sets relation (IVNSS-relation for short).the collection of relations on interval valued neutrosophic soft sets on Ax Bover U is denoted by $\sigma_U(A\text{x } B)$.

Defintion 2.9. Let P, Q ∈ $\sigma_U(Ax\ B)$ and the ordre of their relational matrices are same. Then P ⊆ Q if H $(e_{j,}e_j)$ ⊆ J $(e_{j,}e_j)$ for $(e_{j,}e_j)$ ∈ A x B where P=(H, A x B) and Q = (J, A x B)
Example:
P

U	(e_1 ,e_2)	(e_1 ,e_4)	(e_3 ,e_2)	(e_3 ,e_4)
h_1	([0.2, 0.3],[0.2, 0.3],[0.4, 0.5])	([0.4, 0.6],[0.7, 0.8],[0.1,0.4])	([0.4, 0.6],[0.7, 0.8],[0.1,0.4])	([0.4, 0.6],[0.7, 0.8],[0.1,0.4])
h_2	([0.6, 0.8],[0.3, 0.4],[0.1,0.7])	([1, 1],[0, 0],[0, 0])	([0.1, 0.5],[0.4, 0.7],[0.5,0.6])	([0.1, 0.5],[0.4, 0.7],[0.5,0.6])
h_3	([0.3, 0.6],[0.2, 0.7],[0.3,0.4])	([0.4, 0.7],[0.1, 0.3],[0.2,0.4])	([1, 1],[0, 0],[0, 0])	([0.4, 0.7],[0.1, 0.3],[0.2,0.4])
h_4	([0.6, 0.7],[0.3, 0.4],[0.2,0.4])	([0.3, 0.4],[0.7, 0.9],[0.1,0.2])	([0.3, 0.4],[0.7, 0.9],[0.1,0.2])	([1, 1],[0, 0],[0, 0])

Q

U	(e_1 ,e_2)	(e_1 ,e_4)	(e_3 ,e_2)	(e_3 ,e_4)
h_1	([0.3, 0.4],[0, 0],[0, 0])	([0.4, 0.6],[0.7, 0.8],[0.1,0.4])	([0.4, 0.6],[0.7, 0.8],[0.1,0.4])	([0.4, 0.6],[0.7, 0.8],[0.1,0.4])
h_2	([0.6, 0.8],[0.3, 0.4],[0.1,0.7])	([1, 1],[0, 0],[0, 0])	([0.1, 0.5],[0.4, 0.7],[0.5,0.6])	([0.1, 0.5],[0.4, 0.7],[0.5,0.6])

h_3	([0.3, 0.6],[0.2, 0.7],[0.3,0.4])	([0.4, 0.7],[0.1, 0.3],[0.2,0.4])	([1, 1],[0, 0],[0, 0])	([0.4, 0.7],[0.1, 0.3],[0.2,0.4])
h_4	([0.6, 0.7],[0.3, 0.4],[0.2,0.4])	([0.3, 0.4],[0.7, 0.9],[0.1,0.2])	([0.3, 0.4],[0.7, 0.9],[0.1,0.2])	([1, 1],[0, 0],[0, 0])

Definition 2.10**.**
Let U be an initial universe and (F, A) and (G, B) be two interval valued neutrosophic soft sets. Then a null relation between them is denoted by 0_U and is defined as 0_U =(H_O , A xB) where H_O (e_i, e_j)={<h_k , [0, 0],[1, 1],[1, 1]>; h_k ∈ U} for (e_i, e_j) ∈ A xB.
Example. Consider the interval valued neutrosophic soft sets (F, A) and (G, B). Then a null relation between them is given by

U	(e_1, e_2)	(e_1, e_4)	(e_3, e_2)	(e_3, e_4)
h_1	([0, 0],[1, 1],[1, 1])	([0, 0],[1, 1],[1, 1])	([0, 0],[1, 1],[1, 1])	([0, 0],[1, 1],[1, 1])
h_2	([0, 0],[1, 1],[1, 1])	([0, 0],[1, 1],[1, 1])	([0, 0],[1, 1],[1, 1])	([0, 0],[1, 1],[1, 1])
h_3	([0, 0],[1, 1],[1, 1])	([0, 0],[1, 1],[1, 1])	([0, 0],[1, 1],[1, 1])	([0, 0],[1, 1],[1, 1])
h_4	([0, 0],[1, 1],[1, 1])	([0, 0],[1, 1],[1, 1])	([0, 0],[1, 1],[1, 1])	([0, 0],[1, 1],[1, 1])

Remark. It can be easily seen that P ∪ 0_U =P and P ∩ 0_U =0_U for any P ∈ $\sigma_U(Ax\ B)$
Definition 2.11.
Let U be an initial universe and (F, A) and (G, B) be two interval valued neutrosophic soft sets. Then an absolute relation between them is denoted by I_U and is defined as I_U =(H_I , A xB) where H_I (e_i, e_j)={<h_k , [1, 1],[0, 0],[0, 0]>; h_k ∈ U} for (e_i, e_j) ∈ A xB.

U	(e_1, e_2)	(e_1, e_4)	(e_3, e_2)	(e_3, e_4)
h_1	([1, 1],[0, 0],[0, 0])	([1, 1],[0, 0],[0, 0])	([1, 1],[0, 0],[0, 0])	([1, 1],[0, 0],[0, 0])
h_2	([1, 1],[0, 0],[0, 0])	([1, 1],[0, 0],[0, 0])	([1, 1],[0, 0],[0, 0])	([1, 1],[0, 0],[0, 0])
h_3	([1, 1],[0, 0],[0, 0])	([1, 1],[0, 0],[0, 0])	([1, 1],[0, 0],[0, 0])	([1, 1],[0, 0],[0, 0])
h_4	([1, 1],[0, 0],[0, 0])	([1, 1],[0, 0],[0, 0])	([1, 1],[0, 0],[0, 0])	([1, 1],[0, 0],[0, 0])

Definition.2.12 Let P ∈ $\sigma_U(Ax\ B)$, P= (H, AxB) ,Q = (J, AxB) and the order of their relational matrices are same.Then we define

(i) P ⋃ Q= (H ∘J, AxB) where H∘ J :AxB →IVNS(U) is defined as (H ∘J)(e_i, e_j)= H(e_i, e_j) ∨ J(e_j, e_j) for (e_i, e_j) ∈ A x B, where ∨ denotes the interval valued neutrosophic union.

(ii) P ∩ Q= (H ■J, AxB) where H■J :AxB →IVNS(U) is defined as (H■J)(e_i, e_j)= H(e_i, e_j) ∧ J(e_i, e_j) for (e_j, e_j) ∈ A x B, where ∧ denotes the interval valued neutrosophic intersection

(iii) P^c= (∼H, AxB) , where ∼H :AxB →IVNS(U) is defined as ∼H(e_i, e_j)=[H(e_i, e_j)] ^{c} for (e_i, e_j) ∈ A x B, where c denotes the interval valued neutrosophic complement.

Defintion.2.13.
Let R be an equivalence relation on the universal set U. Then the pair (U, R) is called a Pawlak approximation space. An equivalence class of R containing x will be denoted by $[x]_R$. Now for X ⊆ U, the lower and upper approximation of X with respect to (U, R) are denoted by respectively R ∗X and R* X and are defined by

$R_*X=\{x\in U: [x]_R \subseteq X\}$,
$R^*X=\{ x\in U: [x]_R \cap X \neq\}$.
Now if $R_*X = R^* X$, then X is called definable; otherwise X is called a rough set.

3-Lower and upper soft interval valued neutrosophic rough approximations of an IVNSS-relation

Defntion 3.1 .Let R $\in \sigma_U(A\text{x } A)$ and R=(H, Ax A). Let Θ=(f,B) be an interval valued neutrosophic soft set over U and S= (U, Θ) be the soft interval valued neutrosophic approximation space. Then the lower and upper soft interval valued neutrosophic rough approximations of R with respect to S are denoted by Lwr_S(R)and Upr_S(R) respectively, which are IVNSS- relations over AxB in U given by:
Lwr_S(R)= (J, A xB) and Upr_S(R) =(K, A xB)
J(e_i, e_k**) ={<x,** $[\wedge_{e_j\in A}(\inf \mu_{H(e_i,e_j)}(x) \wedge \inf \mu_{f(e_k)}(x)), \wedge_{e_j\in A}(\sup \mu_{H(e_i,e_j)}(x) \wedge \sup \mu_{f(e_k)}(x))]$,

$[\wedge_{e_j\in A}(\inf \nu_{H(e_i,e_j)}(x) \vee \inf \nu_{f(e_k)}(x)), \wedge_{e_j\in A}(\sup \nu_{H(e_i,e_j)}(x) \vee \sup \nu_{f(e_k)}(x))]$,
$[\wedge_{e_j\in A}(\inf \omega_{H(e_i,e_j)}(x) \vee \inf \omega_{f(e_k)}(x)), \wedge_{e_j\in A}(\sup \omega_{H(e_i,e_j)}(x) \vee \sup \omega_{f(e_k)}(x))]$:x ∈ U}.

K(e_i, e_k**) ={<x,** $[\wedge_{e_i\in A}(\inf \mu_{H(e_i,e_j)}(x) \vee \inf \mu_{f(e_k)}(x)), \wedge_{e_j\in A}(\sup \mu_{H(e_i,e_j)}(x) \vee \sup \mu_{f(e_k)}(x))]$,

$[\wedge_{e_j\in A}(\inf \nu_{H(e_i,e_j)}(x) \wedge \inf \nu_{f(e_k)}(x)), \wedge_{e_j\in A}(\sup \nu_{H(e_i,e_j)}(x) \wedge \sup \nu_{f(e_k)}(x))]$,
$[\wedge_{e_j\in A}(\inf \omega_{H(e_i,e_j)}(x) \wedge \inf \omega_{f(e_k)}(x)), \wedge_{e_j\in A}(\sup \omega_{H(e_i,e_j)}(x) \wedge \sup \omega_{f(e_k)}(x))]$:x ∈ U}.

For $e_i \in$ A , $e_K \in$ B

Theorem 3.2. Let be an interval valued neutrosophic soft over U and S = (U,Θ) be the soft approximation space. Let R_1 , $R_2 \in \sigma_U(A\text{x } A)$ and R_1=(G,Ax A) and R_2=(H,Ax A).Then

(i) $\text{Lwr}_S(0_U)= 0_U$
(ii) $\text{Lwr}_S(1_U)= 1_U$
(iii) $\boldsymbol{R_1} \subseteq \boldsymbol{R_2} \Longrightarrow \text{Lwr}_S(\boldsymbol{R_1}) \subseteq \text{Lwr}_S(\boldsymbol{R_2})$
(iv) $\boldsymbol{R_1} \subseteq \boldsymbol{R_2} \Longrightarrow \text{Upr}_S(\boldsymbol{R_1}) \subseteq \text{Upr}_S(\boldsymbol{R_2}$
(v) $\text{Lwr}_S(\boldsymbol{R_1} \cap \boldsymbol{R_2}) \subseteq \text{Lwr}_S(\boldsymbol{R_1}) \cap \text{Lwr}_S(\boldsymbol{R_2})$
(vi) $\text{Upr}_S(\boldsymbol{R_1} \cap \boldsymbol{R_2}) \subseteq \text{Upr}_S(\boldsymbol{R_1}) \cap \text{Upr}_S(\boldsymbol{R_2})$
(vii) $\text{Lwr}_S(\boldsymbol{R_1}) \cup \text{Lwr}_S(\boldsymbol{R_2}) \subseteq \text{Lwr}_S(\boldsymbol{R_1} \cup \boldsymbol{R_2})$
(viii) $\text{Upr}_S(\boldsymbol{R_1}) \cup \text{Upr}_S(\boldsymbol{R_2}) \subseteq \text{Upr}_S(\boldsymbol{R_1} \cup \boldsymbol{R_2})$

Proof. (i) –(iv) are straight forward.
Let Lwrs($R_1 \cap R_2$) =(S, Ax B).Then for $(\boldsymbol{e_i}, \boldsymbol{e_k}) \in$ A xB , we have

S(e_i, e_k) ={<x, $[\wedge_{e_j\in A}(\inf \mu_{G\circ H(e_i,e_j)}(x) \wedge \inf \mu_{f(e_k)}(x)), \wedge_{e_j\in A}(\sup \mu_{G\circ H(e_i,e_j)}(x) \wedge \sup \mu_{f(e_k)}(x))]$,
$[\wedge_{e_j\in A}(\inf \nu_{G\circ H(e_i,e_j)}(x) \vee \inf \nu_{f(e_k)}(x)), \wedge_{e_j\in A}(\sup \nu_{G\circ H(e_i,e_j)}(x) \vee \sup \nu_{f(e_k)}(x))]$,

$[\wedge_{e_j\in A}(\inf \omega_{G\circ H(e_i,e_j)}(x) \vee \inf \omega_{f(e_k)}(x)), \wedge_{e_j\in A}(\sup \omega_{G\circ H(e_i,e_j)}(x) \vee \sup \omega_{f(e_k)}(x))]$:x ∈ U}

$=\{<x, [\wedge_{e_j \in A}(\min(\inf \mu_{G(e_i,e_j)}(x), \inf \mu_{H(e_i,e_j)}(x)) \wedge \inf \mu_{f(e_k)}(x))$
$, \wedge_{e_j \in A}(\min(\sup \mu_{G(e_i,e_j)}(x), \sup \mu_{H(e_i,e_j)}(x)) \wedge \sup \mu_{f(e_k)}(x))],$

$[\wedge_{e_j \in A}(\max(\inf \nu_{G(e_i,e_j)}(x), \inf \nu_{H(e_i,e_j)}(x)) \vee \inf \nu_{f(e_k)}(x))$
$, \wedge_{e_j \in A}(\max(\sup \nu_{G(e_i,e_j)}(x), \sup \nu_{H(e_i,e_j)}(x)) \vee \sup \nu_{f(e_k)}(x))],$

$[\wedge_{e_j \in A}(\max(\inf \omega_{G(e_i,e_j)}(x), \inf \omega_{H(e_i,e_j)}(x)) \vee \inf \omega_{f(e_k)}(x))$
$, \wedge_{e_j \in A}(\max(\sup \omega_{G(e_i,e_j)}(x), \sup \omega_{H(e_i,e_j)}(x)) \vee \sup \omega_{f(e_k)}(x))] : x \in U\}$

Also for $\mathrm{Lwr}_S(\boldsymbol{R_1}) \cap \mathrm{Lwr}_S(\boldsymbol{R_2})$ =(Z,A x B) and $(\boldsymbol{e_i}, \boldsymbol{e_K}) \in$ A xB ,we have ,
$Z(\boldsymbol{e_i}, \boldsymbol{e_K}) = \{<x, [\,\mathrm{Min}\,(\wedge_{e_j \in A}(\inf \mu_{G(e_i,e_j)}(x) \wedge \inf \mu_{f(e_k)}(x)), \wedge_{e_j \in A}(\inf \mu_{H(e_i,e_j)}(x) \wedge \inf \mu_{f(e_k)}(x))), \mathrm{Min}(\wedge_{e_j \in A}(\sup \mu_{G(e_i,e_j)}(x) \wedge \sup \mu_{f(e_k)}(x)), \wedge_{e_j \in A}(\sup \mu_{H(e_i,e_j)}(x) \wedge \sup \mu_{f(e_k)}(x)))],$
$[\mathrm{Max}\,(\wedge_{e_j \in A}(\inf \nu_{G(e_i,e_j)}(x) \vee \inf \nu_{f(e_k)}(x)), \wedge_{e_j \in A}(\inf \nu_{H(e_i,e_j)}(x) \vee \inf \nu_{f(e_k)}(x))),$
$\mathrm{Max}(\wedge_{e_j \in A}(\sup \nu_{G(e_i,e_j)}(x) \vee \sup \nu_{f(e_k)}(x)), \wedge_{e_j \in A}(\sup \nu_{H(e_i,e_j)}(x) \vee \sup \nu_{f(e_k)}(x)))],$

$[\mathrm{Max}\,(\wedge_{e_j \in A}(\inf \omega_{G(e_i,e_j)}(x) \vee \inf \omega_{f(e_k)}(x)), \wedge_{e_j \in A}(\inf \omega_{H(e_i,e_j)}(x) \vee \inf \omega_{f(e_k)}(x))),$
$\mathrm{Max}(\wedge_{e_j \in A}(\sup \omega_{G(e_i,e_j)}(x) \vee \sup \omega_{f(e_k)}(x)), \wedge_{e_j \in A}(\sup \omega_{H(e_i,e_j)}(x) \vee \sup \omega_{f(e_k)}(x)))] : x \in U\}$

Now since $\min(\inf \mu_{G(e_i,e_j)}, \inf \mu_{H(e_i,e_j)}(x)) \leq \inf \mu_{G(e_i,e_j)}(x)$ and
$\min(\inf \mu_{G(e_i,e_j)}, \inf \mu_{H(e_i,e_j)}(x)) \leq \inf \mu_{H(e_i,e_j)}(x)$ we have
$\wedge_{e_j \in A}(\min(\inf \mu_{G(e_i,e_j)}(x), \inf \mu_{H(e_i,e_j)}(x)) \wedge \inf \mu_{f(e_k)}(x)) \leq \mathrm{Min}\,(\wedge_{e_j \in A}(\inf \mu_{G(e_i,e_j)}(x) \wedge \inf \mu_{f(e_k)}(x)), \wedge_{e_j \in A}(\inf \mu_{H(e_i,e_j)}(x) \wedge \inf \mu_{f(e_k)}(x)))$.
Similarly we can get
$\wedge_{e_j \in A}(\min(\sup \mu_{G(e_i,e_j)}(x), \sup \mu_{H(e_i,e_j)}(x)) \wedge \sup \mu_{f(e_k)}(x)) \leq \mathrm{Min}\,(\wedge_{e_j \in A}(\sup \mu_{G(e_i,e_j)}(x) \wedge \sup \mu_{f(e_k)}(x)), \wedge_{e_j \in A}(\sup \mu_{H(e_i,e_j)}(x) \wedge \sup \mu_{f(e_k)}(x)))$.

Again as $\max(\inf \nu_{G(e_i,e_j)}, \inf \nu_{H(e_i,e_j)}(x)) \geq \inf \nu_{G(e_i,e_j)}(x)$,and
$\max(\inf \nu_{G(e_i,e_j)}, \inf \nu_{H(e_i,e_j)}(x)) \geq \inf \nu_{H(e_i,e_j)}(x)$

we have

$\wedge_{e_j \in A}(\max(\inf \nu_{G(e_i,e_j)}(x), \inf \nu_{H(e_i,e_j)}(x)) \vee \inf \nu_{f(e_k)}(x)) \geq \mathrm{Max}\,(\wedge_{e_j \in A}(\inf \nu_{G(e_i,e_j)}(x) \vee \inf \nu_{f(e_k)}(x)), \wedge_{e_j \in A}(\inf \nu_{H(e_i,e_j)}(x) \vee \inf \nu_{f(e_k)}(x)))$.
Similarly we can get

$\wedge_{e_j \in A}(\max(\sup \nu_{G(e_i,e_j)}(x), \sup \nu_{H(e_i,e_j)}(x)) \vee \sup \nu_{f(e_k)}(x)) \geq \mathrm{Max}\,(\wedge_{e_j \in A}(\sup \nu_{G(e_i,e_j)}(x) \vee \sup \nu_{f(e_k)}(x)), \wedge_{e_j \in A}(\sup \nu_{H(e_i,e_j)}(x) \vee \sup \nu_{f(e_k)}(x)))$.
Again as $\max(\inf \omega_{G(e_i,e_j)}, \inf \omega_{H(e_i,e_j)}(x)) \geq \inf \omega_{G(e_i,e_j)}(x)$,and
$\max(\inf \omega_{G(e_i,e_j)}, \inf \omega_{H(e_i,e_j)}(x)) \geq \inf \omega_{H(e_i,e_j)}(x)$

we have

$\wedge_{e_j \in A}(\max(\inf \omega_{\mathbf{G}(e_i, e_j)}(x), \inf \omega_{\mathbf{H}(e_i, e_j)}(x)) \vee \inf \omega_{\mathbf{f}(e_k)}(x)) \geq \operatorname{Max}(\wedge_{e_j \in A}(\inf \omega_{\mathbf{G}(e_i, e_j)}(x) \vee \inf \omega_{\mathbf{f}(e_k)}(x)), \wedge_{e_j \in A}(\inf \omega_{\mathbf{H}(e_i, e_j)}(x) \vee \inf \omega_{\mathbf{f}(e_k)}(x)))$.

Similarly we can get

$\wedge_{e_j \in A}(\max(\sup \omega_{\mathbf{G}(e_i, e_j)}(x), \sup \omega_{\mathbf{H}(e_i, e_j)}(x)) \vee \sup \omega_{\mathbf{f}(e_k)}(x)) \geq \operatorname{Max}(\wedge_{e_j \in A}(\sup \omega_{\mathbf{G}(e_i, e_j)}(x) \vee \sup \omega_{\mathbf{f}(e_k)}(x)), \wedge_{e_j \in A}(\sup \omega_{\mathbf{H}(e_i, e_j)}(x) \vee \sup \omega_{\mathbf{f}(e_k)}(x)))$.

Consequently, $\mathrm{Lwr}_S(\boldsymbol{R_1} \cap \boldsymbol{R_2}) \subseteq \mathrm{Lwr}_S(\boldsymbol{R_1}) \cap \mathrm{Lwr}_S(\boldsymbol{R_2})$

(vi) Proof is similar to (v)

(vii) Let $\mathrm{Lwr}_S(\boldsymbol{R_1} \cup \boldsymbol{R_2})$ **=(S, A xB).Then for** $(\boldsymbol{e_i}, \boldsymbol{e_k}) \in$ A xB , we have

S$(\boldsymbol{e_i}, \boldsymbol{e_k})$={<x, $[\wedge_{e_j \in A}(\inf \mu_{\mathbf{G \blacksquare H}(e_i, e_j)}(x) \wedge \inf \mu_{\mathbf{f}(e_k)}(x)), \wedge_{e_j \in A}(\sup \mu_{\mathbf{G \blacksquare H}(e_i, e_j)}(x) \wedge \sup \mu_{\mathbf{f}(e_k)}(x))]$,

$[\wedge_{e_j \in A}(\inf \nu_{\mathbf{G \blacksquare H}(e_i, e_j)}(x) \vee \inf \nu_{\mathbf{f}(e_k)}(x)), \wedge_{e_j \in A}(\inf \nu_{\mathbf{G \blacksquare H}(e_i, e_j)}(x) \vee \inf \nu_{\mathbf{f}(e_k)}(x))]$,

$[\wedge_{e_j \in A}(\inf \omega_{\mathbf{G \blacksquare H}(e_i, e_j)}(x) \vee \inf \omega_{\mathbf{f}(e_k)}(x)), \wedge_{e_j \in A}(\inf \omega_{\mathbf{G \blacksquare H}(e_i, e_j)}(x) \vee \inf \omega_{\mathbf{f}(e_k)}(x))]$:x $\in$ U}

={<x, $[\wedge_{e_j \in A}(\max(\inf \mu_{\mathbf{G}(e_i, e_j)}(x), \inf \mu_{\mathbf{H}(e_i, e_j)}(x)) \wedge \inf \mu_{\mathbf{f}(e_k)}(x)), \wedge_{e_j \in A}(\max(\sup \mu_{\mathbf{G}(e_i, e_j)}(x), \sup \mu_{\mathbf{H}(e_i, e_j)}(x)) \wedge \sup \mu_{\mathbf{f}(e_k)}(x))]$,

$[\wedge_{e_j \in A}(\min(\inf \nu_{\mathbf{G}(e_i, e_j)}(x), \inf \nu_{\mathbf{H}(e_i, e_j)}(x)) \vee \inf \nu_{\mathbf{f}(e_k)}(x)), \wedge_{e_j \in A}(\min(\sup \nu_{\mathbf{G}(e_i, e_j)}(x), \sup \nu_{\mathbf{H}(e_i, e_j)}(x)) \vee \sup \nu_{\mathbf{f}(e_k)}(x))]$,

$[\wedge_{e_j \in A}(\min(\inf \omega_{\mathbf{G}(e_i, e_j)}(x), \inf \omega_{\mathbf{H}(e_i, e_j)}(x)) \vee \inf \omega_{\mathbf{f}(e_k)}(x)), \wedge_{e_j \in A}(\min(\sup \omega_{\mathbf{G}(e_i, e_j)}(x), \sup \omega_{\mathbf{H}(e_i, e_j)}(x)) \vee \sup \omega_{\mathbf{f}(e_k)}(x))]$:x $\in$ U}

Also for $\mathrm{Lwrs}(\boldsymbol{R_1}) \cup \mathrm{Lwrs}(\boldsymbol{R_2})$ **= (Z, AxB) and** $(\boldsymbol{e_i}, \boldsymbol{e_k}) \in$ A xB ,we have ,

Z $(\boldsymbol{e_i}, \boldsymbol{e_K})$= {<x, [$\operatorname{Max}(\wedge_{e_j \in A}(\inf \mu_{\mathbf{G}(e_i, e_j)}(x) \wedge \inf \mu_{\mathbf{f}(e_k)}(x)), \wedge_{e_j \in A}(\inf \mu_{\mathbf{H}(e_i, e_j)}(x) \wedge \inf \mu_{\mathbf{f}(e_k)}(x)))$, $\operatorname{Max}(\wedge_{e_j \in A}(\sup \mu_{\mathbf{G}(e_i, e_j)}(x) \wedge \sup \mu_{\mathbf{f}(e_k)}(x)), \wedge_{e_j \in A}(\sup \mu_{\mathbf{H}(e_i, e_j)}(x) \wedge \sup \mu_{\mathbf{f}(e_k)}(x)))$] ,

[$\operatorname{Min}(\wedge_{e_j \in A}(\inf \nu_{\mathbf{G}(e_i, e_j)}(x) \vee \inf \nu_{\mathbf{f}(e_k)}(x)), \wedge_{e_j \in A}(\inf \nu_{\mathbf{H}(e_i, e_j)}(x) \vee \inf \nu_{\mathbf{f}(e_k)}(x)))$,
$\operatorname{Min}(\wedge_{e_j \in A}(\sup \nu_{\mathbf{G}(e_i, e_j)}(x) \vee \sup \nu_{\mathbf{f}(e_k)}(x)), \wedge_{e_j \in A}(\sup \nu_{\mathbf{H}(e_i, e_j)}(x) \vee \sup \nu_{\mathbf{f}(e_k)}(x)))$] ,

[$\operatorname{Min}(\wedge_{e_j \in A}(\inf \omega_{\mathbf{G}(e_i, e_j)}(x) \vee \inf \omega_{\mathbf{f}(e_k)}(x)), \wedge_{e_j \in A}(\inf \omega_{\mathbf{H}(e_i, e_j)}(x) \vee \inf \omega_{\mathbf{f}(e_k)}(x)))$,
$\operatorname{Min}(\wedge_{e_j \in A}(\sup \omega_{\mathbf{G}(e_i, e_j)}(x) \vee \sup \omega_{\mathbf{f}(e_k)}(x)), \wedge_{e_j \in A}(\sup \omega_{\mathbf{H}(e_i, e_j)}(x) \vee \sup \omega_{\mathbf{f}(e_k)}(x)))$] :x $\in$ U}

Now since $\max(\inf \mu_{\mathbf{G}(e_i, e_j)}, \inf \mu_{\mathbf{H}(e_i, e_j)}(x)) \geq \inf \mu_{\mathbf{G}(e_i, e_j)}(x)$ and $\max(\inf \mu_{\mathbf{G}(e_i, e_j)}, \inf \mu_{\mathbf{H}(e_i, e_j)}(x)) \geq \inf \mu_{\mathbf{H}(e_i, e_j)}(x)$ we have

$\wedge_{e_j \in A}(\max(\inf \mu_{\mathbf{G}(e_i, e_j)}(x), \inf \mu_{\mathbf{H}(e_i, e_j)}(x)) \wedge \inf \mu_{\mathbf{f}(e_k)}(x)) \geq \max(\wedge_{e_j \in A}(\inf \mu_{\mathbf{G}(e_i, e_j)}(x) \wedge \inf \mu_{\mathbf{f}(e_k)}(x)), \wedge_{e_j \in A}(\inf \mu_{\mathbf{H}(e_i, e_j)}(x) \wedge \inf \mu_{\mathbf{f}(e_k)}(x)))$.

Similarly we can get

$\wedge_{e_j \in A}(\max(\sup \mu_{\mathbf{G}(e_i, e_j)}(x), \sup \mu_{\mathbf{H}(e_i, e_j)}(x)) \wedge \sup \mu_{\mathbf{f}(e_k)}(x)) \geq \max(\wedge_{e_j \in A}(\sup \mu_{\mathbf{G}(e_i, e_j)}(x) \wedge \sup \mu_{\mathbf{f}(e_k)}(x)), \wedge_{e_j \in A}(\sup \mu_{\mathbf{H}(e_i, e_j)}(x) \wedge \sup \mu_{\mathbf{f}(e_k)}(x)))$.

Again as $\min(\inf \nu_{\mathbf{G}(e_i,e_j)}, \inf \nu_{\mathbf{H}(e_i,e_j)}(x)) \leq \inf \nu_{\mathbf{G}(e_i,e_j)}(x)$,and
$\min(\inf \nu_{\mathbf{G}(e_i,e_j)}, \inf \nu_{\mathbf{H}(e_i,e_j)}(x)) \leq \inf \nu_{\mathbf{H}(e_i,e_j)}(x)$

we have

$\wedge_{e_j \in A}(\min(\inf \nu_{\mathbf{G}(e_i,e_j)}(x), \inf \nu_{\mathbf{H}(e_i,e_j)}(x)) \vee \inf \nu_{\mathbf{f}(e_k)}(x)) \leq \mathrm{Min}\,(\wedge_{e_j \in A}(\inf \nu_{\mathbf{G}(e_i,e_j)}(x) \vee \inf \nu_{\mathbf{f}(e_k)}(x)),\ \wedge_{e_j \in A}(\inf \nu_{\mathbf{H}(e_i,e_j)}(x) \vee \inf \nu_{\mathbf{f}(e_k)}(x)))$.
Similarly we can get

$\wedge_{e_j \in A}(\min(\sup \nu_{\mathbf{G}(e_i,e_j)}(x), \sup \nu_{\mathbf{H}(e_i,e_j)}(x)) \vee \sup \nu_{\mathbf{f}(e_k)}(x)) \leq \mathrm{Min}\,(\wedge_{e_j \in A}(\sup \nu_{\mathbf{G}(e_i,e_j)}(x) \vee \sup \nu_{\mathbf{f}(e_k)}(x)),\ \wedge_{e_j \in A}(\sup \nu_{\mathbf{H}(e_i,e_j)}(x) \vee \sup \nu_{\mathbf{f}(e_k)}(x)))$.

Again as $\min(\inf \omega_{\mathbf{G}(e_i,e_j)}, \inf \omega_{\mathbf{H}(e_i,e_j)}(x)) \leq \inf \omega_{\mathbf{G}(e_i,e_j)}(x)$,and
$\min(\inf \omega_{\mathbf{G}(e_i,e_j)}, \inf \omega_{\mathbf{H}(e_i,e_j)}(x)) \leq \inf \omega_{\mathbf{H}(e_i,e_j)}(x)$

we have

$\wedge_{e_j \in A}(\min(\inf \omega_{\mathbf{G}(e_i,e_j)}(x), \inf \omega_{\mathbf{H}(e_i,e_j)}(x)) \vee \inf \omega_{\mathbf{f}(e_k)}(x)) \leq \mathrm{Min}\,(\wedge_{e_j \in A}(\inf \omega_{\mathbf{G}(e_i,e_j)}(x) \vee \inf \omega_{\mathbf{f}(e_k)}(x)),\ \wedge_{e_j \in A}(\inf \omega_{\mathbf{H}(e_i,e_j)}(x) \vee \inf \omega_{\mathbf{f}(e_k)}(x)))$.
Similarly we can get

$\wedge_{e_j \in A}(\min(\sup \omega_{\mathbf{G}(e_i,e_j)}(x), \sup \omega_{\mathbf{H}(e_i,e_j)}(x)) \vee \sup \omega_{\mathbf{f}(e_k)}(x)) \leq \mathrm{Min}\,(\wedge_{e_j \in A}(\sup \omega_{\mathbf{G}(e_i,e_j)}(x) \vee \sup \omega_{\mathbf{f}(e_k)}(x)),\ \wedge_{e_j \in A}(\sup \omega_{\mathbf{H}(e_i,e_j)}(x) \vee \sup \omega_{\mathbf{f}(e_k)}(x)))$.

Consequently $\mathrm{Lwr}_S(\boldsymbol{R_1}) \cup \mathrm{Lwr}_S(\boldsymbol{R_2}) \subseteq \mathrm{Lwrs}(\boldsymbol{R_1} \cap \boldsymbol{R_2})$
(vii) Proof is similar to (vii).

Conclusion

In the present paper we extend the concept of Lower and upper soft interval valued intuitionstic fuzzy rough approximations of an IVIFSS-relation to the case IVNSS and investigated some of their properties.

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Presented at SISOM & ACOUSTICS 2014 Symposium, Romanian Academy – Institute of Mechanics, Bucharest, 22-23 May 2014, pp. 204-211.

Neutrosophic Crisp Open Set and Neutrosophic Crisp Continuity via Neutrosophic Crisp Ideals

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Abstract—The focus of this paper is to propose a new notion of neutrosophic crisp sets via neutrosophic crisp ideals and to study some basic operations and results in neutrosophic crisp topological spaces. Also, neutrosophic crisp L-openness and neutrosophic crisp L- continuity are considered as a generalizations for a crisp and fuzzy concepts. Relationships between the above new neutrosophic crisp notions and the other relevant classes are investigated. Finally, we define and study two different types of neutrosophic crisp functions.

Index Terms—Neutrosophic Crisp Set; Neutrosophic Crisp Ideals; Neutrosophic Crisp L-open Sets; Neutrosophic Crisp L- Continuity; Neutrosophic Sets.

I. Introduction

The fuzzy set was introduced by Zadeh [20] in 1965, where each element had a degree of membership. In 1983 the intuitionstic fuzzy set was introduced by K. Atanassov [1, 2, 3] as a generalization of fuzzy set, where besides the degree of membership and the degree of non- membership of each element. Salama et al [11] defined intuitionistic fuzzy ideal and neutrosophic ideal for a set and generalized the concept of fuzzy ideal concepts, first initiated by Sarker [19]. Smarandache [16, 17, 18] defined the notion of neutrosophic sets, which is a generalization of Zadeh's fuzzy set and Atanassov's intuitionistic fuzzy set. Neutrosophic sets have been investigated by Salama et al. [4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15]. In this paper is to introduce and study some new neutrosophic crisp notions via neutrosophic crisp ideals. Also, neutrosophic crisp L-openness and neutrosophic crisp L- continuity are considered. Relationships between the above new neutrosophic crisp notions and the other relevant classes are investigated. Recently, we define and study two different types of neutrosophic crisp functions.

The paper unfolds as follows. The next section briefly introduces some definitions related to neutrosophic set theory and some terminologies of neutrosophic crisp set and neutrosophic crisp ideal. Section 3 presents neutrosophic crisp L- open and neutrosophic crisp L- closed sets. Section 4 presents neutrosophic crisp L–continuous functions. Conclusions appear in the last section.

II. Preliminaries

We recollect some relevant basic preliminaries, and in particular, the work of Smarandache in [16, 17, 18], and Salama et al. [4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15].

2.1 Definitions [9].

1) Let X be a non-empty fixed set. A neutrosophic crisp set (NCS for short) A is an object having the form $A=\langle A_1, A_2, A_3\rangle$ where A_1, A_2 and A_3 are subsets of X satisfying $A_1 \cap A_2 = \phi$, $A_1 \cap A_3 = \phi$ and $A_2 \cap A_3 = \phi$.

2) Let $A=\langle A_1, A_2, A_3\rangle$, be a neutrosophic crisp set on a set X, then $p=\langle\{p_1\},\{p_2\},\{p_3\}\rangle$, $p_1 \neq p_2 \neq p_3 \in X$ is called a neutrosophic crisp point. A neutrosophic crisp point (NCP for short) $p=\langle\{p_1\},\{p_2\},\{p_3\}\rangle$, is said to be belong to a neutrosophic crisp set $A=\langle A_1, A_2, A_3\rangle$, of X, denoted by $p \in A$, if may be defined by two types

i) **Type 1:** $\{p_1\} \subseteq A_1, \{p_2\} \subseteq A_2$ and $\{p_3\} \subseteq A_3$,

ii) **Type 2:** $\{p_1\} \subseteq A_1, \{p_2\} \supseteq A_2$ and $\{p_3\} \subseteq A_3$.

3) Let X be non-empty set, and L a non–empty family of NCSs. We call L a neutrosophic crisp ideal (NCL for short) on X if

i. $A \in L$ and $B \subseteq A \Rightarrow B \in L$ [heredity],

ii. $A \in L$ and $B \in L \Rightarrow A \vee B \in L$ [Finite additivity].

A neutrosophic crisp ideal L is called a σ - neutrosophic crisp ideal if $\{A_j\}_{j\in \mathrm{N}} \subseteq L$, implies $\bigcup_{j\in J} A_j \in L$ (countable additivity).

The smallest and largest neutrosophic crisp ideals on a non-empty set X are $\{\phi_N\}$ and the NSs on X. Also, NCL_f, NCL_c are denoting the neutrosophic crisp ideals (NCL for short) of neutrosophic crisp subsets having finite and countable support of X respectively. Moreover, if A is a nonempty NS in X, then $\{B \in NCS : B \subseteq A\}$ is an NCL on X. This is called the principal NCL of all NCSs, denoted by NCL$\langle A \rangle$.

2.1 Proposition [9]

Let $\{L_j : j \in J\}$ be any non - empty family of neutrosophic crisp ideals on a set X. Then $\underset{j\in J}{\cap} L_j$ and $\underset{j\in J}{\cup} L_j$ are neutrosophic crisp ideals on X, where

$\underset{j\in J}{\cap} L_j = \left\langle \underset{j\in J}{\cap} A_{j1}, \underset{j\in J}{\cap} A_{j2}, \underset{j\in J}{\cup} A_{j3} \right\rangle$ or

$\underset{j\in J}{\cap} L_j = \left\langle \underset{j\in J}{\cap} A_{j1}, \underset{j\in J}{\cup} A_{j2}, \underset{j\in J}{\cup} A_{j3} \right\rangle$ and

$\underset{j\in J}{\cup} L_j = \left\langle \underset{j\in J}{\cup} A_{j1}, \underset{j\in J}{\cup} A_{j2}, \underset{j\in J}{\cap} A_{j3} \right\rangle$ or

$\underset{j\in J}{\cup} L_j = \left\langle \underset{j\in J}{\cup} A_{j1}, \underset{j\in J}{\cap} A_{j2}, \underset{j\in J}{\cap} A_{j3} \right\rangle$.

2.2 Proposition [9]

A neutrosophic crisp set $A = \langle A_1, A_2, A_3 \rangle$ in the neutrosophic crisp ideal L on X is a base of L iff every member of L is contained in A.

2.1 Theorem [9]

Let $A = \langle A_1, A_2, A_3 \rangle$, and $B = \langle B_1, B_2, B_3 \rangle$, be neutrosophic crisp subsets of X. Then $A \subseteq B$ iff $p \in A$ implies $p \in B$ for any neutrosophic crisp point p in X.

2.2 Theorem [9]

Let $A = \langle A_1, A_2, A_3 \rangle$, be a neutrosophic crisp subset of X. Then $A = \cup\{p : p \in A\}$.

2.3 Proposition [9]

Let $\{A_j : j \in J\}$ is a family of NCSs in X. Then

(a_1) $p = \langle \{p_1\}, \{p_2\}, \{p_3\} \rangle \in \underset{j\in J}{\cap} A_j$ iff $p \in A_j$ for each $j \in J$.

(a_2) $p \in \underset{j\in J}{\cup} A_j$ iff $\exists j \in J$ such that $p \in A_j$.

2.4 Proposition [9]

Let $A = \langle A_1, A_2, A_3 \rangle$ and $B = \langle B_1, B_2, B_3 \rangle$ be two neutrosophic crisp sets in X. Then

a) $A \subseteq B$ iff for each p we have $p \in A \Leftrightarrow p \in B$ and for each p we have $p \in A \Rightarrow p \in B$.

b) $A = B$ iff for each p we have $p \in A \Rightarrow p \in B$ and for each p we have $p \in A \Leftrightarrow p \in B$.

2.5 Proposition[9]

Let $A = \langle A_1, A_2, A_3 \rangle$ be a neutrosophic crisp set in X. Then $A = \cup < \{p_1 : p_1 \in A_1\}, \{p_2 : p_2 \in A_2\}, \{p_3 : p_3 \in A_3\}$.

2.2 Definition [9]

Let $f : X \to Y$ be a function and p be a neutrosophic crisp point in X. Then the image of p under f, denoted by $f(p)$, is defined by $f(p) = \langle \{q_1\}, \{q_2\}, \{q_3\} \rangle$, where $q_1 = f(p_1), q_2 = f(p_2)$ and $q_3 = f(p_3)$.

It is easy to see that $f(p)$ is indeed a NCP in Y, namely $f(p) = q$, where $q = f(p)$, and it is exactly the same meaning of the image of a NCP under the function f.

2.3 Definition [9]

Let p be a neutrosophic crisp point of a neutrosophic crisp topological space $(X, NC\tau)$. A neutrosophic crisp neighbourhood (NCNBD for short) of a neutrosophic crisp point p if there is a neutrosophic crisp open set(NCOS for short) B in X such that $p \in B \subseteq A$.

2.3 Theorem [9]

Let $(X, NC\tau)$ be a neutrosophic crisp topological space (NCTS for short) of X. Then the neutrosophic crisp set A of X is NCOS iff A is a NCNBD of p for every neutrosophic crisp set $p \in A$.

2.4 Definition [9]

Let (X, τ) be a neutrosophic crisp topological spaces (NCTS for short) and L be neutrosophic crisp ideal (NCL, for short) on X. Let A be any NCS of X. Then the neutrosophic crisp local function $NCA^*(L, \tau)$ of A is the union of all neutrosophic crisp point NCTS(NCP, for short) $P = \langle \{p_1\}, \{p_2\}, \{p_3\} \rangle$, such that if $U \in N((p))$ and

$NA^*(L, \tau) = \cup\{p \in X : A \wedge U \notin L$ for every U nbd of N(P)$\}$

, $NCA^*(L, \tau)$ is called a neutrosophic crisp local function of A with respect to τ and L which it will be denoted by $NCA^*(L, \tau)$, or simply $NCA^*(L)$. The neutrosophic crisp topology generated by $NCA^*(\mathrm{L})$ in [9] we will be denoted by NC*.

2.5 Theorem [9]

Let (X,τ) be a NCTS and L_1, L_2 be two neutrosophic crisp ideals on X. Then for any neutrosophic crisp sets A, B of X. then the following statements are verified

i) $A \subseteq B \Rightarrow NCA^*(L,\tau) \subseteq NCB^*(L,\tau)$,

ii) $L_1 \subseteq L_2 \Rightarrow NCA^*(L_2,\tau) \subseteq NCA^*(L_1,\tau)$,

iii) $NCA^* = NCcl(A^*) \subseteq NCcl(A)$,

iv) $NCA^{**} \subseteq NCA^*$,

v) $NC(A \cup B)^* = NCA^* \cup NCB^*$,

vi) $NC(A \cap B)^*(L) \subseteq NCA^*(L) \cap NCB^*(L)$

vii) $\ell \in L \Rightarrow NC(A \cup \ell)^* = NCA^*$

viii) $NCA^*(L,\tau)$ be a neutrosophic crisp closed set.

2.6 Theorem [9]

Let $NC\tau_1, NC\tau_2$ be two neutrosophic crisp topologies on X. Then for any neutrosophic crisp ideal L on X, $NC\tau_1 \subseteq NC\tau_2$ implies $NCA^*(L,NC\tau_2) \subseteq NCA^*(NCL,NC\tau_1)$, for every $A \in L$ then $NC\tau^*{}_1 \subseteq NC\tau^*{}_2$. A basis $NC\beta(L,\tau)$ for $NC\tau^*(L)$ can be described as follows:
$NC\beta(L,\tau) = \{A - B : A \in NC\tau, B \in NCL\}$. Then we have the following theorem.

2.7 Theorem [9]

$NC\beta(L,\tau) = \{A - B : A \in \tau, B \in L\}$ forms a basis for the generated NCTS of the NCT (X,τ) with neutrosophic crisp ideal L on X.

2.8 Theorem [9]

Let $NC\tau_1, NC\tau_2$ be two neutrosophic crisp topologies on X. Then for any topological neutrosophic crisp ideal L on X, $NC\tau_1 \subseteq NC\tau_2$ implies $NC\tau^*{}_1 \subseteq NC\tau^*{}_2$.

2.9 Theorem [9]

Let (X,τ) be a NCTS and L_1, L_2 be two neutrosophic crisp ideals on X. Then for any neutrosophic crisp set A in X, we have

i) $NCA^*(L_1 \cup L_2,\tau) = NCA^*(L_1, NC\tau^*(L_1)) \cap NCA^*(L_2, NC\tau^*(L_2))$

ii) $NC\tau^*(L_1 \cup L_2) = (NC\tau^*(L_1))^*(L_2) \cap (NC\tau^*(L_2))^*(L_1)$

2.1 Corollary [9]

Let (X,τ) be a NCTS with topological neutrosophic crisp ideal L on X. Then

i) $NCA^*(L,\tau) = NCA^*(L,\tau^*)$ and $NC\tau^*(L) = NC(NC\tau^*(L))^*(L)$,

ii) $NC\tau^*(L_1 \cup L_2) = (NC\tau^*(L_1)) \cup (NC\tau^*(L_2))$.

III. Neutrosophic Crisp L- Open and Neutrosophic Crisp L- Closed Sets

Definition 3.1

Given (X,τ) be a NCTS with neutrosophic crisp ideal L on X, and A is called a neutrosophic crisp L–open set iff there exists $\zeta \in \tau$ such that $A \subseteq \zeta \subseteq NCA^*$.

We will denote the family of all neutrosophic crisp L–open sets by NCLO(X).

Theorem 3.1

Let (X, τ) be a NCTS with neutrosophic crisp ideal L, then $A \in NCLO(X)$ iff $A \subseteq NCint(NCA^*)$.

Proof

Assume that $A \in NCLO(X)$ then by Definition 3.1there exists $\zeta \in \tau$ such that $A \subseteq \zeta \subseteq NCA^*$. But $NCint(NCA^*) \subseteq NCA^*$, put $\zeta = NCint(NCA^*)$. Hence $A \subseteq NCint(NCA^*)$. Conversely $A \subseteq NCint(NCA^*) \subseteq NCA^*$. Then there exists $\zeta = NCint(NCA^*) \in \tau$. Hence $A \in NCLO(X)$.

Remark 3.1

For a NCTS (X,τ) with neutrosophic crisp ideal L and A be a neutrosophic crisp set on X, the following holds: If $A \in NCLO(X)$ then $NCint(A) \subseteq NCA^*$.

Theorem 3.2

Given (X,τ) be a NCTS with neutrosophic crisp ideal L on X and A, B are neutrosophic crisp sets such that $A \in NCLO(X)$, $B \in \tau$ then $A \cap B \in NCLO(X)$

Proof

From the assumption $A \cap B \subseteq NCint(NCA^*) \cap B = NCint(NCA^* \cap B)$, we have $A \cap B \subseteq NCintNC(A \cap B)^*$ and this complete the proof.

Corollary 3.1

If $\{A_j\}_{j\in J}$ is a neutrosophic crisp L–open set in NCTS (X,τ) with neutrosophic crisp ideal L. Then $\cup\{A_j\}_{j\in J}$ is neutrosophic crisp L–open sets.

Corollary 3.2

For a NCTS (X,τ) with neutrosophic crisp ideal L, and neutrosophic crisp set A on X and $A \in NCLO(X)$, then $NCA^* = NC(NCintNC(NCA^*))^*$ and $NCcl^*(A)) = NCint(NCA^*)$.

Proof: It's clear.

Definition 3.2

Given a NCTS (X,τ) with neutrosophic crisp ideal L on X and neutrosophic crisp set A. Then A is said to be:

(i) Neutrosophic crisp τ^* – closed (or NC*-closed) if $NCA^* \leq A$
(ii) Neutrosophic crisp L–dense – in – itself (or NC*– dense – in – itself) if $A \subseteq NCA^*$.
(iii) Neutrosophic crisp * – perfect if A is NC* – closed and NC* – dense – in – itself.

Theorem 3.3

Given a NCTS (X,τ) with neutrosophic crisp ideal L and A is a neutrosophic crisp set on X, then

(i) NC* – closed iff $NCcl^*(A) = A$.
(ii) NC* – dense – in – itself iff $NCcl^*(A) = NCA^*$.
(iii) NC* – perfect iff $NCcl^*(A) = NCA^* = A$.

Proof: Follows directly from the neutrosophic crisp closure operator NCcl* for a neutrosophic crisp topology $\tau^*(L)$ ($NC\tau^*$ for short).

Remark 3.2

One can deduce that

(i) Every NC*–dense – in – itself is neutrosophic crisp dense set.
(ii) Every neutrosophic crisp closed (resp. neutrosophic crisp open) set is N*–closed (resp. $NC\tau*$–open).
(iii) Every neutrosophic crisp L–open set is NC∗ – dense – in – itself.

Corollary 3.3

Given a NCTS (X,τ) with neutrosophic crisp ideal L on X and $A \in \tau$ then we have:

(i) If A is NC* –closed then $A^* \subseteq NCint(A) \subseteq NC\,Cl\,(A)$.
(ii) If A is NC* –dense – in – itself then $Nint(A) \subseteq NCA^*$.
(iii) If A is NC* –perfect then $NCint(A) = NCcl(A) = NCA*$.

Proof: Obvious.

we give the relationship between neutrosophic crisp L–open set and some known neutrosophic crisp openness.

Theorem 3.4

Given a NCTS (X,τ) with neutrosophic crisp ideal L and neutrosophic crisp set A on X then the following holds:

(i) If A is both neutrosophic crisp L – open and NC∗ – erfect then A is neutrosophic crisp open.
(ii) If A is both neutrosophic crisp open and NC∗– dense–in – itself then A is neutrosophic crisp L–open.

Proof. Follows from the definitions.

Corollary 3.4

For a neutrosophic crisp subset A of a NCTS (X,τ) with neutrosophic crisp ideal L on X, we have:

(i) If A is NC∗–closed and NL–open then $NCint(A) = NCint(NCA^*)$.
(ii) If A is NC∗–perfect and NL–open then $A = NC\,int\,(NCA^*)$.

Remark 3.3

One can deduce that the intersection of two neutrosophic crisp L–open sets is neutrosophic crisp L–open.

Corollary 3.5

Given (X,τ) be a NCTS with neutrosophic crisp ideal L and neutrosophic crisp set A on X. The following hold: If $L= \{N^x\}$, then $NCA^*(L) = \phi_N$ and hence A is neutrosophic crisp L–open iff $A = \phi_N$.

Proof: It's clear.

Definition 3.5

Given a NCTS (X,τ) with neutrosophic crisp ideal L and neutrosophic crisp set A then neutrosophic crisp ideal interior of A is defined as largest neutrosophic crisp L–open set contained in A , we denoted by NCL–NCint(A).

Theorem 3.5

If (X,τ) is a NCTS with neutrosophic crisp ideal L and neutrosophic crisp set A then

(i) A ∧ Nint (NCA*) is neutrosophic crisp L–open set.
(ii) NL–Nint (A) =0_N iff Nint (NCA*) = 0_N.

Proof

(i) Since NCint NCA* =NCA* ∩ NCint (NCA*), then NCint NCA* =NCA* ∩ NCint (NCA*) ⊆ NC(A∩ NCA*)*. Thus A ∩ NC A* ⊆ (A ∩ (A ∩ NCint NC(NCA*))* ⊆ NCintNC(A ∩ NCint NC(NCA*)*. Hence A ∩ NCint NCA*∈NCLO(X).
(ii) Let NCL–NCint(A) = ϕ_N , then A ∩ A* = ϕ_N , implies NCcl (A ∩ NCint(NCA*) = ϕ_N and so A ∩ Nint A* = ϕ_N . Conversely assume that NCint NCA*= ϕ_N , then A ∩ NC int(NC A*)= ϕ_N . Hence NCL–NCint (A) = ϕ_N .

Theorem 3.6

If (X,τ) be a NCTS with neutrosophic crisp ideal L and A is aneutrosophic crisp set on X, then NCL–NCint(A) = A ∩ NCint(NCA*).

Proof. The first implication follows from Theorem 3.4, that is A∩NCA* ⊆ NCL-NCint(A) (1)

For the reverse inclusion, if ζ ∈ NCLO(X) and ζ ⊆ A then NCζ* ⊆ NCA* and hence NC int(NCζ*) ⊆ NCint(NCA*). This implies ζ = ζ ∩ NCint(NCζ*) ⊆ A ∩ NCA*.

Thus NCL–NCint(A) ⊆ A ∩ NCint(NCA*) (2)

From (1) and (2) we have the result.

Corollary 3.6

For a NCTS (X,τ) with neutrosophic crisp ideal L and neutrosophic crisp set A on X then the following holds:

(i) If A is NC∗ – closed then NL–Nint (A) ⊆ A.
(ιι) If A is NC∗ – dense – in– itself then NL – Nint (A) ⊆ A*.
(iii) If A is NC∗ – perfect set then NCL – NCint (A) ⊆ NCA*.

Definition 3.6

Given (X,τ) be a NCTS with neutrosophic crisp ideal L and ζ be a neutrosophic crisp set on X, ζ is called neutrosophic crisp L–closed set if its complement is neutrosophic crisp L–open set . We will denote the family of neutrosophic crisp L–closed sets by NLCC(X).

Theorem 3.7

Given (X,τ) be a NCTS with neutrosophic crisp ideal L and ζ be a neutrosophic crisp set on X. ζ is neutrosophic crisp L closed, then NC(NCintζ)* ≤ ζ.

Proof: It's clear.

Theorem 3.8

Let (X,τ) be a NCTS with neutrosophic crisp ideal L on X and ζ be a neutrosophic crisp set on X such that NC(NCint ζ)^{*c} = NCint ζ^{c*} then ζ ∈ NLC(X) iff NC(NCint ζ)* ⊆ ζ.

Proof

(Necessity) Follows immedially from the above theorem (Sufficiency). Let NC) NCintζ (* ⊆ ζ then ζ^c ⊆ NC(NCint ζ)^{*c} = NCint (NCζ)^{c*}. from the hypothesis. Hence ζ^c∈NCLO(X), Thus ζ∈NLCC(X).

Corollary 3.7

For a NCTS (X,τ) with neutrosophic crisp ideal L on X the following holds:

(i) The union of neutrosophic crisp L – closed set and neutrosophic crisp closed set is neutrosophic crisp L–closed set.
(ii) The union of neutrosophic crisp L – closed and neutrosophic crisp L–closed is neutrosophic crisp perfect.

IV. NEUTROSOPHIC CRISP L–CONTINUOUS FUNCTIONS

By utilizing the notion of NL – open sets, we establish in this article a class of neutrosophic crisp L- continuous function. Many characterizations and properties of this concept are investigated.

Definition 4.1

A function f : (X,τ) → (Y,σ) with neutrosophic crisp ideal L on X is said to be neutrosophic crisp L–continuous if for every ζ∈σ, f^{-1}(ζ)∈ NCLO(X).

Theorem 4.1

For a function f : (X,τ) → (Y,σ) with neutrosophic crisp ideal L on X the following are equivalent:

(i.) f is neutrosophic crisp L-continuous. For a neutrosophic crisp point p in X and each ζ ∈ σ containing f (p), there exists A ∈ NCLO(X) containing p such that f (A) ⊆ σ.

(ii.) For each neutrosophic crisp point p in X and $\zeta\in\sigma$ containing f (p), $(f^{-1}(\zeta))^*$ is neutrosophic crisp nbd of p.

(iii.) The inverse image of each neutrosophic crisp closed set in Y is neutrosophic crisp L–closed.

Proof

(i) → (ii).Since $\zeta\in\sigma$ containing f (p), then by (i), $f^{-1}(\zeta) \in NCLO(X)$, by putting $A = f^{-1}(\zeta)$ which containing p, we have $f(A) \subseteq \sigma$

(ii) → (iii). Let $\zeta\in\sigma$ containing f (p). Then by (ii) there exists $A \in NCLO(X)$ containing p such that $f(A) \leq \sigma$, so $p \in A \subseteq NCint(NCA^*) \leq NCint(f^{-1}(\zeta))^* \subseteq (f^{-1}(\zeta))^*$. Hence $(f^{-1}(\zeta))^*$ is neutrosophic crisp nbd of p.

(iii) → (i) Let $\zeta\in\sigma$, since $(f^{-1}(\zeta))$ is neutrosophic crisp nbd of any point $f^{-1}(\zeta)$, every point $x_\varepsilon \in (f^{-1}(\zeta))^*$ is a neutrosophic crisp interior point of $f^{-1}(\zeta)^*$. Then $f^{-1}(\zeta) \subseteq NCint\ NC(f^{-1}(\zeta))^*$ and hence f is neutrosophic crisp L–continuous

(i)→ (iv) Let $\xi\in$ y be a neutrosophic crisp closed set. Then ξ^c is neutrosophic crisp open set, by $f^{-1}(\xi^c) = (f^{-1}(\xi))^c \in NCLO(X)$. Thus $f^{-1}(\xi)$ is neutrosophic crisp L–closed set.

The following theorem establish the relationship between neutrosophic crisp L-continuous and neutrosophic crisp continuous by using the previous neutrosophic crisp notions.

Theorem 4.2

Given f : (X,τ) → (Y,σ) is a function with a neutrosophic crisp ideal L on X then we have. If f is neutrosophic crisp L– continuous of each neutrosophic crisp*– perfect set in X, then f is neutrosophic crisp continuous.

Proof: Obvious.

Corollary 4.1

Given a function f : (X,τ) → (Y,σ) and each member of X is neutrosophic crisp NC*–dense – in – itself. Then we have every neutrosophic crisp continuous function is neutrosophic crisp NCL–continuous.

Proof: It's clear.

We define and study two different types of neutrosophic crisp functions.

Definition 4.2

A function $f:(X,\tau)\rightarrow(Y,\sigma)$ with neutrosophic crisp ideal L on Y is called neutrosophic crisp L-open (resp. neutrosophic crisp NCL- closed), if for each $A\in\tau$ (resp. A is neutrosophic crisp closed in X), $f(A)\in NCLO(Y)$ (resp. $f(A)$ is NCL-closed).

Theorem 4.3

Let a function $f:(X,\tau)\rightarrow(Y,\sigma)$ with neutrosophic crisp ideal L on Y. Then the following are equivalent:

(i.) f is neutrosophic crisp L-open.

(ii.) For each p in X and each neutrosophic crisp ncnbd A of p, there exists a neutrosophic crisp L-open set $B\in I^Y$ containing $f(p)$ such that $B\subseteq f(A)$.

Proof: Obvious.

Theorem 4.4

A neutrosophic crisp function $f:(X,\tau)\rightarrow(Y,\sigma)$ with neutrosophic crisp ideal L on Y be a neutrosophic crisp L-open (resp.neutrosophic crisp L-closed), if A in Y and B in X is a neutrosophic crisp closed (resp. neutrosophic crisp open) set C in Y containing A such that $f^{-1}(C)\subseteq B$.

Proof

Assume that $A=1_Y-(f(1_X-B))$, since $f^{-1}(C)\leq B$ and $A\leq C$ then C is neutrosophic crisp L-closed and $f^{-1}(C)=1_X-f^{-1}(f(1_X-A))\leq B$.

Theorem 4.5

If a function $f:(X,\tau)\rightarrow(Y,\sigma)$ with neutrosophic crisp ideal L on Y is a neutrosophic crisp L-open, then $f^{-1}NC(NC\operatorname{int}(A))^*\leq NC(f^{-1}(A))^*$ such that $f^{-1}(A)$ is neutrosophic crisp*-dense-in-itself and A in Y.

Proof

Since A in Y, $NC(f^{-1}(A))^*$ is neutrosophic crisp closed in X containing $f^{-1}(A)$, f is neutrosophic crisp L-open then by using Theorem 4.4 there is a neutrosophic crisp L-closed set $A\subseteq B$ suchthat, $(f^{-1}(A))^*\supseteq f^{-1}(B)\geq f^{-1}NC(\operatorname{int}(B))^*\supseteq f^{-1}NC(NC\operatorname{int}(\mu))^*$.

Corollary 4.2

For any bijective function $f:(X,\tau)\rightarrow(Y,\sigma)$ with neutrosophic crisp ideal L on Y , the following are equivalent:

(i.) $f^{-1}:(Y,\sigma)\rightarrow(X,\tau)$ is neutrosophic crisp L-continuous.

(ii.) f is neutrosophic crisp L-open.

(iii.) f is neutrosophic crisp L-closed.

Proof: Follows directly from Definitions.

V. CONCLUSION

In our work, we have put forward some new concepts of neutrosophic crisp open set and neutrosophic crisp continuity via neutrosophic crisp ideals. Some related properties have been established with example. It 's hoped that our work will enhance this study in neutrosophic set theory.

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Published in *I.J. Information Engineering and Electronic Business*, No. 3, 2014, pp. 1-8, DOI: 10.5815/ ijieeb.2014.03.01, 8 p., pages 199 - 205.

Neutrosophic Crisp Sets & Neutrosophic Crisp Topological Spaces

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Abstract. In this paper, we generalize the crisp topological space to the notion of neutrosophic crisp topological space, and we construct the basic concepts of the neutrosophic crisp topology. In addition to these, we introduce the definitions of neutrosophic crisp continuous function and neutrosophic crisp compact spaces. Finally, some characterizations concerning neutrosophic crisp compact spaces are presented and one obtains several properties. Possible application to GIS topology rules are touched upon.

Keywords: Neutrosophic Crisp Set; Neutrosophic Topology; Neutrosophic Crisp Topology.

1 Introduction

Neutrosophy has laid the foundation for a whole fami-ly of new mathematical theories generalizing both their crisp and fuzzy counterparts, such as a neutrosophic set theory in [1, 2, 3]. After the introduction of the neutrosoph-ic set concepts in [4, 5, 6, 7, 8, 9, 10, 11, 12] and fter have given the fundamental definitions of neutrosophic set operations we generalize the crisp topological space to the notion of neutrosophic crisp set. Finally, we introduce the definitions of neutrosophic crisp continuous function and neutrosophic crisp compact space, and we obtain several properties and some characterizations concerning the neutrosophic crisp compact space.

2 Terminologies

We recollect some relevant basic preliminaries, and in particular, the work of Smarandache in [1, 2, 3, 11], and Salama et al. [4, 5, 6, 7, 8, 11]. Smarandache introduced the neutrosophic components T, I, F which represent the membership, indeterminacy, and non-membership values respectively, where $]^{-}0, 1^{+}[$ is a non-standard unit interval.

Hanafy and Salama *et al.*[10, 11] considered some possible definitions for basic concepts of the neutrosophic crisp set and its operations. We now improve some results by the following.

3 Neutrosophic Crisp Sets

Definition 3.1 *Let X be a non-empty fixed set. A neutrosophic crisp set (NCS) A is an object having the form $A = \langle A_1, A_2, A_3\rangle$, where A_1, A_2, and A_3 are subsets of X satisfying $A_1 \cap A_2 = \phi$, $A_1 \cap A_3 = \phi$, and $A_2 \cap A_3 = \phi$.*

Remark 3.1 *A neutrosophic crisp set $A = \langle A_1, A_2, A_3\rangle$ can be identified as an ordered triple $\langle A_1, A_2, A_3\rangle$, where A_1, A_2, and A_3 are subsets on X, and one can define several relations and operations between NCSs.*

Since our purpose is to construct the tools for developing neutrosophic crisp sets, we must introduce the types of NCSs ϕ_N and X_N in X as follows:

1) ϕ_N may be defined in many ways as an NCS, as follows

   i) $\phi_N = \langle \phi, \phi, X \rangle$, or
   ii) $\phi_N = \langle \phi, X, X \rangle$, or
   iii) $\phi_N = \langle \phi, X, \phi \rangle$, or
   iv) $\phi_N = \langle \phi, \phi, \phi \rangle$.

2) X_N may also be defined in many ways as an NCS:

   i) $X_N = \langle X, \phi, \phi \rangle$,
   ii) $X_N = \langle X, X, \phi \rangle$,
   iii) $X_N = \langle X, X, X \rangle$.

Every crisp set A formed by three disjoint subsets of a non-empty set X is obviously an NCS having the form $A = \langle A_1, A_2, A_3 \rangle$.

Definition 3.2 *Let $A = \langle A_1, A_2, A_3 \rangle$ an NCS on X, then the complement A^c of the set A may be defined in three different ways:*

(C_1) $A^c = \langle A_1^c, A_2^c, A_3^c \rangle$,

(C_2) $A^c = \langle A_3, A_2, A_1 \rangle$,

(C_3) $A^c = \langle A_3, A_2^c, A_3 \rangle$.

One can define several relations and operations between NCSs as follows:

Definition 3.3 *Let X be a non-empty set, and NCSs A and B in the form $A = \langle A_1, A_2, A_3 \rangle$, $B = \langle B_1, B_2, B_3 \rangle$, then we may consider two possible definitions for subsets ($A \subseteq B$):*

1) $A \subseteq B \Leftrightarrow A_1 \subseteq B_1, A_2 \subseteq B_2$ *and* $A_3 \supseteq B_3$,
or

2) $A \subseteq B \Leftrightarrow A_1 \subseteq B_1, A_2 \supseteq B_2$ *and* $A_3 \supseteq B_3$.

Proposition 3.1 *For any neutrosophic crisp set A the following are hold:*

i) $\phi_N \subseteq A, \phi_N \subseteq \phi_N$,

ii) $A \subseteq X_N, X_N \subseteq X_N$.

Definition 3.4 *Let X is a non-empty set, and the NCSs A and B in the form $A = \langle A_1, A_2, A_3 \rangle$, $B = \langle B_1, B_2, B_3 \rangle$. Then:*

1) $A \cap B$ *may be defined in two ways:*

   i) $A \cap B = \langle A_1 \cap B_1, A_2 \cap B_2, A_3 \cup B_3 \rangle$ *or*
   ii) $A \cap B = \langle A_1 \cap B_1, A_2 \cup B_2, A_3 \cup B_3 \rangle$.

2) $A \cup B$ *may also be defined in two ways:*

   i) $A \cup B = \langle A_1 \cup B_1, A_2 \cap B_2, A_3 \cap B_3 \rangle$ *or*
   ii) $A \cup B = \langle A_1 \cup B_1, A_2 \cup B_2, A_3 \cap B_3 \rangle$.

3) $[\,]A = \langle A_1, A_2, A_1^c \rangle$.

4) $<> A = \langle A_3^c, A_2, A_3 \rangle$.

Proposition 3.2 *For any two neutrosophic crisp sets A and B on X, the followings are true:*

1) $(A \cap B)^c = A^c \cup B^c$.

2) $(A \cup B)^c = A^c \cap B^c$.

We can easily generalize the operations of intersection and union in Definition 3.2 to arbitrary family of neutrosophic crisp subsets as follows:

Proposition 3.3 *Let $\{A_j : j \in J\}$ be arbitrary family of neutrosophic crisp subsets in X. Then*

1) $\cap A_j$ may be defined as the following types:

i) $\cap A_j = \langle \cap A_{J1}, \cap A_{J2}, \cup A_{j3} \rangle$, or

ii) $\cap A_j = \langle \cap A_{J1}, \cup A_{J2}, \cup A_{j3} \rangle$.

2) $\cup A_j$ may be defined as the following types:

i) $\cup A_j = \langle \cup A_{J1}, \cap A_{J2}, \cap A_{j3} \rangle$, or

ii) $\cup A_j = \langle \cup A_{J1}, \cup A_{J2}, \cap A_{j3} \rangle$.

Definition 3.5 *The product of two neutrosophic crisp sets A and B is a neutrosophic crisp set given by*

$$A \times B = \langle A_1 \times B_1, A_2 \times B_2, A_3 \times B_3 \rangle$$

.

4 Neutrosophic crisp Topological Spaces

Here we extend the concepts of topological space and intuitionistic topological space to the case of neutrosophic crisp sets.

Definition 4.1 *A neutrosophic crisp topology (NCT) on a non-empty set X is a family Γ of neutrosophic crisp subsets in X satisfying the following axioms*

i) $\phi_N, X_N \in \Gamma$.

ii) $A_1 \cap A_2 \in \Gamma$ for any A_1 and $A_2 \in \Gamma$.

iii) $\cup A_j \in \Gamma \ \ \forall \{A_j : j \in J\} \subseteq \Gamma$.

In this case the pair (X, Γ) is called a neutrosophic crisp topological space (NCTS) in X. The elements in Γ are called neutrosophic crisp open sets (NCOSs) in Y. A neutrosophic crisp set F is closed if and only if its complement F^c is an open neutrosophic crisp set.

Remark 4.1 *Neutrosophic crisp topological spaces are very natural generalizations of topological spaces and intuitionistic topological spaces, and they allow more general functions to be members of topology:*

$$TS \to ITS \to NCTS$$

Example 4.1 *Let $X = \{a, b, c, d\}$, ϕ_N, X_N be any types of the universal and empty subsets, and A, B two neutrosophic crisp subsets on X defined by $A = \langle \{a\}, \{b, d\}, c \rangle$, $B = \langle \{a\}, \{b\}, \{c\} \rangle$, then the family $\Gamma = \{\phi_N, X_N, A, B\}$ is a neutrosophic crisp topology on X.*

Example 4.2 *Let (X, τ_0) be a topological space such that τ_0 is not indiscrete. Suppose $\{G_i : i \in J\}$ be a family and $\tau_0 = \{X, \phi\} \cup \{G_i : i \in J\}$. Then we can construct the following topologies:*

i) Two intuitionistic topologies

a) $\tau_1 = \{\phi_I, X_I\} \cup \{\langle G_i, \phi \rangle, i \in J\}$.

b) $\tau_2 = \{\phi_I, X_I\} \cup \{\langle \phi, G_i^c \rangle, i \in J\}$.

ii) Four neutrosophic crisp topologies

a) $\Gamma_1 = \{\phi_N, X_N\} \cup \{\langle \phi, \phi, G_i^c \rangle, i \in J\}$.

b) $\Gamma_2 = \{\phi_N, X_N\} \cup \{\langle G_i, \phi, \phi \rangle, i \in J\}$.

c) $\Gamma_3 = \{\phi_N, X_N\} \cup \{\langle G_i, \phi, G_i^c \rangle, i \in J\}$.

d) $\Gamma_4 = \{\phi_N, X_N\} \cup \{\langle G_i^c, \phi, \phi \rangle, i \in J\}$.

Definition 4.2 *Let (X, Γ_1), (X, Γ_2) be two neutrosophic crisp topological spaces on X. Then Γ_1 is said be contained in Γ_2 (in symbols $\Gamma_1 \subseteq \Gamma_2$) if $G \in \Gamma_2$ for each $G \in \Gamma_1$. In this case, we also say that Γ_1 is coarser than Γ_2.*

Proposition 4.1 *Let $\{\Gamma_j : j \in J\}$ be a family of NCTSs on X. Then $\cap\Gamma_j$ is a neutrosophic crisp topology on X. Furthermore, $\cap\Gamma_j$ is the coarsest NCT on X containing all topologies.*

Proof. Obvious

Now, we define the neutrosophic crisp closure and neutrosophic crisp interior operations in neutrosophic crisp topological spaces:

Definition 4.3 *Let (X, Γ) be NCTS and $A = \langle A_1.A_2, A_3\rangle$ be a NCS in X. Then the neutrosophic crisp closure of A ($NCCl(A)$ for short) and neutrosophic crisp interior ($NCInt(A)$ for short) of A are defined by*

$$\begin{aligned} NCCl(A) &= \cap\{K : \text{is an NCS in } X \text{and } A \subseteq K\} \\ NCInt(A) &= \cup\{G : G \text{ is an NCOS in } X \text{and } G \subseteq A\}, \end{aligned}$$

where NCS is a neutrosophic crisp set, and NCOS is a neutrosophic crisp open set.

It can be also shown that $NCCl(A)$ is NCCS (neutrosophic crisp closed set) and $NCInt(A)$ is a CNOS in X.

a) A is in X if and only if $NCCl(A) \supseteq A$.

b) A is an NCCS in X if and only if $NCInt(A) = A$.

Proposition 4.2 *For any neutrosophic crisp set A in (X, Γ) we have*

(a) $NCCl(A^c) = (NCInt(A))^c$.

(b) $NCInt(A^c) = (NCCl(A))^c$.

Proof. Let $A = \langle A_1, A_2, A_3\rangle$ and suppose that the family of neutrosophic crisp subsets contained in A are indexed by the family if NCSs contained in A are indexed by the family $A = \{ < A_{j1}, A_{j2}, A_{j3} >: i \in J\}$.

a) Then we see that we have two types of

$$\begin{aligned} NCInt(A) &= \{< \cup A_{j1}, \cup A_{j2}, \cap A_{j3} >\} \text{ or} \\ NCInt(A) &= \{< \cup A_{j1}, \cap A_{j2}, \cap A_{j3} >\} \text{ hence} \\ (NCInt(A))^c &= \{< \cap A_{j1}, \cap A_{j2}, \cup A_{j3} >\} \text{ or} \\ (NCInt(A))^c &= \{< \cap A_{j1}, \cup A_{j2}, \cup A_{j3} >\}. \end{aligned}$$

b) Hence $NCCl(A^c) = (NCInt(A))^c$ follows immediately, which is analogous to (a).

Proposition 4.3 *Let (X, Γ) be a NCTS and A, B be two neutrosophic crisp sets in X. Then the following properties hold:*

(a) $NCInt(A) \subseteq A$,

(b) $A \subseteq NCCl(A)$,

(c) $A \subseteq B \Rightarrow NCInt(A) \subseteq NCInt(B)$,

(d) $A \subseteq B \Rightarrow NCCl(A) \subseteq NCCl(B)$,

(e) $NCInt(A \cap B) = NCInt(A) \cap NCInt(B)$,

(f) $NCCl(A \cup B) = NCCl(A) \cup NCCl(B)$,

(g) $NCInt(X_N) = X_N$,

(h) $NCCl(\phi_N) = \phi_N$.

Proof. (a), (b) and (e) are obvious; (c) follows from (a) and definitions.

5 Neutrosophic Crisp Continuity

Here come the basic definitions first:

Definition 5.1 *(a) If $B = \langle B_1, B_2, B_3 \rangle$ is a NCS in Y, then the preimage of B under f denoted by $f^{-1}(B)$ is a NCS in X defined by $f^{-1}(B) = \langle f^{-1}(B_1), f^{-1}(B_2), f^{-1}(B_3) \rangle$.*

(b) If $A = \langle A_1, A_2, A_3 \rangle$ is a NCS in X, then the image of A under f denoted by $f(A)$ is the NCS in Y defined by $f(A) = \langle f(A_1), f(A_2), f(A_3)^c \rangle$.

Here we introduce the properties of images and preimages some of which we shall frequently use in the following sections.

Corollary 5.1 *Let A, $\{A_i : i \in J\}$ be NCSs in X, and B, $\{B_j : j \in K\}$ NCS in Y, and $f : X \to Y$ a function. Then*

(a) $A_1 \subseteq A_2 \Leftrightarrow f(A_1) \subseteq f(A_2)$,
$B_1 \subseteq B_2 \Leftrightarrow f^{-1}(B_1) \subseteq f^{-1}(B_2)$,

(b) $A \subseteq f^{-1}(f(A))$ and if f is injective, then $A = f^{-1}(f(A))$.

(c) $f^{-1}(f(B)) \subseteq B$ and if f is surjective, then $f^{-1}(f(B)) = B$.

(d) $f^{-1}(\cup B_i) = \cup f^{-1}(B_i), f^{-1}(\cap B_1) = \cap f^{-1}(B_i)$,

(e) $f(\cup A_i) = \cup f(A_i)$; $f(\cap A_i) \subseteq \cap f(A_i)$; and if f is injective, then $f(\cap A_i) = \cap f(A_i)$;

(f) $f^{-1}(Y_N) = X_N$, $f^{-1}(\phi_N) = \phi_N$.

(g) $f(\phi_N) = \phi_N$, $f(X_N) = Y_N$, if f is subjective.

Proof. Obvius.

Definition 5.2 *Let (X, Γ_1) and (Y, Γ_2) be two NCTSs, and let $f : X \to Y$ be a function. Then f is said to be continuous iff the preimage of each NCS in Γ_2 is an NCS in Γ_1.*

Definition 5.3 *Let (X, Γ_1) and (Y, Γ_2) be two NCTSs and let $f : X \to Y$ be a function. Then f is said to be open iff the image of each NCS in Γ_1 is an NCS in Γ_2.*

Example 5.1 *Let (X, Γ_0) and (Y, ψ_0) be two NCTSs.*

(a) If $f : X \to Y$ is continuous in the usual sense, then in this case, f is continuous in the sense of Definition 5.1 too. Here we consider the NCTs on X and Y, respectively, as follows: $\Gamma_1 = \{\langle G, \phi, G^c \rangle : G \in \Gamma_0\}$ and $\Gamma_2 = \{\langle H, \phi, H^c \rangle : H \in \Psi_0\}$, In this case we have, for each $\langle H, \phi, H^c \rangle \in \Gamma_2$, $H \in \Psi_0$, $f^{-1}\langle H, \phi, H^c \rangle = \langle f^{-1}(H), f^{-1}(\phi), f^{-1}(H^c) \rangle = \langle f^{-1}(H), f(\phi), (f(H))^c \rangle \in \Gamma_1$.

(b) If $f : X \to Y$ is open in the usual sense, then in this case, f is open in the sense of Definition 3.2.

Now we obtain some characterizations of continuity:

Proposition 5.1 *Let $f : (X, \Gamma_1) \to (Y, \Gamma_2)$. f is continuous iff the preimage of each CNCS (crisp neutrosophic closed set) in Γ_2 is a CNCS in Γ_2.*

Proposition 5.2 *The following are equivalent to each other:*

(a) $f : (X, \Gamma_1) \to (Y, \Gamma_2)$ is continuous.

(b) $f^{-1}(CNInt(B) \subseteq CNInt(f^{-1}(B))$ for each CNS B in Y.

(c) $CNCl(f^{-1}(B)) \subseteq f^{-1}(CNCl(B))$ for each CNC B in Y.

Example 5.2 *Let (Y, Γ_2) be an NCTS and $f : X \to Y$ be a function. In this case $\Gamma_1 = \{f^{-1}(H) : H \in \Gamma_2\}$ is a NCT on X. Indeed, it is the coarsest NCT on X which makes the function $f : X \to Y$ continuous. One may call the initial neutrosophic crisp topology with respect to f.*

6 Neutrosophic Crisp Compact Space (NCCS)

First we present the basic concepts:

Definition 6.1 *Let (X, Γ) be a NCTS.*

(a) If a family $\{\langle G_{i_1}, G_{i_2}, G_{i_3}\rangle : i \in J\}$ of NCOSs in X satisfies the condition $\cup\{\langle G_{i_1}, G_{i_2}, G_{i_3}\rangle : i \in J\} = X_N$ then it is called an neutrosophic open cover of X.

(b) A finite subfamily of an open cover $\{\langle G_{i_1}, G_{i_2}, G_{i_3}\rangle : i \in J\}$ on X, which is also a neutrosophic open cover of X, is called a neutrosophic finite subcover $\{\langle G_{i_1}, G_{i_2}, G_{i_3}\rangle : i \in J\}$.

(c) A family $\{\langle K_{i_1}, K_{i_2}, K_{i_3}\rangle : i \in J\}$ of NCCSs in X satisfies the finite intersection property (FIP for short) iff every finite subfamily $\{\langle K_{i_1}, K_{i_2}, K_{i_3}\rangle : i = 1, 2, \ldots, n\}$ of the family satisfies the condition $\cap\{\langle K_{i_1}, K_{i_2}, K_{i_3}\rangle : i \in J\} \neq \phi_N$.

Definition 6.2 *A NCTS (X, Γ) is called neutrosophic crisp compact iff each crisp neutrosophic open cover of X has a finite subcover.*

Example 6.1 *(a) Let $X = N$ and consider the NCSs given below:*

$$\begin{aligned}
A_1 &= \langle\{2,3,4,\ldots\}, \phi, \phi\rangle, \\
A_2 &= \langle\{3,4,5,\ldots\}, \phi, \{1\}\rangle, \\
A_3 &= \langle\{4,5,6,\ldots\}, \phi, \{1,2\}\rangle, \\
&\vdots \\
A_n &= \langle\{n+1, n+2, n+3, \ldots\}, \phi, \{1,2,3,\ldots,n-1\}\rangle.
\end{aligned}$$

Then $\Gamma = \{\phi_N, X_N\} \cup \{A_n = 3, 4, 5, \ldots\}$ is an NCT on X and (X, Γ) is a neutrosophic crisp compact.

(b) Let $X = (0, 1)$ and let's make the NCSs

$$A_n = \left\langle X, \left(\frac{1}{n}, \frac{n-1}{n}\right), \phi, \left(0, \frac{1}{n}\right)\right\rangle,\ n = 3, 4, 5, \ldots \text{ in } X$$

In this case $\Gamma = \{\phi_N, X_n\} \cup \{A_n = 3, 4, 5, \ldots\}$ is a NCT on X, which is not a neutrosophic crisp compact.

Corollary 6.1 *A NCTS (X, Γ) is neutrosophic crisp compact iff every family $\{\langle X, G_{i_1}, G_{j_2}, G_{i_3}\rangle : i \in J\}$ of NCCSs in X having the FIP has nonempty intersection.*

Corollary 6.2 *Let (X, Γ_1), (Y, Γ_2) be NCTSs and $f : X \to Y$ be a continuous surjection. If (X, Γ_1) is a neutrosophic crisp compact, then so is (Y, Γ_2).*

Definition 6.3 *(a) If a family $\{\langle X, G_{i_1}, G_{j_2}, G_{i_3}\rangle : i \in J\}$of NCCSs in X satisfies the condition $A \subseteq \cup\{\langle G_{i_1}, G_{j_2}, G_{i_3}\rangle : i \in J\}$, then it is called a neutrosophic crisp open cover of A.*

(b) Let's consider a finite subfamily of a neutrosophic crisp open subcover of $\{\langle X, G_{i_1}, G_{j_2}, G_{i_3}\rangle : i \in J\}$. A neutrosophic crisp set $A = \langle A_1, A_2, A_3\rangle$ in a NCTS (X, Γ) is called neutrosophic crisp compact iff every neutrosophic crisp open cover of A has a finite neutrosophic crisp open subcover.

Corollary 6.3 *Let (X, Γ_1), (Y, Γ_2) be NCTSs and $f : X \to Y$ is a continuous surjection. If A is a neutrosophic crisp compact in (X, Γ_1), then so is $f(A)$ in (Y, Γ_2).*

7 Conclusion

In this paper we introduced the neutrosophic crisp topology and the neutrosophic crisp compact space. Then we presented several properties for each of them.

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Published in *Neutrosophic Sets and Systems*, 25-30, Vol. 2, 2014.

Neutrosophic Ideal Theory
Neutrosophic Local Function and Generated Neutrosophic Topology

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ABSTRACT

Abstract In this paper we introduce the notion of ideals on neutrosophic set which is considered as a generalization of fuzzy and fuzzy intuitionistic ideals studies in [9,11] , the important neutrosophic ideals has been given in [4]. The concept of neutrosophic local function is also introduced for a neutrosophic topological space. These concepts are discussed with a view to find new nutrosophic topology from the original one in [8]. The basic structure, especially a basis for such generated neutrosophic topologies and several relations between different neutrosophic ideals and neutrosophic topologies are also studied here. Possible application to GIS topology rules are touched upon.

KEYWORDS: Neutrosophic Set, Intuitionistic Fuzzy Ideal, Fuzzy Ideal, Neutrosophic Ideal, Neutrosophic Topology.

1-INTRODUCTION

The neutrosophic set concept was introduced by Smarandache [12, 13]. In 2012 neutrosophic sets have been investigated by Hanafy and Salama at el [4, 5, 6, 7, 8, 9, 10]. The fuzzy set was introduced by Zadeh [14] in 1965, where each element had a degree of membership. In 1983 the intuitionstic fuzzy set was introduced by K. Atanassov [1, 2, 3] as a generalization of fuzzy set, where besides the degree of membership and the degree of non- membership of each element. Salama at el [9] defined intuitionistic fuzzy ideal for a set and generalized the concept of fuzzy ideal concepts, first initiated by Sarker [10]. Neutrosophy has laid the foundation for a whole family of new mathematical theories generalizing both their classical and fuzzy counterparts. In this paper we will introduce the definitions of normal neutrosophic set, convex set, the concept of α-cut and neutrosophic ideals, which can be discussed as generalization of fuzzy and fuzzy intuitionistic studies.

2-TERMINOLOGIES

We recollect some relevant basic preliminaries, and in particular, the work of Smarandache in [12, 13], and Salama at el. [4, 5, 6, 7, 8, 9, 10].

3- NEUTROSOPHIC IDEALS [4].

Definition.3.1

Let X is non-empty set and L a non–empty family of NSs. We will call L is a neutrosophic ideal (NL for short) on X if

- $A \in L$ and $B \subseteq A \Rightarrow B \in L$ [heredity],

- $A \in L$ and $B \in L \Rightarrow A \vee B \in L$ [Finite additivity].

A neutrosophic ideal L is called a σ-neutrosophic ideal if $\{A_j\}_{j \in N} \leq L$, implies $\vee_{j \in J} A_j \in L$ (countable additivity).

The smallest and largest neutrosophic ideals on a non-empty set X are $\{0_N\}$ and NSs on X. Also, $N.L_f$, $N.L_c$ are denoting the neutrosophic ideals (NL for short) of neutrosophic subsets having finite and countable support of X respectively. Moreover, if A is a nonempty NS in X, then $\{B \in NS : B \subseteq A\}$ is an NL on X. This is called the principal NL of all NSs of denoted by NL $\langle A \rangle$.

Remark 3.1

- If $1_N \notin L$, then L is called neutrosophic proper ideal.
- If $1_N \in L$, then L is called neutrosophic improper ideal.
- $O_N \in L$.

Example.3.1

Any Initiutionistic fuzzy ideal ℓ on X in the sense of Salama is obviously and NL in the form $L = \{A : A = \langle x, \mu_A, \sigma_A, \nu_A \rangle \in \ell\}$.

Example.3.2

Let $X = \{a, b, c\}$ $A = \langle x, 0.2, 0.5, 0.6 \rangle$, $B = \langle x, 0.5, 0.7, 0.8 \rangle$, and $D = \langle x, 0.5, 0.6, 0.8 \rangle$, then the family $L = \{O_N, A, B, D\}$ of NSs is an NL on X.

Example.3.3

Let $X = \{a, b, c, d, e\}$ and $A = \langle x, \mu_A, \sigma_A, \nu_A \rangle$ given by:

X	$\mu_A(x)$	$\sigma_A(x)$	$\nu_A(x)$
a	0.6	0.4	0.3
b	0.5	0.3	0.3
c	0.4	0.6	0.4
d	0.3	0.8	0.5
e	0.3	0.7	0.6

Then the family $L = \{O_N, A\}$ is an NL on X.

Definition.3.3

Let L_1 and L_2 be two NL on X. Then L_2 is said to be finer than L_1 or L_1 is coarser than L_2 if $L_1 \leq L_2$. If also $L_1 \neq L_2$. Then L_2 is said to be strictly finer than L_1 or L_1 is strictly coarser than L_2.

Two NL said to be comparable, if one is finer than the other. The set of all NL on X is ordered by the relation L_1 is coarser than L_2 this relation is induced the inclusion in NSs.

The next Proposition is considered as one of the useful result in this sequel, whose proof is clear.

Proposition.3.1

Let $L_j : j \in J$ be any non - empty family of neutrosophic ideals on a set X. Then $\bigcap_{j \in J} L_j$ and $\bigcup_{j \in J} L_j$ are neutrosophic ideal on X,

In fact L is the smallest upper bound of the set of the L_j in the ordered set of all neutrosophic ideals on X.

Remark.3.2

The neutrosophic ideal by the single neutrosophic set O_N is the smallest element of the ordered set of all neutrosophic ideals on X.

Proposition.3.3

A neutrosophic set A in neutrosophic ideal L on X is a base of L iff every member of L contained in A.

Proof

(Necessity)Suppose A is a base of L. Then clearly every member of L contained in A.

(Sufficiency) Suppose the necessary condition holds. Then the set of neutrosophic subset in X contained in A coincides with L by the Definition 4.3.

Proposition.3.4

For a neutrosophic ideal L_1 with base A, is finer than a fuzzy ideal L_2 with base B iff every member of B contained in A.

Proof

Immediate consequence of Definitions

Corollary.3.1

Two neutrosophic ideals bases A, B, on X are equivalent iff every member of A, contained in B and via versa.

Theorem.3.1

Let $\eta = \langle \mu_j, \sigma_j, \gamma_j \rangle : j \in J$ be a non empty collection of neutrosophic subsets of X. Then there exists a neutrosophic ideal L (η) = {A ∈ NSs: A $\subseteq \vee A_j$} on X for some finite collection {A_j: j = 1,2, ..., n $\subseteq \eta$}.

Proof

Clear.

Remark.3.3

ii) The neutrosophic ideal L (η) defined above is said to be generated by η and η is called sub base of L(η).

Corollary.3.2

Let L_1 be an neutrosophic ideal on X and A ∈ NSs, then there is a neutrosophic ideal L_2 which is finer than L_1

and such that A $\in$ L_2 iff

A $\vee$ B $\in$ L_2 for each B$\in$ L_1.

Corollary.3.3

Let $A=\langle x,\mu_A,\sigma_A,\nu_A\rangle \in L_1$ and $B=\langle x,\mu_B,\sigma_B,\nu_B\rangle \in L_2$, where L_1 and L_2 are neutrosophic ideals on the set X. then the neutrosophic set $A*B=\langle \mu_{A*B}(x),\sigma_{A*B}(x),\nu_{A*B}(x)\rangle \in L_1 \vee L_2$ on X where $\mu_{A*B}(x)=\vee(\mu_A(x)\wedge\mu_B(x)):x\in X)$, $\sigma_{A*B}(x)$ may be $=\vee(\sigma_A(x)\wedge\sigma_B(x))$ or $\wedge(\sigma_A(x)\vee\sigma_B(x))$ and $\nu_{A*B}(x)=\wedge(\nu_A(x)\vee\nu_B(x)):x\in X)$.

4. Neutrosophic local Functions

Definition.4.1. Let (X,τ) be a neutrosophic topological spaces (NTS for short) and L be neutrsophic ideal (NL, for short) on X. Let A be any NS of X. Then the neutrosophic local function $NA^*(L,\tau)$ of A is the union of all neutrosophic points(NP, for short) $C(\alpha,\beta,\gamma)$ such that if $U\in N(C(\alpha,\beta,\gamma))$ and $NA^*(L,\tau)=\vee\{C(\alpha,\beta,\gamma)\in X: A\wedge U\notin L$ for every U nbd of $C(\alpha,\beta,\gamma)\}$, $NA^*(L,\tau)$ is called a neutrosophic local function of A with respect to τ and L which it will be denoted by $NA^*(L,\tau)$, or simply $NA^*(L)$.

Example .4.1. One may easily verify that.

If L= $\{0_N\}$, then N $A^*(L,\tau)=Ncl(A)$, for any neutrosophic set $A\in NSs$ on X.

If $L=\{$all NSs on X$\}$ then $NA^*(L,\tau)=0_N$, for any $A\in NSs$ on X .

Theorem.4.1. Let (X,τ) be a NTS and L_1,L_2 be two neutrosophic ideals on X. Then for any neutrosophic sets A, B of X. then the following statements are verified

i) $A\subseteq B\Rightarrow NA^*(L,\tau)\subseteq NB^*(L,\tau)$,

ii) $L_1\subseteq L_2\Rightarrow NA^*(L_2,\tau)\subseteq NA^*(L_1,\tau)$.

iii) $NA^*=Ncl(A^*)\subseteq Ncl(A)$.

iv) $NA^{**}\subseteq NA^*$.

v) $N(A\vee B)^*=NA^*\vee NB^*$.,

vi) $N(A\wedge B)^*(L)\le NA^*(L)\wedge NB^*(L)$.

vii) $\ell\in L\Rightarrow N(A\vee\ell)^*=NA^*$.

viii) $NA^*(L,\tau)$ is neutrosophic closed set .

Proof.

i) Since $A\subseteq B$, let $p=C(\alpha,\beta,\gamma)\in NA^*(L_1)$ then $A\wedge U\notin L$ for every $U\in N(p)$. By hypothesis we get $B\wedge U\notin L$, then $p=C(\alpha,\beta,\gamma)\in NB^*(L_1)$.

ii) Clearly. $L_1\subseteq L_2$ implies $NA^*(L_2,\tau)\subseteq NA^*(L_1,\tau)$ as there may be other IFSs which belong to L_2 so that for GIFP $p=C(\alpha,\beta,\gamma)\in NA^*$ but $C(\alpha,\beta,\gamma)$ may not be contained in $NA^*(L_2)$.

iii) Since $\{O_N\}\subseteq L$ for any NL on X, therefore by (ii) and Example 3.1, $NA^*(L)\subseteq NA^*(O_N)=Ncl(A)$ for any NS A on X. Suppose $p_1=C_1(\alpha,\beta,\gamma)\in Ncl(NA^*(L_1))$. So for every $U\in N(p_1)$, $NA^*\wedge U\neq O_N$, there exists $p_2=C_2(\alpha,\beta)\in A^*(L_1)\wedge U)$ such that for every V nbd of $p_2\in N(p_2)$, $A\wedge U\notin L.$ Since $U\wedge V\in N(p_2)$ then $A\wedge(U\cap V)\notin L$ which leads to $A\wedge U\notin L$, for every $U\in N(C(\alpha,\beta))$ therefore $p_1=C(\alpha,\beta)\in(A^*(L))$

and so $Ncl(NA^*) \leq NA^*$ While, the other inclusion follows directly. Hence $NA^* = Ncl(NA^*)$.But the inequality $NA^* \leq Ncl(NA^*)$.

iv) The inclusion $NA^* \vee NB^* \leq N(A \vee B)^*$ follows directly by (i). To show the other implication, let $p = C(\alpha,\beta,\gamma) \in N(A \vee B)^*$ then for every $U \in N(p)$, $(A \vee B) \wedge U \notin L, i.e, (A \wedge U) \vee (B \wedge U) \notin L.$ then, we have two cases $A \wedge U \notin L$ and $B \wedge U \in L$ or the converse, this means that exist $U_1, U_2 \in N(C(\alpha,\beta,\gamma))$ such that $A \wedge U_1 \notin L$, $B \wedge U_1 \notin L$, $A \wedge U_2 \notin L$ and $B \wedge U_2 \notin L$. Then $A \wedge (U_1 \wedge U_2) \in L$ and $B \wedge (U_1 \wedge U_2) \in L$ this gives $(A \vee B) \wedge (U_1 \wedge U_2) \in L$, $U_1 \wedge U_2 \in N(C(\alpha,\beta,\gamma))$ which contradicts the hypothesis. Hence the equality holds in various cases.

vi) By (iii), we have $NA^{**} = Ncl(NA^*)^* \leq Ncl(NA^*) = NA^*$

Let (X,τ) be a GIFTS and L be GIFL on X . Let us define the neutrosophic closure operator $cl^*(A) = A \cup A^*$ for any GIFS A of X. Clearly, let $Ncl^*(A)$ is a neutrosophic operator. Let $N\tau^*(L)$ be NT generated by Ncl^* .i.e $N\tau^*(L) = \{A : Ncl^*(A^c) = A^c\}$. Now $L = \{O_N\} \Rightarrow Ncl^*(A) = A \cup NA^* = A \cup Ncl(A)$ for every neutrosophic set A. So, $N\tau^*(\{O_N\}) = \tau$. Again $L = \{all$ NSs on X$\} \Rightarrow Ncl^*(A) = A,$ because $NA^* = O_N$, for every neutrosophic set A so $N\tau^*(L)$ is the neutrosophic discrete topology on X. So we can conclude by Theorem 4.1.(ii). $N\tau^*(\{O_N\}) = N\tau^*(L)$ i.e. $N\tau \subseteq N\tau^*$, for any neutrosophic ideal L_1 on X. In particular, we have for two neutrosophic ideals L_1, and L_2 on X, $L_1 \subseteq L_2 \Rightarrow N\tau^*(L_1) \subseteq N\tau^*(L_2)$.

Theorem.4.2. Let τ_1, τ_2 be two neutrosophic topologies on X. Then for any neutrosophic ideal L on X, $\tau_1 \leq \tau_2$ implies $NA^*(L,\tau_2) \subseteq NA^*(L,\tau_1)$, for every A $\in L$ then $N\tau^*_1 \subseteq N\tau^*_2$

Proof. Clear.

A basis $N\beta(L,\tau)$ for $N\tau^*(L)$ can be described as follows:

$N\beta(L,\tau) = \{A - B : A \in \tau, B \in L\}$ Then we have the following theorem

Theorem 4.3. $N\beta(L,\tau) = \{A - B : A \in \tau, B \in L\}$ Forms a basis for the generated NT of the NT (X,τ) with neutrosophic ideal L on X.

Proof. Straight forward.

The relationship between τ and $N\tau^*$(L) established throughout the following result which have an immediately proof .

Theorem 4.4. Let τ_1, τ_2 be two neutrosophic topologies on X. Then for any neutrosophic ideal L on X, $\tau_1 \subseteq \tau_2$ implies $N\tau^*_1 \subseteq N\tau^*_2$.

Theorem 4.5 : Let (X,τ) be a NTS and L_1, L_2 be two neutrosophic ideals on X . Then for any neutrosophic set A in X, we have

i) $NA^*(L_1 \vee L_2,\tau) = NA^*(L_1, N\tau^*(L_1)) \wedge NA^*(L_2, N\tau^*(L_2))$. ii) $N\tau^*(L_1 \vee L_2) = (N\tau^*(L_1))^*(L_2) \wedge N(\tau^*(L_2))^*(L_1)$

Proof Let $p = C(\alpha,\beta) \notin (L_1 \vee L_2, \tau)$, this means that there exists $U_p \in N(P)$ such that $A \wedge U_p \in (L_1 \vee L_2)$ i.e. There exists $\ell_1 \in L_1$ and $\ell_2 \in L_2$ such that $A \wedge U_p \in (\ell_1 \vee \ell_2)$ because of the heredity of L_1, and assuming $\ell_1 \wedge \ell_2 = O_N$.Thus we have $(A \wedge U_p) - \ell_1 = \ell_2$ and $(A \wedge U_p) - \ell_2 = \ell_1$ therefore $(U_p - \ell_1) \wedge A = \ell_2 \in L_2$ and $(U_p - \ell_2) \wedge A = \ell_1 \in L_1$. Hence $p = C(\alpha,\beta,\gamma) \notin NA^*(L_2, N\tau^*(L_1))$ or $p = C(\alpha,\beta,\gamma) \notin NA^*(L_1, N\tau^*(L_2))$ because p must belong to either ℓ_1 or ℓ_2 but not to both. This gives $NA^*(L_1 \vee L_2, \tau) \geq NA^*(L_1, N\tau^*(L_1)) \wedge NA^*(L_2, N\tau^*(L_2))$. .To show the second inclusion, let us assume $p = C(\alpha,\beta,\gamma) \notin NA^*(L_1, N\tau^*(L_2))$. This implies that there exist $U_p \in N(P)$ and $\ell_2 \in L_2$ such that $(U_p - \ell_2) \wedge A \in L_1$. By the heredity of L_2, if we assume that $\ell_2 \leq A$ and define $\ell_1 = (U_p - \ell_2) \wedge A$. Then we have $A \wedge U_p \in (\ell_1 \vee \ell_2) \in L_1 \vee L_2$. Thus,

$NA^*(L_1 \vee L_2, \tau) \leq NA^*(L_1, \tau^*(L_1)) \wedge NA^*(L_2, N\tau^*(L_2))$ and similarly, we can get $A^*(L_1 \vee L_2, \tau) \leq A^*(L_2, \tau^*(L_1))$. This gives the other inclusion, which complete the proof.

Corollary 4.1. Let (X, τ) be a NTS with neutrosophic ideal L on X. Then

i) $NA^*(L,\tau) = NA^*(L,\tau^*)$ and $N\tau^*(L) = N(N\tau^*(L))^*(L)$.

ii) $N\tau^*(L_1 \vee L_2) = (N\tau^*(L_1)) \vee (N\tau^*(L_2))$

Proof. Follows by applying the previous statement.

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Neutrosophic Multirelations and Their Properties

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Abstract

In this paper, the neutrrosophic multi relation (NMR) defined on the neutrosophic multisets [18] is introduced. Various properties like reflexivity,symmetry and transitivity are studied.

Keyword 0.1 *Neutrosophic sets, neutrosophic multisets, neutrosophic multi relations, reflexivity, symmetry, transitivity.*

1 Introduction

Recently, several theories have been proposed to deal with uncertainty, imprecision and vagueness. Theory of probability, fuzzy set theory[40], intuitionistic fuzzy sets[7], rough set theory[25] etc. are consistently being utilized as efficient tools for dealing with diverse types of uncertainties and imprecision embedded in a system. However, All these above theories failed to deal with indeterminate and inconsistent information which exist in beliefs system. In 1995, inspired from the sport games (wining/tie/defeating), from votes (yes/ NA/ No), from decision making (making a decision/ hesitating/not making) etc. and guided by the fact that the law of excluded middle did not work any longer in the modern logics, F.Smarandache[36] developed a new concept called neutrosophic set (NS) which generalizes fuzzy sets and intuitionistic fuzzy sets. NS can be described by membership degree, indeterminate degree and non-membership degree. This theory and their hybrid structures has proven useful in many different fields such as control theory[1], databases[2, 3], medical diagnosis problem[4], decision making problem [16, 21], physics[26], topology [22], etc. The works on neutrosophic set, in theories and applications, have been progressing rapidly (e.g. [5, 6, 10]).

Combining neutrosophic set models with other mathematical models has attracted the attention of many researchers. Maji et al.[23] presented the concept of neutrosophic soft set which is based on a combination of the neutrosophic set and soft set models. Broumi and Smarandache[8, 11] introduced the concept of the intuitionistic neutrosophic soft set by combining the intuitionistic neutrosophic sets set and soft set. Broumi et al. presented the concept of rough neutrosophic set[14] which is based on a combination of the neutrosophic set and rough set models. The works on neutrosophic set combining soft sets, in theories and applications, have been progressing rapidly (e.g. [9, 12, 13, 19]).

The notion of multiset was formulated first in [39] by Yager as generalization of the concept of set theory and then the set developed in [15] by Calude et al. Several authors from time to time made a number of generalization of set theory. For example, Sebastian and Ramakrishnan[34, 33] introduced a new notion is called multi fuzzy set, which is a generalization of multiset. Since then, Several researcher[24, 32, 37, 38] discuussed more properties on multi fuzzy set. [35, 20] made an extension of the concept of Fuzzy multisets by an intuitionstic fuzzy set, which called intuitionstic fuzzy multisets(IFMS). Since then in the study on IFMS , a lot of excellent results have been achieved by researcher [17, 27, 28, 29, 30, 31]. An element of a multi fuzzy sets can occur more than once with possibly the same or different membership values, whereas an element of intuitionistic fuzzy multisets allows the repeated occurrences of membership and non–membership values. The concepts of FMS and IFMS fails to deal with indeterminacy. Therefore Deli et al.[18] give neutrosophic multisets.

The neutrosophic relations are the neutrosophic subsets in a cartesian product of universe. The purpose of this paper is an attempt to extend the neutrosophic relations to neutrosophic multi relations(NMR). This paper is arranged in the following manner. In section 2, we present some definitions and notion about intuitionstic fuzzy set, intuitionstic fuzzy multisets, neutrosophic set and neutrosophic multi set theory which is help us in later section. In section 3, we study the concept of neutrosophic multisets and their operations. In section 4, we present an application of NMR in medical diagnosis. Finally, we conclude the paper.

2 Preliminary

In this section, we mainly recall some notions related to neutrosophic sets[36] relevant to the present work. See especially[2, 3, 4, 5, 6, 10, 16, 21, 22, 26] for further details and background.

Definition 2.1 *[36] Let U be a space of points (objects), with a generic element in U denoted by u. A neutrosophic sets(N-sets) A in U is characterized by a truth-membership function T_A, a indeterminacy-membership function I_A and a falsity-membership function F_A. $T_A(x)$; $I_A(x)$ and $F_A(x)$ are real standard or nonstandard subsets of $[0, 1]$. It can be written as*

$$A = \{< u, (T_A(x), I_A(x), F_A(x)) >: x \in E, \, T_A(x), I_A(x), F_A(x) \in [0, 1]\}.$$

There is no restriction on the sum of $T_A(x)$; $I_A(x)$ and $F_A(x)$, so $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$.

Definition 2.2 *[18] Let E be a universe. A neutrosophic multiset(NMS) A on E can be defined as follows:*

$$\begin{aligned} A \;&= \{< x, (T_A^1(x), T_A^2(x), ..., T_A^P(x)), (I_A^1(x), I_A^2(x), ..., I_A^P(x)), \\ &\qquad (F_A^1(x), F_A^2(x), ..., F_A^P(x)) >: \; x \in E\} \end{aligned}$$

where,

$$T_A^1(x), T_A^2(x), ..., T_A^P(x) : E \to [0, 1],$$
$$I_A^1(x), I_A^2(x), ..., I_A^P(x) : E \to [0, 1],$$

and

$$F_A^1(x), F_A^2(x), ..., F_A^P(x) : E \to [0, 1]$$

such that

$$0 \leq T_A^i(x) + I_A^i(x) + F_A^i(x) \leq 3$$

$(i = 1, 2, ..., P)$ and

$$T_A^1(x) \leq T_A^2(x) \leq ... \leq T_A^P(x)$$

for any $x \in E$.
$(T_A^1(x), T_A^2(x), ..., T_A^P(x))$, $(I_A^1(x), I_A^2(x), ..., I_A^P(x))$ and $(F_A^1(x), F_A^2(x), ..., F_A^P(x))$ is the truth-membership sequence, indeterminacy-membership sequence and falsity-membership sequence of the element x, respectively. Also, P is called the dimension(cardinality) of NMS A. We arrange the truth-membership sequence in decreasing order but the corresponding indeterminacy-membership and falsity-membership sequence may not be in decreasing or increasing order.

The set of all Neutrosophic multisets on E is denoted by NMS(E).

Definition 2.3 *[18] Let $A, B \in NMS(E)$. Then,*

1. *A is said to be NM subset of B is denoted by* $A \tilde{\subseteq} B$ *if* $T_A^i(x) \leq T_B^i(x)$, $I_A^i(x) \geq I_B^i(x)$,$F_A^i(x) \geq F_B^i(x)$, $\forall x \in E$.

2. *A is said to be neutrosophic equal of B is denoted by* $A = B$ *if* $T_A^i(x) = T_B^i(x)$, $I_A^i(x) = I_B^i(x)$,$F_A^i(x) = F_B^i(x)$, $\forall x \in E$.

3. *the complement of A denoted by* $A^{\tilde{c}}$ *and is defined by*

$$A^{\tilde{c}} = \{< x, (F_A^1(x), F_A^2(x), ..., F_A^P(x)), (I_A^1(x), I_A^2(x), ..., I_A^P(x)), (T_A^1(x), T_A^2(x), ..., T_A^P(x)) >: x \in E\}$$

4. *If* $T_A^i(x) = 0$ *and* $I_A^i(x) = F_A^i(x) = 1$ *for all* $x \in E$ *and* $i = 1, 2, ..., P$ *then A is called null* ns-*set and denoted by* $\tilde{\Phi}$.

5. *If* $T_A^i(x) = 1$ *and* $I_A^i(x) = F_A^i(x) = 0$ *for all* $x \in E$ *and* $i = 1, 2, ..., P$, *then A is called universal* ns-*set and denoted by* $\tilde{E}$.

Definition 2.4 *[18] Let* $A, B \in NMS(E)$. *Then,*

1. *the union of A and B is denoted by* $A \tilde{\cup} B = C_1$ *and is defined by*

$$C = \{< x, (T_C^1(x), T_C^2(x), ..., T_C^P(x)), (I_C^1(x), I_C^2(x), ..., I_C^P(x)), (F_C^1(x), F_C^2(x), ..., F_C^P(x)) >: x \in E\}$$

where $T_C^i = T_A^i(x) \vee T_B^i(x)$, $I_C^i = I_A^i(x) \wedge I_B^i(x)$,$F_C^i = F_A^i(x) \wedge F_B^i(x)$, $\forall x \in E$ *and* $i = 1, 2, ..., P$.

2. *the intersection of A and B is denoted by* $A \tilde{\cap} B = D$ *and is defined by*

$$D = \{< x, (T_D^1(x), T_D^2(x), ..., T_D^P(x)), (I_D^1(x), I_D^2(x), ..., I_D^P(x)), (F_D^1(x), F_D^2(x), ..., F_D^P(x)) >: x \in E\}$$

where $T_D^i = T_A^i(x) \wedge T_B^i(x)$, $I_D^i = I_A^i(x) \vee I_B^i(x)$,$F_D^i = F_A^i(x) \vee F_B^i(x)$, $\forall x \in E$ *and* $i = 1, 2, ..., P$.

3. *the addition of A and B is denoted by* $A \tilde{+} B = E_1$ *and is defined by*

$$E_1 = \{< x, (T_{E_1}^1(x), T_{E_1}^2(x), ..., T_{E_1}^P(x)), (I_{E_1}^1(x), I_{E_1}^2(x), ..., I_{E_1}^P(x)), (F_{E_1}^1(x), F_{E_1}^2(x), ..., F_{E_1}^P(x)) >: x \in E\}$$

where $T_{E_1}^i = T_A^i(x) + T_B^i(x) - T_A^i(x).T_B^i(x)$, $I_{E_1}^i = I_A^i(x).I_B^i(x)$,$F_{E_1}^i = F_A^i(x).F_B^i(x)$, $\forall x \in E$ *and* $i = 1, 2, ..., P$.

4. *the multiplication of A and B is denoted by* $A \tilde{\times} B = E_2$ *and is defined by*

$$E_2 = \{< x, (T_{E_2}^1(x), T_{E_2}^2(x), ..., T_{E_2}^P(x)), (I_{E_2}^1(x), I_{E_2}^2(x), ..., I_{E_2}^P(x)), (F_{E_2}^1(x), F_{E_2}^2(x), ..., F_{E_2}^P(x)) >: x \in E\}$$

where $T_{E_2}^i = T_A^i(x).T_B^i(x)$, $I_{E_2}^i = I_A^i(x) + I_B^i(x) - I_A^i(x).I_B^i(x)$,$F_{E_2}^i = F_A^i(x) + F_B^i(x) - F_A^i(x).F_B^i(x)$, $\forall x \in E$ *and* $i = 1, 2, ..., P$.

Here $\vee$, $\wedge$, $+$, $.$, $-$ *denotes maximum, minimum, addition, multiplication, subtraction of real numbers respectively.*

3 Relations on Neutrosophic Multisets

In this section, after given the cartesian products of two neutrosophic multisets, we define a relations on neutrosophic multisets and study their desired properties. The relation extend the concept of intuitionistic multirelation [29] to neutrosophic multirelation. Some of it is quoted from [18, 29, 36].

Definition 3.1 *Let $\emptyset \neq A, B \in NMS(E)$ and $j \in \{1, 2, ..., n\}$. Then, cartesian product of A and B is a neutrosophic multiset in $E \times E$, denoted by $A \times B$, defined as*

$$A \times B = \{< (x, y), T^j_{A\times B}(x, y)), I^j_{A\times B}(x, y), F^j_{A\times B}(x, y) >: (x, y) \in E \times E\}$$

where

$$T^j_{A\times B}(x, y), I^j_{A\times B}(x, y), F^j_{A\times B}(x, y) : E \to [0, 1]$$

,

$$T^j_{A\times B}(x, y) = \min\left\{T^j_A(x), T^j_B(x)\right\},$$

$$I^j_{A\times B}(x, y) = \max\left\{I^j_A(x), I^j_B(x)\right\}$$

and

$$F^j_{A\times B}(x, y) = \max\left\{F^j_A(x), F^j_B(x)\right\}$$

for all $x, y \in E$.

Remark 3.2 *A cartesian product on A is a neutrosophic multiset in $E \times E$, denoted by $A \times A$, defined as*

$$A \times A = \{< (x, y), T^j_{A\times A}(x, y)), I^j_{A\times A}(x, y), F^j_{A\times A}(x, y) >: (x, y) \in E \times E\}$$

where $j = 1, 2, ..., n$ and $T^j_{A\times A}, I^j_{A\times A}, F^j_{A\times A} : E \times E \to [0, 1]$.

Definition 3.3 *Let $\emptyset \neq A, B \in NMS(E)$ and $j \in \{1, 2, ..., n\}$. Then, a neutrosophic multi relation from A to B is a neutrosophic multi subset of $A \times B$. In other words, a neutrosophic multi relation from A to B is of the form (R, C), $(C \subseteq E \times E)$ where $R(x, y) \subseteq A \times B$ $\forall (x, y) \in C$.*

Definition 3.4 *Let $A, B \in NMS(E)$ and, R and S be two neutrosophic multirelation from A to B. Then, the operations $R\tilde{\cup}S$, $R\tilde{\cap}S$, $R\tilde{+}S$ and $R\tilde{\times}S$ are defined as follows;*

1.

$$\begin{aligned} R\tilde{\cup}S = \; & \{< (x, y), (T^1_{R\tilde{\cup}S}(x, y), T^2_{R\tilde{\cup}S}(x, y), ..., T^n_{R\tilde{\cup}S}(x, y)), \\ & (I^1_{R\tilde{\cup}S}(x, y), I^2_{R\tilde{\cup}S}(x, y), ..., I^n_{R\tilde{\cup}S}(x, y)), \\ & (F^1_{R\tilde{\cup}S}(x, y), F^2_{R\tilde{\cup}S}(x, y), ..., F^n_{R\tilde{\cup}S}(x, y)) >: \; x, y \in E\} \end{aligned}$$

where

$$T^i_{R\tilde{\cup}S}(x,y) = T^i_R(x) \vee T^i_S(y),$$
$$I^i_{R\tilde{\cup}S}(x,y) = I^i_R(x) \wedge I^i_S(y),$$
$$F^i_{R\tilde{\cup}S}(x,y) = F^i_R(x) \wedge F^i_S(y)$$

$\forall x, y \in E$ *and* $i = 1, 2, ..., n.$

2.

$$\begin{aligned} R\tilde{\cap}S = \{ &< (x,y), (T^1_{R\tilde{\cap}S}(x,y), T^2_{R\tilde{\cap}S}(x,y), ..., T^n_{R\tilde{\cap}S}(x,y)), \\ &(I^1_{R\tilde{\cap}S}(x,y), I^2_{R\tilde{\cap}S}(x,y), ..., I^n_{R\tilde{\cap}S}(x,y)), \\ &(F^1_{R\tilde{\cap}S}(x,y), F^2_{R\tilde{\cap}S}(x,y), ..., F^n_{R\tilde{\cap}S}(x,y)) >: \ x, y \in E\} \end{aligned}$$

where

$$T^i_{R\tilde{\cap}S}(x,y) = T^i_R(x) \wedge T^i_S(y),$$
$$I^i_{R\tilde{\cap}S}(x,y) = I^i_R(x) \vee I^i_S(y),$$
$$F^i_{R\tilde{\cap}S}(x,y) = F^i_R(x) \vee F^i_S(y)$$

$\forall x, y \in E$ *and* $i = 1, 2, ..., n.$

3.

$$\begin{aligned} R\tilde{+}S = \{ &< (x,y), (T^1_{R\tilde{+}S}(x,y), T^2_{R\tilde{+}S}(x,y), ..., T^n_{R\tilde{+}S}(x,y)), \\ &(I^1_{R\tilde{+}S}(x,y), I^2_{R\tilde{+}S}(x,y), ..., I^n_{R\tilde{+}S}(x,y)), \\ &(F^1_{R\tilde{+}S}(x,y), F^2_{R\tilde{+}S}(x,y), ..., F^n_{R\tilde{+}S}(x,y)) >: \ x, y \in E\} \end{aligned}$$

where

$$T^i_{R\tilde{+}S}(x,y) = T^i_R(x) + T^i_S(y) - T^i_R(x).T^i_S(y),$$
$$I^i_{R\tilde{+}S}(x,y) = I^i_R(x).I^i_S(y),$$
$$F^i_{R\tilde{+}S}(x,y) = F^i_R(x).F^i_S(y)$$

$\forall x, y \in E$ *and* $i = 1, 2, ..., n.$

4.

$$\begin{aligned} R\tilde{\times}S = \{ &< (x,y), (T^1_{R\tilde{\times}S}(x,y), T^2_{R\tilde{\times}S}(x,y), ..., T^n_{R\tilde{\times}S}(x,y)), \\ &(I^1_{R\tilde{\times}S}(x,y), I^2_{R\tilde{\times}S}(x,y), ..., I^n_{R\tilde{\times}S}(x,y)), \\ &(F^1_{R\tilde{\times}S}(x,y), F^2_{R\tilde{\times}S}(x,y), ..., F^n_{R\tilde{\times}S}(x,y)) >: \ x, y \in E\} \end{aligned}$$

where

$$T^i_{R\tilde{\times}S}(x,y) = T^i_R(x).T^i_S(y),$$
$$I^i_{R\tilde{\times}S}(x,y) = I^i_R(x) + I^i_S(y) - I^i_R(x).I^i_S(y),$$
$$F^i_{R\tilde{\times}S}(x,y) = F^i_R(x) + F^i_S(y) - F^i_R(x).F^i_S(y)$$

$\forall x, y \in E$ *and* $i = 1, 2, ..., n.$

Here $\vee$, $\wedge$, $+$, $.$, $-$ denotes maximum, minimum, addition, multiplication, subtraction of real numbers respectively.

Assume that $\emptyset \neq A, B, C \in NMS(E)$. Two neutrosophic multirelations under a suitable composition, could too yield a new neutrosophic multirelation with a useful significance. Composition of relations is important for applications, because of the reason that if a relation on A and B is known and if a relation on B and C is known then the relation on A and C could be computed and defined as follows;

Definition 3.5 *Let R(A→ B) and S (B→ C) be two neutrosophic multirelations. The composition S ∘R is a neutrosophic multirelation from A to C, defined by*

$$S \circ R = \{< (x,z), (T^1_{S\circ R}(x,z), T^2_{S\circ R}(x,z), ..., T^n_{S\circ R}(x,z)), (I^1_{S\circ R}(x,z), I^2_{S\circ R}(x,z), ..., I^n_{S\circ R}(x,z)), (F^1_{S\circ R}(x,z), F^2_{S\circ R}(x,z), ..., F^n_{S\circ R}(x,z)) >: x, z \in E\}$$

where

$$T^j_{S\circ R}(x,z) = \bigvee_y \left\{ T^j_R(x,y) \wedge T^j_S(y,z) \right\}$$

$$I^j_{S\circ R}(x,z) = \bigwedge_y \left\{ I^j_R(x,y) \vee I^j_S(y,z) \right\}$$

and

$$F^j_{S\circ R}(x,z) = \bigwedge_y \left\{ F^j_R(x,y) \vee F^j_S(y,z) \right\}$$

for every (x, z) $E \times E$, for every $y \in E$ and $j = 1, 2, ..., n$.

Definition 3.6 *A neutrosophic multirelation R on A is said to be;*

1. *reflexive if $T^j_R(x,x) = 1$, $I^j_R(x,x) = 0$ and $F^j_R(x,x) = 0$ for all $x \in E$*
2. *symmetric if $T^j_R(x,y) = T^j_R(y,x)$, $I^j_R(x,y) = I^j_R(y,x)$ and $F^j_R(x,y) = F^j_R(y,x)$ for all $x, y \in E$*
3. *transitive if $R \circ R \subseteq R$.*
4. *neutrosophic multi equivalence relation if the relation R satisfies reflexive, symmetric and transitive.*

Definition 3.7 *The transitive closure of a neutrosophic multirelation R on $E \times E$ is $\hat{R} = R\tilde{\cup}R^2\tilde{\cup}R^3\tilde{\cup}...$*

Definition 3.8 *If R is a neutrosophic multirelation from A to B then R^{-1} is the inverse neutrosophic multirelation R from B to A, defined as follows:*

$$R^{-1} = \left\{ \left\langle (y,x), T^j_{R^{-1}}(x,y)), I^j_{R^{-1}}(x,y), F^j_{R^{-1}}(x,y) \right\rangle : (x,y) \in E \times E \right\}$$

where
$T^j_{R^{-1}}(x,y) = T^j_R(y,x)$, $I^j_{R^{-1}}(x,y) = I^j_R(y,x)$, $F^j_{R^{-1}}(x,y) = F^j_R(y,x)$ *and* $j = 1, 2, ..., n$.

Proposition 3.9 *If R and S are two neutrosophic multirelation from A to B and B to C, respectively. Then,*

1. $(R^{-1})^{-1} = R$
2. $(S \circ R)^{-1} = R^{-1} \circ S^{-1}$

Proof

1. Since R^{-1} is a neutrosophic multirelation from B to A, we have $T^j_{R^{-1}}(x,y) = T^j_R(y,x)$, $I^j_{R^{-1}}(x,y) = I^j_R(y,x)$ and $F^j_{R^{-1}}(x,y) = F^j_R(y,x)$ Then,

$$T^j_{(R^{-1})^{-1}}(x,y) = T^j_{R^{-1}}(y,x) = T^j_R(x,y),$$
$$I^j_{(R^{-1})^{-1}}(x,y) = I^j_{R^{-1}}(y,x) = I^j_R(x,y)$$

and

$$F^j_{(R^{-1})^{-1}}(x,y) = F^j_{R^{-1}}(y,x) = F^j_R(x,y)$$

therefore $(R^{-1})^{-1} = R$.

2. If the composition $S \circ R$ is a neutrosophic multirelation from A to C, then the compostion $R^{-1} \circ S^{-1}$ is a neutrosophic multirelation from C to A. Then,

$$\begin{aligned}
T^j_{(S\circ R)^{-1}}(z,x) &= T^j_{(S\circ R)}(x,z) \\
&= \bigvee_y \left\{ T^j_R(x,y) \wedge T^j_S(y,z) \right\} \\
&= \bigvee_y \left\{ T^j_{R^{-1}}(y,x) \wedge T^j_{S^{-1}}(z,y) \right\} \\
&= \bigvee_y \left\{ T^j_{S^{-1}}(z,y) \wedge T^j_{R^{-1}}(y,x) \right\} \\
&= T^j_{R^{-1}\circ S^{-1}}(z,x)
\end{aligned} ,$$

$$\begin{aligned}
I^j_{(S\circ R)^{-1}}(z,x) &= I^j_{(S\circ R)}(x,z) \\
&= \bigwedge_y \left\{ I^j_R(x,y) \vee I^j_S(y,z) \right\} \\
&= \bigwedge_y \left\{ I^j_{R^{-1}}(y,x) \vee I^j_{S^{-1}}(z,y) \right\} \\
&= \bigwedge_y \left\{ I^j_{S^{-1}}(z,y) \vee I^j_{R^{-1}}(y,x) \right\} \\
&= I^j_{R^{-1}\circ S^{-1}}(z,x)
\end{aligned}$$

and

$$\begin{aligned}
F^j_{(S\circ R)^{-1}}(z,x) &= F^j_{(S\circ R)}(x,z) \\
&= \bigwedge_y \left\{ F^j_R(x,y) \vee F^j_S(y,z) \right\} \\
&= \bigwedge_y \left\{ F^j_{R^{-1}}(y,x) \vee F^j_{S^{-1}}(z,y) \right\} \\
&= \bigwedge_y \left\{ F^j_{S^{-1}}(z,y) \vee F^j_{R^{-1}}(y,x) \right\} \\
&= F^j_{R^{-1}\circ S^{-1}}(z,x)
\end{aligned}$$

Finally; proof is valid.

Proposition 3.10 *If R is symmetric ,then $R^{-1}$is also symmetric.*

Proof: Assume that R is Symmetric then we have

$$T_R^j(x,y) = T_R^j(y,x),$$

$$I_R^j(x,y) = I_R^j(y,x)$$

and

$$F_R^j(x,y) = F_R^j(y,x)$$

Also if R^{-1} is an inverse relation, then we have

$$T_{R^{-1}}^j(x,y) = T_R^j(y,x),$$

$$I_{R^{-1}}^j(x,y) = I_R^j(y,x)$$

and

$$F_{R^{-1}}^j(x,y) = F_R^j(y,x)$$

for all $x, y \in E$

To prove R^{-1} is symmetric, it is enough to prove

$$T_{R^{-1}}^j(x,y) = T_{R^{-1}}^j(y,x),$$

$$I_{R^{-1}}^j(x,y) = I_{R^{-1}}^j(y,x)$$

and

$$F_{R^{-1}}^j(x,y) = F_{R^{-1}}^j(y,x)$$

for all $x, y \in E$

Therefore;

$$T_{R^{-1}}^j(x,y) = T_R^j(y,x) = T_R^j(x,y) = T_{R^{-1}}^j(y,x);$$

$$I_{R^{-1}}^j(x,y) = I_R^j(y,x) = I_R^j(x,y) = I_{R^{-1}}^j(y,x)$$

and

$$F_{R^{-1}}^j(x,y) = F_R^j(y,x) = F_R^j(x,y) = F_{R^{-1}}^j(y,x)$$

Finally; proof is valid.

Proposition 3.11 *If R is symmetric ,if and only if $R = R^{-1}$.*

Proof: Let R be symmetric , then

$$T_R^j(x,y) = T_R^j(y,x);$$

$$I_R^j(x,y) = I_R^j(y,x)$$

and

$$F_R^j(x,y) = F_R^j(y,x)$$

and

R^{-1} is an inverse relation, then

$$T^j_{R^{-1}}(x,y) = T^j_R(y,x);$$

$$I^j_{R^{-1}}(x,y) = I^j_R(y,x)$$

and

$$F^j_{R^{-1}}(x,y) = F^j_R(y,x)$$

for all $x, y \in E$

Therefore; $T^j_{R^{-1}}(x,y) = T^j_R(y,x) = T^j_R(x,y)$.

Similarly

$$I^j_{R^{-1}}(x,y) = I^j_R(y,x) = I^j_R(x,y)$$

and

$$F^j_{R^{-1}}(x,y) = F^j_R(y,x) = F^j_R(x,y)$$

for all $x, y \in E$.

Hence $R = R^{-1}$

Conversely, assume that $R = R^{-1}$ then, we have

$$T^j_R(x,y) = T^j_{R^{-1}}(x,y) = T^j_R(y,x).$$

Similarly

$$I^j_R(x,y) = I^j_{R^{-1}}(x,y) = I^j_R(y,x)$$

and

$$F^j_R(x,y) = F^j_{R^{-1}}(x,y) = F^j_R(y,x).$$

Hence R is symmetric.

Proposition 3.12 *If R and S are symmetric neutrosophic multirelations, then*

1. $R\tilde{\cup}S$,
2. $R\tilde{\cap}S$,
3. $R\tilde{+}S$
4. $R\tilde{\times}S$

are also symmetric.

Proof: R is symmetric, then we have;

$$T^j_R(x,y) = T^j_R(y,x),$$

$$I^j_R(x,y) = I^j_R(y,x)$$

and

$$F^j_R(x,y) = F^j_R(y,x)$$

similarly S is symmetric, then we have

$$T^j_S(x,y) = T^j_S(y,x),$$

$$I^j_S(x,y) = I^j_S(y,x)$$

and

$$F^j_S(x,y) = F^j_S(y,x)$$

Therefore,

1.
$$\begin{aligned} T^j_{R\widetilde{\cup}S}(x,y) &= \max\left\{T^j_R(x,y), T^j_S(x,y)\right\} \\ &= \max\left\{T^j_R(y,x), T^j_S(y,x)\right\} \\ &= T^j_{R\widetilde{\cup}S}(y,x) \end{aligned},$$

$$\begin{aligned} I^j_{R\widetilde{\cup}S}(x,y) &= min\left\{I^j_R(x,y), I^j_S(x,y)\right\} \\ &= \min\left\{I^j_R(y,x), I^j_S(y,x)\right\} \\ &= I^j_{R\widetilde{\cup}S}(y,x), \end{aligned}$$

and

$$\begin{aligned} F^j_{R\widetilde{\cup}S}(x,y) &= \min\left\{F^j_R(x,y), F^j_S(x,y)\right\} \\ &= \min\left\{F^j_R(y,x), F^j_S(y,x)\right\} \\ &= F^j_{R\widetilde{\cup}S}(y,x) \end{aligned}$$

therefore, $R\widetilde{\cup}S$ is symmetric.

2.
$$\begin{aligned} T^j_{R\widetilde{\cap}S}(x,y) &= \min\left\{T^j_R(x,y), T^j_S(x,y)\right\} \\ &= \min\left\{T^j_R(y,x), T^j_S(y,x)\right\} \\ &= T^j_{R\widetilde{\cap}S}(y,x), \end{aligned}$$

$$\begin{aligned} I^j_{R\widetilde{\cap}S}(x,y) &= max\left\{I^j_R(x,y), I^j_S(x,y)\right\} \\ &= \max\left\{I^j_R(y,x), I^j_S(y,x)\right\} \\ &= I^j_{R\widetilde{\cap}S}(y,x), \end{aligned}$$

and

$$\begin{aligned} F^j_{R\widetilde{\cap}S}(x,y) &= \max\left\{F^j_R(x,y), F^j_S(x,y)\right\} \\ &= \max\left\{F^j_R(y,x), F^j_S(y,x)\right\} \\ &= F^j_{R\widetilde{\cap}S}(y,x) \end{aligned}$$

therefore; $R\widetilde{\cap}S$ is symmetric.

3.

$$\begin{aligned} T^j_{R\tilde{+}S}(x,y) &= T^j_R(x,y)+T^j_S(x,y)-T^j_R(x,y)T^j_S(x,y) \\ &= T^j_R(y,x)+T^j_S(y,x)-T^j_R(y,x)T^j_S(y,x) \\ &= T^j_{R\tilde{+}S}(y,x) \end{aligned}$$

$$\begin{aligned} I^j_{R\tilde{+}S}(x,y) &= I^j_R(x,y)I^j_S(x,y) \\ &= I^j_R(y,x)I^j_S(y,x) \\ &= I^j_{R\tilde{+}S}(y,x) \end{aligned}$$

and

$$\begin{aligned} F^j_{R\tilde{+}S}(x,y) &= F^j_R(x,y)F^j_S(x,y) \\ &= F^j_R(y,x)F^j_S(y,x) \\ &= F^j_{R\tilde{+}S}(y,x) \end{aligned}$$

therefore, $R\tilde{+}S$ is also symmetric

4.

$$\begin{aligned} T^j_{R\tilde{\times}S}(x,y) &= T^j_R(x,y)T^j_S(x,y) \\ &= T^j_R(y,x)T^j_S(y,x) \\ &= T^j_{R\tilde{\times}tS}(y,x) \end{aligned}$$

$$\begin{aligned} I^j_{R\tilde{\times}S}(x,y) &= I^j_R(x,y)+I^j_S(x,y)-I^j_R(x,y)I^j_S(x,y) \\ &= I^j_R(y,x)+I^j_S(y,x)-I^j_R(y,x)I^j_S(y,x) \\ &= I^j_{R\tilde{\times}S}(y,x) \end{aligned}$$

$$\begin{aligned} F^j_{R\tilde{\times}S}(x,y) &= F^j_R(x,y)+F^j_S(x,y)-F^j_R(x,y)F^j_S(x,y) \\ &= F^j_R(y,x)+F^j_S(y,x)-F^j_R(y,x)F^j_S(y,x) \\ &= F^j_{R\tilde{\times}S}(y,x) \end{aligned}$$

hence, $R\tilde{\times}S$ is also symmetric.

Remark 3.13 *$R\circ S$ in general is not symmetric, as*

$$\begin{aligned} T^j_{(R\circ S)}(x,z) &= \bigvee_y \left\{T^j_S(x,y)\wedge T^j_R(y,z)\right\} \\ &= \bigvee_y \left\{T^j_S(y,x)\wedge T^j_R(z,y)\right\} \\ &\neq T^j_{(R\circ S)}(z,x) \end{aligned}$$

$$\begin{aligned} I^j_{(R\circ S)}(x,z) &= \bigwedge_y \left\{I^j_S(x,y)\vee I^j_R(y,z)\right\} \\ &= \bigwedge_y \left\{I^j_S(y,x)\vee I^j_R(z,y)\right\} \\ &\neq I^j_{(R\circ S)}(z,x) \end{aligned}$$

$$\begin{aligned} F^j_{(R\circ S)}(x,z) &= \bigwedge_y \left\{F^j_S(x,y) \vee F^j_R(y,z)\right\} \\ &= \bigwedge_y \left\{F^j_S(y,x) \vee F^j_R(z,y)\right\} \\ &\neq F^j_{(R\circ S)}(z,x) \end{aligned}$$

but $R\circ S$ is symmetric ,if $R\circ S = S\circ R$, for R and S are symmetric relations.

$$\begin{aligned} T^j_{(R\circ S)}(x,z) &= \bigvee_y \left\{T^j_S(x,y) \wedge T^j_R(y,z)\right\} \\ &= \bigvee_y \left\{T^j_S(y,x) \wedge T^j_R(z,y)\right\} \\ &= \bigvee_y \left\{T^j_R(y,x) \wedge T^j_R(z,y)\right\} \\ &T^j_{(R\circ S)}(z,x) \end{aligned}$$

$$\begin{aligned} I^j_{(R\circ S)}(x,z) &= \bigwedge_y \left\{I^j_S(x,y) \vee I^j_R(y,z)\right\} \\ &= \bigwedge_y \left\{I^j_S(y,x) \vee I^j_R(z,y)\right\} \\ &= \bigwedge_y \left\{I^j_R(y,x) \vee I^j_R(z,y)\right\} \\ &I^j_{(R\circ S)}(z,x) \end{aligned}$$

and

$$\begin{aligned} F^j_{(R\circ S)}(x,z) &= \bigwedge_y \left\{F^j_S(x,y) \vee F^j_R(y,z)\right\} \\ &= \bigwedge_y \left\{F^j_S(y,x) \vee F^j_R(z,y)\right\} \\ &= \bigwedge_y \left\{F^j_R(y,x) \vee F^j_R(z,y)\right\} \\ &F^j_{(R\circ S)}(z,x) \end{aligned}$$

for every $(x,z) \in E \times E$ and for $y \in E$.

Proposition 3.14 *If R is transitive relation, then R^{-1} is also transitive.*

Proof : R is transitive relation, if $R\circ R \subseteq R$, hence if $R^{-1}\circ R^{-1} \subseteq R^{-1}$, then R^{-1} is transitive.

Consider;

$$\begin{aligned} T^j_{R^{-1}}(x,y) &= T^j_R(y,x) \geq T^j_{R\circ R}(y,x) \\ &= \bigvee_z \left\{T^j_R(y,z) \wedge T^j_R(z,x)\right\} \\ &= \bigvee_z \left\{T^j_{R^{-1}}(x,z) \wedge T^j_{R^{-1}}(z,y)\right\} \\ &= T^j_{R^{-1}\circ R^{-1}}(x,y) \end{aligned}$$

$$\begin{aligned} I^j_{R^{-1}}(x,y) &= I^j_R(y,x) \leq I^j_{R\circ R}(y,x) \\ &= \bigwedge_z \left\{I^j_R(y,z) \vee I^j_R(z,x)\right\} \\ &= \bigwedge_z \left\{I^j_{R^{-1}}(x,z) \vee I^j_{R^{-1}}(z,y)\right\} \\ &= I^j_{R^{-1}\circ R^{-1}}(x,y) \end{aligned}$$

and

$$\begin{aligned} F^j_{R^{-1}}(x,y) &= F^j_R(y,x) \leq F^j_{R\circ R}(y,x) \\ &= \underset{z}{\wedge}\left\{F^j_R(y,z) \vee F^j_R(z,x)\right\} \\ &= \underset{z}{\wedge}\left\{F^j_{R^{-1}}(x,z) \vee F^j_{R^{-1}}(z,y)\right\} \\ &= F^j_{R^{-1}\circ R^{-1}}(x,y) \end{aligned}$$

hence, proof is valid.

Proposition 3.15 *If R is transitive relation, then $R \cap S$ is also transitive*

Proof: As R and S are transitive relations, $R \circ R \subseteq R$ and $S \circ S \subseteq S$. also

$$\begin{aligned} T^j_{R\tilde{\cap}S}(x,y) &\geq T^j_{(R\tilde{\cap}S)\circ(R\tilde{\cap}S)}(x,y) \\ I^j_{R\tilde{\cap}S}(x,y) &\leq I^j_{(R\tilde{\cap}S)\circ(R\tilde{\cap}S)}(x,y) \\ F^j_{R\tilde{\cap}S}(x,y) &\leq F^j_{(R\tilde{\cap}S)\circ(R\tilde{\cap}S)}(x,y) \end{aligned}$$

implies $R\tilde{\cap}S) \circ (R\tilde{\cap}S) \subseteq R \cap S$, hence $R \cap S$ is transitive.

Proposition 3.16 *If R and S are transitive relations, then*

1. *$R\tilde{\cup}S$,*
2. *$R\tilde{+}S$*
3. *$R\tilde{\times}S$*

are not transitive.

Proof:

1. As

$$\begin{aligned} T^j_{R\tilde{\cup}S}(x,y) &= \max\left\{T^j_R(x,y), T^j_S(x,y)\right\} \\ I^j_{R\tilde{\cup}S}(x,y) &= \min\left\{I^j_R(x,y), I^j_S(x,y)\right\} \\ F^j_{R\tilde{\cup}S}(x,y) &= \min\left\{F^j_R(x,y), F^j_S(x,y)\right\} \end{aligned}$$

and

$$\begin{aligned} T^j_{(R\tilde{\cup}S)\circ(R\tilde{\cup}S)}(x,y) &\geq T^j_{R\tilde{\cup}S}(x,y) \\ I^j_{(R\tilde{\cup}S)\circ(R\tilde{\cup}S)}(x,y) &\leq I^j_{R\tilde{\cup}S}(x,y) \\ F^j_{(R\tilde{\cup}S)\circ(R\tilde{\cup}S)}(x,y) &\leq F^j_{R\tilde{\cup}S}(x,y) \end{aligned}$$

2. As

$$\begin{aligned} T^j_{R\tilde{+}S}(x,y) &= T^j_R(x,y) + T^j_S(x,y) - T^j_R(x,y)T^j_S(x,y) \\ I^j_{R\tilde{+}S}(x,y) &= I^j_R(x,y)I^j_S(x,y) \\ F^j_{R\tilde{+}S}(x,y) &= F^j_R(x,y)F^j_S(x,y) \end{aligned}$$

and

$$T^j_{(R\tilde{+}S)\circ(R\tilde{+}S)}(x,y) \geq T^j_{R\tilde{+}S}(x,y)$$
$$I^j_{(R\tilde{+}S)\circ(R\tilde{+}S)}(x,y) \leq I^j_{R\tilde{+}S}(x,y)$$
$$F^j_{(R\tilde{+}S)\circ(R\tilde{+}S)}(x,y) \leq F^j_{R\tilde{+}S}(x,y)$$

3. As

$$T^j_{R\tilde{\times}S}(x,y) = T^j_R(x,y)T^j_S(x,y)$$
$$I^j_{R\tilde{\times}S}(x,y) = I^j_R(x,y) + I^j_S(x,y) - I^j_R(x,y)I^j_S(x,y)$$
$$F^j_{R\tilde{\times}S}(x,y) = F^j_R(x,y) + F^j_S(x,y) - F^j_R(x,y)F^j_S(x,y)$$

and

$$T^j_{(R\tilde{\times}S)\circ(R\tilde{\times}S)}(x,y) \geq T^j_{R\tilde{\times}S}(x,y)$$
$$I^j_{(R\tilde{\times}S)\circ(R\tilde{\times}S)}(x,y) \leq I^j_{R\tilde{\times}S}(x,y)$$
$$F^j_{(R\tilde{\times}S)\circ(R\tilde{\times}S)}(x,y) \leq F^j_{R\tilde{\times}S}(x,y)$$

Hence $R\tilde{\cup}S$, $R\tilde{+}S$ and $R\tilde{\times}S$ are not transitive.

Proposition 3.17 *If R is transitive relation, then R^2 is also transitive*

Proof: R is transitive relation, if $R \circ R \subseteq R$, therefore if $R^2 \circ R^{-2} \subseteq R^2$, then R^2 is transitive.

$$T^j_{R\circ R}(y,x) = \bigvee_z \left\{T^j_R(y,z) \wedge T^j_R(z,x)\right\} \geq \bigvee_z \left\{T^j_{R\circ R}(y,z) \wedge T^j_{R\circ R}(z,x)\right\} = T^j_{R^2\circ R^2}(y,x),$$

$$I^j_{R\circ R}(y,x) = \bigwedge_z \left\{I^j_R(y,z) \vee I^j_R(z,x)\right\} \leq \bigwedge_z \left\{I^j_{R\circ R}(y,z) \vee I^j_{R\circ R}(z,x)\right\} = I^j_{R^2\circ R^2}(y,x)$$

and

$$F^j_{R\circ R}(y,x) = \bigwedge_z \left\{F(y,z) \vee F^j_R(z,x)\right\} \leq \bigwedge_z \left\{I^j_{R\circ R}(y,z) \vee F^j_{R\circ R}(z,x)\right\} = F^j_{R^2\circ R^2}(y,x)$$

Finally, the proof is valid.

4 Acknowledgements

The authors would like to thank the anonymous reviewer for their careful reading of this research paper and for their helpful comments.

5 Conclusion

In this paper, we have firstly defined the neutrosophic multirelations(NMR). The NMR are the extension of neutrosophic relation (NR) and intuitionistic multirelation[29]. The notions of inverse, symmerty, reflexivity and transitivity on neutrosophic multirelations are studied.The future work will cover the application of the NMR in decision making, pattern recogntion and in medical diagnosis.

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Neutrosophic Principle of Interconvertibility Matter-Energy-Information (NPI_MEI)

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Abstract
The research aims to reveal and prove the thesis of the neutral and convertibility relationship between constituent constructive elements of the universe: matter, energy and information. The approach perspective is a computationally-communicative-neutrosophic one. We configure a coherent and cohesive ideation line. Matter, energy and information are fundamental elements of the world. Among them, there is an inextricable multiple, elastic and evolutionary connection. The elements are defined by the connections between them. Our hypothesis is that the relationship between matter, energy and information is a neutral one. This relationship is not required by the evidence. At this level, it does not give up in front of the evidence intelligibility. Neutral relationship is revealed as a law connection. First, the premise that matter, energy and information never come into contradiction is taken as strong evidence. Their law-like-reciprocal obligations are non contradictory. Being beyond the contrary, matter, energy and information maintain a neutral relationship. Therefore, on the basis of the establishment and functioning of the universe or multi-verse, there is neutrality. Matter, energy and information are primary-founder neutralities. Matter, energy and information are neutral because they are related to inexorable legitimate. They are neutral because they are perfectly bound to one another. Regularity is the primary form of neutrality. The study further radiographies the relational connections, and it highlights and renders visible the attributes and characteristics of the elements (attributes are essential features of elements and characteristics are their specific features). It explains the bilateral relationships matter-energy, information-matter and energy-information. It finally results that reality is an ongoing and complex process of bilateral and multi-lateral convertibility. Thus, it is formulated the neutrosophic principle of Interconvertibility Matter-Energy-Information (NPI_MEI).

Keywords
matter, energy, information, Neutrosophy Smarandache , Neutrosophic Principle of Interconvertibility Matter-Energy-Information (NPI_MEI)

1. Introduction: properties, constituents, elements or ontological principles

In the last half of past century, there has been issued and acknowledged the idea that the world would be made of matter, energy and information. The axiom of foundation of the world issued by Norbert Wiener has already become canonical. Wiener"s axiom states that "Information is information, not matter or energy" [1]. Everything in the universe/multiverse is based on matter, energy and information.

The material of "construction" of the universe is matter and energy. In Big Bang, the amorphous matter, the vortex, unstructured and volatile was brought to a form by the energy. In other words, since the birth of the universe/ multiverse, there have existed matter, energy, and "construction", the "form" - information. The energy put the matter into the form "in formae" (in Latin), i.e. energy generated "informatio" (in Latin) - information. The movement of the matter to form is performed by energy. The initial impulse of the universe/multiverse is given by power. (D. Deutsch shows that "the physical world is a multiverse" [2]; D. Wallace states that is an "emergent multiverse" [3]).

Therefore, the CERN attempt to simulate the initial phenomenon of the creation of the universe started with a huge amount of energy.

Tom Stonier‟s point of view expressed in "Towards a new theory of information" (1991) is that "information is a basic property of the universe. That is, like matter and energy, information has physical reality. Any system that exhibits organization contains information. Changes in entropy represent changes in the organizational states of systems and, as such, quantify changes in the information content of such systems. Information, like energy, exists in many forms. These are interconvertible. Likewise, energy and information are readily interconverted" [4] [5]. In his turn, Anthony Reading noticed the information as "fundamental property of organized matter" [6].

In the article "Information in the Structure of the World" (2011), Mark Burgin deals with the place of information in the world; he believes that there are four "basic constituents of the World" (...) "matter, energy, mentality and knowledge". He points out that some researchers "relate information only to society", others "include the level of individual human beings", "many presume that information is everywhere in nature". His opinion is that the information is "in the structure of the world" and that the "structure of information processes, as well as relations between information and basic constituents of the world, such as matter, energy, mentality and knowledge" [7] should be taken into account [8]. Mark Burgin and Gordana Dodig-Crnkovic believe that "Information is a basic essence of the world" [9]. On the other hand, they postulate the universality of information: "Information is related to everything and everything is related to information" [10] [11]. David Bawden and Lyn Robinson emphasize that „information is now becoming accepted as a fundamental constituent of the physical universe" [12].

In our opinion, the world is composed of three fundamental elements: element 1-matter, element 2 - energy and element 3 - information. Ontologically, matter and energy are primary natural elements, and information is a secondary element. The Matter and energy are constituent elements. In epistemological order, information is superior, being a constructive element. Information is the computational element of the world. Hans Christian von Baeyer believes that this three are elements [13]. The internal computational principle of information is linked to Wheeler‟s principle [14]: "It from bit", as the Cover-Thomas axiom states: "computation is communication limited, and communication is computation limited" [15]. Information is the computational principle of the world. Information is the first element and then the onto-computational principle.

Rafael Capurro appreciates that that there are not elements, there are not properties, but ontological principles aside others; he lists as ontological principles "energy, matter, spirit, subjectivity, substance, or information" [16]. Without the existence of a direct connection between these principled categories, R. Capurro reveals exponential capacity of information to represent the world: "We would then say: whatever exists can be digitalized. Being is computation" [17] [18] [19] [20]. As ontological principle, information is computational. S. Lloyd emphasizes that "universe is computational" [21].

2. Convertive relationship between matter-energy

The first two elements of the triad are those of the Einstein physical formula of mass-energy equivalence. We are interested, first, in the matter connection (mass)-energy. In principle, this relationship was clarified by Albert Einstein.

On the depth axis of Einsteinian thought, the determination of mass-energy relationship is a synthesis of the major ideas launched in the four articles published in 1905. 1905 is known as the miraculous year "Wunderjahr" (German) or "Miracle Year". In Latin it was called "Annus Mirabilis" and the articles published in Annalen der Physik were called "Annus Mirabilis Papers" [22] [23] [24]. They are considered to have significantly contributed to the foundation of modern physics. We could say more: 1905 is the most important year in the history of physics hitherto.

The first article published on the June 9th 1905 introduced the concept of "energy quanta": „Energy, during the propagation of a ray of light, is not continuously distributed over steadily increasing spaces, but it consists of a finite number of energy quanta localized at points in space, moving without diving and capable of being absorbed or generated only as entities". Albert Einstein notes that "energy quanta" is converted "at least partially into kinetic energy of the electrons". Thus he reveals "photoelectric effect", discovery for which he would win, in 1921, the Nobel Prize for physics. Note that from here, the concern for energy is evident.

The second article, published on July (1905), is a specification of Brownian motion: In this paper, shows A. Einstein, according to the molecular Kinetic theory of heat, "bodies of a microscopically visible size suspended in liquids must, as a result of thermal molecular motion, perform motions of such magnitudes that they can be easily observed with a microscope". The article reveals a high consciousness of scientific honesty: "It is possible that the motions to be discussed here are identical with the so-called Brownian molecular motion; however, the information available to me on regarding the latter is so lacking in precision that I form no judgment in the matter". We notice that in this article the orientation is on matter: liquid, molecules [25].

On September 1905, there is published a study that will be the core of what would later be called the "Special Theory of Relativity" [26]. However, a strong emphasis is placed on the speed of light. Entitled "On the electrodynamics of Moving Bodies", the study analyzes, in context of electricity and magnetism, the major changes that occur in "mechanics", when the speeds are close to the speed of light. A. Einstein shows that the "speed of light" is constant in "all inertial frames of references". Then, he "also introduces another postulate (...) that light is always propagated in empty space with a defined velocity c which is independent of the state of motion of the emitting body". We are interested in the fact that this study is concerned about the speed of light as a constant and that this would be the maximum speed in the universe. In this context it is shown that, as Professor Leonardo F. D. da Motta argues "in 1972, Smarandache proposed there is not a limit speed on the nature" [27]. Initiated in 1972, "Smarandache Hypothesis" was completed by Professor Florentin Smarandache in 1998. Smarandache shows: "We promote the hypothesis that: there is no speed barrier in the universe and one can construct any speed even infinite (instantaneous transmission)" [28]. As Ion Pătraşcu outlined in October 2011, Smarandache‟s hypothesis "has been partially confirmed by the recent CERN results of OPERA team led by Antonio Ereditato that experimentally found that neutrino particles travel faster than light" [29].

The fourth article of "Wundejahr" - 1905 emerges as convergence of the others. Energy, light and matter are brought within a formula. The article is called „Ist die Trägit eines Körpers von seinem Energienhalt abhänging?" "Does the inertia of a body depend upon its energy-content?" It was sent on September 27th 1905 and published on November 21st 1905 [30]. Because in this article we find the phrase "the principle of energy", we consider that the most famous formula in physics and, perhaps, of human knowledge ($E = mc^2$) formulation may be called "the principle of energy". In the article, for energy there are used three symbolic notations, L, H and E, according to the system and measurement. Strictly, the formula itself, in mathematical language ($E = mc^2$) does not appear, it is presented linguistically: „If a body gives off the energy in the form of radiation, its mass diminishes by L/c^2. The fact that the energy withdraws from the body and becomes energy of radiation evidently makes no difference, so that we are led to the more general conclusion that the mass of a body is a measure of its energy-content; if the energy changes by L, the mass changes in the same sense by $L/9x10^{20}$, the energy being measured in ergs, and the mass in grams".

It should result, we show, $m = L/c^2 \leftrightarrow L = mc^2$.

Even if the formula was not canonically marked from the beginning, Albert Einstein underlines the energy equation. Contributions to finalize the formula are also brought, through symbolic using, by Max Planck, Johannes Stark and Louis de Broglie.

In 1946, Albert Einstein published the article "$E = mc^2$: the most urgent problem of our time", accrediting the formula for history. The internal subject of the formula is the relationship between "mass" and "its energy-content". The mass of a body and the energy contained by it are defined mutually and are mutual dependent. The formula $E = mc^2$ is neither mass, nor energy. The formula $E = mc^2$ is information. More precisely, it constitutes scientific information: law-like information, grounded, indisputable in terms of a strengthened conceptual reference system.

Mass and energy are inseparable and mutually convertible. There is no mass without energy and no energy without any mass. All energy has a mass. Energy can be kinetic, chemical, thermal energy given by the position in a field of forces, and so on. When there is added energy to an object, this leads to a gain of mass. While it may seem strange in comparison with common sense, scientifically, the body temperature increase causes the increase of its mass. The mass increases insignificantly, but it increases, because any energy has a mass. The body is a complex mass plus energy.

Einstein‟s formula is available for any type of mass and energy.

Albert Einstein also proved that motion is crucial in the destiny of the world. As far as matter and bodies are concerned, to speak about the "rest mass" and relative mass (motion mass), $E = mc^2$ shows, subsequently, two specific variables.

If we deal with a body at rest $E = mc^2$ becomes $E = m_0c^2$ (m_0 = rest mass). For the rest, the speed is zero. A body has energy also when it is stationary. A solid body, has obviously, at least, one thermal energy.

When the body is in motion, with speed v, $E = mc^2$ becomes $E = m_{rel}c^2$ (where m_{rel} is relative mass,

$$\mathbf{m_{rel}} = \frac{\mathbf{m_0}}{\sqrt{\mathbf{1} - \frac{\mathbf{v^2}}{\mathbf{c^2}}}}$$

).

Another case is the variation of matter and energy: $\Delta E = \Delta m_0c^2$. The formula is valid not only in terms of any type of energy and matter, it is valid in any system. When it is a closed system, there appears a feature: closed systems do not lose mass. On the other hand, in closed systems, the energies are additive, they are cumulative. That means that in closed systems, energy and mass are controlled by each other. Progressively, mass becomes energy and energy becomes mass.

Matter, as it is well known, is defined as something that has mass and volume. Taking into consideration that the mass has as reference system the Earth, and on the planet Earth, the objects are considered in rest, the mass is regularly identified by the rest mass or invariant mass. The volume is measured as the three-dimensional amplitude of the occupied space. Sometimes, the concept of substance is used for the “matter”. Mark Burgin observes that "the matter is the name of all substances" [31].

In relationship with the substance, matter is taken as the substance of which the observed physical objects are constituted. The idea of matter as observed matter is important, because it cannot talk about substance in the case of detected matter only as presence in the fields of forces. In some force fields, besides effects of some visible material elements, there are observed effects of forces due to some objects-matter yet directly unnoticed, even still unknown. Martin H. Krieger states that "matter is matter that is observed" [32]. Scientific discoveries have shown that objects are composed of molecules, atoms, subatomic particles (protons, neutrons, electrons, etc.). At rest, the relationship of matter with energy is measurable, as Martin H. Krieger has demonstrated; at rest, the "matter is energetically stable" [33].

Taking into consideration that in the Universe there are two primary natural elements, element 1 (matter) and element 2 (energy), as we call them, they can be defined, also by one another. Such an understanding of the matter is shown by S. M. Carroll when he asserts that “matter” "contributes to energy" [34]. We observe that the corollary is also true, because energy also "contributes to matter". S. M. Carroll admits that "energy sources are a combination of matter and radiation" [31]. It is generally considered that the radiation is a form of energy. Gary T. Horowitz expresses a similar point of view, contending that "the black hole radiates energy" [36]. Matter and energy are "purely natural elements" fundamental to the universe. They are created and are controlled by each other. It is interesting that $E = mc^2$ has generated along the time no discussion concerning demonstrability, but it has generated debate concerning the positioning. Luce Irigaray shows that $E = mc^2$, as it would favour "the speed of light over the other speeds that is vitally necessary to us", constitutes a "sexed equation" [37].

3. The convert relationship information-energy

Rolf Landauer observed a conversion information-energy: Landauer‟s Principle shows that erasure of one bit of information augments physical entropy, and generates heat [38]. As regards the relations between the elements of the fundamental triad, Mark Burgin and Gordana Dodig-Crnkovic believe that "the most intimate relations exist between information and energy. (...) Energy is a kind of information in the broad sense" [39].

The formulation of the “second law of thermodynamics” by Ludwig Boltzmann was one of the great intellectual challenges of the nineteenth century; the law says that entropy in an isolated system should not decrease [40]. James Clerk Maxwell, Scottish physicist and mathematician, tested foundations of the law, including the foundations of statistical mechanics and thermodynamics. He thought of an event of physical nature that would contradict the content of the law. He imagined a box with two compartments communicating between them through a hatch. In the box there is a gas at a particular temperature. In relation to the average temperature some molecules are cooler and some molecules are hotter. The hotter molecules are moving faster, and the cooler molecules are moving slower. The hatch is activated by a being who decides when the molecules move from a side to other side. After a certain interval and a number of openings of the hatch, the hot molecules will gather in a compartment, cold molecules, in the other. By opening the hatch the being separated the cold molecules from the warm molecules and modified the thermodynamic entropy. That is, initially the gas was a mixture of hot and cold molecules; it was in a state of disorder, it had a higher entropy. Once the molecules were separated and thus a state of order was introduced, it generated a lower entropy. In other words, the entropy of an isolated system was modified. It was proved that, against the law provisions, entropy in an isolated system should decrease. Furthermore, this being was called demon, Maxwell's demon. What we observe today is that the demon decreased the entropy, i.e. it produced information. Apparently the demon contradicts the law, because the functioning of the law compulsorily implies that there is not and there cannot be built a perfect heat engine which can extract energy from an isolated system and use it almost entirely; such a heat engine is not possible because the container itself containing the gas consumes heat to heat as container. The demon has knowledge of the idea of temperature and, without introducing energy into the box, it separates the molecules. The temperature was used as a separator engine, as the perfect heat engine. That is the demon that "seems" to turn information into energy, violating the rules induced by law.

In 1929, in the study "On the reduction of entropy in a thermodynamic system by the intervention of intelligent beings", Leo Szilard proves that the law is not violated. He describes the demon as "intelligent being". He laid aside the qualitative contribution of the demon and put in quantitative terms its activity (intervention of intelligent being). He pointed out that the demon (being) turns the knowledge in thermodynamic energy [41]. Our remark is that Szilard makes from even the intelligence of the demon a consumer of energy: to determine which of molecules are hot and which molecules are cold, the demon exerts some energy. He showed that the law would not be violated if the entropy S of a system increased by an amount $\Delta S = k \ln 2$; k is Boltzmann's constant = $1{,}38 \times 10^{-23}$ joules per degree Kelvin. On the other hand, it is known that the information is the inverse of the entropy. This implies that $\Delta I = \Delta S = - k \ln 2$ [42]. By the description that is made of a dynamic system, a certain observer intervenes in the evolution of the system. The intervention consists in the description of the induced instantaneous dynamics and irreversible discontinuity;

intervention generates a change. In fact, by its description, the observer selects a certain state of the system. The number of unknown states of the system is reduced by choice and the stream of possibilities of the system decreases. So a reduction of entropy takes place. Leo Szilard notes in the observer action “how entropy in a thermodynamic system can be reduced by the intervention of intelligent beings” [43]. With a point of view related to the content of Szilard‟s article, L. Brown, B. Pippard and A. Pais assert that “the decrease of entropy caused through the observation of a thermodynamic system (by an intelligent being) must be compensated by an increase of entropy imposed on the observed system through the procedure of measurement” [44]. Through this demonstration, Leo Szilard formulated a law of relation energy-information, called Szilard‟s engine or “information heat engine” [45].

Today, we say that the law is not violated, since intelligence constitutes energy consumption. Our thesis is that intelligence always converts information into energy and energy into information.

In "A Mathematical Theory of Communication” (published study in numbers 3 and 4, 1948, of the Bell System Technical Journal), Claude E. Shannon defines information based on entropy. The second of the 23 theorems formulated contains one of the most important and most cited formulas in the history of science. It is comparable to Einsteinian $E = mc^2$ or formula of entropy given by L. Boltzmann (and the latter has engraved it on his grave from Vienna). In its development, Shannon starts from the question: „Can we find a measure of how much 'choice' is involved in the selection of the event or of how uncertain we are of the outcome?” [46]. From here, he formulates „theorem 2”: (...) $H = - K\Sigma p_i \log p_i$ [47]; H is entropy. Shannon explains: „Quantities of the form $H = - K\Sigma p_i \log p_i$ (the constant K merely amounts to a choice of a unit of measure) play a central role in information theory as measures of information, choice and uncertainty. The form of H will be recognized as that of entropy, as defined in certain formulations of statistical mechanics where p_i is the probability of a system being in cell of its phase space. H is then, for example, the H of Boltzmann's famous theorem. We shall call $H = - K\Sigma p_i \log p_i$ the entropy of the set of possibilities $p_1, \ldots, p_n$” [48].

Generally, entropy represents the disorder of a system. As we observe H (entropy) is the minus of the information measure - that is information is the reverse of entropy, minus entropy, as such, as we‟ll later see with Louis Brillouin. "Entropy" goes in the same direction with "uncertainty": „this quantity measures how uncertain we are” (...) „entropy (or uncertainty)” [49]. In relation to channel time (continuous, discrete, mixed), "continuous and discrete entropies" are registered: “In the discrete case, the entropy measures in an absolute way the randomness of the chance variable. In the continuous case, the measurement is relative to the coordinate of system” [50].

Later, in 1962, Leon Brillouin (1962) notes that information is "minus entropy", that information is negative entropy, and information means "entropy", i.e. "negentropy" [51]. He formulated the Negentropy Principle of Information, designating the idea that aggregation of information associated to states of a system is directly proportional to the decrease of entropy. Also, he stated that in this situation there is no violation of the second law of thermodynamics; that there is a reduction of the thermodynamic entropy in an area of a system and an increase of entropy in another area of it that do not constitute a violation of the second law of thermodynamics.

On the line of L. Brillouin, S. P. Mahulikar and H. Herwig (2009) consolidated Negentropy Principle of Information. They observed that the reduction of entropy may be understood as a deficiency of entropy; thereby reduction of entropy of a sub-system is a deficiency of entropy in relation to surrounding sub-systems [52].

Further researches cleared the doubts demon entered. Even more, it was demonstrated the possibility of converting information into energy [53] [54] [55] [56]. Starting from Szilárd-type information-to-energy conversion and the Jarzynski equality, S. Toyabe, M. Sagawa, E. Ueda, E. and M. Sano Muneyuki sketched “a new fundamental principle of an „information-to-heat engine‟ that converts information into energy by feedback control” [57].

Mihaela Colhon and N. Tandareanu speak about "sentential form", referring to those forms which include propositional information formulated in a natural language [58]. Thought has several forms: language thought, geometric thought, thought, digital thought, pictorial thought, musical thought etc. Each type of thought has one type of efficiency called intelligence. Efficiency is effectiveness in unit or time interval. Howard Gardner asserts that there are 9 types of intelligence: naturalist intelligence (nature smart), musical intelligence (musical smart), logical-mathematical intelligence (number/reasoning smart), existential intelligence, interpersonal intelligence (people smart), bodily-kinesthetic intelligence (body smart), linguistic intelligence (word smart), intra-personal intelligence (self smart), spatial intelligence (picture smart) [59].

On conditions in which the informational process consists mainly of computation, there is easy to deduce that man is a "computer" that converts energy into information. It is the most efficient converter of the blue planet: "Man is the most complex information-processing system existing on the earth. By some estimates, the total number of bits processed in the human body every second is 3.4×10^{19}, but it uses only 20 watts power" [60].

Information means computation [61] [62] [63] [64] [65] [66]. Intelligence is a computational quality of information converting into energy. Information-energy converters can operate on the principles of computational intelligence.

4. The relationship matter-information

Forms, patterns of the information core are the materials, are material nature. That is in the information core lies matter. Immanent relationship between matter and information is represented by the forms, by patterns. Matter has form, it has information. Rolf Landauer shows that „information is physical" [67]; on the same idea, V. Vedral argues that material, physical „universe" is „quantum information" [68].

Formula $E = mc^2$ is twice impregnated informationally. The first impregnation consists of the fact that the formula which contains the principle is an equation information. Any equation is a piece of information. A second informational impregnation of the formula consists of that c^2 is information. The relationship between energy and matter is informationally mediated.

Information has a quantitative dimension and a qualitative dimension. On the quantitative dimension, information is a function of probability [69] [70]. On the qualitative dimension, information is a function of meaning.

We associate to our opinion about internal form, the position expressed by Anthony Reading related to "intrinsic information". This shows that he caught up Norbert Wiener‟s concept and "intrinsic information" is "the way the various particles, atoms, molecules, and objects in the universe are organized and arranged" [71] [72] [73] [74].

The cognitive organization of the matter, energy and information itself takes place through information. The modelling core of information is represented by the form. The forms are concepts that bring in convergence the observing of the informational object and its structural thought. They are places of objective finding meeting with subjective internal computation. The forms are active patterns. The computation takes place by the designing of forms on informational objectives apparently amorphous and through their structural magnetization. Through information, the mental forms find their modelling resonance in the informational medium [75] [76] [77] [78].

"Intrinsic information" is the finding of a way of organization for the matter, energy and information itself. Like world itself, the informational medium has an intrinsic structure, irrespective of the relation to the informational subject. This objective configuration creates the internal form of the informational object [79] [80] [81].

On the other hand, in its projective approach, the radiant, radiographic, resonator and infusive, informational subject designs on medium of interest the external forms. The external conceptual forms can resonate with the internal forms of informational object or they may not resonate. When the informational medium is structured, the forms resonate and the subject objectifies them. When the informational medium is not structured, the external forms do not find resonance in the amorphous medium. Then the external forms magnetize the amorphous medium and structure it informationally. The subjective is charging the objective. The informational subject infuses itself with forms, the subject "pattern-izes". In the first case, the extrinsic information is brought within range of the intrinsic information. In the second case, the extrinsic information is required as modeller and intrinsic information.

If in the intrinsic information core stands the objective organization discovery of the informational medium, in the centre of extrinsic information there is meaningful structure induced from exterior, an external form. Intrinsic information means discovery of meanings. Extrinsic information is assigning of meanings.

As critical informational tools, forms are active nuclear structure, radiant. Forms are previous informational constructions with which the informational medium is exploring and exploiting. As attested models, the forms are themselves seeking the informational space.

In fact, the forms are forms because they form (form-s). They are inserted in "amorphous and disorganized pyrite" and formally structure it. Like language, information is the discourser of computational process of commensuration with conceptual forms. Information is a putting in discourse on putting in order.

In the intrinsic information, putting in order is noticeable, because organization belongs to inventory domains. In the extrinsic information, putting in order is infused-modelling, because the organization belongs to the implementation order, it is induced by the informational subject. The extrinsic information is of the impression type. The impression-able form is found as impression. In these cases, the forms bring and radiate meanings. Intrinsic information deals with a recognition of meanings-forms. The difference between intrinsic information and extrinsic information comes from central computational operation: recognition, through forms, of meanings organization *vs.* assigning, through forms, of meanings [82]. The extrinsic information is more visible and meaningfully marked. Therefore, it is reasonably thought that they could also be called "meaningful information". Anthony Reading shows that besides intrinsic information there is also meaningful information. He states: "Meaningful information is defined as a detectable pattern of matter or energy that generates a response in a recipient" [83].

The detectable pattern is a "form".

5. Conclusion

Relationally, the neutral relationship between matter, energy and information is not primary, but secondary. The fact that the three elements of the Universe/Multi-verse do not contradict results in a liminal manner from interconvertibility. The primary relationship, principled, law-like, liminal, fundamental to the world is the

Interconvertibility of Matter-Energy-Information (I_MEI). The Matter-Energy-Information interconvertibilility renders the world permanent and dense, more and more dense inter-elements emerging in the conversion process. The unavoidable law that controls any process that takes place in the world is the law of the permanent conversion Matter-Energy-Information. Each process has an index of interconvertibility and a formula of existence, of reality.

The reality is the reality of interconvertibility. Man is the main instrument of conversion and computability. We got to illuminate some of the bilateral and non-contextual (in the absence of the third element) conversions. Reality is the place of the permanent interconversion, simultaneous and multiphase of M-E-I. What happens today contains all the history of interconversion at the beginning of the world. There remains to be investigated how to convert M into I in presence of I (in the context of I), a M in I in the presence of E (in the context of E) and an E in I in the presence of M (in the context of M). Furthermore, there is yet to be clarified how to convert M to E in the presence of I, a M "in the presence of E (in the context of E) and an E in I in the presence of M (in the context of M)" and so on. However, any presence and any context light and shadow the neutral war of the convertibilities.

Acknowledgements

This work was partially supported by the grant number 33C/2014, awarded in the internal grant competition of the University of Craiova.

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Published in *Journal of Information Science*, 2014, pp. 1-9, DOI: 10.1177/0165551510000000, 9 p., pages 237 - 245.

Neutrosophic Refined Relations and Their Properties

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Abstract

In this paper, the neutrosophic refined relation (NRR) defined on the neutrosophic refined sets(multisets) [13] is introduced. Various properties like reflexivity, symmetry and transitivity are studied.

Keyword 0.1 *Neutrosophic sets, neutrosophic refined sets, neutrosophic refined relations, reflexivity, symmetry, transitivity.*

1 Introduction

Recently, several theories have been proposed to deal with uncertainty, imprecision and vagueness. Theory of probability, fuzzy set theory[18], intuitionistic fuzzy sets[17], rough set theory[49] etc. are consistently being utilized as efficient tools for dealing with diverse types of uncertainties and imprecision embedded in a system. But, all these above theories failed to deal with indeterminate and inconsistent information which exist in beliefs system. In 1995, inspired from the sport games (wining/tie/defeating), from votes (yes/ NA/ no), from decision making (making a decision/ hesitating/not making) etc. and guided by the fact that the law of excluded middle did not work any longer in the modern logics, F. Smarandache[10] developed a new concept called neutrosophic set (NS) which generalizes fuzzy sets and intuitionistic fuzzy sets. NS can be described by membership degree, indeterminate degree and non-membership degree. This theory and their hybrid structures have proven useful in many different fields such as control theory[32], databases[20, 21], medical diagnosis problem[1], decision making problem [24, 2], physics[8], topology [9], etc. The works on neutrosophic set, in theories and applications, have been progressing rapidly (e.g. [3, 6, 35, 41, 48, 19]).

Combining neutrosophic set models with other mathematical models has attracted the attention of many researchers. Maji et al.[22] presented the concept of neutrosophic soft sets which is based on a combination of the neutrosophic set and soft set models. Broumi and Smarandache[33, 36] introduced the concept of the intuitionistic neutrosophic soft set by combining the intuitionistic neutrosophic sets and soft sets. Broumi et al. presented the concept of rough neutrosophic set[39] which is based on a combination of neutrosophic sets and rough set models. The works on neutrosophic sets combining with soft sets, in theories and applications, have been progressing rapidly (e.g. [34, 37, 38, 14, 15, 40, 16, 42]).

The notion of multisets was formulated first in [31] by Yager as generalization of the concept of set theory and then the multiset was developed in [7] by Calude et al. Several authors from time to time made a number of generalizations of the multiset theory. For example, Sebastian and Ramakrishnan[46, 45] introduced a new notion called multi fuzzy sets, which is a generalization of the multiset. Since then, Several researchers [30, 44, 4, 5] discussed more properties on multi fuzzy set. And they [47, 23] made an extension of the concept of Fuzzy multisets to an intuitionstic fuzzy set, which was called intuitionstic fuzzy multisets (IFMS). Since then in the study on IFMS , a lot of excellent results have been achieved by researchers [43, 25, 26, 27, 28, 29]. An element of a multi fuzzy set can occur more than once with possibly the same or different membership values, whereas an element of intuitionistic fuzzy multiset allows the repeated occurrences of membership and non–membership values. The concepts of FMS and IFMS fail to deal with indeterminacy. In 2013 Smarandache [11] extended the classical neutrosophic logic to n-valued refined neutrosophic logic, by refining each neutrosophic component T, I, F into respectively T_1, T_2, ..., T_m, and I_1, I_2, ..., I_p, and F_1, F_2, ..., F_r. Recently, Deli et al.[13] used the concept of neutrosophic refined sets and studied some of their basic properties. The concept of neutrosophic refined set (NRS) is a generalization of fuzzy multisets and intuitionistic fuzzy multisets.

The neutrosophic refined relations are the neutrosophic refined subsets in a cartesian product of the universe. The purpose of this paper is an attempt to extend the neutrosophic relations to neutrosophic refined relations (NRR). This paper is arranged in the following manner. In section 2, we present some definitions of neutrosophic set and neutrosophic refined set theory which help us in the later section. In section 3, we study the concept of neutrosophic refined relations and their operations. Finally, we conclude the paper.

2 Preliminary

In this section, we mainly recall some notions related to neutrosophic set[10],single valued neutrosophic set (SVNS)[12] and neutrosophic refined set relevant to the present work. See especially[20, 21, 1, 3, 6, 35, 24, 2, 9, 8, 12] for further details and background.

Smarandache[11] refine T , I, F to T_1, T_2,..., T_m and I_1, I_2,..., I_p and F_1, F_2,..., F_r where all T_m , I_p and F_r can be subset of [0,1]. In the following sections ,we considered only the case when T ,I and F are split into the same number of subcomponents 1,2,...p, and T_A^j I_A^j,F_A^j are single valued neutrosophic number.

Definition 2.1 *[10] Let U be a space of points (objects), with a generic element in U denoted by u. A neutrosophic set (N-set) A in U is characterized by a truth-membership function T_A, a indeterminacy-membership function I_A and a falsity-membership function F_A. $T_A(x)$; $I_A(x)$ and $F_A(x)$ are real standard or nonstandard subsets of $]^-0, 1^+[$. It can be written as*

$$A = \{< u, (T_A(x), I_A(x), F_A(x)) >: x \in E,\ T_A(x), I_A(x), F_A(x) \in]^-0, 1^+[\}.$$

There is no restriction on the sum of $T_A(x)$; $I_A(x)$ and $F_A(x)$, so $^-0 \leq supT_A(x) + supI_A(x) + supF_A(x) \leq 3^+$.

For application in real scientific and engineering areas,Wang et al.[12] proposed the concept of an SVNS, which is an instance of neutrosophic set. In the following, we introduce the definition of SVNS.

Definition 2.2 *[12] Let U be a space of points (objects), with a generic element in U denoted by u. An SVNS A inX is characterized by a truth-membership function $T_A(x)$, a indeterminacy-membership function $I_A(x)$ and a falsity-membership function $F_A(x)$, where $T_A(x)$, $I_A(x)$, and $F_A(x)$ belongs to [0,1] for each point u in U. Then, an SVNS A can be expressed as*

$$A = \{< u, (T_A(x), I_A(x), F_A(x)) >: x \in E,\ T_A(x), I_A(x), F_A(x) \in [0, 1]\}.$$

There is no restriction on the sum of $T_A(x)$; $I_A(x)$ and $F_A(x)$, so $0 \leq supT_A(x) + supI_A(x) + supF_A(x) \leq 3$.

Definition 2.3 *[13] Let E be a universe. A neutrosophic refined set (NRS) A on E can be defined as follows:*

$$\begin{aligned} A \ &= \{< x, (T_A^1(x), T_A^2(x), ..., T_A^P(x)), (I_A^1(x), I_A^2(x), ..., I_A^P(x)), \\ &\qquad (F_A^1(x), F_A^2(x), ..., F_A^P(x)) >:\ x \in E\} \end{aligned}$$

where,

$$T_A^1(x), T_A^2(x), ..., T_A^P(x) : E \to [0, 1],$$

$$I_A^1(x), I_A^2(x), ..., I_A^P(x) : E \to [0,1],$$

and

$$F_A^1(x), F_A^2(x), ..., F_A^P(x) : E \to [0,1]$$

such that

$$0 \leq supT_A^i(x) + supI_A^i(x) + supF_A^i(x) \leq 3$$

($i = 1, 2, ..., P$) and

$$T_A^1(x) \leq T_A^2(x) \leq ... \leq T_A^P(x)$$

for any $x \in E$.

$(T_A^1(x), T_A^2(x), ..., T_A^P(x))$, $(I_A^1(x), I_A^2(x), ..., I_A^P(x))$ *and* $(F_A^1(x), F_A^2(x), ..., F_A^P(x))$ *is the truth-membership sequence, indeterminacy-membership sequence and falsity-membership sequence of the element x, respectively. Also, P is called the dimension(cardinality) of NRS A. We arrange the truth-membership sequence in decreasing order but the corresponding indeterminacy-membership and falsity-membership sequence may not be in decreasing or increasing order.*

The set of all Neutrosophic refined sets on E is denoted by NRS(E).

Definition 2.4 *[13] Let $A, B \in NRS(E)$. Then,*

1. *A is said to be NR subset of B is denoted by $A \tilde{\subseteq} B$ if $T_A^i(x) \leq T_B^i(x)$, $I_A^i(x) \geq I_B^i(x)$,$F_A^i(x) \geq F_B^i(x)$, $\forall x \in E$.*
2. *A is said to be neutrosophic equal of B is denoted by $A = B$ if $T_A^i(x) = T_B^i(x)$, $I_A^i(x) = I_B^i(x)$,$F_A^i(x) = F_B^i(x)$, $\forall x \in E$.*
3. *the complement of A denoted by $A^{\tilde{c}}$ and is defined by*

$$A^{\tilde{c}} = \{< x, (F_A^1(x), F_A^2(x), ..., F_A^P(x)), (I_A^1(x), I_A^2(x), ..., I_A^P(x)), (T_A^1(x), T_A^2(x), ..., T_A^P(x)) >: x \in E\}$$

4. *If $T_A^i(x) = 0$ and $I_A^i(x) = F_A^i(x) = 1$ for all $x \in E$ and $i = 1, 2, ..., P$ then A is called null* ns-*set and denoted by $\tilde{\Phi}$.*
5. *If $T_A^i(x) = 1$ and $I_A^i(x) = F_A^i(x) = 0$ for all $x \in E$ and $i = 1, 2, ..., P$, then A is called universal* ns-*set and denoted by $\tilde{E}$.*

Definition 2.5 *[13] Let $A, B \in NRS(E)$. Then,*

1. *the union of A and B is denoted by $A \tilde{\cup} B = C_1$ and is defined by*

$$C = \{< x, (T_C^1(x), T_C^2(x), ..., T_C^P(x)), (I_C^1(x), I_C^2(x), ..., I_C^P(x)), (F_C^1(x), F_C^2(x), ..., F_C^P(x)) >: x \in E\}$$

where $T_C^i = T_A^i(x) \vee T_B^i(x)$, $I_C^i = I_A^i(x) \wedge I_B^i(x)$,$F_C^i = F_A^i(x) \wedge F_B^i(x)$, $\forall x \in E$ and $i = 1, 2, ..., P$.

2. *the intersection of A and B is denoted by $A\tilde{\cap}B = D$ and is defined by*

$$D = \{< x, (T_D^1(x), T_D^2(x), ..., T_D^P(x)), (I_D^1(x), I_D^2(x), ..., I_D^P(x)), (F_D^1(x), F_D^2(x), ..., F_D^P(x)) >: x \in E\}$$

where $T_D^i = T_A^i(x) \wedge T_B^i(x)$, $I_D^i = I_A^i(x) \vee I_B^i(x)$,$F_D^i = F_A^i(x) \vee F_B^i(x)$, $\forall x \in E$ *and* $i = 1, 2, ..., P$.

3. *the addition of A and B is denoted by $A\tilde{+}B = E_1$ and is defined by*

$$E_1 = \{< x, (T_{E_1}^1(x), T_{E_1}^2(x), ..., T_{E_1}^P(x)), (I_{E_1}^1(x), I_{E_1}^2(x), ..., I_{E_1}^P(x)), (F_{E_1}^1(x), F_{E_1}^2(x), ..., F_{E_1}^P(x)) >: x \in E\}$$

where $T_{E_1}^i = T_A^i(x) + T_B^i(x) - T_A^i(x).T_B^i(x)$, $I_{E_1}^i = I_A^i(x).I_B^i(x)$,$F_{E_1}^i = F_A^i(x).F_B^i(x)$, $\forall x \in E$ *and* $i = 1, 2, ..., P$.

4. *the multiplication of A and B is denoted by $A\tilde{\times}B = E_2$ and is defined by*

$$E_2 = \{< x, (T_{E_2}^1(x), T_{E_2}^2(x), ..., T_{E_2}^P(x)), (I_{E_2}^1(x), I_{E_2}^2(x), ..., I_{E_2}^P(x)), (F_{E_2}^1(x), F_{E_2}^2(x), ..., F_{E_2}^P(x)) >: x \in E\}$$

where $T_{E_2}^i = T_A^i(x).T_B^i(x)$, $I_{E_2}^i = I_A^i(x) + I_B^i(x) - I_A^i(x).I_B^i(x)$,$F_{E_2}^i = F_A^i(x) + F_B^i(x) - F_A^i(x).F_B^i(x)$, $\forall x \in E$ *and* $i = 1, 2, ..., P$.

Here $\vee$, $\wedge$, $+$, $.$, $-$ *denotes maximum, minimum, addition, multiplication, subtraction of real numbers respectively.*

3 Relations on Neutrosophic Refined Sets

In this section, after given the Cartesian product of two neutrosophic refined sets (NRS), we define a relations on neutrosophic refined sets and study their desired properties. The relation extend the concept of intuitionistic multirelation [27] to single valued neutrosophic refined relation. Some of it is quoted from [13, 27, 10].

Definition 3.1 *Let $\emptyset \neq A, B \in NRS(E)$ and $j \in \{1, 2, ..., n\}$. Then, cartesian product of A and B is a neutrosophic refined set in $E \times E$, denoted by $A \times B$, defined as*

$$A \times B = \{< (x, y), T_{A\times B}^j(x, y)), I_{A\times B}^j(x, y), F_{A\times B}^j(x, y) >: (x, y) \in E \times E\}$$

where

$$T_{A\times B}^j(x, y), I_{A\times B}^j(x, y), F_{A\times B}^j(x, y) : E \to [0, 1]$$

,

$$T_{A\times B}^j(x, y) = \min\left\{T_A^j(x), T_B^j(x)\right\},$$

$$I_{A\times B}^j(x, y) = \max\left\{I_A^j(x), I_B^j(x)\right\}$$

and

$$F^j_{A\times B}(x,y) = \max\left\{F^j_A(x), F^j_B(x)\right\}$$

for all $x, y \in E$.

Remark 3.2 *A Cartesian product on* A *is a neutrosophic refined set in* $E \times E$*, denoted by* $A \times A$*, defined as*

$$A \times A = \{< (x,y), T^j_{A\times A}(x,y)), I^j_{A\times A}(x,y), F^j_{A\times A}(x,y) >: (x,y) \in E \times E\}$$

where $j = 1, 2, ..., n$ *and* $T^j_{A\times A}, I^j_{A\times A}, F^j_{A\times A} : E \times E \to [0,1]$.

Example 3.3 *Let* $E = \{x_1, x_2\}$ *be a universal set and* A *and* B *be two Nm-sets over* E *as;*

$$\begin{aligned} A = \quad & \{< x_1, \{0.3, 0.5, 0.6\}, \{0.2, 0.3, 0.4\}, \{0.4, 0.5, 0.9\} >, \\ & < x_2, \{0.4, 0.5, 0.7\}, \{0.4, 0.5, 0.1\}, \{0.6, 0.2, 0.7\} >\} \end{aligned}$$

and

$$\begin{aligned} B = \quad & \{< x_1, \{0.4, 0.5, 0.6\}, \{0.2, 0.4, 0.4\}, \{0.3, 0.8, 0.4\} >, \\ & < x_2, \{0.6, 0.7, 0.8\}, \{0.3, 0.5, 0.7\}, \{0.1, 0.7, 0.6\} >\} \end{aligned}$$

Then, the cartesian product of A *and* B *is obtained as follows*

$$\begin{aligned} A \times B = \quad & \{< (x_1, x_1), \{0.3, 0.5, 0.6\}, \{0.2, 0.4, 0.4\}, \{0.3, 0.8, 0.9\} >, \\ & < (x_1, x_2), \{0.3, 0.7, 0.8\}, \{0.2, 0.5, 0.7\}, \{0.1, 0.7, 0.9\} >, \\ & < (x_2, x_1), \{0.4, 0.5, 0.6\}, \{0.2, 0.5, 0.4\}, \{0.3, 0.8, 0.7\} >, \\ & < (x_2, x_2), \{0.4, 0.7, 0.8\}, \{0.3, 0.5, 0.7\}, \{0.1, 0.7, 0.7\} >\} \end{aligned}$$

Definition 3.4 *Let* $\emptyset \neq A, B \in NRS(E)$ *and* $j \in \{1, 2, ..., n\}$*. Then, a neutrosophic refined relation from* A *to* B *is a neutrosophic refined subset of* $A \times B$*. In other words, a neutrosophic refined relation from* A *to* B *is of the form* (R, C), $(C \subseteq E \times E)$ *where* $R(x,y) \subseteq A \times B$ $\forall (x,y) \in C$.

Example 3.5 *Let us consider the Example 3.3. Then, we define a neutrosophic refined relation* R *and* S*, from* A *to* B*, as follows*

$$\begin{aligned} R = \quad & \{< (x_1, x_1), \{0.2, 0.6, 0.9\}, \{0.2, 0.4, 0.5\}, \{0.3, 0.8, 0.9\} >, \\ & < (x_1, x_2), \{0.3, 0.9, 0.8\}, \{0.2, 0.8, 0.7\}, \{0.1, 0.8, 0.9\} >, \\ & < (x_2, x_1), \{0.1, 0.9, 0.6\}, \{0.2, 0.5, 0.4\}, \{0.2, 0.8, 0.7\} >\} \end{aligned}$$

and

$$\begin{aligned} S = \quad & \{< (x_1, x_1), \{0.1, 0.7, 0.9\}, \{0.2, 0.5, 0.7\}, \{0.1, 0.9, 0.9\} >, \\ & < (x_1, x_2), \{0.3, 0.9, 0.8\}, \{0.2, 0.8, 0.8\}, \{0.1, 0.8, 0.9\} >, \\ & < (x_2, x_1), \{0.1, 0.9, 0.7\}, \{0.2, 0.9, 0.4\}, \{0.2, 0.8, 0.9\} >\} \end{aligned}$$

Definition 3.6 *Let $A, B \in NRS(E)$ and, R and S be two neutrosophic refined relation from A to B. Then, the operations $R\tilde{\cup}S$, $R\tilde{\cap}S$, $R\tilde{+}S$ and $R\tilde{\times}S$ are defined as follows;*

1.

$$\begin{aligned} R\tilde{\cup}S = \ & \{< (x,y), (T^1_{R\tilde{\cup}S}(x,y), T^2_{R\tilde{\cup}S}(x,y), ..., T^n_{R\tilde{\cup}S}(x,y)), \\ & (I^1_{R\tilde{\cup}S}(x,y), I^2_{R\tilde{\cup}S}(x,y), ..., I^n_{R\tilde{\cup}S}(x,y)), \\ & (F^1_{R\tilde{\cup}S}(x,y), F^2_{R\tilde{\cup}S}(x,y), ..., F^n_{R\tilde{\cup}S}(x,y)) >: \ x, y \in E\} \end{aligned}$$

where

$$T^i_{R\tilde{\cup}S}(x,y) = T^i_R(x) \vee T^i_S(y),$$
$$I^i_{R\tilde{\cup}S}(x,y) = I^i_R(x) \wedge I^i_S(y),$$
$$F^i_{R\tilde{\cup}S}(x,y) = F^i_R(x) \wedge F^i_S(y)$$

$\forall x, y \in E$ *and* $i = 1, 2, ..., n$.

2.

$$\begin{aligned} R\tilde{\cap}S \ = & \{< (x,y), (T^1_{R\tilde{\cap}S}(x,y), T^2_{R\tilde{\cap}S}(x,y), ..., T^n_{R\tilde{\cap}S}(x,y)), \\ & (I^1_{R\tilde{\cap}S}(x,y), I^2_{R\tilde{\cap}S}(x,y), ..., I^n_{R\tilde{\cap}S}(x,y)), \\ & (F^1_{R\tilde{\cap}S}(x,y), F^2_{R\tilde{\cap}S}(x,y), ..., F^n_{R\tilde{\cap}S}(x,y)) >: \ x, y \in E\} \end{aligned}$$

where

$$T^i_{R\tilde{\cap}S}(x,y) = T^i_R(x) \wedge T^i_S(y),$$
$$I^i_{R\tilde{\cap}S}(x,y) = I^i_R(x) \vee I^i_S(y),$$
$$F^i_{R\tilde{\cap}S}(x,y) = F^i_R(x) \vee F^i_S(y)$$

$\forall x, y \in E$ *and* $i = 1, 2, ..., n$.

3.

$$\begin{aligned} R\tilde{+}S = \ & \{< (x,y), (T^1_{R\tilde{+}S}(x,y), T^2_{R\tilde{+}S}(x,y), ..., T^n_{R\tilde{+}S}(x,y)), \\ & (I^1_{R\tilde{+}S}(x,y), I^2_{R\tilde{+}S}(x,y), ..., I^n_{R\tilde{+}S}(x,y)), \\ & (F^1_{R\tilde{+}S}(x,y), F^2_{R\tilde{+}S}(x,y), ..., F^n_{R\tilde{+}S}(x,y)) >: \ x, y \in E\} \end{aligned}$$

where

$$T^i_{R\tilde{+}S}(x,y) = T^i_R(x) + T^i_S(y) - T^i_R(x).T^i_S(y),$$
$$I^i_{R\tilde{+}S}(x,y) = I^i_R(x).I^i_S(y),$$
$$F^i_{R\tilde{+}S}(x,y) = F^i_R(x).F^i_S(y)$$

$\forall x, y \in E$ *and* $i = 1, 2, ..., n$.

4.

$$
\begin{aligned}
R\tilde{\times}S = \quad & \{< (x,y), (T^1_{R\tilde{\times}S}(x,y), T^2_{R\tilde{\times}S}(x,y), ..., T^n_{R\tilde{\times}S}(x,y)),\\
& (I^1_{R\tilde{\times}S}(x,y), I^2_{R\tilde{\times}S}(x,y), ..., I^n_{R\tilde{\times}S}(x,y)),\\
& (F^1_{R\tilde{\times}S}(x,y), F^2_{R\tilde{\times}S}(x,y), ..., F^n_{R\tilde{\times}S}(x,y)) >: \ x, y \in E\}
\end{aligned}
$$

where

$$
\begin{aligned}
T^i_{R\tilde{\times}S}(x,y) &= T^i_R(x).T^i_S(y),\\
I^i_{R\tilde{\times}S}(x,y) &= I^i_R(x) + I^i_S(y) - I^i_R(x).I^i_S(y),\\
F^i_{R\tilde{\times}S}(x,y) &= F^i_R(x) + F^i_S(y) - F^i_R(x).F^i_S(y)
\end{aligned}
$$

$\forall x, y \in E$ *and* $i = 1, 2, ..., n$.

Here $\vee$, $\wedge$, $+$, $.$, $-$ *denotes maximum, minimum, addition, multiplication, subtraction of real numbers respectively.*

Example 3.7 *Let us consider the two neutrosophic refined relation* R *and* S*, from* A *to* B*, as follows*

$$
\begin{aligned}
R = \quad & \{< (x_1,x_1), \{0.2, 0.3, 0.4\}, \{0.4, 0.5, 0.6\}, \{0.3, 0.8, 0.9\} >,\\
& < (x_1,x_2), \{0.3, 0.4, 0.6\}, \{0.2, 0.3, 0.4\}, \{0.5, 0.6, 0.7\} >,\\
& < (x_2,x_1), \{0.1, 0.6, 0.3\}, \{0.2, 0.5, 0.6\}, \{0.2, 0.3, 0.4\} >\}
\end{aligned}
$$

and

$$
\begin{aligned}
S = \quad & \{< (x_1,x_1), \{0.1, 0.4, 0.5\}, \{0.3, 0.5, 0.7\}, \{0.2, 0.7, 0.1\} >,\\
& < (x_1,x_2), \{0.2, 0.3, 0.4\}, \{0.5, 0.6, 0.7\}, \{0.2, 0.3, 0.6\} >,\\
& < (x_2,x_1), \{0.4, 0.5, 0.6\}, \{0.2, 0.3, 0.4\}, \{0.1, 0.2, 0.3\} >\}
\end{aligned}
$$

Then,

$$
\begin{aligned}
R\tilde{\cup}S = \quad & \{< (x_1,x_1), \{0.2, 0.3, 0.4\}, \{0.4, 0.5, 0.6\}, \{0.3, 0.7, 0.1\} >,\\
& < (x_1,x_2), \{0.3, 0.3, 0.4\}, \{0.5, 0.3, 0.4\}, \{0.5, 0.3, 0.6\} >,\\
& < (x_2,x_1), \{0.4, 0.5, 0.3\}, \{0.2, 0.3, 0.4\}, \{0.2, 0.2, 0.3\} >\}
\end{aligned}
$$

and

$$
\begin{aligned}
R\tilde{\cap}S = \quad & \{< (x_1,x_1), \{0.1, 0.4, 0.5\}, \{0.3, 0.5, 0.7\}, \{0.2, 0.8, 0.9\} >,\\
& < (x_1,x_2), \{0.2, 0.4, 0.6\}, \{0.2, 0.6, 0.7\}, \{0.2, 0.6, 0.6\} >,\\
& < (x_2,x_1), \{0.1, 0.6, 0.6\}, \{0.2, 0.5, 0.6\}, \{0.1, 0.3, 0.4\} >\}
\end{aligned}
$$

Assume that $\emptyset \neq A, B, C \in NRS(E)$. Two neutrosophic refined relations under a suitable composition, could too yield a new neutrosophic refined relation with a useful significance. Composition of relations is important for applications, because of the reason that if a relation on A and B is known and if a relation on B and C is known then the relation on A and C could be computed and defined as follows;

Definition 3.8 *Let R(A→ B) and S (B→ C) be two neutrosophic refined relations. The composition S ∘R is a neutrosophic refined relation from A to C, defined by*

$$S\circ R = \{<(x,z),(T^1_{S\circ R}(x,z),T^2_{S\circ R}(x,z),...,T^n_{S\circ R}(x,z)), (I^1_{S\circ R}(x,z),I^2_{S\circ R}(x,z),...,I^n_{S\circ R}(x,z)), (F^1_{S\circ R}(x,z),F^2_{S\circ R}(x,z),...,F^n_{S\circ R}(x,z))>:\ x,z\in E\}$$

where

$$T^j_{S\circ R}(x,z)=\underset{y}{\vee}\left\{T^j_R(x,y)\wedge T^j_S(y,z)\right\}$$

$$I^j_{S\circ R}(x,z)=\underset{y}{\wedge}\left\{I^j_R(x,y)\vee I^j_S(y,z)\right\}$$

and

$$F^j_{S\circ R}(x,z)=\underset{y}{\wedge}\left\{F^j_R(x,y)\vee F^j_S(y,z)\right\}$$

for every (x, z) $E\times E$, for every $y\in E$ and $j=1,2,...,n$.

Definition 3.9 *A neutrosophic refined relation R on A is said to be;*

1. *reflexive if $T^j_R(x,x)=1$, $I^j_R(x,x)=0$ and $F^j_R(x,x)=0$ for all $x\in E$*
2. *symmetric if $T^j_R(x,y)=T^j_R(y,x)$, $I^j_R(x,y)=I^j_R(y,x)$ and $F^j_R(x,y)=F^j_R(y,x)$ for all $x,y\in E$*
3. *transitive if $R\circ R\subseteq R$.*
4. *neutrosophic refined equivalence relation if the relation R satisfies reflexive, symmetric and transitive.*

Definition 3.10 *The transitive closure of a neutrosophic refined relation R on $E\times E$ is $\hat{R}=R\tilde{\cup}R^2\tilde{\cup}R^3\tilde{\cup}...$*

Definition 3.11 *If R is a neutrosophic refined relation from A to B then R^{-1} is the inverse neutrosophic refined relation R from B to A, defined as follows:*

$$R^{-1}=\left\{\left\langle(y,x),T^j_{R^{-1}}(x,y)),I^j_{R^{-1}}(x,y),F^j_{R^{-1}}(x,y)\right\rangle:(x,y)\in E\times E\right\}$$

where
$T^j_{R^{-1}}(x,y)=T^j_R(y,x)$, $I^j_{R^{-1}}(x,y)=I^j_R(y,x)$, $F^j_{R^{-1}}(x,y)=F^j_R(y,x)$ *and* $j=1,2,...,n$.

Proposition 3.12 *If R and S are two neutrosophic refined relation from A to B and B to C, respectively. Then,*

1. $(R^{-1})^{-1}=R$

2. $(S \circ R)^{-1} = R^{-1} \circ S^{-1}$

Proof

1. Since R^{-1} is a neutrosophic refined relation from B to A, we have $T^j_{R^{-1}}(x,y) = T^j_R(y,x)$, $I^j_{R^{-1}}(x,y) = I^j_R(y,x)$ and $F^j_{R^{-1}}(x,y) = F^j_R(y,x)$ Then,

$$T^j_{(R^{-1})^{-1}}(x,y) = T^j_{R^{-1}}(y,x) = T^j_R(x,y),$$
$$I^j_{(R^{-1})^{-1}}(x,y) = I^j_{R^{-1}}(y,x) = I^j_R(x,y)$$

and

$$F^j_{(R^{-1})^{-1}}(x,y) = F^j_{R^{-1}}(y,x) = F^j_R(x,y)$$

therefore $(R^{-1})^{-1} = R$.

2. If the composition $S \circ R$ is a neutrosophic refined relation from A to C, then the composition $R^{-1} \circ S^{-1}$ is a neutrosophic refined relation from C to A. Then,

$$\begin{aligned} T^j_{(S\circ R)^{-1}}(z,x) &= T^j_{(S\circ R)}(x,z) \\ &= \bigvee_y \left\{ T^j_R(x,y) \wedge T^j_S(y,z) \right\} \\ &= \bigvee_y \left\{ T^j_{R^{-1}}(y,x) \wedge T^j_{S^{-1}}(z,y) \right\} \quad , \\ &= \bigvee_y \left\{ T^j_{S^{-1}}(z,y) \wedge T^j_{R^{-1}}(y,x) \right\} \\ &= T^j_{R^{-1}\circ S^{-1}}(z,x) \end{aligned}$$

$$\begin{aligned} I^j_{(S\circ R)^{-1}}(z,x) &= I^j_{(S\circ R)}(x,z) \\ &= \bigwedge_y \left\{ I^j_R(x,y) \vee I^j_S(y,z) \right\} \\ &= \bigwedge_y \left\{ I^j_{R^{-1}}(y,x) \vee I^j_{S^{-1}}(z,y) \right\} \\ &= \bigwedge_y \left\{ I^j_{S^{-1}}(z,y) \vee I^j_{R^{-1}}(y,x) \right\} \\ &= I^j_{R^{-1}\circ S^{-1}}(z,x) \end{aligned}$$

and

$$\begin{aligned} F^j_{(S\circ R)^{-1}}(z,x) &= F^j_{(S\circ R)}(x,z) \\ &= \bigwedge_y \left\{ F^j_R(x,y) \vee F^j_S(y,z) \right\} \\ &= \bigwedge_y \left\{ F^j_{R^{-1}}(y,x) \vee F^j_{S^{-1}}(z,y) \right\} \\ &= \bigwedge_y \left\{ F^j_{S^{-1}}(z,y) \vee F^j_{R^{-1}}(y,x) \right\} \\ &= F^j_{R^{-1}\circ S^{-1}}(z,x) \end{aligned}$$

Finally; proof is valid.

Proposition 3.13 *If R is symmetric ,then $R^{-1}$is also symmetric.*

Proof: Assume that R is Symmetric then we have

$$T^j_R(x,y) = T^j_R(y,x),$$

$$I^j_R(x,y) = I^j_R(y,x)$$

and

$$F^j_R(x,y) = F^j_R(y,x)$$

Also if R^{-1} is an inverse relation, then we have

$$T^j_{R^{-1}}(x,y) = T^j_R(y,x),$$

$$I^j_{R^{-1}}(x,y) = I^j_R(y,x)$$

and

$$F^j_{R^{-1}}(x,y) = F^j_R(y,x)$$

for all $x, y \in E$

To prove R^{-1} is symmetric, it is enough to prove

$$T^j_{R^{-1}}(x,y) = T^j_{R^{-1}}(y,x),$$

$$I^j_{R^{-1}}(x,y) = I^j_{R^{-1}}(y,x)$$

and

$$F^j_{R^{-1}}(x,y) = F^j_{R^{-1}}(y,x)$$

for all $x, y \in E$

Therefore;

$$T^j_{R^{-1}}(x,y) = T^j_R(y,x) = T^j_R(x,y) = T^j_{R^{-1}}(y,x);$$

$$I^j_{R^{-1}}(x,y) = I^j_R(y,x) = I^j_R(x,y) = I^j_{R^{-1}}(y,x)$$

and

$$F^j_{R^{-1}}(x,y) = F^j_R(y,x) = F^j_R(x,y) = F^j_{R^{-1}}(y,x)$$

Finally; proof is valid.

Proposition 3.14 *If R is symmetric ,if and only if $R = R^{-1}$.*

Proof: Let R be symmetric , then

$$T^j_R(x,y) = T^j_R(y,x);$$

$$I^j_R(x,y) = I^j_R(y,x)$$

and

$$F^j_R(x,y) = F^j_R(y,x)$$

and

R^{-1} is an inverse relation, then

$$T^j_{R^{-1}}(x,y) = T^j_R(y,x);$$

$$I^j_{R^{-1}}(x,y) = I^j_R(y,x)$$

and

$$F^j_{R^{-1}}(x,y) = F^j_R(y,x)$$

for all $x, y \in E$

Therefore; $T^j_{R^{-1}}(x,y) = T^j_R(y,x) = T^j_R(x,y)$.

Similarly

$$I^j_{R^{-1}}(x,y) = I^j_R(y,x) = I^j_R(x,y)$$

and

$$F^j_{R^{-1}}(x,y) = F^j_R(y,x) = F^j_R(x,y)$$

for all $x, y \in E$.

Hence $R = R^{-1}$

Conversely, assume that $R = R^{-1}$ then, we have

$$T^j_R(x,y) = T^j_{R^{-1}}(x,y) = T^j_R(y,x).$$

Similarly

$$I^j_R(x,y) = I^j_{R^{-1}}(x,y) = I^j_R(y,x)$$

and

$$F^j_R(x,y) = F^j_{R^{-1}}(x,y) = F^j_R(y,x).$$

Hence R is symmetric.

Proposition 3.15 *If R and S are symmetric neutrosophic refined relations, then*

1. $R\tilde{\cup}S$,
2. $R\tilde{\cap}S$,
3. $R\tilde{+}S$
4. $R\tilde{\times}S$

are also symmetric.

Proof: R is symmetric, then we have;

$$T^j_R(x,y) = T^j_R(y,x),$$

$$I^j_R(x,y) = I^j_R(y,x)$$

and

$$F^j_R(x,y) = F^j_R(y,x)$$

similarly S is symmetric, then we have

$$T_S^j(x,y) = T_S^j(y,x),$$

$$I_S^j(x,y) = I_S^j(y,x)$$

and

$$F_S^j(x,y) = F_S^j(y,x)$$

Therefore,

1.
$$\begin{aligned} T^j_{R\widetilde{\cup}S}(x,y) &= \max\left\{T_R^j(x,y), T_S^j(x,y)\right\} \\ &= \max\left\{T_R^j(y,x), T_S^j(y,x)\right\} \\ &= T^j_{R\widetilde{\cup}S}(y,x) \end{aligned},$$

$$\begin{aligned} I^j_{R\widetilde{\cup}S}(x,y) &= min\left\{I_R^j(x,y), I_S^j(x,y)\right\} \\ &= \min\left\{I_R^j(y,x), I_S^j(y,x)\right\} \\ &= I^j_{R\widetilde{\cup}S}(y,x), \end{aligned}$$

and

$$\begin{aligned} F^j_{R\widetilde{\cup}S}(x,y) &= \min\left\{F_R^j(x,y), F_S^j(x,y)\right\} \\ &= \min\left\{F_R^j(y,x), F_S^j(y,x)\right\} \\ &= F^j_{R\widetilde{\cup}S}(y,x) \end{aligned}$$

therefore, $R\widetilde{\cup}S$ is symmetric.

2.
$$\begin{aligned} T^j_{R\widetilde{\cap}S}(x,y) &= \min\left\{T_R^j(x,y), T_S^j(x,y)\right\} \\ &= \min\left\{T_R^j(y,x), T_S^j(y,x)\right\} \\ &= T^j_{R\widetilde{\cap}S}(y,x), \end{aligned}$$

$$\begin{aligned} I^j_{R\widetilde{\cap}S}(x,y) &= max\left\{I_R^j(x,y), I_S^j(x,y)\right\} \\ &= \max\left\{I_R^j(y,x), I_S^j(y,x)\right\} \\ &= I^j_{R\widetilde{\cap}S}(y,x), \end{aligned}$$

and

$$\begin{aligned} F^j_{R\widetilde{\cap}S}(x,y) &= \max\left\{F_R^j(x,y), F_S^j(x,y)\right\} \\ &= \max\left\{F_R^j(y,x), F_S^j(y,x)\right\} \\ &= F^j_{R\widetilde{\cap}S}(y,x) \end{aligned}$$

therefore; $R\widetilde{\cap}S$ is symmetric.

3.

$$\begin{aligned} T^j_{R\tilde{+}S}(x,y) &= T^j_R(x,y) + T^j_S(x,y) - T^j_R(x,y)T^j_S(x,y) \\ &= T^j_R(y,x) + T^j_S(y,x) - T^j_R(y,x)T^j_S(y,x) \\ &= T^j_{R\tilde{+}S}(y,x) \end{aligned}$$

$$\begin{aligned} I^j_{R\tilde{+}S}(x,y) &= I^j_R(x,y)I^j_S(x,y) \\ &= I^j_R(y,x)I^j_S(y,x) \\ &= I^j_{R\tilde{+}S}(y,x) \end{aligned}$$

and

$$\begin{aligned} F^j_{R\tilde{+}S}(x,y) &= F^j_R(x,y)F^j_S(x,y) \\ &= F^j_R(y,x)F^j_S(y,x) \\ &= F^j_{R\tilde{+}S}(y,x) \end{aligned}$$

therefore, $R\tilde{+}S$ is also symmetric

4.

$$\begin{aligned} T^j_{R\tilde{\times}S}(x,y) &= T^j_R(x,y)T^j_S(x,y) \\ &= T^j_R(y,x)T^j_S(y,x) \\ &= T^j_{R\tilde{\times}tS}(y,x) \end{aligned}$$

$$\begin{aligned} I^j_{R\tilde{\times}S}(x,y) &= I^j_R(x,y) + I^j_S(x,y) - I^j_R(x,y)I^j_S(x,y) \\ &= I^j_R(y,x) + I^j_S(y,x) - I^j_R(y,x)I^j_S(y,x) \\ &= I^j_{R\tilde{\times}S}(y,x) \end{aligned}$$

$$\begin{aligned} F^j_{R\tilde{\times}S}(x,y) &= F^j_R(x,y) + F^j_S(x,y) - F^j_R(x,y)F^j_S(x,y) \\ &= F^j_R(y,x) + F^j_S(y,x) - F^j_R(y,x)F^j_S(y,x) \\ &= F^j_{R\tilde{\times}S}(y,x) \end{aligned}$$

hence, $R\tilde{\times}S$ is also symmetric.

Remark 3.16 *$R \circ S$ in general is not symmetric, as*

$$\begin{aligned} T^j_{(R\circ S)}(x,z) &= \bigvee_y \left\{T^j_S(x,y) \wedge T^j_R(y,z)\right\} \\ &= \bigvee_y \left\{T^j_S(y,x) \wedge T^j_R(z,y)\right\} \\ &\neq T^j_{(R\circ S)}(z,x) \end{aligned}$$

$$\begin{aligned} I^j_{(R\circ S)}(x,z) &= \bigwedge_y \left\{I^j_S(x,y) \vee I^j_R(y,z)\right\} \\ &= \bigwedge_y \left\{I^j_S(y,x) \vee I^j_R(z,y)\right\} \\ &\neq I^j_{(R\circ S)}(z,x) \end{aligned}$$

$$\begin{aligned}
F^j_{(R\circ S)}(x,z) &= \bigwedge_y \left\{F^j_S(x,y) \vee F^j_R(y,z)\right\} \\
&= \bigwedge_y \left\{F^j_S(y,x) \vee F^j_R(z,y)\right\} \\
&\neq F^j_{(R\circ S)}(z,x)
\end{aligned}$$

but $R \circ S$ is symmetric, if $R \circ S = S \circ R$, for R and S are symmetric relations.

$$\begin{aligned}
T^j_{(R\circ S)}(x,z) &= \bigvee_y \left\{T^j_S(x,y) \wedge T^j_R(y,z)\right\} \\
&= \bigvee_y \left\{T^j_S(y,x) \wedge T^j_R(z,y)\right\} \\
&= \bigvee_y \left\{T^j_R(y,x) \wedge T^j_R(z,y)\right\} \\
& T^j_{(R\circ S)}(z,x)
\end{aligned}$$

$$\begin{aligned}
I^j_{(R\circ S)}(x,z) &= \bigwedge_y \left\{I^j_S(x,y) \vee I^j_R(y,z)\right\} \\
&= \bigwedge_y \left\{I^j_S(y,x) \vee I^j_R(z,y)\right\} \\
&= \bigwedge_y \left\{I^j_R(y,x) \vee I^j_R(z,y)\right\} \\
& I^j_{(R\circ S)}(z,x)
\end{aligned}$$

and

$$\begin{aligned}
F^j_{(R\circ S)}(x,z) &= \bigwedge_y \left\{F^j_S(x,y) \vee F^j_R(y,z)\right\} \\
&= \bigwedge_y \left\{F^j_S(y,x) \vee F^j_R(z,y)\right\} \\
&= \bigwedge_y \left\{F^j_R(y,x) \vee F^j_R(z,y)\right\} \\
& F^j_{(R\circ S)}(z,x)
\end{aligned}$$

for every $(x,z) \in E \times E$ and for $y \in E$.

Proposition 3.17 *If R is transitive relation, then R^{-1} is also transitive.*

Proof : R is transitive relation, if $R \circ R \subseteq R$, hence if $R^{-1} \circ R^{-1} \subseteq R^{-1}$, then R^{-1} is transitive.

Consider;

$$\begin{aligned}
T^j_{R^{-1}}(x,y) &= T^j_R(y,x) \geq T^j_{R\circ R}(y,x) \\
&= \bigvee_z \left\{T^j_R(y,z) \wedge T^j_R(z,x)\right\} \\
&= \bigvee_z \left\{T^j_{R^{-1}}(x,z) \wedge T^j_{R^{-1}}(z,y)\right\} \\
&= T^j_{R^{-1}\circ R^{-1}}(x,y)
\end{aligned}$$

$$\begin{aligned}
I^j_{R^{-1}}(x,y) &= I^j_R(y,x) \leq I^j_{R\circ R}(y,x) \\
&= \bigwedge_z \left\{I^j_R(y,z) \vee I^j_R(z,x)\right\} \\
&= \bigwedge_z \left\{I^j_{R^{-1}}(x,z) \vee I^j_{R^{-1}}(z,y)\right\} \\
&= I^j_{R^{-1}\circ R^{-1}}(x,y)
\end{aligned}$$

and
$$\begin{aligned} F^j_{R^{-1}}(x,y) &= F^j_R(y,x) \le F^j_{R\circ R}(y,x) \\ &= \underset{z}{\wedge}\left\{F^j_R(y,z) \vee F^j_R(z,x)\right\} \\ &= \underset{z}{\wedge}\left\{F^j_{R^{-1}}(x,z) \vee F^j_{R^{-1}}(z,y)\right\} \\ &= F^j_{R^{-1}\circ R^{-1}}(x,y) \end{aligned}$$

hence, proof is valid.

Proposition 3.18 *If R is transitive relation, then $R \cap S$ is also transitive*

Proof: As R and S are transitive relations, $R \circ R \subseteq R$ and $S \circ S \subseteq S$. also
$$\begin{gathered} T^j_{R\tilde{\cap}S}(x,y) \ge T^j_{(R\tilde{\cap}S)\circ(R\tilde{\cap}S)}(x,y) \\ I^j_{R\tilde{\cap}S}(x,y) \le I^j_{(R\tilde{\cap}S)\circ(R\tilde{\cap}S)}(x,y) \\ F^j_{R\tilde{\cap}S}(x,y) \le F^j_{(R\tilde{\cap}S)\circ(R\tilde{\cap}S)}(x,y) \end{gathered}$$

implies $R\tilde{\cap}S) \circ (R\tilde{\cap}S) \subseteq R \cap S$, hence $R \cap S$ is transitive.

Proposition 3.19 *If R and S are transitive relations, then*

1. $R\tilde{\cup}S$,
2. $R\tilde{+}S$
3. $R\tilde{\times}S$

are not transitive.

Proof:

1. As
$$\begin{gathered} T^j_{R\tilde{\cup}S}(x,y) = \max\left\{T^j_R(x,y), T^j_S(x,y)\right\} \\ I^j_{R\tilde{\cup}S}(x,y) = \min\left\{I^j_R(x,y), I^j_S(x,y)\right\} \\ F^j_{R\tilde{\cup}S}(x,y) = \min\left\{F^j_R(x,y), F^j_S(x,y)\right\} \end{gathered}$$
and
$$\begin{gathered} T^j_{(R\tilde{\cup}S)\circ(R\tilde{\cup}S)}(x,y) \ge T^j_{R\tilde{\cup}S}(x,y) \\ I^j_{(R\tilde{\cup}S)\circ(R\tilde{\cup}S)}(x,y) \le I^j_{R\tilde{\cup}S}(x,y) \\ F^j_{(R\tilde{\cup}S)\circ(R\tilde{\cup}S)}(x,y) \le F^j_{R\tilde{\cup}S}(x,y) \end{gathered}$$
2. As
$$\begin{gathered} T^j_{R\tilde{+}S}(x,y) = T^j_R(x,y) + T^j_S(x,y) - T^j_R(x,y)T^j_S(x,y) \\ I^j_{R\tilde{+}S}(x,y) = I^j_R(x,y)I^j_S(x,y) \\ F^j_{R\tilde{+}S}(x,y) = F^j_R(x,y)F^j_S(x,y) \end{gathered}$$
and

$$T^j_{(R\tilde{+}S)\circ(R\tilde{+}S)}(x,y) \geq T^j_{R\tilde{+}S}(x,y)$$
$$I^j_{(R\tilde{+}S)\circ(R\tilde{+}S)}(x,y) \leq I^j_{R\tilde{+}S}(x,y)$$
$$F^j_{(R\tilde{+}S)\circ(R\tilde{+}S)}(x,y) \leq F^j_{R\tilde{+}S}(x,y)$$

3. As

$$T^j_{R\tilde{\times}S}(x,y) = T^j_R(x,y)T^j_S(x,y)$$
$$I^j_{R\tilde{\times}S}(x,y) = I^j_R(x,y) + I^j_S(x,y) - I^j_R(x,y)I^j_S(x,y)$$
$$F^j_{R\tilde{\times}S}(x,y) = F^j_R(x,y) + F^j_S(x,y) - F^j_R(x,y)F^j_S(x,y)$$

and

$$T^j_{(R\tilde{\times}S)\circ(R\tilde{\times}S)}(x,y) \geq T^j_{R\tilde{\times}S}(x,y)$$
$$I^j_{(R\tilde{\times}S)\circ(R\tilde{\times}S)}(x,y) \leq I^j_{R\tilde{\times}S}(x,y)$$
$$F^j_{(R\tilde{\times}S)\circ(R\tilde{\times}S)}(x,y) \leq F^j_{R\tilde{\times}S}(x,y)$$

Hence $R\tilde{\cup}S$, $R\tilde{+}S$ and $R\tilde{\times}S$ are not transitive.

Proposition 3.20 *If R is transitive relation, then R^2 is also transitive.*

Proof: R is transitive relation, if $R \circ R \subseteq R$, therefore if $R^2 \circ R^{-2} \subseteq R^2$, then R^2 is transitive.

$$T^j_{R\circ R}(y,x) = \bigvee_z \left\{T^j_R(y,z) \wedge T^j_R(z,x)\right\} \geq \bigvee_z \left\{T^j_{R\circ R}(y,z) \wedge T^j_{R\circ R}(z,x)\right\} = T^j_{R^2\circ R^2}(y,x),$$

$$I^j_{R\circ R}(y,x) = \bigwedge_z \left\{I^j_R(y,z) \vee I^j_R(z,x)\right\} \leq \bigwedge_z \left\{I^j_{R\circ R}(y,z) \vee I^j_{R\circ R}(z,x)\right\} = I^j_{R^2\circ R^2}(y,x)$$

and

$$F^j_{R\circ R}(y,x) = \bigwedge_z \left\{F(y,z) \vee F^j_R(z,x)\right\} \leq \bigwedge_z \left\{I^j_{R\circ R}(y,z) \vee F^j_{R\circ R}(z,x)\right\} = F^j_{R^2\circ R^2}(y,x)$$

Finally, the proof is valid.

4 Acknowledgments

The authors would like to thank the anonymous reviewer for their careful reading of this research paper and for their helpful comments.

5 Conclusion

In this paper, we have firstly defined the neutrosophic refined relations(NRR). The NRR are the extension of neutrosophic relation (NR) and intuitionistic multirelation[27]. The notions of inverse, symmetry, reflexivity and transitivity on neutrosophic refined relations are studied. The future work will cover the application of the NRR in decision making, pattern recognition and in medical diagnosis.

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New Distance and Similarity Measures of Interval Neutrosophic Sets

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Abstract: In this paper we proposed a new distance and several similarity measures between interval neutrosophic sets.

Keywords: Neutrosophic set, Interval neutrosohic set, Similarity measure.

I. INTRODUCTION

The neutrsophic set, founded by F.Smarandache [1], has capability to deal with uncertainty, imprecise, incomplete and inconsistent information which exist in the real world. Neutrosophic set theory is a powerful tool in the formal framework, which generalizes the concepts of the classic set, fuzzy set [2], interval-valued fuzzy set [3], intuitionistic fuzzy set [4], interval-valued intuitionistic fuzzy set [5], and so on.

After the pioneering work of Smarandache, in 2005 Wang [6] introduced the notion of interval neutrosophic set (INS for short) which is a particular case of the neutrosophic set. INS can be described by a membership interval, a non-membership interval, and the indeterminate interval. Thus the interval value neutrosophic set has the virtue of being more flexible and practical than single value neutrosophic set. And the Interval Neutrosophic Set provides a more reasonable mathematical framework to deal with indeterminate and inconsistent information.

Many papers about neutrosophic set theory have been done by various researchers [7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20].

A similarity measure for neutrosophic set (NS) is used for estimating the degree of similarity between two neutrosophic sets. Several researchers proposed some similarity measures between NSs, such as S. Broumi and F. Smarandache [26], Jun Ye [11, 12], P. Majumdar and S.K.Smanta [23].

In the literature, there are few researchers who studied the distance and similarity measure of IVNS.

In 2013, Jun Ye [12] proposed similarity measures between interval neutrosophic set based on the Hamming and Euclidean distance, and developed a multicriteria decision–making method based on the similarity degree. S. Broumi and F. Smarandache [10] proposed a new similarity measure, called "cosine similarity measure of interval valued neutrosophic sets". On the basis of numerical computations, S. Broumi and F. Smarandache found out that their similarity measures are stronger and more robust than Ye's measures.

We all know that there are various distance measures in mathematics. So, in this paper, we will extend the generalized distance of single valued neutrosophic set proposed by Ye [12] to the case of interval neutrosophic set and we'll study some new similarity measures.

This paper is organized as follows. In section 2, we review some notions of neutrosophic set andinterval valued neutrosophic set. In section 3, some new similarity measures of interval valued neutrosophic sets and their proofs are introduced. Finally, the conclusions are stated in section 4.

II. PRELIMIAIRIES

This section gives a brief overview of the concepts of neutrosophic set, and interval valued neutrosophic set.

A. Neutrosophic Sets

1) Definition [1]

Let X be a universe of discourse, with a generic element in X denoted by x, then a neutrosophic set A is an object having the form:

A = $\{< x: T_A(x), I_A(x), F_A(x)>, x \in X\}$, where the functions T, I, F : X→ $]^-0, 1^+[$ define respectively the degree of membership (or Truth), the degree of indeterminacy, and the degree of non-membership (or Falsehood) of the element x ∈ X to the set A with the condition:

$$^-0 \le T_A(x)+I_A(x)+F_A(x) \le 3^+. \quad \textbf{(1)}$$

From philosophical point of view, the neutrosophic set takes the value from real standard or non-standard subsets of $]^-0, 1^+[$. Therefore, instead of $]^-0, 1^+[$ we need to take the interval [0, 1] for technical applications, because $]^-0, 1^+[$ will be difficult to apply in the real applications such as in scientific and engineering problems.

For two NSs, A_{NS}= {<x, $T_A(x)$, $I_A(x)$, $F_A(x)$>|x ∈ X} **(2)**

and B_{NS} ={ <x, $T_B(x)$, $I_B(x)$, $F_B(x)$> | x ∈ X } the two relations are defined as follows:

(1) $A_{NS} \subseteq B_{NS}$ if and only if $T_A(x) \le T_B(x), I_A(x) \ge I_B(x), F_A(x) \ge F_B(x)$.

(2) $A_{NS} = B_{NS}$ if and only if , $T_A(x)=T_B(x), I_A(x)=I_B(x), F_A(x)=F_B(x)$.

B. Interval Valued Neutrosophic Sets

In actual applications, sometimes, it is not easy to express the truth-membership, indeterminacy-membership and falsity-membership by crisp value, and they may be easier to be expressed by interval numbers. Wang et al. [6] further defined interval neutrosophic sets (INS) shows as follows:

1) Definition [6]

Let X be a universe of discourse, with generic element in X denoted by x. An interval valued neutrosophic set (for short IVNS) A in X is characterized by truth-membership function$T_A(x)$, indeteminacy-membership function $I_A(x)$, and falsity-membership function $F_A(x)$. For each point x in X, we have that $T_A(x), I_A(x), F_A(x) \in [0,1]$.

For two IVNS, A_{IVNS}={<x,[$T_A^L(x)$,$T_A^U(x)$], [$I_A^L(x), I_A^U(x)$], [$F_A^L(x), F_A^U(x)$]> | x ∈ X } **(3)**

and B_{IVNS}= {<x, [$T_B^L(x)$,$T_B^U(x)$], [$I_B^L(x), I_B^U(x)$], [$F_B^L(x), F_B^U(x)$] > | x ∈ X } the two relations are defined as follows:

(1) $A_{IVNS} \subseteq B_{IVNS}$ if and only if $T_A^L(x) \le T_B^L(x), T_A^U(x) \le T_B^U(x), I_A^L(x) \ge I_B^L(x), I_A^U(x) \ge I_B^U(x), F_A^L(x) \ge F_B^L(x), F_A^U(x) \ge F_B^U(x)$.

(2) $A_{IVNS} = B_{IVNS}$ if and only if $T_A^L(x_i) = T_B^L(x_i)$, $T_A^U(x_i) = T_B^U(x_i)$, $I_A^L(x_i) = I_B^L(x_i)$,$I_A^U(x_i) = I_B^U(x_i)$, $F_A^L(x_i) = F_B^L(x_i)$ and $F_A^U(x_i) = F_B^U(x_i)$ for any x ∈ X.

C. Defintion

Let A and B be two interval valued neutrosophic sets, then

i. $\mathbf{0 \le S(A,B) \le 1}$.

ii. $S(A,B) = S(B,A)$.

iii. $S(A,B) = 1$ if A= B, i.e $T_A^L(x_i) = T_B^L(x_i)$, $T_A^U(x_i) = T_B^U(x_i)$, $I_A^L(x_i) = I_B^L(x_i)$, $I_A^U(x_i) = I_B^U(x_i)$ and $F_A^L(x_i) = F_B^L(x_i)$, $F_A^U(x_i) = F_B^U(x_i)$, for i = 1, 2,…., n.

iv . A⊂ B ⊂ C ⇒ S(A,B) ≤ min (S(A,B), S(B,C).

III. New Distance Measure of Interval Valued Neutrosophic Sets

Let A and B be two single neutrosophic sets, then J. Ye [11] proposed a generalized single valued neutrosophic weighted distance measure between A and B as follows:

$$d_\lambda(A,B) = \left\{\frac{1}{3}\sum_{i=1}^{n} w_i\left[|T_A(x_i) - T_B(x_i)|^\lambda + |I_A(x_i) - I_B(x_i)|^\lambda + |F_A(x_i) - F_B(x_i)|^\lambda\right]\right\}^{\frac{1}{\lambda}} \quad \textbf{(4)}$$

where

$\lambda > 0$ and $T_A(x_i), I_A(x_i), F_A(x_i), T_B(x_i), I_B(x_i), F_B(x_i) \in [0, 1]$.

Based on the geometrical distance model and using the interval neutrosophic sets, we extended the distance (4) as follows:

$$d_\lambda(A,B) = \left\{\frac{1}{6}\sum_{i=1}^{n} w_i\left[|T_A^L(x_i) - T_B^L(x_i)|^\lambda + |T_A^U(x_i) - T_B^U(x_i)|^\lambda + |I_A^L(x_i) - I_B^L(x_i)|^\lambda + |I_A^U(x_i) - I_B^U(x_i)|^\lambda + |F_A^L(x_i) - F_B^L(x_i)|^\lambda + |F_A^U(x_i) - F_B^U(x_i)|^\lambda\right]\right\}^{\frac{1}{\lambda}}. \quad \textbf{(5)}$$

The normalized generalized interval neutrosophic distance is

$$d_\lambda(A,B) = \left\{\frac{1}{6n}\sum_{i=1}^{n} w_i\left[|T_A^L(x_i) - T_B^L(x_i)|^\lambda + |T_A^U(x_i) - T_B^U(x_i)|^\lambda + |I_A^L(x_i) - I_B^L(x_i)|^\lambda + |I_A^U(x_i) - I_B^U(x_i)|^\lambda + |F_A^L(x_i) - F_B^L(x_i)|^\lambda + |F_A^U(x_i) - F_B^U(x_i)|^\lambda\right]\right\}^{\frac{1}{\lambda}}. \quad \textbf{(6)}$$

If w=$\{\frac{1}{n}, \frac{1}{n}, \dots, \frac{1}{n}\}$, the distance (6) is reduced to the following distances:

$$d_\lambda(A,B) = \left\{\frac{1}{6}\sum_{i=1}^{n}\left[|T_A^L(x_i) - T_B^L(x_i)|^\lambda + |T_A^U(x_i) - T_B^U(x_i)|^\lambda + |I_A^L(x_i) - I_B^L(x_i)|^\lambda + |I_A^U(x_i) - I_B^U(x_i)|^\lambda + |F_A^L(x_i) - F_B^L(x_i)|^\lambda + |F_A^U(x_i) - F_B^U(x_i)|^\lambda\right]\right\}^{\frac{1}{\lambda}}. \quad \textbf{(7)}$$

$$d_\lambda(A,B) = \left\{\frac{1}{6n}\sum_{i=1}^{n}\left[|T_A^L(x_i) - T_B^L(x_i)|^\lambda + |T_A^U(x_i) - T_B^U(x_i)|^\lambda + |I_A^L(x_i) - I_B^L(x_i)|^\lambda + |I_A^U(x_i) - I_B^U(x_i)|^\lambda + |F_A^L(x_i) - F_B^L(x_i)|^\lambda + |F_A^U(x_i) - F_B^U(x_i)|^\lambda\right]\right\}^{\frac{1}{\lambda}}. \quad \textbf{(8)}$$

Particular case.

(i) If $\lambda = 1$ then the distances (7) and (8) are reduced to the following Hamming distance and respectively normalized Hamming distance defined by Ye Jun [11]:

$$d_H(A,B) = \left\{\frac{1}{6}\sum_{i=1}^{n}[|T_A^L(x_i) - T_B^L(x_i)| + |T_A^U(x_i) - T_B^U(x_i)| + |I_A^L(x_i) - I_B^L(x_i)| + |I_A^U(x_i) - I_B^U(x_i)| + |F_A^L(x_i) - F_B^L(x_i)| + |F_A^U(x_i) - F_B^U(x_i)|]\right\}, \quad \textbf{(9)}$$

$$d_{NH}(A,B) = \left\{\frac{1}{6n}\sum_{i=1}^{n}[|T_A^L(x_i) - T_B^L(x_i)| + |T_A^U(x_i) - T_B^U(x_i)| + |I_A^L(x_i) - I_B^L(x_i)| + |I_A^U(x_i) - I_B^U(x_i)| + |F_A^L(x_i) - F_B^L(x_i)| + |F_A^U(x_i) - F_B^U(x_i)|]\right\}. \quad \textbf{(10)}$$

(ii) If $\lambda = 2$ then the distances (7) and (8) are reduced to the following Euclidean distance and respectively normalized Euclidean distance defined by Ye Jun [12]:

$$d_E(A,B) = \left\{\frac{1}{6}\sum_{i=1}^{n}[|T_A^L(x_i) - T_B^L(x_i)|^2 + |T_A^U(x_i) - T_B^U(x_i)|^2 + |I_A^L(x_i) - I_B^L(x_i)|^2 + |I_A^U(x_i) - I_B^U(x_i)|^2 + |F_A^L(x_i) - F_B^L(x_i)|^2 + |F_A^U(x_i) - F_B^U(x_i)|^2]\right\}^{\frac{1}{2}}, \quad \textbf{(11)}$$

$$d_{NE}(A,B) = \left\{\frac{1}{6n}\sum_{i=1}^{n}[|T_A^L(x_i) - T_B^L(x_i)|^2 + |T_A^U(x_i) - T_B^U(x_i)|^2 + |I_A^L(x_i) - I_B^L(x_i)|^2 + |I_A^U(x_i) - I_B^U(x_i)|^2 + |F_A^L(x_i) - F_B^L(x_i)|^2 + |F_A^U(x_i) - F_B^U(x_i)|^2]\right\}^{\frac{1}{2}}. \quad \textbf{(12)}$$

IV. NEW SIMILARITY MEASURES OF INTERVAL VALUED NEUTROSOPHIC SET

A. Similarity measure based on the geometric distance model

Based on distance (4), we define the similarity measure between the interval valued neutrosophic sets A and B as follows:

$$S_{DM}(A,B) = 1 - \left\{\frac{1}{6n}\sum_{i=1}^{n}\left[\left|T_A^L(x_i) - T_B^L(x_i)\right|^\lambda + \left|T_A^U(x_i) - T_B^U(x_i)\right|^\lambda + \left|I_A^L(x_i) - I_B^L(x_i)\right|^\lambda + \left|I_A^U(x_i) - I_B^U(x_i)\right|^\lambda + \left|F_A^L(x_i) - F_B^L(x_i)\right|^\lambda + \left|F_A^U(x_i) - F_B^U(x_i)\right|^\lambda\right]\right\}^{\frac{1}{\lambda}}, \quad \textbf{(13)}$$

where $\lambda > 0$ and $S_{DM}(A,B)$ is the degree of similarity of A and B .

If we take the weight of each element $x_i \in X$ into account, then

$$S_{DM}^{w}(A,B) = 1 - \left\{\frac{1}{6}\sum_{i=1}^{n} w_i\left[\left|T_A^L(x_i) - T_B^L(x_i)\right|^\lambda + \left|T_A^U(x_i) - T_B^U(x_i)\right|^\lambda + \left|I_A^L(x_i) - I_B^L(x_i)\right|^\lambda + \left|I_A^U(x_i) - I_B^U(x_i)\right|^\lambda + \left|F_A^L(x_i) - F_B^L(x_i)\right|^\lambda + \left|F_A^U(x_i) - F_B^U(x_i)\right|^\lambda\right]\right\}^{\frac{1}{\lambda}}. \quad \textbf{(14)}$$

If each elements has the same importance, i.e. $w = \{\frac{1}{n}, \frac{1}{n}, \ldots, \frac{1}{n}\}$, then similarity (14) reduces to (13).

By (definition C) it can easily be known that $S_{DM}(A,B)$ satisfies all the properties of the definition..

Similarly, we define another similarity measure of A and B, as:

$$S(A,B) = 1 - \left[\frac{\sum_{i=1}^{n}\left(\left|T_A^L(x_i) - T_B^L(x_i)\right|^\lambda + \left|T_A^U(x_i) - T_B^U(x_i)\right|^\lambda + \left|I_A^L(x_i) - I_B^L(x_i)\right|^\lambda + \left|I_A^U(x_i) - I_B^U(x_i)\right|^\lambda + \left|F_A^L(x_i) - F_B^L(x_i)\right|^\lambda + \left|F_A^U(x_i) - F_B^U(x_i)\right|^\lambda\right)}{\sum_{i=1}^{n}\left(\left|T_A^L(x_i) + T_B^L(x_i)\right|^\lambda + \left|T_A^U(x_i) + T_B^U(x_i)\right|^\lambda + \left|I_A^L(x_i) + I_B^L(x_i)\right|^\lambda + \left|I_A^U(x_i) + I_B^U(x_i)\right|^\lambda + \left|F_A^L(x_i) + F_B^L(x_i)\right|^\lambda + \left|F_A^U(x_i) + F_B^U(x_i)\right|^\lambda\right)}\right]^{\frac{1}{\lambda}}. \quad \textbf{(15)}$$

If we take the weight of each element $x_i \in X$ into account, then

$$S(A,B) = 1 - \left[\frac{\sum_{i=1}^{n} w_i\left(\left|T_A^L(x_i) - T_B^L(x_i)\right|^\lambda + \left|T_A^U(x_i) - T_B^U(x_i)\right|^\lambda + \left|I_A^L(x_i) - I_B^L(x_i)\right|^\lambda + \left|I_A^U(x_i) - I_B^U(x_i)\right|^\lambda + \left|F_A^L(x_i) - F_B^L(x_i)\right|^\lambda + \left|F_A^U(x_i) - F_B^U(x_i)\right|^\lambda\right)}{\sum_{i=1}^{n} w_i\left(\left|T_A^L(x_i) + T_B^L(x_i)\right|^\lambda + \left|T_A^U(x_i) + T_B^U(x_i)\right|^\lambda + \left|I_A^L(x_i) + I_B^L(x_i)\right|^\lambda + \left|I_A^U(x_i) + I_B^U(x_i)\right|^\lambda + \left|F_A^L(x_i) + F_B^L(x_i)\right|^\lambda + \left|F_A^U(x_i) + F_B^U(x_i)\right|^\lambda\right)}\right]^{\frac{1}{\lambda}}. \quad \textbf{(16)}$$

It also has been proved that all conditions of the definition are satisfied. If each elements has the same importance, and then the similarity (16) reduces to (15).

B. Similarity measure based on the interval valued neutrosophic theoretic approach:

In this section, following the similarity measure between two neutrosophic sets defined by P. Majumdar in [24], we extend Majumdar's definition to interval valued neutrosophic sets.

Let A and B be two interval valued neutrosophic sets, then we define a similarity measure between A and B as follows:

$$S_{TA}(A,B)=\frac{\sum_{i=1}^{n}\{\min\{T_A^L(x_i),T_B^L(x_i)\}+\min\{T_A^U(x_i),T_B^U(x_i)\}+\min\{I_A^L(x_i),I_B^L(x_i)\}+\min\{I_A^U(x_i),I_B^U(x_i)\}+\min\{F_A^L(x_i),F_B^L(x_i)\}+\min\{F_A^U(x_i),F_B^U(x_i)\}}{\sum_{i=1}^{n}\{\max\{T_A^L(x_i),T_B^L(x_i)\}+\max\{T_A^U(x_i),T_B^U(x_i)\}+\max\{I_A^L(x_i),I_B^L(x_i)\}+\max\{I_A^U(x_i),I_B^U(x_i)\}+\max\{F_A^L(x_i),F_B^L(x_i)\}+\max\{F_A^U(x_i),F_B^U(x_i)\}} \quad \textbf{(17)}$$

1) Proposition

Let A and B be two interval valued neutrosophic sets, then

iv. $\mathbf{0 \le S_{TA}(A,B) \le 1}$.

v. $S_{TA}(A,B) = S_{TA}(A,B)$.

vi. $S(A,B) = 1$ if A = B i.e.

$T_A^L(x_i) = T_B^L(x_i)$, $T_A^U(x_i) = T_B^U(x_i)$, $I_A^L(x_i) = I_B^L(x_i)$, $I_A^U(x_i) = I_B^U(x_i)$ and $F_A^L(x_i) = F_B^L(x_i)$, $F_A^U(x_i) = F_B^U(x_i)$ for i = 1, 2, …., n.

iv. $A\subset B \subset C \Rightarrow S_{TA}(A,B) \le \min(S_{TA}(A,B), S_{TA}(B,C))$.

Proof. Properties (i) and (ii) follow from the definition.

(iii) It is clearly that if A = B $\Rightarrow S_{TA}(A,B) = 1$

$\Rightarrow \sum_{i=1}^{n}\{\min\{T_A^L(x_i),T_B^L(x_i)\}+\min\{T_A^U(x_i),T_B^U(x_i)\}+\min\{I_A^L(x_i),I_B^L(x_i)\}+\min\{I_A^U(x_i),I_B^U(x_i)\}+\min\{F_A^L(x_i),F_B^L(x_i)\}+\min\{F_A^U(x_i),F_B^U(x_i)\} = \sum_{i=1}^{n}\{max\{T_A^L(x_i),T_B^L(x_i)\}+max\{T_A^U(x_i),T_B^U(x_i)\}+max\{I_A^L(x_i),I_B^L(x_i)\}+max\{I_A^U(x_i),I_B^U(x_i)\}+max\{F_A^L(x_i),F_B^L(x_i)\}+max\{F_A^U(x_i),F_B^U(x_i)\}$

$\Rightarrow \sum_{i=1}^{n}\{[\min\{T_A^L(x_i),T_B^L(x_i)\}-\max\{T_A^L(x_i),T_B^L(x_i)\}]+[\min\{T_A^U(x_i),T_B^U(x_i)\}-\max\{T_A^U(x_i),T_B^U(x_i)\}]+[\min\{I_A^L(x_i),I_B^L(x_i)\}-\max\{I_A^L(x_i),I_B^L(x_i)\}]+[\min\{I_A^U(x_i),I_B^U(x_i)\}-\max\{I_A^U(x_i),I_B^U(x_i)\}]+[\min\{F_A^L(x_i),F_B^L(x_i)\}-\max\{F_A^L(x_i),F_B^L(x_i)\}]+[\min\{F_A^U(x_i),F_B^U(x_i)\}-\max\{F_A^U(x_i),F_B^U(x_i)\}]\} = 0$.

Thus for each x, one has that

$[\min\{T_A^L(x_i),T_B^L(x_i)\}-\max\{T_A^L(x_i),T_B^L(x_i)\}] = 0$
$[\min\{T_A^U(x_i),T_B^U(x_i)\}-\max\{T_A^U(x_i),T_B^U(x_i)\}] = 0$
$[\min\{I_A^L(x_i),I_B^L(x_i)\}-\max\{I_A^L(x_i),I_B^L(x_i)\}] = 0$
$[\min\{I_A^U(x_i),I_B^U(x_i)\}-\max\{I_A^U(x_i),I_B^U(x_i)\}] = 0$
$[\min\{F_A^L(x_i),F_B^L(x_i)\}-\max\{F_A^L(x_i),F_B^L(x_i)\}] = 0$
$[\min\{F_A^U(x_i),F_B^U(x_i)\}-\max\{F_A^U(x_i),F_B^U(x_i)\}] = 0$

hold.

Thus $T_A^L(x_i) = T_B^L(x_i)$, $T_A^U(x_i) = T_B^U(x_i)$, $I_A^L(x_i) = I_B^L(x_i)$, $I_A^U(x_i) = I_B^U(x_i)$, $F_A^L(x_i) = F_B^L(x_i)$ and $F_A^U(x_i) = F_B^U(x_i) \Rightarrow$ A=B

(iv) Now we prove the last result.

Let $A\subset B \subset C$, then we have

$T_A^L(x) \le T_B^L(x) \le T_C^L(x)$, $T_A^U(x) \le T_B^U(x) \le T_C^L(x)$, $I_A^L(x) \ge I_B^L(x) \ge I_C^L(x)$, $I_A^U(x) \ge I_B^U(x) \ge I_C^U(x)$, $F_A^L(x) \ge F_B^L(x) \ge F_C^L(x)$, $F_A^U(x) \ge F_B^U(x) \ge F_C^U(x)$ for all $x \in X$.

Now

$T_A^L(x)+T_A^U(x)+I_A^L(x)+I_A^U(x)+F_B^L(x)+F_B^U(x) \ge T_A^L(x)+T_A^U(x)+I_A^L(x)+I_A^U(x)+F_C^L(x)+F_C^U(x)$

and

$T_B^L(x)+T_B^U(x)+I_B^L(x)+I_B^U(x)+F_A^L(x)+F_A^U(x) \ge T_C^L(x)+T_C^U(x)+I_C^L(x)+I_C^U(x)+F_A^L(x)+F_A^U(x)$.

$$S(A,B)=\frac{T_A^L(x)+T_A^U(x)+I_A^L(x)+I_A^U(x)+F_B^L(x)+F_B^U(x)}{T_B^L(x)+T_B^U(x)+I_B^L(x)+I_B^U(x)+F_A^L(x)+F_A^U(x)} \ge \frac{T_A^L(x)+T_A^U(x)+I_A^L(x)+I_A^U(x)+F_C^L(x)+F_C^U(x)}{T_C^L(x)+T_C^U(x)+I_C^L(x)+I_C^U(x)+F_A^L(x)+F_A^U(x)} = S(A,C).$$

Again, similarly we have

$T_B^L(x)+T_B^U(x)+I_B^L(x)+I_B^U(x)+F_C^L(x)+F_C^U(x) \ge T_A^L(x)+T_A^U(x)+I_A^L(x)+I_A^U(x)+F_C^L(x)+F_C^U(x)$
$T_C^L(x)+T_C^U(x)+I_C^L(x)+I_C^U(x)+F_A^L(x)+F_A^U(x) \ge T_C^L(x)+T_C^U(x)+I_C^L(x)+I_C^U(x)+F_B^L(x)+F_B^U(x)$

$$S(B,C)=\frac{T_B^L(x)+T_B^U(x)+I_B^L(x)+I_B^U(x)+F_C^L(x)+F_C^U(x)}{T_C^L(x)+T_C^U(x)+I_C^L(x)+I_C^U(x)+F_B^L(x)+F_B^U(x)} \ge \frac{T_A^L(x)+T_A^U(x)+I_A^L(x)+I_A^U(x)+F_C^L(x)+F_C^U(x)}{T_C^L(x)+T_C^U(x)+I_C^L(x)+I_C^U(x)+F_A^L(x)+F_A^U(x)} = S(A,C)$$

$\Rightarrow S_{TA}(A,B) \le \min(S_{TA}(A,B), S_{TA}(B,C))$.

Hence the proof of this proposition.

If we take the weight of each element $x_i \in X$ into account, then

$$S(A,B)=\frac{\sum_{i=1}^{n} w_i\{\min\{T_A^L(x_i),T_B^L(x_i)\}+\min\{T_A^U(x_i),T_B^U(x_i)\}+\min\{I_A^L(x_i),I_B^L(x_i)\}+\min\{I_A^U(x_i),I_B^U(x_i)\}+\min\{F_A^L(x_i),F_B^L(x_i)\}+\min\{F_A^U(x_i),F_B^U(x_i)\}}{\sum_{i=1}^{n} w_i\{\max\{T_A^L(x_i),T_B^L(x_i)\}+\max\{T_A^U(x_i),T_B^U(x_i)\}+\max\{I_A^L(x_i),I_B^L(x_i)\}+\max\{I_A^U(x_i),I_B^U(x_i)\}+\max\{F_A^L(x_i),F_B^L(x_i)\}+\max\{F_A^U(x_i),F_B^U(x_i)\}}$$

(18)

Particularly, if each element has the same importance, then (18) is reduced to (17), clearly this also satisfies all the properties of the definition.

C. Similarity measure based on matching function by using interval neutrosophic sets:

Chen [24] and Chen et al. [25] introduced a matching function to calculate the degree of similarity between fuzzy sets. In the following, we extend the matching function to deal with the similarity measure of interval valued neutrosophic sets.

Let A and B be two interval valued neutrosophic sets, then we define a similarity measure between A and B as follows:

$$S_{MF}(A,B) = \frac{\sum_{i=1}^{n}\left(\left(T_A^L(x_i)\cdot T_B^L(x_i)\right)+\left(T_A^U(x_i)\cdot T_B^U(x_i)\right)+\left(I_A^L(x_i)\cdot I_B^L(x_i)\right)+\left(I_A^U(x_i)\cdot I_B^U(x_i)\right)+\left(F_A^L(x_i)\cdot F_B^L(x_i)\right)+\left(F_A^U(x_i)\cdot F_B^U(x_i)\right)\right)}{\max\left(\sum_{i=}^{n}(T_A^L(x_i)^2+T_A^U(x_i)^2+I_A^L(x_i)^2+I_A^U(x_i)^2+F_A^L(x_i)^2+F_A^U(x_i)^2),\ \sum_{i=}^{n}(T_B^L(x_i)^2+T_B^U(x_i)^2+I_B^L(x_i)^2+I_B^U(x_i)^2+F_B^L(x_i)^2+F_B^U(x_i)^2)\right)} \quad \textbf{(19)}$$

Proof.

i. $\mathbf{0 \le S_{MF}(A,B) \le 1}$.

The inequality S_{MF}(A,B) ≥ 0 is obvious. Thus, we only prove the inequality S(A, B) ≤ 1.

S_{MF}(A,B)= $\sum_{i=1}^{n}\left(\left(T_A^L(x_i)\cdot T_B^L(x_i)\right)+\left(T_A^U(x_i)\cdot T_B^U(x_i)\right)+\left(I_A^L(x_i)\cdot I_B^L(x_i)\right)+\left(I_A^U(x_i)\cdot I_B^U(x_i)\right)+\left(F_A^L(x_i)\cdot F_B^L(x_i)\right)+\left(F_A^U(x_i)\cdot F_B^U(x_i)\right)\right)$

$= T_A^L(x_1)\cdot T_B^L(x_1)+T_A^L(x_2)\cdot T_B^L(x_2)+\ldots+T_A^L(x_n)\cdot T_B^L(x_n)+T_A^U(x_1)\cdot T_B^U(x_1)+T_A^U(x_2)\cdot T_B^U(x_2)+\ldots+T_A^U(x_n)\cdot T_B^U(x_n)+$
$I_A^L(x_1)\cdot I_B^L(x_1)+I_A^L(x_2)\cdot I_B^L(x_2)+\ldots+I_A^L(x_n)\cdot I_B^L(x_n)+I_A^U(x_1)\cdot I_B^U(x_1)+I_A^U(x_2)\cdot I_B^U(x_2)+\ldots+I_A^U(x_n)\cdot I_B^U(x_n)+$
$F_A^L(x_1)\cdot F_B^L(x_1)+F_A^L(x_2)\cdot F_B^L(x_2)+\ldots+F_A^L(x_n)\cdot F_B^L(x_n)+F_A^U(x_1)\cdot T_B^U(x_1)+F_A^U(x_2)\cdot F_B^U(x_2)+\ldots+F_A^U(x_n)\cdot F_B^U(x_n)$.

According to the Cauchy–Schwarz inequality:

$(x_1\cdot y_1+x_2\cdot y_2+\cdots+x_n\cdot y_n)^2 \le ({x_1}^2+{x_2}^2+\cdots+{x_n}^2)\cdot({y_1}^2+{y_2}^2+\cdots+{y_n}^2)$
where $(x_1, x_2, \ldots, x_n) \in R^n$ and $(y_1, y_2, \ldots, y_n) \in R^n$
we can obtain
$[S_{MF}(A, B)]^2 \le \sum_{i=1}^{n}\left(T_A^L(x_i)^2+T_A^U(x_i)^2+I_A^L(x_i)^2+I_A^U(x_i)^2+F_A^L(x_i)^2+F_A^U(x_i)^2\right)\cdot$
$\sum_{i=1}^{n}\left(T_B^L(x_i)^2+T_B^U(x_i)^2+I_B^L(x_i)^2+I_B^U(x_i)^2+F_B^L(x_i)^2+F_B^U(x_i)^2\right)$= S(A, A) ∙ S(B, B)

Thus S_{MF}(A,B)≤ $[S(A,A)]^{\frac{1}{2}}\cdot[S(B,B)]^{\frac{1}{2}}$.

Then S_{MF}(A,B)≤ max{S(A,A), S(B,B)].

Therefore S_{MF}(A, B) ≤ 1.

If we take the weight of each element $x_i \in$ X into account, then

$$S_{MF}^{w}(A,B)= \frac{\sum_{i=1}^{n}w_i\left(\left(T_A^L(x_i)\cdot T_B^L(x_i)\right)+\left(T_A^U(x_i)\cdot T_B^U(x_i)\right)+\left(I_A^L(x_i)\cdot I_B^L(x_i)\right)+\left(I_A^U(x_i)\cdot I_B^U(x_i)\right)+\left(F_A^L(x_i)\cdot F_B^L(x_i)\right)+\left(F_A^U(x_i)\cdot F_B^U(x_i)\right)\right)}{\max\left(\sum_{i=}^{n}w_i\,(T_A^L(x_i)^2+T_A^U(x_i)^2+I_A^L(x_i)^2+I_A^U(x_i)^2+F_A^L(x_i)^2+F_A^U(x_i)^2),\ \sum_{i=}^{n}w_i\,(T_B^L(x_i)^2+T_B^U(x_i)^2+I_B^L(x_i)^2+I_B^U(x_i)^2+F_B^L(x_i)^2+F_B^U(x_i)^2)\right)} \quad \textbf{(20)}$$

Particularly, if each element has the same importance, then the similarity (20) is reduced to (19). Clearly this also satisfies all the properties of definition.

The larger the value of S(A,B), the more the similarity between A and B.

V. Comparison of New Similarity Measure of IVNS With The Existing Measures.

Let A and B be two interval valued neutrosophic sets in the universe of discourse X = {x_1, x_2 ,.., x_n}.The new similarity $S_{TA}(A,B)$ of IVNS and the existing similarity measures of interval valued neutrosophic sets (examples 1 and 2) introduced in [10, 12, 23] are listed as follows:

Broumi and Smarandache cosine similarity:

***Pinaki similarity I*:**

this similarity measure was proposed as concept of association coefficient of the neutrosophic sets as follows

$$S_{PI}=\frac{\sum_{i=1}^{n}\{\min\{T_A(x_i),T_B(x_i)\}+\min\{I_A(x_i),I_B(x_i)\}+\min\{F_A(x_i),F_B(x_i)\}\}}{\sum_{i=1}^{n}\{\max\{T_A(x_i),T_B(x_i)\}+\max\{I_A(x_i),I_B(x_i)\}+\max\{F_A(x_i),F_B(x_i)\}\}} \quad \textbf{(21)}$$

$$C_N(A,B)=\frac{1}{n}\sum_{i=1}^{n}\frac{(T_A^L(x_i)+T_A^U(x_i))\,(T_B^L(x_i)+T_B^U(x_i))+(I_A^L(x_i)+I_A^U(x_i))\,(I_B^L(x_i)+I_B^U(x_i))+(F_A^L(x_i)+F_A^U(x_i))\,(F_B^L(x_i)+F_B^U(x_i))}{\sqrt{(T_A^L(x_i)+T_A^U(x_i))^2+(I_A^L(x_i)+I_A^U(x_i))^2+(F_A^L(x_i)+F_A^U(x_i))^2}\ \sqrt{(T_B^L(x_i)+T_B^U(x_i))^2+(I_B^L(x_i)+I_B^U(x_i))^2+(F_B^L(x_i)+F_B^U(x_i))^2}} \quad \textbf{(22)}$$

Ye similarity

$$S_{ye}(A,B)=1-\frac{1}{6}\sum_{i=1}^{n}\ [|\inf T_A(x_i)-\inf T_B(x_i)|+|\sup T_A(x_i)-\sup T_B(x_i)|+|\inf I_A(x_i)-\inf I_B(x_i)|+|\sup I_A(x_i)-\sup I_B(x_i)|+|\inf F_A(x_i)-\inf F_B(x_i)|+|\sup F_A(x_i)-\sup F_B(x_i)|]. \quad \textbf{(23)}$$

Example 1

Let A = {<x, (a, 0.2 , 0.6 , 0.6), (b, 0.5, 0.3 , 0.3), (c, 0.6 , 0.9 , 0.5)>}

and B = {<x, (a, 0.5 , 0.3 , 0.8), (b, 0.6 , 0.2 , 0.5), (c, 0.6 , 0.4 , 0.4)>}.

Pinaki similarity I = 0.6.

S_{ye}(A, B) = 0.38 (With w_i =1).

Cosine similarity $\mathbf{C_N(A,B)}$ = 0.95.

$S_{TA}(A,B)$ = **0.8.**

Example 2:

Let A= {<x, (a, [0.2 , 0.3] ,[0.2, 0.6], [0.6 , 0.8]), (b, [0.5 , 0.7], [0.3, 0.5], [0.3 , 0.6]), (c, [0.6 , 0.9] ,[0.3, 0.9], [0.3, 0.5])>} and

B={<x, (a, [0.5 , 0.3] ,[0.3, 0.6], [0.6 , 0.8]), (b, [0.6, 0.8] ,[0.2, 0.4], [0.5 , 0.6]), (c, [0.6 , 0.9] ,[0.3, 0.4], [0.4 , 0.6])>}.

Pinaki similarity I = NA.

S_{ye}(A, B) = 0.7 (With w_i =1).

Cosine similarity $\mathbf{C_N(A,B)}$ = 0.92.

$S_{TA}(A,B)$ = **0.9.**

On the basis of computational study Jun Ye [12] has shown that their measure is more effective and reasonable. A similar kind of study with the help of the proposed new measure based on theoretic approach, it has been done and it is found that the obtained results are more refined and accurate. It may be observed from the above examples that the values of similarity measures are closer to 1 with $S_{TA}(A,B)$ which is this proposed similarity measure.

VI. CONCLUSIONS

Few distance and similarity measures have been proposed in literature for measuring the distance and the degree of similarity between interval neutrosophic sets. In this paper, we proposed a new method for distance and similarity measure for measuring the degree of similarity between two weighted interval valued neutrosophic sets, and we have extended the work of Pinaki, Majumdar and S. K. Samant and Chen. The results of the proposed similarity measure and existing similarity measure are compared.

In the future, we will use the similarity measures which are proposed in this paper in group decision making

VII. ACKNOWLEDGMENT

The authors are very grateful to the anonymous referees for their insightful and constructive comments and suggestions, which have been very helpful in improving the paper.

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Presented at 17th International Conference on Information Fusion, Salamanca, Spain, 7-10 July 2014.

New Operations on Interval Neutrosophic Sets

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Abstract. An interval neutrosophic set is an instance of a neutrosophic set, which can be used in real scientific and engineering applications. In this paper, three new operations based on the arithmetic mean, geometrical mean, and respectively harmonic mean are defined on interval neutrosophic sets.

Keywords: Neutrosophic Sets, Interval Valued Neutrosophic Sets.

1. Introduction

In recent decades, several types of sets, such as fuzzy sets [1], interval-valued fuzzy sets [2], intuitionistic fuzzy sets [3, 4], interval-valued intuitionistic fuzzy sets [5], type 2 fuzzy sets [6, 7], type n fuzzy sets [6], and hesitant fuzzy sets [8], neutrosophic set theory [9], interval valued neutrosophic set [10] have been introduced and investigated widely. The concept of neutrosophic sets, introduced by Smarandache [6, 9], and is interesting and useful in modeling several real life problems.

The neutrosophic set theory (NS for short), which is a generalization of intuitionistic fuzzy set has three associated defining functions, namely the membership function, the non-membership function and the indeterminacy function, which are completely independent. After the pioneering work of Smarandache [9], Wang, H et al. [10] introduced the notion of interval neutrosophic sets theory (INS for short) which is a special case of neutrosophic sets. This concept is characterized by a membership function, a non-membership function and indeterminacy function, whose values are intervals rather than real numbers. INS is more powerful in dealing with vagueness and uncertainty than NS, also INS is regarded as a useful and practical tool for dealing with indeterminate and inconsistent information in real world.

The theories of both neutrosophic set (NS) and interval neutrosophic set (INS) have achieved great success in various areas such as medical diagnosis [11], database [12, 13], topology[14], image processing [15, 16, 17], and decision making problem[18].

Recently, Ye [19] defined the similarity measures between INSs on the basis of the hamming and Euclidean distances, and a multicriteria decision–making method based on the similarity degree was proposed. Some set theoretic operations such as union, intersection and complement on interval neutrosophic sets have also been proposed by Wang, H. et al. [10].
Later on, S. Broumi and F. Smarandache [20] also defined the correlation coefficient of interval neutrosophic set.

In 2013, Peide Liu [21] have presented some new operational laws for interval neutrosophic sets (INSs) and studied their properties and proposed some aggregation operators, including the interval neutrosophic power generalized weighted aggregation (INPGWA) operator and interval neutrosophic power generalized ordered weighted aggregation (INPGOWA) operator, and gave a decision making method based on these operators.

In this paper, our aim is to propose three new operations on interval neutrosophic sets (INSs) and study their properties.
Therefore, the rest of the paper is set out as follows. In Section 2, some basic definitions related to neutrosophic set and interval valued neutrosophic set are briefly discussed. In Section 3, three new operations on interval neutrosophic sets have been proposed and some properties of the proposed operations on interval neutrosophic sets are proved. In section 4 we conclude the paper.

2. Preliminaries

In this section, we mainly recall some notions related to neutrosophic sets, and interval neutrosophic sets relevant to the present work. See especially [9, 10, and 21] for further details and background.

2.1. Definition ([9]). Let U be an universe of discourse; then the neutrosophic set A is an object having the form A = {< x: $T_A(x)$, $I_A(x)$, $F_A(x)$>, x ∈ U}, where the functions T, I, F : U→]$^-0,1^+$[define respectively the degree of membership, the degree of indeterminacy, and the degree of non-membership of the element x ∈ U to the set A with the condition:

$$^-0 \le T_A(x)+ I_A(x)+ F_A(x)\le 3^+. \quad (1)$$

From philosophical point of view, the neutrosophic set takes the value from real standard or non-standard subsets of]$^-0,1^+$[. So instead of]$^-0,1^+$[we need to take the interval [0,1] for technical applications, because]$^-0,1^+$[will be difficult to apply in the real applications such as in scientific and engineering problems.

2.2 .Definition [10]. Let X be a space of points (objects) with generic elements in X denoted by x. An interval neutrosophic set (for short INS) A in X is characterized by truth-membership function $T_A(x)$, indeteminacy-membership function $I_A(x)$ and falsity-membership function $F_A(x)$. For each point x in X, we have that $T_A(x)$, $I_A(x)$, $F_A(x) \in [0, 1]$.

For convenience, we can use x =([T^L , T^U] , [I^L , I^U] , [F^L , F^U]) to represent an element in INS.

Remark 1. An INS is clearly a NS.

2.3 .Definition [10]. Let A ={([T_A^L , T_A^U] , [I_A^L , I_A^U] , [F_A^L , F_A^U])}

i. An INS A is empty if $T_A^L = T_A^U = 0$, $I_A^L = I_A^U = 1$, $F_A^L = F_A^U = 1$, for all x in A.
ii. Let $\underline{0}$ = <0, 1 ,1> and $\underline{1}$ = <1, 0 ,0>

In the following, we introduce some basic concepts related to INSs.

2.4. Definition [21]. Let $\tilde{n}_1$ ={([T_1^L , T_1^U] , [I_1^L , I_1^U] , [F_1^L , F_1^U])} and $\tilde{n}_2$ = {([T_2^L , T_2^U] , [I_2^L , I_2^U] , [F_2^L , F_2^U])} be two INSs.

i. $\tilde{n}_1 \cup \tilde{n}_2$ =[max(T_1^L, T_2^L), max(T_1^U ,T_2^U)], [min(I_1^L, I_2^L),min(I_1^U, I_2^U)] , [min(F_1^L, F_2^L),min(F_1^U, F_2^U)] }
ii. $\tilde{n}_1 \cap \tilde{n}_2$ =[min(T_1^L, T_2^L), min(T_1^U ,T_2^U)], [max(I_1^L, I_2^L)max(I_1^U, I_2^U)] , [max(F_1^L, F_2^L),max(F_1^U, F_2^U)] }

2.5. Definition. Let $\tilde{n}_1$ ={([T_1^L , T_1^U] , [I_1^L , I_1^U] , [F_1^L , F_1^U])} and $\tilde{n}_2$ = {([T_2^L , T_2^U] , [I_2^L , I_2^U] , [F_2^L , F_2^U])} be two INSs, then the operational laws are defined as follows.

i. $\tilde{n}^c$ = [F^L , F^U] , [$1 - I^L$,$1 - I^U$] , [T^L , T^U]

ii. $\tilde{n}_1 \oplus \quad \tilde{n}_2$ = $([T_1^L + T_2^L - T_1^L T_2^L, T_1^U + T_2^U - T_1^U T_2^U], [I_1^L I_2^L, I_1^U I_2^U], [F_1^L F_2^L, F_1^U F_2^U])$

iii. $\tilde{n}_1 \otimes \tilde{h}_2$ = $[T_1^L T_2^L\ ,\ T_1^U T_2^U)], [I_1^L + I_2^L - I_2^L I_2^L, I_1^U + I_2^U - I_1^U I_2^U], [F_1^L + F_2^L - F_2^L F_2^L\ , F_1^U + F_2^U - F_1^U F_2^U]$

iv. $\lambda\, \tilde{n} = \{([1-(1-T^L)^\lambda, 1-(1-T^U)^\lambda], [(I^L)^\lambda, (I^U)^\lambda], [(F^L)^\lambda, (F^U)^\lambda])\}$.

v.

3. Three New Operations on INSs

3.1. Definition: Let $\tilde{n}_1$ and $\tilde{n}_2$ be two interval neutrosophic sets;, we propose the following operations on INSs as follows:

$\tilde{n}_1$ @ $\tilde{n}_2$ = $\{([\frac{T_1^L + T_2^L}{2}, \frac{T_1^U + T_2^U}{2}], [\frac{I_1^L + I_2^L}{2}, \frac{I_1^U + I_2^U}{2}]\ , [\frac{F_1^L + F_2^L}{2}, \frac{F_1^U + F_2^U}{2}]$, where $< T_1, I_1, F_1 > \in \tilde{n}_1$,$< T_2, I_2, F_2 > \in \tilde{n}_2$ }

$\tilde{n}_1$ \$ $\tilde{n}_2$ = $\{([\sqrt{T_1^L\, T_2^L}, \sqrt{T_1^U\, T_2^U}], [\sqrt{I_1^L\, I_2^L}, \sqrt{I_1^U\, I_2^U}], [\sqrt{F_1^L\, F_2^L}, \sqrt{F_1^U\, F_2^U}]\}$,where $< T_1, I_1, F_1 > \in \tilde{n}_1$,$< T_2, I_2, F_2 > \in \tilde{n}_2$ }

$\tilde{n}_1$ # $\tilde{n}_2$ = $\{([\frac{2\, T_1^L\, T_2^L}{T_1^L + T_2^L}, \frac{2\, T_1^U\, T_2^U}{T_1^U + T_2^U}], [\frac{2\, I_1^L\, I_2^L}{I_1^L + I_2^L}, \frac{2\, I_1^U\, I_2^U}{I_1^U + I_2^U}], [\frac{2\, F_1^L\, F_2^L}{F_1^L + F_2^L}, \frac{2\, F_1^U\, F_2^U}{F_1^U + F_2^U}]\}$, where $< T_1, I_1, F_1 > \in \tilde{n}_1$,$< T_2, I_2, F_2 > \in \tilde{n}_2$ }

With $T_1 = [T_1^L\ , T_1^U]$, $I_1 = [I_1^L\ , I_1^U]$, $F_1 = [F_1^L\ , F_1^U]$ and $T_2 = [T_2^L\ , T_2^U]$, $I_2 = [I_2^L\ , I_2^U]$, $F_2 = [F_2^L\ , F_2^U]$

Obviously, for every two $\tilde{n}_1$ and $\tilde{n}_2$, ($\tilde{n}_1 @ \tilde{n}_2$), ($\tilde{n}_1 \$ \tilde{n}_2$) and ($\tilde{n}_1 \# \tilde{n}_2$) are also INSs.

3.2.Example Let $\tilde{n}_1$ (x)= {([0.2, 0.3], [0.5, 0.6] , [0.2 , 0.4]),([0.5, 0.8], [0.1, 0.2], [0.6 ,0.1])} and $\tilde{n}_2$ (x)= {([0.4, 0.6],[0.3, 0.4], [0.3 , 0.5]), ([0.3, 0.5], [0.1, 0.2], [0.5 ,0.1]) be two interval neutrosophic sets. Then we have

($\tilde{n}_1$ @ $\tilde{n}_2$) = {([0.3, 0.45], [0.4, 0.5],[0.25 ,0.45]),(b, [0.4, 0.65], [0.1, 0.2], [0.55 , 0.1])}

($\tilde{n}_1$ \$ $\tilde{n}_2$) ={(a, [0.28, 0.42], [0.38, 0.48], [0.24 ,0.44]),(b, [0.38, 0.63], [0.1, 0.2], [0.55 ,0.1])}

($\tilde{n}_1 \# \tilde{n}_2$) = {(a, [0.26, 0.4], [0.37, 0.48] , [0.24 , 0.44]), (b, [0.37, 0.61], [0.1, 0.2], [0.54 , 0.1])}

With these operations, several results follow.

3.4. Theorem. For $\tilde{n}_1, \tilde{n}_2 \in$ INSs(X),

(i) $\tilde{n}_1 @ \tilde{n}_2 = \tilde{n}_2 @ \tilde{n}_1$;

(ii) $\tilde{n}_1 \$ \tilde{n}_2 = \tilde{n}_2 \$ \tilde{n}_1$;

(iii) $\tilde{n}_1 \# \tilde{n}_2 = \tilde{n}_2 \# \tilde{n}_1$;

Proof. These also follow from definitions.

3.5. Theorem. For $\tilde{n}_1, \tilde{n}_2 \in$ INSs(X),

$$(\tilde{n}_1{}^c @ \tilde{n}_2{}^c)^c = \tilde{n}_1 @ \tilde{n}_2$$

Proof. $\tilde{n}_1$ @ $\tilde{n}_2$ = $\{([\frac{T_1^L + T_2^L}{2}, \frac{T_1^U + T_2^U}{2}], [\frac{I_1^L + I_2^L}{2}, \frac{I_1^U + I_2^U}{2}]\ , [\frac{F_1^L + F_2^L}{2}, \frac{F_1^U + F_2^U}{2}]$ where $< T_1, I_1, F_1 > \in \tilde{n}_1$,$< T_2, I_2, F_2 > \in \tilde{n}_2$ }

$\tilde{n}_1{}^c$ ={([F_1^L , F_1^U] , [1- I_1^L , $1 - I_1^U$] , [T_1^L , T_1^U])}

$\tilde{n}_2{}^c$ = {([F_2^L , F_2^U] , [1- I_2^L , $1 - I_2^U$] , [T_2^L , T_2^U])}

$\tilde{n}_1{}^c @ \tilde{n}_2{}^c$ ={ $[\frac{F_1^L+F_2^L}{2}, \frac{F_1^U+F_2^U}{2}], [\frac{(1-I_1^L)+(1-I_2^L)}{2}, \frac{(1-I_1^U)+(1-I_2^U)}{2}], [\frac{T_1^L+T_2^L}{2}, \frac{T_1^U+T_2^U}{2}]$ }

$$(\tilde{n}_1{}^c @ \tilde{n}_2{}^c)^c = \left([\frac{F_1^L+F_2^L}{2}, \frac{F_1^U+F_2^U}{2}], [\frac{(1-I_1^L)+(1-I_2^L)}{2}, \frac{(1-I_1^U)+(1-I_2^U)}{2}], [\frac{T_1^L+T_2^L}{2}, \frac{T_1^U+T_2^U}{2}]\right)^c$$

$$= \left([\frac{T_1^L+T_2^L}{2}, \frac{T_1^U+T_2^U}{2}], [1-\frac{(1-I_1^L)+(1-I_2^L)}{2}, 1-\frac{(1-I_1^U)+(1-I_2^U)}{2}], [\frac{F_1^L+F_2^L}{2}, \frac{F_1^U+F_2^U}{2}]\right)$$

$$= \left([\frac{T_1^L+T_2^L}{2}, \frac{T_1^U+T_2^U}{2}], [\frac{2-[2-(I_1^L+I_2^L)]}{2}, \frac{2-[2-(I_1^U+I_2^U)]}{2}], [\frac{F_1^L+F_2^L}{2}, \frac{F_1^U+F_2^U}{2}]\right)$$

$$= \left([\frac{T_1^L+T_2^L}{2}, \frac{T_1^U+T_2^U}{2}], [\frac{(I_1^L+I_2^L)}{2}, \frac{(I_1^U+I_2^U)}{2}], [\frac{F_1^L+F_2^L}{2}, \frac{F_1^U+F_2^U}{2}]\right)$$

Then $(\tilde{n}_1{}^c @ \tilde{n}_2{}^c)^c = \tilde{n}_1 @ \tilde{n}_2$

This proves the theorem.

Note 1: One can easily verify that

(i) $(\tilde{n}_1{}^c \$ \tilde{n}_2{}^c)^c \neq \tilde{n}_1 \$ \tilde{n}_2$
(ii) $(\tilde{n}_1{}^c \# \tilde{n}_2{}^c)^c \neq \tilde{n}_1 \# \tilde{n}_2$

3.6. Theorem. For $\tilde{n}_1, \tilde{n}_2$ and $\tilde{n}_3 \in$ INSs(X) , we have the following identities:

(i) $(\tilde{n}_1 \cup \tilde{n}_2) @ \tilde{n}_3 = (\tilde{n}_1 @ \tilde{n}_3) \cup (\tilde{n}_2 @ \tilde{n}_3)$;

(ii) $(\tilde{n}_1 \cap \tilde{n}_2) @ \tilde{n}_3 = (\tilde{n}_1 @ \tilde{n}_3) \cap (\tilde{n}_2 @ \tilde{n}_3)$

(iii) $(\tilde{n}_1 \cup \tilde{n}_2) \$ \tilde{n}_3 = (\tilde{n}_1 \$ \tilde{n}_3) \cup (\tilde{n}_2 \$ \tilde{n}_3)$;

(iv) $(\tilde{n}_1 \cap \tilde{n}_2) \$ \tilde{n}_3 = (\tilde{n}_1 \$ \tilde{n}_3) \cap (\tilde{n}_2 \$ \tilde{h}_3)$;

(v) $((\tilde{n}_1 \cup \tilde{n}_2)) \# \tilde{n}_3 = (\tilde{n}_1 \# \tilde{n}_3) \cup (\tilde{n}_2 \# \tilde{n}_3)$;

(vi) $(\tilde{n}_1 \cap \tilde{n}_2) \# \tilde{n}_3 = (\tilde{n}_1 \# \tilde{n}_3) \cap (\tilde{n}_2 \# \tilde{n}_3)$;

(vii) $(\tilde{n}_1 @ \tilde{n}_2) \oplus \tilde{n}_3 = (\tilde{n}_1 \oplus \tilde{n}_3) @ (\tilde{n}_2 \oplus \tilde{n}_3)$;

(viii) $(\tilde{n}_1 @ \tilde{n}_2) \otimes \tilde{n}_3 = (\tilde{n}_1 \otimes \tilde{n}_3) @ (\tilde{n}_2 \otimes \tilde{n}_3)$

Proof. We prove (i), (iii), (v), (vii) and (ix), results (ii), (iv), (vi), (viii) and (x) can be proved analogously

(i) Using definitions in 2.4, 2.5 and 3.1,we have

$\tilde{n}_1$ ={([T_1^L , T_1^U] , [I_1^L , I_1^U] , [F_1^L , F_1^U])}

$\tilde{n}_2$ = {([T_2^L , T_2^U] , [I_2^L , I_2^U] , [F_2^L , F_2^U])}

$(\tilde{n}_1 \cup \tilde{n}_2) @ \tilde{n}_3$ = {([max(T_1^L, T_2^L) ,max(T_1^U, T_2^U)] , [min (I_1^L, I_2^L) ,min(I_1^U, I_2^U)] , [min(F_1^L , F_2^L),min(F_1^U, F_2^U)])} @ {([T_3^L , T_3^U] , [I_3^L , I_3^U] , [F_3^L , F_3^U])}

$=\{[\frac{\max(T_1^L,T_2^L)+T_3^L}{2}, \frac{\max(T_1^U,T_2^U)+T_3^U}{2}], [\frac{\min(I_1^L,I_2^L)+I_3^L}{2}, \frac{\min(I_1^U,I_2^U)+I_3^U}{2}], [\frac{\min(F_1^L,F_2^L)+F_3^L}{2}, \frac{\min(F_1^U,F_2^U)+F_3^U}{2}]\}$

$=\{[\max(\frac{T_1^L+T_3^L}{2}, \frac{T_2^L+T_3^U}{2}), \max(\frac{T_1^U+T_3^U}{2}, \frac{T_2^U+T_3^U}{2})], [\min(\frac{I_1^L+I_3^L}{2}, \frac{I_2^L+I_3^L}{2}), \min(\frac{I_1^U+I_3^U}{2}, \frac{I_2^U+I_3^U}{2})], [\min(\frac{F_1^L+F_3^L}{2}, \frac{F_2^L+F_3^L}{2}),$ $\min(\frac{F_1^U+F_3^U}{2}, \frac{F_2^U+F_3^U}{2}]\}$

$= (\tilde{n}_1 @ \tilde{n}_3) \cup (\tilde{n}_2 @ \tilde{n}_3)$

This proves (i)

(iii) From definitions in 2.4, 2.5 and 3.1, we have

$(\tilde{n}_1 \cup \tilde{n}_2) \$ \tilde{n}_3 = \{([\max(T_1^L, T_2^L), \max(T_1^U, T_2^U)], [\min(I_1^L, I_2^L), \min(I_1^U, I_2^U)], [\min(F_1^L, F_2^L), \min(F_1^U, F_2^U)])\} \$ \{([T_3^L, T_3^U], [I_3^L, I_3^U], [F_3^L, F_3^U])\}$

$= \{[\sqrt{max(T_1^L, T_2^L)T_3^L}, \sqrt{\max(T_1^U, T_2^U)T_3^U}], [\sqrt{\min(I_1^L, I_2^L)I_3^L}, \sqrt{\min(I_1^U, I_2^U)I_3^U}], [\sqrt{\min(F_1^L, F_2^L)F_3^L},$ $\sqrt{\min(F_1^U, F_2^U)F_3^U}]\}.$

$=\{[\max(\sqrt{T_1^L T_3^L}, \sqrt{T_2^L T_3^L}), \max(\sqrt{T_1^U T_3^U}, \sqrt{T_2^U T_3^U})], [\min(\sqrt{I_1^L I_3^L}, \sqrt{I_2^L I_3^L}), \min(\sqrt{I_1^U I_3^U},$ $\sqrt{I_2^U I_3^U})], [\min(\sqrt{F_1^L F_3^L}, \sqrt{F_2^L F_3^L}), \min(\sqrt{F_1^U F_3^U}, \sqrt{F_2^U F_3^U})]\}$

$=(\tilde{n}_1 \$ \tilde{n}_3) \cup (\tilde{n}_2 \$ \tilde{n}_3);$

This proves (iii).

(v) Using definitions in 2.4, 2.5 and 3.1, we have

$((\tilde{n}_1 \cup \tilde{n}_2)) \# \tilde{n}_3 = \{([\max(T_1^L, T_2^L), \max(T_1^U, T_2^U)], [\min(I_1^L, I_2^L), \min(I_1^U, I_2^U)], [\min(F_1^L, F_2^L), \min(F_1^U, F_2^U)])\} \# \{([T_3^L, T_3^U], [I_3^L, I_3^U], [F_3^L, F_3^U])\}$

$= \{[\frac{2\max(T_1^L,T_2^L)T_3^L}{\max(T_1^L,T_2^L)+T_3^L}, \frac{2\max(T_1^U,T_2^U)T_3^U}{\max(T_1^U,T_2^U)+T_3^U}], [\frac{2\min(I_1^L,I_2^L)I_3^L}{\min(I_1^L,I_2^L)+I_3^L}, \frac{2\min(I_1^U,I_2^U)I_3^U}{\min(I_1^U,I_2^U)+I_3^U}], [\frac{2\min(F_1^L,F_2^L)F_3^L}{\min(F_1^L,F_2^L)+F_3^L}, \frac{2\min(F_1^U,F_2^U)F_3^U}{\min(F_1^U,F_2^U)+F_3^U}]\}$

$=\{[\max(\frac{2T_1^L T_3^L}{T_1^L+T_3^L}, \frac{2T_2^L T_3^L}{T_2^L+T_3^L}), \max(\frac{2T_1^U T_3^U}{T_1^U+T_3^U}, \frac{2T_2^U T_3^U}{T_2^U+T_3^U})], [\min(\frac{2I_1^L I_3^L}{I_1^L+I_3^L}, \frac{2I_2^L I_3^L}{I_2^L+I_3^L}), \min(\frac{2I_1^U I_3^U}{I_1^U+I_3^U}, \frac{2I_2^U I_3^U}{I_2^U+I_3^U})], [$ $\min(\frac{2F_1^L F_3^L}{F_1^L+F_3^L}, \frac{2F_2^L F_3^L}{F_2^L+F_3^L}), \min(\frac{2F_1^U F_3^U}{F_1^U+F_3^U}, \frac{2F_2^U F_3^U}{F_2^U+F_3^U})]\}$

$=(\tilde{n}_1 \# \tilde{n}_3) \cup (\tilde{n}_2 \# \tilde{n}_3)$

This proves (v)

(vii) Using definitions in 2.4, 2.5 and 3.1, we have

$(\tilde{n}_1 @ \tilde{n}_2) \oplus \tilde{n}_3 = (\tilde{n}_1 \oplus \tilde{n}_3) @ (\tilde{n}_2 \oplus \tilde{n}_3);$

$\tilde{n}_1 = \{([T_1^L, T_1^U], [I_1^L, I_1^U], [F_1^L, F_1^U])\}$

$\tilde{n}_2 = \{([T_2^L, T_2^U], [I_2^L, I_2^U], [F_2^L, F_2^U])\}$

$\tilde{n}_3 = \{([T_3^L, T_3^U], [I_3^L, I_3^U], [F_3^L, F_3^U])\}$

$= \{([\frac{T_1^L+T_2^L}{2}, \frac{T_1^U+T_2^U}{2}], [\frac{I_1^L+I_2^L}{2}, \frac{I_1^U+I_2^U}{2}], [\frac{F_1^L+F_2^L}{2}, \frac{F_1^U+F_2^U}{2}])\} \oplus \{([T_3^L, T_3^U], [I_3^L, I_3^U], [F_3^L, F_3^U])\}$

$=\{[\frac{T_1^L+T_2^L}{2} + T_3^L - \frac{T_1^L+T_2^L}{2} T_3^L, \frac{T_1^U+T_2^U}{2} + T_3^U - \frac{T_1^U+T_2^U}{2} T_3^U], [\frac{I_1^L+I_2^L}{2} I_3^L, \frac{I_1^U+I_2^U}{2} I_3^U], [\frac{F_1^L+F_2^L}{2} F_3^L, \frac{F_1^U+F_2^U}{2} F_3^U]\}$

$=\{ [\frac{(T_1^L+T_3^L- T_1^L T_3^L)+(T_2^L+T_3^L-T_2^L T_3^L)}{2}, \frac{(T_1^U+T_3^U- T_1^U T_3^U)+(T_2^U+T_3^U-T_2^U T_3^U)}{2}], [\frac{I_1^L+ I_2^L}{2} I_3^L, \frac{I_1^U+ I_2^U}{2} I_3^U], [\frac{F_1^L+ F_2^L}{2} F_3^L, \frac{F_1^U+ F_2^U}{2} F_3^U] \}$

$= (\tilde{n}_1 \oplus \tilde{n}_3) @ (\tilde{n}_2 \oplus \tilde{n}_3)$

This proves (vi)

3.7. Theorem. For $\tilde{n}_1$and $\tilde{n}_2 \in$ INSs(X), we have the following identities:

(i) $(\tilde{n}_1 \oplus \tilde{n}_2) \cap (\tilde{n}_1 \otimes \tilde{n}_2) = \tilde{n}_1 \otimes \tilde{n}_2$;

(ii) $(\tilde{n}_1 \oplus \tilde{n}_2) \cup (\tilde{n}_1 \otimes \tilde{n}_2) = \tilde{n}_1 \oplus \tilde{n}_2$;

(iii) $(\tilde{n}_1 \oplus \tilde{n}_2) \cap (\tilde{n}_1 @ \tilde{n}_2) = \tilde{n}_1 @ \tilde{n}_2$;

(iv) $(\tilde{n}_1 \oplus \tilde{n}_2) \cup (\tilde{n}_1 @ \tilde{n}_2) = \tilde{n}_1 \oplus \tilde{n}_2$;

(v) $(\tilde{n}_1 \otimes \tilde{n}_2) \cap (\tilde{n}_1 @ \tilde{n}_2) = \tilde{n}_1 \otimes \tilde{n}_2$;

(vi) $(\tilde{n}_1 \otimes \tilde{n}_2) \cup (\tilde{n}_1 @ \tilde{n}_2) = \tilde{n}_1 @ \tilde{n}_2$;

(vii) $(\tilde{n}_1 \oplus \tilde{n}_2) \cap (\tilde{n}_1 \$ \tilde{n}_2) = \tilde{n}_1 \$ \tilde{n}_2$;

(viii) $(\tilde{n}_1 \oplus \tilde{n}_2) \cup (\tilde{n}_1 \$ \tilde{n}_2) = \tilde{n}_1 \oplus \tilde{n}_2$;

(ix) $(\tilde{n}_1 \otimes \tilde{n}_2) \cap (\tilde{n}_1 \$ \tilde{n}_2) = \tilde{n}_1 \otimes \tilde{n}_2$;

(x) $(\tilde{n}_1 \otimes \tilde{n}_2) \cup (\tilde{n}_1 \$ \tilde{n}_2) = \tilde{n}_1 \$ \tilde{n}_2$;

(xi) $(\tilde{n}_1 \oplus \tilde{n}_2) \cap (\tilde{n}_1 \# \tilde{n}_2) = \tilde{n}_1 \# \tilde{n}_2$;

(xii) $(\tilde{n}_1 \oplus \tilde{n}_2) \cup (\tilde{n}_1 \# \tilde{n}_2) = \tilde{n}_1 \oplus \tilde{n}_2$;

(xiii) $(\tilde{n}_1 \otimes \tilde{n}_2) \cap (\tilde{n}_1 \# \tilde{n}_2) = \tilde{n}_1 \otimes \tilde{n}_2$;

(xiv) $(\tilde{n}_1 \otimes \tilde{n}_2) \cup (\tilde{n}_1 \# \tilde{n}_2) = \tilde{n}_1 \# \tilde{n}_2$

Proof. We prove (i), (iii), (v), (vii), (ix), (xi) and (xii), other results can be proved analogously.

(i) From definitions in 2.4, 2.5 and 3.1, we have

$(\tilde{n}_1 \oplus \tilde{n}_2) \cap (\tilde{n}_1 \otimes \tilde{n}_2)$

$\tilde{n}_1 = \{([T_1^L, T_1^U], [I_1^L, I_1^U], [F_1^L, F_1^U])\}$

$\tilde{n}_2 = \{([T_2^L, T_2^U], [I_2^L, I_2^U], [F_2^L, F_2^U])\}$

$=([T_1^L + T_2^L - T_1^L T_2^L, T_1^U + T_2^U - T_1^U T_2^U], [I_1^L I_2^L, I_1^U I_2^U], [F_1^L F_2^L, F_1^U F_2^U]) \cap \{[T_1^L T_2^L, T_1^U T_2^U], [I_1^L + I_2^L - I_2^L I_2^L, I_1^U + I_2^U - I_1^U I_2^U], [F_1^L + F_2^L - F_2^L F_2^L, F_1^U + F_2^U - F_1^U F_2^U]\}$

$=\{[\min (T_1^L + T_2^L - T_1^L T_2^L, T_1^L T_2^L), \min (T_1^U + T_2^U - T_1^U T_2^U, T_1^U T_2^U)],$

$[\max (I_1^L I_2^L, I_1^L + I_2^L - I_2^L I_2^L), \max (I_1^U I_2^U, I_1^U + I_2^U - I_1^U I_2^U)], [\max (F_1^L F_2^L, F_1^L + F_2^L - F_2^L F_2^L), \max (F_1^U F_2^U, F_1^U + F_2^U - F_1^U F_2^U)]\}$

$=[T_1^L T_2^L, T_1^U T_2^U)], [I_1^L + I_2^L - I_2^L I_2^L, I_1^U + I_2^U - I_1^U I_2^U], [F_1^L + F_2^L - F_2^L F_2^L, F_1^U + F_2^U - F_1^U F_2^U]$

$=\tilde{n}_1 \otimes \tilde{n}_2$

This proves (i)

(iii) Using definitions in 2.4, 2.5 and 3.1, we have

$(\tilde{n}_1 \oplus \tilde{n}_2) \cap (\tilde{n}_1 @ \tilde{n}_2) = \tilde{n}_1 @ \tilde{n}_2$;

$=\{ ([T_1^L + T_2^L - T_1^L T_2^L, T_1^U + T_2^U - T_1^U T_2^U], [I_1^L I_2^L, I_1^U I_2^U], [F_1^L F_2^L, F_1^U F_2^U]) \cap ([\frac{T_1^L + T_2^L}{2}, \frac{T_1^U + T_2^U}{2}], [\frac{I_1^L + I_2^L}{2}, \frac{I_1^U + I_2^U}{2}], [\frac{F_1^L + F_2^L}{2}, \frac{F_1^U + F_2^U}{2}])$

$= \{[\min (T_1^L + T_2^L - T_1^L T_2^L, \frac{T_1^L + T_2^L}{2}), \min (T_1^U + T_2^U - T_1^U T_2^U, \frac{T_1^U + T_2^U}{2}])],$

$[\max (I_1^L I_2^L, \frac{I_1^L + I_2^L}{2}), \max (I_1^U I_2^U, \frac{I_1^U + I_2^U}{2})], [\max (F_1^L F_2^L, \frac{F_1^L + F_2^L}{2}), \max (F_1^U F_2^U, \frac{F_1^U + F_2^U}{2})]\}.$

$=\{ [\frac{T_1^L + T_2^L}{2}, \frac{T_1^U + T_2^U}{2}], [\frac{I_1^L + I_2^L}{2}, \frac{I_1^U + I_2^U}{2}], [\frac{F_1^L + F_2^L}{2}, \frac{F_1^U + F_2^U}{2}] = \tilde{n}_1 @ \tilde{n}_2$

This proves (iii).

(v) From definitions in 2.4, 2.5 and 3.1, we have

$(\tilde{n}_1 \otimes \tilde{n}_2) \cap (\tilde{n}_1 @ \tilde{n}_2) = \tilde{n}_1 \otimes \tilde{n}_2$;

$\{[T_1^L T_2^L, T_1^U T_2^U)], [I_1^L + I_2^L - I_2^L I_2^L, I_1^U + I_2^U - I_1^U I_2^U], [F_1^L + F_2^L - F_2^L F_2^L, F_1^U + F_2^U - F_1^U F_2^U]\} \cap \{ [\frac{T_1^L + T_2^L}{2}, \frac{T_1^U + T_2^U}{2}], [\frac{I_1^L + I_2^L}{2}, \frac{I_1^U + I_2^U}{2}], [\frac{F_1^L + F_2^L}{2}, \frac{F_1^U + F_2^U}{2}]\}.$

$=\{[\min (T_1^L T_2^L, \frac{T_1^L + T_2^L}{2}), \min (T_1^U T_2^U, \frac{T_1^U + T_2^U}{2})], [\max (I_1^L + I_2^L - I_2^L I_2^L, \frac{I_1^L + I_2^L}{2}), \max (I_1^U + I_2^U - I_1^U I_2^U, \frac{I_1^U + I_2^U}{2})], [\max (F_1^L + F_2^L - F_2^L F_2^L, \frac{F_1^L + F_2^L}{2}), \max (F_1^U + F_2^U - F_2^U F_2^U, \frac{F_1^U + F_2^U}{2})]\}.$

$= \{[T_1^L T_2^L, T_1^U T_2^U], [I_1^L + I_2^L - I_2^L I_2^L, I_1^U + I_2^U - I_1^U I_2^U], [F_1^L + F_2^L - F_2^L F_2^L, F_1^U + F_2^U - F_2^U F_2^U]\} = \tilde{n}_1 \otimes \tilde{n}_2$

This proves (v).

(vii) Using definitions in 2.4, 2.5 and 3.1, we have

$(\tilde{n}_1 \oplus \tilde{n}_2) \cap (\tilde{n}_1 \$ \tilde{n}_2) = \tilde{n}_1 \$ \tilde{n}_2$

$= ([T_1^L + T_2^L - T_1^L T_2^L, T_1^U + T_2^U - T_1^U T_2^U], [I_1^L I_2^L, I_1^U I_2^U], [F_1^L F_2^L, F_1^U F_2^U]) \cap \{([\sqrt{T_1^L T_2^L}, \sqrt{T_1^U T_2^U}], [\sqrt{I_1^L I_2^L}, \sqrt{I_1^U I_2^U}], [\sqrt{F_1^L F_2^L}, \sqrt{F_1^U F_2^U}]\}$

$= \{ [\min (T_1^L + T_2^L - T_1^L T_2^L, \sqrt{T_1^L T_2^L}), \min (T_1^U + T_2^U - T_1^U T_2^U, \sqrt{T_1^U T_2^U})], [\max (I_1^L I_2^L, \sqrt{I_1^L I_2^L}), \max (I_1^U I_2^U, \sqrt{I_1^U I_2^U})], [\max (F_1^L F_2^L, \sqrt{F_1^L F_2^L}), \max (F_1^U F_2^U, \sqrt{F_1^U F_2^U})]\}$

$=\{[\sqrt{T_1^L T_2^L}, \sqrt{T_1^U T_2^U}], [\sqrt{I_1^L I_2^L}, \sqrt{I_1^U I_2^U}], [\sqrt{F_1^L F_2^L}, \sqrt{F_1^U F_2^U}]\} = \tilde{n}_1 \$ \tilde{n}_2$

This proves (vii)

(ix) From definitions in 2.4, 2.5 and 3.1, we have

$(\tilde{n}_1 \otimes \tilde{n}_2) \cap (\tilde{n}_1 \$ \tilde{n}_2) = \tilde{n}_1 \otimes \tilde{n}_2$;

$=\{[T_1^L T_2^L \ , \ T_1^U T_2^U)], [I_1^L + I_2^L - I_2^L I_2^L \ , I_1^U + I_2^U - I_1^U I_2^U], [F_1^L + F_2^L - F_2^L F_2^L \ , F_1^U + F_2^U - F_1^U F_2^U \ ]\} \ \cap \{[\sqrt{T_1^L T_2^L} \ , \sqrt{T_1^U T_2^U} \ ], [\ \sqrt{I_1^L I_2^L} \ , \sqrt{I_1^U I_2^U} \ ], [\sqrt{F_1^L F_2^L} \ , \sqrt{F_1^U F_2^U} \ ]\}$

$= \{[\ \min (T_1^L T_2^L \ , \sqrt{T_1^L T_2^L}) \ , \min (\ T_1^U T_2^U, \sqrt{T_1^U T_2^U} \ )], [\max (I_1^L + I_2^L - I_2^L I_2^L \ , \sqrt{I_1^L I_2^L}) \ , \max (I_1^U + I_2^U - I_1^U I_2^U, \sqrt{I_1^U I_2^U})], [\max (F_1^L + F_2^L - F_2^L F_2^L \ , \sqrt{F_1^L F_2^L}) \ , \max (F_1^U + F_2^U - F_1^U F_2^U, \sqrt{F_1^U F_2^U})]\}$

$=\{[\ T_1^L T_2^L \ , T_1^U T_2^U \ ], [I_1^L + I_2^L - I_2^L I_2^L \ , I_1^U + I_2^U - I_1^U I_2^U], [\ F_1^L + F_2^L - F_2^L F_2^L \ , F_1^U + F_2^U - F_1^U F_2^U \ ]\} = \tilde{n}_1 \otimes \tilde{n}_2$

This proves (ix)

(xiii) From definitions in 2.3, 2.5 and 3.1, we have

$(\tilde{n}_1 \otimes \tilde{n}_2) \cap (\ \tilde{n}_1 \# \tilde{n}_2) = \tilde{n}_1 \otimes \tilde{n}_2;$

$=\{[T_1^L T_2^L \ , \ T_1^U T_2^U)], [\ I_1^L + I_2^L - I_2^L I_2^L \ , I_1^U + I_2^U - I_1^U I_2^U], [F_1^L + F_2^L - F_2^L F_2^L \ , F_1^U + F_2^U - F_1^U F_2^U \ ]\} \ \cap \{(\ [\frac{2\, T_1^L T_2^L}{T_1^L + T_2^L}, \frac{2\, T_1^U T_2^U}{T_1^U + T_2^U}], [\frac{2\, I_1^L I_2^L}{I_1^L + I_2^L}, \frac{2\, I_1^U I_2^U}{I_1^U + I_2^U}], [\frac{2\, F_1^L F_2^L}{F_1^L + F_2^L}, \frac{2\, F_1^U F_2^U}{F_1^U + F_2^U}]\}$

$=\{\ [\ \min (T_1^L T_2^L, \frac{2\, T_1^L T_2^L}{T_1^L + T_2^L}) \ , \min (T_1^U T_2^U \ , \frac{2\, T_1^U T_2^U}{T_1^U + T_2^U} \ )\ ], [\ \max (I_1^L + I_2^L - I_2^L I_2^L \ , \frac{2\, I_1^L I_2^L}{I_1^L + I_2^L}) \ , \max(I_1^U + I_2^U - I_1^U I_2^U \ , \frac{2\, I_1^U I_2^U}{I_1^U + I_2^U} \ ], [\ \max (F_1^L + F_2^L - F_2^L F_2^L \ , \frac{2\, F_1^L F_2^L}{F_1^L + F_2^L}) \ , \max(F_1^U + F_2^U - F_1^U F_2^U \ , \frac{2\, F_1^U F_2^U}{F_1^U + F_2^U} \ ]\}$

$=\{\ [T_1^L T_2^L, T_1^U T_2^U], [\ I_1^L + I_2^L - I_2^L I_2^L \ , I_1^U + I_2^U - I_1^U I_2^U], [\ F_1^L + F_2^L - F_2^L F_2^L \ , F_1^U + F_2^U - F_1^U F_2^U \ ]\} \ = \tilde{n}_1 \otimes \tilde{n}_2$

This proves (xiii). This proves the theorem.

3.8. Theorem. For $\tilde{n}_1$ and $\tilde{n}_2 \in$ INSs(X), then following relations are valid:

(i) $(\tilde{n}_1 \# \tilde{n}_2) \$ (\ \tilde{n}_1 \# \tilde{n}_2) = \tilde{n}_1 \# \tilde{n}_2;$

(ii) $(\tilde{n}_1 \oplus \tilde{n}_2) \$ (\ \tilde{n}_1 \oplus \ \tilde{n}_2) = \tilde{n}_1 \oplus \tilde{n}_2;$

(iii) $(\tilde{n}_1 \otimes \tilde{n}_2) \$ (\ \tilde{n}_1 \otimes \tilde{h}_2) = \tilde{n}_1 \otimes \tilde{n}_2;$

(iv) $(\tilde{n}_1 @ \tilde{n}_2) \$ (\ \tilde{n}_1 @ \tilde{n}_2) = \tilde{n}_1 @ \tilde{n}_2;$

(v) $(\tilde{n}_1 \# \tilde{n}_2) @ (\ \tilde{n}_1 \# \tilde{n}_2) = \tilde{n}_1 \# \tilde{n}_2;$

(vi) $(\tilde{n}_1 \oplus \tilde{n}_2) @ (\ \tilde{n}_1 \otimes \tilde{n}_2) = \tilde{n}_1 @ \tilde{n}_2;$

(vii) $(\tilde{n}_1 \cup \tilde{n}_2) @ (\ \tilde{n}_1 \cap \tilde{n}_2) = \tilde{n}_1 @ \tilde{n}_2;$

(viii) $(\tilde{n}_1 \cup \tilde{n}_2) \$ (\ \tilde{n}_1 \cap \tilde{n}_2) = \tilde{n}_1 \$ \tilde{n}_2;$

(ix) $(\tilde{n}_1 \cup \tilde{n}_2) \# (\ \tilde{n}_1 \cap \tilde{n}_2) = \tilde{n}_1 \# \tilde{n}_2;$

Proof. The proofs of these results are the same as in the above proof

3.9. Theorem For every two $\tilde{n}_1$ and $\tilde{n}_2 \in$ INSs(X), we have:

(i) $((\tilde{n}_1 \cup \tilde{n}_2) \oplus (\ \tilde{n}_1 \cap \tilde{n}_2)) @ ((\ \tilde{n}_1 \cup \tilde{n}_2) \otimes (\ \tilde{n}_1 \cap \tilde{n}_2)) = \tilde{n}_1 @ \tilde{n}_2;$

(ii) $((\tilde{n}_1 \cup \tilde{n}_2) \# (\ \tilde{n}_1 \cap \tilde{n}_2)) \$ ((\tilde{n}_1 \cup \tilde{n}_2) @ (\ \tilde{n}_1 \cap \tilde{n}_2)) = \tilde{n}_1 \$ \tilde{n}_2$

(iii) $((\tilde{n}_1 \oplus \tilde{n}_2) \cup (\tilde{n}_1 \otimes \tilde{n}_2))$ @ $((\tilde{n}_1 \oplus \tilde{n}_2) \cap (\tilde{n}_1 \otimes \tilde{n}_2)) = \tilde{n}_1 @ \tilde{n}_2$;

(iv) $((\tilde{n}_1 \oplus \tilde{n}_2) \cup (\tilde{n}_1 @ \tilde{n}_2))$ @ $((\tilde{n}_1 \otimes \tilde{n}_2) \cap (\tilde{n}_1 @ \tilde{n}_2)) = \tilde{n}_1 @ \tilde{n}_2$;

(v) $((\tilde{n}_1 \oplus \tilde{n}_2) \cup (\tilde{n}_1 \# \tilde{n}_2))$ @ $((\tilde{n}_1 \otimes \tilde{n}_2) \cap (\tilde{n}_1 \# \tilde{n}_2)) = \tilde{n}_1 @ \tilde{n}_2$;

(vi) $((\tilde{n}_1 \oplus \tilde{n}_2) \cup (\tilde{n}_1 \$ \tilde{n}_2))$ @ $((\tilde{n}_1 \otimes \tilde{n}_2) \cap (\tilde{n}_1 \$ \tilde{n}_2)) = \tilde{n}_1 @ \tilde{n}_2$;

(vii) $((\tilde{n}_1 \oplus \tilde{n}_2) \cup (\tilde{n}_1 @ \tilde{n}_2))$ @ $((\tilde{n}_1 \oplus \tilde{n}_2) \cap (\tilde{n}_1 \# \tilde{n}_2)) = \tilde{n}_1 \$ \tilde{n}_2$.

Proof. In the following, we prove (i) and (iii), other results can be proved analogously.

(i) From definitions in 2.4, 2.5 and 3.1, we have

$((\tilde{n}_1 \cup \tilde{n}_2) \oplus (\tilde{n}_1 \cap \tilde{n}_2))$ @ $((\tilde{n}_1 \cup \tilde{n}_2) \otimes (\tilde{n}_1 \cap \tilde{n}_2)) =$

$\tilde{n}_1 = \{([T_1^L, T_1^U], [I_1^L, I_1^U], [F_1^L, F_1^U])\}$

$\tilde{n}_2 = \{([T_2^L, T_2^U], [I_2^L, I_2^U], [F_2^L, F_2^U])\}$

$\tilde{n}_3 = \{([T_3^L, T_3^U], [I_3^L, I_3^U], [F_3^L, F_3^U])\}$

$((\tilde{n}_1 \cup \tilde{n}_2) \oplus (\tilde{n}_1 \cap \tilde{n}_2)) =$

$\{ [\max(T_1^L, T_2^L), \max(T_1^U, T_2^U)], [\min(I_1^L, I_2^L), \min(I_1^U, I_2^U)], [\min(F_1^L, F_2^L), \min(F_1^U, F_2^U)] \} \oplus \{ [\min(T_1^L, T_2^L), \min(T_1^U, T_2^U)], [\max(I_1^L, I_2^L), \max(I_1^U, I_2^U)], [\max(F_1^L, F_2^L), \max(F_1^U, F_2^U)] \}.$

$=\{[\max(T_1^L, T_2^L) + \min(T_1^L, T_2^L) - \max(T_1^L, T_2^L)\min(T_1^L, T_2^L),\ \max(T_1^U, T_2^U) + \min(T_1^U, T_2^U) - \max(T_1^U, T_2^U \min(T_1^U, T_2^U)], [\min(I_1^L, I_2^L)\max(I_1^L, I_2^L),\ \min(I_1^U, I_2^U)\max(I_1^U, I_2^U)], [\min(F_1^L, F_2^L)\max(F_1^L, F_2^L),\ \min(F_1^U, F_2^U)\max(F_1^U, F_2^U)]\}.$

$(\tilde{n}_1 \cup \tilde{n}_2) \otimes (\tilde{n}_1 \cap \tilde{n}_2) =$

$\{ [\max(T_1^L, T_2^L), \max(T_1^U, T_2^U)], [\min(I_1^L, I_2^L), \min(I_1^U, I_2^U)], [\min(F_1^L, F_2^L), \min(F_1^U, F_2^U)] \} \otimes \{ [\min(T_1^L, T_2^L), \min(T_1^U, T_2^U)], [\max(I_1^L, I_2^L), \max(I_1^U, I_2^U)], [\max(F_1^L, F_2^L), \max(F_1^U, F_2^U)] \}.$

$= \{[\max(T_1^L, T_2^L)\min(T_1^L, T_2^L), \max(T_1^U, T_2^U)\min(T_1^U, T_2^U)], [\min(I_1^L, I_2^L) + \max(I_1^L, I_2^L) - \min(I_1^L, I_2^L)\max(I_1^L, I_2^L), \min(I_1^U, I_2^U) + \max(I_1^U, I_2^U) - \min(I_1^U, I_2^U)\max(I_1^U, I_2^U)], [\min(F_1^L, F_2^L) + \max(F_1^L, F_2^L) - \min(F_1^L, F_2^L)\max(F_1^L, F_2^L), \min(F_1^U, F_2^U) + \max(F_1^U, F_2^U) - \min(F_1^U, FI_2^U)\max(F_1^U, F_2^U)]\}.$

$((\tilde{n}_1 \cup \tilde{n}_2) \oplus (\tilde{n}_1 \cap \tilde{n}_2))$ @ $((\tilde{n}_1 \cup \tilde{n}_2) \otimes (\tilde{n}_1 \cap \tilde{n}_2)) =$

$$\{[\frac{\max(T_1^L,T_2^L)+\min(T_1^L,T_2^L)-\max(T_1^L,T_2^L)\min(T_1^L,T_2^L)+\max(T_1^L,T_2^L)\min(T_1^L,T_2^L)}{2}, \frac{\max(T_1^U,T_2^U)+\min(T_1^U,T_2^U)-\max(T_1^U,T_2^U)\min(T_1^U,T_2^U)+\max(T_1^U,T_2^U)\min(T_1^U,T_2^U)}{2}],$$

$$[\frac{[\min(I_1^L,I_2^L)\max(I_1^L,I_2^L)+\min(I_1^L,I_2^L)+\max(I_1^L,I_2^L)-\min(I_1^L,I_2^L)\max(I_1^L,I_2^L))}{2}, \frac{\min(I_1^U,I_2^U)\max(I_1^U,I_2^U)+\min(I_1^U,I_2^U)+\max(I_1^U,I_2^U)-\min(I_1^U,I_2^U)\max(I_1^U,I_2^U)}{2}]$$

$$[\frac{[\min(F_1^L,F_2^L)\max(F_1^L,F_2^L),+\min(F_1^L,F_2^L)+\max(F_1^L,F_2^L)-\min(F_1^L,F_2^L)\max(F_1^L,F_2^L))}{2}, \frac{\min(F_1^U,F_2^U)\max(F_1^U,F_2^U)+\min(F_1^U,F_2^U)+\max(F_1^U,F_2^U)-\min(F_1^U,F_2^U)\max(F_1^U,F_2^U)}{2}]\}$$

$$= [\frac{\max(T_1^L,T_2^L)+\min(T_1^L,T_2^L)}{2}, \frac{\max(T_1^U,T_2^U)+\min(T_1^U,T_2^U)}{2}],$$

$[\frac{\min(I_1^L,I_2^L)+\max(I_1^L,I_2^L)}{2}, \frac{\min(I_1^U,I_2^U)+\max(I_1^U,I_2^U)}{2}], [\frac{\min(F_1^L,F_2^L)+\max(F_1^L,F_2^L)}{2}, \frac{\min(I_1^U,I_2^U)+\max(I_1^U,I_2^U)}{2}],$

$=\{[\frac{T_1^L+T_2^L}{2}, \frac{T_1^U+T_2^U}{2}], [\frac{I_1^L+I_2^L}{2}, \frac{I_1^L+I_2^L}{2}], [\frac{F_1^L+F_2^L}{2}, \frac{F_1^L+F_2^L}{2}]\}$

$= \tilde{n}_1 @ \tilde{n}_2$

This proves (i).

(iii) From definitions in 2.4, 2.5 and 3.1, we have

$((\tilde{n}_1 \oplus \tilde{n}_2) \cup (\tilde{n}_1 \otimes \tilde{n}_2)) @ ((\tilde{n}_1 \oplus \tilde{n}_2) \cap (\tilde{n}_1 \otimes \tilde{n}_2)) = \tilde{n}_1 @ \tilde{n}_2;$

$(\tilde{n}_1 \oplus \tilde{n}_2) \cap (\tilde{n}_1 \otimes \tilde{n}_2) = \{([T_1^L+T_2^L-T_1^LT_2^L, T_1^U+T_2^U-T_1^UT_2^U], [I_1^L I_2^L, I_1^U I_2^U], [F_1^LF_2^L, F_1^UF_2^U])\} \cap$

$\{[T_1^L T_2^L, T_1^U T_2^U)], [I_1^L+I_2^L-I_2^LI_2^L, I_1^U+I_2^U-I_1^UI_2^U], [F_1^L+F_2^L-F_2^LF_2^L, F_1^U+F_2^U-F_1^UF_2^U]\}$

$=\{[\min(T_1^L+T_2^L-T_1^LT_2^L, T_1^L T_2^L), \min(T_1^U+T_2^U-T_1^UT_2^U, T_1^U T_2^U)],$

$[\max(I_1^L I_2^L, I_1^L+I_2^L-I_2^LI_2^L), \max(I_1^U I_2^U, I_1^U+I_2^U-I_1^UI_2^U)],$

$[\max(F_1^LF_2^L, F_1^L+F_2^L-F_2^LF_2^L), \max(F_1^UF_2^U, F_1^U+F_2^U-F_1^UF_2^U)]\}.$

$= \{[T_1^L T_2^L, T_1^U T_2^U], [I_1^L+I_2^L-I_2^LI_2^L, I_1^U+I_2^U-I_1^UI_2^U], [F_1^L+F_2^L-F_2^LF_2^L, F_1^U+F_2^U-F_1^UF_2^U]\}$

$(\tilde{n}_1 \oplus \tilde{n}_2) \cup (\tilde{n}_1 \otimes \tilde{n}_2) = \{([T_1^L+T_2^L-T_1^LT_2^L, T_1^U+T_2^U-T_1^UT_2^U], [I_1^L I_2^L, I_1^U I_2^U], [F_1^LF_2^L, F_1^UF_2^U])\} \cup$

$\{[T_1^L T_2^L, T_1^U T_2^U)], [I_1^L+I_2^L-I_2^LI_2^L, I_1^U+I_2^U-I_1^UI_2^U], [F_1^L+F_2^L-F_2^LF_2^L, F_1^U+F_2^U-F_1^UF_2^U]$

$=\{[\max(T_1^L+T_2^L-T_1^LT_2^L, T_1^L T_2^L), \max(T_1^U+T_2^U-T_1^UT_2^U, T_1^U T_2^U)],$

$[\min(I_1^L I_2^L, I_1^L+I_2^L-I_2^LI_2^L), \min(I_1^U I_2^U, I_1^U+I_2^U-I_1^UI_2^U)], [\min(F_1^LF_2^L, F_1^L+F_2^L-F_2^LF_2^L), \min(F_1^UF_2^U, F_1^U+F_2^U-F_1^UF_2^U)]\}$

$=\{[T_1^L+T_2^L-T_1^LT_2^L, T_1^U+T_2^U-T_1^UT_2^U], [I_1^L I_2^L, I_1^U I_2^U], [F_1^LF_2^L, F_1^UF_2^U]\}$

$((\tilde{n}_1 \oplus \tilde{n}_2) \cup (\tilde{n}_1 \otimes \tilde{n}_2)) @ ((\tilde{n}_1 \oplus \tilde{n}_2) \cap (\tilde{n}_1 \otimes \tilde{n}_2)) = \{[\frac{T_1^L T_2^L+T_1^L+T_2^L-T_1^LT_2^L}{2}, \frac{T_1^U T_2^U+T_1^U+T_2^U-T_1^UT_2^U}{2}],$

$[\frac{I_1^L+I_2^L-I_2^LI_2^L+I_1^L I_2^L}{2}, \frac{I_1^U+I_2^U-I_1^UI_2^U+I_1^U I_2^U}{2}], [\frac{F_1^L+F_2^L-F_2^LF_2^L+F_1^LF_2^L}{2}, \frac{F_1^U+F_2^U-F_1^UF_2^U+F_1^UF_2^U}{2}]\}$

$=\{[\frac{T_1^L+T_2^L}{2}, \frac{T_1^U+T_2^U}{2}], [\frac{I_1^L+I_2^L}{2}, \frac{I_1^U+I_2^U}{2}], [\frac{F_1^L+F_2^L}{2}, \frac{F_1^U+F_2^U}{2}]$

Hence, $((\tilde{n}_1 \oplus \tilde{n}_2) \cup (\tilde{n}_1 \otimes \tilde{n}_2)) @ ((\tilde{n}_1 \oplus \tilde{n}_2) \cap (\tilde{n}_1 \otimes \tilde{n}_2)) = \tilde{n}_1 @ \tilde{n}_2$

This proves (iii).

4. Conclusion

In this paper we have defined three new operations on interval neutrosophic sets based on the arithmetic mean, geometrical mean, and respectively harmonic mean, which involve different defining functions. Several related results have been proved and the characteristics of the interval neutrosophic sets revealed.

Acknowledgements

The authors are very grateful to the anonymous referees and to Prof. Piergiulio Corsini for their insightful and constructive comments and suggestions, which have been very helpful in improving the paper**.**

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New Operations over Interval Valued Intuitionistic Hesitant Fuzzy Set

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Abstract Hesitancy is the most common problem in decision making, for which hesitant fuzzy set can be considered as a useful tool allowing several possible degrees of membership of an element to a set. Recently, another suitable means were defined by Zhiming Zhang [1], called interval valued intuitionistic hesitant fuzzy sets, dealing with uncertainty and vagueness, and which is more powerful than the hesitant fuzzy sets. In this paper, four new operations are introduced on interval-valued intuitionistic hesitant fuzzy sets and several important properties are also studied.

Keywords Fuzzy Sets, Intuitionistic Fuzzy Set, Hesitant Fuzzy Sets, Interval-Valued Intuitionistic Hesitant Fuzzy Set, Interval Valued Intuitionistic Fuzzy Sets

1. Introduction

In recent decades, several types of sets, such as fuzzy sets [2], interval-valued fuzzy sets [3], intuitionistic fuzzy sets [4, 5], interval-valued intuitionistic fuzzy sets [6], type 2 fuzzy sets [7, 8], type n fuzzy sets [7], and hesitant fuzzy sets [9], neutrosophic sets, have been introduced and investigated widely. The concept of intuitionistic fuzzy sets, was introduced by Atanassov [4, 5]; it is interesting and useful in modeling several real life problems.

An intuitionistic fuzzy set (IFS for short) has three associated defining functions, namely the membership function, the non-membership function and the hesitancy function. Later, Atanassov and Gargov provided in [6] what they called interval-valued intuitionistic fuzzy sets theory (IVIFS for short), which is a generalization of both interval valued fuzzy sets and intuitionistic fuzzy sets. Their concept is characterized by a membership function and a non-membership function whose values are intervals rather than real number. IVIFS is more powerful in dealing with vagueness and uncertainty than IFS.

Recently, Torra and Narukawa [9] and Torra [10] proposed the concept of hesitant fuzzy sets, a new generalization of fuzzy sets, which allows the membership of an element of a set to be represented by several possible values. They also discussed relationships among hesitant fuzzy sets and other generalizations of fuzzy sets such as intuitionistic fuzzy sets, type-2 fuzzy sets, and fuzzy multisets. Some set theoretic operations such as union, intersection and complement on hesitant fuzzy sets have also been proposed by Torra [9]. Hesitant fuzzy sets can be used as an efficient mathematical tool for modeling people's hesitancy in daily life than the other classical extensions of fuzzy sets. We'll further study the interval valued intuitionistic hesitant fuzzy sets. Xia and Xu [11] made an intensive study of hesitant fuzzy information aggregation techniques and their applications in decision making. They also defined some new operations on hesitant fuzzy sets based on the interconnection between hesitant fuzzy sets and the interval valued intuitionistic fuzzy sets. To aggregate the hesitant fuzzy information under confidence levels, Xia et al. [12] developed a series of confidence induced hesitant fuzzy aggregations operators. Further, Xia and Xu [13, 14] gave a detailed study on distance, similarity and correlation measures for hesitant fuzzy sets and hesitant fuzzy elements respectively. Xu et al. [15] developed several series of aggregation operators for interval valued intuitionistic hesitant fuzzy information such as: the interval valued intuitionistic fuzzy weighted arithmetic aggregation (IIFWA), the interval valued intuitionistic fuzzy ordered weighted aggregation (IIFOWA) and the interval valued intuitionistic fuzzy hybrid aggregation (IIFHA) operator. Wei and Wang [16], Xu et al. [17] introduced the interval valued intuitionistic fuzzy weighted geometric (IIFWG) operator, the interval valued intuitionistic fuzzy ordered weighted geometric (IIFOWG) operator and the interval valued intuitionistic fuzzy hybrid geometric (IIFHG) operator. Recently, Zhiming Zhang [1] have proposed the concept of interval valued intuitionistic hesitant fuzzy set , study their some basic properties and developed several series of aggregation operators for interval valued intuitionistic hesitant fuzzy environment and have applied them to solve multi-attribute group decision making problems.

In this paper, our aim is to propose four new operations on interval valued intuitionistic hesitant fuzzy sets and study their properties.

Therefore, the rest of the paper is set out as follows. In Section 2, some basic definitions related to intuitionistic fuzzy sets, hesitant fuzzy sets and interval valued intuitionistic hesitant fuzzy set are briefly discussed. In Section 3, four new operations on interval valued intuitionistic hesitant fuzzy sets have been proposed and some properties of these operations are proved. In section 4, we conclude the paper.

2. Preliminaries

In this section, we give below some definitions related to intuitionistic fuzzy sets, interval valued intuitionistic fuzzy sets, hesitant fuzzy set and interval valued hesitant fuzzy sets.

Definition 2.1. [4, 5] (Set operations on IFS)

Let IFS(X) denote the family of all intuitionistic fuzzy sets defined on the universe X, and let α, β ∈ IFS(X) be given as

$$\alpha = (\mu_\alpha, \nu_\alpha), \quad \beta = (\mu_\beta, \nu_\beta).$$

Then nine set operations are defined as follows:

(i) $\alpha^c=(\nu_\alpha, \mu_\alpha)$;

(ii) $\alpha \cup \beta = (\max(\mu_\alpha, \mu_\beta), \min(\nu_\alpha, \nu_\beta))$;

(iii) $\alpha \cap \beta = (\min(\mu_\alpha, \mu_\beta), \max(\nu_\alpha, \nu_\beta))$;

(iv) $\alpha \oplus \beta = (\mu_\alpha+\mu_\beta-\mu_\alpha\mu_\beta, \nu_\alpha\nu_\beta)$;

(v) $\alpha \otimes \beta = (\mu_\alpha\mu_\beta, \nu_\alpha+\nu_\beta-\nu_\alpha\nu_\beta)$;

(vi) $\alpha\ @\ \beta = (\frac{\mu_\alpha+\mu_\beta}{2}, \frac{\nu_\alpha+\nu_\beta}{2})$;

(vii) $\alpha\ \$\ \beta = (\sqrt{\mu_\alpha\mu_\beta}, \sqrt{\nu_\alpha\nu_\beta})$;

(viii) $\alpha\ \#\ \beta = (\frac{2\mu_\alpha\mu_\beta}{\mu_\alpha+\mu_\beta}, \frac{2\nu_\alpha\nu_\beta}{\nu_\alpha+\nu_\beta})$;

(ix) $\alpha * \beta = (\frac{\mu_\alpha+\mu_\beta}{2(\mu_\alpha\mu_\beta+1)}, \frac{\nu_\alpha+\nu_\beta}{2(\nu_\alpha\nu_\beta+1)})$;

In the following, we introduce some basic concepts related to IVIFS.

Definition 2.2. [6] (Interval valued intuitionistic fuzzy sets)

An Interval valued intuitionistic fuzzy sets (IVIFS) α in the finite universe X is expressed by the form

α = {<x, μ_α(x), ν_α(x)>| x ∈ X } ,where μ_α(x) = [μ_α^- (x), μ_α^+(x)] ∈ [I] is called membership interval of element to IVIFS α ,while [ν_α^- (x), ν_α^+(x)] ∈ [I] is the non-membership interval of that element to the set α , with the condition 0 ≤ μ_α^+(x) +ν_α^+(x) ≤1 must hold for any x ∈ X .

For convenience, the lower and upper bounds of μ_α (x) and ν_α (x) are denoted by μ_α^- , μ_α^+ , ν_α^-, ν_α^+, respectively. Thus, the IVIFS α may be concisely expressed as

$$\alpha = (\mu_\alpha, \nu_\alpha)= \{<x, [\mu_\alpha^-, \mu_\alpha^+], [\nu_\alpha^-, \nu_\alpha^+]> \mid x \in X\} \quad (1)$$

Where 0 ≤ μ_α^+ +ν_α^+ ≤1

Definition 2.3 [9, 11]

Let X be a fixed set. A hesitant fuzzy set (HFS) on X is in terms of a function that when applied to X returns a subset of [0, 1] the HFS is expressed by a mathematical symbol

$$E=\{<x, h_E(x)> \mid x \in X\} \quad (2)$$

where $h_E(x)$> is a set of some values in[0, 1], denoting the possible membership degree of the element x ∈ X to the set E. For convenience, Xia and Xu [11] called h=$h_E(x)$ a hesitant fuzzy element (HFE) and H be the set of all HFEs.

Given three HFEs represented by h, h_1 ,and h_2 ,Torra [9] defined some operations on them, which can be described as:

1) $h^c=\{1-\gamma \mid \gamma \in h\}$
2) $h_1 \cup h_2=\{\max(\gamma_1, \gamma_2) \mid \gamma_1 \in h_1, \gamma_2 \in h_2\}$
3) $h_1 \cap h_2=\{\min(\gamma_1, \gamma_2) \mid \gamma_1 \in h_1, \gamma_2 \in h_2\}$

Furthermore, in order to aggregate hesitant fuzzy information, Xia and Xu [11] defined some new operations on the HFEs h, h_1 ,and h_2 :

1) $h_1 \oplus h_2=\{\gamma_1+\gamma_2-\gamma_1\gamma_2 \mid \gamma_1 \in h_1, \gamma_2 \in h_2\}$
2) $h_1 \otimes h_2=\{\gamma_1\gamma_2 \mid \gamma_1 \in h_1, \gamma_2 \in h_2\}$
3) $h^\lambda=\{\gamma^\lambda \mid \gamma \in h\}$
4) λ h=$\{1-(1-\gamma)^\lambda \mid \gamma \in h\}$

Definition 2.4 [1] (Interval valued intuitionistic hesitant fuzzy sets)

Let X be a fixed set, an interval-valued intuitionistic hesitant fuzzy set (IVIHFS) on X is given in terms of a function that when applied to X returns a subset of Ω. The IVIHFS is expressed by a mathematical symbol

$$\tilde{E}=\{<x, h_{\tilde{E}}(x)> \mid x \in X\} \quad (3)$$

where $h_{\tilde{E}}(x)$ is a set of some IVIFNs in X , denoting the possible membership degree intervals and non-membership degree intervals of the element x ∈ X to the set $\tilde{E}$.

For convenience, an interval-valued intuitionistic hesitant fuzzy element (IVIHFE) is denoted by $\tilde{h} = h_{\tilde{E}}(x)$ and $\tilde{h}$ be the set of all IVIHFEs. If $\alpha \in \tilde{h}$, then an IVIFN can be denoted by α = (μ_α, ν_α)= ([$\mu_{\alpha_1}^-$, $\mu_{\alpha_1}^+$],[$\nu_{\alpha_1}^-$, $\nu_{\alpha_1}^+$]).

For any ∈ $\tilde{h}$, if α is a real number in [0,1], then $\tilde{h}$ reduces to a hesitant fuzzy element (HFE) [9]; if α is a closed subinterval of the unit interval, then $\tilde{h}$ reduces to an interval-valued hesitant fuzzy element (IVHFE)[1]; if α is an intuitionistic fuzzy number (IFN) , then $\tilde{h}$ reduces to an intuitionistic hesitant fuzzy element (IHFE). Therefore, HFEs, IVHFEs, and IHFEs are special cases of IVIHFEs.

Definition 2.5. [1, 9]

Given three IVIHFEs represented by $\tilde{h}$, $\tilde{h}_1$,and $\tilde{h}_2$, one defines some operations on them, which can be described as:

$\tilde{h}^c = \{\alpha^c \mid \alpha \in \tilde{h}\} = \{([\nu_\alpha^-, \nu_\alpha^+],[\mu_\alpha^-, \mu_\alpha^+]) \mid \alpha \in \tilde{h}\}$,

$\tilde{h}_1 \cup \tilde{h}_2 = \{\max(\alpha_1, \alpha_2) \mid \alpha_1 \in \tilde{h}_1, \alpha_2 \in \tilde{h}_2\}$

$= \{ ([\max(\mu^-_{\alpha_1}, \mu^-_{\alpha_2}), \max(\mu^+_{\alpha_1}, \mu^+_{\alpha_2})],[\min(\nu^-_{\alpha_1}, \nu^-_{\alpha_2}), \min(\nu^+_{\alpha_1}, \nu^+_{\alpha_2})]) \mid \alpha_1 \in \tilde{h}_1, \alpha_2 \in \tilde{h}_2\},$

$\tilde{h}_1 \cap \tilde{h}_2 = \{\min(\alpha_1, \alpha_2) \mid \alpha_1 \in \tilde{h}_1, \alpha_2 \in \tilde{h}_2\}$

$= \{ ([\min(\mu^-_{\alpha_1}, \mu^-_{\alpha_2}), \min(\mu^+_{\alpha_1}, \mu^+_{\alpha_2})],[\max(\nu^-_{\alpha_1}, \nu^-_{\alpha_2}), \max(\nu^+_{\alpha_1}, \nu^+_{\alpha_2})]) \mid \alpha_1 \in \tilde{h}_1, \alpha_2 \in \tilde{h}_2\},$

$\tilde{h}_1 \oplus \tilde{h}_2 = \Big\{\Big(\big([\mu^-_{\alpha_1} + \mu^-_{\alpha_2} - \mu^-_{\alpha_1}\mu^-_{\alpha_2}, \mu^+_{\alpha_1} + \mu^+_{\alpha_2} - \mu^+_{\alpha_1}\mu^+_{\alpha_2}], [\nu^-_{\alpha_1}\nu^-_{\alpha_2}, \nu^+_{\alpha_1}\nu^+_{\alpha_2}] \mid \alpha_1 \in \tilde{h}_1, \alpha_2 \in \tilde{h}_2\big)\Big)\Big\}$

$\tilde{h}_1 \otimes \tilde{h}_2 == \{([\mu^-_{\alpha_1}\mu^-_{\alpha_2}, \mu^+_{\alpha_1}\mu^+_{\alpha_2}],[\nu^-_{\alpha_1} + \nu^-_{\alpha_2} - \nu^-_{\alpha_1}\nu^-_{\alpha_2}, \nu^+_{\alpha_1} + \nu^+_{\alpha_2} - \nu^+_{\alpha_1}\nu^+_{\alpha_2}] \mid \alpha_1 \in \tilde{h}_1, \alpha_2 \in \tilde{h}_2\}$

$\lambda \tilde{h}$

$= \Big\{\Big(\Big[1 - \big(1 - \mu^-_{\alpha}\big)^{\lambda}, 1 - \big(1 - \mu^+_{\alpha}\big)^{\lambda}\Big], [(\nu^-_{\alpha})^{\lambda}, (\nu^+_{\alpha})^{\lambda}]\Big) \mid \alpha \in \tilde{h}\Big\}$

3. Four New Operations on IVIHFEs

Definition 3.1

Let $\tilde{h}_1$ and $\tilde{h}_2 \in$ IVIHFE (X), we propose the following operations on IVIHFEs as follows:

1) $\tilde{h}_1 @ \tilde{h}_2 = \{([\frac{\mu^-_{\alpha_1} + \mu^-_{\alpha_2}}{2}, \frac{\mu^+_{\alpha_1} + \mu^+_{\alpha_2}}{2}],[\frac{\upsilon^-_{\alpha_1} + \upsilon^-_{\alpha_2}}{2}, \frac{\upsilon^+_{\alpha_1} + \upsilon^+_{\alpha_2}}{2}] \mid \alpha_1 \in \tilde{h}_1, \alpha_2 \in \tilde{h}_2\}$

2) $\tilde{h}_1 \$ \tilde{h}_2 = \{([\sqrt{\mu^-_{\alpha_1}\mu^-_{\alpha_2}}, \sqrt{\mu^+_{\alpha_1}\mu^+_{\alpha_2}}],[\sqrt{\upsilon^-_{\alpha_1}\upsilon^-_{\alpha_2}}, \sqrt{\upsilon^+_{\alpha_1}\upsilon^+_{\alpha_2}}] \mid \alpha_1 \in \tilde{h}_1, \alpha_2 \in \tilde{h}_2\}$

3) $\tilde{h}_1 \# \tilde{h}_2 = \{([\frac{2\mu^-_{\alpha_1}\mu^-_{\alpha_2}}{\mu^-_{\alpha_1} + \mu^-_{\alpha_2}}, \frac{2\mu^+_{\alpha_1}\mu^+_{\alpha_2}}{\mu^+_{\alpha_1} + \mu^+_{\alpha_2}}],[\frac{2\upsilon^-_{\alpha_1}\upsilon^-_{\alpha_2}}{\upsilon^-_{\alpha_1} + \upsilon^-_{\alpha_2}}, \frac{2\upsilon^+_{\alpha_1}\upsilon^+_{\alpha_2}}{\upsilon^+_{\alpha_1} + \upsilon^+_{\alpha_2}}] \mid \alpha_1 \in \tilde{h}_1, \alpha_2 \in \tilde{h}_2\}$

4) $\tilde{h}_1 * \tilde{h}_2 = \{([\frac{\mu^-_{\alpha_1} + \mu^-_{\alpha_2}}{2(\mu^-_{\alpha_1}\mu^-_{\alpha_2} + 1)}, \frac{\mu^+_{\alpha_1} + \mu^+_{\alpha_2}}{2(\mu^+_{\alpha_1}\mu^+_{\alpha_2} + 1)}],[\frac{\upsilon^-_{\alpha_1} + \upsilon^-_{\alpha_2}}{2(\upsilon^-_{\alpha_1}\upsilon^-_{\alpha_2} + 1)}, \frac{\upsilon^+_{\alpha_1} + \upsilon^+_{\alpha_2}}{2(\upsilon^+_{\alpha_1}\upsilon^+_{\alpha_2} + 1)}] \mid \alpha_1 \in \tilde{h}_1, \alpha_2 \in \tilde{h}_2\}$

Obviously, for every two IVIHFEs $\tilde{h}_1$ and $\tilde{h}_2$,($\tilde{h}_1 @ \tilde{h}_2$), ($\tilde{h}_1 \$ \tilde{h}_2$), ($\tilde{h}_1 \# \tilde{h}_2$) and ($\tilde{h}_1 * \tilde{h}_2$) are also IVIHFEs.

Example 3.2

Let $\tilde{h}_1$ (x)= {([0.2, 0.3],[0.5, 0.6]),([0.5, 0.8],[0.1, 0.2])} and $\tilde{h}_2$ (x)= {([0.4, 0.6],[0.3, 0.4]), ([0.3, 0.5],[0.1, 0.2]) be two interval valued intuitionistic hesitant fuzzy elements. Then we have

($\tilde{h}_1 @ \tilde{h}_2$) = {([0.3, 0.45],[0.4, 0.5]),([0.4, 0.65],[0.1, 0.2])}

($\tilde{h}_1 \$ \tilde{h}_2$) ={([0.28, 0.42],[0.38, 0.48]),([0.38, 0.63],[0.1, 0.2])}

($\tilde{h}_1 \# \tilde{h}_2$) = {([0.26, 0.4],[0.37, 0.48]),([0.37, 0.61],[0.1, 0.2])}

($\tilde{h}_1 * \tilde{h}_2$) = {([0.27, 0.38],[0.34, 0.40]),([0.34, 0.46],[0.09, 0.19])}

With these operations, several results follow.

Theorem 3.3

For every $\tilde{h} \in$ IVIHFE(X), the following are true,

(i) $\tilde{h} @ \tilde{h} = \tilde{h}$;

(ii) $\tilde{h} \$ \tilde{h} = \tilde{h}$;

(iii) $\tilde{h} \# \tilde{h} = \tilde{h}$;

Proof. we prove only (i) (ii) .

(i) Let $\tilde{h} \in$ IVIHFEs

$\tilde{h} @ \tilde{h} = \{ [\frac{\mu^-_{\alpha} + \mu^-_{\alpha}}{2}, \frac{\mu^+_{\alpha} + \mu^+_{\alpha}}{2}],[\frac{\upsilon^-_{\alpha} + \upsilon^-_{\alpha}}{2}, \frac{\upsilon^+_{\alpha} + \upsilon^+_{\alpha}}{2}] \mid \alpha \in \tilde{h}\}$

$=[\mu^-_{\alpha}, \mu^+_{\alpha}], [\upsilon^-_{\alpha}, \upsilon^+_{\alpha}]$

Then, $\tilde{h} @ \tilde{h} = \tilde{h}$

(ii) Let $\tilde{h} \in$ IVIHFEs

$\tilde{h} \$ \tilde{h} = \{([\sqrt{\mu^-_{\alpha}\mu^-_{\alpha}}, \sqrt{\mu^+_{\alpha}\mu^+_{\alpha}}],[\sqrt{\upsilon^-_{\alpha}\upsilon^-_{\alpha}}, \sqrt{\upsilon^+_{\alpha}\upsilon^+_{\alpha}}] \mid \alpha \in \tilde{h}\}$

$=[\mu^-_{\alpha}, \mu^+_{\alpha}], [\upsilon^-_{\alpha}, \upsilon^+_{\alpha}]$

Then, $\tilde{h} \$ \tilde{h} = \tilde{h}$

Theorem 3.4

For $\tilde{h}_1, \tilde{h}_2 \in$ IVIHFEs,

(i) $\tilde{h}_1 @ \tilde{h}_2 = \tilde{h}_2 @ \tilde{h}_1$;

(ii) $\tilde{h}_1 \$ \tilde{h}_2 = \tilde{h}_2 \$ \tilde{h}_1$;
(iii) $\tilde{h}_1 \# \tilde{h}_2 = \tilde{h}_2 \# \tilde{h}_1$;
(iv) $\tilde{h}_1 * \tilde{h}_2 = \tilde{h}_2 * \tilde{h}_1$;
Proof. These also follow from definitions.

Theorem 3.5

For $\tilde{h}_1, \tilde{h}_2 \in$ IVIHFE(X),

$(\tilde{h}_1^{\,c} @ \tilde{h}_2^{\,c})^c = \tilde{h}_1 @ \tilde{h}_2$

Proof. In the following, we prove (i), (ii) and (iii), results (iv), (v) and (vi) can be proved analogously.

$$(\tilde{h}_1^{\,c} @ \tilde{h}_2^{\,c})^c = \{[\nu_{\alpha_1}^-, \nu_{\alpha_1}^+], [\mu_{\alpha_1}^-, \mu_{\alpha_1}^+] | \alpha_1 \in \tilde{h}_1\} @ \{[\nu_{\alpha_2}^-, \nu_{\alpha_2}^+], [\mu_{\alpha_2}^-, \mu_{\alpha_2}^+] \mid \alpha_2 \in \tilde{h}_2\}$$

$$(\tilde{h}_1^{\,c} @ \tilde{h}_2^{\,c})^c = \left(\{[\nu_{\alpha_1}^-, \nu_{\alpha_1}^+], [\mu_{\alpha_1}^-, \mu_{\alpha_1}^+] | \alpha_1 \in \tilde{h}_1\} @ \{[\nu_{\alpha_2}^-, \nu_{\alpha_2}^+], [\mu_{\alpha_2}^-, \mu_{\alpha_2}^+] | \alpha_2 \in \tilde{h}_2\}\right)^c$$

$$= \left(\{[\frac{\nu_{\alpha_1}^- + \nu_{\alpha_2}^-}{2}, \frac{\nu_{\alpha_1}^+ + \nu_{\alpha_2}^+}{2}], [\frac{\mu_{\alpha_1}^- + \mu_{\alpha_2}^-}{2}, \frac{\mu_{\alpha_1}^+ + \mu_{\alpha_2}^+}{2}] | \alpha_1 \in \tilde{h}_1, \alpha_2 \in \tilde{h}_2\}\right)^c$$

$$= \left(\{[\frac{\mu_{\alpha_1}^- + \mu_{\alpha_2}^-}{2}, \frac{\mu_{\alpha_1}^+ + \mu_{\alpha_2}^+}{2}], [\frac{\nu_{\alpha_1}^- + \nu_{\alpha_2}^-}{2}, \frac{\nu_{\alpha_1}^+ + \nu_{\alpha_2}^+}{2}] | \alpha_1 \in \tilde{h}_1, \alpha_2 \in \tilde{h}_2\}\right)$$

$= \tilde{h}_1 @ \tilde{h}_2$

This proves the theorem.
Note 1: One can easily verify that

1. $(\tilde{h}_1^{\,c} \$ \tilde{h}_2^{\,c})^c \neq \tilde{h}_1 \$ \tilde{h}_2$
2. $(\tilde{h}_1^{\,c} \# \tilde{h}_2^{\,c})^c \neq \tilde{h}_1 \# \tilde{h}_2$
3. $(\tilde{h}_1^{\,c} * \tilde{h}_2^{\,c})^c \neq \tilde{h}_1 * \tilde{h}_2$

Theorem 3.6

For $\tilde{h}_1, \tilde{h}_2$ and $\tilde{h}_3 \in$ IVIHFE(X) , we have the following identities:
(i) $(\tilde{h}_1 \cup \tilde{h}_2) @ \tilde{h}_3 = (\tilde{h}_1 @ \tilde{h}_3) \cup (\tilde{h}_2 @ \tilde{h}_3)$;
(ii) $(\tilde{h}_1 \cap \tilde{h}_2) @ \tilde{h}_3 = (\tilde{h}_1 @ \tilde{h}_3) \cap (\tilde{h}_2 @ \tilde{h}_3)$
(iii) $(\tilde{h}_1 \cup \tilde{h}_2) \$ \tilde{h}_3 = (\tilde{h}_1 \$ \tilde{h}_3) \cup (\tilde{h}_2 \$ \tilde{h}_3)$;
(iv) $(\tilde{h}_1 \cap \tilde{h}_2) \$ \tilde{h}_3 = (\tilde{h}_1 \$ \tilde{h}_3) \cap (\tilde{h}_2 \$ \tilde{h}_3)$;
(v) $((\tilde{h}_1 \cup \tilde{h}_2)) \# \tilde{h}_3 = (\tilde{h}_1 \# \tilde{h}_3) \cup (\tilde{h}_2 \# \tilde{h}_3)$;
(vi) $(\tilde{h}_1 \cap \tilde{h}_2) \# \tilde{h}_3 = (\tilde{h}_1 \# \tilde{h}_3) \cap (\tilde{h}_2 \# \tilde{h}_3)$;
(vii) $(\tilde{h}_1 \cup \tilde{h}_2) * \tilde{h}_3 = (\tilde{h}_1 * \tilde{h}_3) \cup (\tilde{h}_2 * \tilde{h}_3)$;
(viii) $(\tilde{h}_1 \cap \tilde{h}_2) * \tilde{h}_3 = (\tilde{h}_1 * \tilde{h}_3) \cap (\tilde{h}_2 * \tilde{h}_3)$;
(ix) $(\tilde{h}_1 @ \tilde{h}_2) \oplus \tilde{h}_3 = (\tilde{h}_1 \oplus \tilde{h}_3) @ (\tilde{h}_2 \oplus \tilde{h}_3)$;
(x) $(\tilde{h}_1 @ \tilde{h}_2) \otimes \tilde{h}_3 = (\tilde{h}_1 \otimes \tilde{h}_3) @ (\tilde{h}_2 \otimes \tilde{h}_3)$

Proof . We prove (i), (iii), (v), (vii) and (ix), results (ii), (iv), (vi), (viii) and (x) can be proved analogously
Using definitions in 2.3 and 3.1,we have

$$(\tilde{h}_1 \cup \tilde{h}_2) @ \tilde{h}_3 = \{[\max(\mu_{\alpha_1}^-, \mu_{\alpha_2}^-), \max(\mu_{\alpha_1}^+, \mu_{\alpha_2}^+)], [\min(\nu_{\alpha_1}^-, \nu_{\alpha_2}^-), \min(\nu_{\alpha_1}^+, \nu_{\alpha_2}^+)] | \alpha_1 \in \tilde{h}_1, \alpha_2 \in \tilde{h}_2\} @ \{[\mu_{\alpha_3}^-, \mu_{\alpha_3}^+], [\nu_{\alpha_3}^-, \nu_{\alpha_3}^+] | \alpha_3 \in \tilde{h}_3\}$$

$$= \{[\frac{\max(\mu_{\alpha_1}^-, \mu_{\alpha_2}^-) + \mu_{\alpha_3}^-}{2}, \frac{\max(\mu_{\alpha_1}^+, \mu_{\alpha_2}^+) + \mu_{\alpha_3}^+}{2}], [\frac{\min(\nu_{\alpha_1}^-, \nu_{\alpha_2}^-) + \nu_{\alpha_3}^-}{2}, \frac{\min(\nu_{\alpha_1}^+, \nu_{\alpha_2}^+) + \nu_{\alpha_3}^+}{2}] | \alpha_1 \in \tilde{h}_1, \alpha_2 \in \tilde{h}_2, \alpha_3 \in \tilde{h}_3\}$$

$$= \{[\max(\frac{\mu_{\alpha_1}^- + \mu_{\alpha_3}^-}{2}, \frac{\mu_{\alpha_2}^- + \mu_{\alpha_3}^-}{2}), \max(\frac{\mu_{\alpha_1}^+ + \mu_{\alpha_3}^+}{2}, \frac{\mu_{\alpha_2}^+ + \mu_{\alpha_3}^+}{2})], [\min(\frac{\nu_{\alpha_1}^- + \nu_{\alpha_3}^-}{2}, \frac{\nu_{\alpha_2}^- + \nu_{\alpha_3}^-}{2}), \min(\frac{\nu_{\alpha_1}^- + \nu_{\alpha_3}^-}{2}, \frac{\nu_{\alpha_1}^+ + \nu_{\alpha_3}^+}{2})] | \alpha_1 \in \tilde{h}_1, \alpha_2 \in \tilde{h}_2, \alpha_3 \in \tilde{h}_3\}$$

$= (\tilde{h}_1 @ \tilde{h}_3) \cup (\tilde{h}_2 @ \tilde{h}_3)$

This proves (i)
(iii) From definitions in 2.3 and 3.1, we have

$$(\tilde{h}_1 \cup \tilde{h}_2) \$ \tilde{h}_3 = \{[\max(\mu_{\alpha_1}^-, \mu_{\alpha_2}^-), \max(\mu_{\alpha_1}^+, \mu_{\alpha_2}^+)], [\min(\nu_{\alpha_1}^-, \nu_{\alpha_2}^-), \min(\nu_{\alpha_1}^+, \nu_{\alpha_2}^+)] | \alpha_1 \in \tilde{h}_1, \alpha_2 \in \tilde{h}_2\} \$ \{[\mu_{\alpha_3}^-, \mu_{\alpha_3}^+], [\nu_{\alpha_3}^-, \nu_{\alpha_3}^+] | \alpha_3 \in \tilde{h}_3\}$$

$= \{ [\sqrt{max(\mu^-_{\alpha_1}, \mu^-_{\alpha_2})\, \mu^-_{\alpha_3}}, \sqrt{\max(\mu^+_{\alpha_1}, \mu^+_{\alpha_2})\, \mu^+_{\alpha_3}}], [\sqrt{\min(\nu^-_{\alpha_1}, \nu^-_{\alpha_2})\, \nu^-_{\alpha_3}}, \sqrt{\min(\nu^+_{\alpha_1}, \nu^+_{\alpha_2})\, \nu^+_{\alpha_3}}] | \alpha_1 \in \tilde{h}_1, \alpha_2 \in \tilde{h}_2, \alpha_3 \in \tilde{h}_3\}$

$=\{[\max(\sqrt{\mu^-_{\alpha_1}\mu^-_{\alpha_3}}, \sqrt{\mu^-_{\alpha_2}\mu^-_{\alpha_3}}), \max(\sqrt{\mu^+_{\alpha_1}\mu^+_{\alpha_3}}, \sqrt{\mu^+_{\alpha_2}\mu^+_{\alpha_3}})], [\min(\sqrt{\nu^-_{\alpha_1}\nu^-_{\alpha_3}}, \sqrt{\nu^-_{\alpha_2}\nu^-_{\alpha_3}}), \min(\sqrt{\nu^+_{\alpha_1}\nu^+_{\alpha_3}}, \sqrt{\nu^+_{\alpha_2}\nu^+_{\alpha_3}})] | \alpha_1 \in \tilde{h}_1, \alpha_2 \in \tilde{h}_2, \alpha_3 \in \tilde{h}_3\}$

$=(\tilde{h}_1 \$ \tilde{h}_3) \cup (\tilde{h}_2 \$ \tilde{h}_3)$;

This proves (iii).

(v) Using definitions 2.3 and 3.1,we have

$((\tilde{h}_1 \cup \tilde{h}_2)) \# \tilde{h}_3 = \{[\max(\mu^-_{\alpha_1}, \mu^-_{\alpha_2}), \max(\mu^+_{\alpha_1}, \mu^+_{\alpha_2})], [\min(\nu^-_{\alpha_1}, \nu^-_{\alpha_2}), \min(\nu^+_{\alpha_1}, \nu^+_{\alpha_2})] | \alpha_1 \in \tilde{h}_1, \alpha_2 \in \tilde{h}_2\} \# \{[\mu^-_{\alpha_3}, \mu^+_{\alpha_3}], [\nu^-_{\alpha_3}, \nu^+_{\alpha_3}] | \alpha_3 \in \tilde{h}_3\}$

$= \{[\frac{2\max(\mu^-_{\alpha_1}, \mu^-_{\alpha_2})\,\mu^-_{\alpha_3}}{\max(\mu^-_{\alpha_1}, \mu^-_{\alpha_2}) + \mu^-_{\alpha_3}}, \frac{2\max(\mu^+_{\alpha_1}, \mu^+_{\alpha_2})\,\mu^+_{\alpha_3}}{\max(\mu^+_{\alpha_1}, \mu^+_{\alpha_2}) + \mu^+_{\alpha_2}}], [\frac{2\min(\nu^-_{\alpha_1}, \nu^-_{\alpha_2})\,\nu^-_{\alpha_3}}{\min(\nu^-_{\alpha_1}, \nu^-_{\alpha_2}) + \nu^-_{\alpha_3}}, \frac{2\min(\nu^+_{\alpha_1}, \nu^+_{\alpha_2})\,\nu^+_{\alpha_3}}{\min(\nu^+_{\alpha_1}, \nu^+_{\alpha_2}) + \nu^+_{\alpha_3}}] | \alpha_1 \in \tilde{h}_1, \alpha_2 \in \tilde{h}_2, \alpha_3 \in \tilde{h}_3\}$

$=\{[\max(\frac{2\mu^-_{\alpha_1}\mu^-_{\alpha_3}}{\mu^-_{\alpha_1}+\mu^-_{\alpha_3}}, \frac{2\mu^-_{\alpha_2}\mu^-_{\alpha_3}}{\mu^-_{\alpha_2}+\mu^-_{\alpha_3}}), \max(\frac{2\mu^+_{\alpha_1}\mu^+_{\alpha_3}}{\mu^+_{\alpha_1}+\mu^+_{\alpha_3}}, \frac{2\mu^+_{\alpha_2}\mu^+_{\alpha_3}}{\mu^+_{\alpha_2}+\mu^+_{\alpha_3}})], [\min(\frac{2\nu^-_{\alpha_1}\nu^-_{\alpha_3}}{\nu^-_{\alpha_1}+\nu^-_{\alpha_3}}, \frac{2\nu^-_{\alpha_2}\nu^-_{\alpha_3}}{\nu^-_{\alpha_2}+\nu^-_{\alpha_3}}), \min(\frac{2\nu^+_{\alpha_1}\nu^+_{\alpha_3}}{\nu^+_{\alpha_1}+\nu^+_{\alpha_3}}, \frac{2\nu^+_{\alpha_2}\nu^+_{\alpha_3}}{\nu^+_{\alpha_2}+\nu^+_{\alpha_3}})] | \alpha_1 \in \tilde{h}_1, \alpha_2 \in \tilde{h}_2, \alpha_3 \in \tilde{h}_3\}$

$=(\tilde{h}_1 \# \tilde{h}_3) \cup (\tilde{h}_2 \# \tilde{h}_3)$

This proves (v)

(vii) From definitions 2.3 and 3.1, we have

$(\tilde{h}_1 \cup \tilde{h}_2) * \tilde{h}_3 = (\tilde{h}_1 * \tilde{h}_3) \cup (\tilde{h}_2 * \tilde{h}_3)$

$=\{[\max(\mu^-_{\alpha_1}, \mu^-_{\alpha_2}), \max(\mu^+_{\alpha_1}, \mu^+_{\alpha_2})], [\min(\nu^-_{\alpha_1}, \nu^-_{\alpha_2}), \min(\nu^+_{\alpha_1}, \nu^+_{\alpha_2})] | \alpha_1 \in \tilde{h}_1, \alpha_2 \in \tilde{h}_2\} * \{[\mu^-_{\alpha_3}, \mu^+_{\alpha_3}], [\nu^-_{\alpha_3}, \nu^+_{\alpha_3}] | \alpha_3 \in \tilde{h}_3\}$

$= \{[\frac{\max(\mu^-_{\alpha_1}, \mu^-_{\alpha_2}) + \mu^-_{\alpha_3}}{2(\max(\mu^-_{\alpha_1}, \mu^-_{\alpha_2})\,\mu^-_{\alpha_3}+1)}, \frac{\max(\mu^+_{\alpha_1}, \mu^+_{\alpha_2}) + \mu^+_{\alpha_3}}{\max(\mu^+_{\alpha_1}, \mu^+_{\alpha_2})\,\mu^+_{\alpha_2}+1}], [\frac{\min(\nu^-_{\alpha_1}, \nu^-_{\alpha_2}) + \nu^-_{\alpha_3}}{2(\min(\nu^-_{\alpha_1}, \nu^-_{\alpha_2})\,\nu^-_{\alpha_3}+1)}, \frac{\min(\nu^+_{\alpha_1}, \nu^+_{\alpha_2}) + \nu^+_{\alpha_3}}{2(\min(\nu^+_{\alpha_1}, \nu^+_{\alpha_2})\,\nu^+_{\alpha_3}+1)}] | \alpha_1 \in \tilde{h}_1, \alpha_2 \in \tilde{h}_2, \alpha_3 \in \tilde{h}_3\}$

$= \{[\max(\frac{\mu^-_{\alpha_1}+\mu^-_{\alpha_3}}{2(\mu^-_{\alpha_1}\mu^-_{\alpha_3}+1)}, \frac{\mu^-_{\alpha_2}+\mu^-_{\alpha_3}}{2(\mu^-_{\alpha_2}\mu^-_{\alpha_3}+1)}), \max(\frac{\mu^+_{\alpha_1}+\mu^+_{\alpha_3}}{2(\mu^+_{\alpha_1}\mu^+_{\alpha_3}+1)}, \frac{\mu^+_{\alpha_2}+\mu^+_{\alpha_3}}{2(\mu^+_{\alpha_2}\mu^+_{\alpha_3}+1)})], [\min(\frac{\nu^-_{\alpha_1}+\nu^-_{\alpha_3}}{2(\nu^-_{\alpha_1}\nu^-_{\alpha_3}+1)}, \frac{\nu^-_{\alpha_2}+\nu^-_{\alpha_3}}{2(\nu^-_{\alpha_2}\nu^-_{\alpha_3}+1)}), \min(\frac{\nu^+_{\alpha_1}+\nu^+_{\alpha_3}}{2(\nu^+_{\alpha_1}\nu^+_{\alpha_3}+1)}, \frac{\nu^+_{\alpha_2}+\nu^+_{\alpha_3}}{2(\nu^+_{\alpha_2}\nu^+_{\alpha_3}+1)})] | \alpha_1 \in \tilde{h}_1, \alpha_2 \in \tilde{h}_2, \alpha_3 \in \tilde{h}_3\}$

$= (\tilde{h}_1 * \tilde{h}_3) \cup (\tilde{h}_2 * \tilde{h}_3)$

This proves (vii).

(ix) Using definitions 2.3 and 3.1, we have

$(\tilde{h}_1 @ \tilde{h}_2) \oplus \tilde{h}_3 = (\tilde{h}_1 \oplus \tilde{h}_3) @ (\tilde{h}_2 \oplus \tilde{h}_3)$;

$= \{[\frac{\mu^-_{\alpha_1}+\mu^-_{\alpha_2}}{2}, \frac{\mu^+_{\alpha_1}+\mu^+_{\alpha_2}}{2}], [\frac{\upsilon^-_{\alpha_1}+\upsilon^-_{\alpha_2}}{2}, \frac{\upsilon^+_{\alpha_1}+\upsilon^+_{\alpha_2}}{2}] | \alpha_1 \in \tilde{h}_1, \alpha_2 \in \tilde{h}_2\} \oplus \{[\mu^-_{\alpha_3}, \mu^+_{\alpha_3}], [\nu^-_{\alpha_3}, \nu^+_{\alpha_3}] | \alpha_3 \in \tilde{h}_3\}$

$=\{[\frac{\mu^-_{\alpha_1}+\mu^-_{\alpha_2}}{2} + \mu^-_{\alpha_3} - \frac{\mu^-_{\alpha_1}+\mu^-_{\alpha_2}}{2}\mu^-_{\alpha_3}, \frac{\mu^+_{\alpha_1}+\mu^+_{\alpha_2}}{2} + \mu^+_{\alpha_3} - \frac{\mu^+_{\alpha_1}+\mu^+_{\alpha_2}}{2}\mu^+_{\alpha_3}], [\frac{\upsilon^-_{\alpha_1}+\upsilon^-_{\alpha_2}}{2} + \nu^-_{\alpha_3} - \frac{\upsilon^-_{\alpha_1}+\upsilon^-_{\alpha_2}}{2}\nu^-_{\alpha_3}, \frac{\upsilon^+_{\alpha_1}+\upsilon^+_{\alpha_2}}{2} + \nu^+_{\alpha_3} - \frac{\upsilon^+_{\alpha_1}+\upsilon^+_{\alpha_2}}{2}\nu^+_{\alpha_3}] | \alpha_1 \in \tilde{h}_1, \alpha_2 \in \tilde{h}_2, \alpha_3 \in \tilde{h}_3\}$

$=\{[\frac{(\mu^-_{\alpha_1}+\mu^-_{\alpha_3}-\mu^-_{\alpha_1}\mu^-_{\alpha_3})+(\mu^-_{\alpha_2}+\mu^-_{\alpha_3}-\mu^-_{\alpha_2}\mu^-_{\alpha_3})}{2}, \frac{(\mu^+_{\alpha_1}+\mu^+_{\alpha_3}-\mu^+_{\alpha_1}\mu^+_{\alpha_3})+(\mu^+_{\alpha_2}+\mu^+_{\alpha_3}-\mu^+_{\alpha_2}\mu^+_{\alpha_3})}{2}], [\frac{(\upsilon^-_{\alpha_1}+\upsilon^-_{\alpha_3}-\upsilon^-_{\alpha_1}\upsilon^-_{\alpha_3})+(\upsilon^-_{\alpha_2}+\upsilon^-_{\alpha_3}-\upsilon^-_{\alpha_2}\upsilon^-_{\alpha_3})}{2}, \frac{(\upsilon^+_{\alpha_1}+\upsilon^+_{\alpha_3}-\upsilon^+_{\alpha_1}\upsilon^+_{\alpha_3})+(\upsilon^+_{\alpha_2}+\upsilon^+_{\alpha_3}-\upsilon^+_{\alpha_2}\upsilon^+_{\alpha_3})}{2}] | \alpha_1 \in \tilde{h}_1, \alpha_2 \in \tilde{h}_2, \alpha_3 \in \tilde{h}_3\}$

$= (\tilde{h}_1 \oplus \tilde{h}_3) @ (\tilde{h}_2 \oplus \tilde{h}_3)$

This proves (ix)

Theorem 3.7

For $\tilde{h}_1$ and $\tilde{h}_2 \in$ IVIHFS (X), we have the following identities:

(i) $(\tilde{h}_1 \oplus \tilde{h}_2) \cap (\tilde{h}_1 \otimes \tilde{h}_2) = \tilde{h}_1 \otimes \tilde{h}_2$;

(ii) $(\tilde{h}_1 \oplus \tilde{h}_2) \cup (\tilde{h}_1 \otimes \tilde{h}_2) = \tilde{h}_1 \oplus \tilde{h}_2$;
(iii) $(\tilde{h}_1 \oplus \tilde{h}_2) \cap (\tilde{h}_1 @ \tilde{h}_2) = \tilde{h}_1 @ \tilde{h}_2$;
(iv) $(\tilde{h}_1 \oplus \tilde{h}_2) \cup (\tilde{h}_1 @ \tilde{h}_2) = \tilde{h}_1 \oplus \tilde{h}_2$;
(v) $(\tilde{h}_1 \otimes \tilde{h}_2) \cap (\tilde{h}_1 @ \tilde{h}_2) = \tilde{h}_1 \otimes \tilde{h}_2$;
(vi) $(\tilde{h}_1 \otimes \tilde{h}_2) \cup (\tilde{h}_1 @ \tilde{h}_2) = \tilde{h}_1 @ \tilde{h}_2$;
(vii) $(\tilde{h}_1 \oplus \tilde{h}_2) \cap (\tilde{h}_1 \$ \tilde{h}_2) = \tilde{h}_1 \$ \tilde{h}_2$;
(viii) $(\tilde{h}_1 \oplus \tilde{h}_2) \cup (\tilde{h}_1 \$ \tilde{h}_2) = \tilde{h}_1 \oplus \tilde{h}_2$;
(ix) $(\tilde{h}_1 \otimes \tilde{h}_2) \cap (\tilde{h}_1 \$ \tilde{h}_2) = \tilde{h}_1 \otimes \tilde{h}_2$;
(x) $(\tilde{h}_1 \otimes \tilde{h}_2) \cup (\tilde{h}_1 \$ \tilde{h}_2) = \tilde{h}_1 \$ \tilde{h}_2$;
(xi) $(\tilde{h}_1 \oplus \tilde{h}_2) \cap (\tilde{h}_1 \# \tilde{h}_2) = \tilde{h}_1 \# \tilde{h}_2$;
(xii) $(\tilde{h}_1 \oplus \tilde{h}_2) \cup (\tilde{h}_1 \# \tilde{h}_2) = \tilde{h}_1 \oplus \tilde{h}_2$;
(xiii) $(\tilde{h}_1 \otimes \tilde{h}_2) \cap (\tilde{h}_1 \# \tilde{h}_2) = \tilde{h}_1 \otimes \tilde{h}_2$;
(xiv) $(\tilde{h}_1 \otimes \tilde{h}_2) \cup (\tilde{h}_1 \# \tilde{h}_2) = \tilde{h}_1 \# \tilde{h}_2$

Proof. We prove (i), (iii), (v), (vii), (ix), (xi) and (xii), other results can be proved analogously.
From definitions 2.3, 2.5 and 3.1, we have

$(\tilde{h}_1 \oplus \tilde{h}_2) \cap (\tilde{h}_1 \otimes \tilde{h}_2)$

$$=\left([\mu^-_{\alpha_2} + \mu^-_{\alpha_1} - \mu^-_{\alpha_2}\mu^-_{\alpha_1}, \mu^+_{\alpha_2} + \mu^+_{\alpha_1} - \mu^+_{\alpha_2}\mu^+_{\alpha_1}], [\nu^-_{\alpha_2}\nu^-_{\alpha_1}, \nu^+_{\alpha_2}\nu^+_{\alpha_1}]\right) \cap \{[\mu^-_{\alpha_1}\mu^-_{\alpha_2}, \mu^+_{\alpha_1}\mu^+_{\alpha_2}], [\nu^-_{\alpha_1} + \nu^-_{\alpha_2} - \nu^-_{\alpha_1}\nu^-_{\alpha_2}, \nu^+_{\alpha_1} + \nu^+_{\alpha_2} - \nu^+_{\alpha_1}\nu^+_{\alpha_2}] | \alpha_1 \in \tilde{h}_1, \alpha_2 \in \tilde{h}_2\}$$

$=\{[\min(\mu^-_{\alpha_2} + \mu^-_{\alpha_1} - \mu^-_{\alpha_2}\mu^-_{\alpha_1}, \mu^-_{\alpha_1}\mu^-_{\alpha_2}), \min(\mu^+_{\alpha_2} + \mu^+_{\alpha_1} - \mu^+_{\alpha_2}\mu^+_{\alpha_1}, \mu^+_{\alpha_1}\mu^+_{\alpha_2})],$

$[\max(\nu^-_{\alpha_2}\nu^-_{\alpha_1}, \nu^-_{\alpha_1} + \nu^-_{\alpha_2} - \nu^-_{\alpha_1}\nu^-_{\alpha_2}), \max(\nu^+_{\alpha_2}\nu^+_{\alpha_1}, \nu^+_{\alpha_1} + \nu^+_{\alpha_2} - \nu^+_{\alpha_1}\nu^+_{\alpha_2})] | \alpha_1 \in \tilde{h}_1, \alpha_2 \in \tilde{h}_2\}$

$=[\mu^-_{\alpha_1}\mu^-_{\alpha_2}, \mu^+_{\alpha_1}\mu^+_{\alpha_2})], [\nu^-_{\alpha_1} + \nu^-_{\alpha_2} - \nu^-_{\alpha_1}\nu^-_{\alpha_2}, \nu^+_{\alpha_1} + \nu^+_{\alpha_2} - \nu^+_{\alpha_1}\nu^+_{\alpha_2}] | \alpha_1 \in \tilde{h}_1, \alpha_2 \in \tilde{h}_2\}$

$=\tilde{h}_1 \otimes \tilde{h}_2$

This proves (i)
iii) Using definitions 2.3, 2.5 and 3.1, we have

$(\tilde{h}_1 \oplus \tilde{h}_2) \cap (\tilde{h}_1 @ \tilde{h}_2) = \tilde{h}_1 @ \tilde{h}_2$;

$$=\{\left([\mu^-_{\alpha_2} + \mu^-_{\alpha_1} - \mu^-_{\alpha_2}\mu^-_{\alpha_1}, \mu^+_{\alpha_2} + \mu^+_{\alpha_1} - \mu^+_{\alpha_2}\mu^+_{\alpha_1}], [\nu^-_{\alpha_2}\nu^-_{\alpha_1}, \nu^+_{\alpha_2}\nu^+_{\alpha_1}]\right) | \alpha_1 \in \tilde{h}_1, \alpha_2 \in \tilde{h}_2\} \cap \{[\frac{\mu^-_{\alpha_1} + \mu^-_{\alpha_2}}{2}, \frac{\mu^+_{\alpha_1} + \mu^+_{\alpha_2}}{2}], [\frac{\nu^-_{\alpha_1} + \nu^-_{\alpha_2}}{2}, \frac{\nu^+_{\alpha_1} + \nu^+_{\alpha_2}}{2}] | \alpha_1 \in \tilde{h}_1, \alpha_2 \in \tilde{h}_2\}$$

$= \{[\min(\mu^-_{\alpha_2} + \mu^-_{\alpha_1} - \mu^-_{\alpha_2}\mu^-_{\alpha_1}, \frac{\mu^-_{\alpha_1} + \mu^-_{\alpha_2}}{2}), \min(\mu^-_{\alpha_1}, \mu^+_{\alpha_2} + \mu^+_{\alpha_1} - \mu^+_{\alpha_2}\mu^+_{\alpha_1}, \frac{\mu^+_{\alpha_1} + \mu^+_{\alpha_2}}{2})],$

$[\max(\nu^-_{\alpha_2}\nu^-_{\alpha_1}, \frac{\nu^-_{\alpha_1} + \nu^-_{\alpha_2}}{2}), \max(\nu^+_{\alpha_2}\nu^+_{\alpha_1}, \frac{\nu^+_{\alpha_1} + \nu^+_{\alpha_2}}{2})] | \alpha_1 \in \tilde{h}_1, \alpha_2 \in \tilde{h}_2\}$

$=\{[\frac{\mu^-_{\alpha_1} + \mu^-_{\alpha_2}}{2}, \frac{\mu^+_{\alpha_1} + \mu^+_{\alpha_2}}{2}], [\frac{\nu^-_{\alpha_1} + \nu^-_{\alpha_2}}{2}, \frac{\nu^+_{\alpha_1} + \nu^+_{\alpha_2}}{2}] | \alpha_1 \in \tilde{h}_1, \alpha_2 \in \tilde{h}_2\} = \tilde{h}_1 @ \tilde{h}_2$

This proves (iii).
(v) From definitions 2.3, 2.5 and 3.1, we have

$(\tilde{h}_1 \otimes \tilde{h}_2) \cap (\tilde{h}_1 @ \tilde{h}_2) = \tilde{h}_1 \otimes \tilde{h}_2$;

$$=\{[\mu^-_{\alpha_1}\mu^-_{\alpha_2}, \mu^+_{\alpha_1}\mu^+_{\alpha_2}], [\nu^-_{\alpha_1} + \nu^-_{\alpha_2} - \nu^-_{\alpha_1}\nu^-_{\alpha_2}, \nu^+_{\alpha_1} + \nu^+_{\alpha_2} - \nu^+_{\alpha_1}\nu^+_{\alpha_2}] | \alpha_1 \in \tilde{h}_1, \alpha_2 \in \tilde{h}_2\} \cap \{[\frac{\mu^-_{\alpha_1} + \mu^-_{\alpha_2}}{2}, \frac{\mu^+_{\alpha_1} + \mu^+_{\alpha_2}}{2}], [\frac{\nu^-_{\alpha_1} + \nu^-_{\alpha_2}}{2}, \frac{\nu^+_{\alpha_1} + \nu^+_{\alpha_2}}{2}] | \alpha_1 \in \tilde{h}_1, \alpha_2 \in \tilde{h}_2\}$$

$=\{[\min(\mu^-_{\alpha_1}\mu^-_{\alpha_2}, \frac{\mu^-_{\alpha_1} + \mu^-_{\alpha_2}}{2}), \min(\mu^+_{\alpha_1}\mu^+_{\alpha_2}, \frac{\mu^+_{\alpha_1} + \mu^+_{\alpha_2}}{2})],$

$[\max(\nu^-_{\alpha_1} + \nu^-_{\alpha_2} - \nu^-_{\alpha_1}\nu^-_{\alpha_2}, \frac{\nu^-_{\alpha_1} + \nu^-_{\alpha_2}}{2}), \max(\nu^+_{\alpha_1} + \nu^+_{\alpha_2} - \nu^+_{\alpha_1}\nu^+_{\alpha_2}, \frac{\nu^+_{\alpha_1} + \nu^+_{\alpha_2}}{2})] | \alpha_1 \in \tilde{h}_1, \alpha_2 \in \tilde{h}_2\}$

$=\{[\mu^-_{\alpha_1}\mu^-_{\alpha_2}, \mu^+_{\alpha_1}\mu^+_{\alpha_2}], [\nu^-_{\alpha_1} + \nu^-_{\alpha_2} - \nu^-_{\alpha_1}\nu^-_{\alpha_2}, \nu^+_{\alpha_1} + \nu^+_{\alpha_2} - \nu^+_{\alpha_1}\nu^+_{\alpha_2}] | \alpha_1 \in \tilde{h}_1, \alpha_2 \in \tilde{h}_2\} = \tilde{h}_1 \otimes \tilde{h}_2$

This proves (v)

(vii) Using definitions 2.3, 2.5 and 3.1, we have

$(\tilde{h}_1 \oplus \tilde{h}_2) \cap (\tilde{h}_1 \$ \tilde{h}_2) = \tilde{h}_1 \$ \tilde{h}_2$

$= \left([\mu^-_{\alpha_2} + \mu^-_{\alpha_1} - \mu^-_{\alpha_2}\mu^-_{\alpha_1}, \mu^+_{\alpha_2} + \mu^+_{\alpha_1} - \mu^+_{\alpha_2}\mu^+_{\alpha_1}], [\nu^-_{\alpha_2}\nu^-_{\alpha_1}, \nu^+_{\alpha_2}\nu^+_{\alpha_1}]\right) \cap \{[\sqrt{\mu^-_{\alpha_1}\mu^-_{\alpha_2}}, \sqrt{\mu^+_{\alpha_1}\mu^+_{\alpha_2}}], [\sqrt{\upsilon^-_{\alpha_1}\upsilon^-_{\alpha_2}}, \sqrt{\upsilon^+_{\alpha_1}\upsilon^+_{\alpha_2}}] | \alpha_1 \in \tilde{h}_1, \alpha_2 \in \tilde{h}_2\}$

$= \{[\min(\mu^-_{\alpha_2} + \mu^-_{\alpha_1} - \mu^-_{\alpha_2}\mu^-_{\alpha_1}, \sqrt{\mu^-_{\alpha_1}\mu^-_{\alpha_2}}), \min(\mu^+_{\alpha_2} + \mu^+_{\alpha_1} - \mu^+_{\alpha_2}\mu^+_{\alpha_1}, \sqrt{\mu^+_{\alpha_1}\mu^+_{\alpha_2}})], [\max(\nu^-_{\alpha_2}\nu^-_{\alpha_1}, \sqrt{\upsilon^-_{\alpha_1}\upsilon^-_{\alpha_2}}), \max(\nu^+_{\alpha_2}\nu^+_{\alpha_1}, \sqrt{\upsilon^+_{\alpha_1}\upsilon^+_{\alpha_2}})] | \alpha_1 \in \tilde{h}_1, \alpha_2 \in \tilde{h}_2\}$

$= \{[\sqrt{\mu^-_{\alpha_1}\mu^-_{\alpha_2}}, \sqrt{\mu^+_{\alpha_1}\mu^+_{\alpha_2}}], [\sqrt{\upsilon^-_{\alpha_1}\upsilon^-_{\alpha_2}}, \sqrt{\upsilon^+_{\alpha_1}\upsilon^+_{\alpha_2}}] | \alpha_1 \in \tilde{h}_1, \alpha_2 \in \tilde{h}_2\} = \tilde{h}_1 \$ \tilde{h}_2$

This proves (vii)

(ix) From definitions 2.3, 2.5 and 3.1, we have

$(\tilde{h}_1 \otimes \tilde{h}_2) \cap (\tilde{h}_1 \$ \tilde{h}_2) = \tilde{h}_1 \otimes \tilde{h}_2$;

$= \{([\mu^-_{\alpha_1}\mu^-_{\alpha_2}, \mu^+_{\alpha_1}\mu^+_{\alpha_2}], [\nu^-_{\alpha_1} + \nu^-_{\alpha_2} - \nu^-_{\alpha_1}\nu^-_{\alpha_2}, \nu^+_{\alpha_1} + \nu^+_{\alpha_2} - \nu^+_{\alpha_1}\nu^+_{\alpha_2}]) | \alpha_1 \in \tilde{h}_1, \alpha_2 \in \tilde{h}_2\} \cap \{[\sqrt{\mu^-_{\alpha_1}\mu^-_{\alpha_2}}, \sqrt{\mu^+_{\alpha_1}\mu^+_{\alpha_2}}], [\sqrt{\upsilon^-_{\alpha_1}\upsilon^-_{\alpha_2}}, \sqrt{\upsilon^+_{\alpha_1}\upsilon^+_{\alpha_2}}] | \alpha_1 \in \tilde{h}_1, \alpha_2 \in \tilde{h}_2\}$

$= \{[\min(\mu^-_{\alpha_1}\mu^-_{\alpha_2}, \sqrt{\mu^-_{\alpha_1}\mu^-_{\alpha_2}}), \min(\mu^+_{\alpha_1}\mu^+_{\alpha_2}, \sqrt{\mu^+_{\alpha_1}\mu^+_{\alpha_2}})], [\max(\nu^-_{\alpha_1} + \nu^-_{\alpha_2} - \nu^-_{\alpha_1}\nu^-_{\alpha_2}, \sqrt{\upsilon^-_{\alpha_1}\upsilon^-_{\alpha_2}}), \max(\nu^+_{\alpha_1} + \nu^+_{\alpha_2} - \nu^+_{\alpha_1}\nu^+_{\alpha_2}, \sqrt{\upsilon^+_{\alpha_1}\upsilon^+_{\alpha_2}})] | \alpha_1 \in \tilde{h}_1, \alpha_2 \in \tilde{h}_2\}$

$= \{[\mu^-_{\alpha_1}\mu^-_{\alpha_2}, \mu^+_{\alpha_1}\mu^+_{\alpha_2}], [\nu^-_{\alpha_1} + \nu^-_{\alpha_2} - \nu^-_{\alpha_1}\nu^-_{\alpha_2}, \nu^+_{\alpha_1} + \nu^+_{\alpha_2} - \nu^+_{\alpha_1}\nu^+_{\alpha_2}] | \alpha_1 \in \tilde{h}_1, \alpha_2 \in \tilde{h}_2\} = \tilde{h}_1 \otimes \tilde{h}_2$

This proves (ix)

(xiii) From definitions 2.3, 2.5 and 3.1, we have

$(\tilde{h}_1 \otimes \tilde{h}_2) \cap (\tilde{h}_1 \# \tilde{h}_2) = \tilde{h}_1 \otimes \tilde{h}_2$;

$= \{[\mu^-_{\alpha_1}\mu^-_{\alpha_2}, \mu^+_{\alpha_1}\mu^+_{\alpha_2}], [\nu^-_{\alpha_1} + \nu^-_{\alpha_2} - \nu^-_{\alpha_1}\nu^-_{\alpha_2}, \nu^+_{\alpha_1} + \nu^+_{\alpha_2} - \nu^+_{\alpha_1}\nu^+_{\alpha_2}] | \alpha_1 \in \tilde{h}_1, \alpha_2 \in \tilde{h}_2\} \cap \{[\frac{2\mu^-_{\alpha_1}\mu^-_{\alpha_2}}{\mu^-_{\alpha_1} + \mu^-_{\alpha_2}}, \frac{2\mu^+_{\alpha_1}\mu^+_{\alpha_2}}{\mu^+_{\alpha_1} + \mu^+_{\alpha_2}}], [\frac{2\upsilon^-_{\alpha_1}\upsilon^-_{\alpha_2}}{\upsilon^-_{\alpha_1} + \upsilon^-_{\alpha_2}}, \frac{2\upsilon^+_{\alpha_1}\upsilon^+_{\alpha_2}}{\upsilon^+_{\alpha_1} + \upsilon^+_{\alpha_2}}] | \alpha_1 \in \tilde{h}_1, \alpha_2 \in \tilde{h}_2\}$

$= \{[\min(\mu^-_{\alpha_1}\mu^-_{\alpha_2}, \frac{2\mu^-_{\alpha_1}\mu^-_{\alpha_2}}{\mu^-_{\alpha_1} + \mu^-_{\alpha_2}}), \min(\mu^+_{\alpha_1}\mu^+_{\alpha_2}, \frac{2\mu^+_{\alpha_1}\mu^+_{\alpha_2}}{\mu^+_{\alpha_1} + \mu^+_{\alpha_2}})], [\max(\nu^-_{\alpha_1} + \nu^-_{\alpha_2} - \nu^-_{\alpha_1}\nu^-_{\alpha_2}, \frac{2\upsilon^-_{\alpha_1}\upsilon^-_{\alpha_2}}{\upsilon^-_{\alpha_1} + \upsilon^-_{\alpha_2}}), \max(\nu^+_{\alpha_1} + \nu^+_{\alpha_2} - \nu^+_{\alpha_1}\nu^+_{\alpha_2}, \frac{2\upsilon^+_{\alpha_1}\upsilon^+_{\alpha_2}}{\upsilon^+_{\alpha_1} + \upsilon^+_{\alpha_2}})] | \alpha_1 \in \tilde{h}_1, \alpha_2 \in \tilde{h}_2\}$

$= \{[\mu^-_{\alpha_1}\mu^-_{\alpha_2}, \mu^+_{\alpha_1}\mu^+_{\alpha_2}], [\nu^-_{\alpha_1} + \nu^-_{\alpha_2} - \nu^-_{\alpha_1}\nu^-_{\alpha_2}, \nu^+_{\alpha_1} + \nu^+_{\alpha_2} - \nu^+_{\alpha_1}\nu^+_{\alpha_2}] | \alpha_1 \in \tilde{h}_1, \alpha_2 \in \tilde{h}_2\} = \tilde{h}_1 \otimes \tilde{h}_2$

This proves (xiii).

Theorem 3.8

For $\tilde{h}_1$ and $\tilde{h}_2 \in$ IVIHFE(X), then following relations are valid:

(i) $(\tilde{h}_1 \# \tilde{h}_2) \$ (\tilde{h}_1 \# \tilde{h}_2) = \tilde{h}_1 \# \tilde{h}_2$;
(ii) $(\tilde{h}_1 * \tilde{h}_2) \$ (\tilde{h}_1 * \tilde{h}_2) = \tilde{h}_1 * \tilde{h}_2$;
(iii) $(\tilde{h}_1 \oplus \tilde{h}_2) \$ (\tilde{h}_1 \oplus \tilde{h}_2) = \tilde{h}_1 \oplus \tilde{h}_2$;
(iv) $(\tilde{h}_1 \otimes \tilde{h}_2) \$ (\tilde{h}_1 \otimes \tilde{h}_2) = \tilde{h}_1 \otimes \tilde{h}_2$;
(v) $(\tilde{h}_1 @ \tilde{h}_2) \$ (\tilde{h}_1 @ \tilde{h}_2) = \tilde{h}_1 @ \tilde{h}_2$;
(vi) $(\tilde{h}_1 \# \tilde{h}_2) @ (\tilde{h}_1 \# \tilde{h}_2) = \tilde{h}_1 \# \tilde{h}_2$;
(vii) $(\tilde{h}_1 * \tilde{h}_2) @ (\tilde{h}_1 * \tilde{h}_2) = \tilde{h}_1 * \tilde{h}_2$;
(viii) $(\tilde{h}_1 \oplus \tilde{h}_2) @ (\tilde{h}_1 \otimes \tilde{h}_2) = \tilde{h}_1 @ \tilde{h}_2$;
(ix) $(\tilde{h}_1 \cup \tilde{h}_2) @ (\tilde{h}_1 \cap \tilde{h}_2) = \tilde{h}_1 @ \tilde{h}_2$;
(x) $(\tilde{h}_1 \cup \tilde{h}_2) \$ (\tilde{h}_1 \cap \tilde{h}_2) = \tilde{h}_1 \$ \tilde{h}_2$;
(xi) $(\tilde{h}_1 \cup \tilde{h}_2) \# (\tilde{h}_1 \cap \tilde{h}_2) = \tilde{h}_1 \# \tilde{h}_2$;
(xii) $(\tilde{h}_1 \cup \tilde{h}_2) * (\tilde{h}_1 \cap \tilde{h}_2) = \tilde{h}_1 * \tilde{h}_2$;
(xiii) $(\tilde{h}_1 * \tilde{h}_2) @ (\tilde{h}_1 * \tilde{h}_2) = \tilde{h}_1 * \tilde{h}_2$;
(xiv) $(\tilde{h}_1 * \tilde{h}_2) \$ (\tilde{h}_1 * \tilde{h}_2) = \tilde{h}_1 * \tilde{h}_2$.

Proof ,The proofs of these results are the same as in the above proof

Theorem 3.9.

For every two $\tilde{h}_1$ and $\tilde{h}_2 \in$ IVIHFE (X), we have:

(i) $((\tilde{h}_1 \cup \tilde{h}_2) \oplus (\tilde{h}_1 \cap \tilde{h}_2)) @ ((\tilde{h}_1 \cup \tilde{h}_2) \otimes (\tilde{h}_1 \cap \tilde{h}_2)) = \tilde{h}_1 @ \tilde{h}_2$;
(ii) $((\tilde{h}_1 \cup \tilde{h}_2) \# (\tilde{h}_1 \cap \tilde{h}_2)) \$ ((\tilde{h}_1 \cup \tilde{h}_2) @ (\tilde{h}_1 \cap \tilde{h}_2)) = \tilde{h}_1 \$ \tilde{h}_2$
(iii) $((\tilde{h}_1 \oplus \tilde{h}_2) \cup (\tilde{h}_1 \otimes \tilde{h}_2)) @ ((\tilde{h}_1 \oplus \tilde{h}_2) \cap (\tilde{h}_1 \otimes \tilde{h}_2)) = \tilde{h}_1 @ \tilde{h}_2$;
(iv) $((\tilde{h}_1 \oplus \tilde{h}_2) \cup (\tilde{h}_1 @ \tilde{h}_2)) @ ((\tilde{h}_1 \otimes \tilde{h}_2) \cap (\tilde{h}_1 @ \tilde{h}_2)) = \tilde{h}_1 @ \tilde{h}_2$;
(v) $((\tilde{h}_1 \oplus \tilde{h}_2) \cup (\tilde{h}_1 \# \tilde{h}_2)) @ ((\tilde{h}_1 \otimes \tilde{h}_2) \cap (\tilde{h}_1 \# \tilde{h}_2)) = \tilde{h}_1 @ \tilde{h}_2$;
(vi) $((\tilde{h}_1 \oplus \tilde{h}_2) \cup (\tilde{h}_1 \$ \tilde{h}_2)) @ ((\tilde{h}_1 \otimes \tilde{h}_2) \cap (\tilde{h}_1 \$ \tilde{h}_2)) = \tilde{h}_1 @ \tilde{h}_2$;
(vii) $((\tilde{h}_1 \oplus \tilde{h}_2) \cup (\tilde{h}_1 @ \tilde{h}_2)) @ ((\tilde{h}_1 \oplus \tilde{h}_2) \cap (\tilde{h}_1 \# \tilde{h}_2)) = \tilde{h}_1 \$ \tilde{h}_2$.

Proof . In the following, we prove (i) and (iii), other results can be proved analogously.

(i) From definitions 2.3 and 3.1, we have

$((\tilde{h}_1 \cup \tilde{h}_2) \oplus (\tilde{h}_1 \cap \tilde{h}_2)) @ ((\tilde{h}_1 \cup \tilde{h}_2) \otimes (\tilde{h}_1 \cap \tilde{h}_2)) =$

$((\tilde{h}_1 \cup \tilde{h}_2) \oplus (\tilde{h}_1 \cap \tilde{h}_2)) =$

$\{[\max(\mu^-_{\alpha_1}, \mu^-_{\alpha_2}), \max(\mu^+_{\alpha_1}, \mu^+_{\alpha_2})], [\min(\nu^-_{\alpha_1}, \nu^-_{\alpha_2}), \min(\nu^+_{\alpha_1}, \nu^+_{\alpha_2})] | \alpha_1 \in \tilde{h}_1, \alpha_2 \in \tilde{h}_2\} \oplus \{[\min(\mu^-_{\alpha_1}, \mu^-_{\alpha_2}), \min(\mu^+_{\alpha_1}, \mu^+_{\alpha_2})], [\max(\nu^-_{\alpha_1}, \nu^-_{\alpha_2}), \max(\nu^+_{\alpha_1}, \nu^+_{\alpha_2})] | \alpha_1 \in \tilde{h}_1, \alpha_2 \in \tilde{h}_2\}$

$= \{[\max(\mu^-_{\alpha_1}, \mu^-_{\alpha_2}) + \min(\mu^-_{\alpha_1}, \mu^-_{\alpha_2}) - \max(\mu^-_{\alpha_1}, \mu^-_{\alpha_2}) \min(\mu^-_{\alpha_1}, \mu^-_{\alpha_2}), \max(\mu^+_{\alpha_1}, \mu^+_{\alpha_2}) + \min(\mu^+_{\alpha_1}, \mu^+_{\alpha_2}) - \max(\mu^+_{\alpha_1}, \mu^+_{\alpha_2}) + \min(\mu^+_{\alpha_1}, \mu^+_{\alpha_2})], [\min(\nu^-_{\alpha_1}, \nu^-_{\alpha_2}) \max(\nu^-_{\alpha_1}, \nu^-_{\alpha_2}), \min(\nu^+_{\alpha_1}, \nu^+_{\alpha_2}) \max(\nu^+_{\alpha_1}, \nu^+_{\alpha_2})]$

$(\tilde{h}_1 \cup \tilde{h}_2) \otimes (\tilde{h}_1 \cap \tilde{h}_2) =$

$\{[\max(\mu^-_{\alpha_1}, \mu^-_{\alpha_2}), \max(\mu^+_{\alpha_1}, \mu^+_{\alpha_2})], [\min(\nu^-_{\alpha_1}, \nu^-_{\alpha_2}), \min(\nu^+_{\alpha_1}, \nu^+_{\alpha_2})] | \alpha_1 \in \tilde{h}_1, \alpha_2 \in \tilde{h}_2\} \otimes \{[\min(\mu^-_{\alpha_1}, \mu^-_{\alpha_2}), \min(\mu^+_{\alpha_1}, \mu^+_{\alpha_2})], [\max(\nu^-_{\alpha_1}, \nu^-_{\alpha_2}), \max(\nu^+_{\alpha_1}, \nu^+_{\alpha_2})] | \alpha_1 \in \tilde{h}_1, \alpha_2 \in \tilde{h}_2\}$

$= \{[\max(\mu^-_{\alpha_1}, \mu^-_{\alpha_2}) \min(\mu^-_{\alpha_1}, \mu^-_{\alpha_2}), \max(\mu^+_{\alpha_1}, \mu^+_{\alpha_2}) \min(\mu^+_{\alpha_1}, \mu^+_{\alpha_2})], [\min(\nu^-_{\alpha_1}, \nu^-_{\alpha_2}) + \max(\nu^-_{\alpha_1}, \nu^-_{\alpha_2}) - \min(\nu^-_{\alpha_1}, \nu^-_{\alpha_2}) \max(\nu^-_{\alpha_1}, \nu^-_{\alpha_2}), \min(\nu^+_{\alpha_1}, \nu^+_{\alpha_2}) + \max(\nu^+_{\alpha_1}, \nu^+_{\alpha_2}) - \min(\nu^+_{\alpha_1}, \nu^+_{\alpha_2}) \max(\nu^+_{\alpha_1}, \nu^+_{\alpha_2})] | \alpha_1 \in \tilde{h}_1, \alpha_2 \in \tilde{h}_2\}$

$((\tilde{h}_1 \cup \tilde{h}_2) \oplus (\tilde{h}_1 \cap \tilde{h}_2)) @ ((\tilde{h}_1 \cup \tilde{h}_2) \otimes (\tilde{h}_1 \cap \tilde{h}_2)) =$

$$\Big\{\Big[\frac{\{\max(\mu^-_{\alpha_1}, \mu^-_{\alpha_2}) + \min(\mu^-_{\alpha_1}, \mu^-_{\alpha_2}) - \max(\mu^-_{\alpha_1}, \mu^-_{\alpha_2}) \min(\mu^-_{\alpha_1}, \mu^-_{\alpha_2})\} + \max(\mu^-_{\alpha_1}, \mu^-_{\alpha_2}) \min(\mu^-_{\alpha_1}, \mu^-_{\alpha_2})}{2},$$
$$\frac{\{\max(\mu^+_{\alpha_1}, \mu^+_{\alpha_2}) + \min(\mu^+_{\alpha_1}, \mu^+_{\alpha_2}) - \max(\mu^+_{\alpha_1}, \mu^+_{\alpha_2}) + \min(\mu^+_{\alpha_1}, \mu^+_{\alpha_2})\} + \max(\mu^+_{\alpha_1}, \mu^+_{\alpha_2}) \min(\mu^+_{\alpha_1}, \mu^+_{\alpha_2})}{2}\Big],$$
$$\Big[\frac{\min(\nu^-_{\alpha_1}, \nu^-_{\alpha_2}) \max(\nu^-_{\alpha_1}, \nu^-_{\alpha_2}) + \min(\nu^-_{\alpha_1}, \nu^-_{\alpha_2}) + \max(\nu^-_{\alpha_1}, \nu^-_{\alpha_2}) - \min(\nu^-_{\alpha_1}, \nu^-_{\alpha_2}) \max(\nu^-_{\alpha_1}, \nu^-_{\alpha_2})}{2},$$
$$\frac{\min(\nu^+_{\alpha_1}, \nu^+_{\alpha_2}) \max(\nu^+_{\alpha_1}, \nu^+_{\alpha_2}) + \min(\nu^+_{\alpha_1}, \nu^+_{\alpha_2}) + \max(\nu^+_{\alpha_1}, \nu^+_{\alpha_2}) - \min(\nu^+_{\alpha_1}, \nu^+_{\alpha_2}) \max(\nu^+_{\alpha_1}, \nu^+_{\alpha_2})}{2}\Big]$$

$$= \Big[\frac{\{\max(\mu^-_{\alpha_1}, \mu^-_{\alpha_2}) + \min(\mu^-_{\alpha_1}, \mu^-_{\alpha_2})}{2}, \frac{\{\max(\mu^+_{\alpha_1}, \mu^+_{\alpha_2}) + \min(\mu^+_{\alpha_1}, \mu^+_{\alpha_2})\}}{2}\Big],$$
$$\Big[\frac{\min(\nu^-_{\alpha_1}, \nu^-_{\alpha_2}) \max(\nu^-_{\alpha_1}, \nu^-_{\alpha_2}) + \min(\nu^-_{\alpha_1}, \nu^-_{\alpha_2}) + \max(\nu^-_{\alpha_1}, \nu^-_{\alpha_2}) - \min(\nu^-_{\alpha_1}, \nu^-_{\alpha_2}) \max(\nu^-_{\alpha_1}, \nu^-_{\alpha_2})}{2},$$
$$\frac{\min(\nu^+_{\alpha_1}, \nu^+_{\alpha_2}) \max(\nu^+_{\alpha_1}, \nu^+_{\alpha_2}) + \min(\nu^+_{\alpha_1}, \nu^+_{\alpha_2}) + \max(\nu^+_{\alpha_1}, \nu^+_{\alpha_2}) - \min(\nu^+_{\alpha_1}, \nu^+_{\alpha_2}) \max(\nu^+_{\alpha_1}, \nu^+_{\alpha_2})}{2}\Big] \Big| \alpha_1 \in \tilde{h}_1, \alpha_2 \in \tilde{h}_2\Big\}$$

$$= \Big\{\Big[\frac{\mu^-_{\alpha_1} + \mu^-_{\alpha_2}}{2}, \frac{\mu^+_{\alpha_1} + \mu^+_{\alpha_2}}{2}\Big], \Big[\frac{\min(\nu^-_{\alpha_1}, \nu^-_{\alpha_2}) + \max(\nu^-_{\alpha_1}, \nu^-_{\alpha_2})}{2}, \frac{\min(\nu^+_{\alpha_1}, \nu^+_{\alpha_2}) + \max(\nu^+_{\alpha_1}, \nu^+_{\alpha_2})}{2}\Big] \Big| \alpha_1 \in \tilde{h}_1, \alpha_2 \in \tilde{h}_2\Big\}$$

$$= \Big\{\Big[\frac{\mu^-_{\alpha_1} + \mu^-_{\alpha_2}}{2}, \frac{\mu^+_{\alpha_1} + \mu^+_{\alpha_2}}{2}\Big], \Big[\frac{\nu^-_{\alpha_1} + \nu^-_{\alpha_2}}{2}, \frac{\nu^+_{\alpha_1} + \nu^+_{\alpha_2}}{2}\Big] \Big| \alpha_1 \in \tilde{h}_1, \alpha_2 \in \tilde{h}_2\Big\}$$

$= \tilde{h}_1 @ \tilde{h}_2$

This proves (i).

(ii) From definitions 2.3 and 3.1, we have

$((\tilde{h}_1 \oplus \tilde{h}_2) \cup (\tilde{h}_1 \otimes \tilde{h}_2)) @ ((\tilde{h}_1 \oplus \tilde{h}_2) \cap (\tilde{h}_1 \otimes \tilde{h}_2)) = \tilde{h}_1 @ \tilde{h}_2$;

$(\tilde{h}_1 \oplus \tilde{h}_2) \cap (\tilde{h}_1 \otimes \tilde{h}_2) = \{[\mu^-_{\alpha_1} + \mu^-_{\alpha_2} - \mu^-_{\alpha_2}\mu^-_{\alpha_1}, \mu^+_{\alpha_1} + \mu^+_{\alpha_2} - \mu^+_{\alpha_2}\mu^+_{\alpha_1}], [\nu^-_{\alpha_2}\nu^-_{\alpha_1}, \nu^+_{\alpha_2}\nu^+_{\alpha_1}] | \alpha_1 \in \tilde{h}_1, \alpha_2 \in \tilde{h}_2\} \cup$

$\{[\mu^-_{\alpha_1}\mu^-_{\alpha_2}, \mu^+_{\alpha_1}\mu^+_{\alpha_2}], [\nu^-_{\alpha_1} + \nu^-_{\alpha_2} - \nu^-_{\alpha_1}\nu^-_{\alpha_2}, \nu^+_{\alpha_1} + \nu^+_{\alpha_2} - \nu^+_{\alpha_1}\nu^+_{\alpha_2}] | \alpha_1 \in \tilde{h}_1, \alpha_2 \in \tilde{h}_2\}$

$=\{[\min(\mu^-_{\alpha_1}+\mu^-_{\alpha_2}-\mu^-_{\alpha_2}\mu^-_{\alpha_1}, \mu^-_{\alpha_1}\mu^-_{\alpha_2}), \min(\mu^+_{\alpha_1}+\mu^+_{\alpha_2}-\mu^+_{\alpha_2}\mu^+_{\alpha_1}, \mu^+_{\alpha_1}\mu^+_{\alpha_2})],$

$[\max(\nu^-_{\alpha_2}\nu^-_{\alpha_1}, \nu^-_{\alpha_1}+\nu^-_{\alpha_2}-\nu^-_{\alpha_1}\nu^-_{\alpha_2}), \max(\nu^+_{\alpha_2}\nu^+_{\alpha_1}, \nu^+_{\alpha_1}+\nu^+_{\alpha_2}-\nu^+_{\alpha_1}\nu^+_{\alpha_2})] \| \alpha_1 \in \tilde{h}_1, \alpha_2 \in \tilde{h}_2\}$

$=\{[\mu^-_{\alpha_1}\mu^-_{\alpha_2}, \mu^+_{\alpha_1}\mu^+_{\alpha_2}],[\nu^-_{\alpha_1}+\nu^-_{\alpha_2}-\nu^-_{\alpha_1}\nu^-_{\alpha_2}, \nu^+_{\alpha_1}+\nu^+_{\alpha_2}-\nu^+_{\alpha_1}\nu^+_{\alpha_2}] | \alpha_1 \in \tilde{h}_1, \alpha_2 \in \tilde{h}_2\}$

$(\tilde{h}_1 \oplus \tilde{h}_2) \cup (\tilde{h}_1 \otimes \tilde{h}_2) = \{[\mu^-_{\alpha_1}+\mu^-_{\alpha_2}-\mu^-_{\alpha_2}\mu^-_{\alpha_1}, \mu^+_{\alpha_1}+\mu^+_{\alpha_2}-\mu^+_{\alpha_2}\mu^+_{\alpha_1}],[\nu^-_{\alpha_2}\nu^-_{\alpha_1}, \nu^+_{\alpha_2}\nu^+_{\alpha_1}] | \alpha_1 \in \tilde{h}_1, \alpha_2 \in \tilde{h}_2\} \cup$

$\{[\mu^-_{\alpha_1}\mu^-_{\alpha_2}, \mu^+_{\alpha_1}\mu^+_{\alpha_2}],[\nu^-_{\alpha_1}+\nu^-_{\alpha_2}-\nu^-_{\alpha_1}\nu^-_{\alpha_2}, \nu^+_{\alpha_1}+\nu^+_{\alpha_2}-\nu^+_{\alpha_1}\nu^+_{\alpha_2}] | \alpha_1 \in \tilde{h}_1, \alpha_2 \in \tilde{h}_2\}$

$=\{[\max(\mu^-_{\alpha_1}+\mu^-_{\alpha_2}-\mu^-_{\alpha_2}\mu^-_{\alpha_1}, \mu^-_{\alpha_1}\mu^-_{\alpha_2}), \max(\mu^+_{\alpha_1}+\mu^+_{\alpha_2}-\mu^+_{\alpha_2}\mu^+_{\alpha_1}, \mu^+_{\alpha_1}\mu^+_{\alpha_2})],$

$[\min(\nu^-_{\alpha_2}\nu^-_{\alpha_1}, \nu^-_{\alpha_1}+\nu^-_{\alpha_2}-\nu^-_{\alpha_1}\nu^-_{\alpha_2}), \min(\nu^+_{\alpha_2}\nu^+_{\alpha_1}, \nu^+_{\alpha_1}+\nu^+_{\alpha_2}-\nu^+_{\alpha_1}\nu^+_{\alpha_2})] | \alpha_1 \in \tilde{h}_1, \alpha_2 \in \tilde{h}_2\}$

$=\{[\mu^-_{\alpha_1}+\mu^-_{\alpha_2}-\mu^-_{\alpha_2}\mu^-_{\alpha_1}, \mu^+_{\alpha_1}+\mu^+_{\alpha_2}-\mu^+_{\alpha_2}\mu^+_{\alpha_1}],[\nu^-_{\alpha_2}\nu^-_{\alpha_1}, \nu^+_{\alpha_2}\nu^+_{\alpha_1}] | \alpha_1 \in \tilde{h}_1, \alpha_2 \in \tilde{h}_2\}$

$(((\tilde{h}_1 \oplus \tilde{h}_2) \cup (\tilde{h}_1 \otimes \tilde{h}_2)) @ ((\tilde{h}_1 \oplus \tilde{h}_2) \cap (\tilde{h}_1 \otimes \tilde{h}_2)) = \{[\frac{\mu^-_{\alpha_1}\mu^-_{\alpha_2}+\mu^-_{\alpha_1}+\mu^-_{\alpha_2}-\mu^-_{\alpha_2}\mu^-_{\alpha_1}}{2}, \frac{\mu^+_{\alpha_1}\mu^+_{\alpha_2}+\mu^+_{\alpha_1}+\mu^+_{\alpha_2}-\mu^+_{\alpha_2}\mu^+_{\alpha_1}}{2}],$

$[\frac{\nu^-_{\alpha_1}+\nu^-_{\alpha_2}-\nu^-_{\alpha_1}\nu^-_{\alpha_2}+\nu^-_{\alpha_2}\nu^-_{\alpha_1}}{2}, \frac{\nu^+_{\alpha_1}+\nu^+_{\alpha_2}-\nu^+_{\alpha_1}\nu^+_{\alpha_2}+\nu^+_{\alpha_2}\nu^+_{\alpha_1}}{2}] \| \alpha_1 \in \tilde{h}_1, \alpha_2 \in \tilde{h}_2\}$

$=\{[\frac{\mu^-_{\alpha_1}+\mu^-_{\alpha_2}}{2}, \frac{\mu^+_{\alpha_1}+\mu^+_{\alpha_2}}{2}],[\frac{\nu^-_{\alpha_1}+\nu^-_{\alpha_2}}{2}, \frac{\nu^+_{\alpha_1}+\nu^+_{\alpha_2}}{2}] \| \alpha_1 \in \tilde{h}_1, \alpha_2 \in \tilde{h}_2\}$

Hence, $(((\tilde{h}_1 \oplus \tilde{h}_2) \cup (\tilde{h}_1 \otimes \tilde{h}_2)) @ ((\tilde{h}_1 \oplus \tilde{h}_2) \cap (\tilde{h}_1 \otimes \tilde{h}_2)) = \tilde{h}_1 @ \tilde{h}_2$

This proves (ii).

4. Conclusion

In this paper, we have defined four new operations on interval valued intuitionistic hesitant fuzzy sets which involve different defining functions. Some related results have been proved and the characteristics of the interval valued intuitionistic hesitant fuzzy sets have been brought out..

Acknowledgements

The authors are very grateful to the anonymous referees for their insightful and constructive comments and suggestions, which have been very helpful in improving the paper.

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Published in *Mathematics and Statistics*, No. 2(2), 2014, pp. 62-71, DOI: 10.13189/ ms.2014.020202, 10 p.

New Operations on Intuitionistic Fuzzy Soft Sets Based on First Zadeh's Logical Operators

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Abstract – In this paper , we have defined First Zadeh's implication , First Zadeh's intuitionistic fuzzy conjunction and intuitionistic fuzzy disjunction of two intuitionistic fuzzy soft sets and some their basic properties are studied with proofs and examples.

Keywords – *Fuzzy sets, Intuitionistic fuzzy sets, Fuzzy soft sets, Intuitionistic fuzzy soft sets.*

1. Introduction

The concept of the intuitionistic fuzzy (IFS , for short) was introduced in 1983 by Atanassov [1] as an extension of Zadeh's fuzzy set. All operations, defined over fuzzy sets were transformed for the case the IFS case .This concept is capable of capturing the information that includes some degree of hesitation and applicable in various fields of research .For example , in decision making problems, particularly in the case of medial of medical diagnosis ,sales analysis ,new product marketing , financial services, etc. Atanassov et.al [2,3] have widely applied theory of intuitionistic sets in logic programming, Szmidt and Kacprzyk [4] in group decision making, De et al [5] in medical diagnosis etc. Therefore in various engineering application, intuitionstic fuzzy sets techniques have been more popular than fuzzy sets techniques in recent years. After defining a lot of operations over Intuitionstic fuzzy sets during last ten years [6] ,in 2011, Atanassov [7, 8] constructed two new operations based on the First Zadeh's IF-implication which are the first Zadeh's conjunction and disjounction, after that, in 2013, Atanassov[9] introduced the second type of Zadeh 's conjunction and disjunction based on the Second Zadeh's IF-implication.

Acknowledgements

The authors would like to thank the anonymous reviewer for their careful reading of this research paper and for their helpful comments.

Another important concept that addresses uncertain information is the soft set theory originated by Molodtsov [10]. This concept is free from the parameterization inadequacy syndrome of fuzzy set theory, rough set theory, probability theory. Molodtsov has successfully applied the soft set theory in many different fields such as smoothness of functions, game theory, operations research, Riemann integration, Perron integration, and probability. In recent years, soft set theory has been received much attention since its appearance. There are many papers devoted to fuzzify the concept of soft set theory which leads to a series of mathematical models such as fuzzy soft set [11,12,13,14,15], generalized fuzzy soft set [16,17], possibility fuzzy soft set [18] and so on. Thereafter, Maji and his coworker [19] introduced the notion of intuitionstic fuzzy soft set which is based on a combination of the intuitionistic fuzzy sets and soft set models and studied the properties of intuitionistic fuzzy soft set. Later, a lot of extentions of intuitionistic fuzzy soft are appeared such as generalized intuitionistic fuzzy soft set [20], possibility Intuitionistic fuzzy soft set [21] etc.

In this paper, our aim is to extend the three new operations introduced by Atanassov to the case of intuitionistic fuzzy soft and study its properties. This paper is arranged in the following manner. In Section 2, some definitions and notion about soft set, fuzzy soft set and intuitionistic fuzzy soft set and some properties of its. These definitions will help us in later section . In Section 3, we discusses the three operations of intuitionistic fuzzy soft such as first Zadeh's implication, First Zadeh's intuitionistic fuzzy conjunction and first Zadeh intuitionistic fuzzy disjunction. Section 4 concludes the paper.

2. Preliminaries

In this section, some definitions and notions about soft sets and intutionistic fuzzy soft set are given. These will be useful in later sections

Let U be an initial universe, and E be the set of all possible parameters under consideration with respect to U. The set of all subsets of U, i.e. the power set of U is denoted by P(U) and the set of all intuitionistic fuzzy subsets of U is denoted by IFU . Let A be a subset of E.

Definition 2.1 .A pair (F , A) is called a soft set over U , where F is a mapping given by F : A P (U).

In other words, a soft set over U is a parameterized family of subsets of the universe U . For e ∈ A, F (e) may be considered as the set of e-approximate elements of the soft set (F , A).

Definition 2.2. Let U be an initial universe set and E be the set of parameters. Let IF^U denote the collection of all intuitionistic fuzzy subsets of U. Let . A ⊆ E pair (F, A) is called an intuitionistic fuzzy soft set over U where F is a mapping given by F: A→ IF^U .

Definition 2.3. Let F: A→ IF^U then F is a function defined as

$$F\,(\varepsilon) = \{\, x,\ \mu_{F(\varepsilon)}(x),\ \nu_{F(\varepsilon)}(x) : x\ \varepsilon\ U \,\}$$

where μ , ν denote the degree of membership and degree of non-membership respectively.

Definition 2.4 . For two intuitionistic fuzzy soft sets (F , A) and (G, B) over a common universe U , we say that (F , A) is an intuitionistic fuzzy soft subset of (G, B) if

(1) A ⊆ B and
(2) F (ε) ⊆G(ε) for all $\varepsilon \in$ A. i.e $\mu_{F(\varepsilon)}(x) \leq \mu_{G(\varepsilon)}(x)$, $\nu_{F(\varepsilon)}(x) \geq \nu_{G(\varepsilon)}(x)$ for all $\varepsilon \in$ E and

We write (F,A) ⊆ (G, B).

In this case (G, B) is said to be a soft super set of (F , A).

Definition 2.5. Two soft sets (F , A) and (G, B) over a common universe U are said to be soft equal if (F , A) is a soft subset of (G, B) and (G, B) is a soft subset of (F , A).

Definition 2.6. Let U be an initial universe, E be the set of parameters, and A ⊆ E .

(a) (F , A) is called a relative null soft set (with respect to the parameter set A), denoted by $\emptyset_A$, if F (a) = ∅ for all a ∈ A.

(b) (G, A) is called a relative whole soft set (with respect to the parameter set A), denoted by U_A ,if G(e) = U for all e ∈ A.

Definition 2.7. Let (F, A) and (G, B) be two IFSSs over the same universe U. Then the union of (F,A) and (G,B) is denoted by '(F,A)∪(G,B)' and is defined by (F,A) ∪ (G,B)=(H,C), where C=A∪B and the truth-membership, falsity-membership of (H,C) are as follows:

$$H(\varepsilon) = \begin{cases} \{(\mu_{F(\varepsilon)}(x), \nu_{F(\varepsilon)}(x) : x \ U\} & \text{,if } \varepsilon \in \mathrm{A} - \mathrm{B}, \\ \{(\mu_{G(\varepsilon)}(x), \nu_{G(\varepsilon)}(x) : x \ U\} & \text{,if } \varepsilon \in \mathrm{B} - \mathrm{A} \\ \{\max(\mu_{F(\varepsilon)}(x), \mu_{G(\varepsilon)}(x)), \min(\nu_{F(\varepsilon)}(x), \nu_{G(\varepsilon)}(x)) : x \ U\} & \text{if } \varepsilon \in \mathrm{A} \cap \mathrm{B} \end{cases}$$

Where $\mu_{H(\varepsilon)}(x) = \max(\mu_{F(\varepsilon)}(x), \mu_{G(\varepsilon)}(x))$ and $\nu_{H(\varepsilon)}(x) = \min(\nu_{F(\varepsilon)}(x), \nu_{G(\varepsilon)}(x))$

Definition 2.8. Let (F, A) and (G, B) be two IFSS over the same universe U such that A ∩ B ≠ 0. Then the intersection of (F, A) and (G, B) is denoted by '(F, A) ∩ (G, B)' and is defined by (F, A) ∩(G, B) = (K, C),where C =A ∩B and the truth-membership, falsity-membership of (K, C) are related to those of (F, A) and (G, B) by:

$$K(\varepsilon) = \begin{cases} \{(\mu_{F(\varepsilon)}(x), \nu_{F(\varepsilon)}(x) : x \ U\} & \text{,if } \varepsilon \in \mathrm{A} - \mathrm{B}, \\ \{(\mu_{G(\varepsilon)}(x), \nu_{G(\varepsilon)}(x) : x \ U\} & \text{,if } \varepsilon \in \mathrm{B} - \mathrm{A} \\ \{\min(\mu_{F(\varepsilon)}(x), \mu_{G(\varepsilon)}(x)), \max(\nu_{F(\varepsilon)}(x), \nu_{G(\varepsilon)}(x)) : x \ U\} & \text{if } \varepsilon \in \mathrm{A} \cap \mathrm{B} \end{cases}$$

Where $\mu_{K(\varepsilon)}(x) = \min(\mu_{F(\varepsilon)}(x), \mu_{G(\varepsilon)}(x))$ and $\nu_{K(\varepsilon)}(x) = \max(\nu_{F(\varepsilon)}(x), \nu_{G(\varepsilon)}(x))$

3. New Operations on Intuitionstic Fuzzy Soft Sets Based on First Zadeh's Logical Operators

3.1 First Zadeh's Implication of Intuitionistic Fuzzy Soft Sets

Definition 3.1.1. Let (F, A) and (G, B) are two intuitionistic fuzzy soft set s over (U,E) .We define the First Zadeh's intuitionistic fuzzy soft set implication (F, A) $\underset{z,1}{\rightarrow}$(G,B) is defined by

(F, A) $\underset{z,1}{\rightarrow}$ (G,B) = [max $\{\nu_{F(\varepsilon)}(x)$, min $(\mu_{F(\varepsilon)}(x) , \mu_{G(\varepsilon)}(x))\}$, min $(\mu_{F(\varepsilon)}(x) , \nu_{G(\varepsilon)}(x))$

Proposition 3.1.2. Let (F, A) ,(G, B) and (H, C) are three intuitionistic fuzzy soft set s over (U,E). Then the following results hold

(i) (F, A) ∩ (G,B) $\underset{z,1}{\rightarrow}$ (H, C) ⊇ [(F , A) $\underset{z,1}{\rightarrow}$ (H, C)] ∩ [(G , B) $\underset{z,1}{\rightarrow}$ (H, C)]

(ii) (F, A) ∪ (G,B) $\underset{z,1}{\rightarrow}$ (H, C) ⊇ [(F , A) $\underset{z,1}{\rightarrow}$ (H, C)] ∪ [(G , B) $\underset{z,1}{\rightarrow}$ (H, C)]

(iii) (F, A) ∩(G,B) $\underset{z,1}{\rightarrow}$ (H, C) ⊇ [(F , A) $\underset{z,1}{\rightarrow}$ (H, C)] ∪ [(G , B) $\underset{z,1}{\rightarrow}$ (H, C)]

(iv) (F, A) $\underset{z,1}{\rightarrow}$ $(F, A)^{c}$ = $(F, A)^{c}$

(v) (F, A) $\underset{z,1}{\rightarrow}$ (φ, A) =$(F, A)^{c}$

Proof.

(i) (F, A) ∩ (G,B) $\underset{z,1}{\rightarrow}$ (H, C)

$$= \left\{ min\left(\mu_{F(\varepsilon)}(x), \mu_{G(\varepsilon)}(x)\right), \max\left(\nu_{F(\varepsilon)}(x), \nu_{G(\varepsilon)}(x)\right) \right\} \underset{z,1}{\rightarrow} (\mu_{H(\varepsilon)}(x), \nu_{H(\varepsilon)}(x))$$

$$= \begin{bmatrix} \text{MAX}\left\{ \max\left(\nu_{F(\varepsilon)}(x), \nu_{G(\varepsilon)}(x)\right), \min\left(min\left(\mu_{F(\varepsilon)}(x), \mu_{G(\varepsilon)}(x)\right), \mu_{H(\varepsilon)}(x)\right)\right\}, \\ \text{MIN}\left\{ min\left(\mu_{F(\varepsilon)}(x), \mu_{G(\varepsilon)}(x)\right), \nu_{H(\varepsilon)}(x)\right\} \end{bmatrix} \tag{1}$$

[(F , A) $\underset{z,1}{\rightarrow}$ (H, C)] ∩ [(G , B) $\underset{z,1}{\rightarrow}$ (H, C)]

= [max $\{\nu_{F(\varepsilon)}$, min $(\mu_{F(\varepsilon)} , \mu_{H(\varepsilon)})\}$, min $(\mu_{F(\varepsilon)} , \nu_{H(\varepsilon)})$] ∩

[max $\{\nu_{G(\varepsilon)}$, min $(\mu_{G(\varepsilon)} , \mu_{H(\varepsilon)})\}$, min $(\mu_{G(\varepsilon)} , \nu_{H(\varepsilon)})$]

$$= \begin{bmatrix} \text{MIN}\left\{\max\left(\nu_{F(\varepsilon)}(x), min\left(\mu_{F(\varepsilon)}(x), \mu_{H(\varepsilon)}(x)\right)\right), \max\left(\nu_{G(\varepsilon)}(x), min\left(\mu_{G(\varepsilon)}(x), \mu_{H(\varepsilon)}(x)\right)\right)\right\}, \\ \text{MAX}\left\{min\left(\mu_{F(\varepsilon)}(x), \nu_{H(\varepsilon)}(x)\right), min\left(\mu_{G(\varepsilon)}(x), \nu_{H(\varepsilon)}(x)\right)\right\} \end{bmatrix} \tag{2}$$

From (1) and (2) it is clear that (F, A) ∩ (G,B) $\underset{z,1}{\rightarrow}$ (H, C) ⊇ [(F , A) $\underset{z,1}{\rightarrow}$ (H, C)] ∩ [(G , B) $\underset{z,1}{\rightarrow}$ (H, C)]

(ii) And (iii) the proof is similar to (i)

(iv) (F, A) $\underset{z,1}{\rightarrow}$ $(F, A)^{c}$ = $(F, A)^{c}$

$$= \begin{bmatrix} \text{Max}\left\{\nu_{F(\varepsilon)}(x), min\left(\mu_{F(\varepsilon)}(x), \nu_{F(\varepsilon)}(x)\right)\right\}, \\ \text{MIN}\left\{\mu_{F(\varepsilon)}(x), \mu_{F(\varepsilon)}(x)\right\} \end{bmatrix}$$

= $(\nu_{F(\varepsilon)}(x), \mu_{F(\varepsilon)}(x))$

It is shown that the first Zadeh's intuitionistic fuzzy soft implication generate the complement of intuitionistic fuzzy soft set.

(v) The proof is straightforward .

Example 3.1.3.
(F ,A) = {F(e_1) = (a , 0.3 , 0.2)}
(G ,B) = {G(e_1) = (a , 0.4 , 0.5)}
(H ,C) = {H(e_1) = (a , 0.3 , 0.6)}

(F, A) ∩ (G,B) $\underset{z,1}{\rightarrow}$ (H, C) =
[max { (max (0.2, min (0.3,0.4)) , 0.3 } , min { min (0.3,0.5), 0.6))} = (0.5, 0.3)
(F, A) ∩ (G,B) ={(a ,0.3 ,0.5)}

3.2. First Zadeh's Intuitionistic Fuzzy Conjunction of Intuitionistic Fuzzy Soft Set

Definition 3.2.1. Let (F, A) and (G, B) are two intuitionistic fuzzy soft sets over (U,E) .We define the first Zadeh's intuitionistic fuzzy conjunction of (F, A) and (G,B) as the intuitionistic fuzzy soft set (H,C) over (U,E), written as (F, A) $\tilde{\wedge}_{z,1}$ (G,B) =(H ,C). Where C = A ∩ B ≠ ∅ and ∀ ε ∈ C, x ∈ U,

$\mu_{H(\varepsilon)}(x) = MIN\left(\mu_{F(\varepsilon)}(x)\ ,\mu_{G(\varepsilon)}(x)\right)$
$\nu_{H(\varepsilon)}(x) = \ Max\left\{\nu_{F(\varepsilon)}(x)\ ,min\left(\mu_{F(\varepsilon)}(x)\ ,\nu_{G(\varepsilon)}(x)\right)\right\}$

Example 3.2. 2.
Let U={a, b, c} and E ={ e_1 , e_2 , e_3 , e_4} , A ={ e_1 , e_2, e_4} ⊆ E, B={ e_1 , e_2 , e_3} ⊆ E

(F, A) ={ F(e_1) ={((a, 0.5, 0.1), (b, 0.1, 0.8), (c, 0.2, 0.5)},
F(e_2) ={((a, 0.7, 0.1), (b, 0, 0.8), (c, 0.3, 0.5)},
F(e_4) ={((a, 0.6, 0.3), (b, 0.1, 0.7), (c, 0.9, 0.1)}}

(G, B) ={ G(e_1) ={((a, 0.2, 0.6), (b, 0.7, 0.1), (c, 0.8, 0.1)},
G(e_2) ={((a, 0.4, 0.1), (b, 0.5, 0.3), (c, 0.4, 0.5)},
G(e_3) ={((a, 0, 0.6), (b, 0, 0.8), (c, 0.1, 0.5)}}

Let (F, A) $\tilde{\wedge}_z$ (G,B) =(H ,C) ,where C = A ∩ B = { e_1 , e_2 }

(H, C)={H (e_1) ={(a, min(0.5, 0.2), max(0.1, min(0.5, 0.6)))
(b, min(0.1, 0.7), max(0.8, min(0.1, 0.1)))
(c, min(0.2, 0.8), max(0.5, min(0.2, 0.1)))},
H (e_2) ={(a, min(0.7, 0.4), max(0.1, min(0.7, 0.1)))
(b, min(0, 0.5), max(0.8, min(0, 0.3)))
(c, min(0.3, 0.4), max(0.5, min(0.3, 0.5)))}}

(H, C)= { H (e_1)= {(a, min(0.5, 0.2), max(0.1, 0.5)),
(b, min(0.1, 0.7), max(0.8, 0.1)),

(c, min(0.2, 0.8), max(0.5, 0.1))},
H (e_2)= {(a, min(0.7, 0.4), max(0.1, 0.1)),
(b, min(0, 0.5), max(0, 0.8)),
(c, min(0.3, 0.4), max(0.5, 0.3))}}

(H, C)= { H (e_1)= {(a, 0.2, 0.5),(b, 0.1, 0.8), (c,0.2, 0.5)},
H (e_2)= {(a, 0.4, 0.1), (b, 0, 0), (c, 0.3, 0.5)}}

Proposition 3.2. 3. Let (F, A) ,(G, B) and (H, C) are three intuitionistic fuzzy soft set s over (U,E). Then the following result hold

$$(F, A)\ \tilde{\wedge}_{z,1}\ (G,B) \underset{z,1}{\rightarrow} (H, C) \supseteq [(F , A) \underset{z,1}{\rightarrow} (H, C)]\ \tilde{\wedge}_{z,1}\ [(G , B) \underset{z,1}{\rightarrow} (H, C)]$$

Proof. Let (F, A) ,(G, B) and (H,C) are three intuitionistic fuzzy soft set ,then
(F, A) $\tilde{\wedge}_{z,1}$ (G,B) $\underset{z,1}{\rightarrow}$ (H, C) =

$$\begin{bmatrix} \text{Max}\left\{\max\left(\nu_{F(\varepsilon)}(x), min\left(\mu_{F(\varepsilon)}(x), \nu_{G(\varepsilon)}(x)\right)\right), \min\left(min\left(\mu_{F(\varepsilon)}(x), \mu_{G(\varepsilon)}(x)\right), \mu_{H(\varepsilon)}(x)\right)\right\}, \\ \text{MIN}\left\{min\left(\mu_{F(\varepsilon)}(x), \mu_{G(\varepsilon)}(x)\right), \nu_{H(\varepsilon)}(x)\right\} \end{bmatrix} \quad (1)$$

Let [(F , A) $\underset{z,1}{\rightarrow}$ (H, C)] $\tilde{\wedge}_{z,1}$ [(G , B) $\underset{z,1}{\rightarrow}$ (H, C)]

$$(F , A) \underset{z,1}{\rightarrow} (H, C) = \begin{bmatrix} \text{MAX}\left\{\nu_{F(\varepsilon)}(x), min\left(\mu_{F(\varepsilon)}(x), \mu_{H(\varepsilon)}(x)\right)\right\}, \\ \text{MIN}\left\{\mu_{F(\varepsilon)}(x), \nu_{H(\varepsilon)}(x)\right\} \end{bmatrix}$$

$$[(G , B) \underset{z,1}{\rightarrow} (H, C)] = \begin{bmatrix} \text{MAX}\left\{\nu_{G(\varepsilon)}(x), min\left(\mu_{G(\varepsilon)}(x), \mu_{H(\varepsilon)}(x)\right)\right\}, \\ \text{MIN}\left\{\mu_{G(\varepsilon)}(x), \nu_{H(\varepsilon)}(x)\right\} \end{bmatrix}$$

Then [(F , A) $\underset{z,1}{\rightarrow}$ (H, C)] $\tilde{\wedge}_{z,1}$ [(G , B) $\underset{z,1}{\rightarrow}$ (H, C)] =

$$\begin{bmatrix} \text{MIN}\left(max\left\{\nu_{F(\varepsilon)}(x), min\left(\mu_{F(\varepsilon)}(x), \nu_{H(\varepsilon)}(x)\right)\right\}, max\left\{\nu_{G(\varepsilon)}(x), min\left(\mu_{G(\varepsilon)}(x), \mu_{H(\varepsilon)}(x)\right)\right\}\right), \\ \text{MAX}\left(\min\{\mu_{F(\varepsilon)}(x), \nu_{H(\varepsilon)}(x)\}, \min\left\{max\left(\nu_{G(\varepsilon)}(x), min\left(\mu_{G(\varepsilon)}(x), \mu_{H(\varepsilon)}(x)\right)\right), min\left(\mu_{G(\varepsilon)}(x), \nu_{H(\varepsilon)}(x)\right)\right\}\right) \end{bmatrix} \quad (2)$$

From (1) and (2) it is clear that

$$(F, A)\ \tilde{\wedge}_{z,1}(G,B) \underset{z,1}{\rightarrow} (H, C) \supseteq [(F , A) \underset{z,1}{\rightarrow} (H, C)]\ \tilde{\wedge}_{z,1}\ [(G , B) \underset{z,1}{\rightarrow} (H, C)]$$

3. 3. The First Zadeh's Intuitionistic Fuzzy Disjunction of Intuitionstic Fuzzy Soft Set

Definition 3.3.1. Let (F, A) and (G, B) are two intuitionistic fuzzy soft set s over (U,E) .We define the first Zadeh's intuitionistic fuzzy disjunction of (F, A) and (G,B) as the intuitionistic fuzzy soft set (H,C) over (U,E), written as (F, A) $\tilde{\vee}_{z,1}$ (G,B) =(H ,C). Where C = A ∩ B ≠ ∅ and ∀ ε ∈ A , x ∈ U

$$\mu_{H(\varepsilon)}(x) = Max\left\{\mu_{F(\varepsilon)}(x), min\left(\nu_{F(\varepsilon)}(x), \mu_{G(\varepsilon)}(x)\right)\right\}$$

$\nu_{H(\varepsilon)}(x) = Min\big(\nu_{F(\varepsilon)}(x), \nu_{G(\varepsilon)}(x)\big)\big)$

Example 3.3.2. Let U={a, b,c} and E ={ e_1 , e_2 , e_3 , e_4} , A ={ e_1 , e_2, e_4} ⊆ E, B={ e_1 , e_2 , e_3} ⊆ E

(F, A) ={ F(e_1) ={((a, 0.5, 0.1), (b, 0.1, 0.8), (c, 0.2, 0.5)},
F(e_2) ={((a, 0.7, 0.1), (b, 0, 0.8), (c, 0.3, 0.5)},
F(e_4) ={((a, 0.6, 0.3), (b, 0.1, 0.7), (c, 0.9, 0.1)}}

(G, A) ={ G(e_1) ={((a, 0.2, 0.6), (b, 0.7, 0.1), (c, 0.8, 0.1)},
G(e_2) ={((a, 0.4, 0.1), (b, 0.5, 0.3), (c, 0.4, 0.5)},
G(e_3) ={((a, 0, 0.6), (b, 0, 0.8), (c, 0.1, 0.5)}}

Let (F, A) $\widetilde{\vee}_{z,1}$ (G,B) =(H ,C),where C = A ∩ B = { e_1 , e_2 }

(H, C)={H (e_1) ={(a, max(0.5, min(0.1, 0.2)), min(0.1, 0.6))
(b, max(0.1, min(0.8, 0.7)), min(0.8, 0.1))
(c, max(0.2, min(0.5, 0.8)), min(0.5, 0.1)) },
H (e_2) ={(a, max(0.7, min(0.1, 0.4)), min(0.1, 0.1))
(b, max(0, min(0.8, 0.5)), min(0.8, 0.3))
(c, max(0.3, min(0.5, 0.4)), min(0.5, 0.5))}}

(H, C)= { H (e_1)= {(a, max(0.5, 0.1), min(0.1, 0.6)),
(b, max(0.1, 0.7), min(0.8, 0.1)),
(c, max(0.2, 0.5), min(0.5, 0.1))},
H (e_2)= {(a, max(0.7, 0.1), min(0.1, 0.1)),
(b, max(0, 0.5), min(0.8, 0.3)),
(c, max(0.3, 0.4), min(0.5, 0.5))}}

(H, C)= { H (e_1)= {(a, 0.5, 0.1),(b, 0.7, 0.1), (c,0.5, 0.1)},
H (e_2)= {(a, 0.7, 0.1),(b, 0.5, 0.3), (c,0.4, 0.5)}}

Proposition 3.3.3.
(i) (φ ,A) $\widetilde{\wedge}_{z,1}$ (U, A) = (φ ,A)
(ii) (φ ,A) $\widetilde{\vee}_{z,1}$ (U, A) = (U, A)
(iii) (F, A) $\widetilde{\vee}_{z,1}$ (φ ,A) = (F,A)

Proof.
(i) Let (φ ,A) $\widetilde{\wedge}_{z,1}$ (U, A) =(H, A) ,where For all $\varepsilon \in$ A , x ∈ U, we have

$\mu_{H(\varepsilon)}(x)$ =min (0 ,1) = 0
$\nu_{H(\varepsilon)}(x)$= max (1 ,min (0, 0)) =max (1 , 0)= 1

Therefore (H, A)= (0 ,1) , For all $\varepsilon \in$ A , x ∈ U

It follows that ((φ ,A) $\widetilde{\wedge}_{z,1}$ (U, A) = (φ ,A)

(ii) Let (φ ,A) $\widetilde{\vee}_{z,1}$ (U, A) =(H, A) ,where For all $\varepsilon \in$ A , x ∈ U, we have

$\mu_{H(\varepsilon)}(x)$ = max (0 ,min (1, 1)) =max (0 ,1)= 1
$\nu_{H(\varepsilon)}(x)$= min (1 ,0) = 0

Therefore (H, A) = (1,0) , For all $\varepsilon \in$ A , x ∈ U

It follows that ((φ ,A) $\widetilde{\wedge}_{z,1}$ (U, A) = (U, A)

(iii) Let (F, A) $\widetilde{\vee}_{z,1}$ (φ ,A) =(H, A) ,where For all $\varepsilon \in$ A , x ∈ U, we have

$\mu_{H(\varepsilon)}(x)$ = max ($\mu_{F(\varepsilon)}(x)$,min ($\nu_{F(\varepsilon)}(x)$, 0)) = max ($\mu_{F(\varepsilon)}(x)$, 0) = $\mu_{F(\varepsilon)}(x)$
$\nu_{H(\varepsilon)}(x)$= min ($\nu_{H(\varepsilon)}(x)$,1) = $\nu_{H(\varepsilon)}(x)$

Therefore (H, A) = ($\mu_{F(\varepsilon)}(x)$, $\nu_{H(\varepsilon)}(x)$) , For all $\varepsilon \in$ A , x ∈ U

It follows that (F, A) $\widetilde{\vee}_{z,1}$ (φ ,A) =(F, A)

Proposition 3.3.4.
(F, A) $\widetilde{\vee}_{z,1}$ (G,B) $\underset{z,1}{\to}$ (H, C) ⊇ [(F , A) $\underset{z,1}{\to}$ (H, C)] $\widetilde{\vee}_{z,1}$ [(G , B) $\underset{z,1}{\to}$ (H, C)]

Proof. The proof is similar as in proposition 3.2.3

Proposition 3.3.5.
(i) [$(F$, A) $\widetilde{\wedge}_{z,1}$ (G, B)]^c =$(F$, A) ^{c} $\widetilde{\vee}_{z,1}$ $(G$, B)^c
(ii) [$(F$, A) $\widetilde{\vee}_{z,1}$ (G, B)]^c =$(F$, A) ^{c} $\widetilde{\wedge}_{z,1}$ $(G$, B)^c
(iii) [$(F$, A) ^{c} $\widetilde{\wedge}_{z,1}$ $(G$, B) ^{c}]^c = $(F$, A) $\widetilde{\vee}_{z,1}$(G, B)

Proof.
(i) Let [$(F$,A) $\widetilde{\wedge}_{z,1}$ (G, B)]^c =(H, C) ,where For all $\varepsilon \in$ C , x ∈ U, we have

$$[(F,A)\ \widetilde{\wedge}_{z,1}\ (G,B)]^c = \begin{bmatrix} \mathrm{MIN}\{\mu_{F(\varepsilon)}(x), \mu_{G(\varepsilon)}(x)\}, \\ \mathrm{MAX}\left\{\nu_{F(\varepsilon)}(x), min\left(\ \mu_{F(\varepsilon)}(x), \nu_{G(\varepsilon)}(x)\right)\right\} \end{bmatrix}^c$$

$$= \begin{bmatrix} \mathrm{MAX}\left\{\nu_{F(\varepsilon)}(x), min\left(\ \mu_{F(\varepsilon)}(x), \nu_{G(\varepsilon)}(x)\right)\right\}, \\ \mathrm{MIN}\{\mu_{F(\varepsilon)}(x), \mu_{G(\varepsilon)}(x)\} \end{bmatrix}$$

$$= (F,A)^c\ \widetilde{\vee}_{z,1}\ (G,B)^c$$

(ii) Let [$(F$,A) $\widetilde{\vee}_{z,1}$(G, B)]^c =(H, C) ,where For all $\varepsilon \in$ C , x ∈ U , we have

$$[(F,A)\ \widetilde{\vee}_{z,1}(G,B)]^c = \begin{bmatrix} \mathrm{MAX}\left\{\mu_{F(\varepsilon)}(x), min\left(\ \nu_{F(\varepsilon)}(x), \mu_{G(\varepsilon)}(x)\right)\right\}, \\ \mathrm{MIN}\{\nu_{F(\varepsilon)}(x), \nu_{G(\varepsilon)}(x)\} \end{bmatrix}$$

$$= \begin{bmatrix} \mathrm{MIN}\{\nu_{F(\varepsilon)}(x), \nu_{G(\varepsilon)}(x)\}, \\ \mathrm{MAX}\left\{\mu_{F(\varepsilon)}(x), min\left(\ \nu_{F(\varepsilon)}(x), \mu_{G(\varepsilon)}(x)\right)\right\} \end{bmatrix}^c$$

$$= (F,A)^c\ \widetilde{\wedge}_{z,1}\ (G,B)^c$$

(iii) The proof is straightforward.

The following equalities are not valid.

$(F, A)\ \tilde{\vee}_{z,1}(G, B) = (G, B)\ \tilde{\vee}_{z,1}(F, A)$
$(F, A)\ \tilde{\wedge}_{z,1}(G, B) = (G, B)\ \tilde{\wedge}_{z,1}(F, A)$
$[(F, A)\ \tilde{\wedge}_{z,1}(G, B)]\ \tilde{\wedge}_{z,1}(K, C) = (F, A)\ \tilde{\wedge}_{z,1}\ [(G, B)\ \tilde{\wedge}_{z,1}(K, C)]$
$[(F, A)\ \tilde{\vee}_{z,1}(G, B)]\ \tilde{\vee}_{z,1}(K, C) = (F, A)\ \tilde{\vee}_{z,1}\ [(G, B)\ \tilde{\vee}_{z,1}(K, C)]$
$[(F, A)\ \tilde{\wedge}_{z,1}(G, B)]\ \tilde{\vee}_{z,1}(K, C) = [(F, A)\ \tilde{\vee}_{z,1}\ (G, B)]\ \tilde{\wedge}_{z,1}\ [(G, B)\ \tilde{\vee}_{z,1}\ (K, C)]$
$[(F, A)\ \tilde{\vee}_{z,1}(G, B)]\ \tilde{\wedge}_{z,1}(K, C) = [(F, A)\ \tilde{\wedge}_{z,1}\ (G, B)]\ \tilde{\vee}_{z,1}\ [(G, B)\ \tilde{\wedge}_{z,1}\ (K, C)]$

Example 3.3.6. Let U={a, b,c} and E ={ e_1 , e_2 , e_3 , e_4} , A ={ e_1 , e_2, e_4} ⊆ E, B={ e_1 , e_2 , e_3} ⊆ E

(F, A) ={ F(e_1) ={((a, 0.5, 0.1), (b, 0.1, 0.8), (c, 0.2, 0.5)},
F(e_2) ={((a, 0.7, 0.1), (b, 0, 0.8), (c, 0.3, 0.5)},
F(e_4) ={((a, 0.6, 0.3), (b, 0.1, 0.7), (c, 0.9, 0.1)}}

(G, A) ={ G(e_1) ={((a, 0.2, 0.6), (b, 0.7, 0.1), (c, 0.8, 0.1)},
G(e_2) ={((a, 0.4, 0.1), (b, 0.5, 0.3), (c, 0.4, 0.5)},
G(e_3) ={((a, 0, 0.6), (b, 0, 0.8), (c, 0.1, 0.5)}}

Let (F, A) $\tilde{\wedge}_{z,1}$ (G,B) =(H ,C) ,where C = A ∩ B = { e_1 , e_2 }

Then (F, A) $\tilde{\wedge}_{z,1}$ (G,B) = (H, C)= { H (e_1) = {(a, 0.2, 0.5), (b, 0.1, 0.8), (c,0.2, 0.5)},
H (e_2)= {(a, 0.4, 0.1), (b, 0, 0), (c,0.3, 0.5)}}

For (G, B) $\tilde{\wedge}_{z,1}$ (F, A) = (K, C) ,where K = A ∩ B = { e_1 , e_2 }

(K, C)={K (e_1) ={(a, min (0.2, 0.5), max (0.6, min (0.2, 0.1)))
(b, min (0.7, 0.1), max (0.1, min(0.7, 0.8)))
(c, min (0.8, 0.2), max (0.1, min (0.8, 0.5)))},
K (e_2) ={(a, min (0.7 0.4), max(0.1, min (0.4, 0.1)))
(b, min (0.5, 0.), max(0.3, min (0.5, 0.8)))
(c, min (0.4, 0.3), max(0.5, min (0.4, 0.5)))}}

(K, C)= { K (e_1)= {(a, min (0.2, 0.5), max (0.6, 0.1)),
(b, min (0.7, 0.1), max (0.1, 0.7)),
(c, min (0.8, 0.2), max (0.1, 0.5))},
K (e_2)= {(a, min (0.4, 0.7), max (0.1, 0.1)),
(b, min (0.5, 0), max (0.3, 0.5)),
(c, min (0.4, 0.3), max (0.5, 0.4))}}

(K, C)= { K (e_1)= {(a, 0.2, 0.6),(b, 0.1, 0.7), (c,0.2, 0.5)},
K (e_2)= {(a, 0.4, 0.1),(b, 0, 0.5), (c,0.3, 0.5)}}

Then (G, B) $\tilde{\wedge}_{z,1}$ (F, A) = (K, C) = { K (e_1)= {(a, 0.2, 0.6),(b, 0.1, 0.7), (c,0.2, 0.5)},
K (e_2)= {(a, 0.4, 0.1),(b, 0, 0.5), (c,0.3, 0.5)}}

It is obviously that (F, A) $\tilde{\wedge}_{z,1}$ (G,B) $\neq$ (G, B) $\tilde{\wedge}_{z,1}$ (F, A)

Conclusion

In this paper, three new operations have been introduced on intuitionistic fuzzy soft sets. They are based on First Zadeh's implication, conjunction and disjunction operations on intuitionistic fuzzy sets. Some examples of these operations were given and a few important properties were also studied. In our following papers, we will extended the following three operations such as second zadeh's IF-implication, second zadeh's conjunction and second zadeh's disjunction to the intuitionistic fuzzy soft set. We hope that the findings, in this paper will help researcher enhance the study on the intuitionistic soft set theory.

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Published in *Journal of New Results in Science*, No. 4, 2014, 11 p.

Relations on Interval Valued Neutrosophic Soft Sets

Said Broumi, Irfan Deli and Florentin Smarandache

Abstract. Anjan Mukherjee [43] ntroduced the concept of interval valued intuitionstic fuzzy soft relation. In this paper we will extend this concept to the case of interval valued neutrosophic soft relation(IVNSS relation for short) which can be discussed as a generalization of soft relations, fuzzy soft relation, intuitionstic fuzzy soft relation, interval valued intuitionstic fuzzy soft relations and neutrosphic soft relations [44]. Basic operations are presented and the various properties like reflexivity, symmetry, transitivity of IVNSS relations are also studied.

Keywords: Neutrosophic soft sets**,** Interval valued neutrosophic soft sets, Interval valued neutrosophic soft relation.

I.Introduction

In 1999, Florentin Smarandache introduced the theory of neutrosophic set (NS) [1] ,which is the generalization of the classical sets, conventional fuzzy set [2], intuitionistic fuzzy set [3] and interval valued fuzzy set [4]. This concept has been successfully applied to many fields such as Databases [5,6], Medical diagnosis problem [7] ,Decision making problem [8],Topology [9],control theory [10] etc .The concept of neutrosophic set handle indeterminate data whereas fuzzy set theory, and intuitionstic fuzzy set theory failed when the relation are indeterminate.

Presently work on the neutrosophic set theory is progressing rapidly. Bhowmik and M.Pal [11,12] defined “intuitionistic neutrosophic set”. Later on A.A.Salam, S.A.Alblowi [13] introduced another concept called “Generalized neutrosophic set”. Wang et al [14] proposed another extension of neutrosophic set which is” single valued neutrosophic”. Also Wang et al [15] introduced the notion of interval valued neutrosophic set which is an instance of neutrosophic set. It is characterized by an interval membership degree, interval indeterminacy degree and interval non-membership degree. K.Geogiev [16] Ye [17, 18], P. Majumdar and S.K. Samant [19] .S.Broumi and F. Smarandache [20,21 ,22] L.Peid [23 ,] and so on

In 1999 a Russian researcher , Molodotsov proposed an new mathematical tool called” Soft set theory [24], for dealing with uncertainty and how soft set theory is free from the parameterization inadequacy syndrome of fuzzy set theory ,rough set theory, probability theory.

Although there many authors [25,26,27,28,29,32,33] have contributed a lot towards fuzzification which leads to a series of mathematical models such as Fuzzy soft set generalized fuzzy soft set, possibility fuzzy soft set, fuzzy parameterized soft set and so on, intuitionstic fuzzy soft set which is based on a combination of the intuitionstic fuzzy sets and soft set models. Later a lot of extentions of intuitionistic fuzzy soft [34] are appeared such as Generalized intuitionistic fuzzy soft set [35], Possibility Intuitionistic Fuzzy Soft Set [36] and so on . Few studies are focused on neutrosophication of soft set theory. In [37] P.K.Maji, first proposed a new mathematical model called “Neutrosophic Soft Set” and investigate some properties regarding neutrosophic soft union, neutrosophic soft intersection ,complement of a neutrosophic soft set ,De Morgan law etc. Furthermore , in 2013, S.Broumi and F. Smarandache [38] combined the intuitionistic neutrosophic and soft set which lead to a new mathematical model called” intutionistic neutrosophic soft set”. They studied the notions of intuitionistic neutrosophic soft set union, intuitionistic neutrosophic soft set intersection, complement of intuitionistic neutrosophic soft set and several other properties of intuitionistic neutrosophic soft set along with examples and proofs of certain results. Also ,in [39] S.Broumi presented the concept of “Generalized neutrosophic soft set” by combining the Generalized Neutrosophic Sets [40] and Soft set Models ,studied some properties on it, and presented an application of Generalized Neutrosophic Soft Set [39] in decision making problem. S.Broumi and F.smarandache [41] introduced the necessity and possibility operators on intuitionstic neutrosophic and investigated some properties.

Recently, Irfan Deli [42] introduced the concept of interval valued neutrosophic soft set [42] as a combination of interval neutrosophic set and soft set. This concept generalizes the concept of the soft set [24], fuzzy soft set[26], intuitionstic fuzzy soft set [34],interval valued intuitionstic fuzzy soft set[43] ,the concept of neutrosophic soft set[37] and intuitionistic neutrosophic soft set [38].

This paper is an attempt to extend the concept of interval valued intuitionistic fuzzy soft relation (IVIFSS-relations) introduced by A.Mukherjee et al [45]to IVNSS relation .

The organization of this paper is as follow : In section 2, we briefly present some basic definitions and preliminary results are given which will be used in the rest of the paper. In section 3, relation interval neutrosophic soft relation is presented. In section 4 varouis type of interval valued neutrosophic soft relations. In section 5, we concludes the paper.

II.Preliminaries

Throughout this paper, let U be a universal set and E be the set of all possible parameters under consideration with respect to U, usually, parameters are attributes , characteristics, or properties of objects in U. We now recall some basic notions of neutrosophic set , interval neutrosophic set ,soft set , neutrosophic soft set and interval neutrosophic soft set.
For more details, the reader may refer to [5,6,8,9,12].

Definition 1 (see[3]).Neutrosophic set

Let U be an universe of discourse then the neutrosophic set A is an object having the form A = {< x:

$\mu_{A(x)}$, $\nu_{A(x)}$, $\omega_{A(x)}$ >,x ∈ U}, where the functions $\mu, \nu, \omega : U \to]^-0,1^+[$ define respectively the degree of membership , the degree of indeterminacy, and the degree of non-membership of the element x ∈ X to the set A with the condition.

$$^-0 \leqslant \mu_{A(x)} + \nu_{A(x)} + \omega_{A(x)} \leqslant 3^+.$$

From philosophical point of view, the neutrosophic set takes the value from real standard or non-standard subsets of $]^-0,1^+[$.so instead of $]^-0,1^+[$ we need to take the interval [0,1] for technical applications, because $]^-0,1^+[$will be difficult to apply in the real applications such as in scientific and engineering problems.

Definition 2 (see [3]). A neutrosophic set A is contained in another neutrosophic set B i.e. A ⊆ B if ∀x ∈ U, $\mu_A(x) \leq \mu_B(x)$, $\nu_A(x) \leq \nu_B(x)$, $\omega_A(x) \geq \omega_B(x)$.

A complete account of the operations and application of neutrsophic set can be seen in [3] [10].

Definition 3 (see[7]). Interval neutrosophic set

Let X be a space of points (objects) with generic elements in X denoted by x. An interval valued neutrosophic set (for short IVNS) A in X is characterized by truth-membership function $\mu_A(x)$, indeteminacy-membership function $\nu_A(x)$ and falsity-membership function $\omega_A(x)$. For each point x in X, we have that $\mu_A(x), \nu_A(x), \omega_A(x) \in [0,1]$.

For two IVNS , A_{IVNS} ={ <x , $[\mu_A^L(x), \mu_A^U(x)]$, $[\nu_A^L(x), \nu_A^U(x)]$, $[\omega_A^L(x), \omega_A^U(x)]$ > | x ∈ X }

And B_{IVNS} ={ <x , $[\mu_B^L(x), \mu_B^U(x)]$, $[\nu_B^L(x), \nu_B^U(x)]$, $[\omega_B^L(x), \omega_B^U(x)]$> | x ∈ X } the two relations are defined as follows:

(1) $A_{\text{IVNS}} \subseteq B_{\text{IVNS}}$ if and only if $\mu_A^L(x) \leq \mu_B^L(x)$,$\mu_A^U(x) \leq \mu_B^U(x)$, $\nu_A^L(x) \geq \nu_B^L(x)$, $\omega_A^U(x) \geq \omega_B^U(x)$, $\omega_A^L(x) \geq F\omega_B^L(x)$, $\omega_A^U(x) \geq \omega_B^U(x)$

(2) $A_{\text{IVNS}} = B_{\text{IVNS}}$ if and only if , $\mu_A(x) = \mu_B(x)$, $\nu_A(x) = \nu_B(x)$, $\omega_A(x) = \omega_B(x)$ for any x ∈ X

As an illustration ,let us consider the following example.

Example 1. Assume that the universe of discourse U={x_1,x_2,x_3},where x_1 characterizes the capability, x2 characterizes the trustworthiness and x3 indicates the prices of the objects. It may be further assumed that the values of x_1, x_2 and x_3 are in [0,1] and they are obtained from some questionnaires of some experts. The experts may impose their opinion in three components viz. the degree of goodness, the degree of indeterminacy and that of poorness to explain the characteristics of the objects. Suppose A is an interval neutrosophic set (INS) of U, such that,

A = {< x_1,[0.3 0.4],[0.5 0.6],[0.4 0.5] >,< x_2, ,[0.1 0.2],[0.3 0.4],[0.6 0.7]>,< x_3, [0.2 0.4],[0.4 0.5],[0.4 0.6] >}, where the degree of goodness of capability is 0.3, degree of indeterminacy of capability is 0.5 and degree of falsity of capability is 0.4 etc.

Definition 4 (see[4]). **Soft set**

Let U be an initial universe set and E be a set of parameters. Let P(U) denotes the power set of U. Consider a nonempty set A, A ⊂ E. A pair (K, A) is called a soft set over U, where K is a mapping given by K : A → P(U).

As an illustration ,let us consider the following example.

Example 2 . Suppose that U is the set of houses under consideration, say U = {h_1, h_2, . . ., h_5}. Let E be the set of some attributes of such houses, say E = {e_1, e_2, . . ., e_8}, where e_1, e_2, . . ., e_8 stand for the attributes "beautiful", "costly", "in the green surroundings'", "moderate", respectively.

In this case, to define a soft set means to point out expensive houses, beautiful houses, and so on. For example, the soft set (K,A) that describes the "attractiveness of the houses" in the opinion of a buyer, say Thomas, may be defined like this:

A={e_1,e_2,e_3,e_4,e_5};
K(e_1) = {h_2, h_3, h_5}, K(e_2) = {h_2, h_4}, K(e_3) = {h_1}, K(e_4) = U, K(e_5) = {h_3, h_5}.

Definition 5 [ ] (**interval neutrosophic soft set**) .
Let U be an initial universe set and A ⊂ E be a set of parameters. Let IVNS(U) denotes the set of all interval neutrosophic subsets of U. The collection (K,A) is termed to be the soft interval neutrosophic set over U, where F is a mapping given by K : A → IVNS(U).
The interval neutrosophic soft set defined over an universe is denoted by INSS.
To illustrate let us consider the following example:
Let U be the set of houses under consideration and E is the set of parameters (or qualities). Each parameter is a interval neutrosophic word or sentence involving interval neutrosophic words. Consider E = { beautiful, costly, in the green surroundings, moderate, expensive }. In this case, to define a interval neutrosophic soft set means to point out beautiful houses, costly houses, and so on. Suppose that, there are five houses in the universe U given by, U = {h_1,h_2,h_3,h_4,h_5} and the set of parameters A = {e_1,e_2,e_3,e_4}, where each e_i is a specific criterion for houses:

e_1 stands for 'beautiful',
e_2 stands for 'costly',
e_3 stands for 'in the green surroundings',
e_4 stands for 'moderate',

Suppose that,

K(beautiful)={< h_1,[0.5, 0.6], [0.6, 0.7], [0.3, 0.4]>,< h_2,[0.4, 0.5], [0.7 ,0.8], [0.2, 0.3] >, < h_3,[0.6, 0.7],[0.2 ,0.3],[0.3, 0.5] >,< h_4,[0.7 ,0.8],[0.3, 0.4],[0.2, 0.4] >,< h_5,[0.8, 0.4] ,[0.2 ,0.6],[0.3, 0.4] >}.K(costly)={< b_1,[0.5, 0.6], [0.6, 0.7], [0.3, 0.4]>,< h_2,[0.4, 0.5], [0.7 ,0.8], [0.2, 0.3] >, < h_3,[0.6, 0.7],[0.2 ,0.3],[0.3, 0.5] >,< h_4,[0.7 ,0.8],[0.3, 0.4],[0.2, 0.4] >,< h_5,[0.8, 0.4] ,[0.2 ,0.6],[0.3, 0.4] >}.

K(in the green surroundings)= {< h_1,[0.5, 0.6], [0.6, 0.7], [0.3, 0.4]>,< b_2,[0.4, 0.5], [0.7 ,0.8], [0.2, 0.3] >, < h_3,[0.6, 0.7],[0.2 ,0.3],[0.3, 0.5] >,< h_4,[0.7 ,0.8],[0.3, 0.4],[0.2, 0.4] >,< h_5,[0.8, 0.4] ,[0.2 ,0.6],[0.3, 0.4] >}.K(moderate)={< h_1,[0.5, 0.6], [0.6, 0.7], [0.3, 0.4]>,< h_2,[0.4, 0.5], [0.7 ,0.8], [0.2, 0.3] >, < h_3,[0.6, 0.7],[0.2 ,0.3],[0.3, 0.5] >,< h_4,[0.7 ,0.8],[0.3, 0.4],[0.2, 0.4] >,< h_5,[0.8, 0.4] ,[0.2 ,0.6],[0.3, 0.4] >}.

III.Relations on Interval Valued Neutrosophic Soft Sets

Definition 6.

Let U be an initial universe and (F,A) and (G,B) be two interval valued neutrosophic soft set . Then a relation between them is defined as a pair (H, AxB), where H is mapping given by H: AxB→IVNS(U). This is called an interval valued neutrosophic soft sets relation (IVNSS-relation for short).the collection of relations on interval valued neutrosophic soft sets on Ax Bover U is denoted by $\sigma_U(A\text{x}\,B)$

Remark 1: Let U be an initial universe and (F_1, A_1),(F_2, A_2),,...,(F_n, A_n), be n numbers of interval valued neutrosophic soft sets over U. Then a relation σ between them is defined as a pair (H, A_1xA_2x...xA_n), where H is mapping given by H: A_1xA_2x.... xA_n → IVIFS(U)

Example 3. (i) Let us consider an interval valued neutrosophic soft set (F,A) which describes the `attractiveness of the houses' under consideration. Let the universe set U ={h_1 , h_2 , h_3, h_4, h_5}.and the set of parameter A={beautiful(e_1), in the green surroundings (e_3)}.

Then the tabular representation of the interval valued neutrosophic soft set (F, A) is given below:

U	beautiful(e_1)	in the green surroundings (e_3)
h_1	([0.5, 0.6],[0.3 0.8],[0.3,0.4])	([0.2, 0.6],[0.1, 0.3],[0.2,0.8])
h_2	([0.2, 0.5],[0.4, 0.7],[0.5,0.6])	([0.4, 0.5],[0.3, 0.5],[0.2,0.4])
h_3	([0.3, 0.4],[0.7, 0.9],[0.1,0.2])	([0.2, 0.3],[0.1, 0.3],[0.4,0.5])
h_4	([0.1, 0.7],[0.2, 0.4],[0.6,0.7])	([0.5, 0.6],[0.4, 0.5],[0.3,0.4])
h_5	([0.4, 0.5],[0.3, 0.5],[0.2,0.4])	([0.3, 0.6],[0.2, 0.3],[0.5,0.6])

(ii) Now Let us consider an interval valued neutrosophic soft set (G,A) which describes the `cost of the houses' under consideration. Let the universe set U ={h_1 , h_2 , h_3, h_4, h_5}. and the set of parameter A={costly(e_2), moderate (e_4)}.

Then the tabular representation of the interval valued neutrosophic soft set (G, b) is given below:

U	costly(e_2)	moderate (e_4)
h_1	([0.3, 0.4],[0.7, 0.9],[0.1,0.2])	([0.4, 0.6],[0.7, 0.8],[0.1,0.4])
h_2	([0.6, 0.8],[0.3, 0.4],[0.1,0.7])	([0.1, 0.5],[0.4, 0.7],[0.5,0.6])
h_3	([0.3, 0.6],[0.2, 0.7],[0.3,0.4])	([0.4, 0.7],[0.1, 0.3],[0.2,0.4])
h_4	([0.6, 0.7],[0.3, 0.4],[0.2,0.4])	([0.3, 0.4],[0.7, 0.9],[0.1,0.2])
h_5	([0.2, 0.6],[0.2, 0.4],[0.3,0.5])	([0.5, 0.6],[0.6, 0.7],[0.3,0.4])

Let us consider the two IVNSS-relations P and Q on the two given interval valued neutrosophic soft sets given below:

P= (H, A xB)

U	(e_1 ,e_2)	(e_1 ,e_4)	(e_3 ,e_2)	(e_3 ,e_4)
H1	([0.2, 0.4],[0.3, 0.4],[0.1,0.2])	([0.3, 0.4],[0.3, 0.5],[0.3,0.4])	([0.3, 0.5],[0.3, 0.4],[0.3,0.5])	([0.4, 0.5],[0.3, 0.6],[0.2, 1])
h_2	([0.1,0. 3],[0.4, 0.5],[1, 1])	([0.1, 0.2],[0, 0],[0.2,0.4])	([0.4, 0.5],[0.1, 0.3],[0.2,0.4])	([0.3, 0.5],[0.2, 0.4],[0.4, 0.5]
h_3	([0.2, 0.6],[0.1, 0.4],[0.2,0.4])	([0.2, 0.6],[0.1, 0.3],[1, 1])	([0.2, 0.3],[0.1, 0.3],[0.3,0.6])	([0.2, 0.5],[0.2, 0.3],[0,0.4])
h_4	([0.2, 0.4],[0.3, 0.5],[0, 1])	([0.3, 0.4],[0.4, 0.5],[0.1,0.2])	([0.3, 0.4],[0.3, 0.4],[0.4,0.5])	([0, 0.2],[0.4, 0.5],[0.6,0.7])

Q= (J,A xB)

U	(e_1, e_2)	(e_1, e_4)	(e_3, e_2)	(e_3, e_4)
H_1	([0.2, 0.4],[0.3, 0.4],[0.1,0.2])	([0.3, 0.4],[0.3, 0.5],[0.3,0.4])	([0.3, 0.5],[0.3, 0.4],[0.3,0.5])	([0.4, 0.5],[0.3, 0.6],[0.2, 1])
h_2	([0.1,0. 3],[0.4, 0.5],[1, 1])	([0.1, 0.2],[0, 0],[0.2,0.4])	([0.4, 0.5],[0.1, 0.3],[0.2,0.4])	([0.3, 0.5],[0.2, 0.4],[0.4, 0.5]
h_3	([0.2, 0.6],[0.1, 0.4],[0.2,0.4])	([0.2, 0.6],[0.1, 0.3],[1, 1])	([0.2, 0.3],[0.1, 0.3],[0.3,0.6])	([0.2, 0.5],[0.2, 0.3],[0,0.4])
h_4	([0.2, 0.4],[0.3, 0.5],[0, 1])	([0.3, 0.4],[0.4, 0.5],[0.1,0.2])	([0.3, 0.4],[0.3, 0.4],[0.4,0.5])	([0, 0.2],[0.4, 0.5],[0.6,0.7])

The tabular representations of P and Q are called relational matrices for P and Q respectively. From above we have , $\mu_{H(e_1, e_2)}(h_1)$= [0.2 ,0.3] , $\upsilon_{H(e_1, e_2)}(h_2)$ =[0.3 , 0.4] and $\omega_{H(e_i, e_j)} =$. But this intervals lie on the 1st row-1st column and 2nd row -1st column respectively. So we denote $\mu_{H(e_1, e_2)}(h_1)|_{(1,1)}$= [0.2, 0.3] and $\upsilon_{H(e_1, e_2)}(h_2)|_{(1,1)}$ =[0.3 , 0.4] and $\omega_{H(e_i, e_j)}|_{(1,1)} =$ [0.3, 0.4] etc to make the clear concept about what are the positions of the intervals in the relational matrices.

Defintion 7 : The order of the relational matrix is (θ, λ), where θ = number of the universal points and λ = number of pairs of parametrers considered in the relational matrix. In example 3 both the relational matrix for P and Q are of order (5,4). If $\theta = \lambda$,then the relational matrix is called a square matrix

Defintion 8. Let P , Q $\in \sigma_U(Ax\, B)$, P= (H, AxB) ,Q = (J, AxB) and the order of their relational matrices are same.Then we define

(i) P ∪ Q= (H ■J, AxB) where H■ J :AxB →IVNS(U) is defined as (H ■J)(e_i, e_j)= H(e_i, e_j) ∨ J(e_j, e_j) for (e_i, e_j) ∈ A x B, where ∨ denotes the interval valued neutrosophic union.

(ii) P ∩ Q= (H ⋄J, AxB) where H⋄J :AxB →IVNS(U) is defined as (H⋄J)(e_i, e_j)= H(e_i, e_j) ∧ J(e_i, e_j) for (e_j, e_j) ∈ A x B, where ∧ denotes the interval valued neutrosophic intersection

(iii) P^c= (∼H, AxB) , where ∼H :AxB →IVNS(U) is defined as ∼H(e_i, e_j)=[H(e_i, e_j)] ^{c} for (e_i, e_j) ∈ A x B, where c denotes the interval valued neutrosophic complement.

Example 4 .Consider the interval valued neutrosophic soft sets (F,A) and (G,B) given in example 3. Let us consider the two IVNSS-relations P_1 and Q_1 given below:

P_1=(J, A x B):

U	(e_1, e_2)	(e_1, e_4)	(e_3, e_2)	(e_3, e_4)
H1	([0.2, 0.4],[0.3, 0.4],[0.1,0.2])	([0.3, 0.4],[0.3, 0.5],[0.3,0.4])	([0.3, 0.5],[0.3, 0.4],[0.3,0.5])	([0.4, 0.5],[0.3, 0.6],[0.2, 1])
h_2	([0.1,0. 3],[0.4, 0.5],[1, 1])	([0.1, 0.2],[0, 0],[0.2,0.4])	([0.4, 0.5],[0.1, 0.3],[0.2,0.4])	([0.3, 0.5],[0.2, 0.4],[0.4, 0.5]
h_3	([0.2, 0.6],[0.1, 0.4],[0.2,0.4])	([0.2, 0.6],[0.1, 0.3],[1, 1])	([0.2, 0.3],[0.1, 0.3],[0.3,0.6])	([0.2, 0.5],[0.2, 0.3],[0,0.4])
h_4	([0.2, 0.4],[0.3, 0.5],[0, 1])	([0.3, 0.4],[0.4, 0.5],[0.1,0.2])	([0.3, 0.4],[0.3, 0.4],[0.4,0.5])	([0, 0.2],[0.4, 0.5],[0.6,0.7])

Q_1=(J, A x B):

U	(e_1, e_2)	(e_1, e_4)	(e_3, e_2)	(e_3, e_4)
H1	([0.5, 0.8],[0.1, 0.2],[0.1,0.2])	([0.2, 0.3],[0.3, 0.6],[0.3,0.4])	([0.2 0.5],[0.3, 0.5],[0.2,0.4])	([0.2, 0.4],[0.2, 0.3],[1, 1])
h_2	([0.4, 0.5],[0.2, 0.4],[1, 1])	([0.4, 0.6],[0.2, 0.3],[0.2,0.4])	([0.4, 0.5],[0.4, 0.5],[0.2,0.5])	([0.4, 0.5],[0.1, 0.2],[1, 1])
h_3	([0.2, 0.3],[0.5, 0.6],[0.2,0.4])	([0.3, 0.4],[0.4, 0.5],[1, 1])	([0.7, 0.8],[0.1, 0.2],[0.2,0.5])	([0.3, 0.5],[0.3, 0.4],[0,0.4])
h_4	([0.3, 0.5],[0.3, 0.4],[0, 1])	([0.3, 0.5],[0.2, 0.4],[0.1,0.2])	([0.2, 0.4],[0.2, 0.3],[0,0.5])	([0.3, 0.7],[0.1, 0.3],[0.6,0.7])

Then $P_1 \cup Q_1$:

U	(e_1, e_2)	(e_1, e_4)	(e_3, e_2)	(e_3, e_4)
H1	([0.5, 0.8],[0.1, 0.2],[0.1,0.2])	([0.3, 0.4],[0.3, 0.5],[0.3,0.4])	([0.3 0.5],[0.3, 0.4],[0.2,0.4])	([0.4, 0.5],[0.2, 0.3],[0.2, 1])
h_2	([0.4, 0.5],[0.2, 0.4],[1, 1])	([0.4, 0.6],[0.2, 0.3],[0.2,0.4])	([0.4, 0.5],[0.1, 0.3],[0.2,0.4])	([0.4, 0.5],[0.1, 0.2],[0.4, 0.5])
h_3	([0.2, 0.6],[0.1, 0.4],[0.2,0.4])	([0.3, 0.6],[0.1, 0.3],[1, 1])	([0.7, 0.8],[0.1, 0.2],[0.2,0.5])	([0.3, 0.5],[0.3, 0.4],[0,0.4])
h_4	([0.3, 0.5],[0.3, 0.4],[0, 1])	([0.3, 0.5],[0.2, 0.4],[0.1,0.2])	([0.3, 0.4],[0.2, 0.3],[0,0.5])	([0.3, 0.7],[0.1, 0.3],[0.6,0.7])

$P_1 \cap Q_1$:

U	(e_1, e_2)	(e_1, e_4)	(e_3, e_2)	(e_3, e_4)
H1	([0.2, 0.4],[0.3, 0.4],[0.1,0.2])	([0.2, 0.3],[0.3, 0.6],[0.3,0.4])	([0.2 0.5],[0.3, 0.5],[0.3,0.5])	([0.2, 0.4],[0.3, 0.6],[1, 1])
h_2	([0.1, 0.3],[0.4, 0.5],[1, 1])	([0.1, 0.2],[0.2, 0.3],[0.2,0.4])	([0.4, 0.5],[0.4, 0.5],[0.2,0.5])	([0.3, 0.5],[0.2, 0.4],[1, 1])
h_3	([0.2, 0.3],[0.5, 0.6],[0.2,0.4])	([0.2, 0.4],[0.4, 0.5],[1, 1])	([0.7, 0.8],[0.1, 0.3],[0.3,0.6])	([0.2, 0.5],[0.3, 0.4],[0,0.4])
h_4	([0.2, 0.4],[0.3, 0.5],[0, 1])	([0.3, 0.4],[0.4, 0.5],[0.1,0.2])	([0.2, 0.4],[0.3, 0.4],[0.4,0.5])	([0, 0.2],[0.4, 0.5],[0.6,0.7])

${P_1}^c$

U	(e_1, e_2)	(e_1, e_4)	(e_3, e_2)	(e_3, e_4)
H1	([0.1, 0.2],[0.6, 0.7],[0.2,0.4])	([0.3, 0.4],[0.5, 0.7],[0.3,0.4])	([0.3 0.5],[0.6, 0.7],[0.3,0.5])	([0.2, 1],[0.4, 0.7],[0.4, 0.5])
h_2	([1, 1],[0.5, 0.6],[0.1, 0.3])	([0.2, 0.4],[1, 1],[0.1,0.2])	([0.2, 0.4],[0.7, 0.9],[0.4,0.5])	([0.4, 0.5],[0.6, 0.8],[0.3, 0.5])
h_3	([0.2, 0.4],[0.6, 0.9],[0.2,0.6])	([1, 1],[0.7, 0.9],[0.2, 0.6])	([0.3, 0.6],[0.7, 0.9],[0.2,0.3])	([0, 0.4],[0.7, 0.8],[0.2,0.5])
h_4	([0, 0.1],[0.5, 0.7],[0.2, 0.4])	([0.1, 0.2],[0.5, 0.6],[0.3,0.4])	([0.4, 0.5],[0.6, 0.7],[0.3,0.4])	([0.6, 0.7],[0.5, 0.6],[0,0.2])

Theorem 1. Let P, Q , R $\in \sigma_U(Ax\,B)$ and the order of their relational matrices are same.Then the following properties hold:

a) $(P \cup Q)^c = P^c \cap Q^c$
b) $(P \cap Q)^c = P^c \cup Q^c$
c) P ∪ (Q ∪ R)=(P ∪ Q) ∪ R
d) P ∩ (Q ∩ R)=(P ∩ Q) ∩ R
e) P ∩ (Q ∪ R)=(P ∩ Q) ∪ (P ∩ R)
f) P ∪ (Q ∩ R)=(P ∪ Q) ∩ (P ∪ R)

Proof. a) let P= (H, AxB), Q =(J,AxB) .then P ∪ Q = (H ■J ,AxB), where H ■J: Ax B →IVNS(U) is defined as

(H ■J) (e_i, e_j) = H(e_i, e_j) ∨ J(e_i, e_j) for (e_i, e_j)∈ A x B.

So $(P \cup Q)^c$= (~H ■J,AxB),where ~H ■J:A xB→IVNS(U) is defined as (~H ■J) (e_i, e_j)

$=[\mathrm{H}(e_i,e_j) \vee \mathrm{J}(e_i,e_j)]^c$

$=[\{<\mathrm{h_k}, \mu_{\mathrm{H}(e_i,e_j)}(\mathrm{h_k}), \upsilon_{\mathrm{H}(e_i,e_j)}(\mathrm{h_k}), \omega_{\mathrm{H}(e_i,e_j)}(\mathrm{h_k})>: \mathrm{h_k} \in \mathrm{U}\} \vee \{<$
$\mathrm{h_k}, \mu_{\mathrm{J}(e_i,e_j)}(\mathrm{h_k}), \upsilon_{\mathrm{J}(e_i,e_j)}(\mathrm{h_k}), \omega_{\mathrm{J}(e_i,e_j)}(\mathrm{h_k})>: \mathrm{h_k} \in \mathrm{U}\}]^c$

$=\{<\mathrm{h_k}, [\max(\inf \mu_{\mathrm{H}(e_i,e_j)}(\mathrm{h_k}), \inf\mu_{\mathrm{J}(e_i,e_j)}(\mathrm{h_k}), \max(\sup \mu_{\mathrm{H}(e_i,e_j)}(\mathrm{h_k}), \sup\mu_{\mathrm{J}(e_i,e_j)}(\mathrm{h_k})],$

$[\min(\inf \upsilon_{\mathrm{H}(e_i,e_j)}(\mathrm{h_k}), \inf\upsilon_{\mathrm{J}(e_i,e_j)}(\mathrm{h_k}), \min(\sup \upsilon_{\mathrm{H}(e_i,e_j)}(\mathrm{h_k}), \sup\upsilon_{\mathrm{J}(e_i,e_j)}(\mathrm{h_k})],$

$[\min(\inf \omega_{\mathrm{H}(e_i,e_j)}(\mathrm{h_k}), \inf\omega_{\mathrm{J}(e_i,e_j)}(\mathrm{h_k}), \min(\sup \omega_{\mathrm{H}(e_i,e_j)}(\mathrm{h_k}), \sup\omega_{\mathrm{J}(e_i,e_j)}(\mathrm{h_k})]>: \mathrm{h_k} \in \mathrm{U}\}^c$

$=\{<\mathrm{h_k}, [\min(\inf \omega_{\mathrm{H}(e_i,e_j)}(\mathrm{h_k}), \inf\omega_{\mathrm{J}(e_i,e_j)}(\mathrm{h_k}), \min(\sup \omega_{\mathrm{H}(e_i,e_j)}(\mathrm{h_k}), \sup\omega_{\mathrm{J}(e_i,e_j)}(\mathrm{h_k})],$

$[1-\min(\sup \upsilon_{\mathrm{H}(e_i,e_j)}(\mathrm{h_k}), \sup\upsilon_{\mathrm{J}(e_i,e_j)}(\mathrm{h_k}), 1-\min(\inf \upsilon_{\mathrm{H}(e_i,e_j)}(\mathrm{h_k}), \inf\upsilon_{\mathrm{J}(e_i,e_j)}(\mathrm{h_k})],$

$[\max(\inf \mu_{\mathrm{H}(e_i,e_j)}(\mathrm{h_k}), \inf\mu_{\mathrm{J}(e_i,e_j)}(\mathrm{h_k}), \max(\sup \mu_{\mathrm{H}(e_i,e_j)}(\mathrm{h_k}), \sup\mu_{\mathrm{J}(e_i,e_j)}(\mathrm{h_k})] >: \mathrm{h_k} \in \mathrm{U}\}$

Now $P^c \cap Q^c$ = (~H ,A x B) ∩ (~J ,A x B) ,where ~H , ~J: A xB→IVNS(U) are defined as

~ H(e_i, e_j)= $[\mathrm{H}(e_i,e_j)]^c$ and ~ J(e_i, e_j)= $[\mathrm{J}(e_i,e_j)]^c$ for (e_j,e_j) ∈ A x B, we have

(~H ,A x B) ∩ (~J ,A x B) =(~H ⋄~J, A x B) (e_i, e_j)

Now for (e_i, e_j) ∈ A x B., (~H ⋄~J) (e_i, e_j)= ~H(e_j, e_j) ∧~J(e_i, e_j)=

$\{<\mathrm{h_k}, [\inf \omega_{\mathrm{H}(e_i,e_j)}(\mathrm{h_k}), \mathrm{Sup}\omega_{\mathrm{H}(e_i,e_j)}(\mathrm{h_k})], [1-\mathrm{Sup}\ \upsilon_{\mathrm{H}(e_i,e_j)}(\mathrm{h_k}), 1-\inf \upsilon_{\mathrm{H}(e_i,e_j)}(\mathrm{h_k})], [\inf \mu_{\mathrm{H}(e_i,e_j)}(\mathrm{h_k}), \mathrm{Sup}\mu_{\mathrm{H}(e_i,e_j)}(\mathrm{h_k})] >: \mathrm{h_k} \in \mathrm{U}\}$

$\wedge\{<\mathrm{h_k}, [\inf \omega_{J(e_i,e_j)}(\mathrm{h_k}), \mathrm{Sup}\omega_{J(e_i,e_j)}(\mathrm{h_k})], [1-\mathrm{Sup}\, \upsilon_{J(e_i,e_j)}(\mathrm{h_k}), 1-\inf \upsilon_{J(e_i,e_j)}(\mathrm{h_k})], [\inf \mu_{J(e_i,e_j)}(\mathrm{h_k}), \mathrm{Sup}\mu_{J(e_i,e_j)}(\mathrm{h_k})] >: \mathrm{h_k} \in U\}$

$= \{<\mathrm{h_k}, [\min(\inf \omega_{H(e_i,e_j)}(\mathrm{h_k}), \inf \omega_{H(e_i,e_j)}(\mathrm{h_k})), \min(\mathrm{Sup}\omega\mu_{H(e_i,e_j)}(\mathrm{h_k}), \mathrm{Sup}\omega_{J(e_i,e_j)}(\mathrm{h_k}),],$

$[\max((1-\mathrm{Sup}\, \upsilon_{H(e_i,e_j)}(\mathrm{h_k})), (1-\mathrm{Sup}\, \upsilon_{J(e_i,e_j)}(\mathrm{h_k}))), \max((1-\inf \upsilon_{H(e_i,e_j)}(\mathrm{h_k})), (1-\inf \upsilon_{J(e_i,e_j)}(\mathrm{h_k})))],$

$[\max(\inf \mu_{H(e_i,e_j)}(\mathrm{h_k}), \inf \mu_{J(e_i,e_j)}(\mathrm{h_k})), \max(\mathrm{Sup}\, \mu_{H(e_i,e_j)}(\mathrm{h_k}), \mathrm{Sup}\, \mu_{J(e_i,e_j)}(\mathrm{h_k}))] >: \mathrm{h_k} \in U\}$

$=\{<\mathrm{h_k}, [\min(\inf \omega_{H(e_i,e_j)}(\mathrm{h_k}), \inf\omega_{J(e_i,e_j)}(\mathrm{h_k}), \min(\sup \omega_{H(e_i,e_j)}(\mathrm{h_k}), \sup\omega_{J(e_i,e_j)}(\mathrm{h_k})],$

$[1-\min(\sup \upsilon_{H(e_i,e_j)}(\mathrm{h_k}), \sup\upsilon_{J(e_i,e_j)}(\mathrm{h_k}), 1-\min(\inf \upsilon_{H(e_i,e_j)}(\mathrm{h_k}), \inf\upsilon_{J(e_i,e_j)}(\mathrm{h_k})],$

$[\max(\inf \mu_{H(e_i,e_j)}(\mathrm{h_k}), \inf\mu_{J(e_i,e_j)}(\mathrm{h_k}), \max(\sup \mu_{H(e_i,e_j)}(\mathrm{h_k}), \sup\mu_{J(e_i,e_j)}(\mathrm{h_k})] >: \mathrm{h_k} \in U\}$

Then , $(P \cup Q)^c = P^c \cap Q^c$

b) Proof is similar to a)

c) let P= (H, AxB), Q =(J, AxB) and R= (K, AxB).Then $P \cup Q$ = (H ■J, A XB), where

H ■J : A xB→IVNS(U) is defiend as (H■ J) $(e_{i,}e_j)$ =H(e_j, e_j) ∨ J(e_i, e_j) for ($e_{j,}e_j$) ∈ A x B .

So($P \cup Q$) ∪ R=((H ■J) ■K, AxB), where (H ■J) ■K : A xB→IVNS(U) is defined as

for ($e_{j,}e_j$) ∈ A x B (H ■J) ■K) ($e_{j,}e_j$) = H(e_i, e_j) ∨ J(e_i, e_j) ∨ K(e_i, e_j) .Now as

(H(e_i, e_j) ∨ J(e_i, e_j)) ∨ K(e_i, e_j) = H(e_i, e_j) ∨ (J(e_i, e_j) ∨ K(e_i, e_j)).therefore

(H ■J) ■K) ($e_{j,}e_j$) =((H ■ (J ■K)) ($e_{j,}e_j$), Also we have P∪(Q∪R)=(P ∪ Q) ∪ R=(H ■ (J ■K),AxB).consequently, P ∪ (Q ∪ R)=(P ∪ Q) ∪ R

d) Proof is similar to c)

e) let P= (H, AxB), Q =(J,AxB) and R= (K,AxB).Then $Q \cup R$ = (J ■K, A XB), where

J ■K: A xB→IVNS(U) is defiend as (J ■K) $(e_{i,}e_j)$ = J(e_j, e_j) ∨ K(e_i, e_j) for ($e_{j,}e_j$) ∈ A x B .

Then P ∩ (Q ∪ R)=((H⋄ (J ■K), AxB), where H⋄ (J ■K): A xB→IVNS(U) is defined as

for ($e_{j,}e_j$) ∈ A x B, (H⋄ (J ■K)) ($e_{j,}e_j$) = H(e_i, e_j) ∧ (J(e_i, e_j) ∨ K(e_i, e_j)) .

since H(e_i, e_j) ∧ (J(e_i, e_j) ∨ K(e_i, e_j)) =(H(e_i, e_j) ∧ (J(e_i, e_j)) ∨(H(e_i, e_j) ∧ K(e_i, e_j)).We have (H⋄ (J ■K)) ($e_{j,}e_j$) = (H(e_i, e_j) ∧ (J(e_i, e_j)) ∨(H(e_i, e_j) ∧ K(e_i, e_j)).

Also we have (P∩Q) ∪(P∩ R)= (H⋄J, AxB) ∪ (H ⋄ K, AxB) =((H⋄ J) ■(H⋄K),A xB) Now for ($e_{j,}e_j$) ∈ A x B , ((H⋄ J) ■(H⋄K)) ($e_{j,}e_j$) = (H⋄ J)($e_{j,}e_j$) ∨ (H⋄ K) ($e_{j,}e_j$) =(H(e_i, e_j) ∧ J(e_i, e_j)) ∨ (H(e_i, e_j) ∧ K(e_i, e_j)) =(H⋄ (J ■K)) ($e_{j,}e_j$). consequently, P ∩ (Q ∪ R)=(P ∩ Q) ∪ (P ∩ R).

f)Proof is similar to e)

Definition 9. Let P, Q ∈ $\sigma_U(Ax\ B)$ and the ordre of their relational matrices are same. Then

P ⊆ Q if H $(e_{j,}e_j)$ ⊆ J $(e_{j,}e_j)$ for $(e_{j,}e_j)$ ∈ A x B where P=(H, A x B) and Q = (J, A x B)

Example 5:

P

U	(e_1, e_2)	(e_1, e_4)	(e_3, e_2)	(e_3, e_4)
h_1	([0.2, 0.3],[0.2, 0.3],[0.4, 0.5])	([0.2, 0.3],[0.7, 0.8],[0.2,0.4])	([0.3, 0.4],[0.7, 0.8],[0.2,0.5])	([0.4, 0.6],[0.7, 0.8],[0.5,0.6])
h_2	([0.4, 0.5],[0.3, 0.5],[0.2,0.8])	([1, 1],[0, 0],[0, 0])	([0.1, 0.5],[0.4, 0.7],[0.5,0.6])	([0.1, 0.3],[0.4, 0.7],[0.5,0.6])
h_3	([0.2, 0.4],[0.3, 0.4],[0.3,0.4])	([0.3, 0.5,[0.4 0.6],[0.2,0.5])	([1, 1],[0, 0],[0, 0])	([0.1, 0.2],[0.4, 0.5],[0.3,0.5])
h_4	([0.3, 0.5],[0.3, 0.4],[0.3,0.6])	([0.2, 0.3],[0.7, 0.9],[0.4,0.5])	([0.3, 0.4],[0.7, 0.9],[0.3,0.4])	([0.2, 0.3],[0.3, 0.5],[0.5, 0.6])

Q

U	(e_1, e_2)	(e_1, e_4)	(e_3, e_2)	(e_3, e_4)
h_1	([0.3, 0.4],[0.1, 0.2],[0.3, 0.4])	([0.4, 0.6],[0.3, 0.5],[0.1,0.4])	([0.5, 0.6],[0.3, 0.5],[0.1,0.4])	([0.5, 0.7],[0.2, 0.3],[0.3,0.4])
h_2	([0.6, 0.8],[0.3, 0.4],[0.1,0.7])	([1, 1],[0, 0],[0, 0])	([0.3, 0.6],[0.1, 0.3],[0.2,0.3])	([0.3, 0.5],[0.3, 0.5],[0.2,0.4])
h_3	([0.3, 0.6],[0.2, 0.3],[0.1,0.2])	([0.4, 0.7],[0.1, 0.3],[0.2,0.4])	([1, 1],[0, 0],[0, 0])	([0.4, 0.7],[0.1, 0.3],[0.2,0.4])
h_4	([0.6, 0.7],[0.1, 0.2],[0.2,0.4])	([0.3, 0.4],[0.4, 0.6],[0.1,0.2])	([0.4, 0.6],[0.1, 0.4],[0.1,0.2])	([0.4, 0.5],[0.1, 0.2],[0.2, 0.3])

Definition 10 :Let U be an initial universe and (F, A) and (G, B) be two interval valued neutrosophic soft sets. Then a null relation between them is denoted

by 0_U and is defined**ed as** 0_U =(H_O , A xB) where H_O $(e_{i,}e_j)$={<h_k , [0, 0],[1, 1],[1, 1]>; h_k ∈ U} for $(e_{i,}e_j)$ ∈ A xB.

Example 6. Consider the interval valued neutrosophic soft sets (F, A) and (G, B) given in example 3. Then a null relation between them is given by

U	(e_1, e_2)	(e_1, e_4)	(e_3, e_2)	(e_3, e_4)
h_1	([0, 0],[1, 1],[1, 1])	([0, 0],[1, 1],[1, 1])	([0, 0],[1, 1],[1, 1])	([0, 0],[1, 1],[1, 1])
h_2	([0, 0],[1, 1],[1, 1])	([0, 0],[1, 1],[1, 1])	([0, 0],[1, 1],[1, 1])	([0, 0],[1, 1],[1, 1])
h_3	([0, 0],[1, 1],[1, 1])	([0, 0],[1, 1],[1, 1])	([0, 0],[1, 1],[1, 1])	([0, 0],[1, 1],[1, 1])
h_4	([0, 0],[1, 1],[1, 1])	([0, 0],[1, 1],[1, 1])	([0, 0],[1, 1],[1, 1])	([0, 0],[1, 1],[1, 1])

Remark 2 . It can be easily seen that P ∪ 0_U =P and P ∩ 0_U =0_U for any P ∈ $\sigma_U(Ax\ B)$

Definition 11 :Let U be an initial universe and (F, A) and (G, B) be two interval

valued neutrosophic soft sets. Then an absolute relation between them is denoted

by I_U and is defined**ed as** I_U =(H_I , A xB) where H_I $(e_{i,}e_j)$={<h_k , [1, 1],[0, 0],[0, 0]>; h_k ∈ U} for $(e_{i,}e_j)$ ∈ A xB.

Example 7. Consider the interval valued neutrosophic soft sets (F, A) and (G, B) given in example 3. Then an absolute relation between them is given by

U	(e_1, e_2)	(e_1, e_4)	(e_3, e_2)	(e_3, e_4)
h_1	([1, 1],[0, 0],[0, 0])	([1, 1],[0, 0],[0, 0])	([1, 1],[0, 0],[0, 0])	([1, 1],[0, 0],[0, 0])
h_2	([1, 1],[0, 0],[0, 0])	([1, 1],[0, 0],[0, 0])	([1, 1],[0, 0],[0, 0])	([1, 1],[0, 0],[0, 0])
h_3	([1, 1],[0, 0],[0, 0])	([1, 1],[0, 0],[0, 0])	([1, 1],[0, 0],[0, 0])	([1, 1],[0, 0],[0, 0])
h_4	([1, 1],[0, 0],[0, 0])	([1, 1],[0, 0],[0, 0])	([1, 1],[0, 0],[0, 0])	([1, 1],[0, 0],[0, 0])

Remark 3 . It can be easily seen that P ∪ I_U =I_U and P ∩ I_U =P for any P ∈ $\sigma_U(Ax\ B)$

Definition 12 :Let τ be a sub-collection of interval valued neutrosophic soft set relations of the same order belonging to $\sigma_U(Ax\ B)$.Then τ is said to form a relational topology over $\sigma_U(Ax\ B)$ if the following conditions are satisfied:

(i) 0_U , I_U $\in \tau$
(ii) If
(iii) If P_1 , P_2 $\in \tau$,then $P_1 \cap P_2 \in \tau$

Then we say that $(\sigma_U(Ax\ B)\ ,\ \tau)$ is a conditional relational topological space

Example 8: Consider example 3. Then the collection τ={0_U , I_U ,P,Q} forms a relational topology on $\sigma_U(Ax\ B)$.

IV .Various type of interval valued neutrosophic soft relation

In this section , we present some basic properties of IVNSS relation. Let P ∈ $\sigma_U(Ax\ B)$and P=(H, A xB) and Q=(J, A xB) whose relational matrix is a square matrix

Defintion 13. An IVNSS-relation P is said to be reflexive if for (e_i, e_j) ∈ A x B and h_k ∈ U, such that $\mu_{H(e_i,e_j)}(h_k)|_{(m,n)}$ = [1, 1] , $\upsilon_{H(e_i,e_j)}(h_k)|_{(m,n)}$ =[0, 0] and $\omega_{H(e_i,e_j)}(h_k)|_{(m,n)}$ = [0 0] for m = n=k

Example 9: U ={h_1 , h_2 , h_3, h_4}Let us consider the interval valued neutrosophic soft sets (F, A) and (G, B) where A= {e_1, e_3 } and B ={e_2, e_4 } then a reflexive IVNSS-relation between them is

U	(e_1, e_2)	(e_1, e_4)	(e_3, e_2)	(e_3, e_4)
h_1	([1, 1],[0, 0],[0, 0])	([0.4, 0.6],[0.7, 0.8],[0.1,0.4])	([0.4, 0.6],[0.7, 0.8],[0.1,0.4])	([0.4, 0.6],[0.7, 0.8],[0.1,0.4])
h_2	([0.6, 0.8],[0.3, 0.4],[0.1,0.7])	([1, 1],[0, 0],[0, 0])	([0.1, 0.5],[0.4, 0.7],[0.5,0.6])	([0.1, 0.5],[0.4, 0.7],[0.5,0.6])
h_3	([0.3, 0.6],[0.2, 0.7],[0.3,0.4])	([0.4, 0.7],[0.1, 0.3],[0.2,0.4])	([1, 1],[0, 0],[0, 0])	([0.4, 0.7],[0.1, 0.3],[0.2,0.4])
h_4	([0.6, 0.7],[0.3, 0.4],[0.2,0.4])	([0.3, 0.4],[0.7, 0.9],[0.1,0.2])	([0.3, 0.4],[0.7, 0.9],[0.1,0.2])	([1, 1],[0, 0],[0, 0])

Defintion 14. An IVNSS-relation P is said to be anti- reflexive if for $(e_i, e_j) \in$ A x B and $h_k \in$ U, such that $\mu_{H(e_i,e_j)}(h_k)|_{(m,n)} = [0, 0]$, $\upsilon_{H(e_i,e_j)}(h_k)|_{(m,n)} = [0, 0]$ and $\omega_{H(e_i,e_j)}(h_k)|_{(m,n)} = [1\ 1]$ for m = n=k

Example 10: let U $=\{h_1 , h_2 , h_3, h_4\}$. Let us consider the interval valued neutrosophic soft sets (F, A) and (G, B) where A= $\{e_1, e_3\}$ an B $=\{e_2, e_4\}$ then an anti-reflexive IVNSS-relation between them is

U	(e_1 ,e_2)	(e_1 ,e_4)	(e_3 ,e_2)	(e_3 ,e_4)
h_1	([0, 0],[0, 0],[1, 1])	([0.4, 0.6],[0.7, 0.8],[0.1,0.4])	([0.4, 0.6],[0.7, 0.8],[0.1,0.4])	([0.4, 0.6],[0.7, 0.8],[0.1,0.4])
h_2	([0.6, 0.8],[0.3, 0.4],[0.1,0.7])	([0, 0],[0, 0],[1, 1])	([0.1, 0.5],[0.4, 0.7],[0.5,0.6])	([0.1, 0.5],[0.4, 0.7],[0.5,0.6])
h_3	([0.3, 0.6],[0.2, 0.7],[0.3,0.4])	([0.4, 0.7],[0.1, 0.3],[0.2,0.4])	([0, 0],[0, 0],[1, 1])	([0.4, 0.7],[0.1, 0.3],[0.2,0.4])
h_4	([0.6, 0.7],[0.3, 0.4],[0.2,0.4])	([0.3, 0.4],[0.7, 0.9],[0.1,0.2])	([0.3, 0.4],[0.7, 0.9],[0.1,0.2])	([0, 0],[0, 0],[1, 1])

Defintion 15. An IVNSS-relation P is said to be symmetric if for $(e_i, e_j) \in$ A x B and $h_k \in$ U, $\exists$ $(e_i, e_j) \in$ A x B and $h_l \in$ U such that $\mu_{H(e_i,e_j)}(h_k)|_{(m,n)} = \mu_{H(e_p,e_q)}(h_l)|_{(n,m)}$, $\upsilon_{H(e_i,e_j)}(h_k)|_{(m,n)} = \upsilon_{H(e_p,e_q)}(h_l)|_{(n,m)}$ and $\omega_{H(e_i,e_j)}(h_k)|_{(m,n)} = \omega_{H(e_p,e_q)}(h_l)|_{(n,m)}$

Example 11: let U $=\{h_1 , h_2 , h_3, h_4\}$. Let us consider the interval valued neutrosophic soft sets (F, A) and (G, B) where A= $\{e_1, e_3\}$ an B $=\{e_2, e_4\}$ then a symmetric IVNSS-relation between them is

U	(e_1 ,e_2)	(e_1 ,e_4)	(e_3 ,e_2)	(e_3 ,e_4)
h_1	([0.3, 0.4],[0.7, 0.9],[0.1,0.2])	([0.5, 0.6],[0.6, 0.7],[0.3,0.4])	([0.3, 0.6],[0.5, 0.7],[0.2,0.4])	([0.4, 0.6],[0.3 0.4],[0.3,0.4])
h_2	([0.5, 0.6],[0.6, 0.7],[0.3,0.4])	([0, 0],[1, 1],[1, 1])	([0.4, 0.7],[0.1, 0.3],[0.2,0.4])	([0.3, 0.4],[0.7, 0.9],[0.1,0.2])
h_3	([0.3, 0.6],[0.5, 0.7],[0.2,0.4])	([0.4, 0.7],[0.1, 0.3],[0.2,0.4])	([0.4, 0.6],[0.1, 0.3],[0.2,0.5])	([0.4, 0.5],[0.3, 0.4],[0.1,0.4])
h_4	([0.4, 0.6],[0.3 0.4],[0.3,0.4])	([0.3, 0.4],[0.7, 0.9],[0.1,0.2])	([0.4, 0.5],[0.3, 0.4],[0.1,0.4])	([0.2, 0.7],[0.3, 0.4],[0.6,0.7])

Defintion 16. An IVNSS-relation P is said to be anti-symmetric if for each $(e_i, e_j) \in$ A x B and $h_k \in$ U, $\exists$ $(e_i, e_j) \in$ A x B and $h_l \in$ U such that either $\mu_{H(e_i,e_j)}(h_k)|_{(m,n)} \neq \mu_{H(e_p,e_q)}(h_l)|_{(n,m)}$, $\upsilon_{H(e_i,e_j)}(h_k)|_{(m,n)} \neq \upsilon_{H(e_p,e_q)}(h_l)|_{(n,m)}$ and $\omega_{H(e_i,e_j)}(h_k)|_{(m,n)} \neq \omega_{H(e_p,e_q)}(h_l)|_{(n,m)}$ or $\mu_{H(e_i,e_j)}(h_k)|_{(m,n)} = \mu_{H(e_p,e_q)}(h_l)|_{(n,m)}$ [0, 0] , $\upsilon_{H(e_i,e_j)}(h_k)|_{(m,n)} = \upsilon_{H(e_p,e_q)}(h_l)|_{(n,m)}$ =[0 ,0] and $\omega_{H(e_i,e_j)}(h_k)|_{(m,n)} = \omega_{H(e_p,e_q)}(h_l)|_{(n,m)}$ =[1 , 1]

Example 12: let U $=\{h_1 , h_2 , h_3, h_4\}$. Let us consider the interval valued neutrosophic soft sets (F, A) and (G, B) where A= $\{e_1, e_3\}$ an B $=\{e_2, e_4\}$ then an anti-symmetric IVNSS-relation between them is

U	(e_1, e_2)	(e_1, e_4)	(e_3, e_2)	(e_3, e_4)
h_1	([0.3, 0.4],[0.7, 0.9],[0.1,0.2])	([0.5, 0.6],[0.6, 0.7],[0.3,0.4])	([0.3, 0.6],[0.5, 0.7],[0.2,0.4])	([0, 0],[0, 0],[1, 1])
h_2	([0.5, 0.6],[0.6, 0.7],[0.3,0.4])	([0, 0],[1, 1],[1, 1])	([0.4, 0.7],[0.1, 0.3],[0.2,0.4])	([0.3, 0.4],[0.7, 0.9],[0.1,0.2])
h_3	([0.3, 0.6],[0.5, 0.7],[0.2,0.4])	([0.4, 0.7],[0.1, 0.3],[0.2,0.4])	([0.4, 0.6],[0.1, 0.3],[0.2,0.5])	([0.4, 0.5],[0.3, 0.4],[0.1,0.4])
h_4	([0, 0],[0, 0],[1, 1])	([0.3, 0.4],[0.7, 0.9],[0.1,0.2])	([0, 0],[0, 0],[1, 1])	([0.2, 0.7],[0.3, 0.4],[0.6,0.7])

Defintion 17 . An IVNSS-relation P is said to be perfectly anti-symmetric if for each $(e_i, e_j) \in$ A x B and $h_k \in$ U, $\exists$ $(e_i, e_j) \in$ A x B and $h_l \in$ U such that whenever inf $\mu_{H(e_i,e_j)}(h_k)|_{(m,n)} > 0$, inf $\upsilon_{H(e_i,e_j)}(h_k)|_{(m,n)} > 0$ and inf $\omega_{H(e_i,e_j)}(h_k)|_{(m,n)} > 0$, $\mu_{H(e_p,e_q)}(h_l)|_{(n,m)} = [0, 0]$, $\upsilon_{H(e_p,e_q)}(h_l)|_{(n,m)} = [0, 0]$ and $\omega_{H(e_p,e_q)}(h_l)|_{(n,m)} = [1, 1]$

Example 13: let U $=\{h_1, h_2, h_3, h_4\}$. Let us consider the interval valued neutrosophic soft sets (F, A) and (G, B) where A= $\{e_1, e_3\}$ an B $=\{e_2, e_4\}$ then a perfectly anti-symmetric IVNSS-relation between them is

U	(e_1, e_2)	(e_1, e_4)	(e_3, e_2)	(e_3, e_4)
H1	([0.3, 0.4],[0.7, 0.9],[0.1,0.2])	([0.5, 0.6],[0.6, 0.7],[0.3,0.4])	([0.3, 0.6],[0.5, 0.7],[0.2,0.4])	([0, 0],[0, 0],[1, 1])
h_2	([0, 0],[0, 0],[1, 1])	([0.4, 0.7],[0.1, 0.3],[0.2,0.4])	([0.4, 0.6],[0.1, 0.3],[0.2,0.5])	([0, 0],[0, 0],[1, 1])
h_3	([0.3, 0.6],[0.5, 0.7],[0.2,0.4])	([0, 0],[0, 0],[1, 1])	([0.4, 0.6],[0.1, 0.3],[0.2,0.5])	([0, 0.5],[0, 0.4],[0,0.4])
h_4	([0, 0.6],[0, 0.2],[0, 1])	([0.3, 0.4],[0.7, 0.9],[0.1,0.2])	([0, 0.6],[0, 0.3],[0,0.5])	([0.2, 0.7],[0.3, 0.4],[0.6,0.7])

In the following, we define two composite of interval valued neutrosophic soft relation.

Definition 18 : Let P ,Q $\in \sigma_U(Ax\,A)$ and P =(H,AxA), Q=(J,AxA) and the order of their relational matrices are same.Then the compostion of P and Q, denoted by P*Q is defined by P*Q =(H$\circ$ J,AxA) where H$\circ$ J :AxA $\to$ IVNS(U)

Is defined as $(H\circ J)(e_i,e_j) = \{<h_k, \mu_{(H\circ J)(e_i,e_j)}(h_k), \upsilon_{(H\circ J)(e_i,e_j)}(h_k), \omega_{(H\circ J)(e_i,e_j)}(h_k)>: h_k \in U\}$

Where

$\mu_{(H\circ J)(e_i,e_j)}(h_k) = [\max_l(\min(inf\ \mu_{H(e_i,e_l)}(h_k), \inf \mu_{J(e_l,e_j)}(h_k))), \max_l(\min(sup\ \mu_{H(e_i,e_l)}(h_k), \sup \mu_{J(e_l,e_j)}(h_k)))]$,

$\upsilon_{(H\circ J)(e_i,e_j)}(h_k) = [\min_l(\max(inf\ \upsilon_{H(e_i,e_l)}(h_k), \inf \upsilon_{J(e_l,e_j)}(h_k))), \min_l(\max(sup\ \upsilon_{H(e_i,e_l)}(h_k), \sup \upsilon_{J(e_l,e_j)}(h_k)))]$,

And

$\omega_{(H\circ J)(e_i,e_j)}(h_k) = [\min_l(\max(inf\ \omega_{H(e_i,e_l)}(h_k), \inf \omega_{J(e_l,e_j)}(h_k))), \min_l(\max(sup\ \omega_{H(e_i,e_l)}(h_k), \sup \omega_{J(e_l,e_j)}(h_k)))]$

For $(e_i,e_j) \in$ A x A

Example 14: U $=\{h_1, h_2, h_3, h_4\}$.let us consider the interval valued neutrosophic soft sets (F,A) and (G,A) where A=$\{e_1, e_2\}$ .Let P ,Q $\in \sigma_U(Ax\,A)$ and P =(H, AxA), Q=(J,AxA) where P:

U	(e_1, e_2)	(e_1, e_4)	(e_3, e_2)	(e_3, e_4)
H1	([0.3, 0.4],[0.3, 0.4],[0.1,0.2])	([0.2, 0.4],[0.3, 0.5],[0.3,0.4])	([0.2 0.5],[0.3, 0.4],[0.3,0.4])	([0.2, 0.3],[0.3, 0.6],[0.2, 0.3])
h_2	([1, 1],[0, 0],[1, 1])	([0.1, 0.2],[0, 0],[0.2,0.5])	([0.4, 0.5],[0.1, 0.3],[0.3,0.5])	([0.4, 0.7],[0.1, 0.3],[1, 1])
h_3	([0.2, 0.6],[0.1, 0.4],[0.3,0.4])	([0.2, 0.6],[0.1, 0.3],[1, 1])	([0.2, 0.3],[0.1, 0.3],[0.2,0.5])	([0.2, 0.5],[0.2, 0.3],[0,0.4])
h_4	([0.2, 0.4],[0.3, 0.5],[0, 1])	([0.3, 0.4],[0.4, 0.5],[0.3,0.4])	([0.3, 0.4],[0.2, 0.3],[0,0.5])	([0, 0.2],[0.4, 0.5],[0.6,0.7])

Q:

U	(e_1, e_2)	(e_1, e_4)	(e_3, e_2)	(e_3, e_4)
H1	([0.5, 0.8],[0.1, 0.2],[0.1,0.2])	([0.2, 0.3],[0.3, 0.6],[0.3,0.4])	([0.2 0.5],[0.3, 0.5],[0.2,0.4])	([0.2, 0.4],[0.2, 0.3],[1, 1])
h_2	([0.4, 0.5],[0.2, 0.4],[1, 1])	([0.4, 0.6],[0.2, 0.3],[0.2,0.4])	([0.4, 0.5],[0.4, 0.5],[0.2,0.5])	([0.4, 0.5],[0.1, 0.2],[1, 1])
h_3	([0.2, 0.3],[0.5, 0.6],[0.2,0.4])	([0.3, 0.4],[0.4, 0.5],[1, 1])	([0.7, 0.8],[0.1, 0.2],[0.2,0.5])	([0.3, 0.5],[0.3, 0.4],[0,0.4])
h_4	([0.3, 0.5],[0.3, 0.4],[0, 1])	([0.3, 0.5],[0.2, 0.4],[0.1,0.2])	([0.2, 0.4],[0.2, 0.3],[0,0.5])	([0.3, 0.7],[0.1, 0.3],[0.6,0.7])

Then

P*Q

U	(e_1, e_2)	(e_1, e_4)	(e_3, e_2)	(e_3, e_4)
H1	([0.3, 0.4],[0.3, 0.4],[0.1,0.2])	([0.2, 0.4],[0.3, 0.5],[0.2,0.3])	([0.2 0.5],[0.3, 0.4],[0.2,0.4])	([0.2, 0.3],[0.2, 0.6],[0.3, 0.4])
h_2	([0.4, 0.5],[0.2, 0.4],[0.3, 0.5])	([0.1, 0.6],[0.1, 0.2],[0.2,0.5])	([0.4, 0.5],[0.2, 0.4],[0.2,0.5])	([0.4, 0.5],[0.1, 0.3],[0.3, 0.5])
h_3	([0.2, 0.6],[0.1, 0.3],[0,0.4])	([0.2, 0.5],[0.3, 0.4],[0.1, 0.4])	([0.2, 0.5],[0.2, 0.3],[0.2,0.4])	([0.2, 0.5],[0.3, 0.4],[0.2,0.5])
h_4	([0.2, 0.4],[0.3, 0.5],[0, 0.2])	([0.3, 0.4],[0.2, 0.5],[0.3,0.4])	([0.3, 0.4],[0.2, 0.4],[0.2,0.5])	([0.3, 0.4],[0.3, 0.4],[0.2,0.5])

Definition 19 : Let P ,Q $\in \sigma_U(Ax\ A)$ and P =(H,AxA), Q=(J,AxA) and the order of their relational matrices are same.Then the compostion of P and Q, denoted by P∘ Q is defined by P∘ Q =(H∘ J,AxA) where H∘ J :AxA → IVNS(U)

Is defined as $(H\circ J)\,(e_i,e_j) = \{<h_k, \mu_{(H\circ J)\,(e_i,e_j)}(h_k), \upsilon_{(H\circ J)(e_i,e_j)}(h_k), \omega_{(H\circ J)(e_i,e_j)}(h_k)>: h_k \in U\}$

Whre

$\mu_{(H\circ J)\,(e_i,e_j)}(h_k) = [\min_l(\max(inf\ \mu_{H(e_i,e_l)}(h_k), \inf \mu_{J(e_l,e_j)}(h_k))), \min_l(\max(sup\ \mu_{H(e_i,e_l)}(h_k), \sup \mu_{J(e_l,e_j)}(h_k)))]$,

$\upsilon_{(H\circ J)(e_i,e_j)}(h_k) = [\max_l(\min(inf\ \upsilon_{H(e_i,e_l)}(h_k), \inf \upsilon_{J(e_l,e_j)}(h_k))), \max_l(\min(sup\ \upsilon_{H(e_i,e_l)}(h_k), \sup \upsilon_{J(e_l,e_j)}(h_k)))]$,

And

$\omega_{(H\circ J)(e_i,e_j)}(h_k) = [\max_l(\min(inf\ \omega_{H(e_i,e_l)}(h_k), \inf \omega_{J(e_l,e_j)}(h_k))), \max_l(\min(sup\ \omega_{H(e_i,e_l)}(h_k), \sup \omega_{J(e_l,e_j)}(h_k)))]$

For $(e_i,e_j) \in$ A x A

Example 15 :Let U $=\{h_1, h_2, h_3, h_4\}$.let us consider the interval valued neutrosophic soft sets (F,A) and (G,A) where A=$\{e_1, e_2\}$ .Let P ,Q $\in \sigma_U(Ax\ A)$ and P =(H, AxA), Q=(J,AxA) where P:

U	(e_1, e_2)	(e_1, e_4)	(e_3, e_2)	(e_3, e_4)
H1	([0.3, 0.4],[0.3, 0.4],[0.1,0.2])	([0.2, 0.4],[0.3, 0.5],[0.3,0.4])	([0.2 0.5],[0.3, 0.4],[0.3,0.4])	([0.2, 0.3],[0.3, 0.6],[0.2, 0.3])
h_2	([1, 1],[0, 0],[1, 1])	([0.1, 0.2],[0, 0],[0.2,0.5])	([0.4, 0.5],[0.1, 0.3],[0.3,0.5])	([0.4, 0.7],[0.1, 0.3],[1, 1])
h_3	([0.2, 0.6],[0.1, 0.4],[0.3,0.4])	([0.2, 0.6],[0.1, 0.3],[1, 1])	([0.2, 0.3],[0.1, 0.3],[0.2,0.5])	([0.2, 0.5],[0.2, 0.3],[0,0.4])
h_4	([0.2, 0.4],[0.3, 0.5],[0, 1])	([0.3, 0.4],[0.4, 0.5],[0.3,0.4])	([0.3, 0.4],[0.2, 0.3],[0,0.5])	([0, 0.2],[0.4, 0.5],[0.6,0.7])

Q:

U	(e_1, e_2)	(e_1, e_4)	(e_3, e_2)	(e_3, e_4)
H1	([0.5, 0.8],[0.1, 0.2],[0.1,0.2])	([0.2, 0.3],[0.3, 0.6],[0.3,0.4])	([0.2 0.5],[0.3, 0.5],[0.2,0.4])	([0.2, 0.4],[0.2, 0.3],[1, 1])
h_2	([0.4, 0.5],[0.2, 0.4],[1, 1])	([0.4, 0.6],[0.2, 0.3],[0.2,0.4])	([0.4, 0.5],[0.4, 0.5],[0.2,0.5])	([0.4, 0.5],[0.1, 0.2],[1, 1])
h_3	([0.2, 0.3],[0.5, 0.6],[0.2,0.4])	([0.3, 0.4],[0.4, 0.5],[1, 1])	([0.7, 0.8],[0.1, 0.2],[0.2,0.5])	([0.3, 0.5],[0.3, 0.4],[0,0.4])
h_4	([0.3, 0.5],[0.3, 0.4],[0, 1])	([0.3, 0.5],[0.2, 0.4],[0.1,0.2])	([0.2, 0.4],[0.2, 0.3],[0,0.5])	([0.3, 0.7],[0.1, 0.3],[0.6,0.7])

Then

P∘Q

U	(e_1, e_2)	(e_1, e_4)	(e_3, e_2)	(e_3, e_4)
H1	([0.2, 0.5],[0.3, 0.4],[0.3,0.4])	([0.2, 0.4],[0.3, 0.4],[0.3,0.4])	([0.2 0.4],[0.3, 0.4],[0.2,0.4])	([0.2, 0.3],[0.2, 0.6],[0.3, 0.4])
h_2	([0.4, 0.5],[0.1, 0.3],[0.2, 0.5])	([0.4, 0.5],[0.1, 0.3],[0.3,0.5])	([0.4, 0.5],[0.1, 0.3],[0.2,0.5])	([0.4, 0.5],[0.1, 0.3],[1, 1])
h_3	([0.2, 0.3],[0.2, 0.3],[1,1])	([0.2, 0.4],[0.2, 0.4],[0.3, 0.5])	([0.2, 0.5],[0.2, 0.4],[0.2,0.5])	([0.2, 0.5],[0.1 0.3],[1,1])
h_4	([0.3, 0.4],[0.3, 0.4],[0.3, 0.7])	([0.2, 0.4],[0.3, 0.5],[0.2,0.5])	([0.2, 0.5],[0.4, 0.5],[0.2,0.5])	([0.2, 0.4],[0.2, 0.3],[0.6,0.7])

Definition 20 : **Let P** ∈ $\sigma_U(Ax\ A)$ and P =(H,AxA).Then P is called transitive IVNSS-relation if P*P ⊆ P, i.e $\cup(\mathrm{H}(e_{i,}e_l) \cap \mathrm{H}(e_{l,}e_j)) \subseteq \mathrm{H}(e_{i,}e_j)$, i.e,

Max(inf $\mu_{\mathrm{H}(e_{i,}e_l)}(\mathrm{h_k})$, inf $\mu_{\mathrm{H}(e_{l,}e_j)}(\mathrm{h_k})$) ≤ inf $\mu_{\mathrm{H}(e_{i,}e_j)}(\mathrm{h_k})$,

Max(sup $\mu_{\mathrm{H}(e_{i,}e_l)}(\mathrm{h_k})$, sup $\mu_{\mathrm{H}(e_{l,}e_j)}(\mathrm{h_k})$) ≤ sup $\mu_{\mathrm{H}(e_{i,}e_j)}(\mathrm{h_k})$,

Min(inf $\upsilon_{\mathrm{H}(e_{i,}e_l)}(\mathrm{h_k})$, inf $\upsilon_{\mathrm{H}(e_{l,}e_j)}(\mathrm{h_k})$) ≤ inf $\upsilon_{\mathrm{H}(e_{i,}e_j)}(\mathrm{h_k})$,

Min(sup $\upsilon_{\mathrm{H}(e_{i,}e_l)}(\mathrm{h_k})$, sup $\upsilon_{\mathrm{H}(e_{l,}e_j)}(\mathrm{h_k})$) ≤ sup $\upsilon_{\mathrm{H}(e_{i,}e_j)}(\mathrm{h_k})$,

Min(inf $\omega_{\mathrm{H}(e_{i,}e_l)}(\mathrm{h_k})$, inf $\omega_{\mathrm{H}(e_{l,}e_j)}(\mathrm{h_k})$) ≤ inf $\omega_{\mathrm{H}(e_{i,}e_j)}(\mathrm{h_k})$,

Min(sup $\omega_{\mathrm{H}(e_{i,}e_l)}(\mathrm{h_k})$, sup $\omega_{\mathrm{H}(e_{l,}e_j)}(\mathrm{h_k})$) ≤ sup $\omega_{\mathrm{H}(e_{i,}e_j)}(\mathrm{h_k})$,

Example 16: Let U ={h_1 , h_2 , h_3, h_4}.let us consider the interval valued neutrosophic soft sets (F,A) and (G,A) where A={e_1 , e_2 } .Let P ,Q ∈ $\sigma_U(Ax\ A)$ and P =(H, AxA), Q=(J,AxA) where P:

U	(e_1, e_2)	(e_1, e_4)	(e_3, e_2)	(e_3, e_4)
H1	([0.3, 0.4],[0.3, 0.4],[0.1,0.2])	([0.2, 0.4],[0.3, 0.5],[0.3,0.4])	([0.2 0.5],[0.3, 0.4],[0.2,0.4])	([0.2, 0.3],[0.3, 0.6],[1, 1])
h_2	([1, 1],[0, 0],[1, 1])	([0.1, 0.2],[0, 0],[0.2,0.4])	([0.4, 0.5],[0.1, 0.3],[0.2,0.5])	([0.4, 0.7],[0.1, 0.3],[1, 1])
h_3	([0.2, 0.6],[0.1, 0.4],[0.2,0.4])	([0.2, 0.6],[0.1, 0.3],[1, 1])	([0.2, 0.3],[0.1, 0.3],[0.2,0.5])	([0.2, 0.5],[0.2, 0.3],[0,0.4])
h_4	([0.2, 0.4],[0.3, 0.5],[0, 1])	([0.3, 0.4],[0.4, 0.5],[0.1,0.2])	([0.3, 0.4],[0.2, 0.3],[0,0.5])	([0, 0.2],[0.4, 0.5],[0.6,0.7])

Then P*P

U	(e_1, e_2)	(e_1, e_4)	(e_3, e_2)	(e_3, e_4)
H1	([0.3, 0.4],[0.3, 0.4],[0.1,0.2])	([0.2, 0.4],[0.3, 0.5],[0.3,0.4])	([0.2 0.5],[0.3, 0.4],[0.2,0.4])	([0.2, 0.3],[0.3, 0.6],[1, 1])
h_2	([1, 1],[0, 0],[1, 1])	([0.1, 0.2],[0, 0],[0.2,0.4])	([0.4, 0.5],[0.1, 0.3],[0.2,0.5])	([0.4, 0.7],[0.1, 0.3],[1, 1])
h_3	([0.2, 0.6],[0.1, 0.4],[0.2,0.4])	([0.2, 0.6],[0.1, 0.3],[1, 1])	([0.2, 0.3],[0.1, 0.3],[0.2,0.5])	([0.2, 0.5],[0.2, 0.3],[0,0.4])
h_4	([0.2, 0.4],[0.3, 0.5],[0, 1])	([0.3, 0.4],[0.4, 0.5],[0.1,0.2])	([0.3, 0.4],[0.2, 0.3],[0,0.5])	([0, 0.2],[0.4, 0.5],[0.6,0.7])

Thus, P * P ⊆ P and so P is a transitive IVNSS-relation.

Definition 21 . Let P ∈ $\sigma_U(Ax\ A)$ and P =(H,AxA).Then P is called equivalence IVNSS-relation if P satisfies the following conditions:

1) **Reflexivity (see definition 13).**
2) **Symmetry (see definition 15).**
3) **Transitivity (see definition 20).**

Example 17: Let U ={h_1 , h_2 , h_3}.let us consider the interval valued neutrosophic soft sets (F,A) where A={e_1 , e_2 } .Let P ,Q ∈ $\sigma_U(Ax\ A)$ and P =(H, AxA), where P:

U	(e_1, e_2)	(e_1, e_4)	(e_3, e_2)
h_1	([1, 1],[0, 0],[0, 0])	([0.2, 0.3],[0.2, 0.4],[0.3,0.4])	([0.1, 0.5],[0.2, 0.4],[0.2,0.3])
h_2	([0.2, 0.3],[0.4, 0.6],[0.3,0.4])	([1, 1],[0, 0],[0, 0])	([0.2, 0.3],[0.1, 0.5],[0.2,0.3])
h_3	([0.1, 0.5],[0.2, 0.4],[0.2,0.3])	([0.2, 0.3],[0.1, 0.5],[0.2,0.3])	([1, 1],[0, 0],[0, 0])

P*P

U	(e_1, e_2)	(e_1, e_4)	(e_3, e_2)
h_1	([1, 1],[0, 0],[0, 0])	([0.2, 0.3],[0.2, 0.4],[0.3,0.4])	([0.1, 0.5],[0.2, 0.4],[0.2,0.3])
h_2	([0.2, 0.3],[0.4, 0.6],[0.3,0.4])	([1, 1],[0, 0],[0, 0])	([0.2, 0.3],[0.1, 0.5],[0.2,0.3])
h_3	([0.1, 0.5],[0.2, 0.4],[0.2,0.3])	([0.2, 0.3],[0.1, 0.5],[0.2,0.3])	([1, 1],[0, 0],[0, 0])

Then P is equivalence IVNSS-relation

Conclusions

In this paper we have defined, for the first time, the notion of interval neutrosophic soft relation. We have studied some properties for interval neutrosophic soft relation.We hope that this paper will promote the future study on IVNSS and IVNSS relation to carry out a general framework for their application in practical life.

Acknowledgements

The authors are very grateful to the anonymous referees for their insightful and constructive comments and suggestions, which have been very helpful in improving the paper.

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Several Similarity Measures of Neutrosophic Sets

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Abstract- Smarandache (1995) defined the notion of neutrosophic sets, which is a generalization of Zadeh's fuzzy set and Atanassov's intuitionistic fuzzy set. In this paper, we first develop some similarity measures of neutrosophic sets. We will present a method to calculate the distance between neutrosophic sets (NS) on the basis of the Hausdorff distance. Then we will use this distance to generate a new similarity measure to calculate the degree of similarity between NS. Finally we will prove some properties of the proposed similarity measures.

Keywords- **Neutrosophic Set, Matching Function, Hausdorff Distance, Similarity Measure.**

I-Introduction

Smarandache introduced a concept of neutrosophic set which has been a mathematical tool for handling problems involving imprecise, indeterminacy, and inconsistent data [1, 2].The concept of similarity is fundamentally important in almost every scientific field. Many methods have been proposed for measuring the degree of similarity between fuzzy sets (Chen, [11]; Chen et al., [12]; Hyung, Song, & Lee, [14]; Pappis&Karacapilidis, [10]; Wang, [13]...). But these methods are unsuitable for dealing with the similarity measures of neutrosophic set (NS). Few researchers have dealt with similarity measures for neutrosophic set ([3, 4]). Recently, Jun [3] discussed similarity measures on interval neutrosophic set (which an instance of NS) based on Hamming distance and Euclidean distance and showed how these measures may be used in decision making problems. Furthermore, A.A.Salama [4] defined the correlation coefficient, on the domain of neutrosophic sets, which is another kind of similarity measurement. In this paper we first extend the Hausdorff distance to neutrosophic set which plays an important role in practical application, especially in many visual tasks, computer assisted surgery and so on. After that a new series of similarity measures has been proposed for neutrosophic set using different approaches.

Similarity measures have extensive application in several areas such as pattern recognition, image processing, region extraction, psychology [5], handwriting recognition [6], decision making [7], coding theory etc.

This paper is organized as follows: Section2 briefly reviews the definition of Hausdorff distance and the neutrosophic set. Section 3 presents the new extended Hausdorff distance between neutrosophic sets. Section 4 provides the new series of similarity measure between neutrosophic sets, some of its properties are discussed. In section 5 a comparative study was done. Finally the section 6 outlines some conclusions.

II-Preliminaries

In this section we briefly review some definitions and examples which will be used in rest of the paper.

Definition 2.1: Hausdorff Distance

The Hausdorff distance (Nadler, 1978) is the maximum distance of a set to the nearest point in the other set. More formal description is given by the following

Given two finite sets $A = \{a_1, ..., a_p\}$ and $B = \{b_1, ..., b_q\}$, the Hausdorff distance H (A, B) is defined as:

$$H(A, B) = \max \{h(A, B), h(B, A)\}$$

where

$$H(A, B) = \max_{a \in A} \min_{b \in B} d(a, b)$$

a and b are elements of sets A and B respectively; d (a, b) is any metric between these elements.

The two distances h (A, B) and h (B, A) are called directed Hausdorff distances.

The function h (A, B) (the directed Hausdorff distance from A to B) ranks each element of A based on its distance to the nearest element of B, and then the largest ranked such element (the most mismatched element of A) specifies the value of the distance. Intuitively, if h(A, B) = c, then each element of A must be within distance c of some element of B, and there also is some element of A that is exactly distance c from the nearest element of B (the most mismatched element). In general h (A, B) and h (B, A) can attain very different values (the directed distances are not symmetric).

Let us consider the real space R, for any two intervals A= $[a_1,a_2]$ and B= $[b_1,b_2]$, the Hausdorff distance H(A,B) is given by

$$H(A, B) = \max\{|a_1 - b_1|, |a_2 - b_2|\}$$

Definition 2.2 (see [2]). Let U be an universe of discourse then the neutrosophic set A is an object having the form A = {< x: $T_{A(x)}, I_{A(x)}, F_{A(x)}$ >,x ∈ U}, where the functions T, I, F : U→$]^-0,1^+[$ define respectively the degree of membership (or Truth) , the degree of indeterminacy, and the degree of non-membership (or Falsehood) of the element x ∈ U to the set A with the condition.

$$^-0 \leq T_{A(x)} + I_{A(x)} + F_{A(x)} \leq 3^+.$$

From philosophical point of view, the neutrosophic set takes the value from real standard or non-standard subsets of $]^-0,1^+[$. So instead of $]^-0,1^+[$ we need to take the interval [0,1] for technical applications, because $]^-0,1^+[$will be difficult to apply in the real applications such as in scientific and engineering problems.

Definition 2.3 (see [2]). A neutrosophic set A is contained in another neutrosophic set B i.e. A ⊆ B if ∀x ∈ U, $T_A(x) \leq T_B(x)$, $I_A(x) \geq I_B(x)$, $F_A(x) \geq F_B(x)$.

Definition 2.4 (see [2]). The complement of a neutrosophic set A is denoted by A^c and is defined as $T_{A^c}(x) = T_{A(x)}$, $I_{A^c}(x) = I_{A(x)}$, and $F_{A^c}(x) = F_{A(x)}$ for every x in X.

A complete study of the operations and application of neutrosophic set can be found in [1] [2].

In this paper we are concerned with neutrosophic sets whose T_A, I_A and F_A values are single points in [0, 1] instead of subintervals/subsets in [0, 1].

III. Extended Hausdorff Distance Between Two Neutrosophic Sets

Based on the Hausdorff metric, Eulalia Szmidt and Janusz Kacprzyk defined a new distance between intuitionistic fuzzy sets and/or interval-valued fuzzy sets in[8], taking into account three parameter representation (membership, non-membership values, and the hesitation margins) of A-IFSs which fulfill the properties of the Hausdorff distances. Their definition is defined by:

$$H_3(A,B) = \frac{1}{n}\sum_{i=1}^{n} max\{|\mu_A(x) - \mu_B(x)|, |\nu_A(x) - \nu_B(x)|, |\pi_A(x) - \pi_B(x)|\}$$

where A = {< x, $\mu_A(x)$, $\nu_A(x)$, $\pi_A(x)$ >} and B = {< x, $\mu_B(x)$, $\nu_B(x)$, $\pi_B(x)$>}.

The terms and symbols used in [8] are changed so that they are consistent with those in this section.

In this paper we are interested in extending the Hausdorff distance formulation in constructing a new distance for neutrosophic set due to its simplicity in the calculation.

Let X={$x_1, x_2, \ldots, x_n$} be a discrete finite set. Consider a neutrosophic set A in X, where $T_{A(xi)}$, $I_{A(xi)}$, $F_{A(xi)}$ ∈ [0, 1], for every x_i ∈ X, represent its membership, indeterminacy, and non-membership values respectively denoted by A = {< x, $T_{A(xi)}$, $I_{A(xi)}$, $F_{A(xi)}$ >}.

Then we propose a new distance between A ∈ NS and B ∈ NS defined by

$$d_H(A,B) = \frac{1}{n}\sum_{i=1}^{n} \max\{|T_A(x_i) - T_B(x_i)|, |I_A(x_i) - I_B(x_i)|, |F_A(x_i) - F_B(x_i)|\}$$

Where $d_H(A,B) =$ H (A, B) denote the extended Hausdorff distance between two neutrosophic sets A and B.

Let A, B and C be three neutrosophic sets for all x_i ∈ X we have:

$d_H(A,B)$ = H (A, B) = max $\{|T_A(x_i) - T_B(x_i)|, |I_A(x_i) - I_{B(x_i)}|, |F_A(x_i) - F_B(x_i)|\}$

The same between A and C are written as:

For all x_i ∈ X

H (A, C) = max $\{|T_A(x_i) - T_C(x_i)|, |I_A(x_i) - I_C(x_i)|, |F_A(x_i) - F_C(x_i)|\}$

and between B and C is written as:

For all $x_i \in X$

H (B , C) = max $\{|T_B(x_i) - T_C(x_i)|, |I_B(x_i) - I_C(x_i)|, |F_B(x_i) - F_C(x_i)|\}$

Proposition 3.1:

The above defined distance $d_H(A,B)$ between NS A and B satisfies the following properties (D1-D4):

(D1) $d_H(A,B) \geq 0$.

(D2) $d_H(A,B) = 0$ if and only if A = B; for all A, B ∈ NS.

(D3) $d_H(A,B) = d_H(B,A)$.

(D4) If A⊆B⊆C, C is an NS in X, then

$$d_H(A,C) \geq d_H(A,B)$$

And

$$d_H(A,C) \geq d_H(B,C)$$

Remark: Let A, B ∈ NS, A⊆ B if and only if , for all x_i in X

$$T_A(x_i) \leq T_B(x_i), I_A(x_i) \geq I_B(x_i), F_A(x_i) \geq F_B(x_i)$$

It is easy to see that the defined measure $d_H(A,B)$ satisfies the above properties (D1)-(D3). Therefore, we only prove (D4).

Proof of (D4) for the extended Hausdorff distance between two neutrosophic sets. Since

A⊆ B ⊆ C implies , for all x_i in X $T_A(x_i) \leq T_B(x_i) \leq T_C(x_i), I_A(x_i) \geq I_B(x_i) \geq I_C(x_i), F_A(x_i) \geq F_B(x_i) \geq F_C(x_i)$

We prove that $d_H(A,B) \leq d_H(A,C)$

α - If $|T_A(x_i) - T_C(x_i)| \geq |I_A(x_i) - I_C(x_i)| \geq |F_A(x_i) - F_C(x_i)|$

Then

H (A, C) = $|T_A(x_i) - T_C(x_i)|$ but we have

(i) For all x_i in X, $|I_A(x_i) - I_B(x_i)| \leq |I_A(x_i) - I_C(x_i)| \leq |T_A(x_i) - T_C(x_i)|$

And , for all x_i in X $|F_A(x_i) - F_B(x_i)| \leq |F_A(x_i) - F_C(x_i)| \leq |T_A(x_i) - T_C(x_i)|$

(ii) For all x_i in X, $|I_B(x_i) - I_C(x_i)| \leq |I_A(x_i) - I_C(x_i)| \leq |T_A(x_i) - T_C(x_i)|$

And ,for all x_i in X $|F_B(x_i) - F_C(x_i)| \leq |F_A(x_i) - F_C(x_i)| \leq |T_A(x_i) - T_C(x_i)|$

On the other hand we have, for all x_i in X

(iii) $|T_A(x_i) - T_B(x_i)| \leq |T_A(x_i) - T_C(x_i)|$
and $|T_B(x_i) - T_C(x_i)| \leq |T_A(x_i) - T_C(x_i)|$

Combining (i), (ii), and (iii) we obtain

Therefore, for all x_i in X

$\frac{1}{n}\sum_1^n \max\{|T_A(x_i) - T_B(x_i)|, |I_A(x_i) - I_B(x_i)|, |F_A(x_i) - F_B(x_i)|\} \leq \frac{1}{n}\sum_1^n \max\{|T_A(x_i) - T_C(x_i)|, |I_A(x_i) - I_C(x_i)|, |F_A(x_i) - F_C(x_i)|\}$

And

$\frac{1}{n}\sum_1^n \max\{|T_B(x_i) - T_C(x_i)|, |I_B(x_i) - I_C(x_i)|, |F_B(x_i) - F_C(x_i)|\} \leq \frac{1}{n}\sum_1^n \max\{|T_A(x_i) - T_C(x_i)|, |I_A(x_i) - I_C(x_i)|, |F_A(x_i) - F_C(x_i)|\}$

That is

$$d_H(A,B) \leq d_H(A,C) \text{ and } d_H(B,C) \leq d_H(A,C).$$

β - If $|\mathbf{T_A(x_i)} - \mathbf{T_C(x_i)}| \leq |\mathbf{F_A(x_i)} - \mathbf{F_C(x_i)}| \leq |\mathbf{I_A(x_i)} - \mathbf{I_C(x_i)}|$

Then

H (A, C) = $|I_A(x_i) - I_C(x_i)|$ but we have for all x_i in X

(a) $|T_A(x_i) - T_B(x_i)| \le |T_A(x_i) - T_C(x_i)|$
$\le |I_A(x_i) - I_C(x_i)|$

And $|F_A(x_i) - F_B(x_i)| \le |F_A(x_i) - F_C(x_i)|$
$\le |I_A(x_i) - I_C(x_i)|$

(b) $|T_B(x_i) - T_C(x_i)| \le |T_A(x_i) - T_C(x_i)|$
$\le |I_A(x_i) - I_C(x_i)|$

And $|F_B(x_i) - F_C(x_i)| \le |F_A(x_i) - F_C(x_i)|$
$\le |I_A(x_i) - I_C(x_i)|$

On the other hand we have for all $x_i \in X$:

(c) $|I_A(x_i) - I_B(x_i)| \le |I_A(x_i) - I_C(x_i)|$ and
$|I_B(x_i) - I_C(x_i)| \le |I_A(x_i) - I_C(x_i)|$

Combining (a) and (c) we obtain:

Therefore, for all x_i in X

$$\frac{1}{n}\sum_1^n \max\{|T_A(x_i) - T_B(x_i)|, |I_A(x_i) - I_B(x_i)|, |F_A(x_i) - F_B(x_i)|\} \le \frac{1}{n}\sum_1^n \max\{|T_A(x_i) - T_C(x_i)|, |I_A(x_i) - I_C(x_i)|, |F_A(x_i) - F_C(x_i)|\}$$

And

$$\frac{1}{n}\sum_1^n \max\{|T_B(x_i) - T_C(x_i)|, |I_B(x_i) - I_C(x_i)|, |F_B(x_i) - F_C(x_i)|\} \le \frac{1}{n}\sum_1^n \max\{|T_A(x_i) - T_C(x_i)|, |I_A(x_i) - I_C(x_i)|, |F_A(x_i) - F_C(x_i)|\}$$

That is

$d_H(A,B) \le d_H(A,C)$ and $d_H(B,C) \le d_H(A,C)$

$\boldsymbol{\gamma}$ - If $|T_A(x_i) - T_C(x_i)| \le |I_A(x_i) - I_C(x_i)| \le |F_A(x_i) - F_C(x_i)|$

Then

H (A, C) = $|F_A(x_i) - F_C(x_i)|$ but we have for all x_i in X

(a) $|T_A(x_i) - T_B(x_i)| \le |T_A(x_i) - T_C(x_i)|$
$\le |F_A(x_i) - F_C(x_i)|$

and $|I_A(x_i) - I_B(x_i)| \le |I_A(x_i) - I_C(x_i)|$
$\le |F_A(x_i) - F_C(x_i)|$

(b) for all x_i in X $|T_B(x_i) - T_C(x_i)| \le |T_A(x_i) - T_C(x_i)|$
$\le |F_A(x_i) - F_C(x_i)|$

and for all x_i in X $|I_B(x_i) - I_C(x_i)| \le |I_A(x_i) - I_C(x_i)|$
$\le |F_A(x_i) - F_C(x_i)|$

On the other hand we have for all x_i in X

(c) $|F_A(x_i) - F_B(x_i)| \le |F_A(x_i) - F_C(x_i)|$ and
$|F_B(x_i) - F_C(x_i)| \le |F_A(x_i) - F_C(x_i)|$

Combining (a), (b), and (c) we obtain

Therefore, for all x_i in X

$$\frac{1}{n}\sum_1^n \max\{|T_A(x_i) - T_B(x_i)|, |I_A(x_i) - I_B(x_i)|, |F_A(x_i) - F_B(x_i)|\} \le \frac{1}{n}\sum_1^n \max\{|T_A(x_i) - T_C(x_i)|, |I_A(x_i) - I_C(x_i)|, |F_A(x_i) - F_C(x_i)|\}.$$

And

$$\frac{1}{n}\sum_1^n \max\{|T_B(x_i) - T_C(x_i)|, |I_B(x_i) - I_C(x_i)|, |F_B(x_i) - F_C(x_i)|\} \le \frac{1}{n}\sum_1^n \max\{|T_A(x_i) - T_C(x_i)|, |I_A(x_i) - I_C(x_i)|, |F_A(x_i) - F_C(x_i)|\}$$

That is

$d_H(A,B) \le d_H(A,C)$ and $d_H(B,C) \le d_H(A,C)$.

From α, β , and γ, we can obtain the property (D4).

3.2 Weighted Extended Hausdorff Distance Between Two Neutrosophic Sets.

In many situations the weight of the element $x_i \in X$ should be taken into account. Usually the elements have different importance. We need to consider the weight of the element so that we have the following weighted distance between NS. Assume that the weight of $x_i \in X$ is w_i where $X=\{x_1, x_2,.., x_n\}$, $w_i \in [0,1]$, $i=\{1,2,3,.., n\}$ and $\sum_1^n w_i = 1$. Then the weighted extended Hausdorff distance between NS A and B is defined as:

$$d_{Hw}(A,B) = \sum_1^n w_i \, d_H(A(x_i), B(x_i))$$

It is easy to check that $d_{Hw}(A,B)$ satisfies the four properties D1-D4 defined above.

IV. Some new similarity measures for neutrosophic sets

The distance measure between two NS is used in finding the similarity between neutrosophic sets.

We found in the literature different similarity measures, and we extend them to neutrosophic sets (NS), several of them defined below:

Liu [9] also gave an axiom definition for the similarity measure of fuzzy sets, which also can be expressed for neutrosophic sets (NS) as follow:

Definition 4.1*: *Axioms of a Similarity Measure

A mapping $S:NS(X)\times NS(X)\rightarrow[0,1]$, NS(X) denotes the set of all NS in $X=\{x_1,x_2,\ldots,x_n\}$, S(A, B) is said to be the degree of similarity between $A\in NS$ and $B \in NS$, if S(A,B) satisfies the properties of conditions (P1-P4):

(**P1**) S (A, B) = S (B, A).
(**P2**) S(A,B) = (1,0,0) = $\underline{1}$ .If A = B for all A,B ∈ NS.
(**P3**) $S_T(A,B) \geq 0$, $S_I(A,B) \geq 0$, $S_F(A,B) \geq 0$.
(**P4**) If A B C for all A, B, C ∈ NS, then S (A, B) ≥S (A, C) and S (B, C) ≥ S (A, C).

Numerical Example:

Let $A \leq B \leq C$. with $T_A \leq T_B \leq T_C$ and $I_A \geq I_B \geq I_C$ and $F_A \geq F_B \geq F_C$ for each $x_i \in$ NS.

For example:
A= { x_1 (0.2, 0.5, 0.6); x_2 (0.2, 0.4, 0.4) }
B= { x_1 (0.2, 0.4, 0.4); x_2 (0.4, 0.2, 0.3) }
C= { x_1 (0.3, 0.3, 0.4); x_2 (0.5, 0.0, 0.3) }
In the following we define a new similarity measure of neutrosophic set and discuss its properties.

4.2 Similarity Measures Based on the Set –Theoretic Approach.

In this section we extend the similarity measure for intuitionistic and fuzzy set defined by Hung and Yung [16] to neutrosophic set which is based on set-theoretic approach as follow.

Definition 4.2: Let A,B be two neutrosophic sets in $X=\{x_1,x_2,.., x_n\}$, if $A = \{< x, T_A(x_i), I_A(x_i), F_A(x_i) >\}$ and $B= \{< x, T_B(x_i), I_B(x_i), F_B(x_i) >\}$ are neutrosophic values of X in A and B respectively, then the similarity measure between the neutrosophic sets A and B can be evaluated by the function

For all x_i in X

$$S_T(A,B) = \left(\sum_1^N \left[\frac{min(T_A(x_i),T_B(x_i))}{Max(T_A(x_i),T_B(x_i))}\right]\right)/n$$

$$S_I(A,B) = 1 - \left(\sum_1^N \left[\frac{min(I_A(x_i),I_B(x_i))}{Max(I_A(x_i),I_B(x_i))}\right]\right)/n$$

$$S_F(A,B) = 1 - \left(\sum_1^N \left[\frac{min(F_A(x_i),F_B(x_i))}{Max(F_A(x_i),F_B(x_i))}\right]\right)/n$$

and $S(A,B) = (S_T(A,B), S_I(A,B), S_F(A,B))$ **eq. (1)**

where

$S_T(A,B)$ denote the degree of similarity (where we take only the T's).
$S_I(A,B)$ denote the degree of indeterminate similarity (where we take only the I's).
$S_F(A,B)$ denote degree of nonsimilarity (where we take only the F's).
Min denotes the minimum between each element of A and B.
Max denotes the minimum between each element of A and B.

Proof of (P4) for the eq. (1).

Since A⊆B⊆C implies, for all x_i in X

$T_A(x_i) \leq T_B(x_i) \leq T_C(x_i), I_A(x_i) \geq I_B(x_i) \geq I_C(x_i), F_A(x_i) \geq F_B(x_i) \geq F_C(x_i)$

Then, for all x_i in X

$$\frac{min\big(T_A(x_i), T_B(x_i)\big)}{Max\big(T_A(x_i), T_B(x_i)\big)} = \frac{T_A(x_i)}{T_B(x_i)}$$
$$\frac{min\big(T_A(x_i), T_C(x_i)\big)}{Max\big(T_A(x_i), T_C(x_i)\big)} = \frac{T_A(x_i)}{T_C(x_i)}$$
$$\frac{min\big(T_B(x_i), T_C(x_i)\big)}{Max\big(T_B(x_i), T_C(x_i)\big)} = \frac{T_B(x_i)}{T_C(x_i)}$$

Therefore, for all x_i in X

$$\frac{T_A(x_i)}{T_C(x_i)} = \frac{T_B(x_i)}{T_C(x_i)} + \frac{T_A(x_i)-T_B(x_i)}{T_C(x_i)} \leq \frac{T_B(x_i)}{T_C(x_i)} \quad (1)$$

(since $T_A(x_i) \leq T_B(x_i)$)

Furthermore, for all x_i in X

$$\frac{\boldsymbol{min}\big(\boldsymbol{T_A(x_i)}, \boldsymbol{T_B(x_i)}\big)}{\boldsymbol{Max}\big(\boldsymbol{T_A(x_i)}, \boldsymbol{T_B(x_i)}\big)} \geq \frac{\boldsymbol{min}\big(\boldsymbol{T_A(x_i)}, \boldsymbol{T_C(x_i)}\big)}{\boldsymbol{Max}\big(\boldsymbol{T_A(x_i)}, \boldsymbol{T_C(x_i)}\big)} \quad (2)$$

Or

$$\frac{\boldsymbol{T_A(x_i)}}{\boldsymbol{T_B(x_i)}} \geq \frac{\boldsymbol{T_A(x_i)}}{\boldsymbol{T_C(x_i)}} \text{ or } T_B(x_i) \leq T_C(x_i)$$

(since $T_C(x_i) \geq T_B(x_i)$)

Inequality (2) implies that, for all x_i in X

$$\frac{\boldsymbol{T_A(x_i)}}{\boldsymbol{T_C(x_i)}} \leq \frac{\boldsymbol{T_A(x_i)}}{\boldsymbol{T_B(x_i)}} \qquad (3)$$

From the inequalities (1) and (3), the property (P4) for $S_T(A,B) \geq S_T(A,C)$ is proven.

In a similar way we can prove that $S_I(A,B)$ and $S_F(A,B)$.

We will to prove that $S_I(A,C) \geq S_I(A,B)$. For all $x_i \in$ X we have:

$S_I(A,C) = 1 - \frac{min\big(I_A(x_i), I_C(x_i)\big)}{Max\big(I_A(x_i), I_C(x_i)\big)} = 1 - \frac{I_C(x_i)}{I_A(x_i)} \geq 1 - \frac{I_B(x_i)}{I_A(x_i)}$

Since $I_C(x_i) \leq I_B(x_i)$

Similarly we prove $S_F(A,C) \geq S_F(A,B)$ for all x_i in X

$\boldsymbol{S_F(A,C) = 1 - \frac{min\big(F_A(x_i), F_C(x_i)\big)}{Max\big(F_A(x_i), F_C(x_i)\big)} = 1 - \frac{F_C(x_i)}{F_A(x_i)} \geq 1 - \frac{F_B(x_i)}{F_A(x_i)}}$

Since $F_C(x_i) \leq F_B(x_i)$

Then $S(A,C) \leq S(A,B)$ where $S(A,C)=(S_T(A,C), S_I(A,C), S_F(A,C))$ and $S(A,B) = (S_T(A,B), S_I(A,B), S_F(A,B))$.

In a similar way we can prove that S (B, C) ≥ S (A, C). If A⊆B⊆C therefore S (A, B) satisfies (P4) of definition 4.1.

By applying eq. (1), the degree of similarity between the neutrosophic sets (A, B), (A, C) and (B, C) are:

S(A, B) =($S_T(A,B)$, $S_I(A,B)$, $S_F(A,B)$)= (0.75, 0.35, 0.30)
S (A, C) = ($S_T(A,C)$, $S_I(A,C)$, $S_F(A,C)$)= (0.53, 0.7, 0.30)
S (B, C) = $S_T(B,C)$, $S_I(B,C)$, $S_F(B,C)$)= (0.73, 0.63, 0)

Then eq. (1) satisfies property P4: S(A, C) ≤ S(A, B) and S(A, C) ≤ S(B, C).

Usually, the weight of the element $x_i \in$ X should be taken into account, then we present the following weighted similarity between NS. Assume that the weight of $x_i \in$ X={1,2,…,n} is w_i (i=1,2,…, n) when $w_i \in [0,1], \sum_1^n w_i = 1$.

Denote $S_w^T(A,B) = (\sum_1^N w_i \left[\frac{min\big(T_A(x_i), T_B(x_i)\big)}{Max\big(T_A(x_i), T_B(x_i)\big)}\right])/n$

$$S_w^I(A,B) = 1 - (\sum_1^N w_i \left[\frac{min\big(I_A(x_i), I_B(x_i)\big)}{Max\big(I_A(x_i), I_B(x_i)\big)}\right])/n$$

$$S_w^F(A,B) = 1 - (\sum_1^N w_i \left[\frac{min\big(F_A(x_i), F_B(x_i)\big)}{Max\big(F_A(x_i), F_B(x_i)\big)}\right])/n$$

and $S_w(A,B) = (S_w^T((A,B), S_w^I((A,B), S_w^F((A,B))$

It is easy to check that $S_w(A,B)$ satisfies the four properties P1-P4 defined above.

4.3 Similarity Measure Based on the Type1 Geometric Distance Model

In the following, we express the definition of similarity measure between fuzzy sets based on the model of geometric distance proposed by Pappis and Karacapilidis in [10] to similarity of neutrosophic set.

Definition 4.3 : Let A,B be two neutrosophic sets in X={x_1, x_2,..., x_n}, if A = {< x, $T_A(x_i)$, $I_A(x_i)$, $F_A(x_i)$ >} and B= {< x, $T_B(x_i)$, $I_B(x_i)$, $F_B(x_i)$ >} are neutrosophic values of X in A and B respectively, then the similarity measure between the neutrosophic sets A and B can be evaluated by the function

For all x_i in X

$$L_T(A,B) = 1 - \frac{\sum_1^n |\mathrm{T_A(x_i)} - \mathrm{T_B(x_i)}|}{\sum_1^n (\mathrm{T_A(x_i)} + \mathrm{T_B(x_i)})}$$
$$L_I(A,B) = \frac{\sum_1^n |\mathrm{I_A(x_i)} - \mathrm{I_B(x_i)}|}{\sum_1^n (\mathrm{I_A(x_i)} + \mathrm{I_B(x_i)})}$$
$$L_F(A,B) = \frac{\sum_1^n |F_A(x_i) - F_B(x_i)|}{\sum_1^n (F_A(x_i) + F_B(x_i))} \quad \text{and}$$
$$L(A,B) = (L_T(A,B),\ L_I(A,B),\ L_F(A,B)) \quad \text{eq. (2)}$$

We will prove this similarity measure satisfies the properties 1-4 as above. The property (P1) for the similarity measure eq. (2) is obtained directly from the definition 4.1.

Proof: obviously, eq. (2) satisfies P1-P3-P4 of definition 4.1. In the following L (A, B) will be proved to satisfy (P2) and (P4).

Proof of (P2) for the eq. 2

For all x_i in X

First of all, $L_T(A,B) = 1 \leftrightarrow \frac{\sum_1^n |\mathrm{T_A(x_i)} - \mathrm{T_B(x_i)}|}{\sum_1^n (\mathrm{T_A(x_i)} + \mathrm{T_B(x_i)})} = 0$

$$\leftrightarrow |\mathrm{T_A(x_i)} - \mathrm{T_B(x_i)}| = 0$$
$$\leftrightarrow \mathrm{T_A(x_i)} = \mathrm{T_B(x_i)}$$

$$L_I(A,B) = 0 \leftrightarrow \frac{\sum_1^n |\mathrm{I_A(x_i)} - \mathrm{I_B(x_i)}|}{\sum_1^n (\mathrm{I_A(x_i)} + \mathrm{I_B(x_i)})} = 0$$
$$\leftrightarrow |\mathrm{I_A(x_i)} - \mathrm{I_B(x_i)}| = 0 \quad \leftrightarrow \mathrm{I_A(x_i)} = \mathrm{I_B(x_i)}$$
$$L_F(A,B) = 0 \leftrightarrow \frac{\sum_1^n |\mathrm{F_A(x_i)} - \mathrm{F_B(x_i)}|}{\sum_1^n (\mathrm{F_A(x_i)} + \mathrm{F_B(x_i)})} = 0$$
$$\leftrightarrow |\mathrm{F_A(x_i)} - \mathrm{F_B(x_i)}| = 0 \leftrightarrow \mathrm{F_A(x_i)} = \mathrm{F_B(x_i)}$$

Then $\boldsymbol{L}(\mathrm{A,B}) = (\mathrm{L_T(A,B)},\ \mathrm{L_I(A,B)},\ \mathrm{L_F(A,B)})$ = (1, 0, 0) if A=B for all A, B ∈ NS.

Proof of P3 for the eq .(2) is obvious.

By applying eq.2 the degree of similarity between the neutrosophic sets (A, B), (A, C) and (B, C) are:

L (A, B) =$(L_T(A,B), L_I(A,B), L_F(A,B))$= (0.8, 0.2, 0.17).
L (A, C) = $(L_T(A,C), L_I(A,C), L_F(A,C))$= (0.67, 0.5, 0.17).
L (B, C) = $(L_T(B,C), L_I(B,C), L_F(B,C))$= (0.85, 0.33, 0).

The result indicates that the degree of similarity between neutrosophic sets A and B ∈ [0, 1]. Then Eq.(2)satisfies property P4**:** L(A, C) ≤ L(A, B) and L(A, C) ≤ L(B, C).

4.4 Similarity Measure Based on the Type 2 Geometric Distance model

In this section we extend the similarity measure proposed by Yang and Hang [16] to neutrosophic set as follow:

Definition 4.4**:** Let A, B be two neutrosophic set in X={x_1,x_2,.., x_n}, if A = {< x, $T_A(x_i)$, $I_A(x_i)$, $F_A(x_i)$ >} and B= {<x, $T_B(x_i)$, $I_B(x_i)$, $F_B(x_i)$ >} are neutrosophic values of X in A and B respectively, then the similarity measure between the neutrosophic set A and B can be evaluated by the function:

For all x_i in X

M_T (A, B)= $\frac{1}{n}\sum_1^n (1 - \frac{|\mathrm{T_A(x_i)} - \mathrm{T_B(x_i)}|}{2})$.
M_I (A, B)= $\frac{1}{n}\sum_1^n (\frac{|\mathrm{I_A(x_i)} - \mathrm{I_B(x_i)}|}{2})$.
M_F (A, B)= $\frac{1}{n}\sum_1^n (\frac{|\mathrm{F_A(x_i)} - \mathrm{F_B(x_i)}|}{2})$.
And $\mathrm{M_{T,I,F}} = (\mathrm{M_T(A,B)}, \mathrm{M_I(A,B)}, \mathrm{M_F(A,B)})$ for all i={x_1,x_2 ,.., x_n} eq. (3)

The proofs of the properties P1-P2-P3 in definition 4.1 (Axioms of a Similarity Measure) of the similarity measure in definition 4.4 are obvious.

Proof of (P4) for the eq. (3).

Since for all x_i in X

$T_A(x_i) \le T_B(x_i) \le T_C(x_i), I_A(x_i) \ge I_B(x_i) \ge I_C(x_i), F_A(x_i) \ge F_B(x_i) \ge F_C(x_i)$ Then for all x_i in X

$$1 - \frac{|T_C(x_i) - T_A(x_i)|}{2} = 1 - \frac{(T_C(x_i) - T_A(x_i))}{2}$$
$$= 1 - \left(\frac{(T_C(x_i)-T_B(x_i))}{2} + \frac{(T_B(x_i)-T_A(x_i))}{2}\right)$$
$$\le 1 - \left(\frac{(T_C(x_i)-T_B(x_i))}{2}\right)$$
$$= 1 - \frac{|T_C(x_i)-T_B(x_i)|}{2}$$

Then M_T(A, C) $\le$ M_T(B, C).
Similarly, M_T(A, C) $\le$ M_T(A, B) can be proved easily.
For M_I(A, C) $\ge$ M_I(B, C) and M_F(A, C) $\ge$ M_F(B, C) the proof is easy.
Then by the definition 4.4, (P4) for definition 4.1, is satisfied as well.
By applying eq. (3), the degree of similarity between the neutrosophic sets (A, B), (A, C) and (B, C) are:

M(A,B)=(M_T (A,B), M_I (A,B), M_F (A,B))=(0.95 , 0.075 , 0.075)
M(A,C)= (M_T (A,C), M_I (A,C), M_F (A,C))=(0.9, 0.15 , 0.075)
M(B,C)= (M_T (B,C), M_I (B,C), M_F (B,C))=(0.9, 0.075 , 0)

Then eq. (3) satisfies property P4**:**
M (A, C) $\le$ M (A, B) and M (A, C) $\le$ M (B, C).

Another way of calculating similarity (degree) of neutrosophic sets is based on their distance. There are more approaches on how the relation between the two notions in form of a function can be expressed. Two of them are presented below (in section 4.5 and 4.6).

4.5 Similarity Measure Based on the Type3 Geometric Distance Model.

In the following we extended the similarity measure proposed by Koczy in [15] to neutrosophic set (NS).

Definition 4.5: Let A, B be two neutrosophic sets in X={x_1,x_2,.., x_n}, if A = {< x, $T_A(x_i)$, $I_A(x_i)$, $F_A(x_i)$ >} and B= {< x, $T_B(x_i)$, $I_B(x_i)$, $F_B(x_i)$ >} are neutrosophic values of x in A and B respectively, then the similarity measure between the neutrosophic sets A and B can be evaluated by the function

$\mathbf{H_T(A,B)} = \frac{\mathbf{1}}{\mathbf{1+d_\infty^T(A,B)}}$ denotes the degree of similarity.

$\mathbf{H_I(A,B)} = \mathbf{1} - \frac{\mathbf{1}}{\mathbf{1+d_\infty^I(A,B)}}$ denotes the degree of indeterminate similarity.

$\mathbf{H_F(A,B)} = \mathbf{1} - \frac{\mathbf{1}}{\mathbf{1+d_\infty^F(A,B)}}$ denotes degree of non-similarity

where $d_\infty^T(A,B)$, $d_\infty^I(A,B)$, and $d_\infty^F(A,B)$ are the distance measure of two neutrosophic sets **A** and B.

For all x_i in X

$d_\infty^T(A,B) = \max\{|T_A(x_i) - T_B(x_i)|\}$.

$d_\infty^I(A,B) = \max\{|I_A(x_i) - I_B(x_i)|\}$.

$d_\infty^F(A,B) = \max\{|F_A(x_i) - F_B(x_i)|\}$.

and H (A, B) = (H_T(A, B), H_I(A, B), H_F(A, B)). Eq. (4)

By applying the Eq. (4) in numerical example we obtain:

$d_\infty(A,B)$= (0.2, 0.2, 0.2), then H (A, B) = (0.83, 0.17, 0.17).

$d_\infty(A,C)$= (0.3, 0.4, 0.1), then H (A, C) = (0.76, 0.29, 0.17).

$d_\infty(B,C)$= (0.1, 0.2, 0), then H (B, C) = (0.90, 0.17, 0).

It can be verified that H (A, B) also has the properties (P1)-(P4).

4.6 Similarity Measure Based on Extended Hausdorff Distance

It is well known that similarity measures can be generated from distance measures. Therefore, we may use the proposed distance measure based on extended Hausdorff distance to define similarity measures. Based on the relationship of similarity measures and distance measures, we can define a new similarity measure between NS A and B as follows:

$$N(A,B) = 1 - d_H(A,B) \quad \text{eq. (5)}$$

where $d_H(A,B)$ represent the extended Hausdorff distance between neutrosophic sets (NS) A and B.

According to the above distance properties (D1-D4).It is easy to check that the similarity measure eq. (5) satisfies the four properties of axiom similarity defined in 4.1

By applying the eq. (5) in numerical example we obtain:

$N(A,B)$ =0.8
$N(A,C)$ =0.7
$N(B,C)$ =0.85
Then eq. (5) satisfies property P4**:**
N(A, C) $\leq$ N(A, B) and N(A, C) $\leq$ N(B, C)

Remark: It is clear that the larger the value of N(A, B), the more the similarity between NS A and B.
Next we define similarity measure between NS A and B using a matching function.

4.7 Similarity Measure of two Neutrosophic Sets Based on Matching Function.

Chen [11] and Chen et al. [12] introduced a matching function to calculate the degree of similarity between fuzzy sets. In the following, we extend the matching function extend to deal with the similarity measure of NS.

Definition 4.7 Let F and E be two neutrosophic sets over U. Then the similarity between them, denoted by K (F, G) or $K_{F,G}$ has been defined based on the matching function as:

For all x_i in X

$$K(F,G) = K_{F,G} = \frac{\sum_1^n (T_F(x_i)\cdot T_G(x_i) + I_F(x_i)\cdot I_G(x_i) + F_F(x_i)\cdot F_G(x_i))}{\max\left(\sum_1^n ((T_F(x_i))^2 + (I_F(x_i))^2 + (F_F(x_i))^2), \sum_1^n ((T_G(x_i))^2 + (I_G(x_i))^2 + (F_G(x_i))^2)\right)}$$

Eq. (6)

Considering the weight $w_j \in [0, 1]$ of each element $x_i \in X$, we get the weighting similarity measure between NS as:

For all x_i in X

$$K_w(F,G) = \frac{\sum_1^n w_i(T_F(x_i)\cdot T_G(x_i) + I_F(x_i)\cdot I_G(x_i) + F_F(x_i)\cdot F_G(x_i))}{\max\left(\sum_1^n w_i((T_F(x_i))^2 + (I_F(x_i))^2 + (F_F(x_i))^2), \sum_1^n w_i((T_G(x_i))^2 + (I_G(x_i))^2 + (F_G(x_i))^2)\right)}$$

Eq. (7)

If each element $x_i \in$ X has the same importance, then Eq.(7) is reduced to eq. (6). The larger the value of $K(F,G)$ the more the similarity between F and G. Here $K(F,G)$ has all the properties described as listed in the definition 4.1.

By applying the eq. (6) in numerical example we obtain:

$K(A,B) = 0.75$, $K(A,C) = 0.66$, and $K(B,C) = 0.92$

Then Eq. (6) satisfies property P4: K(A, C) $\leq$ K(A, B) and K(A, C) $\leq$ K(B, C)

V. Comparison of various similarity measures

In this section, we make a comparison among similarity measures proposed in the paper. Table I show the comparison of various similarity measures between two neutrosophic sets respectively.

Table I . Example results obtained from the similarity measures between neutrosophic sets A , B and C.

	A, B	A, C	B, C
Eq. (1)	(0.75, 0.35, 0.3)	(0.53, 0.7, 0.3)	(0.73, 0.63, 0)
Eq. (2)	(0.8, 0.2,0.17)	(0.67, 0.5, 0.17)	(0.85, 0.33, 0)
Eq. (3)	(0.95, 0.075, 0.075)	(0.9, 0.15, 0.075)	(0.9, 0.075, 0)
Eq. (4)	(0.83, 0.17, 0.17)	(0.76, 0.29, 0.17)	(0.9, 0.17, 0)
Eq. (5)	0.8	0.7	0.85
Eq. (6)	0.75	0.66	0.92

Each similarity measure expression has its own measuring, they all evaluate the similarities in neutrosophic sets, and they can meet all or most of the properties of similarity measure.

In definition 4.1, that is P1-P4. It seem from the table above that from the results of similarity measures between neutrosophic sets can be classified in two type of similarity measures: the first type which we called “crisp similarity measure” is illustrated by similarity measures (N and K) and the second type called “neutrosophic similarity measures” illustrated by similarity measures (S, L, M and H). The computation of measure **H , N** and **S** are much simpler than that of **L, M** and **K**

CONCLUSIONS

In this paper we have presented a new distance called "extended Hausdorff distance for neutrosophic sets" or "neutrosophic Hausdorff distance", then we defined a new series of similarity measures to calculate the similarity between neutrosophic sets. It’s hoped that our findings will help enhancing this study on neutrosophic set for researchers.

ACKNOWLEDGMENT

The authors are thankful to the anonymous referee for his valuable and constructive remarks that helped to improve the clarity and the completeness of this paper.

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Published in *Neutrosophic Sets and Systems*, 54-62, Vol. 1, 2013.

Soft Neutrosophic Left Almost Semigroup

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Abstract
In this paper we have extended neutrosophic LA-semigroup, neutrosophic sub LA-semigroup, neutrosophic ideals, neutosophic prime ideals, neutrosophic semiprime ideals, neutrosophic strong irreducible ideals to soft neutrosophic LA-semigroup,soft neutosophic sub LA-semigroup,soft neutrosophic ideals,soft neutrosophic prime ideals,soft neutrosophic semiprime ideals and soft strong irreducible neutrosophic ideals respectively. We have found some new notions related to the strong or pure part of neutrosophy and we give explaination with necessary illustrative examples. We have also given rigorious theorems and propositions. The notion of soft neutrosophic homomorphism is presented at the end.

Keywords
Neutrosophic LA-semigroup, neutrosophic sub LA-semigroup, neutrosophic ideal, soft LA-semigroup, soft LA-subsemigroup, soft ideal, soft neutrosophic LA-semigroup, soft sub-neutrosophic LA-semigroup, soft neutrosophic ideal.

1. INTRODUCTION

In 1995, Florentin Smarandache introduced the concept of neutrosophy. In neutrosophic logiceach proposition is approximated to have the percentage of truth in a subset T, the percentage of indeterminacy in a subset I, and the percentage of falsity in a subset F, so that this neutrosophic logic is called an extension of fuzzy logic. In fact neutrosophic set is the generalization of classical sets,conventional fuzzy set $\left[1\right]$, intuitionistic fuzzy set $\left[2\right]$ and interval valued fuzzy set $\left[3\right]$.This mathematical tool is used to handle problems like imprecise,indeterminacy and inconsistent data etc. By utilizing neutrosophic theory, Vasantha Kandasamy and Florentin Smarandache introduced neutrosophic algebraic structures in $\left[11\right]$. Some of them are neutrosophic fields, neutrosophic vector spaces, neutrosophic groups, neutrosophic bigroups, neutrosophic N-groups, neutrosophic semigroups, neutrosophic bisemigroups, neutrosophic N-semigroup, neutrosophic loops, neutrosophic biloops, neutrosophic N-loops, neutrosophic groupoids, and neutrosophic bigroupoids and so on.

Neutrosophic LA-semigroup is already introduced. It is basically a midway algebraic structure between neutrosophic groupoid and commutative neutrosophic semigroups. This is in fact a generalization of neutrosophic
semigroup theory. In neutrosophic LA-semigroup we have two basic types of notions and they are traditional notions as well as strong or pure neutrosophic notions. It is also an extension of LA-semigroup and involves the origin of neutralities orindeterminacy factor in LA-semigroup structure. This is a rich structure because of the indeterminacy's presence in all the corresponding notions of LA-semigroup and this property makes the differences between approaches of an LA-semigroup and a neutrosophic LA-semigroup. Molodstov introduced the concept of soft set theory which is free from the problems of parameterization inadequacy.

In his paper [11], he presented the fundamental results of new theory and successfully applied it into several directions such as smoothness of functions, game theory, operations research, Riemann-integration, Perron integration, theory of probability. After getting a high attention of researchers, soft set theory is applied in many fields successfully and so as in the field of LA-semigroup theory. A soft LA-semigroup means the parameterized collection of sub LA-semigroup over an LA-semigroup. It is more general concept than the concept of LA-semigroup.

We have further generalized this idea by adding neutrosophy and extended operations of soft set theory. In this paper we introduced the basic concepts of soft neutrosophic LA-semigroup. In the proceeding section we define soft neutrosophic LA-semigroup and characterized with some of their properties. Soft neutrosophic ideal over a neutrosophic LA-semigroup and soft neutrosophic ideal of a neutrosophic LA-semigroup is given in the further sections and studied some of their related results. In the last section, the concept of soft homomorphism of a soft LA-semigroup is extended to soft neutrosophic homomorphism of soft neutrosophic LA-semigroup.

2. PRELIMINARIES

2.1. Definition 1

Let $(S,*)$ be an LA-semigroup and let $\langle S \cup I \rangle = \{a+bI : a,b \in S\}$. The neutrosophic LA-semigroup is generated by S and I under $*$ denoted as $N(S) = \{\langle S \cup I \rangle, *\}$, where I is called theneutrosophic element with property $I^2 = I$ .For an integer n , $n+I$ and nI are neutrosophic elements and $0.I = 0$. I^{-1}, the inverse of I is not defined and hence does not exist.Similarly we can define neutrosophic RA-semigroup on the same lines.

Definition2 Let $N(S)$ be a neutrosophic LA-semigroup and $N(H)$ be a proper subset of $N(S)$. Then $N(H)$ is called a neutrosophic sub LA-semigroup if $N(H)$ itself is a neutrosophic LA-semigroup under the operation of $N(S)$.

Definition3A neutrosophic sub LA-semigroup $N(H)$ is called strong neutrosophic sub LA-semigroup or pure neutrosophic sub LA-semigroup if all the elements of $N(H)$ are neutrosophic elements.

Definition4 Let $N(S)$ be a neutrosophic LA-semigroup and $N(K)$ be a subset of $N(S)$. Then $N(K)$ is called Left (right) neutrosophic ideal of $N(S)$ if $N(S)N(K) \subseteq N(K)$, { $N(K)N(S) \subseteq N(K)$ }.If $N(K)$ is both left and right neutrosophic ideal, then $N(K)$ is called a two sided neutrosophic ideal or simply a neutrosophic ideal.

Definition5A neutorophic ideal $N(P)$ of a neutrosophic LA-semigroup $N(S)$ with left identity e is called prime neutrosophic ideal if $N(A)N(B) \subseteq N(P)$ implies either $N(A) \subseteq N(P)$ or $N(B) \subseteq N(P)$, where $N(A), N(B)$ are neutrosophic ideals of $N(S)$.

Definition 6A neutrosophic LA-semigroup $N(S)$ is called fully prime neutrosophic LA-semigroup if all of its neutrosophic ideals are prime neutrosophic ideals.

Definition7A neutrosophic ideal $N(P)$ is called semiprime neutrosophic ideal if $N(T).N(T) \subseteq N(P)$ implies $N(T) \subseteq N(P)$ for any neutrosophic ideal $N(T)$ of $N(S)$.

Definition8A neutrosophic LA-semigroup $N(S)$ is called fully semiprime neutrosophic LA-semigroup if every neutrosophic ideal of $N(S)$ is semiprime neutrosophic ideal.

Definition9 A neutrosophic ideal $N(R)$ of a neutrosophic LA-semigroup $N(S)$ is called strongly irreducible neutrosophic ideal if for any neutrosophic ideals $N(H),N(K)$ of $N(S)$ $N(H)\cap N(K)\subseteq N(R)$ implies $N(H)\subseteq N(R)$ or $N(K)\subseteq N(R)$.

Definition 10 Let S,T be two LA-semigroups and $\phi: S\to T$ be a mapping from S to T. Let $N(S)$ and $N(T)$ be the corresponding neutrosophic LA-semigroups of S and T respectively. Let $\theta: N(S)\to N(T)$ be another mapping from $N(S)$ to $N(T)$. Then θ is called neutrosophic homomorphis if ϕ is homomorphism from S to T.

2.2 Soft Sets

Throughout this subsection U refers to an initial universe, E is a set of parameters, $P(U)$ is the power set of U, and $A \subset E$. Molodtsov [12] defined the soft set in the following manner:

Definition 11 A pair (F,A) is called a soft set over U where F is a mapping given by

$F : A \to P(U)$.

In other words, a soft set over U is a parameterized family of subsets of the universe U. For $a \in A$, $F(a)$ may be considered as the set of a -elements of the soft set (F,A), or as the set of a-approximate elements of the soft set.

Definition 12 For two soft sets (F,A) and (H,B) over U, (F,A) is called a soft subset of (H,B) if

1) $A \subseteq B$ and
2) $F(a) \subseteq H(a)$, for all $a \in A$.

This relationship is denoted by $(F,A)\subset(H,B)$. Similarly (F,A) is called a soft superset of (H,B) if (H,B) is a soft subset of (F,A) which is denoted by $(F,A)\supset(H,B)$.

Definition 13 Let (F,A) and (G,B) be two soft sets over a common universe U such that $A\cap B\neq\phi$. Then their restricted intersection is denoted by $(F,A)\cap_R (G,B)=(H,C)$ where (H,C) is defined as $H(c)=F(c)\cap G(c)$ for all $c\in C=A\cap B$.

Definition 14 The extended intersection of two soft sets (F,A) and (G,B) over a common universe U is the soft set (H,C), where $C = A\cup B$, and for all $c\in C$, $H(c)$ is defined as

$$H(c)=\begin{cases} F(c) & \text{if c} \in A-B \\ G(c) & \text{if c} \in B-A \\ F(c)\cap G(c) & \text{if c} \in A\cap B. \end{cases}$$

We write $(F,A)\cap_\varepsilon (G,B)=(H,C)$.

Definition 15 The restricted union of two soft sets (F,A) and (G,B) over a common universe U is the soft set (H,C), where $C = A\cup B$, and for all $c\in C$, $H(c)$ is defined as the soft set $(H,C)=(F,A)\cup_R (G,B)$ where $C=A\cap B$ and $H(c)=F(c)\cup G(c)$ for all $c\in C$.

Definition 16 The extended union of two soft sets $\left(F,A\right)$ and $\left(G,B\right)$ over a common universe U is the soft set $\left(H,C\right)$, where $C=A\cup B$, and for all $c\in C$, $H\left(c\right)$ is defined as $H(c)=\begin{cases} F(c) & \text{if c} \in A-B \\ G(c) & \text{if c} \in B-A \\ F(c)\cup G(c) & \text{if c} \in A\cap B. \end{cases}$

We write $(F,A)\cup_{\varepsilon}(G,B)=(H,C)$.

2.3 Soft LA-semigroup

Definition 17 The restricted product $\left(H,C\right)$ of two soft sets $\left(F,A\right)$ and $\left(G,B\right)$ over an LA-semigroup S is dfined as the soft set $\left(H,C\right)=\left(F,A\right)\odot\left(G,B\right)$, where $H\left(c\right)=F\left(c\right)G\left(c\right)$ for all $c\in C=A\cap B$.

Definition 18 A soft set $\left(F,A\right)$ over S is called soft LA-semigroup over S if $\left(F,A\right)\odot\left(F,A\right)\subseteq\left(F,A\right)$.

Definition19 A soft LA-semigroup $\left(F,A\right)$ over S is said to be soft LA-semigroup with left identity e if $F\left(a\right)\neq\phi$ is a sub LA-semigroup with left identiy e ,where e is the left identity of S for all $a\in A$.

Definition 20 Let $\left(F,A\right)$ and $\left(G,B\right)$ be two soft LA-semigroups over S. Then the operation $*$ for soft sets is
defined as $\left(F,A\right)*\left(G,B\right)=\left(H,A\times B\right)$, where $H\left(a,b\right)=F\left(a\right)G\left(b\right)$ for all $a\in A$, $b\in B$ and $A\times B$ is the Cartesian product of A,B.

Definition 21A soft set $\left(F,A\right)$ over an LA-semigroup S is called a soft left (right) ideal over S if $\widetilde{A}_S\odot\left(F,A\right)\subseteq\left(F,A\right),\left(\left(F,A\right)\odot\widetilde{A}_S\subseteq\left(F,A\right)\right)$ where $\widetilde{A}_S$ is the absolute soft LA-semigroup over S.

Definition 22 Let $\left(F,A\right)$ and $\left(G,B\right)$ be two soft LA-semigroups over S. Then the Cartesian product is defined
as $\left(F,A\right)\times\left(G,B\right)=\left(H,A\times B\right)$, where $H\left(a,b\right)=F\left(a\right)\times G\left(b\right)$ for all $a\in A$ and $b\in B.$

Definition 23 Let $\left(G,B\right)$ be a soft subset of $\left(F,A\right)$ over S. Then $\left(G,B\right)$ is called a soft ideal of $\left(F,A\right)$, if $G\left(b\right)$ is an ideal of $F\left(b\right)$ for all $b\in B$.

3. SOFT NEUTROSOPHIC LA-SEMIGROUPS

The definition of soft neutrosophic LA-semigroup isintroduced in this section and we also examine some of their properties.Throughout this section $N\left(S\right)$ will dnote a neutrosophic LA-semigroup unless stated otherwise.

Definition 24 Let $\left(F,A\right)$ be a soft set over $N\left(S\right)$. Then $\left(F,A\right)$ over $N\left(S\right)$ is called soft neutrosophic LA-semigroup if $\left(F,A\right)\odot\left(F,A\right)\subseteq\left(F,A\right)$.

Proposition 1 A soft set $\left(F,A\right)$ over $N\left(S\right)$ is a soft neutrosophic LA-semigroup if and only if $\phi\neq F\left(a\right)$ is a
neutrosophic sub LA-semigroup of $N\left(S\right)$ for all $a\in A$.

Example 1 Let $N\left(S\right)=\left\{1,2,3,4,1I,2I,3I,4I\right\}$ be a neutrosophic LA-semigroup with the following table.

*	1	2	3	4	1I	2I	3I	4I
1	1	4	2	3	1I	4I	2I	3I
2	3	2	4	1	3I	2I	4I	1I
3	4	1	3	2	4I	1I	3I	2I
4	2	3	1	4	2I	3I	1I	4I
1I	1I	4I	2I	3I	1I	4I	2I	3I
2I	3I	2I	4I	1I	3I	2I	4I	1I
3I	4I	1I	3I	2I	4I	1I	3I	2I
4I	2I	3I	1I	4I	2I	3I	1I	4I

Let (F,A) be a soft set over $N(S)$. Then clearly (F,A) is a soft neutrosophic LA-semigroup over $N(S)$,
where

$$F(a_1)=\{1,1I\},\ F(a_2)=\{2,2I\},$$

$$F(a_3)=\{3,3I\},\ F(a_4)=\{4,4I\}.$$

Theorem 1A soft LA-semigroup over an LA-semigroup S is contained in a soft neutrosophic LA-semigroup over $N(S)$.

Proposition 2 Let (F,A) and (H,B) be two soft neutronsophic LA-semigroup over $N(S)$. Then

1) Their extended intersection $(F,A)\cap_{\varepsilon}(H,B)$ is a soft neutrosophic LA-semigroup over $N(S)$
2) Their restricted intersection $(F,A)\cap_{R}(H,B)$ is also soft neutrosophic LA-semigroup over $N(S)$.

Remark 1 Let (F,A) and (H,B) be two soft neutrosophic LA-semigroup over $N(S)$. Then

1) Their extended union $(F,A)\cup_{\varepsilon}(H,B)$ is not a soft neutrosophic LA-semigroup over $N(S)$.
2) Their restricted union$(F,A)\cup_{R}(H,B)$ is nota soft neutrosophic LA-semigroup over $N(S)$.

Proposition 3 Let (F,A) and (G,B) be two soft neutrosophic LA-semigroup over $N(S)$. Then $(F,A)\wedge(H,B)$ is also soft neutrosophic LA-semigroup if it is non-empty.

Proposition 4 Let (F,A) and (G,B) be two soft neutrosophic LA-semigroup over the neutosophic LA-semigroup $N(S)$. If $A\cap B=\phi$ Then their extended union $(F,A)\cup_{\varepsilon}(G,B)$ is a softneutrosophic LA-semigroup over $N(S)$.

Definition 25 A soft neutrosophic LA-semigroup (F,A) over $N(S)$ is said to be a soft neutosophic LA-semigroup with left identity e if for all $a\in A$, the parameterized set $F(a)$ is aneutrosophic sub LA-semigroup with left identity e where e is the left identity of $N(S)$.

Lemma 1 Let (F,A) be a soft neutrosophic LA-semigroup with left identity e over $N(S)$, then

$$(F,A)\odot(F,A)=(F,A).$$

Proposition 5 Let (F,A) and (G,B) be two soft neutronsophic LA-semigroups over $N(S)$. Then the cartesian product of (F,A) and (G,B) is also soft neutrosophic LA-semigroup over $N(S)$.

Definition 26 A soft neutosophic LA-semigroup (F,A) over $N(S)$ is called soft strong neutrosophic LA-semigroup or soft pure neutrosophic LA-semigroup if each $F(a)$ is a strong or pure neutrosophic sub LA-semigroup for all $a\in A$.

Theorem 2 All soft strong neutrosophic LA-semigroups or pure neutrosophic LA-semigroups are trivially soft neutrosophic LA-semigroups but the converse is not true in general.

Definition 28 Let (F,A) be a soft neutrosophic LA-semigroup over $N(S)$. Then (F,A) is called an absolute soft neutrosophic LA-semigroup if $F(a)=N(S)$ for all $a\in A$. We denote it by $\widetilde{A}_{N(S)}$.

Definition 29 Let (F,A) and (G,B) be two soft neutrosophic LA-semigroup over $N(S)$. Then (G,B) is soft sub neutrosophic LA-semigroup of (F,A), if

1) $B\subseteq A$, and
2) $H(b)$ is a neutrosophic sub LA-semigroup of $F(b)$, for all $b\in B$.

Theorem 3 Every soft LA-semigroup over S is a soft sub neutrosophic LA-semigroup of a soft neutrosophic LA-
semigroup over $N(S)$.

Definition 30 Let (G,B) be a soft sub-neutrosophic LA-smigroup of a soft neutrosophic LA-semigroup (F,A) over $N(S)$. Then (G,B) is said to be soft strong or pure sub-neutrosophic LA-semigroup of (F,A) if each $G(b)$is strong or pure neutrosophic sub LA-semigroup of $F(b)$, for all $b\in B$.

Theorem 4 A soft neutrosophic LA-semigroup (F,A)over $N(S)$ can have soft sub LA-semigroups, soft sub-neutrosophic LA-semigroups and soft strong or pure sub-neutrosophic LA-semigroups.

Theorem 5 If (F,A) over $N(S)$is a soft strong or pureneutrosophic LA-semigroup, then every soft sub-neutrosophic LA-semigroup of (F,A) is a soft strong or pure sub-neutrosophic LA-semigroup.

4. SOFT NEUTROSOPHIC IDEALS OVER A NEUTROSOPHIC LA-SEMIGROUP

Definition 31A soft set (F,A) over a neutrosophic LA-semigroup $N(S)$ is called a soft neutrosophic left (right) ideal over $N(S)$ if $\widetilde{A}_{N(S)}\odot(F,A)\subseteq(F,A),\left((F,A)\odot\widetilde{A}_{N(S)}\subseteq(F,A)\right)$ where $\widetilde{A}_{N(S)}$ is the absolute soft neutrosophic LA-semigroup over $N(S)$. A soft set (F,A) over $N(S)$ is a soft neutrosophic ideal if it is soft neutrosophic left ideal as well as soft neutrosophic right ideal over $N(S)$.

Proposition 5 Let (F,A) be a soft set over $N(S)$. Then (F,A) is a soft neutrosophic ideal over $N(S)$ if and only if $F(a)\neq\phi$ is a neutrosophic ideal of $N(S)$, for all $a\in A$.

Proposition 6 Let (F,A) and (G,B) be two soft neutrosophic ideals over $N(S)$. Then

1) Their restricted union $(F,A)\cup_R(G,B)$ is a soft neutrosophic ideal over $N(S)$.

2) Their restricted intersection $(F,A)\cap_R(G,B)$ is a soft neutrosophic ideal over $N(S)$.
3) Their extended union $(F,A)\cup_\varepsilon(G,B)$ is also a soft neutrosophic ideal over $N(S)$.
4) Their extended intersection $(F,A)\cap_\varepsilon(G,B)$ is asoft neutrosophic ideal over $N(S)$.

Proposition 7 Let (F,A) and (G,B) be two soft neutrosophic ideals over $N(S)$. Then

1. Their OR operation $(F,A)\vee(G,B)$ is a soft neutrosophic ideal over $N(S)$.
2. Their AND operation $(F,A)\wedge(G,B)$ is a soft neutrosophic ideal over $N(S)$.

Proposition 8 Let (F,A) and (G,B) be two soft neutrosophic ideals over $N(S)$, where $N(S)$ is a neutrosophic LA-semigroup with left identity e. Then $(F,A)*(G,B)=(H,A\times B)$ is also a soft neutrosophic ideal over $N(S)$.

Proposition 9 Let (F,A) and (G,B) be two soft neutrosophic ideals over $N(S)$ and $N(T)$.Then the cartesian product $(F,A)\times(G,B)$ is a soft neutrosophic ideal over $N(S)\times N(T)$.

Definition 32A soft neutrosophic ideal (F,A) over $N(S)$ is called soft strong or pure neutrosophic ideal over $N(S)$ if $F(a)$ is a strong or pure neutrosophicideal of $N(S)$, for all $a\in A$.

Theorem 6 All soft strong or pure neutrosophic ideals over $N(S)$ are trivially soft neutrosophic ideals but the converse is not true.

Proposition 8 Let (F,A) and (G,B) be two soft strong or pure neutrosophic ideals over $N(S)$. Then

1) Their restricted union $(F,A)\cup_R(G,B)$ is a soft strong or pure neutrosophic ideal over $N(S)$.
2) Their restricted intersection $(F,A)\cap_R(G,B)$ is a soft strong or pure neutrosophic ideal over $N(S)$.
3) Their extended union $(F,A)\cup_\varepsilon(G,B)$ is also a soft strong or pure neutrosophic ideal over $N(S)$.
4) Their extended intersection $(F,A)\cap_\varepsilon(G,B)$ is a soft strong or pure neutrosophic ideal over $N(S)$.

Proposition 9 Let (F,A) and (G,B) be two soft strong or pure neutrosophic ideals over $N(S)$. Then

1) Their OR operation $(F,A)\vee(G,B)$ is a soft strong or pure neutrosophic ideal over $N(S)$.
2) Their AND operation $(F,A)\wedge(G,B)$ is a soft strong or pure neutrosophic ideal over $N(S)$.

Proposition 10 Let (F,A) and (G,B) be two soft strong or pure neutrosophic ideals over $N(S)$, where $N(S)$ is a neutrosophic LA-semigroup with left identity e. Then $(F,A)*(G,B)=(H,A\times B)$ is also a softstrong or pure neutrosophic ideal over $N(S)$.

Proposition 11 Let (F,A) and (G,B) be two soft strong or pure neutrosophic ideals over $N(S)$ and $N(T)$ respectively. Then the cartesian product $(F,A)\times(G,B)$ is a soft strong or pure neutrosophic ideal over $N(S)\times N(T)$.

5. SOFT NEUTROSOPHIC IDEAL OF SOFT NEUTROSOPHIC LA-SEMIGROUP

Definition 33 Let (F,A) and (G,B) be a soft neutrosophic LA-semigroups over $N(S)$. Then (G,B) is soft neutrosophic ideal of (F,A), if

1) $B \subseteq A$, and
2) $H(b)$ is a neutrosophic ideal of $F(b)$, for all $b \in B$.

Proposition 12 If $(F^{'},A^{'})$ and $(G^{'},B^{'})$ are soft neutrosophic ideals of soft neutrosophic LA-semigroup (F,A) and (G,B) over neutrosophic LA-semigroups $N(S)$ and $N(T)$respectively.
Then$(F^{'},A^{'})\times(G^{'},B^{'})$ is a soft neutrosophic ideal of soft neutrosophic LA-semigroup $(F,A)\times(G,B)$ over $N(S)\times N(T)$.

Theorem 17 Let (F,A) be a soft neutrosophic LA-semigroup over $N(S)$ and $\{(H_j,B_j): j \in J\}$ be a non-empty family of soft neutrosophic sub LA-semigroups of (F,A). Then

1) $\bigcap_{j\in J} {}_R (H_j,B_j)$ is a soft neutrosophic sub LA-semigroup of (F,A).
2) $\bigwedge_{j\in J} {}_R (H_j,B_j)$ is a soft neutrosophic sub LA-semigroup of (F,A).
3) $\bigcup_{j\in J} {}_\varepsilon (H_j,B_j)$ is a soft neutrosophic sub LA-semigroup of (F,A) if $B_j \cap B_k = \phi$ for all $j,k \in J$

Theorem 8 Let (F,A) be a soft neutrosophic LA-semigroup over $N(S)$ and $\{(H_j,B_j): j \in J\}$ be a non-empty family of soft neutrosophic ideals of (F,A). Then

1) $\bigcap_{j\in J} {}_R (H_j,B_j)$ is a soft neutrosophic ideal of (F,A).
2) $\bigwedge_{j\in J} (H_j,B_j)$ is a soft neutrosophic ideal of (F,A).
3) $\bigcup_{j\in J} {}_\varepsilon (H_j,B_j)$ is a soft neutrosophic ideal of (F,A).
4) $\bigvee_{j\in J} (H_j,B_j)$ is a soft neutrosophic ideal of(F,A).

Proposition 13 Let (F,A) be a soft neutrosophic LAsemigroup with left identity e over $N(S)$ and (G,B) be a soft neutrosophic right ideal of (F,A). Then (G,B) is also soft neutrosophic left ideal of (F,A).

Lemma 2 Let (F,A) be a soft neutrosophic LA-semigroup with left identity e over $N(S)$ and (G,B) be a soft neutrosophic right ideal of (F,A). Then $(G,B)\odot(G,B)$is a soft neutrosophic ideal of (F,A).

Definition 34 A soft neutrosophic ideal (G,B) of a soft neutrosophic LA-semigroup (F,A) is called soft strong or pure neutrosophic ideal if $G(b)$ is a strong or pure neutrosophic ideal of $F(b)$ for all $b \in B$.

Theorem 9 Every soft strong or pure neutrosophic ideal of a soft neutrosophic LA-semigroup is trivially a soft
neutrosophic ideal but the converse is not true.

Definition 35A soft neutrosophic ideal (G,B) of a soft neutrosophic LA-semigroup (F,A) over $N(S)$ is
called soft prime neutrosophic ideal if $(H,C)\odot(J,D)\subseteq(G,B)$ implies either $(H,C)\subseteq(G,B)$ or $(J,D)\subseteq(G,B)$ for soft neutosophic ideals (H,C) and (J,D) of (F,A).

Definition 36A soft neutrosophic LA-semigroup (F,A) over $N(S)$ is called soft fully prime neutrosophic LA-semigroup if all the soft neutrosophic ideals of (F,A) are soft prime neutrosophic ideals.

Definition37 A soft neutrosophic ideal (G,B) of a soft neutrosophic LA-semigroup (F,A) over $N(S)$ is called soft semiprime neutrosophic ideal if $(H,C)\odot(H,C)\subseteq(G,B)$ implies that $(H,C)\subseteq(G,B)$ for any soft neutrosophic ideal $(H,C)$of (F,A).

Definition38A soft neutrosophic LA-semigroup (F,A) over $N(S)$ is called soft fully semiprime neutrosophic LA-semigroup if all the soft neutrosophic ideals of (F,A) are soft semiprime neutrosophic ideals.

Definition39A soft neutrosophic ideal (G,B) of a soft neutrosophic LA-semigroup (F,A) over $N(S)$ is called soft strongly irreducible neutrosophic ideal if $(H,C)\cap_R(J,D)\subseteq(G,B)$ implies either $(H,C)\subseteq(G,B)$ or $(J,D)\subseteq(G,B)$for soft neutrosophic ideals (H,C) and (J,D) of (F,A).

6. SOFT NEUTROSOPHIC HOMOMORPHISM

Definition 40 Let (F,A) and (G,B) be two soft neutrosophic LA-semigroups over $N(S)$ and $N(T)$ respectively. Let $f:N(S)\to N(T)$ and $g:A\to B$ be two mappings. Then $(f,g):(F,A)\to(G,B)$ is called soft neutrosophic homomorphism, if

1) f is a neutrosophic homomorphism from $N(S)$ onto $N(T)$.
2) g is a maping from A onto B.
3) $f(F(a))=G(g(a))$ for all $a\in A$.

If f is a neutrosophic isomorphism from $N(S)$ to $N(T)$ and g is one to one mapping from A onto B. Then (f,g) is called soft neutrosophic isomorphism from (F,A) to (G,B).

CONCLUSION

The literature shows us that soft LA-semigroup is a general framework than LA-semigroup but in this paper we can see that there exista more general structure which we call soft neutrosophic LA-semigroup.A soft LA-semigroup becomes soft sub-neutrosophic LA-semigroup of the corresponding soft neutrosophic LA-semigroup.Soft neutrosophic LA-semigroup points out the indeterminacy factor involved in soft LA-semigroup.Soft neutrosophic LA-semigroup can be characterized by soft neutrosophic ideals over a soft neutrosophic LA-semigroup. We can also extend soft homomorphism of soft LA-semigroup to soft neutrosophic homomorphism of soft neutrosophic LA-semigroup. It is also mentioned here that there is still a space to much more work in this field and explorations of further results has still to be done, this is just a beginning.

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Presented at SISOM & ACOUSTICS 2014 International Conference, Bucharest, 22-23 May 2014.

Soft Neutrosophic Loop, Soft Neutrosophic Biloop and Soft Neutrosophic N-Loop

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Abstract. Soft set theory is a general mathematical tool for dealing with uncertain, fuzzy, not clearly defined objects. In this paper we introduced soft neutrosophic loop,soft neutosophic biloop, soft neutrosophic N-loop with the discuission of some of their characteristics. We also introduced a new type of soft neutrophic loop, the so called soft strong neutrosophic loop which is of pure neutrosophic character. This notion also foound in all the other corresponding notions of soft neutrosophic thoery. We also given some of their properties of this newly born soft structure related to the strong part of neutrosophic theory.

Key words and phrases. Neutrosophic loop, neutrosophic biloop, neutrosophic N-loop, soft set, soft neutrosophic loop,soft neutrosophic biloop, soft neutrosophic N-loop.

1. Introduction

Florentin Smarandache for the first time intorduced the concept of neutrosophy in 1995, which is basically a new branch of philosophy which actually studies the origion, nature, and scope of neutralities. The neutrosophic logic came into being by neutrosophy. In neutrosophic logic each proposition is approximated to have the percentage of truth in a subset T, the percentage of indeterminacy in a subset I, and the percentage of falsity in a subset F. Neutrosophic logic is an extension of fuzzy logic. In fact the neutrosophic set is the generalization of classical set, fuzzy conventional set, intuitionistic fuzzy set, and interal valued fuzzy set. Neutrosophic logic is used to overcome the problems of imperciseness, indeterminate, and inconsistentness of date etc. The theoy of neutrosophy is so applicable to every field of agebra. W.B Vasantha Kandasamy and Florentin Smarandache introduced neutrosophic fields, neutrosophic rings,neutrosophic vectorspaces,neutrosophic groups,neutrosophic bigroups and neutrosophic N-groups, neutrosophic semigroups, neutrosophic bisemigroups, and neutrsosophic N-semigroups, neutrosophic loops, nuetrosophic biloops, and neutrosophic N-loops, and so on. Mumtaz ali *et al* introduced neutrosophic LA-semigoups.

Molodtsov intorduced the theory of soft set. This mathematical tool is free from parameterization inadequacy, syndrome of fuzzy set theory, rough set theory, probability theory and so on. This theory has been applied successfully in many fields such as smoothness of functions, game theory, operation reaserch, Riemann integration, Perron integration, and probability. Recently soft set theory attained much attention of the researchers since its appearance and the work based on several operations of soft set introduced in [2, 9, 10]. Some properties and algebra may be found in [1] . Feng *et al.* introduced soft semirings in [5]. By means of level soft sets an adjustable approach to fuzzy soft set can be seen in [6]. Some other concepts together with fuzzy set and rough set were shown in [7, 8].

This paper is about to introduced soft nuetrosophic loop, soft neutrosphic biloop, and soft neutrosophic N-loop and the related strong or pure part of neutrosophy with the notions of soft set theory. In the proceeding section, we define soft neutrosophic loop, soft neutrosophic strong loop, and some of their properties are discuissed. In the next section, soft neutrosophic biloop are presented with their strong neutrosophic part. Also in this section some of their characterization have been made. In the last section soft neutrosophic N-loop and their coresponding strong theory have been constructed with some thier properties.

2. Neutrosophic Loop

Definition 1. *A neutrosophic loop is generated by a loop L and I denoted by $\langle L \cup I \rangle$. A neutrosophic loop in general need not be a loop for $I^2 = I$ and I may not have an inverse but every element in a loop has an inverse.*

Definition 2. *Let $\langle L \cup I \rangle$ be a neutrosophic loop. A proper subset $\langle P \cup I \rangle$ of $\langle L \cup I \rangle$ is called the neutrosophic subloop, if $\langle P \cup I \rangle$ is itself a neutrosophic loop under the operations of $\langle L \cup I \rangle$.*

Definition 3. *Let $(\langle L \cup I \rangle, o)$ be a neutrosophic loop of finite order. A proper subset P of $\langle L \cup I \rangle$ is said to be Lagrange neutrosophic subloop, if P is a neutrosophic subloop under the operation 'o' and $o(P)/o\langle L \cup I \rangle$.*

If every neutrosophic subloop of $\langle L \cup I \rangle$ is Lagrange then we call $\langle L \cup I \rangle$ to be a Lagrange neutrosophic loop.

Definition 4. *If $\langle L \cup I \rangle$ has no Lagrange neutrosophic subloop then we call $\langle L \cup I \rangle$ to be a Lagrange free neutrosophic loop.*

Definition 5. *If $\langle L \cup I \rangle$ has atleast one Lagrange neutrosophic subloop then we call $\langle L \cup I \rangle$ a weakly Lagrange neutrosophic loop.*

3. Neutrosophic Biloops

Definition 6. *Let $(\langle B \cup I \rangle, *_1, *_2)$ be a non empty neutrosophic set with two binary operations $*_1, *_2$, $\langle B \cup I \rangle$ is a neutrosophic biloop if the following conditions are satisfied.*

(1) $\langle B \cup I \rangle = P_1 \cup P_2$ where P_1 and P_2 are proper subsets of $\langle B \cup I \rangle$.
(2) $(P_1, *_1)$ is a neutrosophic loop.
(3) $(P_2, *_2)$ is a group or a loop.

Definition 7. *Let $(\langle B \cup I \rangle, *_1, *_2)$ be a neutrosophic biloop. A proper subset P of $\langle B \cup I \rangle$ is said to be a neutrosophic subbiloop of $\langle B \cup I \rangle$ if $(P, *_1, *_2)$ is itself a neutrosophic biloop under the operations of $\langle B \cup I \rangle$.*

Definition 8. *Let $(B = B1 \cup B2, *_1, *_2)$ be a finite neutrosophic biloop. Let $P = (P_1 \cup P_2, *_1, *_2)$ be a neutrosophic biloop. If $o(P)/o(B)$ then we call P a Lagrange neutrosophic subbiloop of B.*

If every neutrosophic subbiloop of B is Lagrange then we call B to be a Lagrange neutrosophic biloop.

Definition 9. *If B has atleast one Lagrange neutrosophic subbiloop then we call B to be a weakly Lagrange neutrosophic biloop.*

Definition 10. *If B has no Lagrange neutrosophic subbiloops then we call B to be a Lagrange free neutrosophic biloop.*

4. Neutrosophic N-loop

Definition 11. *Let $S(B) = \{S(B1) \cup \ldots \cup S(BN), *_1, \ldots, *_N\}$ be a non empty neutrosophic set with N binary operations. $S(B)$ is a neutrosophic N-loop if $S(B) = S(B_1) \cup \ldots \cup S(B_N)$, $S(B_i)$ are proper subsets of $S(B)$ for $1 \leq i \leq N$) and some of $S(B_i)$ are neutrosophic loops and some of the $S(B_j)$ are groups.*

Definition 12. *Let $S(B) = \{S(B_1) \cup S(B_2) \cup \ldots \cup S(B_N), *_1, \ldots, *_N\}$ be a neutrosophic N-loop. A proper subset $(P, *_1, \ldots, *_N)$ of $S(B)$ is said to be a neutrosophic sub N loop of $S(B)$ if P itself is a neutrosophic N-loop under the operations of $S(B)$.*

Definition 13. *Let $(L = L_1 \cup L_2 \cup \ldots \cup L_N, *_1, *_2, \ldots, *_N\}$ be a neutrosophic N-loop of finite order. Suppose P is a proper subset of L, which is a neutrosophic sub N-loop. If $o(P)/o(L)$ then we call P a Lagrange neutrosophic sub N-loop.*

If every neutrosophic sub N-loop is Lagrange then we call L to be a Lagrange neutrosophic N-loop.

Definition 14. *If L has atleast one Lagrange neutrosophic sub N-loop then we call L to be a weakly Lagrange neutrosophic N-loop.*

Definition 15. *If L has no Lagrange neutrosophic sub N-loop then we call L to be a Lagrange free neutrosophic N-loop.*

5. Soft Set

Throughout this subsection U refers to an initial universe, E is a set of parameters, $P(U)$ is the power set of U, and $A \subset E$. Molodtsov [10] defined the soft set in the following manner:

Definition 16. *A pair (F, A) is called a soft set over U where F is a mapping given by $F : A \longrightarrow P(U)$.*

In other words, a soft set over U is a parameterized family of subsets of the universe U. For $a \in A$, $F(a)$ may be considered as the set of a-elements of the soft set (F, A), or as the set of a-approximate elements of the soft set.

Example 1. *Suppose that U is the set of shops. E is the set of parameters and each parameter is a word or senctence. Let $E = \{$high rent,normal rent,in good condition,in bad condition$\}$. Let us consider a soft set (F, A) which describes the "attractiveness of shops" that Mr.Z is taking on rent. Suppose that there are five houses in the universe $U = \{h_1, h_2, h_3, h_4, h_5\}$ under consideration, and that $A = \{e_1, e_2, e_3\}$ be the set of parameters where*

a_1 stands for the parameter 'high rent,
a_2 stands for the parameter 'normal rent,
a_3 stands for the parameter 'in good condition.
Suppose that
$F(a_1) = \{h_1, h_4\}$,
$F(a_2) = \{h_2, h_5\}$,
$F(a_3) = \{h_3\}$.

The soft set (F, A) is an approximated family $\{F(a_i), i = 1, 2, 3\}$ of subsets of the

set U which gives us a collection of approximate description of an object. Thus, we have the soft set (F,A) as a collection of approximations as below:

$(F, A) = \{$high rent $= \{h_1, h_4\}$,normal rent $= \{h_2, h_5\}$,in good condition $= \{h_3\}\}$.

Definition 17. *For two soft sets (F, A) and (H, B) over U, (F, A) is called a soft subset of (H, B) if*

(1) $A \subseteq B$ and
(2) $F(a) \subseteq G(a)$ for all $a \in A$.

This relationship is denoted by $(F, A) \tilde{\subset} (H, B)$. Similarly (F, A) is called a soft superset of (H, B) if (H, B) is a soft subset of (F, A) which is denoted by $(F, A) \tilde{\supset} (H, B)$.

Definition 18. *Two soft sets (F, A) and (H, B) over U are called soft equal if (F, A) is a soft subset of (H, B) and (H, B) is a soft subset of (F, A).*

Definition 19. *(F, A) over U is called an absolute soft set if $F(a) = U$ for all $a \in A$ and we denote it by $\mathcal{F}_U$.*

Definition 20. *Let (F, A) and (G, B) be two soft sets over a common universe U such that $A \cap B \neq \phi$. Then their restricted intersection is denoted by$(F, A) \cap_R (G, B) = (H, C)$ where (H, C) is defined as $H(c) = F(c) \cap G(c)$ for all $c \in C = A \cap B$.*

Definition 21. *The extended intersection of two soft sets (F, A) and (G, B) over a common universe U is the soft set (H, C), where $C = A \cup B$, and for all $c \in C$, $H(c)$ is defined as*

$$H(c) = \begin{cases} F(c) & \text{if } c \in A - B \\ G(c) & \text{if } c \in B - A \\ F(c) \cap G(c) & \text{if } c \in A \cap B. \end{cases}$$

We write $(F, A) \cap_\varepsilon (G, B) = (H, C)$.

Definition 22. *The resticted union of two soft sets (F, A) and (G, B) over a common universe U is the soft set (H, C), where $C = A \cup B$, and for all $c \in C$, $H(e)$ is defined as the soft set $(H, C) = (F, A) \cup_R (G, B)$ where $C = A \cap B$ and $H(c) = F(c) \cup G(c)$ for all $c \in C$.*

Definition 23. *The extended union of two soft sets (F, A) and (G, B) over a common universe U is the soft set (H, C), where $C = A \cup B$, and for all $c \in C$, $H(c)$ is defined as*

$$H(c) = \begin{cases} F(c) & \text{if } c \in A - B \\ G(c) & \text{if } c \in B - A \\ F(c) \cup G(c) & \text{if } c \in A \cap B. \end{cases}$$

We write $(F, A) \cup_\varepsilon (G, B) = (H, C)$.

6. Soft Neutrosophic Loop

Definition 24. *Let $\langle L \cup I \rangle$ be a neutrosophic loop and (F, A) be a soft set over $\langle L \cup I \rangle$. Then (F, A) is called soft neutrosophic loop if and only if $F(a)$ is neutrosophic subloop of $\langle L \cup I \rangle$, for all $a \in A$.*

Example 2. *Let $\langle L\cup I\rangle = \langle L_7(4)\cup I\rangle$ be a neutrosophic loop where $L_7(4)$ is a loop. $\langle e, eI, 2, 2I\rangle, \langle e, 3\rangle$ and $\langle e, eI\rangle$ are neutrosophic subloops of $L_7(4)$. Then (F, A) is a soft neutrosophic loop over $\langle L \cup I\rangle$, where*

$$\begin{aligned} F(a_1) &= \{\langle e, eI, 2, 2I\rangle\}, F(a_2) = \{\langle e, 3\rangle\}, \\ F(a_3) &= \{\langle e, eI\rangle\}. \end{aligned}$$

Theorem 1. *Every soft neutrosophic loop over $\langle L\cup I\rangle$ contains a soft loop over L.*

Proof. The proof is straight forward. □

Theorem 2. *Let (F, A) and (H, A) be two soft neutrosophic loops over $\langle L \cup I\rangle$. Then their intersection $(F, A)\cap(H, A)$ is again a soft neutrosophic loop over $\langle L\cup I\rangle$.*

Proof. The proof is staight forward. □

Theorem 3. *Let (F, A) and (H, B) be two soft neutrosophic loops over $\langle L\cup I\rangle$. If $A \cap B = \phi$, then $(F, A) \cup (H, B)$ is a soft neutrosophic loop over $\langle L \cup I\rangle$.*

Theorem 4. *Let (F, A) and (H, A) be two soft neutrosophic loops over $\langle L\cup I\rangle$. If $F(a) \subseteq H(a)$ for all $a \in A$, then (F, A) is a soft neutrosophic subloop of (H, A).*

Theorem 5. *Let (F, A) and (K, B) be two soft neutrosophic loops over $\langle L \cup I\rangle$. Then*

(1) Their extended union $(F, A)\cup_{\varepsilon} (K, B)$ over $\langle L\cup I\rangle$ is not soft neutrosophic loop over $\langle L \cup I\rangle$.
(2) Their extended intersection $(F, A)\cap_{\varepsilon}(K, B)$ over $\langle L\cup I\rangle$ is soft neutrosophic loop over $\langle L \cup I\rangle$.
(3) Their restricted union $(F, A)\cup_R (K, B)$ over $\langle L\cup I\rangle$ is not soft neutrosophic loop over $\langle L \cup I\rangle$.
(4) Their restricted intersection $(F, A)\cap_{\varepsilon}(K, B)$ over $\langle L\cup I\rangle$ is soft neutrosophic soft loop over $\langle L \cup I\rangle$.

Theorem 6. *Let (F, A) and (H, B) be two soft neutrosophic loops over $\langle L \cup I\rangle$. Then*

(1) Their AND operation $(F, A)\wedge(H, B)$ is soft neutrosophic loop over $\langle L\cup I\rangle$.
(2) Their OR operation $(F, A)\vee(H, B)$ is not soft neutrosophic loop over $\langle L\cup I\rangle$.

Definition 25. *Let $\langle L_n(m) \cup I\rangle = \{e, 1, 2, \ldots, n, e.I, 1I, \ldots, nI\}$ be a new class of neutrosophic loop and (F, A) be a soft neutrosophic loop over $\langle L_n(m) \cup I\rangle$. Then (F, A) is called soft new class neutrosophic loop if $F(a)$ is neutrosophic subloop of $\langle L_n(m) \cup I\rangle$, for all $a \in A$.*

Example 3. *Let $\langle L_5(3)\cup I\rangle = \{e, 1, 2, 3, 4, 5, eI, 1I, 2I, 3I, 4I, 5I\}$ be a new class of neutrosophic loop and $\{e, eI, 1, 1I\}, \{e, eI, 2, 2I\}, \{e, eI, 3, 3I\}, \{e, eI, 4, 4I\}, \{e, eI, 5, 5I\}$ are neutrosophic subloops of $L_5(3)$. Then (F, A) is soft new class neutrosophic loop over $L_5(3)$, where*

$$\begin{aligned} F(a_1) &= \{e, eI, 1, 1I\}, F(a_2) = \{e, eI, 2, 2I\}, \\ F(a_3) &= \{e, eI, 3, 3I\}, F(a_4) = \{e, eI, 4, 4I\}, \\ F(a_5) &= \{e, eI, 5, 5I\}. \end{aligned}$$

Theorem 7. *Every soft new class neutrosophic loop over $\langle L_n(m) \cup I\rangle$ is a soft neutrosophic loop over $\langle L_n(m) \cup I\rangle$ but the converse is not true.*

Theorem 8. *Let (F, A) and (K, B) be two soft new class neutrosophic loops over $\langle L_n(m) \cup I \rangle$. Then*

(1) Their extended union $(F, A) \cup_{\varepsilon} (K, B)$ over $\langle L_n(m) \cup I \rangle$ is not soft new class neutrosophic loop over $\langle L_n(m) \cup I \rangle$.
(2) Their extended intersection $(F, A) \cap_{\varepsilon} (K, B)$ over $\langle L_n(m) \cup I \rangle$ is soft new class neutrosophic loop over $\langle L_n(m) \cup I \rangle$.
(3) Their restricted union $(F, A) \cup_{R} (K, B)$ over $\langle L_n(m) \cup I \rangle$ is not soft new class neutrosophic loop over $\langle L_n(m) \cup I \rangle$.
(4) Their restricted intersection $(F, A) \cap_{\varepsilon} (K, B)$ over $\langle L_n(m) \cup I \rangle$ is soft new class neutrosophic soft loop over $\langle L_n(m) \cup I \rangle$.

Theorem 9. *Let (F, A) and (H, B) be two soft new class neutrosophic loops over $\langle L_n(m) \cup I \rangle$. Then*

(1) Their AND operation $(F, A) \wedge (H, B)$ is soft new class neutrosophic loop over $\langle L_n(m) \cup I \rangle$.
(2) Their OR operation $(F, A) \vee (H, B)$ is not soft new class neutrosophic loop over $\langle L_n(m) \cup I \rangle$.

Definition 26. *Let (F, A) be a soft neutrosophic loop over $\langle L \cup I \rangle$, then (F, A) is called the identity soft neutrosophic loop over $\langle L \cup I \rangle$ if $F(a) = \{e\}$, for all $a \in A$, where e is the identity element of L.*

Definition 27. *Let (F, A) be a soft neutrosophic loop over $\langle L \cup I \rangle$, then (F, A) is called Full-soft neutrosophic loop over $\langle L \cup I \rangle$ if $F(a) = \langle L \cup I \rangle$, for all $a \in A$.*

Definition 28. *Let (F, A) and (H, B) be two soft neutrosophic loops over $\langle L \cup I \rangle$. Then (H, B) is soft neutrosophic subloop of (F, A), if*

(1) $B \subset A$.
(2) $H(a)$ is neutrosophic subloop of $F(a)$, for all $a \in A$.

Example 4. *Consider the neutrosophic loop $\langle L_{15}(2) \cup I \rangle = \{e, 1, 2, 3, 4, \ldots, 15, eI, 1I, 2I, \ldots, 14I, 15I\}$ of order 32. It is easily verified $P = \{e, 2, 5, 8, 11, 14, eI, 2I, 5I, 8I, 11I, 14I\}$, $Q = \{e, 2, 5, 8, 11, 14\}$ and $T = \{e, 3, eI, 3I\}$ are neutrosophic subloops of $\langle L_{15}(2) \cup I \rangle$. Then (F, A) is a soft neutrosophic loop over $\langle L_{15}(2) \cup I \rangle$, where*

$$\begin{aligned} F(a_1) &= \{e, 2, 5, 8, 11, 14, eI, 2I, 5I, 8I, 11I, 14I\}, \\ F(a_2) &= \{e, 2, 5, 8, 11, 14\}, \\ F(a_3) &= \{e, 3, eI, 3I\}. \end{aligned}$$

Hence (G, B) is a soft neutrosophic subloop of (F, A) over $\langle L_{15}(2) \cup I \rangle$, where

$$\begin{aligned} G(a_1) &= \{e, eI, 2I, 5I, 8I, 11I, 14I\}, \\ G(e_3) &= \{e, 3\}. \end{aligned}$$

Theorem 10. *Every soft loop over L is a soft neutrosophic subloop over $\langle L \cup I \rangle$.*

Theorem 11. *Every absolute soft loop over L is a soft neutrosophic subloop of Full-soft neutrosophic loop over $\langle L \cup I \rangle$.*

Definition 29. *Let $\langle L \cup I \rangle$ be a neutrosophic loop and (F, A) be a soft set over $\langle L \cup I \rangle$. Then (F, A) is called normal soft neutrosophic loop if and only if $F(a)$ is normal neutrosophic subloop of $\langle L \cup I \rangle$, for all $a \in A$.*

Example 5. *Let $\langle L_5(3)\cup I\rangle = \{e,1,2,3,4,5,eI,1I,2I,3I,4I,5I\}$ be a neutrosophic loop and $\{e,eI,1,1I\}$, $\{e,eI,2,2I\}$, $\{e,eI,3,3I\}$ are normal neutrosophic subloops of $\langle L_5(3)\cup I\rangle$. Then Clearly (F,A) is normal soft neutrosophic loop over $\langle L_5(3)\cup I\rangle$, where*

$$\begin{aligned} F(a_1) &= \{e,eI,1,1I\}, F(a_2) = \{e,eI,2,2I\}, \\ F(a_3) &= \{e,eI,3,3I\}. \end{aligned}$$

Theorem 12. *Every normal soft neutrosophic loop over $\langle L\cup I\rangle$ is a soft neutrosophic loop over $\langle L\cup I\rangle$ but the converse is not true.*

Theorem 13. *Let (F,A) and (K,B) be two normal soft neutrosophic loops over $\langle L\cup I\rangle$. Then*

(1) Their extended union $(F,A)\cup_{\varepsilon}(K,B)$ over $\langle L\cup I\rangle$ is not normal soft neutrosophic loop over $\langle L\cup I\rangle$.
(2) Their extended intersection $(F,A)\cap_{\varepsilon}(K,B)$ over $\langle L\cup I\rangle$ is normal soft neutrosophic loop over $\langle L\cup I\rangle$.
(3) Their restricted union $(F,A)\cup_{R}(K,B)$ over $\langle L\cup I\rangle$ is not normal soft neutrosophic loop over $\langle L\cup I\rangle$.
(4) Their restricted intersection $(F,A)\cap_{\varepsilon}(K,B)$ over $\langle L\cup I\rangle$ is normal soft neutrosophic soft loop over $\langle L\cup I\rangle$.

Theorem 14. *Let (F,A) and (H,B) be two normal soft neutrosophic loops over $\langle L\cup I\rangle$. Then*

(1) Their AND operation $(F,A)\wedge(H,B)$ is normal soft neutrosophic loop over $\langle L\cup I\rangle$.
(2) Their OR operation $(F,A)\vee(H,B)$ is not normal soft neutrosophic loop over $\langle L\cup I\rangle$.

Definition 30. *Let $\langle L\cup I\rangle$ be a neutrosophic loop and (F,A) be a soft neutrosophic loop over $\langle L\cup I\rangle$. Then (F,A) is called Lagrange soft neutrosophic loop if each $F(a)$ is lagrange neutrosophic subloop of $\langle L\cup I\rangle$, for all $a\in A$.*

Example 6. *In (example 1), (F,A) is lagrange soft neutrosophic loop over $\langle L\cup I\rangle$.*

Theorem 15. *Every lagrange soft neutrosophic loop over $\langle L\cup I\rangle$ is a soft neutrosophic loop over $\langle L\cup I\rangle$ but the converse is not true.*

Theorem 16. *If $\langle L\cup I\rangle$ is lagrange neutrosophic loop, then (F,A) over $\langle L\cup I\rangle$ is lagrange soft neutrosophic loop but the converse is not true.*

Theorem 17. *Every soft new class neutrosophic loop over $\langle L_n(m)\cup I\rangle$ is lagrange soft neutrosophic loop over $\langle L_n(m)\cup I\rangle$ but the converse is not true.*

Theorem 18. *If $\langle L\cup I\rangle$ is a new class neutrosophic loop, then (F,A) over $\langle L\cup I\rangle$ is lagrange soft neutrosophic loop.*

Theorem 19. *Let (F,A) and (K,B) be two lagrange soft neutrosophic loops over $\langle L\cup I\rangle$. Then*

(1) Their extended union $(F,A)\cup_{\varepsilon}(K,B)$ over $\langle L\cup I\rangle$ is not lagrange soft neutrosophic loop over $\langle L\cup I\rangle$.
(2) Their extended intersection $(F,A)\cap_{\varepsilon}(K,B)$ over $\langle L\cup I\rangle$ is not lagrange soft neutrosophic loop over $\langle L\cup I\rangle$.

(3) Their restricted union $(F,A) \cup_R (K,B)$ over $\langle L \cup I \rangle$ is not lagrange soft neutrosophic loop over $\langle L \cup I \rangle$.
(4) Their restricted intersection $(F,A) \cap_\varepsilon (K,B)$ over $\langle L \cup I \rangle$ is not lagrange soft neutrosophic soft loop over $\langle L \cup I \rangle$.

Theorem 20. *Let (F,A) and (H,B) be two lagrange soft neutrosophic loops over $\langle L \cup I \rangle$. Then*

(1) Their AND operation $(F,A) \wedge (H,B)$ is not lagrange soft neutrosophic loop over $\langle L \cup I \rangle$.
(2) Their OR operation $(F,A) \vee (H,B)$ is not lagrange soft neutrosophic loop over $\langle L \cup I \rangle$.

Definition 31. *Let $\langle L \cup I \rangle$ be a neutrosophic loop and (F,A) be a soft neutrosophic loop over $\langle L \cup I \rangle$. Then (F,A) is called weak Lagrange soft neutrosophic loop if atleast one $F(a)$ is lagrange neutrosophic subloop of $\langle L \cup I \rangle$, for some $a \in A$.*

Example 7. *Consider the neutrosophic loop $\langle L_{15}(2) \cup I \rangle = \{e, 1, 2, 3, 4, \ldots, 15, eI, 1I, 2I, \ldots, 14I, 15I\}$ of order 32. It is easily verified $P = \{e, 2, 5, 8, 11, 14, eI, 2I, 5I, 8I, 11I, 14I\}$, $Q = \{e, 2, 5, 8, 11, 14\}$ and $T = \{e, 3, eI, 3I\}$ are neutrosophic subloops of $\langle L_{15}(2) \cup I \rangle$. Then (F,A) is a weak lagrange soft neutrosophic loop over $\langle L_{15}(2) \cup I \rangle$, where*

$$\begin{aligned} F(a_1) &= \{e, 2, 5, 8, 11, 14, eI, 2I, 5I, 8I, 11I, 14I\}, \\ F(a_2) &= \{e, 2, 5, 8, 11, 14\}, \\ F(a_3) &= \{e, 3, eI, 3I\}. \end{aligned}$$

Theorem 21. *Every weak lagrange soft neutrosophic loop over $\langle L \cup I \rangle$ is a soft neutrosophic loop over $\langle L \cup I \rangle$ but the converse is not true.*

Theorem 22. *If $\langle L \cup I \rangle$ is weak lagrange neutrosophic loop, then (F,A) over $\langle L \cup I \rangle$ is also weak lagrange soft neutrosophic loop but the converse is not true.*

Theorem 23. *Let (F,A) and (K,B) be two weak lagrange soft neutrosophic loops over $\langle L \cup I \rangle$. Then*

(1) Their extended union $(F,A) \cup_\varepsilon (K,B)$ over $\langle L \cup I \rangle$ is not weak lagrange soft neutrosophic loop over $\langle L \cup I \rangle$.
(2) Their extended intersection $(F,A) \cap_\varepsilon (K,B)$ over $\langle L \cup I \rangle$ is not weak lagrange soft neutrosophic loop over $\langle L \cup I \rangle$.
(3) Their restricted union $(F,A) \cup_R (K,B)$ over $\langle L \cup I \rangle$ is not weak lagrange soft neutrosophic loop over $\langle L \cup I \rangle$.
(4) Their restricted intersection $(F,A) \cap_\varepsilon (K,B)$ over $\langle L \cup I \rangle$ is not weak lagrange soft neutrosophic soft loop over $\langle L \cup I \rangle$.

Theorem 24. *Let (F,A) and (H,B) be two weak lagrange soft neutrosophic loops over $\langle L \cup I \rangle$. Then*

(1) Their AND operation $(F,A) \wedge (H,B)$ is not weak lagrange soft neutrosophic loop over $\langle L \cup I \rangle$.
(2) Their OR operation $(F,A) \vee (H,B)$ is not weak lagrange soft neutrosophic loop over $\langle L \cup I \rangle$.

Definition 32. *Let $\langle L \cup I \rangle$ be a neutrosophic loop and (F,A) be a soft neutrosophic loop over $\langle L \cup I \rangle$. Then (F,A) is called Lagrange free soft neutrosophic loop if $F(a)$ is not lagrange neutrosophic subloop of $\langle L \cup I \rangle$, for all $a \in A$.*

Theorem 25. *Every lagrange free soft neutrosophic loop over $\langle L \cup I\rangle$ is a soft neutrosophic loop over $\langle L \cup I\rangle$ but the converse is not true.*

Theorem 26. *If $\langle L\cup I\rangle$ is lagrange free neutrosophic loop, then (F, A) over $\langle L\cup I\rangle$ is also lagrange free soft neutrosophic loop but the converse is not true.*

Theorem 27. *Let (F, A) and (K, B) be two lagrange free soft neutrosophic loops over $\langle L \cup I\rangle$. Then*

(1) Their extended union $(F, A) \cup_{\varepsilon} (K, B)$ over $\langle L \cup I\rangle$ is not lagrange free soft neutrosophic loop over $\langle L \cup I\rangle$.
(2) Their extended intersection $(F, A) \cap_{\varepsilon} (K, B)$ over $\langle L \cup I\rangle$ is not lagrange free soft neutrosophic loop over $\langle L \cup I\rangle$.
(3) Their restricted union $(F, A) \cup_{R} (K, B)$ over $\langle L\cup I\rangle$ is not lagrange free soft neutrosophic loop over $\langle L \cup I\rangle$.
(4) Their restricted intersection $(F, A) \cap_{\varepsilon} (K, B)$ over $\langle L \cup I\rangle$ is not lagrange free soft neutrosophic soft loop over $\langle L \cup I\rangle$.

Theorem 28. *Let (F, A) and (H, B) be two lagrange free soft neutrosophic loops over $\langle L \cup I\rangle$. Then*

(1) Their AND operation $(F, A) \wedge (H, B)$ is not lagrange free soft neutrosophic loop over $\langle L \cup I\rangle$.
(2) Their OR operation $(F, A) \vee (H, B)$ is not lagrange free soft neutrosophic loop over $\langle L \cup I\rangle$.

7. Soft Neutrosophic Biloop

Definition 33. *Let $(\langle B \cup I\rangle, *_1, *_2)$ be a neutrosophic biloop and (F, A) be a soft set over $(\langle B \cup I\rangle, *_1, *_2)$. Then (F, A) is called soft neutrosophic biloop if and only if $F(a)$ is neutrosophic subbiloop of $(\langle B \cup I\rangle, *_1, *_2)$, for all $a \in A$.*

Example 8. *Let $(\langle B \cup I\rangle, *_1, *_2) = (\{e, 1, 2, 3, 4, 5, eI, 1I, 2I, 3I, 4I, 5I\} \cup \{g \mid g^6 = e\}, *_1, *_2)$ be a neutrosophic biloop and $\{e, 2, eI, 2I\} \cup \left\{g^2, g^4, e\right\}$, $\{e, 3, eI, 3I\} \cup \left\{g^3, e\right\}$ are two neutrosophic subbiloops of $(\langle B \cup I\rangle, *_1, *_2)$. Then (F, A) is clearly soft neutrosophic biloop over $(\langle B \cup I\rangle, *_1, *_2)$, where*

$$\begin{aligned} F(a_1) &= \{e, 2, eI, 2I\} \cup \left\{g^2, g^4, e\right\}, \\ F(a_2) &= \{e, 3, eI, 3I\} \cup \left\{g^3, e\right\}. \end{aligned}$$

Theorem 29. *Let (F, A) and (H, A) be two soft neutrosophic biloops over $(\langle B \cup I\rangle, *_1, *_2)$. Then their intersection $(F, A)\cap(H, A)$ is again a soft neutrosophic biloop over $(\langle B \cup I\rangle, *_1, *_2)$.*

Proof. Straight forward. □

Theorem 30. *Let (F, A) and (H, B) be two soft neutrosophic biloops over $(\langle B \cup I\rangle, *_1, *_2)$ such that $A \cap B = \phi$, then their union is soft neutrosophic biloop over $(\langle B \cup I\rangle, *_1, *_2)$.*

Proof. Straight forward. □

Theorem 31. *Let (F, A) and (K, B) be two soft neutrosophic biloops over $(\langle B \cup I\rangle, *_1, *_2)$. Then*

(1) Their extended union $(F,A)\cup_{\varepsilon}(K,B)$ over $(\langle B\cup I\rangle,*_1,*_2)$ is not soft neutrosophic biloop over $(\langle B\cup I\rangle,*_1,*_2)$.
(2) Their extended intersection $(F,A)\cap_{\varepsilon}(K,B)$ over $(\langle B\cup I\rangle,*_1,*_2)$ is soft neutrosophic biloop over

$(\langle B\cup I\rangle,*_1,*_2)$.

(1) Their restricted union $(F,A)\cup_{R}(K,B)$ over $(\langle B\cup I\rangle,*_1,*_2)$ is not soft neutrosophic biloop over $(\langle B\cup I\rangle,*_1,*_2)$.
(2) Their restricted intersection $(F,A)\cap_{\varepsilon}(K,B)$ over $(\langle B\cup I\rangle,*_1,*_2)$ is soft neutrosophic biloop over $(\langle B\cup I\rangle,*_1,*_2)$.

Theorem 32. *Let (F,A) and (H,B) be two soft neutrosophic biloops over $(\langle B\cup I\rangle,*_1,*_2)$. Then*

(1) Their AND operation $(F,A)\wedge(H,B)$ is soft neutrosophic biloop over $(\langle B\cup I\rangle,*_1,*_2)$.
(2) Their OR operation $(F,A)\vee(H,B)$ is not soft neutrosophic biloop over $(\langle B\cup I\rangle,*_1,*_2)$.

Definition 34. *Let $B=(\langle L_n(m)\cup I\rangle\cup B_2,*_1,*_2)$ be a new class neutrosophic biloop and (F,A) be a soft set over $B=(\langle L_n(m)\cup I\rangle\cup B_2,*_1,*_2)$. Then (F,A) is called soft new class neutrosophic subbiloop if and only if $F(a)$ is neutrosophic subbiloop of $B=(\langle L_n(m)\cup I\rangle\cup B_2,*_1,*_2)$, for all $a\in A$.*

Example 9. *Let $B=(\langle B_1\cup B_2,*_1,*_2)$ be a new class neutrosophic biloop $B_1=(\langle L_5(3)\cup I\rangle=\{e,1,2,3,4,5,eI,1I,2I,3I,4I,5I\}$ be a new class of neutrosophic loop and $B_2=\{g:g^{12}=1\}$ is a group. $\{e,eI,1,1I\}\cup\{1,g^6\}$, $\{e,eI,2,2I\}\cup\{1,g^2,g^4,g^6,g^8,g^{10}\}$, $\{e,eI,3,3I\}\cup\{1,g^3,g^6,g^9\}$, $\{e,eI,4,4I\}\cup\{1,g^4,g^8\}$ are neutrosophic subloops of B. Then (F,A) is soft new class neutrosophic biloop over B, where*

$$\begin{aligned}
F(a_1) &= \{e,eI,1,1I\}\cup\{1,g^6\},\\
F(a_2) &= \{e,eI,2,2I\}\cup\{1,g^2,g^4,g^6,g^8,g^{10}\},\\
F(a_3) &= \{e,eI,3,3I\}\cup\{1,g^3,g^6,g^9\},\\
F(a_4) &= \{e,eI,4,4I\}\cup\{1,g^4,g^8\}.
\end{aligned}$$

Theorem 33. *Every soft new class neutrosophic biloop over $B=(\langle L_n(m)\cup I\rangle\cup B_2,*_1,*_2)$ is a soft neutrosophic biloop over but the converse is not true.*

Theorem 34. *Let (F,A) and (K,B) be two soft new class neutrosophic biloops over $B=(\langle L_n(m)\cup I\rangle\cup B_2,*_1,*_2)$. Then*

(1) Their extended union $(F,A)\cup_{\varepsilon}(K,B)$ over $\langle L_n(m)\cup I\rangle$ is not soft new class neutrosophic biloop over $B=(\langle L_n(m)\cup I\rangle\cup B_2,*_1,*_2)$.
(2) Their extended intersection $(F,A)\cap_{\varepsilon}(K,B)$ over $B=(\langle L_n(m)\cup I\rangle\cup B_2,*_1,*_2)$ is soft new class neutrosophic biloop over $B=(\langle L_n(m)\cup I\rangle\cup B_2,*_1,*_2)$.
(3) Their restricted union $(F,A)\cup_{R}(K,B)$ over $B=(\langle L_n(m)\cup I\rangle\cup B_2,*_1,*_2)$ is not soft new class neutrosophic biloop over $B=(\langle L_n(m)\cup I\rangle\cup B_2,*_1,*_2)$.
(4) Their restricted intersection $(F,A)\cap_{\varepsilon}(K,B)$ over $B=(\langle L_n(m)\cup I\rangle\cup B_2,*_1,*_2)$ is soft new class neutrosophic soft biloop over $B=(\langle L_n(m)\cup I\rangle\cup B_2,*_1,*_2)$.

Theorem 35. *Let (F,A) and (H,B) be two soft new class neutrosophic biloops over $B=(\langle L_n(m)\cup I\rangle\cup B_2,*_1,*_2)$. Then*

(1) Their AND operation $(F,A)\wedge(H,B)$ is soft new class neutrosophic biloop over $B=(\langle L_n(m)\cup I\rangle\cup B_2,*_1,*_2)$.
(2) Their OR operation $(F,A)\vee(H,B)$ is not soft new class neutrosophic biloop over $B=(\langle L_n(m)\cup I\rangle\cup B_2,*_1,*_2)$.

Definition 35. *Let (F,A) be a soft neutrosophic biloop over $B=(\langle B_1\cup I\rangle\cup B_2,*_1,*_2)$, then (F,A) is called the identity soft neutrosophic biloop over $B=(\langle B_1\cup I\rangle\cup B_2,*_1,*_2)$ if $F(a)=\{e_1,e_2\}$, for all $a\in A$, where e_1,e_2 are the identities element of $B=(\langle B_1\cup I\rangle\cup B_2,*_1,*_2)$ respectively.*

Definition 36. *Let (F,A) be a soft neutrosophic biloop over $B=(\langle B_1\cup I\rangle\cup B_2,*_1,*_2)$, then (F,A) is called Full-soft neutrosophic biloop over $B=(B_1\cup B_2,*_1,*_2)$ if $F(a)=B=(\langle B_1\cup I\rangle\cup B_2,*_1,*_2)$, for all $a\in A$.*

Definition 37. *Let (F,A) and (H,B) be two soft neutrosophic biloops over $B=(\langle B_1\cup I\rangle\cup B_2,*_1,*_2)$. Then (H,B) is soft neutrosophic subbiloop of (F,A), if*

(1) $B\subset A$.
(2) $H(a)$ is neutrosophic subbiloop of $F(a)$, for all $a\in A$.

Example 10. *Let $B=(\langle B_1\cup I\rangle\cup B_2,*_1,*_2)$ be a neutrosophic biloop where $B_1=(\langle L_5(3)\cup I\rangle=\{e,1,2,3,4,5,eI,1I,2I,3I,4I,5I\}$ be a neutrosophic loop and $B_2=\left\{g:g^{12}=1\right\}$ is a group. $\{e,eI,1,1I\}\cup\left\{1,g^6\right\}$, $\{e,eI,2,2I\}\cup\left\{1,g^2,g^4,g^6,g^8,g^{10}\right\}$, $\{e,eI,3,3I\}\cup\left\{1,g^3,g^6,g^9\right\}$, $\{e,eI,4,4I\}\cup\left\{1,g^4,g^8\right\}$ are neutrosophic subbiloops of B. Then (F,A) is soft neutrosophic biloop over B, where*

$$\begin{aligned}
F(a_1) &= \{e,eI,1,1I\}\cup\left\{1,g^6\right\},\\
F(a_2) &= \{e,eI,2,2I\}\cup\left\{1,g^2,g^4,g^6,g^8,g^{10}\right\},\\
F(a_3) &= \{e,eI,3,3I\}\cup\left\{1,g^3,g^6,g^9\right\},\\
F(a_4) &= \{e,eI,4,4I\}\cup\{1,g^4,g^8\}.
\end{aligned}$$

(H,B) is soft neutrosophic subbiloop of (F,A), where

$$\begin{aligned}
H(a_2) &= \{e,2,\}\cup\left\{1,g^6\right\},\\
H(a_3) &= \{e,eI,3I\}\cup\left\{1,g^6\right\}.
\end{aligned}$$

Definition 38. *Let $B=(\langle B_1\cup I\rangle\cup B_2,*_1,*_2)$ be a neutrosophic biloop and (F,A) be a soft set over $B=(\langle B_1\cup I\rangle\cup B_2,*_1,*_2)$. Then (F,A) is called soft neutrosophic Moufang biloop if and only if $F(a)=(P_1\cup P_2,*_1,*_2$, where P_1 is a proper neutrosophic Moufang loop of $B_1)$ is neutrosophic subbiloop of $B=(\langle B_1\cup I\rangle\cup B_2,*_1,*_2)$, for all $a\in A$.*

Example 11. *Let $B=(\langle B_1\cup I\rangle\cup B_2),*_1,*_2)$ be a neutrosophic biloop where $B_1=\langle L_5(3)\cup I\rangle$ and $B_2=S_3$. Let $P=\{e,2,eI,2I\}\cup\{e,(12)\}$ and $Q=\{e,3,eI,3I\}\cup\{e,(123),(132)\}$ are neutrosophic subbiloops of B in which $\{e,2,eI,2I\}$ and $\{e,3,eI,3I\}$ are proper neutrosophic Moufang loops. Then clearly (F,A) is soft neutrosophic Moufang biloop over B, where*

$$\begin{aligned}
F(a_1) &= \{e,2,eI,2I\}\cup\{e,(12)\},\\
F(a_2) &= \{e,3,eI,3I\}\cup\{e,(123),(132)\}.
\end{aligned}$$

Theorem 36. *Every soft neutrosophic Moufang biloop over* $B = (\langle B_1 \cup I \rangle \cup B_2, *_1, *_2)$ *is a soft neutrosophic biloop but the converse is not true.*

Theorem 37. *Let* (F, A) *and* (K, B) *be two soft neutrosophic Moufang biloops over* $B = (\langle B_1 \cup I \rangle \cup B_2, *_1, *_2)$. *Then*

(1) Their extended union $(F, A) \cup_\varepsilon (K, B)$ over B is not soft neutrosophic Moufang biloop over B.
(2) Their extended intersection $(F, A) \cap_\varepsilon (K, B)$ over B is soft neutrosophic Moufang biloop over B.
(3) Their restricted union $(F, A) \cup_R (K, B)$ over B is not soft neutrosophic Moufang biloop over B.
(4) Their restricted intersection $(F, A) \cap_\varepsilon (K, B)$ over B is soft neutrosophic Moufang biloop over B.

Theorem 38. *Let* (F, A) *and* (H, B) *be two soft neutrosophic Moufang biloops over* $B = (\langle B_1 \cup I \rangle \cup B_2, *_1, *_2)$. *Then*

(1) Their AND operation $(F, A) \wedge (H, B)$ is soft neutrosophic Moufang biloop over B.
(2) Their OR operation $(F, A) \vee (H, B)$ is not soft neutrosophic Moufang biloop over B.

Definition 39. *Let* $B = (\langle B_1 \cup I \rangle \cup B_2, *_1, *_2)$ *be a neutrosophic biloop and* (F, A) *be a soft set over* $B = (\langle B_1 \cup I \rangle \cup B_2, *_1, *_2)$. *Then* (F, A) *is called soft neutrosophic Bol biloop if and only if* $F(a) = (P_1 \cup P_2, *_1, *_2$, *where* P_1 *is a proper neutrosophic Bol loop of* $B_1)$ *is neutrosophic subbiloop of* $B = (\langle B_1 \cup I \rangle \cup B_2, *_1, *_2)$, *for all* $a \in A$.

Example 12. *Let* $B = (\langle B_1 \cup I \rangle \cup B_2), *_1, *_2)$ *be a neutrosophic biloop where* $B_1 = \langle L_5(3) \cup I \rangle$ *and* $B_2 = S_3$. *Let* $P = \{e, 3, eI, 3I\} \cup \{e, (12)\}$ *and* $Q = \{e, 2, eI, 2I\} \cup \{e, (123), (132)\}$ *are neutrosophic subbiloops of* B *in which* $\{e, 3, eI, 3I\}$ *and* $\{e, 2, eI, 2I\}$ *are proper neutrosophic Bol loops. Then clearly* (F, A) *is soft neutrosophic Bol biloop over* B, *where*

$$\begin{aligned} F(a_1) &= \{e, 3, eI, 3I\} \cup \{e, (12)\}, \\ F(a_2) &= \{e, 2, eI, 2I\} \cup \{e, (123), (132)\}. \end{aligned}$$

Theorem 39. *Every soft neutrosophic Bol biloop over* $B = (\langle B_1 \cup I \rangle \cup B_2, *_1, *_2)$ *is a soft neutrosophic biloop but the converse is not true.*

Theorem 40. *Let* (F, A) *and* (K, B) *be two soft neutrosophic Bol biloops over* $B = (\langle B_1 \cup I \rangle \cup B_2, *_1, *_2)$. *Then*

(1) Their extended union $(F, A) \cup_\varepsilon (K, B)$ over B is not soft neutrosophic Bol biloop over B.
(2) Their extended intersection $(F, A) \cap_\varepsilon (K, B)$ over B is soft neutrosophic Bol biloop over B.
(3) Their restricted union $(F, A) \cup_R (K, B)$ over B is not soft neutrosophic Bol biloop over B.
(4) Their restricted intersection $(F, A) \cap_\varepsilon (K, B)$ over B is soft neutrosophic Bol biloop over B.

Theorem 41. *Let* (F, A) *and* (H, B) *be two soft neutrosophic Bol biloops over* $B = (\langle B_1 \cup I \rangle \cup B_2, *_1, *_2)$. *Then*

(1) Their AND operation $(F,A)\wedge(H,B)$ is soft neutrosophic Bol biloop over B.
(2) Their OR operation $(F,A)\vee(H,B)$ is not soft neutrosophic Bol biloop over B.

Definition 40. *Let $(\langle B\cup I\rangle,*_1,*_2)$ be a neutrosophic biloop and (F,A) be a soft set over $(\langle B\cup I\rangle,*_1,*_2)$. Then (F,A) is called soft Lagrange neutrosophic biloop if and only if $F(a)$ is Lagrange neutrosophic subbiloop of $(\langle B\cup I\rangle,*_1,*_2)$, for all $a\in A$.*

Example 13. *Let $B=(B_1\cup B_2,*_1,*_2)$ be a neutrosophic biloop of order 20, where $B_1=\{\langle L_5(3)\cup I\rangle,*_1\}$ and $B_2=\{g|\ g^8=1\}$. Let $(P=P_1\cup P_2,*_1,*_2)$ where $P_1=\{e,eI,2,2I\}\subset B_1$ and $P_2=\{1\}\subset B2$ and $(Q=Q_1\cup Q_2,*_1,*_2)$ where $Q_1=\{e,eI,3,3I\}\subset B_1$ and $Q_2=\{1\}\subset B_2$ are Lagrange neutrosophic subbiloops of B. Then clearly (F,A) is a soft Lagrange neutrosophic biloop over B, where*

$$\begin{aligned} F(a_1) &= \{e,eI,2,2I\}\cup\{1\},\\ F(a_2) &= \{e,eI,3,3I\}\cup\{1\}. \end{aligned}$$

Theorem 42. *Every soft Lagrange neutrosophic biloop over $B=(\langle B_1\cup I\rangle\cup B_2,*_1,*_2)$ is a soft neutrosophic biloop but the converse is not true.*

Theorem 43. *Let (F,A) and (K,B) be two soft Lagrange neutrosophic biloops over $B=(\langle B_1\cup I\rangle\cup B_2,*_1,*_2)$. Then*

(1) Their extended union $(F,A)\cup_\varepsilon(K,B)$ over B is not soft Lagrange neutrosophic biloop over B.
(2) Their extended intersection $(F,A)\cap_\varepsilon(K,B)$ over B is not soft Lagrange neutrosophic biloop over B.
(3) Their restricted union $(F,A)\cup_R(K,B)$ over B is not soft Lagrange neutrosophic biloop over B.
(4) Their restricted intersection $(F,A)\cap_\varepsilon(K,B)$ over B is not soft Lagrange neutrosophic biloop over B.

Theorem 44. *Let (F,A) and (H,B) be two soft Lagrange neutrosophic biloops over $B=(\langle B_1\cup I\rangle\cup B_2,*_1,*_2)$. Then*

(1) Their AND operation $(F,A)\wedge(H,B)$ is not soft Lagrange neutrosophic biloop over B.
(2) Their OR operation $(F,A)\vee(H,B)$ is not soft Lagrange neutrosophic biloop over B.

Definition 41. *Let $(\langle B\cup I\rangle,*_1,*_2)$ be a neutrosophic biloop and (F,A) be a soft set over $(\langle B\cup I\rangle,*_1,*_2)$. Then (F,A) is called soft weakly Lagrange neutrosophic biloop if atleast one $F(a)$ is not Lagrange neutrosophic subbiloop of $(\langle B\cup I\rangle,*_1,*_2)$, for some $a\in A$.*

Example 14. *Let $B=(B_1\cup B_2,*_1,*_2)$ be a neutrosophic biloop of order 20, where $B_1=\{\langle L_5(3)\cup I\rangle,*_1\}$ and $B_2=\{g|\ g^8=1\}$. Let $(P=P_1\cup P_2,*_1,*_2)$ where $P_1=\{e,eI,2,2I\}\subset B_1$ and $P_2=\{1\}\subset B2$ is a Lagrange neutrosophic subbiloop of B and $(Q=Q_1\cup Q_2,*_1,*_2)$ where $Q_1=\{e,eI,3,3I\}\subset B_1$ and $Q_2=\{1,g^4\}\subset B_2$ is not Lagrange neutrosophic subbiloop of B. Then clearly (F,A) is a soft weakly*

Lagrange neutrosophic biloop over B*, where*

$$\begin{aligned} F(a_1) &= \{e, eI, 2, 2I\} \cup \{1\}, \\ F(a_2) &= \{e, eI, 3, 3I\} \cup \{1, g^4\}. \end{aligned}$$

Theorem 45. *Every soft weakly Lagrange neutrosophic biloop over* $B = (\langle B_1 \cup I\rangle \cup B_2, *_1, *_2)$ *is a soft neutrosophic biloop but the converse is not true.*

Theorem 46. *If* $B = (\langle B_1 \cup I\rangle \cup B_2, *_1, *_2)$ *is a weakly Lagrange neutrosophic biloop, then* (F, A) *over* B *is also soft weakly Lagrange neutrosophic biloop but the converse is not holds.*

Theorem 47. *Let* (F, A) *and* (K, B) *be two soft weakly Lagrange neutrosophic biloops over* $B = (\langle B_1 \cup I\rangle \cup B_2, *_1, *_2)$*. Then*

1. Their extended union $(F, A) \cup_\varepsilon (K, B)$ over B is not soft weakly Lagrange neutrosophic biloop over B.
2. Their extended intersection $(F, A) \cap_\varepsilon (K, B)$ over B is not soft weakly Lagrange neutrosophic biloop over B.
3. Their restricted union $(F, A) \cup_R (K, B)$ over B is not soft weakly Lagrange neutrosophic biloop over B.
4. Their restricted intersection $(F, A) \cap_\varepsilon (K, B)$ over B is not soft weakly Lagrange neutrosophic biloop over B.

Theorem 48. *Let* (F, A) *and* (H, B) *be two soft weakly Lagrange neutrosophic biloops over* $B = (\langle B_1 \cup I\rangle \cup B_2, *_1, *_2)$*. Then*

1. Their AND operation $(F, A) \wedge (H, B)$ is soft not weakly Lagrange neutrosophic biloop over B.
2. Their OR operation $(F, A) \vee (H, B)$ is not soft weakly Lagrange neutrosophic biloop over B.

Definition 42. *Let* $(\langle B \cup I\rangle, *_1, *_2)$ *be a neutrosophic biloop and* (F, A) *be a soft set over* $(\langle B \cup I\rangle, *_1, *_2)$*. Then* (F, A) *is called soft Lagrange free neutrosophic biloop if and only if* $F(a)$ *is not Lagrange neutrosophic subbiloop of* $(\langle B \cup I\rangle, *_1, *_2)$*, for all* $a \in A$*.*

Example 15. *Let* $B = (B_1 \cup B_2, *_1, *_2)$ *be a neutrosophic biloop of order* 20*, where* $B_1 = \{\langle L_5(3) \cup I\rangle, *_1\}$ *and* $B_2 = \{g |\ g^8 = 1\}$*. Let* $(P = P_1 \cup P_2, *_1, *_2)$ *where* $P_1 = \{e, eI, 2, 2I\} \subset B_1$ *and* $P_2 = \{1, g^2, g^4, g^6\} \subset B2$ *and* $(Q = Q_1 \cup Q_2, *_1, *_2)$ *where* $Q_1 = \{e, eI, 3, 3I\} \subset B_1$ *and* $Q_2 = \{1, g^4\} \subset B_2$ *are not Lagrange neutrosophic subbiloop of* B*. Then clearly* (F, A) *is a soft Lagrange free neutrosophic biloop over* B*, where*

$$\begin{aligned} F(a_1) &= \{e, eI, 2, 2I\} \cup \{1, g^2, g^4, g^6\}, \\ F(a_2) &= \{e, eI, 3, 3I\} \cup \{1, g^4\}. \end{aligned}$$

Theorem 49. *Every soft Lagrange free neutrosophic biloop over* $B = (\langle B_1 \cup I\rangle \cup B_2, *_1, *_2)$ *is a soft neutrosophic biloop but the converse is not true.*

Theorem 50. *If* $B = (\langle B_1 \cup I\rangle \cup B_2, *_1, *_2)$ *is a Lagrange free neutrosophic biloop, then* (F, A) *over* B *is also soft Lagrange free neutrosophic biloop but the converse is not holds.*

Theorem 51. *Let* (F, A) *and* (K, B) *be two soft Lagrange free neutrosophic biloops over* $B = (\langle B_1 \cup I\rangle \cup B_2, *_1, *_2)$*. Then*

(1) Their extended union $(F,A) \cup_{\varepsilon} (K,B)$ over B is not soft Lagrange free neutrosophic biloop over B.
(2) Their extended intersection $(F,A) \cap_{\varepsilon} (K,B)$ over B is not soft Lagrange free neutrosophic biloop over B.
(3) Their restricted union $(F,A) \cup_{R} (K,B)$ over B is not soft Lagrange free neutrosophic biloop over B.
(4) Their restricted intersection $(F,A) \cap_{\varepsilon} (K,B)$ over B is not soft Lagrange free neutrosophic biloop over B.

Theorem 52. *Let (F,A) and (H,B) be two soft Lagrange free neutrosophic biloops over $B = (\langle B_1 \cup I\rangle \cup B_2, *_1, *_2)$. Then*

(1) Their AND operation $(F,A) \wedge (H,B)$ is not soft Lagrange free neutrosophic biloop over B.
(2) Their OR operation $(F,A) \vee (H,B)$ is not soft Lagrange free neutrosophic biloop over B.

Definition 43. *Let $B = (B_1 \cup B_2, *_1, *_2)$ be a neutrosophic biloop where B_1 is a neutrosopphic biloop and B_2 is a neutrosophic group and (F,A) be soft set over B. Then (F,A) over B is called soft strong neutrosophic biloop if and only if $F(a)$ is a neutrosopchic subbiloop of B, for all $a \in A$.*

Example 16. *Let $(B = B_1 \cup B_2, *_1, *_2)$ where $B_1 = \langle L_5(2) \cup I\rangle$ is a neutrosophic loop and $B_2 = \{1,2,3,4,I,2I,3I,4I\}$ under multiplication modulo 5 is a neutrosophic group. Let $P = \{e,2,eI,2I\} \cup \{1,I,4I\}$ and $Q = \{e,3,eI,3I\} \cup \{1,I\}$ are neutrosophic subbiloops of B. Then (F,A) is soft strong neutrosophic biloop of B, where*

$$\begin{aligned} F(a_1) &= \{e,2,eI,2I\} \cup \{1,I,4I\}, \\ F(a_2) &= \{e,3,eI,3I\} \cup \{1,I\}. \end{aligned}$$

Theorem 53. *Every soft strong neutrosophic biloop over $B = (\langle B_1 \cup I\rangle \cup B_2, *_1, *_2)$ is a soft neutrosophic biloop but the converse is not true.*

Theorem 54. *If $B = (\langle B_1 \cup I\rangle \cup B_2, *_1, *_2)$ is a strong neutrosophic biloop, then (F,A) over B is also soft strong neutrosophic biloop but the converse is not holds.*

Theorem 55. *Let (F,A) and (K,B) be two soft soft neutrosophic biloops over $B = (\langle B_1 \cup I\rangle \cup B_2, *_1, *_2)$. Then*

(1) Their extended union $(F,A) \cup_{\varepsilon} (K,B)$ over B is not soft strong neutrosophic biloop over B.
(2) Their extended intersection $(F,A) \cap_{\varepsilon} (K,B)$ over B is soft strong neutrosophic biloop over B.
(3) Their restricted union $(F,A) \cup_{R} (K,B)$ over B is not soft strong neutrosophic biloop over B.
(4) Their restricted intersection $(F,A) \cap_{\varepsilon} (K,B)$ over B is soft strong neutrosophic biloop over B.

Theorem 56. *Let (F,A) and (H,B) be two soft strong neutrosophic biloops over $B = (\langle B_1 \cup I\rangle \cup B_2, *_1, *_2)$. Then*

(1) Their AND operation $(F,A) \wedge (H,B)$ is soft strong neutrosophic biloop over B.

(2) Their OR operation $(F,A)\vee(H,B)$ is not soft strong neutrosophic biloop over B.

Definition 44. *Let $B=(B1\cup B2,*1,*2)$ be a neutrosophic biloop of type II and (F,A) be a soft set over B. Then (F,A) over B is called soft neutrosophic biloop of type II if and only if $F(a)$ is a neutrosopchic subbiloop of B, for all $a\in A$.*

Example 17. *Let $B=(B1\cup B2,*1,*2)$ where $B1=\langle L7(3)\cup I\rangle$ and $B2=L5(2)$, then B is a neutrosophic biloop of type II. Hence (F,A) over B is a soft neutrosophic biloop of type II.*

All the properties defined for soft neutrosophic biloop can easily be extend to soft neutrosophic biloop of type II.

8. Soft Neutrosophic N-loop

Definition 45. *Let $S(B)=\{S(B_1)\cup S(B_2)\cup\ldots\cup S(B_n),*_1,\ldots,*_N\}$ be a neutrosophic N-loop and (F,A) be a soft set over $S(B)$. Then (F,A) over $S(B)$ is called soft neutrosophic N-loop if and only if $F(a)$ is a neutrosopchic sub N-loop of $S(B)$, for all $a\in A$.*

Example 18. *Let $S(B)=\{S(B_1\cup S(B_2)\cup S(B_3),*_1,*_2,*_3\}$ where $S(B_1)=\{\langle L_5(3)\cup I\rangle\}$, $S(B_2)=\langle g|g^{12}=1\rangle$ and $S(B_3)=S_3$, is a neutrosophic 3-loop. Let $P=\{e,eI,2,2I,1,g^6,e,(12)\}$ and $\{e,eI,3,3I,1,g^4,g^8,e,(13)\}$ are neutrosophic sub N-loops of $S(B)$. Then (F,A) is sof neutrosophic N-loop over $S(B)$, where*

$$\begin{aligned}F(a_1)&=\{e,eI,2,2I,1,g^6,e,(12)\},\\F(a_2)&=\{e,eI,3,3I,1,g^4,g^8,e,(13)\}.\end{aligned}$$

Theorem 57. *Let (F,A) and (H,A) be two soft neutrosophic N-loops over $S(B)$. Then their intersection $(F,A)\cap(H,A)$ is again a soft neutrosophic biloop over $S(B)$.*

Proof. Straight forward. □

Theorem 58. *Let (F,A) and (H,C) be two soft neutrosophic N-loops over $S(B)$ such that $A\cap C=\phi$, then their union is soft neutrosophic biloop over $S(B)$.*

Proof. Straight forward. □

Theorem 59. *Let (F,A) and (K,C) be two soft neutrosophic N-loops over $S(B)=(S(B_1)\cup S(B_2)\cup,...,\cup S(B_N),*_1,...,*_N)$. Then*

(1) Their extended union $(F,A)\cup_\varepsilon(K,C)$ over $S(B)$ is not soft neutrosophic N-loop over $S(B)$.
(2) Their extended intersection $(F,A)\cap_\varepsilon(K,C)$ over $S(B)$ is soft neutrosophic N-loop over $S(B)$.
(3) Their restricted union $(F,A)\cup_R(K,C)$ over $S(B)$ is not soft neutrosophic N-loop over $S(B)$.
(4) Their restricted intersection $(F,A)\cap_\varepsilon(K,C)$ over $S(B)$ is soft neutrosophic N-loop over $S(B)$.

Theorem 60. *Let (F,A) and (H,C) be two soft neutrosophic N-loops over $S(B)$. Then*

(1) Their AND operation $(F, A) \wedge (H, B)$ is soft neutrosophic N-loop over $S(B)$.
(2) Their OR operation $(F, A) \vee (H, B)$ is not soft neutrosophic N-loop over $S(B)$.

Definition 46. *Let $S(L) = \{L_1 \cup L_2 \cup \ldots \cup L_N, *_1, \ldots, *_N\}$ be a neutrosophic N-loop of level II and (F, A) be a soft set over $S(L)$. Then (F, A) over $S(L)$ is called soft neutrosophic N-loop of level II if and only if $F(a)$ is a neutrosopchic sub N-loop of $S(L)$, for all $a \in A$.*

Example 19. *Let $S(L) = \{L_1 \cup L_2 \cup L_3 \cup L_4, *_1, *_2, *_3, *_4\}$ be a neutrosophic 4-loop of level II where $L_1 = \{\langle L_5(3) \cup I \rangle\}$, $L_2 = \{e, 1, 2, 3\}$, $L_3 = S_3$ and $L_4 = N(Z_3)$, under multiplication modulo 3. Let $P = \{e, eI, 2, 2I\} \cup \{e, 1\} \cup \{e, (12)\} \cup \{1, I\}$ and $\{e, eI, 3, 3I\} \cup \{e, 2\} \cup \{e, (13)\} \cup \{1, 2\}$ are neutrosophic sub N-loops of $S(L)$. Then (F, A) is sof neutrosophic N-loop of level II over $S(L)$, where*

$$\begin{aligned} F(a_1) &= \{e, eI, 2, 2I\} \cup \{e, 1\} \cup \{e, (12)\} \cup \{1, I\}, \\ F(a_2) &= \{e, eI, 3, 3I\} \cup \{e, 2\} \cup \{e, (13)\} \cup \{1, 2\}. \end{aligned}$$

Theorem 61. *Every soft neutrosophic N-loop of level II over $S(L) = \{L_1 \cup L_2 \cup \ldots \cup L_N, *_1, \ldots, *_N\}$ is a soft neutrosophic N-loop but the converse is not true.*

Theorem 62. *Let (F, A) and (K, C) be two soft neutrosophic N-loops of level II over $S(L) = \{L_1 \cup L_2 \cup \ldots \cup L_N, *_1, \ldots, *_N\}$. Then*

(1) Their extended union $(F, A) \cup_\varepsilon (K, C)$ over $S(L)$ is not soft neutrosophic N-loop of level II over $S(L)$.
(2) Their extended intersection $(F, A) \cap_\varepsilon (K, C)$ over $S(L)$ is soft neutrosophic N-loop of level II over $S(L)$.
(3) Their restricted union $(F, A) \cup_R (K, C)$ over $S(L)$ is not soft neutrosophic N-loop of level II over $S(L)$.
(4) Their restricted intersection $(F, A) \cap_\varepsilon (K, C)$ over $S(L)$ is soft neutrosophic N-loop of level II over $S(L)$.

Theorem 63. *Let (F, A) and (H, C) be two soft neutrosophic N-loops of level II over $S(L) = \{L_1 \cup L_2 \cup \ldots \cup L_N, *_1, \ldots, *_N\}$. Then*

(1) Their AND operation $(F, A) \wedge (H, B)$ is soft neutrosophic N-loop of level II over $S(L)$.
(2) Their OR operation $(F, A) \vee (H, B)$ is not soft neutrosophic N-loop of level 11 over $S(L)$.

Now what all we define for neutrosophic N-loops will be carried out to neutrosophic N-loops of level II with appropriate modifications.

Definition 47. *Let (F, A) be a soft neutrosophic N-loop over $S(B) = (S(B_1) \cup S(B_2) \cup, ..., \cup S(B_N), *_1, ..., *_N)$, then (F, A) is called the identity soft neutrosophic N-loop over $S(B)$ if $F(a) = \{e_1, e_2, ..., e_N\}$, for all $a \in A$, where e_1,e_2, ..., e_N are the identities element of $S(B_1), S(B_2), ..., S(B_N)$ respectively.*

Definition 48. *Let (F, A) be a soft neutrosophic N-loop over $S(B) = (S(B_1) \cup S(B_2) \cup, ..., \cup S(B_N), *_1, ..., *_N)$, then (F, A) is called Full-soft neutrosophic N-loop over $S(B)$ if $F(a) = S(B)$, for all $a \in A$.*

Definition 49. *Let (F, A) and (H, C) be two soft neutrosophic N-loops over $S(B) = (S(B_1) \cup S(B_2) \cup, ..., \cup S(B_N), *_1, ..., *_N)$. Then (H, C) is soft neutrosophic sub N-loop of (F, A), if*

(1) $B \subset A$.
(2) $H(a)$ is neutrosophic sub N-loop of $F(a)$, for all $a \in A$.

Definition 50. *Let $S(B) = (S(B_1) \cup S(B_2) \cup, ..., \cup S(B_N), *_1, ..., *_N)$ be a neutrosophic N-loop and (F, A) be a soft set over $S(B)$. Then (F, A) is called soft Lagrange neutrosophic N-loop if and only if $F(a)$ is Lagrange neutrosophic sub N-loop of $S(B)$, for all $a \in A$.*

Theorem 64. *All soft Lagrange neutrosophic N-loops are soft neutrosophic N-loops but the converse is not true.*

Theorem 65. *Let (F, A) and (K, C) be two soft Lagrange neutrosophic N-loops over $S(B) = (S(B_1) \cup S(B_2) \cup, ..., \cup S(B_N), *_1, ..., *_N)$. Then*

(1) Their extended union $(F, A) \cup_\varepsilon (K, C)$ over $S(B)$ is not soft Lagrange neutrosophic N-loop over $S(B)$.
(2) Their extended intersection $(F, A) \cap_\varepsilon (K, C)$ over $S(B)$ is not soft Lagrange neutrosophic N-loop over $S(B)$.
(3) Their restricted union $(F, A) \cup_R (K, C)$ over $S(B)$ is not soft Lagrange neutrosophic N-loop over $S(B)$.
(4) Their restricted intersection $(F, A) \cap_\varepsilon (K, C)$ over $S(B)$ is not soft Lagrange neutrosophic N-loop over $S(B)$.

Theorem 66. *Let (F, A) and (H, C) be two soft Lagrange neutrosophic N-loops over $S(B)$. Then*

(1) Their AND operation $(F, A) \wedge (H, B)$ is not soft Lagrane neutrosophic N-loop over $S(B)$.
(2) Their OR operation $(F, A) \vee (H, B)$ is not soft Lagrange neutrosophic N-loop over $S(B)$.

Definition 51. *Let $S(B) = (S(B_1) \cup S(B_2) \cup, ..., \cup S(B_N), *_1, ..., *_N)$ be a neutrosophic N-loop and (F, A) be a soft set over $S(B)$. Then (F, A) is called soft weakly Lagrange neutrosophic N-loop if atleast one $F(a)$ is not Lagrange neutrosophic sub N-loop of $S(B)$, for all $a \in A$.*

Theorem 67. *All soft weakly Lagrange neutrosophic N-loops are soft neutrosophic N-loops but the converse is not true.*

Theorem 68. *Let (F, A) and (K, C) be two soft weakly Lagrange neutrosophic N-loops over $S(B) = (S(B_1) \cup S(B_2) \cup, ..., \cup S(B_N), *_1, ..., *_N)$. Then*

(1) Their extended union $(F, A) \cup_\varepsilon (K, C)$ over $S(B)$ is not soft weakly Lagrange neutrosophic N-loop over $S(B)$.
(2) Their extended intersection $(F, A) \cap_\varepsilon (K, C)$ over $S(B)$ is not soft weakly Lagrange neutrosophic N-loop over $S(B)$.
(3) Their restricted union $(F, A) \cup_R (K, C)$ over $S(B)$ is not soft weakly Lagrange neutrosophic N-loop over $S(B)$.
(4) Their restricted intersection $(F, A) \cap_\varepsilon (K, C)$ over $S(B)$ is not soft weakly Lagrange neutrosophic N-loop over $S(B)$.

Theorem 69. *Let (F, A) and (H, C) be two soft weakly Lagrange neutrosophic N-loops over $S(B)$. Then*

(1) Their AND operation $(F, A) \wedge (H, B)$ is not soft weakly Lagrane neutrosophic N-loop over $S(B)$.
(2) Their OR operation $(F, A) \vee (H, B)$ is not soft weakly Lagrange neutrosophic N-loop over $S(B)$.

Definition 52. *Let $S(B) = (S(B_1) \cup S(B_2) \cup, ..., \cup S(B_N), *_1, ..., *_N)$ be a neutrosophic N-loop and (F, A) be a soft set over $S(B)$. Then (F, A) is called soft Lagrange free neutrosophic N-loop if and only if $F(a)$ is not Lagrange neutrosophic sub N-loop of $S(B)$, for all $a \in A$.*

Theorem 70. *All soft Lagrange free neutrosophic N-loops are soft neutrosophic N-loops but the converse is not true.*

Theorem 71. *Let (F, A) and (K, C) be two soft Lagrange free neutrosophic N-loops over $S(B) = (S(B_1) \cup S(B_2) \cup, ..., \cup S(B_N), *_1, ..., *_N)$. Then*

(1) Their extended union $(F, A) \cup_{\varepsilon} (K, C)$ over $S(B)$ is not soft Lagrange free neutrosophic N-loop over $S(B)$.
(2) Their extended intersection $(F, A) \cap_{\varepsilon} (K, C)$ over $S(B)$ is not soft Lagrange free neutrosophic N-loop over $S(B)$.
(3) Their restricted union $(F, A) \cup_R (K, C)$ over $S(B)$ is not soft Lagrange free neutrosophic N-loop over $S(B)$.
(4) Their restricted intersection $(F, A) \cap_{\varepsilon} (K, C)$ over $S(B)$ is not soft Lagrange free neutrosophic N-loop over $S(B)$.

Theorem 72. *Let (F, A) and (H, C) be two soft Lagrange free neutrosophic N-loops over $S(B)$. Then*

(1) Their AND operation $(F, A) \wedge (H, B)$ is not soft Lagrane free neutrosophic N-loop over $S(B)$.
(2) Their OR operation $(F, A) \vee (H, B)$ is not soft Lagrange free neutrosophic N-loop over $S(B)$.

Definition 53. *Let $\{\langle L \cup I \rangle = L_1 \cup L_2 \cup L_3, *_1, \ldots, *_N\}$ be a neutrosophic N-loop and (F, A) be a soft set over $\{\langle L \cup I \rangle = L_1 \cup L_2 \cup L_3, *_1, ..., *_N\}$. Then (F, A) over $\{\langle L \cup I \rangle = L_1 \cup L_2 \cup L_3, *_1, ..., *_N\}$ is called soft strong neutrosophic N-loop if and only if $F(a)$ is strong neutrosopchic sub N-loop of $\{\langle L \cup I \rangle = L_1 \cup L_2 \cup L_3, *_1, ..., *_N\}$, for all $a \in A$.*

Example 20. *Let $\{\langle L \cup I \rangle = L_1 \cup L_2 \cup L_3, *_1, *_2, *_3\}$ where $L_1 = \langle L_5(3) \cup I \rangle, L_2 = \langle L_7(3) \cup I \rangle$ and $L_2 = \{1, 2, I, 2I\}$. $\{\langle L \cup I \rangle\}$ is a strong neutrosophic 3-loop. Then (F, A) is a soft strong neutrosophic N-loop over $\langle L \cup I \rangle$, where*

$$\begin{aligned} F(a_1) &= \{e, 2, eI, 2I\} \cup \{e, 2, eI, 2I\} \cup \{1, I\}, \\ F(a_2) &= \{e, 3, eI, 3I\} \cup \{e, 3, eI, 3I\} \cup \{1, 2, 2I\}. \end{aligned}$$

Theorem 73. *All soft strong neutrosophic N-loops are soft neutrosophic N-loops but the converse is not true.*

Theorem 74. *Let (F, A) and (K, C) be two soft strong neutrosophic N-loops over $\{\langle L \cup I \rangle = L_1 \cup L_2 \cup L_3, *_1, ..., *_N\}$. Then*

(1) Their extended union $(F,A)\cup_{\varepsilon}(K,C)$ over $\{\langle L\cup I\rangle = L_1\cup L_2\cup L_3, *_1, ..., *_N\}$ is not soft Lagrange free neutrosophic N-loop.
(2) Their extended intersection $(F,A)\cap_{\varepsilon}(K,C)$ over $\{\langle L\cup I\rangle = L_1\cup L_2\cup L_3, *_1, ..., *_N\}$ is soft Lagrange free neutrosophic N-loop.
(3) Their restricted union $(F,A)\cup_{R}(K,C)$ over $\{\langle L\cup I\rangle = L_1\cup L_2\cup L_3, *_1, ..., *_N\}$ is not soft Lagrange free neutrosophic N-loop over.
(4) Their restricted intersection $(F,A)\cap_{\varepsilon}(K,C)$ over $\{\langle L\cup I\rangle = L_1\cup L_2\cup L_3, *_1, ..., *_N\}$ is soft Lagrange free neutrosophic N-loop over.

Theorem 75. *Let (F,A) and (H,C) be two soft strong neutrosophic N-loops over $\{\langle L\cup I\rangle = L_1\cup L_2\cup L_3, *_1, ..., *_N\}$. Then*

(1) Their AND operation $(F,A)\wedge(H,B)$ is soft strong neutrosophic N-loop over $\{\langle L\cup I\rangle = L_1\cup L_2\cup L_3, *_1, ..., *_N\}$.
(2) Their OR operation $(F,A)\vee(H,B)$ is not soft strong neutrosophic N-loop over $\{\langle L\cup I\rangle = L_1\cup L_2\cup L_3, *_1, ..., *_N\}$.

Definition 54. *Let (F,A) and (H,C) be two soft strong neutrosophic N-loops over $\{\langle L\cup I\rangle = L_1\cup L_2\cup L_3, *_1, ..., *_N\}$. Then (H,C) is soft strong neutrosophic sub N-loop of (F,A), if*

(1) $B\subset A$.
(2) $H(a)$ is neutrosophic sub N-loop of $F(a)$, for all $a\in A$.

Definition 55. *Let $\{\langle L\cup I\rangle = L_1\cup L_2\cup L_3, *_1, ..., *_N\}$ be a strong neutrosophic N-loop and (F,A) be a soft set over $\{\langle L\cup I\rangle = L_1\cup L_2\cup L_3, *_1, ..., *_N\}$. Then (F,A) is called soft strong Lagrange neutrosophic N-loop if and only if $F(a)$ is strong Lagrange neutrosophic sub N-loop of $\{\langle L\cup I\rangle = L_1\cup L_2\cup L_3, *_1, ..., *_N\}$, for all $a\in A$.*

Theorem 76. *All soft strong Lagrange neutrosophic N-loops are soft Lagrange neutrosophic N-loops but the converse is not true.*

Theorem 77. *All soft strong Lagrange neutrosophic N-loops are soft neutrosophic N-loops but the converse is not true.*

Theorem 78. *Let (F,A) and (K,C) be two soft strong Lagrange neutrosophic N-loops over $\{\langle L\cup I\rangle = L_1\cup L_2\cup L_3, *_1, ..., *_N\}$. Then*

(1) Their extended union $(F,A)\cup_{\varepsilon}(K,C)$ over $\{\langle L\cup I\rangle = L_1\cup L_2\cup L_3, *_1, ..., *_N\}$ is not soft strong Lagrange neutrosophic N-loop.
(2) Their extended intersection $(F,A)\cap_{\varepsilon}(K,C)$ over $\{\langle L\cup I\rangle = L_1\cup L_2\cup L_3, *_1, ..., *_N\}$ is not soft strong Lagrange neutrosophic N-loop.
(3) Their restricted union $(F,A)\cup_{R}(K,C)$ over $\{\langle L\cup I\rangle = L_1\cup L_2\cup L_3, *_1, ..., *_N\}$ is not soft strong Lagrange neutrosophic N-loop over.
(4) Their restricted intersection $(F,A)\cap_{\varepsilon}(K,C)$ over $\{\langle L\cup I\rangle = L_1\cup L_2\cup L_3, *_1, ..., *_N\}$ is not soft strong Lagrange neutrosophic N-loop over.

Theorem 79. *Let (F,A) and (H,C) be two soft strong Lagrange neutrosophic N-loops over $\{\langle L\cup I\rangle = L_1\cup L_2\cup L_3, *_1, ..., *_N\}$. Then*

(1) Their AND operation $(F,A)\wedge(H,B)$ is not soft strong Lagrange neutrosophic N-loop over $\{\langle L\cup I\rangle = L_1\cup L_2\cup L_3, *_1, ..., *_N\}$.
(2) Their OR operation $(F,A)\vee(H,B)$ is not soft strong Lagrange neutrosophic N-loop over $\{\langle L\cup I\rangle = L_1\cup L_2\cup L_3, *_1, ..., *_N\}$.

Definition 56. *Let $\{\langle L \cup I \rangle = L_1 \cup L_2 \cup L_3, *_1, ..., *_N\}$ be a strong neutrosophic N-loop and (F, A) be a soft set over $\{\langle L \cup I \rangle = L_1 \cup L_2 \cup L_3, *_1, ..., *_N\}$. Then (F, A) is called soft strong weakly Lagrange neutrosophic N-loop if atleast one $F(a)$ is not strong Lagrange neutrosophic sub N-loop of $\{\langle L \cup I \rangle = L_1 \cup L_2 \cup L_3, *_1, ..., *_N\}$, for some $a \in A$.*

Theorem 80. *All soft strong weakly Lagrange neutrosophic N-loops are soft weakly Lagrange neutrosophic N-loops but the converse is not true.*

Theorem 81. *All soft strong weakly Lagrange neutrosophic N-loops are soft neutrosophic N-loops but the converse is not true.*

Theorem 82. *Let (F, A) and (K, C) be two soft strong weakly Lagrange neutrosophic N-loops over $\{\langle L \cup I \rangle = L_1 \cup L_2 \cup L_3, *_1, ..., *_N\}$. Then*

(1) Their extended union $(F, A) \cup_\varepsilon (K, C)$ over $\{\langle L \cup I \rangle = L_1 \cup L_2 \cup L_3, *_1, ..., *_N\}$ is not soft strong weakly Lagrange neutrosophic N-loop.
(2) Their extended intersection $(F, A) \cap_\varepsilon (K, C)$ over $\{\langle L \cup I \rangle = L_1 \cup L_2 \cup L_3, *_1, ..., *_N\}$ is not soft strong weakly Lagrange neutrosophic N-loop.
(3) Their restricted union $(F, A) \cup_R (K, C)$ over $\{\langle L \cup I \rangle = L_1 \cup L_2 \cup L_3, *_1, ..., *_N\}$ is not soft strong weakly Lagrange neutrosophic N-loop.
(4) Their restricted intersection $(F, A) \cap_\varepsilon (K, C)$ over $\{\langle L \cup I \rangle = L_1 \cup L_2 \cup L_3, *_1, ..., *_N\}$ is not soft strong weakly Lagrange neutrosophic N-loop.

Theorem 83. *Let (F, A) and (H, C) be two soft strong weakly Lagrange neutrosophic N-loops over $\{\langle L \cup I \rangle = L_1 \cup L_2 \cup L_3, *_1, ..., *_N\}$. Then*

(1) Their AND operation $(F, A) \wedge (H, B)$ is not soft strong weakly Lagrange neutrosophic N-loop over $\{\langle L \cup I \rangle = L_1 \cup L_2 \cup L_3, *_1, ..., *_N\}$.
(2) Their OR operation $(F, A) \vee (H, B)$ is not soft strong weakly Lagrange neutrosophic N-loop over $\{\langle L \cup I \rangle = L_1 \cup L_2 \cup L_3, *_1, ..., *_N\}$.

Definition 57. *Let $\{\langle L \cup I \rangle = L_1 \cup L_2 \cup L_3, *_1, ..., *_N\}$ be a strong neutrosophic N-loop and (F, A) be a soft set over $\{\langle L \cup I \rangle = L_1 \cup L_2 \cup L_3, *_1, ..., *_N\}$. Then (F, A) is called soft strong Lagrange free neutrosophic N-loop if and only if $F(a)$ is not strong Lagrange neutrosophic sub N-loop of $\{\langle L \cup I \rangle = L_1 \cup L_2 \cup L_3, *_1, ..., *_N\}$, for all $a \in A$.*

Theorem 84. *All soft strong Lagrange free neutrosophic N-loops are soft Lagrange free neutrosophic N-loops but the converse is not true.*

Theorem 85. *All soft strong Lagrange free neutrosophic N-loops are soft neutrosophic N-loops but the converse is not true.*

Theorem 86. *Let (F, A) and (K, C) be two soft strong Lagrange free neutrosophic N-loops over $\{\langle L \cup I \rangle = L_1 \cup L_2 \cup L_3, *_1, ..., *_N\}$. Then*

(1) Their extended union $(F, A) \cup_\varepsilon (K, C)$ over $\{\langle L \cup I \rangle = L_1 \cup L_2 \cup L_3, *_1, ..., *_N\}$ is not soft strong Lagrange free neutrosophic N-loop.
(2) Their extended intersection $(F, A) \cap_\varepsilon (K, C)$ over $\{\langle L \cup I \rangle = L_1 \cup L_2 \cup L_3, *_1, ..., *_N\}$ is not soft strong Lagrange free neutrosophic N-loop.
(3) Their restricted union $(F, A) \cup_R (K, C)$ over $\{\langle L \cup I \rangle = L_1 \cup L_2 \cup L_3, *_1, ..., *_N\}$ is not soft strong Lagrange free neutrosophic N-loop.
(4) Their restricted intersection $(F, A) \cap_\varepsilon (K, C)$ over $\{\langle L \cup I \rangle = L_1 \cup L_2 \cup L_3, *_1, ..., *_N\}$ is not soft strong Lagrange free neutrosophic N-loop.

Theorem 87. *Let (F, A) and (H, C) be two soft strong Lagrange free neutrosophic N-loops over $\{\langle L \cup I \rangle = L_1 \cup L_2 \cup L_3, *_1, ..., *_N\}$. Then*

(1) Their AND operation $(F, A) \wedge (H, B)$ is not soft strong Lagrange free neutrosophic N-loop over $\{\langle L \cup I \rangle = L_1 \cup L_2 \cup L_3, *_1, ..., *_N\}$.
(2) Their OR operation $(F, A) \vee (H, B)$ is not soft strong Lagrange free neutrosophic N-loop over $\{\langle L \cup I \rangle = L_1 \cup L_2 \cup L_3, *_1, ..., *_N\}$.

Conclusion 1. *This paper is an extension of neutrosphic loop to soft neutrosophic loop. We also extend neutrosophic biloop, neutrosophic N-loop to soft neutrosophic biloop, and soft neutrosophic N-loop. Their related properties and results are explained with many illustrative examples. The notions related with strong part of neutrosophy also established within soft neutrosophic loop.*

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Published in *Neutrosophic Sets and Systems*, Vol. 4, 55-75, 2014.

Soft neutrosophic semigroups and their generalization

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Abstract Soft set theory is a general mathematical tool for dealing with uncertain, fuzzy, not clearly defined objects. In this paper we introduced soft neutrosophic semigroup,soft neutosophic bisemigroup, soft neutrosophic N-semigroup with the discuissionf of some of their characteristics. We also introduced a new type of soft neutrophic semigroup, the so called soft strong neutrosophic semigoup which is of pure neutrosophic character. This notion also foound in all the other corresponding notions of soft neutrosophic thoery. We also given some of their properties of this newly born soft structure related to the strong part of neutrosophic theory.

Keywords Neutrosophic semigroup, neutrosophic bisemigroup, neutrosophic N-semigroup, soft set, soft semigroup, soft neutrosophic semigroup, soft neutrosophic bisemigroup, soft neutrosophic N-semigroup.

§1. Introduction and preliminaries

Florentin Smarandache for the first time introduced the concept of neutrosophy in 1995, which is basically a new branch of philosophy which actually studies the origin, nature, and scope of neutralities. The neutrosophic logic came into being by neutrosophy. In neutrosophic logic each proposition is approximated to have the percentage of truth in a subset T, the percentage of indeterminacy in a subset I, and the percentage of falsity in a subset F. Neutrosophic logic is an extension of fuzzy logic. In fact the neutrosophic set is the generalization of classical set, fuzzy conventional set, intuitionistic fuzzy set, and interval valued fuzzy set. Neutrosophic logic is used to overcome the problems of impreciseness, indeterminate, and inconsistencies of date etc. The theory of neutrosophy is so applicable to every field of algebra. W. B. Vasantha Kandasamy and Florentin Smarandache introduced neutrosophic fields, neutrosophic rings,neutrosophic vector spaces,neutrosophic groups,neutrosophic bigroups and neutrosophic N-groups, neutrosophic semigroups, neutrosophic bisemigroups, and neutrosophic

N-semigroups, neutrosophic loops, nuetrosophic biloops, and neutrosophic N-loops, and so on. Mumtaz ali et al. introduced nuetrosophic LA-semigroups.

Molodtsov introduced the theory of soft set. This mathematical tool is free from parameterization inadequacy, syndrome of fuzzy set theory, rough set theory, probability theory and so on. This theory has been applied successfully in many fields such as smoothness of functions, game theory, operation research, Riemann integration, Perron integration, and probability. Recently soft set theory attained much attention of the researchers since its appearance and the work based on several operations of soft set introduced in [2, 9, 10]. Some properties and algebra may be found in [1] . Feng et al. introduced soft semirings in [5]. By means of level soft sets an adjustable approach to fuzzy soft set can be seen in [6]. Some other concepts together with fuzzy set and rough set were shown in [7, 8].

This paper is about to introduced soft nuetrosophic semigroup, soft neutrosophic group, and soft neutrosophic N-semigroup and the related strong or pure part of neutrosophy with the notions of soft set theory. In the proceeding section, we define soft neutrosophic semigroup, soft neutrosophic strong semigroup, and some of their properties are discussed. In the next section, soft neutrosophic bisemigroup are presented with their strong neutrosophic part. Also in this section some of their characterization have been made. In the last section soft neutrosophic N-semigroup and their corresponding strong theory have been constructed with some of their properties.

§2. Definition and properties

Definition 2.1. Let S be a semigroup, the semigroup generated by S and I i.e. $S \cup I$ denoted by $\langle S \cup I \rangle$ is defined to be a neutrosophic semigroup where I is indeterminacy element and termed as neutrosophic element.

It is interesting to note that all neutrosophic semigroups contain a proper subset which is a semigroup.

Example 2.1. Let $Z = \{$the set of positive and negative integers with zero$\}$, Z is only a semigroup under multiplication. Let $N(S) = \{\langle Z \cup I \rangle\}$ be the neutrosophic semigroup under multiplication. Clearly $Z \subset N(S)$ is a semigroup.

Definition 2.2. Let $N(S)$ be a neutrosophic semigroup. A proper subset P of $N(S)$ is said to be a neutrosophic subsemigroup, if P is a neutrosophic semigroup under the operations of $N(S)$. A neutrosophic semigroup $N(S)$ is said to have a subsemigroup if $N(S)$ has a proper subset which is a semigroup under the operations of $N(S)$.

Theorem 2.1. Let $N(S)$ be a neutrosophic semigroup. Suppose P_1 and P_2 be any two neutrosophic subsemigroups of $N(S)$ then $P_1 \cup P_2$ (i.e. the union) the union of two neutrosophic subsemigroups in general need not be a neutrosophic subsemigroup.

Definition 2.3. A neutrosophic semigroup $N(S)$ which has an element e in $N(S)$ such that $e * s = s * e = s$ for all $s \in N(S)$, is called as a neutrosophic monoid.

Definition 2.4. Let $N(S)$ be a neutrosophic monoid under the binary operation $*$. Suppose e is the identity in $N(S)$, that is $s * e = e * s = s$ for all $s \in N(S)$. We call a proper subset P of $N(S)$ to be a neutrosophic submonoid if

1. P is a neutrosophic semigroup under $*$.

2. $e \in P$, i.e., P is a monoid under $*$.

Definition 2.5. Let $N(S)$ be a neutrosophic semigroup under a binary operation $*$. P be a proper subset of $N(S)$. P is said to be a neutrosophic ideal of $N(S)$ if the following conditions are satisfied.

1. P is a neutrosophic semigroup.

2. For all $p \in P$ and for all $s \in N(S)$ we have $p * s$ and $s * p$ are in P.

Definition 2.6. Let $N(S)$ be a neutrosophic semigroup. P be a neutrosophic ideal of $N(S)$, P is said to be a neutrosophic cyclic ideal or neutrosophic principal ideal if P can be generated by a single element.

Definition 2.7. Let $(BN(S), *, o)$ be a nonempty set with two binary operations $*$ and o. $(BN(S), *, o)$ is said to be a neutrosophic bisemigroup if $BN(S) = P1 \cup P2$ where atleast one of $(P1, *)$ or $(P2, o)$ is a neutrosophic semigroup and other is just a semigroup. $P1$ and $P2$ are proper subsets of $BN(S)$, i.e. $P1 \subsetneq P2$.

If both $(P1, *)$ and $(P2, o)$ in the above definition are neutrosophic semigroups then we call $(BN(S), *, o)$ a strong neutrosophic bisemigroup. All strong neutrosophic bisemigroups are trivially neutrosophic bisemigroups.

Example 2.2. Let $(BN(S), *, o) = \{0, 1, 2, 3, I, 2I, 3I, S(3), *, o\} = (P_1, *) \cup (P_2, o)$ where $(P_1, *) = \{0, 1, 2, 3, I, 2I, 3I, *\}$ and $(P_2, o) = (S(3), o)$. Clearly $(P_1, *)$ is a neutrosophic semigroup under multiplication modulo 4. (P_2, o) is just a semigroup. Thus $(BN(S), *, o)$ is a neutrosophic bisemigroup.

Definition 2.8. Let $(BN(S) = P1 \cup P2; o, *)$ be a neutrosophic bisemigroup. A proper subset $(T, o, *)$ is said to be a neutrosophic subbisemigroup of $BN(S)$ if

1. $T = T1 \cup T2$ where $T1 = P1 \cap T$ and $T2 = P2 \cap T$.

2. At least one of $(T1, o)$ or $(T2, *)$ is a neutrosophic semigroup.

Definition 2.9. Let $(BN(S) = P_1 \cup P_2, o, *)$ be a neutrosophic strong bisemigroup. A proper subset T of $BN(S)$ is called the strong neutrosophic subbisemigroup if $T = T_1 \cup T_2$ with $T_1 = P_1 \cap T$ and $T_2 = P_2 \cap T$ and if both $(T_1, *)$ and (T_2, o) are neutrosophic subsemigroups of $(P_1, *)$ and (P_2, o) respectively. We call $T = T_1 \cup T_2$ to be a neutrosophic strong subbisemigroup, if atleast one of $(T_1, *)$ or (T_2, o) is a semigroup then $T = T_1 \cup T_2$ is only a neutrosophic subsemigroup.

Definition 2.10. Let $(BN(S) = P_1 \cup P_2 *, o)$ be any neutrosophic bisemigroup. Let J be a proper subset of $B(NS)$ such that $J_1 = J \cap P_1$ and $J_2 = J \cap P_2$ are ideals of P_1 and P_2 respectively. Then J is called the neutrosophic bi-ideal of $BN(S)$.

Definition 2.11. Let $(BN(S), *, o)$ be a strong neutrosophic bisemigroup where $BN(S) = P_1 \cup P_2$ with $(P_1, *)$ and (P_2, o) be any two neutrosophic semigroups. Let J be a proper subset of $BN(S)$ where $I = I_1 \cup I_2$ with $I_1 = J \cap P_1$ and $I_2 = J \cap P_2$ are neutrosophic ideals of the neutrosophic semigroups P_1 and P_2 respectively. Then I is called or defined as the strong neutrosophic bi-ideal of $B(N(S))$.

Union of any two neutrosophic bi-ideals in general is not a neutrosophic bi-ideal. This is true of neutrosophic strong bi-ideals.

Definition 2.12. Let $\{S(N), *_1, \ldots, *_N\}$ be a non empty set with N-binary operations defined on it. We call $S(N)$ a neutrosophic N-semigroup (N a positive integer) if the following conditions are satisfied.

1. $S(N) = S_1 \cup \ldots \cup S_N$ where each S_i is a proper subset of $S(N)$ i.e. $S_i \subsetneq S_j$ or $S_j \subsetneq S_i$ if $i \neq j$.

2. $(S_i, *_i)$ is either a neutrosophic semigroup or a semigroup for $i = 1, 2, \ldots, N$.

If all the N-semigroups $(Si, *i)$ are neutrosophic semigroups (i.e. for $i = 1, 2, \ldots, N$) then we call $S(N)$ to be a neutrosophic strong N-semigroup.

Example 2.3. Let $S(N) = \{S_1 \cup S_2 \cup S_3 \cup S_4, *_1, *_2, *_3, *_4\}$ be a neutrosophic 4-semigroup where

$S_1 = \{Z_{12}$, semigroup under multiplication modulo $12\}$.

$S_2 = \{0, 1, 2, 3, I, 2I, 3I$, semigroup under multiplication modulo $4\}$, a neutrosophic semigroup.

$S_3 = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix}; a, b, c, d \in \langle R \cup I \rangle \right\}$, neutrosophic semigroup under matrix multiplication and $S_4 = \langle Z \cup I \rangle$, neutrosophic semigroup under multiplication.

Definition 2.13. Let $S(N) = \{S_1 \cup S_2 \cup \ldots \cup S_N, *_1, \ldots, *_N\}$ be a neutrosophic N-semigroup. A proper subset $P = \{P_1 \cup P_2 \cup \ldots \cup P_N, *_1, *_2, \ldots, *_N\}$ of $S(N)$ is said to be a neutrosophic Nsubsemigroup if $P_i = P \cap S_i, i = 1, 2, \ldots, N$ are subsemigroups of S_i in which atleast some of the subsemigroups are neutrosophic subsemigroups.

Definition 2.14. Let $S(N) = \{S_1 \cup S_2 \cup \ldots \cup S_N, *_1, \ldots, *_N\}$ be a neutrosophic strong N-semigroup. A proper subset $T = \{T_1 \cup T_2 \cup \ldots \cup T_N, *_1, \ldots, *_N\}$ of $S(N)$ is said to be a neutrosophic strong sub N-semigroup if each $(T_i, *_i)$ is a neutrosophic subsemigroup of $(S_i, *_i)$ for $i = 1, 2, \ldots, N$ where $T_i = T \cap S_i$.

If only a few of the $(Ti, *i)$ in T are just subsemigroups of $(Si, *i)$ (i.e. $(Ti, *i)$ are not neutrosophic subsemigroups then we call T to be a sub N-semigroup of $S(N)$.

Definition 2.15. Let $S(N) = \{S_1 \cup S_2 \cup \ldots \cup S_N, *_1, \ldots, *_N\}$ be a neutrosophic N-semigroup. A proper subset $P = \{P_1 \cup P_2 \cup \ldots \cup P_N, *_1, \ldots, *_N\}$ of $S(N)$ is said to be a neutrosophic N-subsemigroup, if the following conditions are true,

i. P is a neutrosophic sub N-semigroup of $S(N)$.

ii. Each $P_i = P \cap S_i, i = 1, 2, \ldots, N$ is an ideal of S_i.

Then P is called or defined as the neutrosophic N-ideal of the neutrosophic N-semigroup $S(N)$.

Definition 2.16. Let $S(N) = \{S_1 \cup S_2 \cup \ldots \cup S_N, *_1, \ldots, *_N\}$ be a neutrosophic strong N-semigroup. A proper subset $J = \{I_1 \cup I_2 \cup \ldots \cup I_N\}$ where $I_t = J \cap S_t$ for $t = 1, 2, \ldots, N$ is said to be a neutrosophic strong N-ideal of $S(N)$ if the following conditions are satisfied.

1. Each is a neutrosophic subsemigroup of $S_t, t = 1, 2, \ldots, N$ i.e. It is a neutrosophic strong N-subsemigroup of $S(N)$.

2. Each is a two sided ideal of S_t for $t = 1, 2, \ldots, N$.

Similarly one can define neutrosophic strong N-left ideal or neutrosophic strong right ideal of $S(N)$.

A neutrosophic strong N-ideal is one which is both a neutrosophic strong N-left ideal and N-right ideal of $S(N)$.

Throughout this subsection U refers to an initial universe, E is a set of parameters, $P(U)$ is the power set of U, and $A \subset E$. Molodtsov [12] defined the soft set in the following manner:

Definition 2.17. A pair (F, A) is called a soft set over U where F is a mapping given by $F : A \longrightarrow P(U)$.

In other words, a soft set over U is a parameterized family of subsets of the universe U. For $e \in A$, $F(e)$ may be considered as the set of e-elements of the soft set (F, A), or as the set of e-approximate elements of the soft set.

Example 2.4. Suppose that U is the set of shops. E is the set of parameters and each parameter is a word or sentcence. Let E={high rent, normal rent, in good condition, in bad condition}. Let us consider a soft set (F, A) which describes the attractiveness of shops that Mr. Z is taking on rent. Suppose that there are five houses in the universe $U = \{h_1, h_2, h_3, h_4, h_5\}$ under consideration, and that $A = \{e_1, e_2, e_3\}$ be the set of parameters where

e_1 stands for the parameter high rent.

e_2 stands for the parameter normal rent.

e_3 stands for the parameter in good condition.

Suppose that

$F(e_1) = \{h_1, h_4\}$.

$F(e_2) = \{h_2, h_5\}$.

$F(e_3) = \{h_3, h_4, h_5\}$.

The soft set (F, A) is an approximated family $\{F(e_i), i = 1, 2, 3\}$ of subsets of the set U which gives us a collection of approximate description of an object. Thus, we have the soft set (F, A) as a collection of approximations as below:

$(F, A) = \{$high rent $= \{h_1, h_4\}$, normal rent $= \{h_2, h_5\}$, in good condition $= \{h_3, h_4, h_5\}\}$.

Definition 2.18. For two soft sets (F, A) and (H, B) over U, (F, A) is called a soft subset of (H, B) if

1. $A \subseteq B$.

2. $F(e) \subseteq G(e)$, for all $e \in A$.

This relationship is denoted by $(F, A) \tilde{\subset} (H, B)$. Similarly (F, A) is called a soft superset of (H, B) if (H, B) is a soft subset of (F, A) which is denoted by $(F, A) \tilde{\supset} (H, B)$.

Definition 2.19. Two soft sets (F, A) and (H, B) over U are called soft equal if (F, A) is a soft subset of (H, B) and (H, B) is a soft subset of (F, A).

Definition 2.20. (F, A) over U is called an absolute soft set if $F(e) = U$ for all $e \in A$ and we denote it by U.

Definition 2.21. Let (F, A) and (G, B) be two soft sets over a common universe U such that $A \cap B \neq \phi$. Then their restricted intersection is denoted by$(F, A) \cap_R (G, B) = (H, C)$ where (H, C) is defined as $H(c) = F(c) \cap G(c)$ for all $c \in C = A \cap B$.

Definition 2.22. The extended intersection of two soft sets (F, A) and (G, B) over a common universe U is the soft set (H, C), where $C = A \cup B$, and for all $e \in C$, $H(e)$ is defined as

$$H(e) = \begin{cases} F(e), & \text{if } e \in A - B, \\ G(e), & \text{if } e \in B - A, \\ F(e) \cap G(e), & \text{if } e \in A \cap B. \end{cases}$$

We write $(F, A) \cap_{\varepsilon} (G, B) = (H, C)$.

Definition 2.23. The resticted union of two soft sets (F, A) and (G, B) over a common universe U is the soft set (H, C), where $C = A \cup B$, and for all $e \in C$, $H(e)$ is defined as the soft set $(H, C) = (F, A) \cup_R (G, B)$ where $C = A \cap B$ and $H(c) = F(c) \cup G(c)$ for all $c \in C$.

Definition 2.24. The extended union of two soft sets (F, A) and (G, B) over a common universe U is the soft set (H, C), where $C = A \cup B$, and for all $e \in C$, $H(e)$ is defined as

$$H(e) = \begin{cases} F(e), & \text{if } e \in A - B, \\ G(e), & \text{if } e \in B - A, \\ F(e) \cup G(e), & \text{if } e \in A \cap B. \end{cases}$$

We write $(F, A) \cup_{\varepsilon} (G, B) = (H, C)$.

Definition 2.25. A soft set (F, A) over S is called a soft semigroup over S if $(F, A) \overset{\curlywedge}{\circ} (F, A) \subseteq (F, A)$.

It is easy to see that a soft set (F, A) over S is a soft semigroup if and only if $\phi \neq F(a)$ is a subsemigroup of S.

Definition 2.26. A soft set (F, A) over a semigroup S is called a soft left (right) ideal over S, if $(S, E) \subseteq (F, A), ((F, A) \subseteq (S, E))$.

A soft set over S is a soft ideal if it is both a soft left and a soft right ideal over S.

Proposition 2.1. A soft set (F, A) over S is a soft ideal over S if and only if $\phi \neq F(a)$ is an ideal of S.

Definition 2.27. Let (G, B) be a soft subset of a soft semigroup (F, A) over S, then (G, B) is called a soft subsemigroup (ideal) of (F, A) if $G(b)$ is a subsemigroup (ideal) of $F(b)$ for all $b \in A$.

§3. Soft neutrosophic semigroup

Definition 3.1. Let $N(S)$ be a neutrosophic semigroup and (F, A) be a soft set over $N(S)$. Then (F, A) is called soft neutrosophic semigroup if and only if $F(e)$ is neutrosophic subsemigroup of $N(S)$, for all $e \in A$.

Equivalently (F, A) is a soft neutrosophic semigroup over $N(S)$ if $(F, A) \overset{\curlywedge}{\circ} (F, A) \subseteq (F, A)$, where $\tilde{N}_{(N(S),A)} \neq (F, A) \neq \overset{\sim}{\phi}$.

Example 3.1. Let $N(S) = \langle Z^+ \cup \{0\}^+ \cup \{I\} \rangle$ be a neutrosophic semigroup under $+$. Consider $P = \langle 2Z^+ \cup I \rangle$ and $R = \langle 3Z^+ \cup I \rangle$ are neutrosophic subsemigroup of $N(S)$. Then clearly for all $e \in A$, (F, A) is a soft neutrosophic semigroup over $N(S)$, where $F(x_1) = \{\langle 2Z^+ \cup I \rangle\}$, $F(x_2) = \{\langle 3Z^+ \cup I \rangle\}$.

Theorem 3.1. A soft neutrosophic semigroup over $N(S)$ always contain a soft semigroup over S.

Proof. The proof of this theorem is straight forward.

Theorem 3.2. Let (F, A) and (H, A) be two soft neutrosophic semigroups over $N(S)$. Then their intersection $(F, A) \cap (H, A)$ is again soft neutrosophic semigroup over $N(S)$.

Proof. The proof is staight forward.

Theorem 3.3. Let (F, A) and (H, B) be two soft neutrosophic semigroups over $N(S)$. If $A \cap B = \phi$, then $(F, A) \cup (H, B)$ is a soft neutrosophic semigroup over $N(S)$.

Remark 3.1. The extended union of two soft neutrosophic semigroups (F, A) and (K, B) over $N(S)$ is not a soft neutrosophic semigroup over $N(S)$.

We take the following example for the proof of above remark.

Example 3.2. Let $N(S) = \langle Z^+ \cup I \rangle$ be the neutrosophic semigroup under $+$. Take $P_1 = \{\langle 2Z^+ \cup I \rangle\}$ and $P_2 = \{\langle 3Z^+ \cup I \rangle\}$ to be any two neutrosophic subsemigroups of $N(S)$. Then clearly for all $e \in A$, (F, A) is a soft neutrosophic semigroup over $N(S)$, where $F(x_1) = \{\langle 2Z^+ \cup I \rangle\}, F(x_2) = \{\langle 3Z^+ \cup I \rangle\}$.

Again Let $R_1 = \{\langle 5Z^+ \cup I \rangle\}$ and $R_2 = \{\langle 4Z^+ \cup I \rangle\}$ be another neutrosophic subsemigroups of $N(S)$ and (K, B) is another soft neutrosophic semigroup over $N(S)$, where $K(x_1) = \{\langle 5Z^+ \cup I \rangle\}, K(x_3) = \{\langle 4Z^+ \cup I \rangle\}$.

Let $C = A \cup B$. The extended union $(F, A) \cup_\varepsilon (K, B) = (H, C)$ where $x_1 \in C$, we have $H(x_1) = F(x_1) \cup K(x_1)$ is not neutrosophic subsemigroup as union of two neutrosophic subsemigroup is not neutrosophic subsemigroup.

Proposition 3.1. The extended intersection of two soft neutrosophic semigroups over $N(S)$ is soft neutrosophic semigruop over $N(S)$.

Remark 3.2. The restricted union of two soft neutrosophic semigroups (F, A) and (K, B) over $N(S)$ is not a soft neutrosophic semigroup over $N(S)$.

We can easily check it in above example.

Proposition 3.2. The restricted intersection of two soft neutrosophic semigroups over $N(S)$ is soft neutrosophic semigroup over $N(S)$.

Proposition 3.3. The AND operation of two soft neutrosophic semigroups over $N(S)$ is soft neutrosophic semigroup over $N(S)$.

Proposition 3.4. The OR operation of two soft neutosophic semigroup over $N(S)$ may not be a soft nuetrosophic semigroup over $N(S)$.

Definition 3.2. Let $N(S)$ be a neutrosophic monoid and (F, A) be a soft set over $N(S)$. Then (F, A) is called soft neutrosophic monoid if and only if $F(e)$ is neutrosophic submonoid of $N(S)$, for all $x \in A$.

Example 3.3. Let $N(S) = \langle Z \cup I \rangle$ be a neutrosophic monoid under $+$. Let $P = \langle 2Z \cup I \rangle$ and $Q = \langle 3Z \cup I \rangle$ are neutrosophic submonoids of $N(S)$. Then (F, A) is a soft neutrosophic monoid over $N(S)$, where $F(x_1) = \{\langle 2Z \cup I \rangle\}, F(x_2) = \{\langle 3Z \cup I \rangle\}$.

Theorem 3.4. Every soft neutrosophic monoid over $N(S)$ is a soft neutrosophic semigroup over $N(S)$ but the converse is not true in general.

Proof. The proof is straightforward.

Proposition 3.5. Let (F, A) and (K, B) be two soft neutrosophic monoids over $N(S)$. Then

1. Their extended union $(F,A) \cup_{\varepsilon} (K,B)$ over $N(S)$ is not soft neutrosophic monoid over $N(S)$.

2. Their extended intersection $(F,A) \cap_{\varepsilon} (K,B)$ over $N(S)$ is soft neutrosophic monoid over $N(S)$.

3. Their restricted union $(F,A) \cup_{R} (K,B)$ over $N(S)$ is not soft neutrosophic monoid over $N(S)$.

4. Their restricted intersection $(F,A) \cap_{\varepsilon} (K,B)$ over $N(S)$ is soft neutrosophic monoid over $N(S)$.

Proposition 3.6. Let (F,A) and (H,B) be two soft neutrosophic monoid over $N(S)$. Then

1. Their AND operation $(F,A) \wedge (H,B)$ is soft neutrosophic monoid over $N(S)$.

2. Their OR operation $(F,A) \vee (H,B)$ is not soft neutrosophic monoid over $N(S)$.

Definition 3.3. Let (F,A) be a soft neutrosophic semigroup over $N(S)$, then (F,A) is called Full-soft neutrosophic semigroup over $N(S)$ if $F(x) = N(S)$, for all $x \in A$. We denote it by $N(S)$.

Theorem 3.5. Every Full-soft neutrosophic semigroup over $N(S)$ always contain absolute soft semigroup over S.

Proof. The proof of this theorem is straight forward.

Definition 3.4. Let (F,A) and (H,B) be two soft neutrosophic semigroups over $N(S)$. Then (H,B) is a soft neutrosophic subsemigroup of (F,A), if

1. $B \subset A$.

2. $H(a)$ is neutrosophic subsemigroup of $F(a)$, for all $a \in B$.

Example 3.4. Let $N(S) = \langle Z \cup I \rangle$ be a neutrosophic semigroup under $+$. Then (F,A) is a soft neutrosophic semigroup over $N(S)$, where $F(x_1) = \{\langle 2Z \cup I \rangle\}, F(x_2) = \{\langle 3Z \cup I \rangle\}, F(x_3) = \{\langle 5Z \cup I \rangle\}$.

Let $B = \{x_1, x_2\} \subset A$. Then (H,B) is soft neutrosophic subsemigroup of (F,A) over $N(S)$, where $H(x_1) = \{\langle 4Z \cup I \rangle\}, H(x_2) = \{\langle 6Z \cup I \rangle\}$.

Theorem 3.6. A soft neutrosophic semigroup over $N(S)$ have soft neutrosophic subsemigroups as well as soft subsemigroups over $N(S)$.

Proof. Obvious.

Theorem 3.7. Every soft semigroup over S is always soft neutrosophic subsemigroup of soft neutrosophic semigroup over $N(S)$.

Proof. The proof is obvious.

Theorem 3.8. Let (F,A) be a soft neutrosophic semigroup over $N(S)$ and $\{(H_i, B_i); i \in I\}$ is a non empty family of soft neutrosophic subsemigroups of (F,A) then

1. $\cap_{i \in I} (H_i, B_i)$ is a soft neutrosophic subsemigroup of (F,A).

2. $\wedge_{i \in I} (H_i, B_i)$ is a soft neutrosophic subsemigroup of $\wedge_{i \in I} (F,A)$.

3. $\cup_{i \in I} (H_i, B_i)$ is a soft neutrosophic subsemigroup of (F,A) if $B_i \cap B_j = \phi$, for all $i \neq j$.

Proof. Straightforward.

Definition 3.5. A soft set (F,A) over $N(S)$ is called soft neutrosophic left (right) ideal over $N(S)$ if $N(S) \overset{\curlywedge}{\circ} (F,A) \subseteq (F,A)$, where $\tilde{N}_{(N(S),A)} \neq (F,A) \neq \widetilde{\phi}$ and $N(S)$ is Full-soft neutrosophic semigroup over $N(S)$.

A soft set over $N(S)$ is a soft neutrosophic ideal if it is both a soft neutrosophic left and a soft neutrosophic right ideal over $N(S)$.

Example 3.5. Let $N(S) = \langle Z \cup I \rangle$ be the neutrosophic semigroup under multiplication. Let $P = \langle 2Z \cup I \rangle$ and $Q = \langle 4Z \cup I \rangle$ are neutrosophic ideals of $N(S)$. Then clearly (F, A) is a soft neutrosophic ideal over $N(S)$, where $F(x_1) = \{\langle 2Z \cup I \rangle\}, F(x_2) = \{\langle 4Z \cup I \rangle\}$.

Proposition 3.7. (F, A) is soft neutrosophic ideal if and only if $F(x)$ is a neutrosophic ideal of $N(S)$, for all $x \in A$.

Theorem 3.9. Every soft neutrosophic ideal (F, A) over $N(S)$ is a soft neutrosophic semigroup but the converse is not true.

Proposition 3.8. Let (F, A) and (K, B) be two soft neutrosophic ideals over $N(S)$. Then

1. Their extended union $(F, A) \cup_\varepsilon (K, B)$ over $N(S)$ is soft neutrosophic ideal over $N(S)$.
2. Their extended intersection $(F, A) \cap_\varepsilon (K, B)$ over $N(S)$ is soft neutrosophic ideal over $N(S)$.
3. Their restricted union $(F, A) \cup_R (K, B)$ over $N(S)$ is soft neutrosophic ideal over $N(S)$.
4. Their restricted intersection $(F, A) \cap_\varepsilon (K, B)$ over $N(S)$ is soft neutrosophic ideal over $N(S)$.

Proposition 3.9.

1. Let (F, A) and (H, B) be two soft neutrosophic ideal over $N(S)$.
2. Their AND operation $(F, A) \wedge (H, B)$ is soft neutrosophic ideal over $N(S)$.
3. Their OR operation $(F, A) \vee (H, B)$ is soft neutrosophic ideal over $N(S)$.

Theorem 3.10. Let (F, A) and (G, B) be two soft semigroups (ideals) over S and T respectively. Then $(F, A) \times (G, B)$ is also a soft semigroup (ideal) over $S \times T$.

Proof. The proof is straight forward.

Theorem 3.11. Let (F, A) be a soft neutrosophic semigroup over $N(S)$ and $\{(H_i, B_i); i \in I\}$ is a non empty family of soft neutrosophic ideals of (F, A) then

1. $\cap_{i \in I} (H_i, B_i)$ is a soft neutrosophic ideal of (F, A).
2. $\wedge_{i \in I} (H_i, B_i)$ is a soft neutrosophic ideal of $\wedge_{i \in I} (F, A)$.
3. $\cup_{i \in I} (H_i, B_i)$ is a soft neutrosophic ideal of (F, A).
4. $\vee_{i \in I} (H_i, B_i)$ is a soft neutrosophic ideal of $\vee_{i \in I} (F, A)$.

Definition 3.6. A soft set (F, A) over $N(S)$ is called soft neutrosophic principal ideal or soft neutrosophic cyclic ideal if and only if $F(x)$ is a principal or cyclic neutrosophic ideal of $N(S)$, for all $x \in A$.

Proposition 3.10. Let (F, A) and (K, B) be two soft neutrosophic principal ideals over $N(S)$. Then

1. Their extended union $(F, A) \cup_\varepsilon (K, B)$ over $N(S)$ is not soft neutrosophic principal ideal over $N(S)$.
2. Their extended intersection $(F, A) \cap_\varepsilon (K, B)$ over $N(S)$ is soft neutrosophic principal ideal over $N(S)$.
3. Their restricted union $(F, A) \cup_R (K, B)$ over $N(S)$ is not soft neutrosophic principal ideal over $N(S)$.

4. Their restricted intersection $(F, A) \cap_{\varepsilon} (K, B)$ over $N(S)$ is soft neutrosophic principal ideal over $N(S)$.

Proposition 3.11. Let (F, A) and (H, B) be two soft neutrosophic principal ideals over $N(S)$. Then

1. Their AND operation $(F, A) \wedge (H, B)$ is soft neutrosophic principal ideal over $N(S)$.
2. Their OR operation $(F, A) \vee (H, B)$ is not soft neutrosophic principal ideal over $N(S)$.

§3. Soft neutrosophic bisemigroup

Definition 3.1. Let $\{BN(S), *_1, *_2\}$ be a neutrosophic bisemigroup and let (F, A) be a soft set over $BN(S)$. Then (F, A) is said to be soft neutrosophic bisemigroup over $BN(G)$ if and only if $F(x)$ is neutrosophic subbisemigroup of $BN(G)$ for all $x \in A$.

Example 3.1. Let $BN(S) = \{0, 1, 2, I, 2I, \langle Z \cup I\rangle, \times, +\}$ be a neutosophic bisemigroup. Let $T = \{0, I, 2I, \langle 2Z \cup I\rangle, \times, +\}, P = \{0, 1, 2, \langle 5Z \cup I\rangle, \times, +\}$ and $L = \{0, 1, 2, Z, \times, +\}$ are neutrosophic subbisemigroup of $BN(S)$. The (F, A) is clearly soft neutrosophic bisemigroup over $BN(S)$, where $F(x_1) = \{0, I, 2I, \langle 2Z \cup I\rangle, \times, +\}, F(x_2) = \{0, 1, 2, \langle 5Z \cup I\rangle, \times, +\}, F(x_3) = \{0, 1, 2, Z, \times, +\}$.

Theorem 3.1. Let (F, A) and (H, A) be two soft neutrosophic bisemigroup over $BN(S)$. Then their intersection $(F, A) \cap (H, A)$ is again a soft neutrosophic bisemigroup over $BN(S)$.

Proof. Straightforward.

Theorem 3.2. Let (F, A) and (H, B) be two soft neutrosophic bisemigroups over $BN(S)$ such that $A \cap B = \phi$, then their union is soft neutrosophic bisemigroup over $BN(S)$.

Proof. Straightforward.

Proposition 3.1. Let (F, A) and (K, B) be two soft neutrosophic bisemigroups over $BN(S)$. Then

1. Their extended union $(F, A) \cup_{\varepsilon} (K, B)$ over $BN(S)$ is not soft neutrosophic bisemigroup over $BN(S)$.
2. Their extended intersection $(F, A) \cap_{\varepsilon} (K, B)$ over $BN(S)$ is soft neutrosophic bisemigroup over $BN(S)$.
3. Their restricted union $(F, A) \cup_{R} (K, B)$ over $BN(S)$ is not soft neutrosophic bisemigroup over $BN(S)$.
4. Their restricted intersection $(F, A) \cap_{\varepsilon} (K, B)$ over $BN(S)$ is soft neutrosophic bisemigroup over $BN(S)$.

Proposition 3.2. Let (F, A) and (K, B) be two soft neutrosophic bisemigroups over $BN(S)$. Then

1. Their AND operation $(F, A) \wedge (K, B)$ is soft neutrosophic bisemigroup over $BN(S)$.
2. Their OR operation $(F, A) \vee (K, B)$ is not soft neutrosophic bisemigroup over $BN(S)$.

Definition 3.2. Let (F, A) be a soft neutrosophic bisemigroup over $BN(S)$, then (F, A) is called Full-soft neutrosophic bisemigroup over $BN(S)$ if $F(x) = BN(S)$, for all $x \in A$. We denote it by $BN(S)$.

Definition 3.3. Let (F, A) and (H, B) be two soft neutrosophic bisemigroups over $BN(S)$. Then (H, B) is a soft neutrosophic subbisemigroup of (F, A), if

1. $B \subset A$.
2. $H(x)$ is neutrosophic subbisemigroup of $F(x)$, for all $x \in B$.

Example 3.2. Let $BN(S) = \{0, 1, 2, I, 2I, \langle Z \cup I \rangle, \times, +\}$ be a neutosophic bisemigroup. Let $T = \{0, I, 2I, \langle 2Z \cup I \rangle, \times, +\}, P = \{0, 1, 2, \langle 5Z \cup I \rangle, \times, +\}$ and $L = \{0, 1, 2, Z, \times, +\}$ are neutrosophic subbisemigroup of $BN\,(S)$. The (F, A) is clearly soft neutrosophic bisemigroup over $BN\,(S)$, where $F\,(x_1) = \{0, I, 2I, \langle 2Z \cup I \rangle, \times, +\}, F\,(x_2) = \{0, 1, 2, \langle 5Z \cup I \rangle, \times, +\}, F\,(x_3) = \{0, 1, 2, Z, \times, +\}$.

Then (H, B) is a soft neutrosophic subbisemigroup of (F, A), where $H\,(x_1) = \{0, I, \langle 4Z \cup I \rangle, \times, +\}, H\,(x_3) = \{0, 1, 4Z, \times, +\}$.

Theorem 3.3. Let (F, A) be a soft neutrosophic bisemigroup over $BN\,(S)$ and $\{(H_i, B_i)\,; i \in I\}$ be a non-empty family of soft neutrosophic subbisemigroups of (F, A) then

1. $\cap_{i \in I}\,(H_i, B_i)$ is a soft neutrosophic subbisemigroup of (F, A).
2. $\wedge_{i \in I}\,(H_i, B_i)$ is a soft neutrosophic subbisemigroup of $\wedge_{i \in I}\,(F, A)$.
3. $\cup_{i \in I}\,(H_i, B_i)$ is a soft neutrosophic subbisemigroup of (F, A) if $B_i \cap B_j = \phi$, for all $i \neq j$.

Proof. Straightforward.

Theorem 3.4. (F, A) is called soft neutrosophic biideal over $BN\,(S)$ if $F(x)$ is neutrosophic biideal of $BN(S)$, for all $x \in A$.

Example 3.3. Let $BN(S) = (\{\langle Z \cup I \rangle, 0, 1, 2, I, 2I, +, \times\}(\times$ under multiplication modulo 3)). Let $T = \{\langle 2Z \cup I \rangle, 0, I, 1, 2I, +, \times\}$ and $J = \{\langle 8Z \cup I \rangle, \{0, 1, I, 2I\}, +\times\}$ are ideals of $BN\,(S)$. Then (F, A) is soft neutrosophic biideal over $BN\,(S)$, where $F\,(x_1) = \{\langle 2Z \cup I \rangle, 0, I, 1, 2I, +, \times\}, F\,(x_2) = \{\langle 8Z \cup I \rangle, \{0, 1, I, 2I\}, +\times\}$.

Theorem 3.5. Every soft neutrosophic biideal (F, A) over $BS\,(N)$ is a soft neutrosophic bisemigroup but the converse is not true.

Proposition 3.3. Let (F, A) and (K, B) be two soft neutrosophic biideals over $BN(S)$. Then

1. Their extended union $(F, A) \cup_{\varepsilon} (K, B)$ over $BN(S)$ is not soft neutrosophic biideal over $BN(S)$.
2. Their extended intersection $(F, A) \cap_{\varepsilon} (K, B)$ over $BN(S)$ is soft neutrosophic biideal over $BN(S)$.
3. Their restricted union $(F, A) \cup_R (K, B)$ over $BN(S)$ is not soft neutrosophic biideal over $BN(S)$.
4. Their restricted intersection $(F, A) \cap_{\varepsilon} (K, B)$ over $BN(S)$ is soft neutrosophic biideal over $BN(S)$.

Proposition 3.4. Let (F, A) and (H, B) be two soft neutrosophic biideal over $BN(S)$. Then

1. Their AND operation $(F, A) \wedge (H, B)$ is soft neutrosophic biideal over $BN(S)$.
2. Their OR operation $(F, A) \vee (H, B)$ is not soft neutrosophic biideal over $BN(S)$.

Theorem 3.6.

Let (F, A) be a soft neutrosophic bisemigroup over $BN\,(S)$ and $\{(H_i, B_i)\,; i \in I\}$ is a non empty family of soft neutrosophic biideals of (F, A) then

1. $\cap_{i \in I}\,(H_i, B_i)$ is a soft neutrosophic biideal of (F, A).
2. $\wedge_{i \in I}\,(H_i, B_i)$ is a soft neutrosophic biideal of $\wedge_{i \in I}\,(F, A)$.

§4. Soft neutrosophic strong bisemigroup

Definition 4.1. Let (F, A) be a soft set over a neutrosophic bisemigroup $BN(S)$. Then (F, A) is said to be soft strong neutrosophic bisemigroup over $BN(G)$ if and only if $F(x)$ is neutrosophic strong subbisemigroup of $BN(G)$ for all $x \in A$.

Example 4.1. Let $BN(S) = \{0, 1, 2, I, 2I, \langle Z \cup I \rangle, \times, +\}$ be a neutrosophic bisemigroup. Let $T = \{0, I, 2I, \langle 2Z \cup I \rangle, \times, +\}$ and $R = \{0, 1, I, \langle 4Z \cup I \rangle, \times, +\}$ are neutrosophic strong subbisemigroups of $BN(S)$. Then (F, A) is soft neutrosophic strong bisemigroup over $BN(S)$, where $F(x_1) = \{0, I, 2I, \langle 2Z \cup I \rangle, \times, +\}, F(x_2) = \{0, I, 1, \langle 4Z \cup I \rangle, \times, +\}$.

Theorem 4.1. Every soft neutrosophic strong bisemigroup is a soft neutrosophic bisemigroup but the converse is not true.

Proposition 4.1. Let (F, A) and (K, B) be two soft neutrosophic strong bisemigroups over $BN(S)$. Then

1. Their extended union $(F, A) \cup_\varepsilon (K, B)$ over $BN(S)$ is not soft neutrosophic strong bisemigroup over $BN(S)$.
2. Their extended intersection $(F, A) \cap_\varepsilon (K, B)$ over $BN(S)$ is soft neutrosophic stong bisemigroup over $BN(S)$.
3. Their restricted union $(F, A) \cup_R (K, B)$ over $BN(S)$ is not soft neutrosophic stong bisemigroup over $BN(S)$.
4. Their restricted intersection $(F, A) \cap_\varepsilon (K, B)$ over $BN(S)$ is soft neutrosophic strong bisemigroup over $BN(S)$.

Proposition 4.2. Let (F, A) and (K, B) be two soft neutrosophic strong bisemigroups over $BN(S)$. Then

1. Their AND operation $(F, A) \wedge (K, B)$ is soft neutrosophic strong bisemigroup over $BN(S)$.
2. Their OR operation $(F, A) \vee (K, B)$ is not soft neutrosophic strong bisemigroup over $BN(S)$.

Definition 4.2. Let (F, A) and (H, B) be two soft neutrosophic strong bisemigroups over $BN(S)$. Then (H, B) is a soft neutrosophic strong subbisemigroup of (F, A), if

1. $B \subset A$.
2. $H(x)$ is neutrosophic strong subbisemigroup of $F(x)$, for all $x \in B$.

Example 4.2. Let $BN(S) = \{0, 1, 2, I, 2I, \langle Z \cup I \rangle, \times, +\}$ be a neutrosophic bisemigroup. Let $T = \{0, I, 2I, \langle 2Z \cup I \rangle, \times, +\}$ and $R = \{0, 1, I, \langle 4Z \cup I \rangle, \times, +\}$ are neutrosophic strong subbisemigroups of $BN(S)$. Then (F, A) is soft neutrosophic strong bisemigroup over $BN(S)$, where $F(x_1) = \{0, I, 2I, \langle 2Z \cup I \rangle, \times, +\}, F(x_2) = \{0, I, \langle 4Z \cup I \rangle, \times, +\}$.

Then (H, B) is a soft neutrosophic strong subbisemigroup of (F, A), where $H(x_1) = \{0, I, \langle 4Z \cup I \rangle, \times, +\}$.

Theorem 4.2. Let (F, A) be a soft neutrosophic strong bisemigroup over $BN(S)$ and $\{(H_i, B_i); i \in I\}$ be a non empty family of soft neutrosophic strong subbisemigroups of (F, A) then

1. $\cap_{i \in I} (H_i, B_i)$ is a soft neutrosophic strong subbisemigroup of (F, A).
2. $\wedge_{i \in I} (H_i, B_i)$ is a soft neutrosophic strong subbisemigroup of $\wedge_{i \in I} (F, A)$.

3. $\cup_{i\in I}(H_i, B_i)$ is a soft neutrosophic strong subbisemigroup of (F, A) if $B_i \cap B_j = \phi$, for all $i \neq j$.

Proof. Straightforward.

Definition 4.3. (F, A) over $BN(S)$ is called soft neutrosophic strong biideal if $F(x)$ is neutosophic strong biideal of $BN(S)$, for all $x \in A$.

Example 4.3. Let $BN(S) = (\{\langle Z \cup I\rangle, 0, 1, 2, I, 2I\}, +, \times(\times$ under multiplication modulo 3)). Let $T = \{\langle 2Z \cup I\rangle, 0, I, 1, 2I, +, \times\}$ and $J = \{\langle 8Z \cup I\rangle, \{0, 1, I, 2I\}, +\times\}$ are neutrosophic strong ideals of $BN(S)$. Then (F, A) is soft neutrosophic strong biideal over $BN(S)$, where $F(x_1) = \{\langle 2Z \cup I\rangle, 0, I, 1, 2I, +, \times\}, F(x_2) = \{\langle 8Z \cup I\rangle, \{0, 1, I, 2I\}, +\times\}$.

Theorem 4.3. Every soft neutrosophic strong biideal (F, A) over $BS(N)$ is a soft neutrosophic bisemigroup but the converse is not true.

Theorem 4.4. Every soft neutrosophic strong biideal (F, A) over $BS(N)$ is a soft neutrosophic strong bisemigroup but the converse is not true.

Proposition 4.3. Let (F, A) and (K, B) be two soft neutrosophic strong biideals over $BN(S)$. Then

1. Their extended union $(F, A) \cup_\varepsilon (K, B)$ over $BN(S)$ is not soft neutrosophic strong biideal over $BN(S)$.

2. Their extended intersection $(F, A) \cap_\varepsilon (K, B)$ over $BN(S)$ is soft neutrosophic strong biideal over $BN(S)$.

3. Their restricted union $(F, A) \cup_R (K, B)$ over $BN(S)$ is not soft neutrosophic strong biideal over $BN(S)$.

4. Their restricted intersection $(F, A) \cap_\varepsilon (K, B)$ over $BN(S)$ is soft neutrosophic stong biideal over $BN(S)$.

Proposition 4.4. Let (F, A) and (H, B) be two soft neutrosophic strong biideal over $BN(S)$. Then

1. Their AND operation $(F, A) \wedge (H, B)$ is soft neutrosophic strong biideal over $BN(S)$.

2. Their OR operation $(F, A) \vee (H, B)$ is not soft neutrosophic strong biideal over $BN(S)$.

Theorem 4.5. Let (F, A) be a soft neutrosophic strong bisemigroup over $BN(S)$ and $\{(H_i, B_i); i \in I\}$ is a non empty family of soft neutrosophic strong biideals of (F, A) then

1. $\cap_{i\in I}(H_i, B_i)$ is a soft neutrosophic strong biideal of (F, A).

2. $\wedge_{i\in I}(H_i, B_i)$ is a soft neutrosophic strong biideal of $\wedge_{i\in I}(F, A)$.

§5. Soft neutrosophic N-semigroup

Definition 5.1. Let $\{S(N), *_1, \ldots, *_N\}$ be a neutrosophic N-semigroup and (F, A) be a soft set over $\{S(N), *_1, \ldots, *_N\}$. Then (F, A) is termed as soft neutrosophic N-semigroup if and only if $F(x)$ is neutrosophic sub N-semigroup, for all $x \in A$.

Example 5.1. Let $S(N) = \{S_1 \cup S_2 \cup S_3 \cup S_4, *_1, *_2, *_3, *_4\}$ be a neutrosophic 4-semigroup where

$S_1 = \{Z_{12}$, semigroup under multiplication modulo 12$\}$.

$S_2 = \{0, 1, 2, 3, I, 2I, 3I$, semigroup under multiplication modulo 4$\}$, a neutrosophic semigroup.

$S_3 = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix}; a, b, c, d \in \langle R \cup I \rangle \right\}$, neutrosophic semigroup under matrix multiplication.

$S_4 = \langle Z \cup I \rangle$, neutrosophic semigroup under multiplication. Let $T = \{T_1 \cup T_2 \cup T_3 \cup T_4, *_1, *_2, *_3, *_4\}$ is a neutosophic sub 4-semigroup of $S(4)$, where $T_1 = \{0, 2, 4, 6, 8, 10\} \subseteq Z_{12}$, $T_2 = \{0, I, 2I, 3I\} \subset S_2$, $T_3 = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix}; a, b, c, d \in \langle Q \cup I \rangle \right\} \subset S_3$, $T4 = \{\langle 5Z \cup I \rangle\} \subset S_4$, the neutrosophic semigroup under multiplication. Also let $P = \{P_1 \cup P_2 \cup P_3 \cup P_4, *_1, *_2, *_3, *_4\}$ be another neutrosophic sub 4-semigroup of $S(4)$, where $P_1 = \{0, 6\} \subseteq Z_{12}$, $P_2 = \{0, 1, I\} \subset S_2$, $P_3 = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix}; a, b, c, d \in \langle Z \cup I \rangle \right\} \subset S_3$, $P_4 = \{\langle 2Z \cup I \rangle\} \subset S_4$. Then (F, A) is soft neutrosophic 4-semigroup over $S(4)$, where

$$F(x_1) = \{0, 2, 4, 6, 8, 10\} \cup \{0, I, 2I, 3I\} \cup \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix}; a, b, c, d \in \langle Q \cup I \rangle \right\} \cup \{\langle 5Z \cup I \rangle\},$$

$$F(x_2) = \{0, 6\} \cup \{0, 1, I\} \cup \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix}; a, b, c, d \in \langle Z \cup I \rangle \right\} \cup \{\langle 2Z \cup I \rangle\}.$$

Theorem 5.1. Let (F, A) and (H, A) be two soft neutrosophic N-semigroup over $S(N)$. Then their intersection $(F, A) \cap (H, A)$ is again a soft neutrosophic N-semigroup over $S(N)$.

Proof. Straightforward.

Theorem 5.2. Let (F, A) and (H, B) be two soft neutrosophic N-semigroups over $S(N)$ such that $A \cap B = \phi$, then their union is soft neutrosophic N-semigroup over $S(N)$.

Proof. Straightforward.

Proposition 5.1. Let (F, A) and (K, B) be two soft neutrosophic N-semigroups over $S(N)$. Then

1. Their extended union $(F, A) \cup_{\varepsilon} (K, B)$ over $S(N)$ is not soft neutrosophic N-semigroup over $S(N)$.
2. Their extended intersection $(F, A) \cap_{\varepsilon} (K, B)$ over $S(N)$ is soft neutrosophic N-semigroup over $S(N)$.
3. Their restricted union $(F, A) \cup_R (K, B)$ over $S(N)$ is not soft neutrosophic N-semigroup over $S(N)$.
4. Their restricted intersection $(F, A) \cap_{\varepsilon} (K, B)$ over $S(N)$ is soft neutrosophic N-semigroup over $S(N)$.

Proposition 5.2. Let (F, A) and (K, B) be two soft neutrosophic N-semigroups over $S(N)$. Then

1. Their AND operation $(F, A) \wedge (K, B)$ is soft neutrosophic N-semigroup over $S(N)$.
2. Their OR operation $(F, A) \vee (K, B)$ is not soft neutrosophic N-semigroup over $S(N)$.

Definition 5.2. Let (F, A) be a soft neutrosophic N-semigroup over $S(N)$, then (F, A) is called Full-soft neutrosophic N-semigroup over $S(N)$ if $F(x) = S(N)$, for all $x \in A$. We denote it by $S(N)$.

Definition 5.3. Let (F, A) and (H, B) be two soft neutrosophic N-semigroups over $S(N)$. Then (H, B) is a soft neutrosophic sub N-semigroup of (F, A), if

1. $B \subset A$.
2. $H(x)$ is neutrosophic sub N-semigroup of $F(x)$, for all $x \in B$.

Example 5.2. Let $S(N) = \{S_1 \cup S_2 \cup S_3 \cup S_4, *_1, *_2, *_3, *_4\}$ be a neutrosophic 4-semigroup where

$S_1 = \{Z_{12}$, semigroup under multiplication modulo 12$\}$.

$S_2 = \{0, 1, 2, 3, I, 2I, 3I$, semigroup under multiplication modulo 4$\}$, a neutrosophic semigroup.

$S_3 = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix}; a, b, c, d \in \langle R \cup I \rangle \right\}$, neutrosophic semigroup under matrix multiplication.

$S_4 = \langle Z \cup I \rangle$, neutrosophic semigroup under multiplication. Let $T = \{T_1 \cup T_2 \cup T_3 \cup T_4, *_1, *_2, *_3, *_4\}$ is a neutosophic sub 4-semigroup of $S(4)$, where $T_1 = \{0, 2, 4, 6, 8, 10\} \subseteq Z_{12}$, $T_2 = \{0, I, 2I, 3I\} \subset S_2$, $T_3 = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix}; a, b, c, d \in \langle Q \cup I \rangle \right\} \subset S_3$, $T_4 = \{\langle 5Z \cup I \rangle\} \subset S_4$, the neutrosophic semigroup under multiplication. Also let $P = \{P_1 \cup P_2 \cup P_3 \cup P_4, *_1, *_2, *_3, *_4\}$ be another neutrosophic sub 4-semigroup of $S(4)$, where $P_1 = \{0, 6\} \subseteq Z_{12}$, $P_2 = \{0, 1, I\} \subset S_2$, $P_3 = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix}; a, b, c, d \in \langle Z \cup I \rangle \right\} \subset S_3$, $P_4 = \{\langle 2Z \cup I \rangle\} \subset S_4$. Also let $R = \{R_1 \cup R_2 \cup R_3 \cup R_4, *_1, *_2, *_3, *_4\}$ be a neutrosophic sub 4-semigroup os $S(4)$ where $R_1 = \{0, 3, 6, 9\}, R_2 = \{0, I, 2I\}, R_3 = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix}; a, b, c, d \in \langle 2Z \cup I \rangle \right\}, R_4 = \{\langle 3Z \cup I \rangle\}$. Then (F, A) is soft neutrosophic 4-semigroup over $S(4)$, where

$$F(x_1) = \{0, 2, 4, 6, 8, 10\} \cup \{0, I, 2I, 3I\} \cup \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix}; a, b, c, d \in \langle Q \cup I \rangle \right\} \cup \{\langle 5Z \cup I \rangle\},$$

$$F(x_2) = \{0, 6\} \cup \{0, 1, I\} \cup \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix}; a, b, c, d \in \langle Z \cup I \rangle \right\} \cup \{\langle 2Z \cup I \rangle\},$$

$$F(x_3) = \{0, 3, 6, 9\} \cup \{0, I, 2I\} \cup \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix}; a, b, c, d \in \langle 2Z \cup I \rangle \right\} \cup \{\langle 3Z \cup I \rangle\}.$$

Clearly (H, B) is a soft neutrosophic sub N-semigroup of (F, A), where

$$H(x_1) = \{0, 4, 8\} \cup \{0, I, 2I\} \cup \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix}; a, b, c, d \in \langle Z \cup I \rangle \right\} \cup \{\langle 10Z \cup I \rangle\},$$

$$H(x_3) = \{0, 6\} \cup \{0, I\} \cup \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix}; a, b, c, d \in \langle 4Z \cup I \rangle \right\} \cup \{\langle 6Z \cup I \rangle\}.$$

Theorem 5.3. Let (F, A) be a soft neutrosophic N-semigroup over $S(N)$ and $\{(H_i, B_i); i \in I\}$ is a non empty family of soft neutrosophic sub N-semigroups of (F, A) then

1. $\cap_{i \in I}(H_i, B_i)$ is a soft neutrosophic sub N-semigroup of (F, A).

2. $\wedge_{i \in I}(H_i, B_i)$ is a soft neutrosophic sub N-semigroup of $\wedge_{i \in I}(F, A)$.

3. $\cup_{i \in I}(H_i, B_i)$ is a soft neutrosophic sub N-semigroup of (F, A) if $B_i \cap B_j = \phi$, for all $i \neq j$.

Proof. Straightforward.

Definition 5.4. (F, A) over $S(N)$ is called soft neutrosophic N-ideal if $F(x)$ is neutosophic N-ideal of $S(N)$, for all $x \in A$.

Theorem 5.4. Every soft neutrosophic N-ideal (F, A) over $S(N)$ is a soft neutrosophic N-semigroup but the converse is not true.

Proposition 5.3. Let (F, A) and (K, B) be two soft neutrosophic N-ideals over $S(N)$. Then

1. Their extended union $(F, A) \cup_{\varepsilon} (K, B)$ over $S(N)$ is not soft neutrosophic N-ideal over $S(N)$.

2. Their extended intersection $(F, A) \cap_{\varepsilon} (K, B)$ over $S(N)$ is soft neutrosophic N-ideal over $S(N)$.

3. Their restricted union $(F, A) \cup_R (K, B)$ over $S(N)$ is not soft neutrosophic N-ideal over $S(N)$.

4. Their restricted intersection $(F, A) \cap_{\varepsilon} (K, B)$ over $S(N)$ is soft neutrosophic N-ideal over $S(N)$.

Proposition 5.4. Let (F, A) and (H, B) be two soft neutrosophic N-ideal over $S(N)$. Then

1. Their AND operation $(F, A) \wedge (H, B)$ is soft neutrosophic N-ideal over $S(N)$.

2. Their OR operation $(F, A) \vee (H, B)$ is not soft neutrosophic N-ideal over $S(N)$.

Theorem 5.5. Let (F, A) be a soft neutrosophic N-semigroup over $S(N)$ and $\{(H_i, B_i);\ i \in I\}$ is a non empty family of soft neutrosophic N-ideals of (F, A) then

1. $\cap_{i \in I}(H_i, B_i)$ is a soft neutrosophic N-ideal of (F, A).

2. $\wedge_{i \in I}(H_i, B_i)$ is a soft neutrosophic N-ideal of $\wedge_{i \in I}(F, A)$.

§6. Soft neutrosophic strong N-semigroup

Definition 6.1. Let $\{S(N), *_1, \ldots, *_N\}$ be a neutrosophic N-semigroup and (F, A) be a soft set over $\{S(N), *_1, \ldots, *_N\}$. Then (F, A) is called soft neutrosophic strong N-semigroup if and only if $F(x)$ is neutrosophic strong sub N-semigroup, for all $x \in A$.

Example 6.1. Let $S(N) = \{S_1 \cup S_2 \cup S_3 \cup S_4, *_1, *_2, *_3, *_4\}$ be a neutrosophic 4-semigroup where

$S_1 = \langle Z_6 \cup I \rangle$, a neutrosophic semigroup.

$S_2 = \{0, 1, 2, 3, I, 2I, 3I$, semigroup under multiplication modulo $4\}$, a neutrosophic semigroup.

$S_3 = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix}; a, b, c, d \in \langle R \cup I \rangle \right\}$, neutrosophic semigroup under matrix multiplication.

$S_4 = \langle Z \cup I\rangle$, neutrosophic semigroup under multiplication. Let $T = \{T_1 \cup T_2 \cup T_3 \cup T_4, *_1, *_2, *_3, *_4\}$ is a neutosophic strong sub 4-semigroup of $S(4)$, where $T_1 = \{0, 3, 3I\} \subseteq \langle Z_6 \cup I\rangle$, $T_2 = \{0, I, 2I, 3I\} \subset S_2$, $T_3 = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} ; a, b, c, d \in \langle Q \cup I\rangle \right\} \subset S_3$, $T_4 = \{\langle 5Z \cup I\rangle\} \subset S_4$, the neutrosophic semigroup under multiplication. Also let $P = \{P_1 \cup P_2 \cup P_3 \cup P_4, *_1, *_2, *_3, *_4\}$ be another neutrosophic strong sub 4-semigroup of $S(4)$, where $P_1 = \{0, 2I, 4I\} \subseteq \langle Z_6 \cup I\rangle$, $P_2 = \{0, 1, I\} \subset S_2$, $P_3 = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} ; a, b, c, d \in \langle Z \cup I\rangle \right\} \subset S_3$, $P_4 = \{\langle 2Z \cup I\rangle\} \subset S_4$. Then (F, A) is soft neutrosophic strong 4-semigroup over $S(4)$, whereThen (F, A) is soft neutrosophic 4-semigroup over $S(4)$, where

$$\begin{aligned} F(x_1) &= \{0, 3, 3I\} \cup \{0, I, 2I, 3I\} \cup \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} ; a, b, c, d \in \langle Q \cup I\rangle \right\} \cup \{\langle 5Z \cup I\rangle\}, \\ F(x_2) &= \{0, 2I, 4I\} \cup \{0, 1, I\} \cup \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} ; a, b, c, d \in \langle Z \cup I\rangle \right\} \cup \{\langle 2Z \cup I\rangle\}. \end{aligned}$$

Theorem 6.1. Every soft neutrosophic strong N-semigroup is trivially a soft neutrosophic N-semigroup but the converse is not true.

Proposition 6.1. Let (F, A) and (K, B) be two soft neutrosophic strong N-semigroups over $S(N)$. Then

1. Their extended union $(F, A) \cup_\varepsilon (K, B)$ over $S(N)$ is not soft neutrosophic strong N-semigroup over $S(N)$.
2. Their extended intersection $(F, A) \cap_\varepsilon (K, B)$ over $S(N)$ is soft neutrosophic strong N-semigroup over $S(N)$.
3. Their restricted union $(F, A) \cup_R (K, B)$ over $S(N)$ is not soft neutrosophic strong N-semigroup over $S(N)$.
4. Their restricted intersection $(F, A) \cap_\varepsilon (K, B)$ over $S(N)$ is soft neutrosophic strong N-semigroup over $S(N)$.

Proposition 6.2. Let (F, A) and (K, B) be two soft neutrosophic strong N-semigroups over $S(N)$. Then

1. Their AND operation $(F, A) \wedge (K, B)$ is soft neutrosophic strong N-semigroup over $S(N)$.
2. Their OR operation $(F, A) \vee (K, B)$ is not soft neutrosophic strong N-semigroup over $S(N)$.

Definition 6.2. Let (F, A) and (H, B) be two soft neutrosophic strong N-semigroups over $S(N)$. Then (H, B) is a soft neutrosophic strong sub N-semigroup of (F, A), if

1. $B \subset A$.
2. $H(x)$ is neutrosophic strong sub N-semigroup of $F(x)$, for all $x \in B$.

Theorem 6.2.

1. Let (F, A) be a soft neutrosophic strong N-semigroup over $S(N)$ and $\{(H_i, B_i); i \in I\}$ is a non empty family of soft neutrosophic stong sub N-semigroups of (F, A) then
2. $\cap_{i \in I} (H_i, B_i)$ is a soft neutrosophic strong sub N-semigroup of (F, A).

3. $\wedge_{i\in I}(H_i, B_i)$ is a soft neutrosophic strong sub N-semigroup of $\wedge_{i\in I}(F, A)$.

4. $\cup_{i\in I}(H_i, B_i)$ is a soft neutrosophic strong sub N-semigroup of (F, A) if $B_i \cap B_j = \phi$, for all $i \neq j$.

Proof. Straightforward.

Definition 6.3. (F, A) over $S(N)$ is called soft neutrosophic strong N-ideal if $F(x)$ is neutosophic strong N-ideal of $S(N)$, for all $x \in A$.

Theorem 6.3. Every soft neutrosophic strong N-ideal (F, A) over $S(N)$ is a soft neutrosophic strong N-semigroup but the converse is not true.

Theorem 6.4. Every soft neutrosophic strong N-ideal (F, A) over $S(N)$ is a soft neutrosophic N-semigroup but the converse is not true.

Proposition 6.3. Let (F, A) and (K, B) be two soft neutrosophic strong N-ideals over $S(N)$. Then

1. Their extended union $(F, A) \cup_{\varepsilon} (K, B)$ over $S(N)$ is not soft neutrosophic strong N-ideal over $S(N)$. 2. Their extended intersection $(F, A) \cap_{\varepsilon} (K, B)$ over $S(N)$ is soft neutrosophic strong N-ideal over $S(N)$.

3. Their restricted union $(F, A) \cup_R (K, B)$ over $S(N)$ is not soft neutrosophic strong N-ideal over $S(N)$.

4. Their restricted intersection $(F, A) \cap_{\varepsilon} (K, B)$ over $S(N)$ is soft neutrosophic strong N-ideal over $S(N)$.

Proposition 6.4. Let (F, A) and (H, B) be two soft neutrosophic strong N-ideal over $S(N)$. Then

1. Their AND operation $(F, A) \wedge (H, B)$ is soft neutrosophic strong N-ideal over $S(N)$.

2. Their OR operation $(F, A) \vee (H, B)$ is not soft neutrosophic strong N-ideal over $S(N)$.

Theorem 6.5. Let (F, A) be a soft neutrosophic strong N-semigroup over $S(N)$ and $\{(H_i, B_i); i \in I\}$ is a non empty family of soft neutrosophic strong N-ideals of (F, A) then

1. $\cap_{i\in I}(H_i, B_i)$ is a soft neutrosophic strong N-ideal of (F, A).

2. $\wedge_{i\in I}(H_i, B_i)$ is a soft neutrosophic strong N-ideal of $\wedge_{i\in I}(F, A)$.

Conclusion

This paper is an extension of neutrosphic semigroup to soft semigroup. We also extend neutrosophic bisemigroup, neutrosophic N-semigroup to soft neutrosophic bisemigroup, and soft neutrosophic N-semigroup. Their related properties and results are explained with many illustrative examples, the notions related with strong part of neutrosophy also established within soft semigroup.

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Published in *Scientia Magna*, Vol. 10 (2014), No. 1, pp. 93-111, 19 p.

Rough Neutrosophic Sets

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Abstract. Both neutrosophic sets theory and rough sets theory are emerging as powerful tool for managing uncertainty, indeterminate, incomplete and imprecise information. In this paper we develop an hybrid structure called *rough neutrosophic sets* and studied their properties.

Keywords: Rough set, rough neutrosophic set.

1. Introduction

In 1982, Pawlak [1] introduced the concept of rough set (RS), as a formal tool for modeling and processing incomplete information in information systems. There are two basic elements in rough set theory, crisp set and equivalence relation, which constitute the mathematical basis of RSs. The basic idea of rough set is based upon the approximation of sets by a pair of sets known as the lower approximation and the upper approximation of a set. Here, the lower and upper approximation operators are based on equivalence relation. After Pawlak, there has been many models built upon different aspect, i.e., universe, relations, object, operators by many scholars [2], [3], [4], [5], [6], [7]. Various notions that combine rough sets and fuzzy sets, vague set and intuitionistic fuzzy sets are introduced, such as rough fuzzy sets, fuzzy rough sets, generalize fuzzy rough, intuitionistic fuzzy rough sets, rough intuitionistic fuzzy sets, rough vagues sets. The theory of rough sets is based upon the classification mechanism, from which the classification can be viewed as an equivalence relation and knowledge blocks induced by it be a partition on universe.

One of the interesting generalizations of the theory of fuzzy sets and intuitionistic fuzzy sets is the theory of neutrosophic sets introduced by F. Smarandache [8], [9]. Neutrosophic sets described by three functions: a membership function indeterminacy function and a non-membership function that are independently related. The theory of neutrosophic set have achieved great success in various areas such as medical diagnosis [10], database [11], [12], topology [13], image processing [14], [15], [16], and decision making problem [17]. While the neutrosophic set is a powerful tool to deal with indeterminate and inconsistent data, the theory of rough sets is a powerful mathematical tool to deal with incompleteness.

Neutrosophic sets and rough sets are two different topics, none conflicts the other. Recently many researchers applied the notion of neutrosophic sets to relations, group theory, ring theory, soft set theory [23], [24], [25], [26], [27], [28], [29], [30], [31], [32] and so on. The main objective of this study was to introduce a new hybrid intelligent structure called *rough neutrosophic sets*. The significance of introducing hybrid set structures is that the computational techniques based on any one of theses structures alone will not always yield the best results but a fusion of two or more of them can often give better results.

The rest of this paper is organized as follows. Some preliminary concepts required in our work are briefly recalled in Section 2. In Section 3, the concept of rough neutrosophic sets is investigated. Section 4 concludes the paper.

2. Preliminaries

In this section we present some preliminaries which will be useful to our work in the next section. For more details the reader may refer to [1], [8], [9].

Definition 2.1. [8] Let X be an universe of discourse, with a generic element in X denoted by x, the neutrosophic (NS) set is an object having the form

$$A = \{\langle x : \mu_A(x), \nu_A(x), \omega_A(x)\rangle\,,\ x \in X\},$$

where the functions $\mu, \nu, \omega : X \to]^-0, 1^+[$ define respectively the degree of membership (or Truth), the degree of indeterminacy, and the degree of non-membership (or Falsehood) of the element $x \in X$ to the set A with the condition

$$(1) \qquad {}^-0 \leq \mu_A(x) + \nu_A(x) + \omega_A(x) \leq 3^+.$$

From a philosophical point of view, the neutrosophic set takes the value from real standard or non-standard subsets of $]^-0, 1^+[$. So, instead of $]^-0, 1^+[$ we need to take the interval $[0, 1]$ for technical applications, because $]^-0, 1^+[$ will be difficult to apply in the real applications such as in scientific and engineering problems.

For two NS,

$$A = \{\langle x, \mu_A(x), \nu_A(x), \omega_A(x)\rangle \mid x \in X\} \text{ and}$$
$$B = \{\langle x, \mu_B(x), \nu_B(x), \omega_B(x)\rangle \mid x \in X\},$$

the relations are defined as follows:

(i) $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$, $\nu_A(x) \geq \mu_B(x)$, $\omega_A(x) \geq \omega_B(x)$,

(ii) $A = B$ if and only if $\mu_A(x) = \mu_B(x)$, $\nu_A(x) = \mu_B(x)$, $\omega_A(x) = \omega_B(x)$,

(iii) $A \cap B = \{\langle x, \min(\mu_A(x), \mu_B(x)), \max(\nu_A(x), \nu_B(x)), \max(\omega_A(x), \omega_B(x))\rangle \mid x \in X\}$,

(iv) $A \cup B = \{\langle x, \max(\mu_A(x), \mu_B(x)), \min(\nu_A(x), \nu_B(x)), \min(\omega_A(x), \omega_B(x))\rangle \mid x \in X\}$,

(v) $A^C = \{\langle x, \omega_A(x), 1 - \nu_A(x), \mu_A(x)\rangle \mid x \in X\}$

(vi) $0_n = (0, 1, 1)$ and $1_n = (1, 0, 0)$.

As an illustration, let us consider the following example.

Example 2.2. Assume that the universe of discourse $U = \{x_1, x_2, x_3\}$, where x_1 characterizes the capability, x_2 characterizes the trustworthiness and x_3 indicates the prices of the objects. It may be further assumed that the values of x_1, x_2 and x_3 are in $[0, 1]$ and they are obtained from some questionnaires of some experts. The experts may impose their opinion in three components viz. the degree of goodness, the degree of indeterminacy and that of poorness to explain the characteristics of the objects. Suppose A is a neutrosophic set (NS) of U, such that,

$$A = \{\langle x_1, (0.3, 0.5, 0.6)\rangle, \langle x_2, (0.3, 0.2, 0.3)\rangle, \langle x_3, (0.3, 0.5, 0.6)\rangle\},$$

where the degree of goodness of capability is 0.3, degree of indeterminacy of capability is 0.5 and degree of falsity of capability is 0.6 etc.

Definition 2.3. [1] Let U be any non-empty set. Suppose R is an equivalence relation over U. For any non-null subset X of U, the sets

$$A_1(x) = \{x : [x]_R \subseteq X\} \text{ and } A_2(x) = \{x : [x]_R \cap X \neq \emptyset\}$$

are called the lower approximation and upper approximation, respectively of X, where the pair $S = (U, R)$ is called an approximation space. This equivalent relation R is called indiscernibility relation.

The pair $A(X) = (A_1(x), A_2(x))$ is called the rough set of X in S. Here $[x]_R$ denotes the equivalence class of R containing x.

Definition 2.4. [1] Let $A = (A_1, A_2)$ and $B = (B_1, B_2)$ be two rough sets in the approximation space $S = (U, R)$. Then,

$$\begin{aligned}
&A \cup B = (A_1 \cup B_1, A_2 \cup B_2),\\
&A \cap B = (A_1 \cap B_1, A_2 \cap B_2),\\
&A \subseteq B \text{ if } A \cap B = A,\\
&\sim A = \{U - A_2, U - A_1\}.
\end{aligned}$$

3. Rough neutrosophic sets

In this section we introduce the notion of rough neutrosophic sets by combining both rough sets and nuetrosophic sets. and some operations viz. union, intersection, inclusion and equalities over them. Rough neutrosophic set are the generalization of rough fuzzy sets [2] and rough intuitionistic fuzzy sets [22].

Definition 3.1. Let U be a non-null set and R be an equivalence relation on U. Let F be neutrosophic set in U with the membership function μ_F, indeterminacy function ν_F and non-membership function ω_F. The lower and the upper approximations of F in the approximation (U, R) denoted by $\underline{N}(F)$ and $\overline{N}(F)$ are respectively defined as follows:

$$\begin{aligned}
&\underline{N}(F) = \{< x, \mu_{\underline{N}(F)}(x), \nu_{\underline{N}(F)}(x), \omega_{\underline{N}(F)}(x) >|\ y \in [x]_R,\ x \in U\},\\
&\overline{N}(F)) = \{< x, \mu_{\overline{N}(F)}(x), \nu_{\overline{N}(F)}(x), \omega_{\overline{N}(F)}(x) >|\ y \in [x]_R,\ x \in U\},
\end{aligned}$$

where:

$$\begin{aligned}
&\mu_{\underline{N}(F)}(x) = \bigwedge_{y \in [x]_R} \mu_F(y),\ \nu_{\underline{N}(F)}(x) = \bigvee_{y \in [x]_R} \nu_F(y),\ \omega_{\underline{N}(F)}(x) = \bigvee_{y \in [x]_R} \omega_F(y),\\
&\mu_{\overline{N}(F)}(x) = \bigvee_{y \in [x]_R} \mu_F(y),\ \nu_{\overline{N}(F)}(x) = \bigwedge_{y \in [x]_R} \nu_F(y),\ \omega_{\overline{N}(F)}(x) = \bigwedge_{y \in [x]_R} \omega_F(y).
\end{aligned}$$

So

$$0 \leq \mu_{\overline{N}(F)}(x) + \nu_{\overline{N}(F)}(x) + \omega_{\overline{N}(F)}(x) \leq 3$$

and

$$\mu_{\underline{N}(F)}(x) + \nu_{\underline{N}(F)}(x) + \omega_{\underline{N}(F)}(x) \leq 3,$$

where "$\vee$" and "$\wedge$" mean "max" and "min" operators respectively, $\mu_F(x)$, $\nu_F(y)$ and $\omega_F(y)$ are the membership, indeterminacy and non-membership of y with respect to F. It is easy to see that $\overline{N}(F)$ and $\underline{N}(F)$ are two neutrosophic sets in U, thus the NS mappings $\overline{N}, \underline{N} : N(U \to N(U)$ are, respectively, referred to as the upper and lower rough NS approximation operators, and the pair $(\underline{N}(F), \overline{N}(F))$ is called the rough neutrosophic set in (U, R).

From the above definition, we can see that $\underline{N}(F)$ and $\overline{N}(F)$ have constant membership on the equivalence classes of U, if $\underline{N}(F) = \overline{N}(F)$; i.e.,

$$\mu_{\underline{N}(F)} = \mu_{\overline{N}(F)},$$
$$\nu_{\underline{N}(F)} = \nu_{\overline{N}(F)},$$
$$\omega_{\underline{N}(F)} = \omega_{\overline{N}(F)}.$$

For any $x \in U$, we call F a definable neutrosophic set in the approximation (U, R). It is easily to be proved that Zero O_N neutrosophic set and unit neutrosophic sets 1_N are definable neutrosophic sets. Let us consider a simple example in the following.

Example 3.2. Let $U = \{p_1, p_2, p_3, p_4, p_5, p_6, p_7, p_8\}$ be the universe of discourse. Let R be an equivalence relation its partition of U is given by

$$U/R = \{\{p_1, p_4\}, \{p_2, p_3, p_6\}, \{p_5\}, \{p_7, p_8\}\}.$$

Let

$$N(F)=\{(p_1, (0.2, 0.3, 0.4), (p_4, (0.3, 0.5, 0.4)), (p_5, (0.4, 0.6, 0.2)), (p_7, (0.1, 0.3, 0.5))\}$$

be a neutrosophic set of U. By Definition 3.1, we obtain:

$$\begin{aligned} \underline{N}(F) &= \{(p_1, (0.2, 0.5, 0.4)), (p_4, (0.2, 0.5, 0.4)), (p_5, (0.4, 0.6, 0.2))\}; \\ \overline{N}(F) &= \{(p_1, (0.2, 0.3, 0.4)), (p_4, (0.2, 0.3, 0.4)), (p_5, (0.4, 0.6, 0.2)), \\ &\qquad (p_7, (0.1, 0.3, 0.5)), (p_8, (0.1, 0.3, 0.5))\}. \end{aligned}$$

For another neutrosophic sets

$$N(G) = \{(p_1, (0.2, 0.3, 0.4)), (p_4, (0.2, 0.3, 0.4)), (p_5, (0.4, 0.6, 0.2))\}.$$

The lower approximation and upper approximation of $N(G)$ are calculated as

$$\underline{N}(G) = \{(p_1, (0.2, 0.3, 0.4)), (p_4, (0.2, 0.3, 0.4)), (p_5, (0.4, 0.6, 0.2))\};$$
$$\overline{N}(G) = \{(p_1, (0.2, 0.3, 0.4)), (p_4, (0.2, 0.3, 0.4)), (p_5, (0.4, 0.6, 0.2))\}.$$

Obviously $\underline{N}(G) = \overline{N}(G)$ is a definable neutrosophic set in the approximation space (U, R).

Definition 3.3. If $N(F) = (\underline{N}(F), \overline{N}(F))$ is a rough neutrosophic set in (U, R), the rough complement of $N(F)$ is the rough neutrosophic set denoted $\sim N(F) = (\underline{N}(F)^c, \overline{N}(F)^c)$, where $\underline{N}(F)^c, \overline{N}(F)^c$ are the complements of neutrosophic sets $\underline{N}(F)$ and $\overline{N}(F)$, respectively,

$$\underline{N}(F)^c = \{< x, \omega_{\underline{N}(F)}, 1 - \nu_{\underline{N}(F)}(x), \mu_{\underline{N}(F)}(x) >| \; x \in U\},$$

and

$$\overline{N}(F)^c = \{< x, \omega_{\overline{N}(F)}, 1 - \nu_{\overline{N}(F)}(x), \mu_{\overline{N}(F)}(x) >| \; x \in U\}.$$

Definition 3.4. If $N(F_1)$ and $N(F_2)$ are two rough neutrosophic set of the neutrosophic sets F_1 and F_2 respectively in U, then we define the following:

(i) $N(F_1) = N(F_2)$ iff $\underline{N}(F_1) = \underline{N}(F_2)$ and $\overline{N}(F_1) = \overline{N}(F_2)$.

(ii) $N(F_1) \subseteq N(F_2)$ iff $\underline{N}(F_1) \subseteq \underline{N}(F_2)$ and $\overline{N}(F_1) \subseteq \overline{N}(F_2)$.

(iii) $N(F_1) \cup N(F_2) = \left\langle \underline{N}(F_1) \cup \underline{N}(F_2), \overline{N}(F_1) \cup \overline{N}(F_2) \right\rangle$.

(iv) $N(F_1) \cap N(F_2) = \left\langle \underline{N}(F_1) \cap \underline{N}(F_2), \overline{N}(F_1) \cap \overline{N}(F_2) \right\rangle$.

(v) $N(F_1) + N(F_2) = \left\langle \underline{N}(F_1) + \underline{N}(F_2), \overline{N}(F_1) + \overline{N}(F_2) \right\rangle$.

(vi) $N(F_1) \cdot N(F_2) = \left\langle \underline{N}(F_1) \cdot \underline{N}(F_2), \overline{N}(F_1) \cdot \overline{N}(F_2) \right\rangle$.

If N, M, L are rough neutrosophic set in (U, R), then the results in the following proposition are straightforward from definitions.

Proposition 3.5.

(i) $\sim N(\sim N) = N$

(ii) $N \cup M = M \cup N,\ N \cap M = M \cap N$

(iii) $(N \cup M) \cup L = N \cup (M \cup L)$ and $(N \cap M) \cap L = N \cap (M \cap L)$

(iv) $(N \cup M) \cap L = (N \cup M) \cap (N \cup L)$ and $(N \cap M) \cup L = (N \cap M) \cup (N \cap L)$.

De Morgan 's Laws are satisfied for neutrosophic sets:

Proposition 3.6.

(i) $\sim (N(F_1) \cup N(F_2)) = (\sim N(F_1)) \cap (\sim N(F_2))$

(ii) $\sim (N(F_1) \cap N(F_2)) = (\sim N(F_1)) \cup (\sim N(F_2))$.

Proof. (i) $(N(F_1) \cup N(F_2)) = \sim (\{\underline{N}(F_1) \cup \underline{N}(F_2)\}, \{\overline{N}(F_1) \cup \overline{N}(F_2)\}) = (\sim\{\underline{N}(F_1) \cup \underline{N}(F_2)\}, \sim\{\overline{N}(F_1) \cup \overline{N}(F_2)\}) = (\{\underline{N}(F_1) \cup \underline{N}(F_2)\}^c, \{\overline{N}(F_1) \cup \overline{N}(F_2)\}^c) = (\sim\{\underline{N}(F_1) \cap \underline{N}(F_2)\}, \sim\{\overline{N}(F_1) \cap \overline{N}(F_2)\}) = (\sim N(F_1)) \cap (\sim N(F_2))$.

(ii) Similar to the proof of (i).

Proposition 3.7. *If F_1 and F_2 are two neutrosophic sets in U such that $F_1 \subseteq F_2$, then $N(F_1) \subseteq N(F_2)$*

(i) $N(F_1 \cup F_2) \supseteq N(F_1) \cup N(F_2)$,

(ii) $N(F_1 \cap F_2) \subseteq N(F_1) \cap N(F_2)$.

Proof.

$$\begin{aligned}\mu_{\underline{N}(F_1\cup F_2)}(x) &= \inf\{\mu_{F_1\cup F_2)}(x) \mid x\in X_i\} \\ &= \inf(\max\{\mu_{F_1}(x),\mu_{F_2}(x) \mid x\in X_i\}) \\ &\geq \max\{\inf\{\mu_{F_1}(x)\mid x\in X_i\},\inf\{\mu_{F_2}(x)\mid x\in X_i\}\} \\ &= \max\{\mu_{\underline{N}(F_1)}(x_i),\mu_{\underline{N}(F_2)}(x_i)\} \\ &= \mu_{\underline{N}(F_1)}\cup\mu_{\underline{N}(F_2)}(x_i).\end{aligned}$$

Similarly,

$$\nu_{\underline{N}(F_1\cup F_2)}(x_i)\leq(\nu_{\underline{N}(F_1)}\cup\nu_{\underline{N}(F_2)})(x_i)$$
$$\omega_{\underline{N}(F_1\cup F_2)}(x_i)\leq(\omega_{\underline{N}(F_1)}\cup\omega_{\underline{N}(F_2)})(x_i)$$

Thus,

$$\underline{N}(F_1\cup F_2)\supseteq\underline{N}(F_1)\cup\underline{N}(F_2).$$

We can also see that

$$\overline{N}(F_1\cup F_2)=\overline{N}(F_1)\cup\overline{N}(F_2).$$

Hence,

$$N(F_1\cup F_2)\supseteq N(F_1)\cup N(F_2).$$

(ii) The proof of (ii) is similar to the proof of (i).

Proposition 3.8.

(i) $N(F) = \sim\overline{N}(\sim F)$

(ii) $\overline{N}(F) = \sim\underline{N}(\sim F)$

(iii) $\underline{N}(F)\supseteq\overline{N}(F)$.

Proof. According to Definition 3.1, we can obtain

(i) $F=\{\langle x,\mu_F(x),\nu_F(x),\omega_F(x)\rangle \mid x\in X\}$

$\sim F=\{\langle x,\omega_F(x),1-\nu_F(x),\mu_F(x)\rangle \mid |x\in X\}$

$$\overline{N}(\sim F)=\left\{\left\langle x,\omega_{\overline{N}(\sim F)}(x),1-\nu_{\overline{N}(\sim F)}(x),\mu_{\overline{N}(\sim F)}(x)\right\rangle \mid y\in[x]_R,\ x\in U\right\}$$

$$\begin{aligned}\sim\overline{N}(\sim F)&=\left\{\left\langle x,\mu_{\overline{N}(\sim F)}(x),1-(1-\nu_{\overline{N}(\sim F)}(x)),\omega_{\overline{N}(\sim F)}(x)\right\rangle |y\in[x]_R,\ x\in U\right\}\\ &=\left\{\left\langle x,\mu_{\overline{N}(\sim F)}(x),\nu_{\overline{N}(\sim F)}(x),\omega_{\overline{N}(\sim F)}(x)\right\rangle \mid y\in[x]_R,\ x\in U\right\}\end{aligned}$$

where

$$\mu_{\overline{N}(\sim F)}(x)=\bigwedge_{y\in[x]_R}\mu_F(y),\ \nu_{\overline{N}(\sim F)}(x)=\bigvee_{y\in[x]_R}\nu_F(y),\ \omega_{\overline{N}(\sim F)}(x)=\bigvee_{y\in[x]_R}\omega_F(y).$$

Hence $\underline{N}(F)=\sim\overline{N}(\sim F)$.

(ii) The proof is similar to the proof of (i).

(iii) For any $y \in \underline{N}(F)$, we can have
$\mu_{\underline{N}(F)}(x) = \bigwedge_{y\in[x]_R} \mu_F(y) \leq \bigvee_{y\in[x]_R} \mu_F(y)$, $\nu_{\underline{N}(F)}(x) = \bigvee_{y\in[x]_R} \nu_F(y) \geq \bigwedge_{y\in[x]_R} \nu_F(y)$
and $\omega_{\underline{N}(F)}(x) = \bigvee_{y\in[x]_R} \omega_F(y) \geq \bigwedge_{y\in[x]_R} \omega_F(y)$.
Hence $\underline{N}(F) \subseteq \overline{N}(F)$.

4. Conclusion

In this paper we have defined the notion of rough neutrosophic sets. We have also studied some properties on them and proved some propositions. The concept combines two different theories which are rough sets theory and neutrosophic theory. While neutrosophic set theory is mainly concerned with, indeterminate and inconsistent information, rough set theory is with incompleteness; but both the theories deal with imprecision. Consequently, by the way they are defined, it is clear that rough neutrosophic sets can be utilized for dealing with both of indeterminacy and incompleteness.

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Some Types of Neutrosophic Crisp Sets and Neutrosophic Crisp Relations

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***Abstract*—**The purpose of this paper is to introduce a new types of crisp sets are called the neutrosophic crisp set with three types 1, 2, 3. After given the fundamental definitions and operations, we obtain several properties, and discussed the relationship between neutrosophic crisp sets and others. Finally, we introduce and study the notion of neutrosophic crisp relations.

***Index Terms*—**Neutrosophic set, neutrosophic crisp sets, neutrosophic crisp relations, generalized neutrosophic set, Intuitionistic neutrosophic Set.

I. Introduction

Since the world is full of indeterminacy, the neutrosophics found their place into contemporary research. The fundamental concepts of neutrosophic set, introduced by Smarandache in [16, 17, 18], and Salama et al. in [4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15], provides a natural foundation for treating mathematically the neutrosophic phenomena which exist pervasively in our real world and for building new branches of neutrosophic mathematics. Neutrosophy has laid the foundation for a whole family of new mathematical theories generalizing both their classical and fuzzy counterparts [1, 2, 3, 19] such as a neutrosophic set theory. In this paper we introduce a new types of crisp sets are called the neutrosophic crisp set with three types 1, 2, 3. After given the fundamental definitions and operations, we obtain several properties, and discussed the relationship between neutrosophic crisp sets and others. Finally, we introduce and study the notion of neutrosophic crisp relations.

The paper unfolds as follows. The next section briefly introduces some definitions related to neutrosophic set theory and some terminologies of neutrosophic crisp set. Section 3 presents new types of neutrosophic crisp sets and studied some of their basic properties. Section 4 presents the concept of neutrosophic crisp relations . Finally we concludes the paper.

II. Preliminaries

We recollect some relevant basic preliminaries, and in particular, the work of Smarandache in [16 , 17, 18], and Salama et al. [4,5]. Smarandache introduced the neutrosophic components T, I, F which represent the membership, indeterminacy, and non-membership values respectively, where $]^-0, 1^+[$ is nonstandard unit interval.

Definition 2.1 *[9, 13, 15]*

A neutrosophic crisp set (NCS for short) $A = \langle A_1, A_2, A_3 \rangle$ can be identified to an ordered triple $\langle A_1, A_2, A_3 \rangle$ are subsets on X, and every crisp event in X is obviously an NCS having the form $\langle A_1, A_2, A_3 \rangle$, Salama et al. constructed the tools for developed neutrosophic crisp set, and introduced the NCS ϕ_N, X_N in X as follows:

1) ϕ_N may be defined as four types:

i)Type1: $\phi_N = \langle \phi, \phi, X \rangle$, or

ii)Type2: $\phi_N = \langle \phi, X, X \rangle$, or

iii)Type3: $\phi_N = \langle \phi, X, \phi \rangle$, or

iv)Type4: $\phi_N = \langle \phi, \phi, \phi \rangle$

2) X_N may be defined as four types

i) Type1: $X_N = \langle X, \phi, \phi \rangle$,

ii) Type2: $X_N = \langle X, X, \phi \rangle$,

iii) Type3: $X_N = \langle X, X, \phi \rangle$,

iv) Type4: $X_N = \langle X, X, X \rangle$,

Definition 2.2 *[9, 13, 15]*

Let $A = \langle A_1, A_2, A_3 \rangle$ a NCE or UNCE on X, then the complement of the set A (A^c, for short) maybe defined as three kinds of complements

(C_1) Type1: $A^c = \langle A^c{}_1, A^c{}_2, A^c{}_3 \rangle$,

(C_2) Type2: $A^c = \langle A_3, A_2, A_1 \rangle$

(C_3) Type3: $A^c = \langle A_3, A^c{}_2, A_1 \rangle$

One can define several relations and operations between NCS as follows:

Definition 2.3 *[9, 13, 15]*

Let X be a non-empty set, and NCSS A and B in the form $A = \langle A_1, A_2, A_3 \rangle, B = \langle B_1, B_2, B_3 \rangle$, then we may consider two possible definitions for subsets ($A \subseteq B$)

($A \subseteq B$) may be defined as two types:

1)Type1:

$A \subseteq B \Leftrightarrow A_1 \subseteq B_1, A_2 \subseteq B_2$ and $A_3 \supseteq B_3$ or

2)Type2:

$A \subseteq B \Leftrightarrow A_1 \subseteq B_1, A_2 \supseteq B_2$ and $A_3 \supseteq B_3$

Definition 2.4 *[9, 13, 15]*

Let X be a non-empty set, and NCSS A and B in the form $A = \langle A_1, A_2, A_3 \rangle, B = \langle B_1, B_2, B_3 \rangle$ are NCSS Then

1) $A \cap B$ may be defined as two types:

i) Type1:

$A \cap B = \langle A_1 \cap B_1, A_2 \cap B_2, A_3 \cup B_3 \rangle$ or

ii) Type2:

$A \cap B = \langle A_1 \cap B_1, A_2 \cup B_2, A_3 \cup B_3 \rangle$

2) $A \cup B$ may be defined as two types:

i) Type 1: $A \cup B = \langle A_1 \cup B_1, A_2 \cap B_2, A_3 \cup B_3 \rangle$

or

ii) Type 2: $A \cup B = \langle A_1 \cup B_1, A_2 \cap B_2, A_3 \cap B_3 \rangle$

Proposition 2.1 *[9, 13, 15]*

Let $\{A_j : j \in J\}$ be arbitrary family of neutrosophic crisp subsets in X, then

1) $\cap A_j$ may be defined two types as :

i)Type1: $\cap A_j = \langle \cap Aj_1, \cap A_{j2}, \cup A_{j3} \rangle$,or

ii)Type2: $\cap A_j = \langle \cap Aj_1, \cup A_{j2}, \cup A_{j3} \rangle$.

2) $\cup A_j$ may be defined two types as :

i)Type1: $\cup A_j = \langle \cup Aj_1, \cap A_{j2}, \cap A_{j3} \rangle$ or

ii)Type2: $\cup A_j = \langle \cup Aj_1, \cup A_{j2}, \cap A_{j3} \rangle$.

III. New Types of Neutrosophic Crisp Sets

We shall now consider some possible definitions for some types of neutrosophic crisp sets

Definition 3.1

Let X be a non-empty fixed sample space. A neutrosophic crisp set (NCS for short) A is an object having the form $A = \langle A_1, A_2, A_3 \rangle$ where A_1, A_2 and A_3 are subsets of X .

Definition 3.2

The object having the form $A = \langle A_1, A_2, A_3 \rangle$ is called

1) (Neutrosophic Crisp Set with Type 1) If satisfying $A_1 \cap A_2 = \phi$, $A_1 \cap A_3 = \phi$ and $A_2 \cap A_3 = \phi$. (NCS-Type1 for short).

2) (Neutrosophic Crisp Set with Type 2) If satisfying $A_1 \cap A_2 = \phi$, $A_1 \cap A_3 = \phi$ and $A_2 \cap A_3 = \phi$ and $A_1 \cup A_2 \cup A_3 = X$. (NCS-Type2 for short).

3) (Neutrosophic Crisp Set with Type 3) If satisfying

$A_1 \cap A_2 \cap A_3 = \phi$ and

$A_1 \cup A_2 \cup A_3 = X$. (NCS-Type3 for short).

Definition 3.3

1) (Neutrosophic Set [7]): Let X be a non-empty fixed set. A neutrosophic set (NS for short) A is an object having the form $A = \langle x, \mu_A(x), \sigma_A(x), \nu_A(x) \rangle$ where $\mu_A(x), \sigma_A(x)$ and $\nu_A(x)$ which represent the degree of member ship function (namely $\mu_A(x)$), the degree of indeterminacy (namely $\sigma_A(x)$), and the degree of non-member ship (namely $\nu_A(x)$) respectively of each element $x \in X$ to the set A where $0^- \le \mu_A(x), \sigma_A(x), \nu_A(x) \le 1^+$ and $0^- \le \mu_A(x) + \sigma_A(x) + \nu_A(x) \le 3^+$.

2) (Generalized Neutrosophic Set [8]): Let X be a non-empty fixed set. A generalized neutrosophic (GNS for short) set A is an object having the form $A = \langle x, \mu_A(x), \sigma_A(x), \nu_A(x) \rangle$ where $\mu_A(x), \sigma_A(x)$ and $\nu_A(x)$ which represent the degree of member ship function (namely $\mu_A(x)$), the degree of indeterminacy (namely $\sigma_A(x)$), and the degree of non-member ship (namely $\nu_A(x)$) respectively of each element $x \in X$ to the set A where $0^- \le \mu_A(x), \sigma_A(x), \nu_A(x) \le 1^+$ and the functions satisfy the condition $\mu_A(x) \wedge \sigma_A(x) \wedge \nu_A(x) \le 0.5$ and $0^- \le \mu_A(x) + \sigma_A(x) + \nu_A(x) \le 3^+$.

3) (Intuitionistic Neutrosophic Set [16]). Let X be a non-empty fixed set. An intuitionistic neutrosophic set A (INS for short) is an object having the form $A = \langle \mu_A(x), \sigma_A(x), \nu_A(x) \rangle$ where $\mu_A(x), \sigma_A(x)$ and $\nu_A(x)$ which represent the degree of member ship function (namely $\mu_A(x)$), the degree of indeterminacy (namely $\sigma_A(x)$), and the degree of non-member ship (namely $\nu_A(x)$) respectively of each element $x \in X$ to the set A where $0.5 \le \mu_A(x), \sigma_A(x), \nu_A(x)$ and the functions satisfy the condition $\mu_A(x) \wedge \sigma_A(x) \le 0.5$, $\mu_A(x) \wedge \nu_A(x) \le 0.5$, $\sigma_A(x) \wedge \nu_A(x) \le 0.5$, and ${}^-0 \le \mu_A(x) + \sigma_A(x) + \nu_A(x) \le 2^+$.

A neutrosophic crisp with three types the object $A = \langle A_1, A_2, A_3 \rangle$ can be identified to an ordered triple $\langle A_1, A_2, A_3 \rangle$ are subsets on X, and every crisp set in X is obviously a NCS having the form $\langle A_1, A_2, A_3 \rangle$.Every neutrosophic set $A = \langle \mu_A(x), \sigma_A(x), \nu_A(x) \rangle$ on X is obviously on NS having the form $\langle \mu_A(x), \sigma_A(x), \nu_A(x) \rangle$.

Remark 3.1

1) The neutrosophic set not to be generalized neutrosophic set in general.
2) The generalized neutrosophic set in general not intuitionistic NS but the intuitionistic NS is generalized NS.

Intuitionistic NS $\longrightarrow$ Generalized NS $\longrightarrow$ NS

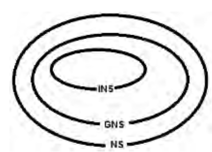

Fig.1: Represents the relation between types of NS

Corollary 3.1

Let X non-empty fixed set and $A = \langle \mu_A(x), \sigma_A(x), \nu_A(x) \rangle$ be INS on X

Then:

1) Type1- A^c of INS be a GNS.
2) Type2- A^c of INS be a INS.
3) Type3- A^c of INS be a GNS.

Proof

Since A INS then $0.5 \le \mu_A(x), \sigma_A(x), \nu_A(x)$, and $\mu_A(x) \wedge \sigma_A(x) \le 0.5, \nu_A(x) \wedge \mu_A(x) \le 0.5$ $\nu_A(x) \wedge \sigma_A(x) \le 0.5$ Implies $\mu^c{}_A(x), \sigma^c{}_A(x), \nu^c{}_A(x) \le 0.5$ then is not to be Type1- A^c INS. On other hand the Type 2- A^c, $A^c = \langle \nu_A(x), \sigma_A(x), \mu_A(x) \rangle$ be INS and Type 3- A^c, $A^c = \langle \nu_A(x), \sigma^c{}_A(x), \mu_A(x) \rangle$ and $\sigma^c{}_A(x) \le 0.5$ implies to $A^c = \langle \nu_A(x), \sigma^c{}_A(x), \mu_A(x) \rangle$ GNS and not to be INS

Example 3.1
Let $X = \{a, b, c\}$, and A, B, C are neutrosophic sets on X,

$A = \langle 0.7, 0.9, 0.8) \setminus a, (0.6, 0.7, 0.6) \setminus b, (0.9, 0.7, 0.8 \setminus c \rangle$,

$B = \langle 0.7, 0.9, 0.5) \setminus a, (0.6, 0.4, 0.5) \setminus b, (0.9, 0.5, 0.8 \setminus c \rangle$

$C = \langle 0.7, 0.9, 0.5) \setminus a, (0.6, 0.8, 0.5) \setminus b, (0.9, 0.5, 0.8 \setminus c \rangle$

By the Definition 3.3 no.3
$\mu_A(x) \wedge \sigma_A(x) \wedge \nu_A(x) \geq 0.5$, A be not GNS and INS,
$B = \langle 0.7, 0.9, 0.5) \setminus a, (0.6, 0.4, 0.5) \setminus b, (0.9, 0.5, 0.8 \setminus c \rangle$
not INS, where $\sigma_A(b) = 0.4 < 0.5$. Since $\mu_B(x) \wedge \sigma_B(x) \wedge \nu_B(x) \leq 0.5$ then B is a GNS but not INS.
$A^c = \langle 0.3, 0.1, 0.2) \setminus a, (0.4, 0.3, 0.4) \setminus b, (0.1, 0.3, 0.2 \setminus c \rangle$
be a GNS, but not INS.

$B^c = \langle 0.3, 0.1, 0.5) \setminus a, (0.4, 0.6, 0.5) \setminus b, (0.1, 0.5, 0.2 \setminus c \rangle$
be a GNS, but not INS, C be INS and GNS,
$C^c = \langle 0.3, 0.1, 0.5) \setminus a, (0.4, 0.2, 0.5) \setminus b, (0.1, 0.5, 0.2 \setminus c \rangle$
be a GNS but not INS.

Definition 3.4
A neutrosophic crisp set (NCS for short) $A = \langle A_1, A_2, A_3 \rangle$ can be identified to an ordered triple $\langle A_1, A_2, A_3 \rangle$ are subsets on X, and every crisp set in X is obviously an NCS having the form $\langle A_1, A_2, A_3 \rangle$,
Salama et al in [6,13] constructed the tools for developed neutrosophic crisp set, and introduced the NCS ϕ_N, X_N in X as follows:

1) ϕ_N may be defined as four types:
   i) Type1: $\phi_N = \langle \phi, \phi, X \rangle$, or
   ii)Type2: $\phi_N = \langle \phi, X, X \rangle$, or
   iii) Type3: $\phi_N = \langle \phi, X, \phi \rangle$, or
   iv) Type4: $\phi_N = \langle \phi, \phi, \phi \rangle$

2) X_N may be defined as four types
   i) Type1: $X_N = \langle X, \phi, \phi \rangle$,
   ii) Type2: $X_N = \langle X, X, \phi \rangle$,
   v) Type3: $X_N = \langle X, X, \phi \rangle$,
   vi) Type4: $X_N = \langle X, X, X \rangle$,

Definition 3.5
A NCS-Type1 ϕ_{N_1}, X_{N_1} in X as follows:

1) ϕ_{N1} may be defined as three types:
   i) Type1: $\phi_{N_1} = \langle \phi, \phi, X \rangle$, or
   ii) Type2: $\phi_{N_1} = \langle \phi, X, \phi \rangle$, or
   iii) Type3: $\phi_N = \langle \phi, \phi, \phi \rangle$.

2) X_{N1} may be defined as one type
   Type1: $X_{N_1} = \langle X, \phi, \phi \rangle$.

Definition 3.6
A NCS-Type2, ϕ_{N_2}, X_{N2} in X as follows:

1) ϕ_{N2} may be defined as two types:
   i) Type1: $\phi_{N_2} = \langle \phi, \phi, X \rangle$, or
   ii) Type2: $\phi_{N_2} = \langle \phi, X, \phi \rangle$

2) X_{N2} may be defined as one type
   Type1: $X_{N_2} = \langle X, \phi, \phi \rangle$

Definition 3.7
a NCS-Type 3, ϕ_{N3}, X_{N3} in X as follows:

1) ϕ_{N3} may be defined as three types:
   i) Type1: $\phi_{N3} = \langle \phi, \phi, X \rangle$, or
   ii) Type2: $\phi_{N3} = \langle \phi, X, \phi \rangle$, or
   iii) Type3: $\phi_{N3} = \langle \phi, X, X \rangle$.

2) X_{N3} may be defined as three types
   i)Type1: $X_{N3} = \langle X, \phi, \phi \rangle$,
   ii)Type2: $X_{N3} = \langle X, X, \phi \rangle$,
   iii)Type3: $X_{N3} = \langle X, \phi, X \rangle$,

Corollary 3.1
In general
1-Every NCS-Type 1, 2, 3 are NCS.
2-Every NCS-Type 1 not to be NCS-Type2, 3.
3-Every NCS-Type 2 not to be NCS-Type1, 3.
4-Every NCS-Type 3 not to be NCS-Type2, 1, 2.
5-Every crisp set be NCS.

The following Venn diagram represents the relation between NCSs

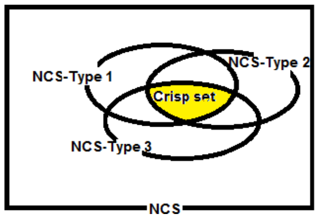

Fig. 2: Venn diagram represents the relation between NCSs

Example 3.2

Let A, B, C, D are NCSs on $X = \{a,b,c,d,e,f\}$, the following types of neutrosophic crisp sets

i) $A = \langle \{a\}, \{b\}, \{c\} \rangle$ be a NCS-Type 1, but not NCS-Type 2 and Type 3

ii) $B = \langle \{a,b\}, \{c,d\}, \{f,e\} \rangle$ be a NCS-Type 1, 2, 3

iii) $C = \langle \{a,b,c,d\}, \{e\}, \{a,b,f\} \rangle$ be a NCS-Type 3 but not NCS-Type 1, 2.

iv) $D = \langle \{a,b,c,d\}, \{a,b,c\}, \{a,b,d,f\} \rangle$ be a NCS but not NCS-Type 1, 2, 3.

The complement for A, B, C, D may be equals

The complement of A

i)Type 1: $A^c = \langle \{b,c,d,e,f\}, \{a,c,d,e,f\}, \{a,b,d,e,f\} \rangle$ be a NCS but not NCS—Type1, 2,3

ii)Type 2: $A^c = \langle \{c\}, \{b\}, \{a\} \rangle$ be a NCS-Type 3 but not NCS—Type1, 2

iii)Type 3: $A^c = \langle \{c\}, \{a,c,d,e,f\}, \{a\} \rangle$ be a NCS-Type 1 but not NCS—Type 2, 3.

The complement of B may be equals

i)Type 1:

$B^c = \langle \{c,d,e,f\}, \{a,b,e,f\}, \{a,b,c,d\} \rangle$ be NCS-Type 3 but not NCS-Type 1, 2.

ii) Type 2: $B^c = \langle \{e,f\}, \{c,d\}, \{a,b\} \rangle$ be NCS-Type 1, 2, 3.

iii)Type 3: $B^c = \langle \{e,f\}, \{a,b,e,f\}, \{a,b\} \rangle$ be NCS-Type 3, but not NCS-Type 1, 2.

The complement of C may be equals

i)Type 1: $C^c = \langle \{e,f\}, \{a,b,c,d,f\}, \{c,d,e\} \rangle$.

ii)Type 2: $C^c = \langle \{a,b,f\}, \{e\}, \{a,b,c,d\} \rangle$,

iii)Type 3:

$$C^c = \langle \{a,b,f\}, \{a,b,c,d\}, \{a,b,c,d\} \rangle,$$

The complement of D may be equals

i)Type 1: $D^c = \langle \{e,f\}, \{d,e,f\}, \{c,e\} \rangle$ be NCS-Type 3 but not NCS-Type 1, 2.

ii)Type 2: $D^c = \langle \{a,b,d,f\}, \{a,b,c\}, \{a,b,c,d\} \rangle$ be NCS-Type 3 but not NCS-Type 1, 2.

iii)Type 3: $D^c = \langle \{a,b,d,f\}, \{d,e,f\}, \{a,b,c,d\} \rangle$ be NCS-Type 3 but not NCS-Type 1, 2.

Definition 3.8

Let X be a non-empty set, $A = \langle A_1, A_2, A_3 \rangle$

1) If A be a NCS-Type1 on X, then the complement of the set A (A^c, for short) maybe defined as one kind of complement Type1: $A^c = \langle A_3, A_2, A_1 \rangle$.

2) If A be a NCS-Type 2 on X, then the complement of the set A (A^c, for short) maybe defined as one kind of complement $A^c = \langle A_3, A_2, A_1 \rangle$.

3)If A be NCS-Type3 on X, then the complement of the set A (A^c, for short) maybe defined as one kind of complement defined as three kinds of complements

(C_1) Type1: $A^c = \langle A^c{}_1, A^c{}_2, A^c{}_3 \rangle$,

(C_2) Type2: $A^c = \langle A_3, A_2, A_1 \rangle$

(C_3) Type3: $A^c = \langle A_3, A^c{}_2, A_1 \rangle$

Example 3.3

Let $X = \{a,b,c,d,e,f\}$, $A = \langle \{a,b,c,d\}, \{e\}, \{f\} \rangle$ be a NCS-Type 2, $B = \langle \{a,b,c\}, \{\phi\}, \{d,e\} \rangle$ be a NCS-Type1., $C = \langle \{a,b\}, \{c,d\}, \{e,f\} \rangle$ NCS-Type 3, then the complement $A = \langle \{a,b,c,d\}, \{e\}, \{f\} \rangle$, $A^c = \langle \{f\}, \{e\}, \{a,b,c,d\} \rangle$ NCS-Type 2, the complement of $B = \langle \{a,b,c\}, \{\phi\}, \{d,e\} \rangle$, $B^c = \langle \{d,e\}, \{\phi\}, \{a,b,c\} \rangle$ NCS-Type1. The complement of $C = \langle \{a,b\}, \{c,d\}, \{e,f\} \rangle$ may be defined as three types:

Type 1: $C^c = \langle \{c,d,e,f\}, \{a,b,e,f\}, \{a,b,c,d\} \rangle$.

Type 2: $C^c = \langle \{e,f\}, \{c,d\}, \{a,b\} \rangle$,

Type 3: $C^c = \langle \{e,f\}, \{a,b,e,f\}, \{a,b\} \rangle$,

Proposition 3.1

Let $\{A_j : j \in J\}$ be arbitrary family of neutrosophic crisp subsets on X, then

1) $\cap A_j$ may be defined two types as :

Type1: $\cap A_j = \left\langle \cap Aj_1, \cap A_{j_2}, \cup A_{j_3} \right\rangle$, or

Type2: $\cap A_j = \left\langle \cap Aj_1, \cup A_{j_2}, \cup A_{j_3} \right\rangle$.

2) $\cup A_j$ may be defined two types as :

Type1: $\cup A_j = \left\langle \cup Aj_1, \cap A_{j_2}, \cap A_{j_3} \right\rangle$ or

Type2: $\cup A_j = \left\langle \cup Aj_1, \cup A_{j_2}, \cap A_{j_3} \right\rangle$.

Definition 3.9

(a) If $B = \langle B_1, B_2, B_3 \rangle$ is a NCS in Y, then the preimage of B under f, denoted by $f^{-1}(B)$, is a NCS in X defined by $f^{-1}(B) = \langle f^{-1}(B_1), f^{-1}(B_2), f^{-1}(B_3) \rangle$.

(b) If $A = \langle A_1, A_2, A_3 \rangle$ is a NCS in X, then the image of A under f, denoted by $f(A)$, is the a NCS in Y defined by $f(A) = \langle f(A_1), f(A_2), f(A_3)^c) \rangle$.

Here we introduce the properties of images and preimages some of which we shall frequently use in the following.

Corollary 3.2

Let A, $\{A_i : i \in J\}$, be a family of NCS in X, and B, $\{B_j : j \in K\}$ NCS in Y, and $f : X \to Y$ a function. Then

(a) $A_1 \subseteq A_2 \Leftrightarrow f(A_1) \subseteq f(A_2)$,
$B_1 \subseteq B_2 \Leftrightarrow f^{-1}(B_1) \subseteq f^{-1}(B_2)$,

(b) $A \subseteq f^{-1}(f(A))$ and if f is injective, then $A = f^{-1}(f(A)\ )$,

(c) $f^{-1}(f(B)) \subseteq B$ and if f is surjective, then $f^{-1}(f(B)\ ) = B$,

(d) $f^{-1}(\cup B_i)\ ) = f^{-1}(B_i)$, $f^{-1}(\cap B_i)\ ) = \cap f^{-1}(B_i)$,

(e) $f(\cup A_{ii})\ = \cup f(A_{ii})$; $f(\cap A_{ii})\ \subseteq \cap f(A_{ii})$; and if f is injective, then $f(\cap A_{ii})\ = \cap f(A_{ii})$;

(f) $f^{-1}(Y_N)\ = X_N$, $f^{-1}(\phi_N)\ = \phi_N$.

(g) $f(\phi_N)\ = \phi_N$, $f(X_N)\ = Y_N$, if f is subjective.

Proof

Obvious

IV. Neutrosophic Crisp Relations

Here we give the definition relation on neutrosophic crisp sets and study of its properties.

Let X, Y and Z be three ordinary nonempty sets

Definition 4.1

Let X and Y are two non-empty crisp sets and NCSS A and B in the form $A = \langle A_1, A_2, A_3 \rangle$ on X, $B = \langle B_1, B_2, B_3 \rangle$ on Y. Then

i) The product of two neutrosophic crisp sets A and B is a neutrosophic crisp set $A \times B$ given by $A \times B = \langle A_1 \times B_1, A_2 \times B_2, A_3 \times B_3 \rangle$ on $X \times Y$.

ii) We will call a neutrosophic crisp relation $R \subseteq A \times B$ on the direct product $X \times Y$.

The collection of all neutrosophic crisp relations on $X \times Y$ is denoted as $NCR(X \times Y)$

Definition 4.2

Let R be a neutrosophic crisp relation on $X \times Y$, then the inverse of R is denoted by R^{-1} where $R \subseteq A \times B$ on $X \times Y$ then $R^{-1} \subseteq B \times A$ on $Y \times X$.

Example 4.1

Let $X = \{a, b, c, d\}$, $A = \langle \{a, b\}, \{c\}, \{d\} \rangle$ and $B = \langle \{a\}, \{c\}, \{d, b\} \rangle$ then the product of two neutrosophic crisp sets given by

$A \times B = \langle \{(a,a), (b,a)\}, \{(c,c)\}, \{(d,d), (d,b)\} \rangle$

and

$B \times A = \langle \{(a,a), (a,b)\}, \{(c,c)\}, \{(d,d), (b,d)\} \rangle$,

and

$R_1 = \langle \{(a,a)\}, \{(c,c)\}, \{(d,d)\} \rangle$, $R_1 \subseteq A \times B$ on $X \times X$,

$R_2 = \langle \{(a,b)\}, \{(c,c)\}, \{(d,d), (b,d)\} \rangle$

$R_2 \subseteq B \times A$ on $X \times X$.

Example 4.2

From the Example 3.1

$R_1^{-1} = \langle \{(a,a)\}, \{(c,c)\}, \{(d,d)\} \rangle \subseteq B \times A$ and

$R_2^{-1} = \langle \{(b,a)\}, \{(c,c)\}, \{(d,d), (d,b)\} \rangle \subseteq B \times A$.

Example 4.3

Let $X = \{a, b, c, d, e, f\}$,

$A = \langle \{a, b, c, d\}, \{e\}, \{f\} \rangle$,

$D = \langle \{a, b\}, \{e, c\}, \{f, d\} \rangle$ be a NCS-Type 2,

$B = \langle \{a,b,c\},\{\phi\},\{d,e\} \rangle$ be a NCS-Type1.

$C = \langle \{a,b\},\{c,d\},\{e,f\} \rangle$ be a NCS-Type 3. Then

$$A \times D = \left\langle \begin{matrix} \{(a,a),(a,b),(b,a),(b,b),(b,b),(c,a),(c,b) \\ ,(d,a),(d,b)\},\{(e,e),(e,c)\},\{(f,f),(f,d)\} \end{matrix} \right\rangle$$

$$D \times C = \left\langle \begin{matrix} \{(a,a),(a,b),(b,a),(b,b)\},\{(e,c),(e,d), \\ (c,c),(c,d)\},\{(f,e),(f,f),(d,e),(d,f)\} \end{matrix} \right\rangle$$

we can construct many types of relations on products. We can define the operations of neutrosophic crisp relation.

Definition 4.4

Let R and S be two neutrosophic crisp relations between X and Y for every $(x,y) \in X \times Y$ and NCSS A and B in the form $A = \langle A_1, A_2, A_3 \rangle$ on X, $B = \langle B_1, B_2, B_3 \rangle$ on Y, Then we can defined the following operations

i) $R \subseteq S$ may be defined as two types

a)Type1: $R \subseteq S \Leftrightarrow A_{1_R} \subseteq B_{1_S}$, $A_{2_R} \subseteq B_{2_S}$, $A_{3R} \supseteq B_{3S}$

b)Type2: $R \subseteq S \Leftrightarrow A_{1_R} \subseteq B_{1_S}$, $A_{2R} \supseteq B_{2S}$, $B_{3S} \subseteq A_{3R}$

ii) $R \cup S$ may be defined as two types

a)Type1:

$R \cup S = \langle A_{1R} \cup B_{1S}, A_{2R} \cup B_{2S}, A_{3R} \cap B_{3S} \rangle$,

b)Type2:

$R \cup S$
$= \langle A_{1R} \cup B_{1S}, A_{2R} \cap B_{2S}, A_{3R} \cap B_{3S} \rangle$.

iii) $R \cap S$ may be defined as two types

a)Type1: $R \cap S$
$= \langle A_{1R} \cap B_{1S}, A_{2R} \cup B_{2S}, A_{3R} \cup B_{3S} \rangle$,

b)Type2:

$R \cap S$
$= \langle A_{1R} \cap B_{1S}, A_{2R} \cap B_{2S}, A_{3R} \cup B_{3S} \rangle$.

Theorem 4.1

Let R, S and Q be three neutrosophic crisp relations between X and Y for every $(x,y) \in X \times Y$, then

i) $R \subseteq S \Rightarrow R^{-1} \subseteq S^{-1}$.

ii) $(R \cup S)^{-1} \Rightarrow R^{-1} \cup S^{-1}$.

iii) $(R \cap S)^{-1} \Rightarrow R^{-1} \cap S^{-1}$.

iv) $(R^{-1})^{-1} = R$.

v) $R \cap (S \cup Q) = (R \cap S) \cup (R \cap Q)$.

vi) $R \cup (S \cap Q) = (R \cup S) \cap (R \cup Q)$.

vii)If $S \subseteq R$, $Q \subseteq R$, then $S \cup Q \subseteq R$

Proof

Clear

Definition 4.5

The neutrosophic crisp relation $I \in NCR(X \times X)$, the neutrosophic crisp relation of identity may be defined as two types

i)Type1: $I = \{< \{A \times A\}, \{A \times A\}, \phi >\}$

ii)Type2: $I = \{< \{A \times A\}, \phi, \phi >\}$

Now we define two composite relations of neutrosophic crisp sets.

Definition 4.6

Let R be a neutrosophic crisp relation in $X \times Y$, and S be a neutrosophic crisp relation in $Y \times Z$. Then the composition of R and S, $R \circ S$ be a neutrosophic crisp relation in $X \times Z$ as a definition may be defined as two types

i)Type1:

$R \circ S \leftrightarrow (R \circ S)(x,z)$
$= \cup\{< \{(A_1 \times B_1)_R \cap (A_2 \times B_2)_S\},$
$\{(A_2 \times B_2)_R \cap (A_2 \times B_2)_S\},$
$\{(A_3 \times B_3)_R \cap (A_3 \times B_3)_S\} >$.

ii)Type2 :

$R \circ S \leftrightarrow (R \circ S)(x,z)$
$= \cap\{< \{(A_1 \times B_1)_R \cup (A_2 \times B_2)_S\},$
$\{(A_2 \times B_2)_R \cup (A_2 \times B_2)_S\},$
$\{(A_3 \times B_3)_R \cup (A_3 \times B_3)_S\} >$.

Example 4.5

Let $X = \{a,b,c,d\}$, $A = \langle \{a,b\},\{c\},\{d\} \rangle$ and $B = \langle \{a\},\{c\},\{d,b\} \rangle$ then the product of two events given by $A \times B = \langle \{(a,a),(b,a)\},\{(c,c)\},\{(d,d),(d,b)\} \rangle$, and $B \times A = \langle \{(a,a),(a,b)\},\{(c,c)\},\{(d,d),(b,d)\} \rangle$, and

$R_1 = \langle \{(a,a)\},\{(c,c)\},\{(d,d)\} \rangle$, $R_1 \subseteq A \times B$ on $X \times X$,

$R_2 = \langle \{(a,b)\},\{(c,c)\},\{(d,d),(b,d)\} \rangle$

$R_2 \subseteq B \times A$ on $X \times X$.

$R_1 \circ R_2 = \cup\langle\{(a,a)\} \cap \{(a,b)\},\{(c,c)\},\{(d,d)\}\rangle$
$= \langle\{\phi\},\{(c,c)\},\{(d,d)\}\rangle$ and
$I_{A1} = \langle\{(a,a).(a,b).(b.a)\},\{(a,a).(a,b).(b,a)\},\{\phi\}\rangle$
$, I_{A2} = \langle\{(a,a).(a,b).(b.a)\},\{\phi\},\{\phi\}\rangle$

Theorem 4.2

Let R be a neutrosophic crisp relation in $X \times Y$, and S be a neutrosophic crisp relation in $Y \times Z$ then $(R \circ S)^{-1} = S^{-1} \circ R^{-1}$.

Proof

Let $R \subseteq A \times B$ on $X \times Y$ then $R^{-1} \subseteq B \times A$, $S \subseteq B \times D$ on $Y \times Z$ then $S^{-1} \subseteq D \times B$, from Definition 3.6 and similarly we can $I_{(R\circ S)^{-1}}(x,z) = I_{S^{-1}}(x,z)$ and $I_{R^{-1}}(x,z)$ then $(R \circ S)^{-1} = S^{-1} \circ R^{-1}$.

V. Conclusion

In our work, we have put forward some new types of neutrosophic crisp sets and neutrosophic crisp continuity relations. Some related properties have been established with example. It 's hoped that our work will enhance this study in neutrosophic set theory.

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Published in *I.J. Information Engineering and Electronic Business*, 2014, 9 p.

New Results of Intuitionistic Fuzzy Soft Set

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Abstract—In this paper, three new operations are introduced on intuitionistic fuzzy soft sets .They are based on concentration, dilatation and normalization of intuitionistic fuzzy sets. Some examples of these operations were given and a few important properties were also studied.

Index Terms—Soft Set, Intuitionistic Fuzzy Soft Set, Concentration, Dilatation, Normalization.

I. INTRODUCTION

The concept of the intuitionistic fuzzy (IFS, for short) was introduced in 1983 by K. Aanassov [1] as an extension of Zadeh's fuzzy set. All operations, defined over fuzzy sets were transformed for the case the IFS case .This concept is capable of capturing the information that includes some degree of hesitation and applicable in various fields of research. For example, in decision making problems, particularly in the case of medical diagnosis, sales analysis, new product marketing, financial services, etc. Atanassov et.al [2,3] have widely applied theory of intuitionistic sets in logic programming, Szmidt and Kacprzyk [4] in group decision making , De et al [5] in medical diagnosis etc. Therefore in various engineering application, intuitionistic fuzzy sets techniques have been more popular than fuzzy sets techniques in recent years. Another important concept that addresses uncertain information is the soft set theory originated by Molodtsov [6]. This concept is free from the parameterization inadequacy syndrome of fuzzy set theory, rough set theory, probability theory. Molodtsov has successfully applied the soft set theory in many different fields such as smoothness of functions, game theory, operations research, Riemann integration, Perron integration, and probability. In recent years, soft set theory has been received much attention since its appearance. There are many papers devoted to fuzzify the concept of soft set theory which leads to a series of mathematical models such as fuzzy soft set [7,8,9,10,11], generalized fuzzy soft set [12,13], possibility fuzzy soft set [14] and so on. Thereafter, P.K.Maji and his coauthor [15] introduced the notion of intuitionistic fuzzy soft set which is based on a combination of the intuitionistic fuzzy sets and soft set models and they studied the properties of intuitionistic fuzzy soft set. Then, a lot of extensions of intuitionistic fuzzy soft have appeared such as generalized intuitionistic fuzzy soft set [16], possibility intuitionistic fuzzy soft set [17] etc.

In this paper our aim is to extend the two operations defined by Wang et al. [18] on intuitionistic fuzzy set to the case of intuitionistic fuzzy soft sets, then we define the concept of normalization of intuitionistic fuzzy soft sets and we study some of their basic properties.

This paper is arranged in the following manner .In section 2, some definitions and notions about soft set, fuzzy soft set, intuitionistic fuzzy soft set and several properties of them are presented. In section 3, we discuss the normalization intuitionistic fuzzy soft sets. In section 4, we conclude the paper.

II. PRELIMINARIES

In this section, some definitions and notions about soft sets and intutionistic fuzzy soft set are given. These will be useful in later sections.

Let U be an initial universe, and E be the set of all possible parameters under consideration with respect to U. The set of all subsets of U, i.e. the power set of U is denoted by P(U) and the set of all intuitionistic fuzzy subsets of U is denoted by IF^U. Let A be a subset of E.

2.1 Definition

A pair (F, A) is called a soft set over U, where F is a mapping given by F: A→ P (U).

In other words, a soft set over U is a parameterized family of subsets of the universe U. For $\varepsilon \in$ A, F (ε) may be considered as the set of ε -approximate elements of the soft set (F, A).

2.2 Definition

Let U be an initial universe set and E be the set of parameters. Let IF^U denote the collection of all intuitionistic fuzzy subsets of U. Let. A ⊆ E pair (F, A) is called an intuitionistic fuzzy soft set over U where F is a mapping given by F: A→ IF^U.

2.3 Defintion

Let F: A→ IF^U then F is a function defined as F (ε) ={ x, $\boldsymbol{\mu_{F(\varepsilon)}(x)}$, $\boldsymbol{\nu_{F(\varepsilon)}(x) : x \in U}$, $\varepsilon \in \boldsymbol{E}\}$ where μ , ν denote the degree of membership and degree of non-membership respectively.

2.4 Definition

For two intuitionistic fuzzy soft sets (F , A) and (G, B) over a common universe U , we say that (F , A) is an intuitionistic fuzzy soft subset of (G, B) if

(1) A ⊆ B and

(2) F (ε) ⊆ G(ε) for all $\varepsilon \in$ A. i.e $\boldsymbol{\mu_{F(\varepsilon)}(x) \leq \mu_{G(\varepsilon)}(x)}$, $\boldsymbol{\nu_{F(\varepsilon)}(x) \geq \nu_{G(\varepsilon)}(x)}$ for all $\varepsilon \in$ E and
We write (F, A) ⊆ (G, B).

2.5 Definition

Two intuitionistic fuzzy soft sets (F, A) and (G, B) over a common universe U are said to be soft equal if (F, A) is a soft subset of (G, B) and (G, B) is a soft subset of (F, A).

2.6 Definition

Let U be an initial universe, E be the set of parameters, and A ⊆ E.

(a) (F, A) is called a null intuitionistic fuzzy soft set (with respect to the parameter set A), denoted by φ_A, if F (a) = φ for all a ∈ A.

(b) (G, A) is called an absolute intuitionistic fuzzy soft set (with respect to the parameter set A), denoted by U_A, if G(e) = U for all e ∈ A.

2.7Definition

Let (F, A) and (G, B) be two IFSSs over the same universe U. Then the union of (F, A) and (G, B) is denoted by ‗(F, A) ∪ (G, B)' and is defined by (F, A) ∪ (G, B) = (H, C), where C=A ∪ B and the truth-membership, falsity-membership of (H, C) are as follows:

$$\boldsymbol{H}(\varepsilon) = \begin{cases} \{(\boldsymbol{\mu_{F(\varepsilon)}(x), \nu_{F(\varepsilon)}(x) : x \in U}\} & \text{, if } \varepsilon \in \text{A} - \text{B}, \\ \{(\boldsymbol{\mu_{G(\varepsilon)}(x), \nu_{G(\varepsilon)}(x) : x \in U}\} & \text{, if } \varepsilon \in \text{B} - \text{A} \\ \left\{\max\left(\boldsymbol{\mu_{F(\varepsilon)}(x), \mu_{G(\varepsilon)}(x)}\right), \min\left(\boldsymbol{\nu_{F(\varepsilon)}(x), \nu_{G(\varepsilon)}(x)}\right) : \boldsymbol{x \in U}\right\}, & \text{if } \varepsilon \in \text{A} \cap \text{B} \end{cases}$$

Where $\boldsymbol{\mu_{H(\varepsilon)}(x)}$ = $\max(\boldsymbol{\mu_{F(\varepsilon)}(x), \mu_{G(\varepsilon)}(x)})$ and $\boldsymbol{\nu_{H(\varepsilon)}(x)}$ = $\min\left(\boldsymbol{\nu_{F(\varepsilon)}(x), \nu_{G(\varepsilon)}(x)}\right)$

2.8 Definition

Let (F, A) and (G, B) be two IFSSs over the same universe U such that A ∩ B≠0. Then the intersection of (F, A) and (G, B) is denoted by ‗(F, A) ∩ (G, B)' and is defined by (F, A) ∩ (G, B) = (K, C),where C =A ∩B and the truth-membership, falsity-membership of (K, C) are related to those of (F, A) and (G, B) by:

$$\boldsymbol{K}(\varepsilon) = \begin{cases} \{(\boldsymbol{\mu_{F(\varepsilon)}(x), \nu_{F(\varepsilon)}(x) : x \in U}\} & \text{, if } \varepsilon \in \text{A} - \text{B}, \\ \{(\boldsymbol{\mu_{G(\varepsilon)}(x), \nu_{G(\varepsilon)}(x) : x \in U}\} & \text{, if } \varepsilon \in \text{B} - \text{A} \\ \left\{\min\left(\boldsymbol{\mu_{F(\varepsilon)}(x), \mu_{G(\varepsilon)}(x)}\right), \max\left(\boldsymbol{\nu_{F(\varepsilon)}(x), \nu_{G(\varepsilon)}(x)}\right) : \boldsymbol{x \in U}\right\}, & \text{if } \varepsilon \in \text{A} \cap \text{B} \end{cases}$$

III. CONCENTRATION OF INTUITIONISTIC FUZZY SOFT SET

3.1 Definition

The concentration of an intuitionistic fuzzy soft set (F, A) of universe U, denoted by **CON** (F, A), and is defined as a unary operation on IF^U:

Con: $IF^U \rightarrow IF^U$

Con (F, A) =
{Con {F(ε) } = {<x, $\boldsymbol{\mu_{F(\varepsilon)}}(x)$, 1- (1 $\boldsymbol{\nu_{(F(\varepsilon))}(x)})^2$ >| ∈ U and ε∈ A}. where

From 0 $_{(\varepsilon)}(x), \nu_{F(\varepsilon)}(x) \leq 1$

and $_{(\varepsilon)}(x) + \nu_{F(\varepsilon)}(x) \leq 1$,

we obtain 0 $_{(\varepsilon)}(x)$ $_{(\varepsilon)}(x)$

1- (1 $\boldsymbol{\nu_{(F(\varepsilon))}(x)})$ $_{(\varepsilon)}(x)$

and **Con** (F, A) ∈ IF^U, i.e **Con** (F, A) ⊆ (F, A) this means that concentration of a intuitionistic fuzzy soft set leads to a reduction of the degrees of membership.

In the following theorem, The operator ―Con ―reveals nice distributive properties with respect to intuitionistic union and intersection.

3.2 Theorem

i. **Con** (F, A) (F, A)

ii. **Con** ((F, A) ∪ (G,B)) = **Con** (F, A) ∪ **Con** (G, B)

iii. **Con** ((F, A) ∩ (G,B)) = **Con** (F, A) ∩ **Con** (G, B)

iv. **Con** ((F, A) ⊗ (G,B))= **Con** (F, A) ⊗ Con (G,B)

v. **Con** (F, A) ⊕ **Con** (G, B) ⊆ **Con** ((F, A) ⊕ (G,B))

vi. (F, A) (G, B) **Con** (F, A) ⊆ Con(G, B)

Proof , we prove only (v) ,i.e

$_{(\varepsilon)}(x) +$ $_{(\varepsilon)}(x)$ - $_{(\varepsilon)}(x)\ \mu^2_{G(\varepsilon)}(x) \leq (\ \mu_{F(\varepsilon)}(x)$ $_{(\varepsilon)}(x)$ $_{(\varepsilon)}(x)$ $_{(\varepsilon)}(x))^2$,

(1- (1 $_{(\varepsilon)}\ (x))^2$). (1- (1 $_{(\varepsilon)}(x))^2$) ≥ 1- (1 $_{(\varepsilon)}(x).\nu_{G(\varepsilon)}(x))^2$ or, putting

a= $_{(\varepsilon)}(x)$, b= $\mu_{G(\varepsilon)}(x)$, c = $\nu_{F(\varepsilon)}(x)$, d = $\nu_{G(\varepsilon)}(x)$

\+ - ($b)^2$,

(1- (1 $c)^2$) . (1- (1 $d)^2$) 1- (1 $d)$

The last inequality follows from 0 ≤ a, b, c, d ≤ 1.

Example

Let U={a, b, c} and E ={ e_1, , , e_4} , A ={ e_1, e_2, e_4} E, B={ e_1, e_2 , e_3} ⊆ E

(F, A) ={ **F(e_1)** ={((a, 0.5, 0.1), (b, 0.1, 0.8), (c, 0.2, 0.5)}, **F(e_2)** ={((a, 0.7, 0.1), (b, 0, 0.8), (c, 0.3, 0.5)}, **F(e_4)** ={((a, 0.6, 0.3), (b, 0.1, 0.7), (c, 0.9, 0.1)}}

(G, B) ={ **G(e_1)** ={((a, 0.2, 0.6), (b, 0.7, 0.1), (c, 0.8, 0.1)}, **G(e_2)** ={((a, 0.4, 0.1), (b, 0.5, 0.3), (c, 0.4, 0.5)}, **G(e_3)** ={((a, 0, 0.6), (b, 0, 0.8), (c, 0.1, 0.5)}}

Con (F, A)={ **con(F(e_1))** ={ (a, 0.25, 0.19), (b, 0.01, 0.96), (c, 0.04, 0.75)}, **con(F(e_2)**)={ (a, 0.49, 0.19), (b, 0, 0.96), (c, 0.09, 0.75) }, **con(F(e_4)**)={ (a, 0.36, 0.51), (b, 0.01, 0.91), (c, 0.81, 0.19) }

Con (G, B)={ **con(G(**e_1))={ (a, 0.04, 0.84), (b, 0.49, 0.19), (c, 0.64, 0.75)},

con(G(e_2))={ (a, 0.16, 0.19), (b, 0.25, 0.51), (c, 0.16, 0.51) }, **con(G(e_3)**)={ (a, 0, 0.84), (b, 0, 0.96), (c, 0.01, 0.75) }

(F, A) ∩ (G, B) = (H, C) = {**H (e_1)** ={(a, 0.2, 0.6), (b, 0.1, 0.8), (c, 0.2, 0. 5)}, **H (e_2)** ={(a, 0.4, 0.1), (b, 0, 0.8), (c, 0.3, 0. 5)}}

Con ((F, A) ∩ (G,B))= {**con H(e_1)** ={(a, 0.04, 0.84), (b, 0.01, 0.96), (c, 0.04, 0. 75)}, **con H(e_2)** ={(a, 0.16, 0.19), (b, 0, 0.96), (c, 0.09, 0. 75)}}

Con (F, A) ∩ **Con** (G, B) =(K,C) ={**con K(e_1)** ={(a, 0.04, 0.84), (b, 0.01, 0.96), (c, 0.04, 0. 75)}, **conK(e_2)** ={(a, 0.16, 0.19), (b, 0, 0.96), (c, 0.09, 0. 75)}}.

Then

Con ((F, A) ∩ (G,B)) = **Con** (F, A) ∩ **Con** (G, B)

IV. DILATATION OF INTUITIONISTIC FUZZY SOFT SET

4.1 Definition

The dilatation of an intuitionistic fuzzy soft set (F, A) of universe U, denoted by **DIL** (F, A), and is defined as a unary operation on IF^U:

$$\mathbf{DIL}\text{: } IF^U \rightarrow IF^U$$

(F, A)= {<x, $_{(\varepsilon)}(x), \nu_{F(\varepsilon)}(x)$ >|$\boldsymbol{x}$ U and A}.

DIL(F, A) ={ **DIL** {F(ε) } =

{<x, $\boldsymbol{\mu}^{\frac{1}{2}}_{\boldsymbol{F}(\varepsilon)}(x)$, 1- (1 $\boldsymbol{\nu}_{(\boldsymbol{F}(\varepsilon))}(\boldsymbol{x}))^{\frac{1}{2}}$ >|$\boldsymbol{x}$ U and A}.

where
From 0 ≤ μ $_{(\varepsilon)}(x), \nu_{F(\varepsilon)}(x) \leq 1$,

and $_{(\varepsilon)}(x)$ +$\nu_{F(\varepsilon)}(x) \leq 1$,

we obtain 0 $_{(\varepsilon)}(x)$ $^{-}_{(\varepsilon)}(x)$

0 ≤ (1 $\boldsymbol{\nu}_{(\boldsymbol{F}(\varepsilon))}(\boldsymbol{x}))^{-}$ $_{(\varepsilon)}(x)$

and **DIL**(F, A) ∈ IF^U, i.e (F, A) ⊆ **DIL**(F, A) this means that dilatation of an intuitionistic fuzzy soft set leads to an increase of the degrees of membership.

4.2 Theorem

i. (F, A) **DIL**(F, A)

ii. **DIL** ((F, A) ∪ (G, B)) = **DIL** (F, A) ∪ **DIL** (G, B)

iii. **DIL** ((F, A) ∩ (G, B)) = **DIL**(F, A) ∩ **DIL** (G, B)

iv. **DIL**((F, A) ⊗ (G, B))= **DIL** (F, A) ⊗ **DIL** (G,B)

v. **DIL** (F, A) ⊕ **DIL** (G, B) ⊆ **DIL** ((F, A) ⊕ (G,B))

vi. **CON** (**DIL** (F, A)) = (F,A)

vii. **DIL** (**CON** (F, A) = (F,A)

viii. (F, A) (G, B) ⇒ **DIL** (F, A) ⊆ DIL(G, B)

Proof .we prove only (v), i.e

$\bar{\ }_{(\varepsilon)}(x)$ + $\bar{\ }_{G(\varepsilon)}(x)$ - $\bar{\ }_{F(\varepsilon)}(x)$ $\bar{\ }_{G(\varepsilon)}(x) \geq (\mu_{F(\varepsilon)}(x)$ $_{(\varepsilon)}(x)$ $_{(\varepsilon)}(x)$ $_{(\varepsilon)}(x))^{\bar{2}}$,

(1- (1 $_{(\varepsilon)}(x))^{\bar{2}}$). (1- $(1 - \nu_{G(\varepsilon)}(x))^{\bar{2}}$) ≤ 1- (1 ν $_{(\varepsilon)}(x).\nu_{G(\varepsilon)}(x))^{\bar{2}}$ or, putting

a= $_{(\varepsilon)}(x)$, b= $\mu_{G(\varepsilon)}(x)$, c = $\nu_{F(\varepsilon)}(x)$, d = $\nu_{G(\varepsilon)}(x)$

$^{-}$ + $b^{\bar{2}}$ - $^{-}$ $^{-}$ ($b)^{\bar{2}}$,

(1- (1 $c)^{\bar{2}}$). (1- (1 $d)^{\bar{2}}$) ≤ 1- (1 $d)^{\bar{2}}$, or equivalently : a+ b – a b ≤ 1 ,$\sqrt{1}$ 1.

The last inequality follows from 0 ≤ a, b,c,d ≤ 1.

Example

Let U={a, b, c} and E ={ e_1, , , e_4}, A ={ e_1, e_2, e_4} E, B={ e_1, e_2 , e_3} ⊆ E

(F, A) ={ $\mathbf{F(e_1)}$ ={((a, 0.5, 0.1), (b, 0.1, 0.8), (c, 0.2, 0.5)}, $\mathbf{F(e_2)}$ ={((a, 0.7, 0.1), (b, 0, 0.8), (c, 0.3, 0.5)}, $\mathbf{F(e_4)}$ ={((a, 0.6, 0.3), (b, 0.1, 0.7), (c, 0.9, 0.1)}} and

(G, B) ={ $\mathbf{G(e_1)}$ ={((a, 0.2, 0.6), (b, 0.7, 0.1), (c, 0.8, 0.1)}, $\mathbf{G(e_2)}$ ={((a, 0.4, 0.1), (b, 0.5, 0.3), (c, 0.4, 0.5)}, $\mathbf{G(e_3)}$ ={((a, 0, 0.6), (b, 0, 0.8), (c, 0.1, 0.5)}}

DIL(F, A)={ **DIL**(F(e_1))={ (a, 0.70, 0.05), (b, 0.31, 0.55), (c, 0.44, 0.29)}, DIL (F(e_2))={ (a, 0.83, 0.05), (b, 0, 0.55), (c, 0.54, 0.29) }, **DIL**(F(e_4))={ (a, 0.77, 0.05), (b, 0.31, 0.45), (c, 0.94, 0.05) } and

DIL (G, B) = {**DIL** (G (e_1)) = {(a, 0.44, 0.36), (b, 0.83, 0.05), (c, 0.89, 0.05)},

DIL(G(e_2)) ={ (a, 0.63, 0.05), (b, 0.70, 0.05), (c, 0.63, 0.29) }, **DIL**(G(e_3))={ (a, 0, 0.36), (b, 0, 0.55), (c, 0.31, 0.29) }

(F, A) ∩ (G, B) = (H, C) = {$\mathbf{H\ (e_1)}$ = {(a, 0.2, 0.6), (b, 0.1, 0.8), (c, 0.2, 0. 5)}, $\mathbf{H\ (e_2)}$ = {(a, 0.4, 0.1), (b, 0, 0.8), (c, 0.3, 0. 5)}}

DIL ((F, A) ∩ (G,B))= {$\mathbf{DILH(e_1)}$ ={(a, 0.44, 0.36), (b, 0.31, 0.55), (c, 0.44, 0. 29)}, $\mathbf{DILH(e_2)}$ ={(a, 0.63, 0.05), (b, 0, 0.55), (c, 0.54, 0. 29)}}

DIL (F, A) ∩ **DIL** (G, B) =(K,C) ={ **DIL** $\mathbf{K(e_1)}$ ={(a, 0.04, 0.84), (b, 0.01, 0.96), (c, 0.04, 0. 75)}, **DIL** $\mathbf{K(e_2)}$ ={(a, 0.16, 0.19), (b, 0, 0.96), (c, 0.09, 0. 75)}}

Then

DIL ((F, A) ∩ (G,B)) = **DIL**(F, A) ∩ **DIL** (G, B)

V. NORMALIZATION OF INTUITIONISTIC FUZZY SOFT SET

In this section, we shall introduce the normalization operation on intuitionistic fuzzy soft set.

5.1 Definition:

The normalization of an intuitionistic fuzzy soft set (F, A) of universe U ,denoted by

NORM (F, A) is defined as:

NORM (F, A) ={ Norm {F(ε)} = {<x, $_{(F(\varepsilon))}(x)$, $_{(F(\varepsilon))}(x)$, > |x ∈ U and A}. where

$_{(F(\varepsilon))}(x) = \frac{_{(\varepsilon)}(x)}{\sup(\mu_{F(\varepsilon)}(x))}$ and $_{(F(\varepsilon))}(x) = \frac{_{(\varepsilon)}(x) - nf(\nu_{F(\varepsilon)}(x))}{f(\nu_{F(\varepsilon)}(x))}$ and

Inf $(\nu_{F(\varepsilon)}(x)) \neq 0$**.**

Example. Let there are five objects as the universal set where U = {x_1, x_2, x_3, x_4, x_5} and the set of parameters as E = {beautiful, moderate, wooden, muddy, cheap, costly} and Let A = {beautiful, moderate, wooden}. Let the attractiveness of the objects represented by the intuitionistic fuzzy soft sets (F, A) is given as

F(beautiful)={$x_{1/(.6,.4)}$, $x_{2/(.7,\ .3)}$, $x_{3/(.5,\ .5)}$, $x_{4/(.8,\ .2)}$, $x_{5/(.9,\ .1)}$},

F(moderate)={$x_{1/(.3,\ .7)}$, $x_{2/(6,\ .4)}$, $x_{3/(.8,\ .2)}$, $x_{4/(.3,\ .7)}$, $x_{5/(1,\ .9)}$} and

F(wooden) ={ $x_{1/(.4,\ .6)}$, $x_{2/(.6,\ .4)}$, $x_{3/(.5,\ .5)}$, $x_{4/(.2,\ .8)}$, $x_{5/(.3,\ .7,)}$}.

Then,

sup($\mu_{F(}$ $_{)}(x)$) = 0.9, inf($\nu_{F(}$ $_{)}(x)$ = 0.1. We have

$rm\ (F$(be $_{))}(x_1)$ = — = 0.66,

$rm\ (F$(be $_{))}(x_2)$ = — = 0.77,

$rm\ (F$(be $_{))}(x_3)$ = — = 0.55,

$rm\ (F$(bea $_{))}(x_4)$ = — =0.88,

$rm\ (F$(bea $_{))}(x_5)$ = — = 1 and

$rm\ (F$(bea $_{))}(x_1)$ = — = 0.33,

$rm\ (F$(bea $_{))}(x_2)$ = — = 0.22,

$rm\,(F(\text{be}\quad))(x_3)$ = — = 0.44

$rm\,(F(\text{be}\quad))(x_4)$ = — = 0.11,

$rm\,(F(\text{be}\quad))(x_5)$ = — = 0.

Norm(F(beautiful)) ={ $x_{1/(.66,.33)}$, $x_{2/(.77,\ .22)}$, $x_{3/(.55,\ .44)}$, $x_{4/(.88,\ .11)}$, $x_{5/(1,\ 0)}$ }.

sup($\mu_{F(\quad)}(x)$) = 0.8, f($\nu_{F(\quad)}(x)$ = 0.2. We have

$rm\,(F(\text{mo}\quad))(x_1)$ = — = 0.375,

$rm\,(F(\text{mo}\quad))(x_2)$ = — = 0.75,

$rm\,(F(\text{mo}\quad))(x_3)$ = — = 1,

$rm\,(F(\text{mo}\quad))(x_4)$ = — =0.375,

$rm\,(F(\text{mo}\quad))(x_5)$ = — = 0.125 And

$rm\,(F(\text{mo}\quad))(x_1)$ = — = 0.625,

$rm\,(F(\text{mo}\quad))(x_2)$ = — = 0.25,

$rm\,(F(\text{mo}\quad))(x_3)$ = — = 0,

$rm\,(F(\text{mo}\quad))(x_4)$ = — = 0.625,

$rm\,(F(\text{mo}\quad))(x_5)$ = — = 0.875.

Norm(F(mod ate)) ={ $x_{1/(.375,.625)}$, $x_{2/(.75,\ .25)}$, $x_{3/(1,\ 0)}$, $x_{4/(.375,\ .625)}$, $x_{5/(0.125,\ 0.875)}$ }.

sup($\mu_{F(\quad)}(x)$) = 0.6, f($\nu_{F(\quad)}(x)$ = 0.4. We have

$rm\,(F(\text{w}\quad))(x_1)$ = — = 0.66,

$rm\,(F(\text{w}\quad))(x_2)$ = — = 1,

$rm\,(F(\quad))(x_3)$ = — = 0.83,

$rm\,(F(\text{w}\quad))(x_4)$ = — = 0.34,

$rm\,(F(\quad))(x_5)$ = — = 0.5 and

$rm\,(F(\text{woo}\quad))(x_1)$ = — = 0.34,

$rm\,(F(\text{woo}\quad))(x_2)$ = — = 0,

$rm\,(F(\text{woo}\quad))(x_3)$ = — = 0.17,

$rm\,(F(\text{woo}\quad))(x_4)$ = — = 0.66,

$rm\,(F(\text{w}\quad))(x_5)$ = — = 0.5.

Norm(F(n)) ={ $x_{1/(.66,.34)}$, $x_{2/(1,\ .0)}$, $x_{3/(0.83,\ 0.17)}$, $x_{4/(.34,\ .66)}$, $x_{5/(0.5,\ 0.\ 5)}$ }.

Then, **Norm** (F, A) = {Norm F (be utiful), Norm F(moderate), Norm F(n)}

Norm (F,A)={ F(be utiful) ={ $x_{1/(.66,.33)}$, $x_{2/(.77,\ .22)}$, $x_{3/(.55,\ .44)}$, $x_{4/(.88,\ .11)}$, $x_{5/(1,\ 0)}$ }, F(moderate)={ $x_{1/(.375,.625)}$, $x_{2/(.75,\ .25)}$, $x_{3/(1,\ 0)}$, $x_{4/(.375,\ .625)}$, $x_{5/(0.125,\ 0.875)}$ }, F(wooden) ={ $x_{1/(.66,.34)}$, $x_{2/(1,\ .0)}$, $x_{3/(0.83,\ 0.17)}$, $x_{4/(.34,\ .66)}$, $x_{5/(0.5,\ 0.\ 5)}$ }}

Clearly, $\mu_{Norm\,(F(\varepsilon))}(x) + \nu_{\quad(F(\varepsilon))}(x)$ = 1, for i = 1, 2, 3, 4, 5 which satisfies the property of intuitionistic fuzzy soft set. Therefore, **Norm** (F, A) is an intuitionistic fuzzy soft set.

VI. Conclusion

In this paper, we have extended the two operations of intuitionistic fuzzy set introduced by Wang et al.[18] to the case of intuitionistic fuzzy soft sets. Then we have introduced the concept of normalization of intuitionistic fuzzy soft sets and studied several properties of these operations.

Acknowledgments

The authors are highly grateful to the referees for their valuable comments and suggestions for improving the paper and finally to God who made all the things possible.

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Published in I.J. Information Engineering and Electronic Business, No. 2, 2014, pp. 47-52, DOI: 10.5815/ijieeb.2014.02.06, 6 p.

On Fuzzy Soft Matrix Based on Reference Function

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***Abstract*—**In this paper we study fuzzy soft matrix based on reference function.Firstly, we define some new operations such as fuzzy soft complement matrix and trace of fuzzy soft matrix based on reference function.Then, we introduced some related properties, and some examples are given. Lastly, we define a new fuzzy soft matrix decision method based on reference function.

***Index Terms*—**Soft set, fuzzy soft set, fuzzy soft set based on reference function, fuzzy soft matrix based on reference function.

I. INTRODUCTION

Fuzzy set theory was proposed by LotfiA.Zadeh[1] in 1965,where each element (real valued) [0, 1] had a degree of membership defined on the universe of discourse X, the theory has been found extensive application in various field to handle uncertainty. Therefore,several researches were conducted on the generalization on the notions of fuzzy sets such as intuitionistic fuzzy set proposed by Atanassov[2,3], interval valued fuzzy set[5]. In the literature we found many well –known theories to describe uncertainty: rough set theory[6]..etc, but all of these theories have their inherit difficulties as pointed by Molodtsov in his pioneer work[7].The concept introduced by Molodtsov is called "soft set theory" which is set valued mapping. This new mathematical model is free from the difficulties mentioned above.Since its introduction, the concept of soft set has gained considerable attention and this concept has resulted in a series of work [8, 9,10,11,12, 13, 14].

Also as we know, matrices play an important role in science and technology. However, the classical matrix theory sometimes fails to solve the problems involving uncertainties,occurring in an imprecise environment. In [4] Thomason, introduced the fuzzy matrices to represent fuzzy relation in a system based on fuzzy set theory and discussed about the convergence of powers of fuzzy matrix. In [15,16,17],some important results on determinant of a square fuzzy matrices are discussed .Also,Ragab et al. [18,19] presented some properties of the min-max composition of fuzzy matrices. Later on, several studies and some applications of fuzzy matrices are defined in [20,21].

In 2010,Cagmanet al [13] defined soft matrix which is representation of soft set, to make operations in theoretical studies in soft set more functional. This representation has several advantages, it's easy to store and manipulate matrices and hence the soft sets represented by them in a computer.

Recently severalresearch have been studied the connection between soft set and soft matrices [13,14,22]. Later,Maji et al [9] introduced the theory of fuzzy soft set and applied it to decision making problem. In 2011, Yang and C.Ji[22],defined fuzzy soft matrix (FSM) which is very useful in representing and computing the data involving fuzzy soft sets.

The concept of fuzzy set based on reference function was first introduced by Baruah[23,24,25] in the following manner - According to him, to define a fuzzy set, two functions namely fuzzy membership function and fuzzy reference function are necessary. Fuzzy membership value is the difference between fuzzy membership function and reference function. Fuzzy membership function and fuzzy membership value are two different things. In [26, 27] M.Dhar applied this concept to fuzzy square matrix and developed some interesting properties as determinant, trace and so on. Thereafter, in [28], T.J.Neog, D. K.Sutwere extended this new concept to soft set theory, introducing a new concept called "fuzzy soft set based on fuzzy reference function". Recently,Neog. T.J, Sut D.K,M.Bora[29] combinedfuzzy soft set based on reference function with soft matrices. The paper unfolds as follows. The next section briefly introduces some definitions related tosoft set,fuzzy soft set, and fuzzy soft setbased on reference function. Section 3 presents fuzzy soft complement matrix based on reference function. Section 4presentstrace of fuzzy soft matrix based on reference function..Section5presentsnew fuzzy soft matrix theory in decision making.Conclusions appear in the last section.

II. PRELIMINARIES

In this section first we review some concepts and definitions of soft set,fuzzy soft set, and fuzzy soft set based on reference functionfrom [9,12,13,29], which will be needed in the sequel.

Remark:

For the sake of simplicity we adopt the following notation of fuzzy soft set based on reference function defined in our way as: Fuzzy soft set based on reference =$(F, A)_{rf}$

To make the difference between the notation (F, A) defined for classical soft set or its variants as fuzzy soft set.

2.1.Definition (Soft Set [13])

Suppose that U is an initial universe set and E is a set of parameters, let P(U) denotes the power set of U.A pair(F,E) is called a soft set over U where F is a mapping given by F: E→P(U).Clearly, a soft set is a mapping from parameters to P(U),and it is not a set, but a parameterized family of subsets of the universe.

2.2. Example.

Suppose that U={s1,s2,s3,s4} is a set of students and E={e1,e2,e3} is a set of parameters, which stand for result, conduct and sports performances respectively. Consider the mapping from parameters set E to the set of all subsets of power set U.Then soft set (F,E) describes the character of the students with respect to the given parameters, for finding the best student of an academic year.

(F, E) = {{result = s1, s3, s4} {conduct = s1,s2 } {sports performances =s2,s3,s4 }}

*2.3. Definition (FuzzySoft Set [**9**, 1**2**])*

Let U be an initial universe set and E be the set of parameters. Let A⊆E .A pair (F,A) is called fuzzy soft set over U where F is a mapping given by F: $A \to F^u$,whereF^udenotes the collection of all fuzzy subsets of U.

2.4.Example.

Consider the example2.2,in soft set(F,E),if s1 is medium in studies, we cannot expressed with only the two numbers 0 and 1,we can characterize it by a membership function instead of the crisp number 0 and 1,which associates with each element a real number in the interval [0,1].Then fuzzy soft set can describe as

(F, A)={F(e1) = {(s1,0.9), (s2,0.3), (s3,0.8), (s4,0.9)}, F(e2) = {(s1,0.8), (s2,0.9), (s3,0.4), (s4,0.3)}},where A={ e1,e2}.

In the following, Neog et al. [29] showed by an example that this definition sometimes gives degenerate cases and revised the above definition as follows:

2.5 .Definition [29]

Let A (μ_1,μ_2)={x,μ_1(x) , μ_2(x) ; x ∈ U } and B (μ_3,μ_4) ={x, μ_3(x), μ_4(x) ; x ∈ U } be two fuzzy sets defined over the same universe U.

Then the operations intersection and union are defined asA (μ_1 , μ_2) ∩ B (μ_3 , μ_4)= {x, min(μ_1(x),μ_3(x)) ,max(μ_2(x) ,μ_4(x)) ; x ∈ U } and A (μ_1 , μ_2) ∪ B (μ_3 , μ_4)= {x, max(μ_1(x) ,μ_3(x)) ,min(μ_2(x),μ_4(x)) ; x ∈ U }

2.6.Definition [29]

Let A (μ_1,μ_2)={x, $\mu_1(x)$, $\mu_2(x)$; x ∈ U } and B (μ_3,μ_4)={x, $\mu_3(x)$, $\mu_4(x)$; x ∈ U } be two fuzzy sets defined over the same universe U.To avoid degenerate cases we assume that min($\mu_1(x)$, $\mu_3(x)$) ≥ max($\mu_2(x)$,$\mu_4(x)$) for all x∈ U.

Then the operations intersection and union are defined as A (μ_1 , μ_2) ∩ B (μ_3 , μ_4)= {x, min(μ_1(x),μ_3(x)) ,max(μ_2(x),μ_4(x)) ; x ∈ U } and A (μ_1 , μ_2) ∪ B (μ_3 , μ_4)= {x, max($\mu_1(x)$,$\mu_3(x)$) ,min($\mu_2(x)$,$\mu_4(x)$) ; x ∈ U }

2.7. Definition[29]

For usual fuzzy setsA (μ, 0)={x, $\mu(x)$,0 ; x ∈ U }and B (1, μ)={x, 1 , $\mu(x)$; x ∈ U } defined over the same universe U, we have A (μ , 0) ∩ B (1, μ)= {x, min((x), 1) ,max(0 ,$\mu(x)$) ; x ∈ U }= {x, $\mu(x)$, $\mu(x)$; x ∈ U }, which is nothing but the null setφ and A (μ, 0) ∪ B (1, μ)= {x, max($\mu(x)$, 1) ,min(0, $\mu(x)$) ; x ∈ U } = {x,1, 0 ; x ∈ U }, which is nothing but the universal set U.

This means if we define a fuzzy set(A (μ, 0))^c ={x, 1, $\mu(x)$; x ∈ U } it is nothing but the complement ofA (μ, 0)={x, $\mu(x)$,0 ; x ∈ U }.

2.8. Definition[29]

Let A (μ_1,μ_2)={x, $\mu_1(x)$, $\mu_2(x)$; x ∈ U } and B (μ_3,μ_4)={x, $\mu_3(x)$, $\mu_4(x)$; x ∈ U } be two fuzzy sets defined over the same universe U.The fuzzy setA $(\mu_1,\mu_2)$is a subset of the fuzzy set B (μ_3,μ_4)if for all x ∈ U , $\mu_1(x) \le \mu_3(x)$ and $\mu_4(x) \le \mu_2(x)$.

Two fuzzy setsC={x, $\mu_C(x)$x ∈ U }and D={x, $\mu_D(x)$; x ∈ U } in the usual definition would be expressed asC(μ_C, 0)={x, $\mu_C(x)$, 0; x ∈ U }and D(μ_D, 0)={x, $\mu_D(x)$,0; x ∈ U }

Accordingly, we have C(μ_C, 0) ⊆ D(μ_D, 0) if for all x ∈ U, $\mu_C(x) \le \mu_D(x)$, which can be obtained by putting$\mu_2(x) = \mu_4(x)$=0 in the new definition.

2.9 .Defintion [29] (Fuzzy soft matrices (FSMs) based on reference function)

Let U be an initial universe, E be the set of parameters and A ⊆ E. Let (f_A , E) be fuzzy soft set (FS) over U. Then a subset of U ×E is uniquely defined by R_A = {(u, e); e∈ A, u∈ $f_A(e)$} which is called a relation form of (f_A , E).

2.10. Example

Assume that U ={u_1 ,u_2 ,u_3 ,u_4 } is a universal setand E ={ e_1 , e ,e_3 ,e_4 ,e_5 } be the set of parameters and A={e_1 ,e_2 ,e_3 } ⊆ E
and

$f_A(e_1)$ ={ $u_1/(0.7,0)$, $u_2/(0.1,0)$, $u_3/(0.2,0)$, $u_4/(0.6,0)$ }
$f_A(e_2)$ ={ $u_1/(0.8,0)$, $u_2/(0.6,0)$, $u_3/(0.1,0)$, $u_4/(0.5,0)$ }
$f_A(e_3)$ ={ $u_1/(0.1,0)$, $u_2/(0.2,0)$, $u_3/(0.7,0)$, $u_4/(0.3,0)$ }

Then the fuzzy soft set (f_A , E) is a parameterized family { $f_A(e_1)$, $f_A(e_2)$,$f_A(e_3)$ } of all fuzzy soft sets over U. Then the relation form of (f_A , E) is written as

TABLE 1.The relation form of (f_A , E)

R_A	e_1	e_2	e_3	e_4
u_1	(0.7, 0)	(0.8, 0)	(0.1, 0)	(0, 0)
u_2	(0.1, 0)	(0.6, 0)	(0.2, 0)	(0, 0)
u_3	(0.2, 0)	(0.1, 0)	(0.7, 0)	(0, 0)
u_4	(0.6, 0)	(0.5, 0)	(0.3, 0)	(0, 0)

Hence,the fuzzy soft matrix representing this fuzzy soft set would be represented as

$$A = \begin{bmatrix} (0.7,0)(0.8,0)(0.1,0)(0,0) \\ (0.1,0)(0.6,0)(0.2,0)(0,0) \\ (0.2,0)(0.1,0)(0.7,0)(0,0) \\ (0.6,0)(0.5,0)(0.3,0)(0,0) \end{bmatrix}$$

2.11. Definition[29]

We define the membership value matrix corresponding to the matrix A as MV(A) =[$\delta_{ji}(c_i)$] Where $\delta_{(A)ij}$ = $\mu_{j1}(c_i)$- $\mu_{j2}(c_i)$ i= 1,2,3,…,m and j =1,2,3…..,n ,where $\mu_{j1}(c_i)$ and $\mu_{j2}(c_i)$ represent the fuzzy membership function and fuzzy reference function respectively ofc_i in the fuzzy set F(e_j).

2.12. Definition [29]

Let the fuzzy soft matrices corresponding to the fuzzy soft sets (F,E), and (G,E) beA=[a_{ij}] ∈ $FSM_{m\times n}$, B=[b_{ij}]where a_{ij} = ($\mu_{j1}(c_i)$, $\mu_{j2}(c_i)$) and b_{ij} =($\chi_{j1}(c_i)$,$\chi_{j2}(c_i)$),i =1,2,3,…, m ;j =1,2,3,…,n ;Then Aand Bare called fuzzy soft equal matrices denoted byA=B, if $\mu_{j1}(c_i) = \chi_{j1}(c_i)$ and $\mu_{j1}(c_i) = \chi_{j1}(c_i)$ for all i,j.

In [13], the 'addition (+)' operation between two fuzzy soft matrices is defined as follows

2.13. Definition [29]

Let U={$c_1,c_2,c_3,\dots,c_m$} be the universal set and Ebe the set of parameters given by E={$e_1,c_2,e_3,\dots,e_n$}.Let the set of all m× n fuzzy soft matrices over U be $FSM_{m\times n}$.

Let A , B ∈ $FSM_{m\times n}$,where A= $[a_{ij}]_{m\times n}$, a_{ij} = ($\mu_{j1}(c_i)$, $\mu_{j2}(c_i)$) and B= $[b_{ij}]_{m\times n}$, b_{ij} =($\chi_{j1}(c_i)$,$\chi_{j2}(c_i)$).To avoid degenerate cases we assume that**min** ($\mu_{j1}(c_i)$,$\chi_{j1}(c_i)$) ≥**max** ($\mu_{j2}(c_i)$,$\chi_{j2}(c_i)$) for all i and j .The operation of 'addition (+)' between A and B is defined as **A+ B=C ,where** C= $[c_{ij}]_{m\times n}$,c_{ij} =(max ($\mu_{j1}(c_i)$,$\chi_{j1}(c_i)$), min ($\mu_{j2}(c_i)$, $\chi_{j2}(c_i)$))

2.14. Example

Let U={c_1,c_2,c_3,c_4 } be the universal set and Ebe the set of parameters given by E={e_1,e_2,e_3 }
We consider the fuzzy soft sets based on reference function.

(F,E)={F(e_1)={(c_1,0.3, 0)),(c_2,0.5, 0), (c_3, 0.6, 0), (c_4,0.5,0)},F(e_2)={(c_1,0.7, 0)),(c_2,0.9, 0), (c_3, 0.7, 0), (c_4,0.8,0)},F(e_3)={(c_1,0.6, 0)),(c_2,0.7, 0), (c_3, 0.7, 0), (c_4,0.3,0)}}.

(G,E)={G(e_1)={(c_1,0.8, 0)),(c_2,0.7, 0), (c_3, 0.5, 0), (c_4,0.4,0)}, G(e_2)={(c_1,0.9, 0)),(c_2,0.9, 0), (c_3, 0.8, 0), (c_4,0.7,0)},G(e_3)={(c_1,0.5, 0)),(c_2,0.9, 0), (c_3, 0.6, 0), (c_4,0.8,0)}}.

The fuzzy soft matrices based on reference function representing these two fuzzy soft sets are respectively

$$A = \begin{bmatrix} (0.3,0)(0.7,0)(0.6,0) \\ (0.5,0)(0.9,0)(0.7,0) \\ (0.6,0)(0.7,0)(0.7,0) \\ (0.5,0)(0.8,0)(0.3,0) \end{bmatrix}, B = \begin{bmatrix} (0.8,0)(0.9,0)(0.5,0) \\ (0.7,0)(0.9,0)(0.9,0) \\ (0.5,0)(0.8,0)(0.6,0) \\ (0.4,0)(0.7,0)(0.8,0) \end{bmatrix}$$

$$\text{Here}A+B = \begin{bmatrix} (0.8,0)(0.9,0)(0.6,0) \\ (0.7,0)(0.9,0)(0.9,0) \\ (0.6,0)(0.8,0)(0.7,0) \\ (0.5,0)(0.8,0)(0.8,0) \end{bmatrix}$$

III. FUZZY SOFT COMPLEMENT MATRIX BASED ON REFERENCE FUNCTION

In this section ,westart by introducing the notion of the fuzzy soft complement matrix based on reference function,and we prove some formal properties.

3.1. Definition

Let A= $[(a_{ij},0)]_{m\times n}$ ∈ $FSM_{m\times n}$ according to the definition in [26], then A^c is calledfuzzy soft complement matrix if $A^c = [(1, a_{ij})]_{m\times n}$ for all a_{ij} ∈ [0 , 1].

3.2 .Example

Let A =$\begin{bmatrix} (0.7,0)(0.8,0) \\ (0.1,0)(0.6,0) \end{bmatrix}$ be fuzzy soft matrix based on reference function, then the complement of this matrix is $A^c = \begin{bmatrix} (1,0.7)(1,0.8) \\ (1,0.1)(1,0.6) \end{bmatrix}$.

3.3. Proposition

Let A, B be two fuzzy soft matrix based on fuzzy reference function .Then

(i)$(A^c)^T = (A^T)^c$ (1)

(ii) $(A^c + B^c)^T = (A^T)^c + (B^T)^c$ (2)

Proof:

To show (i)

$(A^c)^T = (A^T)^c$

We have, let A $\in FSM_{m\times n}$, then

$$A = [(\mu_{j1}(c_i), \mu_{j2}(c_i)]$$
$$A^c = [1, \mu_{j1}(c_i)]$$
$$(A^c)^T = [1, \mu_{i1}(c_j)]$$
$$\text{For } A^T = [(\mu_{i1}(c_j), \mu_{i2}(c_j)],$$

we have

$$(A^T)^c = [1, \mu_{i1}(c_j)]$$
$$\text{Hence } (A^c)^T = (A^T)^c$$

The proof of (ii) follows similar lines as above.

3.4. Example

$$\text{Let } A=\begin{bmatrix}(\mathbf{0.2,0})(\mathbf{0.3,0})\\(\mathbf{0.1,0})(\mathbf{0.4,0})\end{bmatrix},\ B=\begin{bmatrix}(\mathbf{0.5,0})(\mathbf{0.4,0})\\(\mathbf{0.6,0})(\mathbf{0.2,0})\end{bmatrix}$$

$$A^c=\begin{bmatrix}(\mathbf{1,0.2})(\mathbf{1,0.3})\\(\mathbf{1,0.1})(\mathbf{1,0.4})\end{bmatrix},\ B^c=\begin{bmatrix}(\mathbf{1,0.5})(\mathbf{1,0.4})\\(\mathbf{1,0.6})(\mathbf{1,0.2})\end{bmatrix}$$

$$(A^c)^T=\begin{bmatrix}(1,0.2)(1,0.1)\\(1,0.3)(1,0.4)\end{bmatrix},\ (B^c)^T=\begin{bmatrix}(1,0.5)(1,0.6)\\(1,0.4)(1,0.2)\end{bmatrix},\ (A^T)^c+(B^T)^c=\begin{bmatrix}(\mathbf{1,0.2})(\mathbf{1,0.1})\\(\mathbf{1,0.3})(\mathbf{1,0.2})\end{bmatrix}$$

$$A^c+B^c=\begin{bmatrix}(\mathbf{1,0.2})(\mathbf{1,0.3})\\(\mathbf{1,0.1})(\mathbf{1,0.2})\end{bmatrix},\ (A^c+B^c)^T=\begin{bmatrix}(\mathbf{1,0.2})(\mathbf{1,0.1})\\(\mathbf{1,0.3})(\mathbf{1,0.2})\end{bmatrix}$$

Then

$$(A^c + B^c)^T = (A^T)^c + (B^T)^c.$$

IV. TRACE OF FUZZY SOFT MATRIXBASED ON REFERENCE FUNCTION

In this section we extend the concept of trace of fuzzy square matrix proposed M. Dhar[26] to fuzzy soft square matrix based on reference function, and we prove some formal properties.

4.1. Definition

Let A be a square matrix. Then the trace ofthe matrix A is denoted by tr A and is defined as:

trA= (max(μ_{ii}),min(r_{ii})) (3)

where μ_{ii} stands for the membership functions lying along the principal diagonal and r_{ii} refers to the reference function of the corresponding membership functions.

4.2. Proposition

Let A and B be two fuzzy softsquare matrices each of order n.

Then

tr (A+B) =trA+ trB (4)

proof.

We have from the proposed definition of trace of fuzzy soft matrices

$$\text{trA} = (\max a_{ii}, \min r_{ii})$$

and

$$\text{trB} = (\max b_{ii}, \min r_{ii}')$$

then

$$A+B = C \text{ where } C=[c_{ij}]$$

Following the definition of addition of twofuzzy soft matrices, we have

$$C_{ij} = (\max(a_{ii}, b_{ii}), \min(r_{ii}, r_{ii}'))$$

According to definition 4.1 the trace of fuzzy soft matrixbased on reference function would be:

tr (C) = [max {max(a_{ii} , b_{ii}) }, min {min(r_{ii} ,r_{ii}')}]

= [max {max(a_{ii}) ,max(b_{ii})} , min {min(r_{ii}), min(r_{ii}')}]

= trA+trB,

Conversely,

trA+trB = [max {max(a_{ii}) ,max(b_{ii})} , min {(min(r_{ii}), min(r_{ii}'))}]

= [max {max(a_{ii} , b_{ii}), min (min(r_{ii} ,r_{ii}')}]

=tr(A+B)

hence the resulttrA+trB =tr(A+B)

4.3. Example:

Let us consider the following two fuzzy soft matrices A and Bbased on reference function for illustration purposes

$$A=\begin{bmatrix}(\mathbf{0.3,0})(\mathbf{0.7,0})(\mathbf{0.8,0})\\(\mathbf{0.4,0})(\mathbf{0.5,0})(\mathbf{0.3,0})\\(\mathbf{0.6,0})(\mathbf{0.1,0})(\mathbf{0.4,0})\end{bmatrix}\text{and}B=\begin{bmatrix}(\mathbf{1,0})(\mathbf{0.2,0})(\mathbf{0.3,0})\\(\mathbf{0.8,0})(\mathbf{0.5,0})(\mathbf{0.2,0})\\(\mathbf{0.5,0})(\mathbf{1,0})(\mathbf{0.8,0})\end{bmatrix}$$

The addition of two soft matrices would be:

$$A+B=\begin{bmatrix}(\mathbf{1,0})(\mathbf{0.7,0})(\mathbf{0.8,0})\\(\mathbf{0.8,0})(\mathbf{0.5,0})(\mathbf{0.3,0})\\(\mathbf{0.6,0})(\mathbf{1,0})(\mathbf{0.8,0})\end{bmatrix}$$

Using the definition of trace of fuzzy soft matrices, we see the following results:

tr A = { max(0.3, 0.5, 0.4), min (0, 0, 0)}=(0.5 , 0)

tr B = { max(1, 0.5, 0.8), min (0, 0, 0)}=(1 , 0)

Thus we have

trA+trB = { max(1, 0.5, 0.8) , min (0, 0, 0)}=(1 , 0)

And

tr (A+B) = { max(1, 0.5, 0.8) , min (0, 0, 0)}= (1, 0)

Hence the result

$$\text{trA+trB} = \text{tr(A+B)}$$

4.4.Proposition

Let A = $[a_{ij}, r_{ij}] \in \text{FSM}_{m\times n}$ be fuzzy soft squarematrix of order n, if λ is a scalar such that $0 \leq \lambda \leq 1$. Then tr(λ A)= λ tr(A) (5)

proof.
to prove
tr(λ A) = λtrA $0 \leq \lambda \leq 1$
we have
tr(λ A) ={ max(λa_{ii}), min (λr_{ii})}
=λ{ max(a_{ii}), min(r_{ii})}
= λtr (A)

4.5. Example

$$\text{Let } A = \begin{vmatrix} (\mathbf{0.3,0})(\mathbf{0.7,0})(\mathbf{0.8,0}) \\ (\mathbf{0.4,0})(\mathbf{0.5,0})(\mathbf{0.3,0}) \\ (\mathbf{0.6,0})(\mathbf{0.1,0})(\mathbf{0.4,0}) \end{vmatrix} \text{and } \lambda = 0.5$$

Then

$$\lambda A = \begin{vmatrix} (\mathbf{0.15,0})(\mathbf{0.35,0})(\mathbf{0.40,0}) \\ (\mathbf{0.20,0})(\mathbf{0.25,0})(\mathbf{0.15,0}) \\ (\mathbf{0.30,0})(\mathbf{0.05,0})(\mathbf{0.20,0}) \end{vmatrix}$$

tr(λ A) = { max(0.15, 0.25, 0.20), min ((0, 0, 0)}
= (0.25,0)
Again
tr A=(0.5,0)
and hence
tr(λ A) =0.5 (0.5, 0) = (0.25,0)

4.6. Proposition:

Let A=[a_{ij}, r_{ij}] $\in FSM_{m\times n}$ be fuzzy soft square matrices each of order n .
Then
trA=tr(A^t),where A^t is the transpose of A

4.7.Example

$$\text{Let } A^t = \begin{vmatrix} (\mathbf{0.3,0})(\mathbf{0.4,0})(\mathbf{0.6,0}) \\ (\mathbf{0.7,0})(\mathbf{0.5,0})(\mathbf{0.1,0}) \\ (\mathbf{0.8,0})(\mathbf{0.3,0})(\mathbf{0.4,0}) \end{vmatrix}$$

Then
tr(A^t)= { max(0.3, 0.5, 0.4) , min (0, 0, 0)}
=(0.5, 0)
HencetrA=tr(A^t)

The same result will hold if we consider the complements of fuzzy soft square matrices.

$$A^c = \begin{vmatrix} (\mathbf{1,0.3})(\mathbf{1,0.7})(\mathbf{1,0.8}) \\ (\mathbf{1,0.4})(\mathbf{1,0.5})(\mathbf{1,0.3}) \\ (\mathbf{1,0.6})(\mathbf{1,0.1})(\mathbf{1,0.4}) \end{vmatrix}$$

tr A^c ={max (1, 1, 1), min (0.3, 0.5, 0.4)}=(1, 0.3)
If we consider another fuzzy soft matrice B:

$$B = \begin{vmatrix} (\mathbf{1,0})(\mathbf{0.2,0})(\mathbf{0.3,0}) \\ (\mathbf{0.8,0})(\mathbf{0.5,0})(\mathbf{0.2,0}) \\ (\mathbf{0.5,0})(\mathbf{1,0})(\mathbf{0.8,0}) \end{vmatrix}$$

$$B^c = \begin{vmatrix} (\mathbf{1,1})(\mathbf{1,0.2})(\mathbf{1,0.3}) \\ (\mathbf{1,0.8})(\mathbf{1,0.5})(\mathbf{1,0.2}) \\ (\mathbf{1,0.5})(\mathbf{1,1})(\mathbf{1,0.8}) \end{vmatrix}$$

Then the trace of B^c will be the following:
tr(B^c)={max(1,1,1),min (1, 0.5, 0.8)}=(1, 0.5)

Following the definition 2.13 of addition of two fuzzy soft matrices based on reference function, we have.

$$A^c + B^c = \begin{vmatrix} (\mathbf{1,0.3})(\mathbf{1,0.2})(\mathbf{1,0.3}) \\ (\mathbf{1,0.4})(\mathbf{1,0.5})(\mathbf{1,0.2}) \\ (\mathbf{1,0.5})(\mathbf{1,0.1})(\mathbf{1,0.4}) \end{vmatrix}$$

tr (A^c +B^c) ={max(1,1,1),min (0.3, 0.5, 0.4)}
= (1, 0.3)

V. NEW FUZZY SOFT MATRIX THEORY IN DECISION MAKING

In this section we adopted the definition of fuzzy soft matrix decision method proposed by P. Rajarajeswari,P. Dhanalakshmi in [30] to the case of fuzzy soft matrix based on reference function in order to define a new fuzzy soft matrix decision method based on reference function.

5.1. Definition: (Value Matrix)

Let A=[a_{ij}, 0] $\in [FSM]_{m\times n}$ .Then we define the value matrix of fuzzy soft matrix A based on reference function as V(A)=[a_{ij}] =[a_{ij} - r_{ij}], i=1, 2, ,….,m , j= 1,2,3,…, n, where r_{ij} = $[0]_{m\times n}$.

5.2. Definition:(Score Matrix)

If A=[a_{ij}] $\in$ FSM,B=[b_{ij}] $\in [FSM]_{m\times n}$ .Then we define score matrix of A and B as:

$s_{A,B}$=$[d_{ij}]_{mxn}$ where [d_{ij}]=V(A)-V(B)

5.3. Definition:(Total Score)

If A=[a_{ij}, 0] $\in [FSM]_{m\times n}$,B=[b_{ij}, 0] $\in [FSM]_{m\times n}$.Let the corresponding value matrices be V(A),V(B) and their score matrix is $s_{A,B}$ =$[d_{ij}]_{mxn}$then we define total score for each c_i in U as $s_i = \sum_{j=1}^{n} d_{ij}$.

Methodology and algorithm

Assume that there is a set of candidates(programmer), U={c_1 , c_2, ,…,c_n} is a set of candidates to be recruited by software development organization in programmer post.Let E is a set of parameters related to innovative attitude of the programmer. We construct fuzzy soft set (F,E)over U represent the selection of candidate by field expert X,where F is a mapping F:E→ F^u,F^u is the collection of all fuzzy subsets of U. We further construct another fuzzy soft set (G,E)over Urepresent the selection of candidate by field expert Y,where G is a mapping G:E→ F^u ,F^u is the collection of all fuzzy subsets of U.The matricesA and B corresponding to the fuzzy softsets (F,E) and (G,E) are constructed,we compute the complementsand theirmatrices A^c and B^c corresponding to (F, E) ^{c} and (G, E) ^{c} respectively. Compute A+B which is themaximum membership of selection of candidates by the judges. Compute $A^c + B^c$ which is the maximum membership of non selection of candidates by the judges. usingdef (5.1) ,Compute V(A+B),V($A^c + B^c$) $s_{((A+B),(A^c+B^c))}$ and the total score S_i for each candidate in U.Finally find S_j = max(S_i),then conclude that the candidate c_j has selected by the judges.If $S_j$has more than one value the process is repeated by reassessing the parameters.

Now, using definitions 5-1, 5-2 and 5-3 we can construct a fuzzy soft matrix decision making method based on reference function by the following algorithm.

Algorithm

Step1 :Input the fuzzy soft set (F, E), (G,E) and obtain the fuzzy soft matrices A,B corresponding to (F,E) and (G, E) respectively.

Step2 :Write the fuzzy soft complement set $(F,E)^c$, $(G,E)^c$ and obtain the fuzzy soft matrices A^c , B^ccorresponding to $(F,E)^c$,and $(G,E)^c$respectively.

Step3 :Compute (A+B),(A^c+B^c), V(A+B),V(A^c+B^c) and $s_{((A+B),(A^c+B^c))}$.

Step4 :Compute the total score $S_i$for each $c_i$inU.

Step5 :Find c_i for which max (S_i) .

Then we conclude that the candidate $c_i$is selected for the post.

Incase max $S_i$occurs for more than one value, then repeatthe process by reassessing the parameters.

Case Study

Let (F,E) and (G,E) be two fuzzy soft set based on reference function representing the selection of four candidates from the universal set U= {c_1,c_2,c_3,c_4} by the experts X,and Y.Let E = {e_1, e_2,e_3} be the set of parameters which stand for intelligence,innovative and analysis .

(F,E)={F(e_1)={(c_1,0.1, 0)),(c_2,0.5, 0), (c_3, 0.1, 0), (c_4,0.4,0)},F(e_2)={(c_1,0.6, 0)),(c_2,0.4, 0), (c_3, 0.5, 0), (c_4,0.7,0)},F(e_3)={(c_1,0.5, 0)),(c_2,0.7, 0), (c_3, 0.6, 0), (c_4, 0.5, 0)}}.

(G,E)={G(e_1)={(c_1,0.2, 0)),(c_2,0.6, 0), (c_3, 0.2, 0), (c_4,0.3,0)}, G(e_2)={(c_1,0.6, 0)),(c_2,0.5, 0), (c_3, 0.6, 0), (c_4,0.8,0)}, ,G(e_3)={(c_1,0.5, 0)),(c_2,0.8, 0), (c_3, 0.7, 0), (c_4,0.5,0)}}.

These two fuzzy soft sets based on reference function are represented by the following fuzzy soft matrices based on reference function respectively

$$A = \begin{bmatrix} (0.1,0) & (0.6,0) & (0.5,0) \\ (0.5,0) & (0.4,0) & (0.7,0) \\ (0.1,0) & (0.5,0) & (0.6,0) \\ (0.4,0) & (0.7,0) & (0.5,0) \end{bmatrix} B = \begin{bmatrix} (0.2,0.) & (0.6,0) & (0.5,0) \\ (0.6,0) & (0.5,0) & (0.8,0) \\ (0.2,0) & (0.6,0) & (0.7,0) \\ (0.3,0) & (0.8,0) & (0.5,0) \end{bmatrix}$$

Then, the fuzzy soft complement matricesbased on reference function are

$$A^c = \begin{bmatrix} (1,0.1) & (1,0.6) & (1,0.5) \\ (1,0.5) & (1,0.4) & (1,0.7) \\ (1,0.1) & (1,0.5) & (1,0.6) \\ (1,0.4) & (1,0.7) & (1,0.5) \end{bmatrix}, B^c = \begin{bmatrix} (1,0.2) & (1,0.6) & (1,0.5) \\ (1,0.6) & (1,0.5) & (1,0.8) \\ (1,0.2) & (1,0.6) & (1,0.7) \\ (1,0.3) & (1,0.8) & (1,0.5) \end{bmatrix}$$

Then the addition matrices are

$$A+B = \begin{bmatrix} (0.2,0) & (0.6,0) & (0.5,0) \\ (0.6,0) & (0.5,0) & (0.8,0) \\ (0.2,0) & (0.6,0) & (0.7,0) \\ (0.4,0) & (0.8,0) & (0.5,0) \end{bmatrix}, A^c+B^c = \begin{bmatrix} (1,0.1) & (1,0.6) & (1,0.5) \\ (1,0.5) & (1,0.4) & (1,0.7) \\ (1,0.1) & (1,0.5) & (1,0.6) \\ (1,0.3) & (1,0.7) & (1,0.5) \end{bmatrix}$$

$$V(A+B) = \begin{bmatrix} 0.2 & 0.6 & 0.5 \\ 0.6 & 0.5 & 0.8 \\ 0.2 & 0.6 & 0.7 \\ 0.4 & 0.8 & 0.5 \end{bmatrix}, V(A^c+B^c) = \begin{bmatrix} 0.9 & 0.4 & 0.5 \\ 0.5 & 0.6 & 0.3 \\ 0.9 & 0.5 & 0.4 \\ 0.7 & 0.3 & 0.5 \end{bmatrix}$$

Calculate the score matrix and the total score for selection

$$s_{((A+B),(A^c+B^c))} = \begin{bmatrix} -0.7 & 0.2 & 0 \\ 0.1 & -0.1 & 0.5 \\ -0.7 & 0.1 & 0.3 \\ -0.3 & 0.5 & 0 \end{bmatrix}$$

$$\text{Total score} = \begin{bmatrix} -0.5 \\ 0.5 \\ -0.3 \\ 0.2 \end{bmatrix}$$

We see that the second candidate has the maximum value and thus conclude that from both the expert's opinion, candidate$c_2$is selected for the post.

VI. CONCLUSIONS

In our work, we have put forward some new concepts such as complement, trace of fuzzy soft matrix based on reference function. Some related properties have been established with example. Finally an application of fuzzy soft matrix based on reference function in decision making problem is given. It 's hoped that our work will enhance this study in fuzzy soft matrix.

VII. ACKNOWLEDGMENTS

The authors would like to thank the anonymous reviewers for their careful reading of this paper and for their helpful comments.

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Published in *I.J. Information Engineering and Electronic Business*, No. 2, 2013, pp. 52-59, DOI: 10.5815/ijieeb.2013.02.08, 8 p.

Neutrosophic Parametrized Soft Set Theory and Its Decision Making

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Abstract: In this work, we present definition of neutrosophic parameterized (NP) soft set and its operations. Then we define NP-aggregation operator to form NP-soft decision making method which allows constructing more efficient decision processes. We also dive an example which shows that they can be successfully applied to problem that contain indeterminacy.

Keywords: Soft set, neutrosophic set, neutrosophic soft set, neutrosophic parameterized soft set, aggregation operator.

1. Introduction

In 1999, Smarandache firstly proposed the theory of neutrosophic set (NS) [28], which is the generalization of the classical sets, conventional fuzzy set [30] and intuitionistic fuzzy set [5]. After Smarandache, neutrosophic sets has been successfully applied to many fields such as;control theory [1], databases [2,3], medical diagnosis problem [4], decision making problem [21], topology [22], and so on.

In 1999 a Russian researcher [27] firstly gave the soft set theory as a general mathematical tool for dealing with uncertainty and vagueness and how soft set theory is free from the parameterization inadequacy syndrome of fuzzy set theory, rough set theory, probability theory. Then, many interesting results of soft set theory have been studied on fuzzy soft sets [8,12,23], on intuitionistic fuzzy soft set theory [14,25], on possibility fuzzy soft set [7], on generalized fuzzy soft sets [26,29], on generalized intuitionistic fuzzy soft [6], on interval-valued intuitionistic fuzzy soft sets [20], on intuitionistic neutrosophic soft set [9], on generalized neutrosophic soft set [10], on fuzzy parameterized soft set theory [17,18], on fuzzy parameterized fuzzy soft set theory [13], on intuitionistic fuzzy parameterized soft set theory [15], on IFP−fuzzy soft set theory [16],on neutrosophic soft set [24].interval-valued neutrosophic soft set [11,19].

In this paper our main objective is to introduce the notion of neutrosophic parameterized soft set which is a generalization of fuzzy parameterized soft set and intuitionistic fuzzy parameterized soft set.The paper is structured as follows. In section 2, we first recall the necessary background on neutrosophic and soft set. In section 3, we give neutrosophic parameterized soft set theoryand their respective properties. In section 4, we present a neutrosophic parameterized aggregation operator. In section 5,a neutrosophic parameterized decision methods is presented with example. Finally we conclude the paper.

2. Preliminaries

Throughout this paper, let U be a universal set and E be the set of all possible parameters under consideration with respect to U, usually, parameters are attributes, characteristics, or properties of objects in U.

We now recall some basic notions of neutrosophic set and soft set. For more details, the reader could refer to [33, 37].

Definition 1.**[37] Let U be a universe of discourse then the neutrosophic set A is an object having the form**

$A = \{< x: \mu_{A(x)}, \nu_{A(x)}, \omega_{A(x)}>, x \in U\}$

where the functions $\mu, \nu, \omega : U \rightarrow]^-0,1^+[$ define respectively the degree of membership, the degree of indeterminacy, and the degree of non-membership of the element $x \in X$ to the set A with the condition.

$$^-0 \leq \mu_{A(x)} + \nu_{A(x)} + \omega_{A(x)} \leq 3^+. \qquad (1)$$

From philosophical point of view, the neutrosophic set takes the value from real standard or non-standard subsets of $]-0,1^+[$. So instead of $]^-0,1^+[$ we need to take the interval [0,1] for technical applications, because $]^-0,1^+[$ will be difficult to apply in the real applications such as in scientific and engineering problems.

For two NS,

$A_{NS} = \{<x, \mu_A(x), \nu_A(x), \omega_A(x)> \mid x \in X\}$

and

$B_{NS} = \{<x, \mu_B(x), \nu_B(x), \omega_B(x)> \mid x \in X\}$

Then,

$A_{NS} \subseteq B_{NS}$ if and only if

$\mu_A(x) \leq \mu_B(x), \nu_A(x) \geq \nu_B(x), \omega_A(x) \geq \omega_B(x)$.

$A_{NS} = B_{NS}$ if and only if,

$\mu_A(x) = \mu_B(x), \nu_A(x) = \nu_B(x), \omega_A(x) = \omega_B(x)$ for any $x \in X$.

The complement of A_{NS} is denoted by A^o_{NS} and is defined by

$A^o_{NS} = \{<x, \omega_A(x), 1 - \nu_A(x), \mu_A(x) \mid x \in X\}$

$A \cap B = \{<x, \min\{\mu_A(x), \mu_B(x)\}, \max\{\nu_A(x), \nu_B(x)\}, \max\{\omega_A(x), \omega_B(x)\}>: x \in X\}$

$A \cup B = \{<x, \max\{\mu_A(x), \mu_B(x)\}, \min\{\nu_A(x), \nu_B(x)\}, \min\{\omega_A(x), \omega_B(x)\}>: x \in X\}$

As an illustration, let us consider the following example.

Example 1. Assume that the universe of discourse U= $\{x_1, x_2, x_3, x_4\}$. It may be further assumed that the values of x_1, x_2, $x_3$and $x_4$are in [0, 1] Then, A is a neutrosophic set (NS) of U, such that,

A={< x_1, 0.4, 0.6, 0.5>, < x_2, 0.3, 0.4, 0.7>, < x_3, 0.4,0.4, 0.6] >,< x_4, 0.5,0.4, 0.8 >}

Definition 2.[33]

Let U be an initial universe set and E be a set of parameters. Let P(U) denotes the power set of U. Consider a nonempty set A, A ⊂ E. A pair (K, A) is called a soft set over U, where K is a mapping given by K: A → P(U).

As an illustration, let us consider the following example.

Example 2. Suppose that U is the set of houses under consideration, say U= $\{h_1, h_2, h_3, h_4, h_5\}$. Let E be the set of some attributes of such houses, say E = $\{e_1, e_2, e_3, e_4, e_5\}$, where e_1, e_2, e_3, e_4, e_5stand for the attributes "beautiful", "costly", "in the green surroundings", "moderate" and technically, respectively. In this case, to define a soft set means to point out expensive houses, beautiful houses, and so on. For example, the soft set (K, A) that describes the "attractiveness of the houses" in the opinion of a buyer, says Thomas, and may be defined like this:

A= $\{e_1, e_2, e_3, e_4, e_5\}$;

K(e_1) = $\{h_2, h_3, h_5\}$, K(e_2) = $\{h_2, h_4\}$, K(e_3) = $\{h_1\}$, K(e_4) = U, K(e_5) = $\{h_3, h_5\}$.

3. Neutrosophic Parameterized Soft Set Theory

In this section, we define neutrosophic parameterized soft set and their operations.

Definition 3.1. Let U be an initial universe, P (U) be the power set of U,E be a set of all parameters and K be a neutrosophic set over E. Then a neutrosophic parameterized soft sets

$\Psi_k = \{(< x, \mu_K(x), \nu_K(x), \omega_K(x) >, f_K(x)): x \in E\}$

where μ_K:E → [0, 1], ν_K:E → [0, 1],ω_K:E → [0, 1] and f_K:E →P(U) such that $f_K(x)$ =Φ if $\mu_K(x) = 0$,$\nu_K(x) = 1$ and$\omega_K(x) = 1$.

Here, the function μ_K , $\nu_K$and $\omega_K$called membership function, indeterminacy function and non-membership function of neutrosophic parameterized soft set (NP-soft set), respectively.

Example 3.2. Assume that U= $\{u_1, u_2, u_3\}$ is a universal set and E= $\{x_1, x_2\}$ is a set of parameters.If

K= {<x_1,0.2, 0.3, 0.4> , <x_2,0.3, 0.5, 0.4>}

and

$f_K(x_1)$= {u_2 ,u_3},$f_K(x_2)$=U

Then a neutrosophic parameterized soft set $\Psi_k$is written by

Ψ_k={(<x_1,0.2, 0.3, 0.4>,{u_2,u_3}),(<x_2,0.3, 0.5, 0.4>,U)}

Definition 3.3. LetΨ_k ∈ NP-soft set. if $f_K(x)$ = U , $\mu_K(x)$ =0 , $\nu_K(x) = 1$ and $\omega_K(x)= 1$ all x ∈ E. then Ψ_k is called K-empty NP-soft set, denoted by Ψ_{Φ_k}.

If K=Φ,then the K-emptyNP-soft set is called empty NP-soft set, denoted byΨ_{Φ}.

Definition 3.4. Let Ψ_k ∈ NP-soft set. if $f_K(x)$ = U , $\mu_K(x)$ =1 , $\nu_K(x)$ =0and $\omega_K(x)$=0 all x ∈ E. then Ψ_k is called K-universal NP-soft set, denoted by$\Psi_{\tilde{K}}$.

If K= E, then the K-universal NP-soft set is called universal NP-soft set, denoted by $\Psi_{\tilde{E}}$.

Definition 3.5.$\Psi_k$andΩ_L are two NP-soft set. Then,Ψ_k is NP-subset of Ω_L, denoted by $\Psi_k \sqsubseteq \Omega_L$ if and only if $\mu_K(x) \le \mu_L(x)$,$\nu_K(x) \ge \nu_L(x)$ and $\omega_K(x) \ge \omega_L(x)$ and $f_K(x) \subseteq g_L(x)$ for all x ∈ E.

Definition 3.6.$\Psi_k$andΩ_L are two NP-soft set. Then, Ψ_k=Ω_L, if and only if $\Psi_k \sqsubseteq \Omega_L$ and $\Omega_L \sqsubseteq \Psi_k$ for all x ∈ E.

Definition 3.7. Let Ψ_k ∈ NP-soft set. Then, the complement of Ψ_k, denoted by Ψ_K^c , is defined by

Ψ_K^c={(< $x, \omega_K(x) ,\nu_K(x) ,\mu_K(x)$ >, $f_{K^c}(x)$): x ∈ E}

Where $f_{K^c}(x)$ =U \\$f_K(x)$

Definition 3.8.Let Ψ_k and Ω_L are two NP-soft set. Then ,union of Ψ_k and Ω_L,denoted by $\Psi_k \sqcup \Omega_L$, is defined by

$\Psi_k \sqcup \Omega_L$={(< x, max{ $\mu_K(x),\mu_L(x)$} ,min{$\nu_K(x),\nu_L(x)$} ,min{$\omega_K(x),\omega_L(x)$} >, $f_{K\cup L}(x)$): x ∈ E }

where$f_{K\cup L}(x)$ =$f_K(x) \cup f_L(x)$.

Definition 3.9. Let Ψ_k and Ω_L are two NP-soft set. Then, intersection of $\Psi_K$and Ω_L,denoted by $\Psi_K \sqcap \Omega_L$, is defined by

$\Psi_k \sqcap \Omega_L=$
$\{(< x, \min\{ \mu_K(x), \mu_L(x)\}, \max\{\nu_K(x), \nu_L(x)\}, \max\{\omega_K(x), \omega_L(x)\} >, f_K(x) \cap f_L(x)) : x \in E\}$

where $f_{K\cap L}(x) = f_K(x) \cap f_L(x)$.

Example 3.10. Let $U = \{u_1, u_2, u_3, u_4\}$, $E = \{x_1, x_2, x_3\}$. Then,

$\Psi_K = \{(<x_1, 0.2, 0.3, 0.4>, \{u_1, u_2\}), (<x_2, 0.3, 0.5, 0.4>, \{u_2, u_3\})\}$

$\Omega_L = \{(<x_2, 0.1, 0.2, 0.4>, \{u_3, u_4\}), (<x_3, 0.5, 0.2\ 0.3>, \{u_3\})\}$

Then

$\Psi_K \sqcup \Omega_L = \{(<x_1, 0.2, 0.3, 0.4>, \{u_1, u_2\}), (<x_2, 0.3, 0.2, 0.4>, \{u_2, u_3, u_4\}), (<x_3, 0.5, 0.2\ 0.3>, \{u_3\})\}$

$\Psi_K \sqcap \Omega_L = \{(<x_2, 0.1, 0.5, 0.4>, \{u_3, u_4\})\}$.

$\Psi_K^c = \{(<x_1, 0.4, 0.3, 0.2>, \{u_3, u_4\}), (<x_2, 0.4, 0.5, 0.3>, \{u_1, u_4\})\}$

Remark 3.11. $\Psi_K \sqsubseteq \Omega_L$ does not imply that every element of Ψ_K is an element of Ω_L as in the definition of classical subset. For example assume that $U = \{u_1, u_2, u_3, u_4\}$ is a universal set of objects and $E = \{x_1, x_2, x_3\}$ is a set of all parameters, if NP-soft sets Ψ_K and Ω_L are defined as

$\Psi_K = \{(<x_1, 0.2, 0.3, 0.4>, \{u_1, u_2\}), (<x_2, 0.3, 0.5, 0.4>, \{u_2\})\}$

$\Omega_L = \{(<x_1, 0.1, 0.2, 0.4>, U), (<x_2, 0.5, 0.2\ 0.3>, \{u_1, u_4\})\}$

It can be seen that $\Psi_K \sqsubseteq \Omega_L$, but every element of Ψ_K is not an element of Ω_L

Proposition 3.12. Let $\Psi_K, \Omega_L \in$ NP-soft set. Then

$\Psi_K \sqsubseteq \Psi_{\tilde{E}}$

$\Psi_{\Phi} \sqsubseteq \Psi_K$

$\Psi_K \sqsubseteq \Psi_K$

Proof. It is clear from Definition 3.3-3.5.

Proposition 3.13. Let Ψ_K, Ω_L and $\Upsilon_M \in$ NP-soft set, Then

$\Psi_K = \Omega_L$ and $\Omega_L = \Upsilon_M \Leftrightarrow \Psi_K = \Upsilon_M$

$\Psi_K \sqsubseteq \Omega_L$ and $\Omega_L \sqsubseteq \Psi_K \Leftrightarrow \Psi_K \quad \Omega_L$

$\Psi_K \sqsubseteq \Omega_L$ and $\Omega_L \sqsubseteq \Upsilon_M \Rightarrow \Psi_K \sqsubseteq \Upsilon_M$

Proof .It can be proved by Definition 3.3-3.5

Proposition 3.14 Let $\Psi_K \in$ NP-soft set. Then

$(\Psi_K^c)^c = \Psi_k$

$\Psi_{\tilde{\Phi}}^c = \Psi_{\tilde{E}}$

$\Psi_{\tilde{E}}^c = \Psi_{\tilde{\Phi}}$

Proof. It is trial.

Proposition 3.15. LetΨ_K,Ω_L and $\Upsilon_M \in$ NP-soft set, Then

$\Psi_K \sqcup \Psi_K = \Psi_K$

$\Psi_K \sqcup \Psi_{\tilde{\Phi}} = \Psi_K$

$\Psi_K \sqcup \Psi_{\tilde{E}} = \Psi_{\tilde{E}}$

$\Psi_K \sqcup \Omega_L = \Omega_L \sqcup \Psi_K$

$(\Psi_K \sqcup \Omega_L) \sqcup \Upsilon_M = \Psi_K \sqcup (\Omega_L \sqcup \Upsilon_M)$

Proof.It is clear

Proposition 3.16. Let Ψ_K,Ω_L and $\Upsilon_M \in$ NP-soft set, Then

$\Psi_K \sqcap \Psi_K = \Psi_K$

$\Psi_K \sqcap \Psi_{\tilde{\Phi}} = \Psi_{\tilde{\Phi}}$

$\Psi_K \sqcap \Psi_{\tilde{E}} = \Psi_K$

$\Psi_K \sqcap \Omega_L = \Omega_L \sqcap \Psi_K$

$(\Psi_K \sqcap \Omega_L) \sqcap \Upsilon_M = \Psi_K \sqcap (\Omega_L \sqcap \Upsilon_M)$

Proof.It is clear

Proposition 3.17. Let Ψ_K,Ω_L and $\Upsilon_M \in$ NP-soft set, Then

$\Psi_K \sqcup (\Omega_L \sqcap \Upsilon_M) = (\Psi_K \sqcup \Omega_L) \sqcap (\Psi_K \sqcup \Upsilon_M)$

$\Psi_K \sqcap(\Omega_L \sqcup \Upsilon_M=(\Psi_K \sqcap \Omega_L) \sqcup(\Psi_K \sqcap \Upsilon_M)$

Proof .It can be proved by definition 3.8 and 3.9

Proposition 3.18. LetΨ_K,Ω_L $\in$ NP-soft set, Then

$(\Psi_K \sqcup \Omega_L)^c=\Psi_K^c \sqcap \Omega_L^c$

$(\Psi_K \sqcap \Omega_L)^c=\Psi_K^c \sqcup \Omega_L^c$

Proof. It is clear.

Definition 3.19. LetΨ_K,Ω_L $\in$ NP-soft set, Then

OR-product of $\Psi_K$and Ω_L denoted by $\Psi_K \veebar \Omega_L$,is defined as following

$\Psi_K \veebar \Omega_L$={ $(<(x,y),(\max\{\mu_K(x),\mu_L(y)\},\min\{\nu_K(x),\nu_L(x)\},\min\{\omega_K(x),\omega_L(y)\}>,f_{K\cup L}(x,y)) :x,y \in E$ }

where$f_{K\cup L}(x,y)=f_K(x) \cup f_L(y)$.

AND-product of $\Psi_K$and Ω_L denoted by $\Psi_K \barwedge \Omega_L$is defined as following

$\Psi_K \barwedge \Omega_L$={ $(<(x,y),(\min\{\mu_K(x),\mu_L(y)\},\max\{\nu_K(x),\nu_L(y)\},\max\{\omega_K(x),\omega_L(y)\}>,f_{K\cap L}(x,y)) :x,y \in E$ }

where$f_{K\cap L}(x,y)=f_K(x) \cap f_L(y)$.

Proposition 3.20. LetΨ_K,Ω_L and Υ_M $\in$ NP-soft set, Then

$\Psi_K \barwedge \Psi_{\Phi}=\Psi_{\Phi}$

$(\Psi_K \barwedge \Omega_L)\barwedge \Upsilon_M=\Psi_K \barwedge(\Omega_L \barwedge \Upsilon_M)$

$(\Psi_K \veebar \Omega_L)\veebar \Upsilon_M=\Psi_K \veebar(\Omega_L \veebar \Upsilon_M)$

Proof .It can be proved by definition 3.15

4. NP-aggregation operator

In this section, we define NP-aggregation operator of an NP-soft set to construct a decisionmethod by which approximate functions of a soft set are combined to produce a single neutrosophic set that can be used to evaluate each alternative.

Definition 4.1. $\Psi_K \in$ NP-soft set. Then a NP-aggregation operatorof Ψ_K , denotedby$\Psi_K{}^{agg}$, is definedby

$\Psi_K{}^{agg}$={ (<u, $\mu_K{}^{agg}(u)$,$\nu_K{}^{agg}(u)$,$\omega_K{}^{agg}$(u): u ∈ U }

which is a neutrosophic set over U,

where

$\mu_K{}^{agg}$:U→ [0,1] $\mu_K{}^{agg}(u) = \frac{1}{|U|}\sum_{x\in E, u\in U} \mu_K(x)\gamma_{f_{K(x)}}(u)$,

$\nu_K{}^{agg}$ ::U→ [0,1] $v_K{}^{agg}(u) = \frac{1}{|U|}\sum_{x\in E, u\in U} v_K(x)\gamma_{f_{K(x)}}(u)$

and

$\omega_K{}^{agg}$::U→ [0,1] $\omega_K{}^{agg}(u) = \frac{1}{|U|}\sum_{x\in E, u\in U} \omega_K(x)\gamma_{f_{K(x)}}(u)$

And where

$$\gamma_{f_{K(x)}}(u) = \begin{cases} 1, & x \in f_K(x) \\ 0, & other\ wise \end{cases}$$

|U|is the cardinality of U.

Definition 4.2 Let $\Psi_K \in$ NP-soft set and $\Psi_K{}^{agg}$ an aggregation neutrosophic parameterized soft set ,then a reduced fuzzy set of $\Psi_K{}^{agg}$ is a fuzzy set over U denoted by

$$\Psi_K{}^{agg} = \left\{\frac{\mu_{\Psi_{Kf}{}^{agg}}(u)}{u} : u \in U\right\}$$

where $\mu_{\Psi_{Kf}{}^{agg}}(u)$: U→ [0,1] and $\mu_{\Psi_{Kf}{}^{agg}}(u)$=$\mu_K{}^{agg}(u)$+$v_K{}^{agg}(u)$-$\omega_K{}^{agg}(u)$.

NP-Decision Methods

Inspired by the decision making methods regard in [12-19].In this section, we also present NP-decision method to neutrosophic parameterized soft set. Based on definition 4.1 and 4.2 we construct an NP-decision making method by the following algorithm.

Now, we construct a NP-soft decision making method by the following algorithm to produce a decision fuzzy set from a crisp set of the alternatives.

According to the problem, decision maker

i. constructs a feasible Neutrosophic subsets K over the parameters set E,

ii. constructs a NP-soft set Ψ_K over the alternatives set U,

iii. computes the aggregation neutrosophic parameterized soft set $\Psi_K{}^{agg}$ of Ψ_K,

iv. computes the reduced fuzzy set $\mu_{\Psi_{Kf}{}^{agg}(u)}$ of $\Psi_K{}^{agg}$,

v. chooses the element of $\mu_{\Psi_{Kf}{}^{agg}(u)}$ that has maximum membership degree.

Now, we can give an example for the NP-soft decision making method

Example. Assume that a company wants to fill a position. There are four candidates who fill in a form in order to apply formally for the position. There is a decision maker (DM) that is from the department of human resources. He wants to interview the candidates, but it is very difficult to make it all of them. Therefore, by using the NP-soft decision making method, the number of candidates are reduced to a suitable one. Assume that the set of candidates U=$\{u_1, u_2, u_3, u_4\}$ which may be characterized by a set of parameters E=$\{x_1, x_2, x_3\}$ For i=1,2,3 the parameters i stand for experience, computer knowledge and young age, respectively. Now, we can apply the method as follows:

Step i. Assume that DM constructs a feasible neutrosophic subsets K over the parameters set E as follows;

K={<x_1,0.2, 0.3, 0.4>,<x_2,0.3, 0.2, 0.4>,<x_3,0.5, 0.2 0.3>}

Step ii. DM constructs an NP-soft set Ψ_K over the alternatives set U as follows;

Ψ_K={(<x_1,0.2,0.3,0.4>,$\{u_1, u_2\}$),(<x_2,0.3,0.2,0.4>,$\{u_2,u_3,u_4\}$),(<x_3,0.5,0.2,0.3>,$\{u_3\}$)}

Step iii. DM computes the aggregation neutrosophic parameterized soft set $\Psi_K{}^{agg}$ of Ψ_K as follows;

$\Psi_K{}^{agg}$={<u_1,0.05, 0.075, 0.1>,<u_2,0.1, 0.125, 0.2>,<u_3,0.2, 0.1 0.175>,<u_4,0.125, 0.05 0.075>}

Step iv .computes the reduced fuzzy set $\mu_{\Psi_{Kf}{}^{agg}(u)}$ of $\Psi_K{}^{agg}$ as follows;

$\mu_{\Psi_{Kf}{}^{agg}(u_1)}$=0.025

$\mu_{\Psi_{Kf}{}^{agg}(u_2)}$=0.025

$\mu_{\Psi_{Kf}{}^{agg}(u_3)}$=0.125

$\mu_{\Psi_{Kf}{}^{agg}(u_4)}$=0.1

Step v . Finally, DM chooses u_3 for the position from $\mu_{\Psi_{K_f}}{}^{agg}{}_{(u)}$ since it has the maximum degree 0.125 among the others.

Conclusion

In this work, we have introduced the concept of neutrosophic parameterized soft set and studied some of its properties. The complement, union and intersection operations have been defined on the neutrosophic parameterized soft set. The definition of NP-aggregation operator is introduced with application of this operation in decision making problems.

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Interval Valued Neutrosophic Parameterized Soft Set Theory and its Decision Making

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Abstract – In this work, we present definition of interval valued neutrosophic parameterized (IVNP-)soft set and its operations. Then we define parameter reduction method for IVNP-soft set.We also give an example which shows that they can be successfully applied to problem that contains indeterminacy.

Keywords –
soft set, neutrosophic set, neutrosophic soft set.

1. Introduction

In 1999, Smarandache firstly proposed the theory of neutrosophic set (NS) [34], which is the generalization of the classical sets, conventional fuzzy set [40] and intuitionistic fuzzy set [5]. In recent years, neutrosophic sets has been successfully applied to many fields such as;control theory [1], databases [3,4], clustering [36], medical diagnosis problem [2], decision making problem [25,37], topology [26],and so on.

Presently work on the neutrosophic set theory is progressing rapidly such as; Bhowmik and Pal defined intuitionistic neutrosophic set [9] and intuitionistic neutrosophic relations [10]. Later on Salam, Alblowi [33] introduced another concept called generalized neutrosophic set. Wang et al. [38] proposed another extension of neutrosophic set which is single valued neutrosophic. Also Wang etal. [39] introduced the notion of interval valued neutrosophic set which is an instance of neutrosophic set. It is characterized by an interval membership degree, interval indeterminacy degree and interval non-membership degree.Many applications of neutrosophic theory have been worked by Geogiev [23],Ye

[36,37], Majumdar and Samanta [31], P.D. Liu [41,42] and Broumi and Smarandache [14] and so on.

In 1999 a Russian researcher [32] firstly gave the soft set theory as a general mathematical tool for dealing with uncertainty and vagueness and how soft set theory is free from the parameterization inadequacy syndrome of fuzzy set theory, rough set theory, probability theory. Then, many interesting results of soft set theory have been studied on fuzzy soft sets [15,,27], on FP-soft sets [20,21], on intuitionistic fuzzy soft set theory [8,17,28], on intuitionistic fuzzy parameterized soft set theory [18], oninterval valued intuitionistic fuzzy soft set [24], on generalized fuzzy soft sets [30,35], on generalized intuitionistic fuzzy soft [6], on possibility intuitionistic fuzzy soft set [7], on intuitionistic neutrosophic soft set [11], on generalized neutrosophic soft set [12], on fuzzy parameterized fuzzy soft set theory [16], on IFP−fuzzy soft set theory [19], on neutrosophic soft set [29]. Recently, Deli [22] introduced the concept of interval valued neutrosophic soft set as a combination of interval neutrosophic set and soft sets.

In this paper our main objective is to introduce the notion of interval valued neutrosophic parameterized soft set which is a generalization of neutrosophic parameterized soft sets [13].The paper is structured as follows. In Section 2, we first recall the necessary background on neutrosophic sets, interval neutrosophic sets and soft sets. In Section 3,we present interval valued neutrosophic parameterized soft set theory and examines their respective properties. In section 4, we present a interval valued neutrosophic parameterized aggregation operator. Section 5, interval valued neutrosophic parameterized decision methods is presented with example. Finally we conclude the paper.

2. Preliminaries

Throughout this paper, let U be a universal set and E be the set of all possible parameters under consideration with respect to U, usually, parameters are attributes, characteristics, or properties of objects in U.

We now recall some basic notions of neutrosophic set, interval valued neutrosophic set and soft set. For more details, the reader could refer to [29, 32, 34, 39].

Definition 2.1. [34] Let U be a universe of discourse then the neutrosophic set A is an object having the form

$$A = \{< x: \mu_{A(x)}, \nu_{A(x)}, \omega_{A(x)} >, x \in U\}$$

where the functions $\mu, \nu, \omega : U \rightarrow]^-0,1^+[$ define respectively the degree of membership, the degree of indeterminacy, and the degree of non-membership of the element $x \in X$ to the set A with the condition.

$$^-0 \leq \mu_{A(x)} + \nu_{A(x)} + \omega_{A(x)} \leq 3^+. \quad (1)$$

From philosophical point of view, the neutrosophic set takes the value from real standard or non-standard subsets of $]-0,1^+[$. So instead of $]^-0,1^+[$ we need to take the interval [0,1] for technical applications, because $]^-0,1^+[$ will be difficult to apply in the real applications such as in scientific and engineering problems.

Definition 2.2. [39] Let X be a space of points (objects) with generic elements in X denoted by x. An interval valued neutrosophic set (for short IVNS) A in X is characterized by truth-membership function), indeteminacy-membership function and falsity-membership function $(\mathbf{x})$. For each point x in X, we have that $\boldsymbol{\mu}_{\mathbf{A}}(\mathbf{x})$,$\boldsymbol{\nu}_{\mathbf{A}}(\mathbf{x})$,$\boldsymbol{\omega}$ ∈[0,1].

For two IVNS

= {<x, [μ),)] , > | x ∈ X }

and

= {<x, [μ),)] ,]> | x ∈ X }

Then,

1. if and only if

),μ ,ω ,
,ω).

2. ,

=μ ,ν =ν ,ω =ω for anyx ∈ X.

3. The complement of is denoted by and is defined by

={<x,]>,], [μ),)] | x ∈ X }

4. A∩B = { <x, [min( ),μ)), min(μ),μ))], [max(ν),ν)),
max(ν),ν [max(ω),ω)), max(ω),ω))] >: x ∈ X }

5. A∪B = {<x, [max( ),μ)), max(μ),μ))], [min(ν),ν)),
min (ν),ν [min(ω),ω)), min(ω),ω))] >: x ∈ X }

As an illustration, let us consider the following example.

Example 2.3. Assume that the universe of discourse U= {x }. Then, A is a interval valued neutrosophic set (IVNS) of U such that,

A = {< x_1, [0.1 0.8], [0.2 0.6], [0.8 0.9] >, < x_2, [0.2 0.5], [0.3 0.5], [0.6 0.8]>,
< x_3, [0.5 0.8], [0.4 0.5], [0.45 0.6] >, < x_4, [0.1 0.4], [0.1 0.5], [0.4 0.8] >}

Definition 2.4. [32] Let U be an initial universe set and E be a set of parameters. Let P(U) denotes the power set of U. Consider a nonempty set A, A ⊂ E. A pair (K, A) is called a soft set over U, where K is a mapping given by K: A → P (U).

As an illustration, let us consider the following example.
Example 2.5 . Suppose that U is the set of houses under consideration, say U = $\{h_1,h_2. . ., h_8\}$. Let E be the set of some attributes of such houses, say E = {e_1, e_2, . . ., e_4}, where e_1, e_2, . . ., e_4 stand for the attributes "beautiful", "costly", "in the green surroundings'", "moderate", respectively.

In this case, to define a soft set means to point out expensive houses, moderate houses, and so on. For example, the soft set (K, A) that describes the "attractiveness of the houses" in the opinion of a buyer, saysMr. X, and may be defined like this:

A= E,(K, A) ={(e_1, $\{h_1,h_2\}$), (e_2, $\{h_3\}$), (e_3, $\{h_1,h_2.h_5\}$), (e_4, U)}.

3. Interval Neutrosophic Parameterized Soft Set Theory

In this section, we define interval neutrosophic parameterized soft set and their operations.

Definition 3.1. Let U be an initial universe, P(U) be the power set of U,E be a set of all parameters and K be an interval valued neutrosophic set over E . Then an interval neutrosophic parameterized soft sets(IVNP-soft sets), denoted byΨ_K, defined as;

={ ∈
:E → [0, 1], ν_K:E → [0, 1],ω_K:E → [0, 1] and :E →P(U) such that =Φ if
1.

Here, theμ_K, $\nu_K$and called truth-membership function, indeteminacy-membership function and falsity-membership function of (IVNP-soft set), respectively.

Example 3.2. Assume that U= {u_1,u } is a universal set and E= {x_1,x_2} is a set of parameters. If

K= {(<x_1, [0.2, 0.3],[0.3,0.5],[0.4 ,0.5]>) , (<x_2,[0.3 ,0.4], [0.5,0.6],[0.4 ,0.5]>)}

and

)= {u_2 ,u_3},f)=U

then a IVNP-soft set is written by

Ψ = {(<x_1, [0.2, 0.3], [0.3,0.5], [0.4 ,0.5]>,{u_2,u_3}),(<x_2,[0.3 ,0.4], [0.5,0.6],[0.4 ,0.5]>, U)}

Definition 3.3. Let ∈IVNP-soft sets. If = Φ,)=μ =0 ,)=ν and ω_K^L(x)=ω =1 all x ∈ E. then is called empty IVNP-soft set, denoted by .

Definition 3.4.Let Ψ_K ∈IVNP-soft sets. If)=U,μ)=μ)=1,)=ν and)=ω_K^U(x)=0 all x ∈ E. Then is called K-universal IVNP-soft set, denoted by$\Psi_{\tilde{K}}$. If K= E, then the K-universal IVNP-soft set is called universal IVNP-soft set, denoted by $\Psi_{\tilde{E}}$.

Definition 3.5. and are two IVNP-soft set. Then, $=\Omega_M$,if and only if and for all x ∈ E.

Definition 3.6. and are two IVNP-soft set. Then,Ψ is IVNP-subset ofΩ , denoted by Ψ if and only if ,μ (x),ν ( ),),ω ,ω andf for all x ∈ E.

Definition 3.7. Let ∈IVNP-soft set. Then, the complement of , denoted by , is defined by

=\{(<x, ,f ∈ E\}

where =U

Definition 3.8. Let and are two IVNP-soft set. Then ,union of and ,denoted by , is defined by

= \{(<x, [max \{μ ,max \{μ , [min \{ν min \{ν], [min\{ω min\{ω]>, ∈ E\}

Where =f

Definition 3.9. Let and are two IVNP-soft set. Then, intersection of andΩ_M,denoted by , is defined by

= \{(<x, [min \{μ ,min \{μ , [max \{ν max \{ν], [max\{ω max\{ω]>, ∈ E\}

wheref =f

Example 3.10. Let U = $\{u_1, u_2, u_3, u_4\}$, E=$\{x_1, x_2, x_3\}$. Then,
Ψ = $\{(<x_1,[0.1,0.5], [0.4, 0.5], [0.2, 0.3]>, \{u_1, u_2\}),$
$(<x_2,[0.2,0.3], [0.5, 0.7], [0.1, 0.3]>, \{u_2, u_3\})\}$
Ω = $\{(<x_2, [0.1,0.6], [0.2, 0.3], [0.2, 0.4]>, \{u_3, u_4\}),$
$(<x_3, [0.4,0.7], [0.1, 0.2], [0.3, 0.4]>, \{u_3\})\}$

Then

$=\{(<x_1, [0.1,0.5], [0.4, 0.5], [0.2, 0.3]>, \{u_1, u_2\}), (<x \ , [0.2,0.6], [0.2, 0.3], [0.1, 0.3]>, \{u_2, u_3, u \ \}), (<x_3, [0.4,0.7], [0.1, 0.2], [0.3, 0.4]>, \{u_3\})\}$
$= \{(<x_2, [0.1,0.3], [0.5, 0.7], [0.2, 0.4]>, \{u_3\})\}.$
Ψ $=\{(<x_1, [0.2,0.3], [0.4, 0.5], [0.1, 0.5]>, \{u_3, u_4\}),$
$(<x_2, [0.1,0.3], [0.5, 0.7], [0.2, 0.3]>, \{u_1, u_4\})\}$

Remark 3.11. $\Psi_K \sqsubseteq \Omega_M$ does not imply that every element of Ψ_K is an element of Ω_M as in the definition of classical subset. For example assume that U={u_1,u_2,u_3,u_4}is a universal set of objects and E={x_1,x_2,x_3}is a set of all parameters, if IVNP-soft sets Ψ_K and Ω_M are defined as

Ψ = {(<x_1,[0.1,0.3], [0.5, 0.5] ,[0.2 ,0.3]>,{u_1,u_2}),(<x_2, [0.3,0.4], [0.4, 0.5] , [0.3, 0.5]>,{u_2})}
Ω = {(<x_1,[0.2,0.6], [0.3, 0.4] ,[0.1 ,0.2]>,U),(<x_2, [0.4,0.7], [0.1, 0.3] , [0.2, 0.3]>,{u_1,u_4})}

It can be seen that , but every element of is not an element ofΩ .

Proposition 3.12. LetΨ_K,Ω ∈IVNP-soft set .Then
i.
ii.
iii.

Proof. It is clear from Definition 3.3-3.5.

Proposition 3.13. LetΨ_K,Ω and ∈IVNP-soft set. Then
i. Ψ_K = and = = ϒ
ii. and =
iii. and

Proof. It can be proved by Definition 3.3-3.5

Proposition 3.14 Let ∈ IVNP-soft set. Then
i. $(\Psi_K^c)^c$ =Ψ
ii. =Ψ
iii. =Ψ

Proposition 3.15. Let ,Ω and ∈IVNP-soft set. Then
i. =Ψ
ii. =Ψ
iii. =Ψ
iv. =Ω
v. ( ) =Ψ ⊔(Ω)

Proof. It is clear

Proposition 3.16. Let ,Ω and ∈IVNP-soft set, Then
i. =Ψ
ii. =Ψ
iii. =Ψ
iv. =Ω
v. ( ) =Ψ ⊓(Ω)

Proof. It is clear

Proposition 3.17. LetΨ_K,Ω and ∈IVNP-soft set, Then

i. ⊔(Ω)= (Ψ Ω) ⊓ (Ψ_K ⊔ Υ_N)
ii. ⊓(Ω =(Ψ) ⊔(Ψ_K ⊓ Υ_N)

Proof. It can be proved by definition 3.8 and 3.9

Proposition 3.18. LetΨ_K,Ω ∈IVNP-soft set, Then

i. =Ψ
ii. =Ψ

Proof. It is clear.

Definition 3.19. LetΨ ∈IVNP-soft set, Then

i. OR-product of and denoted by _ ,is defined as following
_ (x, y)={ (<(x,y),([max {μ ,$\mu_M^L(y)$},max{μ ,$\mu_M^U(y)$}],
[min {ν ,$\nu_M^L(y)$},min{ν ,$\nu_M^U(y)$],[min{ω ,$\omega_M^L(y)$},
min {ω ,ω (y)]) >,Ψ (x,y)) :x,y ∈
where (x,y)= (x) (y)
ii. AND-product of and denoted by is defined as following
(x,y)={ (<(x,y), ([min{μ (x) ,$\mu_M^L(y)$},min{μ (x) ,$\mu_M^U(y)$}],
[max {ν ,$\nu_M^L(y)$},max{ν ,$\nu_M^U(y)$], [max{ω ,$\omega_M^L(y)$},
max {ω ,ω (y)]) >,Ψ (x,y)) :x,y ∈
where (x,y)= (x) (y)

Proposition 3.20. LetΨ_K,Ω and ∈IVNP-soft set. Then

i. =Ψ
ii. =Ω
iii. _ =Ω _
iv. ( )$\overline{\wedge}$ =Ψ $\overline{\wedge}$(Ω)
v. (_)$\underline{\vee}$ =Ψ $\underline{\vee}$(Ω _)

Proof . It can be proved by definition 3.15

4. Parameter Reduction Method

In this section, we have defined a parameter reduction method of an IVNP-soft set, that produce a soft set from an IVNP-soft set. For this, we define level set for IVNP-soft set. This concept presents an adjustable approach to IVNP–soft sets based decision making problems.

Throughout this section we will accept that the parameter set E and the initial universe U are finite sets.

Definition 4.1 Let ∈ IVNPS. Then for as= [s^-,s^+], t= [t^-,t^+],q= [q ,q^+]⊆ [0, 1], the (s, t, q) –level soft set of is a crisp soft set,denoted by (Ψ_K; (s, t , q)), defined by

(; (s,t,q)) = {((x_i, (x_i)) :x_i ∈ E}

where

),s),t),t and),q

Remark In Definition 4.1, s ⊆ [0,1] can be viewed as a given least threshold on degrees of truth-membership, t⊆[0, 1] can be viewed as a given greatest threshold on degrees of indeterminacy-membership and q [0, 1] can be viewed as a given greatest threshold on degrees of falsity-membership. If),$s^+ \leq \mu_K^U(x_i)$,$t^- \geq \nu_K^L(x_i)$,$t^+ \geq \nu_K^U(x_i)$ and),q), it shows that the degree of the truth-membership of x with respect to the u is not less than s, and the degree of the indeterminacy-membership of u with respect to the parameter x is not more than t and the degree of the falsity-membership of u with respect to the parameter x is not more than q. In practical applications of IVNP-soft sets, the thresholds s,t,q⊆[0, 1] is pre-established by decision makers and reflect decision makers requirements on "truth-membership levels", "indeterminacy-membership levels "and "falsity-membership levels".

Definition 4.2 LetΨ ∈ IVNPS and an = [s ,s], = [t ,t], = [q_{min}^-,q]⊆[0, 1] which is called a threshould of IVNSP-soft set. The level soft set of with respect to (s ,t_{min} ,q_{min}) is a crisp soft set , denoted by

(Ψ_K; (s ,t ,q)) ,

defined by;

(Ψ_K; (s ,t ,q)) = {((x_i, (x_i)) :x_i ∈ E}

where,

,s ,t ,t and),

= inf {μ :x_i ∈ E}, = inf {μ :x_i ∈ E}
= inf {ν :x_i ∈ E}, = inf {ν :x_i ∈ E}
= inf {ω :x_i ∈ E}, = inf {ω :x_i ∈ E}

The (s_{min},t_{min}, q_{min}) is called the mmm-threshold of the IVNP-soft set Ψ_K . In the following discussions, the mmm-level decision rule will mean using the mmm-threshold and considering the mmm-level soft set in IVNP-soft sets based decision making.

Definition 4.3 Let ∈ IVNPS and an = [,s], = [t ,t], = [q_{mid}^-,q]⊆ [0,1] which is called a threshould of IVNSP-soft set . The level soft set of with respect to (s ,t_{mid},q_{mid}) is a crisp soft set, denoted by (Ψ_K; (s ,t_{mid},)),defined by;

(; (s ,t ,q)) = {((x_i, (x_i)) :x_i ∈ E}

where,

),s),t),t)and),q

=$\sum_{x\in}$ |, =$\sum_{x\in}$

=$\sum_{x\in}$ |, =$\sum_{x\in}$

=∑x∈ |, =∑x∈

For all $x_i \in E^*$, where if $x_i \in E/E^*$ then $\Psi_K(x_i) = \emptyset$.
The (s_{mid},t_{mid} ,q_{mid}) is called the mmm-threshold of the IVNP-soft set Ψ_K. In the following discussions, the mid-level decision rule will mean using the mid-threshold and considering the mid-level soft set in IVNP-soft sets based decision making.

Definition 4.4 Let ∈ IVNPS and an = [,s], = [,t], = [,q^+_{min}]⊆ [0, 1] which is called a threshould of IVNSP-soft set . The level soft set of Ψ with respect to (s_{max},t_{min} ,q_{min}) is a crisp soft set, denoted by (Ψ_K; (s ,t_{min} ,q_{min})),defined by;

(; (s ,t ,q)) = {((x_i, (x_i)) :x_i ∈ E}

where,

,s ,t ,t and ,q

= sup{μ :x_i ∈ E}, = sup{ :x_i ∈ E}
= inf{ν :x_i ∈ E}, = inf{ :x_i ∈ E}
= inf{ω :x_i ∈ E}, = inf{ :x_i ∈ E}

The (s_{max},t_{min} ,q_{min}) is called the Mmm-threshold of the IVNP-soft set Ψ_K . In the following discussions, the Mmm-level decision rule will mean using the Mmm-threshold and considering the Mmm-level soft set in IVNP-soft sets based decision making.

Definition 4.5 Let Ψ_K ∈ IVNPS. The threshold based on median could be expressed as a function
: A→$[0,1]^3$, i.e. s_{med}=[s^-_{med},s^+_{med}], t_{med}= [t^-_{med},t^+_{med}],q_{med}= [q^-_{med},q^+_{med}] ⊆ [0,1] for all ε ∈ A, where for ∀ε ∈ A,s^-_{med},s^+_{med} is the median by ranking the degree of interval truth membership of all alternatives according to order from large to small (or from small to large), namely

=

=

,t is the median by ranking the degree of interval indeterminacy membership of all alternatives according to order from large to small (or from small to large), namely

=

=

And q^{-}_{med} ,q is the median by ranking the interval degree of falsity membership of all alternatives according to order from large to small (or from small to large), namely

=

=

The (s_{med},t_{med} ,q_{med}) is called themed-threshold of the IVNP-soft set Ψ_K . In the following discussions, the Med-level decision rule will mean using the Med-threshold and considering the Med-level soft set in IVNP-soft sets based decision making.

Example 4.6 Ψ_K= {(<x_1 ,[0.1,0.5], [0.4, 0.5] ,[0.2 ,0.3]>,{u ,u_2 }),(<x_2 ,[0.2,0.3], [0.5, 0.7], [0.1, 0.3]>,{u_2 ,u_3 }),(<x_3,[0.1,0.3], [0.1, 0.7] ,[0.3 ,0.4]>,{u_1 ,u ,u_3 })}
Then

= [0.13, 0.36], = [0.33, 0.63], = [0.2 0.43]
= [0.1, 0.3], = [0.1, 0.5], q = [0.1,0.3]
= [0.2, 0.5], = [0.1, 0.5],q = [0.1,0.3]
= [0.2, 0.3], t = [0.5, 0.7], q = [0.1, 0.3]

Theorem 4.7. Let ∈ IVNP-soft set (Ψ_K; (s_{mid},t_{mid} ,q_{mid})), (Ψ_K; (s_{min},t_{min} ,q_{min}))and (; (s ,t_{min} ,q_{min})) be the mid-level soft set, max –level soft set and min –level soft set of ,respectively. Then,

1. (; (s ,t ,q)) ⊆(; (s ,t ,q))
2. (; (s ,t ,q)) (; (s ,t ,q))

Proof. Let ∈ IVNPSS.From definition, definition and definition,it can be seen that
, $t^{-}_{min} \leq t^{-}_{mid} \leq t^{-}_{max}$ and $q^{-}_{min} \leq q^{-}_{mid} \leq q^{-}_{max}$.

Thus

i. for all ∈ E which providing the inequalities

, $(x_i, \Psi_K(x_i) \notin (\Psi_K; (s_{max}, t\ \ , q\ \ ))$.

So, $(\Psi_K; (s\ \ , t\ \ , q\ \ )) \subseteq(\ \ ; (s\ \ , t\ \ , q\ \ ))$

ii. it can be proved similar way

Now, we construct an IVNP –soft sets decision making method by the following algorithm;

Algorithm:

Step 1. Input the IVNP –soft sets -soft set Ψ

Step 2. Input a threshold ($s_{mid}, t_{mid}, q_{mid}$) (or ($s_{max}, t_{min}, q_{min}$),($s_{min}, t_{min}, q_{min}$)) by using mid –level decision rule (or Mmm-level decision rule, mmm-level decision rule) for decision making.

Step 3. Compute mid-level soft set (; (s ,t ,q)) (or Mmm-level soft set (; (s ,t ,q)) ,mmm –level soft set (; (s ,t ,q_{min}), Med –level soft set (; (s ,t ,q))

Step 4. Present the level soft set (Ψ_K;(s ,t ,q)) (or the level soft set (Ψ_K; (s ,t ,q)), the level soft set (Ψ_K;(s ,t ,q), Med–level soft set (Ψ_K; (s ,t ,q)))in tabular form.

Step 5. Compute the choice value c_i of for any u_i∈ U ,

Step 6. The optimal decision is to select$u$ if c_k = $\max_{u_i\ \in}$

Remark If k has more than one value then any one of u_k may be chosen. If there are too many optimal choices in Step 6, we may go back to the second step and change the threshold (or decision rule) such that only one optimal choice remains in the end.

Example 4.8. Assume that a company wants to fill a position. There are 4 candidates who fill in a form in order to apply formally for the position. There is a decision maker (DM) that is from the department of human resources. He wants to interview the candidates, but it is very difficult to make it all of them. Therefore, by using the parameter reduction method, the numbers of candidates are reduced to a suitable one. Assume that the set of candidates U={u_1, u_2, u_3, u_4,u_5, u_6, u_7} which may be characterized by a set of parameters E={x_1, x_2, x ,x_4, x_5, x_6} For i=1,2,3,4,5,6 the parameters i stand for experience, computer knowledge, training, young age, diction and flexible working hours compatible, respectively. Now, we can apply the method as follows:

Step 1. After thinking thoroughly, he/she evaluates the alternative according to choosing parameters and constructs an IVNP-soft set $\Psi_K$as follows

= {<x_1, ([0.6, 0.8], [0.1, 0.2], [0.3, 0.5])>,{u_1,u_4,u_5,u_7}), (<x_2,([0.5, 0.6], [0.3, 0.4], [0.2, 0.3])>,{u_2,u_5,u_6,u_8}),(<x_3, ([0.4, 0.5], [0.3, 0.4], [0.1 ,0.4])>, {u_2,u_3,u_6,u_7}), (<x_4, ([0.1, 0.2], [0.4, 0.8], [0.4 ,0.5])>, {u_1,u_4,u_5,u_8}}, (<x_5, ([0.4, 0.5], [0.2, 0.4], [0.3, 0.6])>, {u_3,u_6,u_7})},(<x_6, ([0.7, 0. 7], [0.1, 0.3], [0.1, 0.5])>, {u_2,u_4,u_8}})}

Step 2. Then, we have

, = [0.45, 0.655],t = [0.23, 0.41] ,q = [0.23, 0.46]

, = [0.7, 0.8],t = [0.1, 0.2] ,q = [0.1, 0.3]

, = [0.1, 0.2], = [0.1, 0.2], q = [0.1, 0.3]
, = [0.25, 0.35], = [0.35, 0.6] ,q = [0.25, 0.45]

Step 3. Thus, the (s ,t_{med},q_{med})-level soft set of is (after the necessary calculations, they can be seen that (s ,t ,q_{min})-level soft set,(s ,t ,q)-level soft set, and (s_{mid},t ,q)- level soft set of are not suitable for decision making in this problem.)
(Ψ_K; (s ,t ,q)) = {(<x_2,([0.5, 0.6], [0.3, 0.4], [0.2 ,0.3])>,{u_2,u_5,u_6,u_8}),

(<x_3, ([0.4, 0.5], [0.3, 0.4], [0.1, 0.4])>, {u_2,u_3,u_6,u_7})}

Step 4. Tabular form of (Ψ_K; (s ,t ,q)) is

u								
	0	1	0	0	1	1	0	1
	0	1	1	0	0	1	1	0

Step 5. Then, we have the choice value c_i for i = 1, 2, 3,…,8
= 0,c_2= 2,c_3= 1,c_4= 0,c_5= 1,c_6= 2,c_7= 1 andc_8=1

Step 6. So,the optimal decision is $u_2$or u
Note that this decision making method can be applied for group decision making easily with help of the definition 3.19.

5. Conclusions

In this work, we have introduced the concept of interval valued neutrosophic parameterized soft set and studied some of its properties. The complement, union and intersection operations have been defined on the interval valued neutrosophic parameterized soft set. The definition of parameter reduction method is introduced with application of this operation in decision making problems.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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Published in *Journal of New Research in Science*, No. 7 (2014), pp. 58-71, 14 p.

To be and Not to be – An introduction to Neutrosophy: A Novel Decision Paradigm

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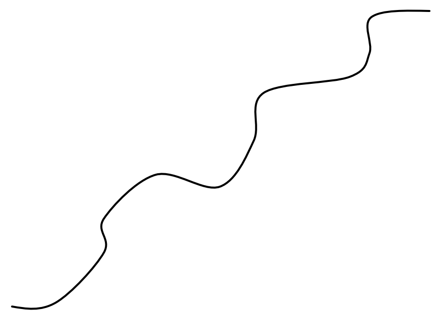
The Need for a Novel Decision Paradigm in Management

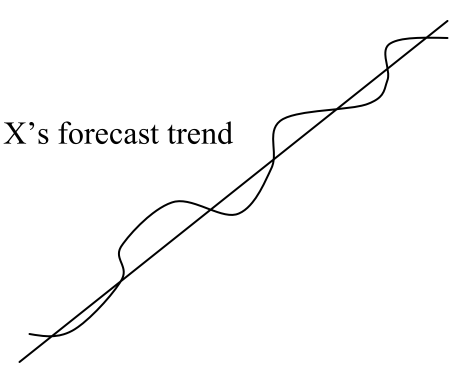
- The process of scientific decision-making necessarily follows and input-output system
- The primary input is in the form of raw data (quantitative, qualitative, or both)
- This raw data is subsequently "cleaned", "filtered", and "organized" to yield information
- The available information is then processed accordingly to either (a) very well-structures, "hard" rules, or (b) partially-structured "semi-soft" rules, or (c) almost completely unstructures "soft" rules
- The output is the final decision which may be a relatively simple and routine one such as deciding on an optimal inventory re-ordering level of a much more complex and involved such as discounting a product line or establishing a new SBU. It has been observed that most of these complex and involved decision problems are those that need to be worked out using the "soft" rules of information processing
- Besides being largely subjective, "soft" decision rules are often ambiguous, inconsistent and even contradictory
- The main reason is that the event spaces governing complex decision problems are not completely known. However, the human mind abhors incompleteness when it comes to complex cognitive processing. The mind invariably tries to "fill in the blanks" whenever it encounters incompleteness
- Therefore, when different people form their own opinions from a given set of incomplete information, it is only to be expected that there will be areas of inconsistency, because everybody will try to „complete the set" in their own individual ways, governed by their own subjective utility preferences

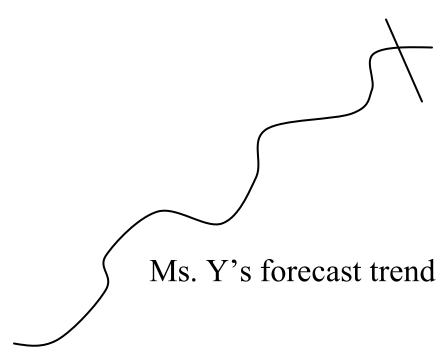
- Looking at the following temporal trajectory of the market price of a share in ABC Corp. over the past thirty days, would it be considered advisable to invest in this asset?
- The "hard" decision rule applicable in this case is that "one should buy and asset when its price is going up and one should sell an asset when its price is going down"



- The share price as shown above is definitely trending in a particular direction. But will the observred trend over the past thirty days continue in the future? It is really very hard to say because most financial analysts will find this information rather inadequate to arrive at an informal judgement
- Although this illustration is purely anecdotal, it is nevertheless a matter of fact that the world of managerial decision-making is fraught with such inadequacies and „complete information" is often an unaffordable luxury
- The more statiscally minded decison-takers would try to forecast the future direction of the price trend of a share in ABC Corp. from the given (historical) information
- The implied logic is that the more accurate this forecast the more profitable will be the oucome resulting from the decision
- Let us take two financial analyst Mr. X and Ms Y trying to forecast the price of a share in ABC Corp. To fit their respective trendlines, Mr. X considers the entire thirty days of data while Ms Y (who knows about Markovian property of stock prices) considers only the price movement over a single day

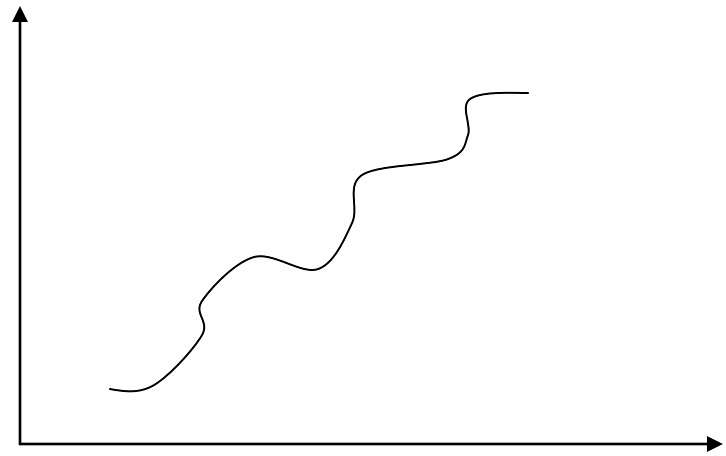
Mr. X's forecast trend





Ms. Y's forecast trend

- Who do you think is more likely to make the greater profit?



- Most people will have formed their opinions after having made spontaneous assumption about the orientation of the coordinate axes i.e. the temporal order of the price data! This is an example of how our minds sub-consciously complete an “incomplete set” of information prior to cognitive processing
- Obviously, without a definite knowledge about the orientatopm of the axws it is impossible to tell who is more likely to make a greater profit. This has nothing to do with which one of Mr. X or Ms. Y has the better forecasting model. In fact it is a somewhat paradoxical situation – we may know who among Mr. X and Ms. Y has a technically better forecasting model and yet not know who will make more profit! That will remain indeterminate as long as the exact orientation of the two coordinate axes is unknown!
- The neutrosophic probability approach makes a distinction between “relative sure event”, event that is true only in certain world(s) and “absolute sure event”, event that is true for all possible world(s)
- Similar relations can be drawn for “relative impossible event” / “absolute impossible event” and “relative indeterminate event” / “absolute indeterminate event”
- In case where the truth- and falsity- components are complimentary i.e. they sum up the unity and there is no indeterminacy, then one is reduced to classical probability. Therefore, neutrosophic probability may be viewed as a *three-way generalization* of classical and imprecise probabilities

- In our little anecdotal illustration, we may visualize a world where stock prices follow a Markovian path *and* Ms. Y knows the correct orientation of the coordinate axes. That Ms. Y will make a greater profit thereby becomes a *relative sure event* and that Mr. X will make a greater profit becomes a *relative impossible event*.
- Similarly we may visualize a different world where stock prices follow a linear path *and* Mr. X knows the correct orientation of the coordinate axes. That Mr. X will make a greater profit thereby becomes a *relative sure event* and that Ms. Y will make a greater profit becomes a *relative impossible event*
- Then there is our present world where we have no knowledge at all as to the correct orientation of the coordinate axes and hence both become relative indeterminate events!
- Because real-life managers have to mostly settle for "incomplete sets" of information, the arena of managerial decision-making is replete with such instances of paradoxes and inconsistencies. This is where neutrosophy can play a very significant role as a novel addition to the managerial decision paradigm!

Neutrosophy

A new branch of philosophy which studiues the origin, nature, and scope of neutralities, as well as their interactions with different ideational spectra (1995);

Extension of dialectics;

The Fundamental Theory: Every idea <A> tends to be neutralized, diminished, balanced by <NonA> ideas (not only <AntiA> as Hegel asserted) – as a state of equilibrium

<NonA> = what is not <A>

<AntiA> = the opposite of <A>

<NeutA> = what is neither <A> nor <AntiA>;

Basement for Neutrosophical Logic, Neutrosophic Set, Neutrosophic Probability

Applications of Neutrosophy to Indian Philosophy

In India's VIIIth – IXth centuries one promulgated the Non-Duality (*Advaita*) through the non-differentiation between Individual Being (*Atman*) and Supreme Being (*Brahman*). The philosopher Sankaracharya (782-814 AC) was then considered the savior of Hinduism, just in the moment when the Buddhism and the Jainism were in a severe turmoil and India was in a spiritual crisis. Non-Duality means elimination of ego, in order to blend yourself with the Supreme Being (to reach the happiness).

Or, arriving to the Supreme was done by Prayer (*Bhakti*) or Cognition (*Jnana*). It is part of Sankaracharya's huge merit (*charya* means teacher) the originality of interpreting and synthesizing the Source of Cognition (*Vedas*, IVth century BC), the Epic (with many stories), and the *Upanishads* (principles of Hindu philosophy) concluding in Non-Duality.

Then Special Duality (Visishta Advaita) follows, which asserts that Individual Being and Supreme Being are different in the beginning, but end to blend themselves (Ramanujacharya, XIth century).

And later, to see that the neutrosophic scheme perfectly functions, Duality (Dvaita) ensues, through whom the Individual Being and Supreme Being were differentiated (Madvacharya, XIIIth – XIVth centuries).

Thus, Non-Duality converged to Duality, i.e. <NonA> converges through <NeutA> to <A>.

Introduction to Nonstandard Analysis

- Abraham Robinson developed the nonstandard analysis (1960s)
- x is called *infinitesimal* if |x|<1/n for any positive n
- A *left monad* (^-a) = {a-x: x∈R*, x>0 infinitesimal} = a-ε
  and a *right monad* (b^-) = {a+x: x∈R*, x>0 infinitesimal} = b+ε
  where ε>0 is infinitesimal;
  a, b called *standard parts*, ε called *nonstandard part*.
- <u>Operations with nonstandard finite real numbers</u>:

$^-a*b = {}^-(a*b)$, $a*b^+ = (a*b)^+$, $^-a*b^+ = {}^-(a*b)^+$,
$^-a*^-b = {}^-(a*b)$ [the left monads absorb themselves],
$a^+*b^+ = (a*b)^+$ [the right monads absorb themselves],
where "*" can be addition, subtraction, multiplication, division, power.

Operations with Classical Sets

S1 and S2 two real standard or nonstandard sets.

→ Addition: $S_1 \oplus S_2 = \{x \mid x=s_1+s_2$, where $s_1 \in S_1$ and $s_2 \in S_2\}$

→ Substraction: $S_2 = \{x \mid x=s_1+s_2$, where $s_1 \in S_1$ and $s_2 \in S_2\}$

→ Multiplication: $S_1 \ominus S_2 = \{x \mid x=s_1-s_2$, where $s_1 \in S_1$ and $s_2 \in S_2\}$

Let $k \in \mathbb{R}^*$, then $S_1 \oslash k = \{x \mid x=s_1/k$, where $s_1 \in S_1\}$

Neutrosophic Logic

- Consider the *nonstandard unit interval* $]^-0, 1^+[$, with left and right borders vague, imprecise
- Let T, I, F be standard or nonstandard subsets of $]^-0, 1^+[$
- Neutrosophic Logic (NL) is a logic in which each proposition is T% true, I% indeterminate, and F% false
- $^-0 \leq \inf T + \inf I + \inf F \leq \sup T + \sup I + \sup F \leq 3^+$
- T, I, F are not necessary intervals, but any sets (discrete, continuous, open or closed or half-open/half-closed interval, intersections or unions of the previous sets, etc.)
- Example: proposition P is between 30-40% or 45-50% true, 20% indeterminate, and 60% or between 66-70% false (according to various analyzers or parameters)
- NL is a generalization of Zadeh's *fuzzy logic* (FL), especially of Atanassov's *intuitionistic fuzzy logic* (IFL), and other logics

Differences between Neutrosophic Logic and Intuitionistic Fuzzy Logic

- In NL there is no restriction on T, I, F, while in IFL the sum of components (or their superior limits) = 1;
  thus NL can characterize the *incomplete information* (sum < 1), *paraconsistent information* (sum > 1).
- NL can distinguish, in philosophy, between *absolute truth* [NL(absolute truth)=1^+] and *relative truth* [NL(relative truth)=1], while IFL cannot;
  absolute truth is truth in all possible worlds (Leibniz),
  relative truth is truth in at least one world.
- In NL the components can be nonstandard, in IFL they don't.
- NL, like *dialetheism* [some contradictions are true], can deal with paradoxes, NL(paradox) = (1,I,1), while IFL cannot.

Neutrosophic Logic generalizes many Logics - I

Let the components reduced to scalar numbers, t,i,f, with t+i+f=n; NL generalizes:

- the *Boolean logic* (for n = 1 and i = 0, with t, f either 0 or 1);
- the *multi-valued logic*, which supports the existence of many values between true and false [Lukasiewicz, 3 values; Post, m values] (for n = 1, i = 0, $0 \le t, f \le 1$);
- the *intuitionistic logic*, which supports incomplete theories, where $A \vee \neg A$ not always true, and $\exists x P(x)$ needs an algorithm constructing x [Brouwer, 1907]
  (for $0 < n < 1$ and i = 0, $0 \le t, f < 1$);
- the *fuzzy logic*, which supports degrees of truth [Zadeh, 1965] (for n = 1 and i = 0, $0 \le t, f \le 1$);
- the *intuitionistic fuzzy logic*, which supports degrees of truth and degrees of falsity while what's left is considered indeterminacy [Atanassov, 1982] (for n = 1);
- the *paraconsistent logic*, which supports conflicting information, and 'anything follows from contradictions' fails, i.e. $A \wedge \neg A \supset B$ fails; $A \wedge \neg A$ is not always false
  (for n > 1 and i = 0, with both $0 < t, f < 1$);
- the *dialetheism*, which says that some contradictions are true, $A \wedge \neg A$=true (for t = f = 1 and i = 0; some paradoxes can be denoted this way too);
- the *faillibilism*, which says that uncertainty belongs to every proposition (for i > 0);

Neutrosophic Logic Connectors

$A_1(T_1, I_1, F_1)$ and $A_2(T_2, I_2, F_2)$ are two propositions.

1. Negation:

$NL(\neg A_1) = (\{1\}\ominus T_1, \{1\}\ominus I_1, \{1\}\ominus F_1)$

2. Conjunction:

$NL(A_1 \wedge A_2) = (T_1\odot T_2, I_1\odot I_2, F_1\odot F_2)$.
(And, in a similar way, generalized for n propositions.)

3. Weak or inclusive disjunction:

$NL(A_1 \vee A_2) = (T_1\oplus T_2\ominus T_1\odot T_2, I_1\oplus I_2\ominus I_1\odot I_2, F_1\oplus F_2\ominus F_1\odot F_2)$.
(And, in a similar way, generalized for n propositions.)

4. Strong or exclusive disjunction:

$$\begin{aligned} NL(A_1 \veebar A_2) = \; & (T_1\odot(\{1\}\ominus T_2)\oplus T_2\odot(\{1\}\ominus T_1)\ominus T_1\odot T_2\odot(\{1\}\ominus T_1)\odot(\{1\}\ominus T_2), \\ & I_1\odot(\{1\}\ominus I_2)\oplus I_2\odot(\{1\}\ominus I_1)\ominus I_1\odot I_2\odot(\{1\}\ominus I_1)\odot(\{1\}\ominus I_2), \\ & F_1\odot(\{1\}\ominus F_2)\oplus F_2\odot(\{1\}\ominus F_1)\ominus F_1\odot F_2\odot(\{1\}\ominus F_1)\odot(\{1\}\ominus F_2)). \end{aligned}$$

(And, in a similar way, generalized for n propositions.)

5. Material conditional (implication):

$NL(A_1 \mapsto A_2) = (\{1\}\ominus T_1\oplus T_1\odot T_2, \{1\}\ominus I_1\oplus I_1\odot I_2, \{1\}\ominus F_1\oplus F_1\odot F_2)$.

6. Material biconditional (equivalence):

$$\begin{aligned} NL(A_1 \leftrightarrow A_2) = (& (\{1\}\ominus T_1\oplus T_1\odot T_2)\odot(\{1\}\ominus T_2\oplus T_1\odot T_2), \\ & (\{1\}\ominus I_1\oplus I_1\odot I_2)\odot(\{1\}\ominus I_2\oplus I_1\odot I_2), \\ & (\{1\}\ominus F_1\oplus F_1\odot F_2)\odot(\{1\}\ominus F_2\oplus F_1\odot F_2)) \end{aligned}$$

7. Sheffer's connector:

$NL(A_1 \mid A_2) = NL(\neg A_1 \vee \neg A_2) = (\{1\}\ominus T_1\odot T_2, \{1\}\ominus I_1\odot I_2, \{1\}\ominus F_1\odot F_2)$.

8. Peirce's connector:

$$\begin{aligned} NL(A_1 \downarrow A_2) &= NL(\neg A_1 \wedge \neg A_2) = \\ &= ((\{1\}\ominus T_1)\odot(\{1\}\ominus T_2), (\{1\}\ominus I_1)\odot(\{1\}\ominus I_2), (\{1\}\ominus F_1)\odot(\{1\}\ominus F_2)). \end{aligned}$$

Many properties of the classical logic operators do not apply in neutrosophic logic.

Neutrosophic logic operators (connectors) can be defined in many ways according to the needs of applications or of the problem solving.

Neutrosophic Set (NS)

- Let U be a universe of discourse, M a set included in U. An element x from U is noted with respect to the **neutrosophic set** M as x(T, I, F) and belongs to M in the following way:
it is t% true in the set (*degree of membership*),
i% indeterminate (unknown if it is in the set) (*degree of indeterminacy*),
and f% false (*degree of non-membership*),
where t varies in T, i varies in I, f varies in F.
- Definition analogue to NL
- Generalizes the fuzzy set (FS), especially the intuitionistic fuzzy set (IFS), intuitionistic set (IS), paraconsistent set (PS)
- <u>Example</u>: x(50,20,40)∈A means: with a believe of 50% x is in A, with a believe of 40% x is not in A, and the 20% is undecidable

Neutrosophic Set Operators

A and B two sets over the universe U.
An element $x(T_1, I_1, F_1) \in A$ and $x(T_2, I_2, F_2) \in B$ [*neutrosophic membership appurtenance* to A and respectively to B].
NS operators (similar to NL connectors) can also be defined in many ways.

1. Complement of A:

If $x(T_1, I_1, F_1) \in A$,
then $x(\{1^+\} \ominus T_1, \{1^+\} \ominus I_1, \{1^+\} \ominus F_1) \in C(A)$.

2. Intersection:

If $x(T_1, I_1, F_1) \in A$, $x(T_2, I_2, F_2) \in B$,
then $x(T_1 \odot T_2, I_1 \odot I_2, F_1 \odot F_2) \in A \cap B$.

3. Union:

If $x(T_1, I_1, F_1) \in A$, $x(T_2, I_2, F_2) \in B$,
then $x(T_1 \oplus T_2 \ominus T_1 \odot T_2, I_1 \oplus I_2 \ominus I_1 \odot I_2, F_1 \oplus F_2 \ominus F_1 \odot F_2) \in A \cup B$.

4. Difference:

If $x(T_1, I_1, F_1) \in A$, $x(T_2, I_2, F_2) \in B$,
then $x(T_1 \ominus T_1 \odot T_2, I_1 \ominus I_1 \odot I_2, F_1 \ominus F_1 \odot F_2) \in A \setminus B$,
because $A \setminus B = A \cap C(B)$.

Differences between Neutrosophic Set and Intuitionistic Fuzzy Set

- In NS there is no restriction on T, I, F, while in IFS the sum of components (or their superior limits) = 1;
  thus NL can characterize the *incomplete information* (sum < 1), *paraconsistent information* (sum > 1).
- NS can distinguish, in philosophy, between *absolute membership* [NS(absolute membership)=1^+] and *relative membership* [NS(relativemembership)=1], while IFS cannot;
  absolute membership is membership in all possible worlds,
  relative membership is membership in at least one world.
- In NS the components can be nonstandard, in IFS they don't.
- NS, like *dialetheism* [some contradictions are true], can deal with paradoxes, NS(paradox element) = (1,I,1), while IFS cannot.
- NS operators can be defined with respect to T,I,F while IFS operators are defined with respect to T and F only
- I can be split in NS in more subcomponents (for example in Belnap's four-valued logic (1977) indeterminacy is split into uncertainty and contradiction), but in IFS it cannot

Applications of Neutrosophic Logic - I

Voting (pro, contra, neuter):

- The candidate C, who runs for election in a metropolis M of p people with right to vote, will win.

  This proposition is, say, 20-25% true (percentage of people voting for him), 35-45% false (percentage of people voting against him), and 40% or 50% indeterminate (percentage of people not coming to the ballot box, or giving a blank vote - not selecting anyone, or giving a negative vote - cutting all candidates on the list).

Epistemic/subjective uncertainty (which has hidden/unknown parameters).

- Tomorrow it will rain.

  This proposition is, say, 50% true according to meteorologists who have investigated the past years' weather, between 20-30% false according to today's very sunny and droughty summer, and 40% undecided.

Paradoxes:

- This is a heap (Sorites Paradox).

  We may now say that this proposition is 80% true, 40% false, and 25-35% indeterminate (the neutrality comes for we don't know exactly where is the difference between a heap and a non-heap; and, if we approximate the border, our 'accuracy' is subjective). Vagueness plays here an important role.
- The Medieval paradox, called Buridan's Ass after Jean Buridan (near 1295-1356), is a perfect example of complete indeterminacy. An ass, equidistantly from two quantitatively and qualitatively heaps of grain, starves to death because there is no ground for preferring one heap to another.

  The neutrosophic value of ass's decision, NL = (0, 1, 0).

Games (win, defeated, tied).

Electrical charge, temperature, altitude, numbers, and other 3-valued systems (positive, negative, zero)

Business (M. Khoshnevisan, S. Bhattacharya):

- Investors who are: Conservative and security-oriented (*risk shy*), Chance-oriented and progressive (*risk happy*), or Growth-oriented and dynamic (*risk neutral*).

Applications of Neutrosophic Sets

<u>Philosophical Applications</u>:

- Or, how to calculate the truth-value of Zen (in Japanese) / Chan (in Chinese) doctrine philosophical proposition: the present is eternal and comprises in itself the past and the future?
- In Eastern Philosophy the contradictory utterances form the core of the Taoism and Zen/Chan (which emerged from Buddhism and Taoism) doctrines.
- How to judge the truth-value of a metaphor, or of an ambiguous statement, or of a social phenomenon which is positive from a standpoint and negative from another standpoint?

<u>Physics Applications</u>:

- How to describe a particle ξ in the infinite micro-universe of Quantum Physics that belongs to two distinct places P_1 and P_2 in the same time? $\xi \in P_1$ and $\xi \notin P_1$ as a true contradiction, or $\xi \in P_1$ and $\xi \in \neg P_1$.
- Don't we better describe, using the attribute "neutrosophic" than "fuzzy" and others, a quantum particle that neither exists nor non-exists? [high degree of indeterminacy]
- In Schroedinger's Equation on the behavior of electromagnetic waves and "matter waves" in Quantum Theory, the wave function Psi which describes the superposition of possible states may be simulated by a neutrosophic function, i.e. a function whose values are not unique for each argument from the domain of definition (the vertical line test fails, intersecting the graph in more points).
- A cloud is a neutrosophic set, because its borders are ambiguous, and each element (water drop) belongs with a neutrosophic probability to the set (e.g. there are a kind of separated water drops, around a compact mass of water drops, that we don't know how to consider them: in or out of the cloud).

More Applications of Neutrosophic Logic and Neutrosophic Set

- Mohammad Khoshnevisan and Sukanto Bhattacharya in finance, business
- Haibin Wang, F. Smarandache, Yanqing Zhang, Rajshekhar Sunderraman in engineering
- A. Tchamova, F. Smarandache, J. Dezert in information fusion, cybernetics, medicine, military
- Anne-Laure Jousselme, Patrick Maupin in situation analysis

A few specific applications of neutrosophics in business and economics

- **Application of Neutrosophics as a situation analysis tool**
- **Application of Neutrosophics in the reconciliation of financial market information**
- **Application of Neutrosophics in Production Facility Layout Planning and Design**

Neutrosophics as a situation analysis tool

- In situation analysis (SA), an agent observing a scene receives information from heterogeneous sources of information including for example remote sensing devices, human reports and databases. The aim of this agent is to reach a certain awareness about the situation in order to take decisions
- Considering the logical connection between belief and knowledge, the challenge for the designer is to transform the raw, imprecise, conflicting and often paradoxical information received from the different sources into statements understandable by both man and machines
- Hence, two levels of processing coexist in SA: measuring of the world and reasoning about the world. Another great challenge in SA is the reconciliation of both aspects. As a consequence, SA applications need frameworks general enough to take into account the different types of uncertainty present in the SA context, doubled with a semantics allowing reasoning on situations
- A particularity of SA is that most of the time it is impossible to list every possible situation that can occur. Corresponding frames of discernment cannot, thus, be considered as exhaustive
- Furthermore, in SA situations are not clear-cut elements of the frames of discernment. Considering these particular aspects of SA, a neutrosophic logic paradigm incorporating the *Dezert-Smarandache theory* (DSmT) appears as an appropriate modeling tool
- It has been recently shown that the neutrosophic logic paradigm does have the capacity to cope with the epistemic and uncertainty-related problems of SA
- In particular, it has been formally demonstrated that the neutrosophic logic paradigm incorporating DSmT has the ability to process symbolic and numerical statements on belief and knowledge using the *possible worlds* semantics (Jousselme and Maupin, 2004)

Applications of Neutrosophics in the reconciliation of financial market information

- In the classical, *Black-Scholes* world, if stock price volatility is known *a priori*, the prices of long-term option contracts are completely determinable and any deviations are deemed to be quickly arbitraged away. Therefore, when a long-term option priced by the collective action of the market agents is observed to be deviating from the theoretical price, the following three possibilities must be considered:

- (1) The theoretical price is obtained by an inadequate pricing model, which means that the observed market price may well *be* the true price, i.e. ***Observed Price* = *Theoretical Price*** $\pm \varepsilon$; (where ε is the ***systematic error*** in valuation that will result whenever any option is evaluated by the inadequate pricing model)

- (2) The theoretical pricing model is valid but a largely irrational buying/selling behavior by a group of market agents has temporarily pushed the market price of a particular option j 'out of sync' with its true price, i.e. ***(Observed Price)***$_j$ **= *(True Price)***$_j \pm \varepsilon_j$; (where ε_j is the ***unsystematic error*** in valuation specific to option j) or

- (3) The nature of the deviation is indeterminate and could be due to either (a) or (b) or even a super-position of both (a) and (b) and/or due to some random white noise

- With T, I, F as the neutrosophic components, let us now define the following two apparently mutually inconsistent events:

H = {p: p is the true option price determined by the theoretical pricing model}; and

M = {p: p is the true option price determined by the prevailing market price}

- Then there is a t% chance that the event ($H \cap M^c$) is true, or corollarily, the corresponding complimentary event ($H^c \cap M$) is untrue i.e. the best dětereminant of the true market price is the theoretical pricing model

- There is a f% chance that the event ($M^c \cap H$) is untrue, or corollarily, the complimentary event ($M \cap H^c$) is true i.e. the best detereminant of the true market price is the observed market price and;

- There is a i% chance that neither ($H \cap M^c$) nor ($M \cap H^c$) is true/untrue; i.e. the best determinant of the true market price is indeterminate!

- Illustratively, a set of AR1 models used to extract the mean reversion parameter driving the volatility process over time have *coefficients of determination* in the range say between 50%-70%, then we can say that t varies in the set T (50% - 70%).

- If the subjective probability assessments of well-informed market agents about the weight of the current excursions in implied volatility on short-term options lie in the range say between 40%-60%, then f varies in the set F (40% - 60%).

- The unexplained variation in the temporal volatility driving process together with the subjective assessment by the market agents will make the event indeterminate say by either 30% or 40%

- Then the neutrosophic probability of the true price of the option being determined by the theoretical pricing model is formally representable as follows: **NP ($H \cap M^c$) = [(50 – 70), (40 – 60), {30, 40}]**. The *Dezert-Smarandache formula* can be used in cases like these to *fuse* the conflicting sources of information and arrive at a correct and computable probabilistic assessment of the true price of the long-term option

Applications of Neutrosophics in Production Facility Layout Planning and Design

- The original CRAFT (Computerized Relative Allocation of Facilities Technique) model for cost-optimal relative allocation of production facilities as well as many of its later extensions tend to be quite "heavy" in terms of CPU engagement time due to their heuristic nature

- A Modified Assignment (MASS) model (first proposed by Bhattacharya and Khoshnevisan in 2003) increases the computational efficiency by developing the facility layout problem as primarily a *Hungarian assignment* problem but becomes indistinguishable from the earlier CRAFT-type models beyond the initial configuration

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- However, some amount of introspection will reveal that the production facilities layout problem is basically one of achieving *best interconnectivity by optimal fusion of spatial information*. In that sense, the problem may be better modeled in terms of mathematical information theory whereby *the best layout is obtainable as the one that maximizes relative entropy of the spatial configuration*

- Going a step further, one may hypothesize a neutrosophic dimension to the problem. Given a combination rule like the *Dezert-Smarandache formula*, the layout optimization problem may be formulated as a *normalized basic probability assignment* for optimally comparing between several alternative interconnectivities

- The neutrosophic argument can be justified by considering the very practical possibility of conflicting bodies of evidence for the structure of the load matrix possibly due to conflicting assessments of two or more design engineers

- If for example we consider two mutually conflicting bodies of evidence Ξ_1 and Ξ_2, characterized respectively by their basic probability assignments μ_1 and μ_2 and their cores k (μ_1) and k (μ_2) then one has to look for the optimal combination rule which maximizes the joint entropy of the two conflicting information sources

- Mathematically, it boils down to the general optimization problem of evaluating **– min**$_\mu$ **[–H (μ)]** subject to the constraints that (a) the marginal basic probability assignments μ_1 (.) and μ_2 (.) are obtainable by the summation over each column and summation over each row respectively of the relevant information matrix and (b) the sum of all cells of the information matrix is unity

Dialectics and the Dao:
On Both, A and Non-A in Neutrosophy and Chinese Philosophy

Feng Liu

Florentin Smarandache

This paper introduces readers to a new approach to dialectical logic: neutrosophy. Specifically it proposes a multi-valued logic in which the statement "both A and Non-A," historically rejected as logically incoherent, is treated as meaningful. This unity of opposites constitutes both the objective world and the subjective world –a view with deep roots in Buddhism and Daoism, including the *I-Ching*. This leads in turn to the presentation of a framework for the development of a contradiction oriented learning philosophy inspired by the Later Trigrams of King Wen in the *I-Ching*. We show that although A and Non-A are *logically* inconsistent, they can be understood to be *philosophically* consistent. Indeed, recognition of their consistency is the basis for freeing ourselves from the mental confusion which results from taking as real what are in fact just mental impressions.

1. Neutrosophy

Neutrosophy is a new branch of philosophy that studies the origin, nature, and scope of neutralities, as well as their interactions with different ideational spectra. It is the basis of *neutrosophic logic*, a multi-valued logic that generalizes fuzzy logic and deals with paradoxes, contradictions, antitheses, and antinomies. The characteristics of this mode of thinking are as follows: Neutrosophy

- reveals that world is full of indeterminacy;
- interprets the uninterpretable;
- regards, from many different angles, old concepts and systems, showing that an idea which is true in a given system of reference , may be false in another one, and vice versa,
- attempts to make peace in the war of ideas and to make war on peaceful ideas, and
- measures the stability of unstable systems the and instability of stable systems.

Let's denote by <A> an idea, or proposition, theory, event, concept, entity, by <Non-A> what is not <A>, and by <Anti-A> the opposite of <A>. Also, <Neut-A> means what is neither <A> nor <Anti-A>, i.e. neutrality in between the two extremes. <A'> is a version of <A>. Note that <Non-A> is different from <Anti-A>.

The Main Principle of Neutrosophy:

Between an idea <A> and its opposite <Anti-A>, there is a continuum-power spectrum of neutralities <Neut-A>.

The Fundamental Thesis of Neutrosophy:

Any idea <A> is T% true, I% indeterminate, and F% false, where T, I, F are subsets in $]^-0, 1^+[$.

The Main Laws of Neutrosophy:

Let <α> be an attribute, and (T, I, F) belongs to $]^-0, 1^+[^3$. Then:

- There is a proposition <P> and a referential system {R}, such that <P> is T% <α>, I% indeterminate or <Neut-α >, and F% <Anti-α >.
- For any proposition <P>, there is a referential system {R}, such that <P> is T% <α >, I% indeterminate or <Neut-α >, and F% <Anti-α >.
- <α > is at some degree <Anti-α >, while <Anti-α > is at some degree <α >.

2. The Objective world and subjective world

These ideas can shed light on the relationship between the objective world and our subjective impressions, leading to insights which, we shall see, find important echoes in the Buddhist and Daoist traditions. It is commonly assumed that the objective world consists simply of the totality of things which we can see or otherwise experience. This is, however, very wrong. In fact, this is rather a belief than an objective reflection, and cannot be proven. In his paper “To be or not to be, A multidimensional logic approach” Carlos Gershenson [2] has generalized proofs for the following claims:

- Everything both is and is not to a certain degree (i.e., there is no absolute truth or falsehood).
- Nothing can be proven definitively to exist or not exist, i.e., no one can prove that his consciousness is right.
- I believe, therefore I am (i.e., I take it true, because I believe so).

What I believe *is* something, but it is not the figure I have in my mind.

This is, interestingly enough, the starting point in Daoism (F. Liu [2]). The *Daodejing* begins with the following saying:

> Dao, Daoable, but not the normal Dao; name, namable, but not the normal name.

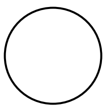
We can say that something is *Dao*, but this doesn’t mean what we intend for it to mean. Whenever we mention the *Dao*, it somehow slips beyond the limits of what we meant in mentioning it.

The *Daodejing* deals with the common problem: “What/who creates everything in the world we see and feel?” It is *Dao*: like a mother that bears things with shape and form. But what/who is the Dao? It is just unimaginable, because whenever we try to imagine it, our imagination can never *be* it. We can never completely describe it. The more we describe it, more wrong we are. It is also unnamable, because whenever we name it, our concept based on the name can never be adequate to it.

Daoism illustrates the origin of everything in a form which doesn’t show in any form we can perceive. This is the reason why it says that everything comes from nothingness, or that nothingness creates every form through dynamic change. Whatever we can perceive is merely the created form, rather than its genuine nature, as if we were to distinguish people by their outer clothing. Even great scientists like Einstein are far from really understanding nature.

3. Creativity and implementation

Once we have understood the inconceivability of the Dao, we can model our mind in the alternation of yin and yang that is universal in everything (Feng Liu): Yang pertains to dynamic change, and directs great beginnings of things; yin to relatively static stage, and gives those exhibited by yang their completion. In the course of development and evolution of everything yang acts as the creativity (Feng Liu) that brings new beginnings to it, whereas yin implements it in the forms as we perceive as temporary states. It is in this infinite parallelism that things inherit modifications and adapt to changes.



If, when asked what the figure at the left represents, we answer that is a circle, we are inhibiting our creativity. Nor should we hold that it is a cake, a dish, a bowl, a balloon, the moon, or the sun, for we also spoil our creativity in this way. Then, what is it? "It is nothing." Is it correct? It is, if we do not hold on to the assumption "it is something". It is also wrong, if we persist in the doctrine that "the figure is something we call nothing." This nothing has in this way become something that inhibits our creativity. How ridiculous!

Whenever we hold the belief "it is …", we are loosing our creativity. Whenever we hold that "it is not …", we are also loosing our creativity. Our true intelligence requires that we completely free our mind — that we adhere neither to any extremity nor to "adhering to no assumption or belief". This is a kind of genius or gift rather than a logical rule, acquired largely after birth, e.g., through Buddhist practice. Note that our creativity lies just between intentionality and unintenationality (F. Liu [2]).

Not (it is) and not (it is not),
It seems nothing, but creates everything,
Including our true consciousness,
The power of genius to understand all.

Considerable insight regarding contradiction-compatible **learning philosophy** can be garnered from the Later Trigrams of King Wen in the *I-Ching.* When something (controversial) is perceived (in Zhen), it is referred (in Xun) to various knowledge models and, by assembling the fragments perceived from these models, we reach a general pattern to which fragments attach (in Li), as leading to the formation of an hypothesis, which needs to be nurtured and to grow up (Kun) in a particular environment. When the hypothesis is mature enough, it needs to be represented (in Dui) in diverse situations, and to expand and contradict older knowledge (in Qian) to update, renovate, reform or even revolutionize the existing knowledge base. In this way the new thought is verified, modified and substantialized. When the novel thought takes the principal role (dominant position) in the conflict, we should have a rest (in Kan) to avoid being trapped into depth (it would be too partial of us to persist in any kind of logic, to adapt to the outer changes). Finally, we reach the end of the cycle (in Gen).

I-Ching [in Chinese: Yi Jing] means: *Yi* = change, *Jing* = scripture. It deals with the creation and evolution (up and down) of everything in such a perspective that everything is an outer form of a void existence, and that everything always exists in the form of a unity of opposites, whether that unity is understood as compensatory or complementary. This philosophy shows that contradiction acts as the momentum or impetus to learning and evolution. Without controversy there is no innovation. This is essentially the principal thesis of neutrosophy (Florentin Smarandache). In the cycle there is unintentionality implied throughout it:

- Where do the reference models relating to the present default model come from? They are different objectively.
- How can we assemble the model from different or even incoherent or inconsistent fragments?
- If we always do it intentionally, how does the hypothesis grow on its own, as if we study something without sleep?
- How can our absolute intention be complemented without contradiction?

- Is it right that we always hold our intention?

- There is only one step between truth and prejudice — when the truth is overbelieved regardless of constraint in situations, it becomes prejudice.

- Is there no end for the intention? Then, how can we obtain a concept that is never finished? If there is an end, then it should be the beginning of unintentionally, as yin and yang in the *Tai Chi* figure.

4. Completeness and incompleteness: knowledge and practice

There always is contradiction between completeness and incompleteness of knowledge. Various papers presented by Carlos Gershenson prove this point. The same point is developed in the Daoist and Buddhist traditions. This contradiction is shown by the fact that people are satisfied with their knowledge relative to a default, well-defined domain. But later on, they get fresh insight in it. They face contradictions and new challenges in their practice and further development. As a result of this we are forced to ask:

- Do we understand ourselves?

- Do we understand the universe?

- What do we mean by knowledge, complete whole or incomplete?

Our silliness prompts us to try to find complete specifications, but where on earth are they (Gershenson [1])? Meanwhile, our effort would be nothing more than a static imitation of some dynamic process (Liu [1]), since humans understand the world through the interaction of the inter-contradictory and inter-complementary effects of two kinds of knowledge: perceptual knowledge and rational knowledge - they can’t be split apart.

- In discovering knowledge there are merely strictly limited conditions that focus our eyes to a local domain rather than on a open extension, therefore our firsthand knowledge is only relative to our default referential system, and possibly extremely subjective.

- Is it possible to reach a relatively complete piece at first? No, unless we are gods.

- Then we need to perceive the rightness, falseness, flexibility, limitation, etc. of our ideas and arrive at a more realistic conception --and an understanding the real meaning of our previous knowledge.

- Having done that, we may have less subjective minds, based on which our original concept is modified, revised, and adapted as further proposals.

- Again through practice, proposals are verified and improved.

- This cycle recurs to the infinite, in each of which our practice is extended in a more comprehensive way. The same is true of our knowledge.

- We discover the truth through practice, and again through practice verify and develop the truth. We start from perceptual knowledge and actively develop it into rational knowledge; then start from rational knowledge and actively guide revolutionary practice to change in both the subjective and the objective world. Practice, knowledge, again practice, and again knowledge. This form repeats itself in endless cycles, and with each cycle the content of practice and knowledge rises to a higher level.

❖ Through practice, we can verify our knowledge, find the inconsistencies and incompleteness in it, and face new problems, and new challenges as well, maintaining a critical outlook. Knowledge is based on an infinite cycle of critiques and negations(partial or revolutionary) which constantly transform our subjective world. We are never too old to learn.

5. Conclusion

Whenever we say "it is", we refer it to both subjective and objective worlds. We can creatively use the philosophical expression "both A and non-A" to describe both our subjective world and the objective world, and possibly the neutrality of both. Whenever we say "it is", there is a subjective world, in the sense that concepts always include subjectivity. So our problem becomes: is "it" really "it"? A real story from the Chinese Tang dynasty recorded in a sutra (adapted from Yan Kuanhu Culture and Education Fund) illustrates this principle nicely:

> Huineng arrived at a Temple in Guangzhou where a pennant was being blown by wind. Two monks who happened to see the pennant were debating what was in motion, the wind or the pennant. Huineng heard their discussion and said: "It was neither the wind nor the pennant. What actually moved were your own minds." Overhearing this conversation, the assembly (a lecture was to begin) were startled at Huineng's knowledge and outstanding views.

When we see the pennant and wind we will naturally believe we are right in our consciousness, however it is subjective. In other words, what we call "the objective world" can never absolutely be objective at all. Whenever we believe we are objective, this belief is subjective too. In fact, all these things are merely our mental creations (called illusions in Buddhism) that in turn cheat our consciousness: There is neither pennant nor wind, but only our mental creations. The world is made up of our subjective beliefs that in turn cheat our consciousness. This is in fact a cumulative cause-effect phenomenon.

Everyone can extricate himself out of this maze, said Sakyamuni and all the Buddhas. Bodhisattvas abound in the universe. Their number is as many as that of the sands in the Ganges (Limitless Life Sutra).

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The Fifth Function of University: "Neutrosophic E-function" of Communication-Collaboration-Integration of University in the Information Age

Florentin Smarandache Stefan Vladutescu

The study is based on the following hypothesis with practical foundation:
- Premise 1 - if two members of university on two continents meet on the Internet and initiate interdisciplinary scientific communication;
- Premise 2 - subsequently, if within the curricular interests they develop an academic scientific collaboration;
- Premise 3 - if the so-called collaboration integrates the interests of other members of the university;
- Premise 4 - finally, if the university allows, accepts, validates and promotes such an approach;
- Conclusion: then it means the university as a system (the global academic system) has, and it is, exerting a potential function to provide communication, collaboration and integration of research and of academic scientific experience.

We call this function of the university "neutrosophic e-function" because it mixes heterogeneous notions. It is specialized, according to the functions of "teaching-learning, researching, the public interest and entrepreneurial interest," as the fifth function. As the other four have structured and shaped university paradigms, this one configures one as well. E-function makes visible a functional structure in a scientific scan: the communicative-collaborative-integrative paradigm.

Beyond the practical and inferential logic arguments, the research bases the hypothesis on historical and systemic-operational arguments. The foundation consists of the fundamental contributions of some academics (Y. Takahara, C. Brătianu, M. Păun, R. Carraz, Y. Harayama, I. Jianu, A. Marga, M. Castells, H. Etzkowitz, A. Ghicov, T. Callo, and S. Naidu), and our contribution is apprehending the strong tendency of the university system to exercise an e-function and to move toward a global university e-system.

Keywords: university, system, e-function, communication, collaboration, integration.

I. The concept of university. Axis 1

In relation to the requirements of accuracy, the side resonances turn the idea of university into an elusive and vague concept. This does not come from the specialists' lack of concern for the radiography of such a major social agent. University is, from all existing institutions, the organization with the oldest, most solid and most thorough history. As a place of knowledge, it is also a medium of self-understanding. From this perspective, it is paradoxical that in the house of knowledge is not found a thorough and robust self-understanding. It seems that the university does not have a clear and lucid self-awareness. Epistemologically, the university is the fountain, the criteria and the archive of knowledge. Any knowledge, it appears, implies a lack of knowledge. And maybe, once the status of knowledge is accepted, ignorance can be considered as the foundation of knowledge. Therefore, an explanation of the elusiveness of the concept of universality comes from the uncertainty about the content of the ignorance. In a way, the meaning of university is the unknown. The awareness of the unknown and the awareness of the need for developing knowledge forms the energetic poles that feed the university system.

Another line of explanation is to understand current university as moving quickly in relation to the subject of knowledge and to the actors of knowledge. University is the most agile, insidious and versatile of all the institutions of knowledge.

Thirdly, the fact that it knows itself better and better, while rapidly changing, makes visible knowledge variable itself. Variability is the subject of entropy and thus of negentropy and information. Therefore, the accuracy of self-knowledge induces an effect of vagueness that reinforces the impression of elusiveness.

Practically and conceptually, the university is all right. The first axis of understanding the university is this conceptual elusive understanding.

II. University as an organization. Axis 2

On a second axis of preliminary understanding-explaining, the university is specialized, as shown by Professor Constantin Brătianu as "a very complex organization" (2005, pp. 43-55). Generically, the organization is founded as a social group dedicated to a specific task. Subsequently, Norman Goodman shows it has a "formal structure that tries to accomplish the task" (1998, p. 71). In accomplishing the defining task, it exploits some of the statutes and potential roles of its members. Related, it generates status and roles arising from the title of member and of organizational actor.

The genesis of organization is not conceptual, but social. Through it, society solves social problems. Essentially, traditionally, university solves two categories of problems: knowledge and education. The first category includes the production and transfer of knowledge. The other includes ethical, political, medical, economic-entrepreneurial education etc.

Organizations are defined not by the tasks they propose, by the objectives they set or by the mottos they are acting under, but by the problems they solve. They are not ends but means. Organization is a social tool for solving problems. The word organization comes from the French vocable “organisation” and etymologically comes from the Greek “organon” which means “instrument.” Basically, the organization carries out activities that lead to solving social problems. The first feature of the organization is to be an association of people interacting in the idea of preparing a group engaged in cultural, social, educational, and administrative activities. Underlying features are linked to it. Members related to a set of values, are subjected to rules and accomplish shared tasks when performing roles and statutes.

Organizations may be firms, companies, associations, governmental or non-governmental entities, foundations, etc. The most important organizations have legal grounds. When the activities of an organization and the social relations established by it acquire state importance, they are regulated by law. The organizations that acquire state importance or have national or supranational interest are legally recognized as institutions.

University is a fundamental scientific and educational institution of a state. Organizations have a social profile not because of the accomplishment of “specific objectives,” as S.P. Robbins, D. A. DeCenzo and M. Coulter deem (2010), but due to the problems they solve. In our opinion, the role of the organization as an intelligent operator is to perform activities that solve problems.

III. University as a system. Axis 3

3.1. A third axis of comprehension is to address the university as a system. As shown by Yasuhito Takahara, “An organizational system is a complex of interconnected human and nonliving machines” (2004, p. 3).

As a system, the organization has inputs and outputs. The inputs would be of two kinds: “The first type is a resource input such as personnel, material, money, energy, and information. The second is external managerial information related to customer demands, consumer behaviors, marketing conditions, economic situations, etc.“ (Takahara Y., 2004, p. 4). The organizational mechanism “transforms the resource inputs into products or services and transmits them to environments as an output” (Takahara Y., 2004, p. 4). The Japanese specialist understands the organization as being “formed for a purpose” (Takahara Y., 2004, p. 3) and as performing activities in this regard. About the transformation of input resources into output products or services is stated: “The transformation, which usually requires support of a specific technology, is the primary activity of an organization” (Takahara Y., 2004, p. 4). The professors Constantin Brătianu, Simona Vasilache and Ionela Jianu conceive the organization similarly. They emphasize that any organization is made up of “resources,” “processes” and “products” (Brătianu C., S. Vasilache, Jianu I., 2006). In a later article, Constantin Brătianu highlights: “In any organization all activities can be grouped together in two basic processes: the production process and the management process” (2007, p. 376). The production process (technological process) leads to achieving tangible final results of the organization that can be “objects or services” (as Y. Takahara asserted in 2004). The organizational system develops management activities as well: “management activity is to control the primary activity of transformation so that the organizational goal is realized” (Takahara Y., 2004, p. 4). The management process is connected with the production process and together they made up a systemic unit. It is focused on ensuring the production performing “effectively and efficiently”: the fulfillment of tasks correctly and obtaining products with a minimum allocation of resources and execution of those activities that lead to achieving goals. In the same context, Professor Constantin Brătianu explains: “The process of management can be performed through its main functions: planning, organizing, leading and controlling” (2007, p. 376).

3.2. Topologically, the organization as a system is defined by several modules. The above mentioned specialists identify the input, the output and the processes (Constanin Brătianu) or the transformation (Yasuhito Takahara). Collaterally, in order to designate activities performed between the input module and the output module we will use the concept of throughput. David Besanko, David Dranove, Mark Stanley and Scott Schaefer use the term “throughput” to conceptualize a phenomenon that conditions the successful businesses. Throughput is “the movement of inputs and outputs through the production process” (2010, p. 100).

So by throughput it is understood the module of activities which ensures the conversion of input (resources) to output (products and/or services).

3.3. Besides the topological coordinate the system has two more coordinates: the structural and the functional.

The entirety, the “multitude of elements” of a system with the connections, the “relations between them” “form the system structure” (Dima I.C. Cucui I., Petrescu M., Stegăroiu I., Năbârjoiu N., 2007, p. 11). The structure is emerging as a configuration of the moment. The system has potential for structural changes. It remains valid even when structural changes occur. In this coordinate, the system seems to be capable of allowing the evolution of elements and relationships, of components. At one point, the system has a structure, a state and a set of possibilities for transformation and development. The structure is the specific internal way of organizing the system elements. It is a configuration currently stable and qualitatively determined of the connections between elements.

3.4. The functional coordinate of the system is inextricably linked to the structural coordinate. Between the system structure and the functions performed by the system, a strong connection exists. The structure determines the function and the functioning modifies the structure. As the functioning is the prerogative of managers, it is at the same time, subjected to the power of the management strategies. As Peter F. Drucker shows, "structure follows strategy" (2010, p. 94). The functional connections, on the other hand, determine in time the variations in input and output. The state system is a functional problem. It appears as a constant of the connection's parameters within certain time. State is the measure of the system characteristics of the moment. The functional coordinate consists of the processes by which the system performs its functions. The transition from one functional state to another is the transformation.

The components of an organization are employees, managers, leaders, clients, beneficiaries etc. This is the structural capital of the organization. Systemic social connections appear as relations. In its relational capital, a system may include relationships of cooperation, collaboration, exchange, determination, influence, and communication. They may be hierarchical, vertical, horizontal, etc. Relations are those that ensure the system stability and allow its operation and adaptation to internal and external environments (natural, social, financial, economic, strategic, etc.). Relationships vary in time and give the dynamic character of the system. Effective systems seek to maintain stability. In general, however, systems have a strong inertia. As S.P. Robbins argues, "Organizations, by their very nature, are conservative" (2008, p. 187).

Structural-functional internal stability can be maintained in two ways. Adapting to the environment, systems tend to preserve internal steady states and perform its functions. First of all, W. R. Ashby states, the actions of the system "as varied as they are have one goal, to maintain constant conditions in the internal environment" (1958, p. 121). The more structurally elements are more independent of each other the more each one develops a greater capability to adapt. A better flexibility of the elements, namely a lower interdependence, is a premise for higher functional stability of the system. The second manner that the system preserves its stability in is feedback. Yasuhito Takahara speaks of two types of stability:

“behavior stability and structural stability” (2004, p. 4). “Behavior stability” is achieved through “feedback mechanism” and “structural stability” (or “the practice of keeping characteristic parameters of an organization constant”) is achieved ”by higher level management activities” (2004, p. 4).

In the article “Interactions among components of the university system,” Mihaela Păun (from Louisiana Tech University) and Miltiade Stanciu (from ASE Bucharest) start from the assumption of the university as system and institution. Zetetic stake is finding a revealing answer to the question: “Which is the most important component/resource in a university?” (2008, p. 94). Research is moving toward the components/resources of the university. The perspective is, implicitly, topological, structural and functional. The referred components are students, teachers and infrastructure. Resources are put into the equation to conclude about an intangible resultant. The unknown is defined: the human components (students, teachers) and the infrastructure are crucial in the university performance and competitiveness. They are equally important. From other perspective, we mention that there are “teaching oriented” universities and “researching oriented” universities. It is also recalled the existence of components of “teaching” and “researching” in most universities (Păun M., Stanciu M., 2008, p. 98).

Students and teachers appear to be defining systemic academic components (M. Trow, 1975). Professor Constantin Brătianu considers that “professors and students represent the most important resources” (2009, p.67). In higher education, teachers and students are defined as actors who have specific functions. Social actors exercising functions become system factors. Functional actors, ontological factors of the university, are the students and teachers (including teachers who have managerial responsibilities). They are engaged in an academic contract of didactic communication. The rights and obligations of the academic actors bear the mark of university functions. In turn, academic institution exists through its factors and through didactic teaching and research actions carried out in the university.

IV. The four institutionalized functions of the university

4.1. The first functions: “Teaching-learning” and “Researching.”

Generations of universities, the Humboldtian university paradigm:

Today, university is at the end of an evolution and in a transformation process that takes into account the forecasting, the foresight and the normative future. The functioning of the system means conducting specific activities. This happens within some processes. As Yasuhito Takahara (2004), Constantin Brătianu, S. Vasilache and Ionela Jianu (2006) argue, any organization runs two types of processes: processes of production (or technology) and management processes. The set of academic technological processes is subsumed to some functions undertaken by the university institutions. On the other hand, an effective university management process will be in line with technological processes, first of all and defining, regarding the functions of the university system. This university management process is supported by a structure with a clear profile, which Yuko Harayama and René Carraz would call "the university management structure" (2008, p. 93).

In 2003, Parliament of Australia retained that the "core functions of university" are "teaching, learning, and research" (2003, p. 21440). The one who diachronically has implemented this academic and functional model was Wilhelm von Humboldt, founder of the University of Berlin. "His university model," professor Gerd Hohendorf (Hohendorf G., 1993, pp. 617-618) argues, "is characterized by the unity of teaching and research. It was to be a special feature of the higher science establishments that they treated science as a problem which is never completely solved and therefore engaged in constant research."

Professor Constantin Brătianu and professor Yuko Harayama agree with the idea that Wilhelm von Humboldt introduced a "new university paradigm" (incidentally in Greek "paradigm" meant "modeled"). In addition, the Romanian specialist found that the two functions were also complementary. "The new university paradigm... is founded on the unity and the complementarity of the functions of teaching and research" (Brătianu C., 2009, p. 63).

The core of the functional Humboldtian model is that scientific issues are never "completely solved" and that, therefore, the university must remain "engaged in constant research." Understanding the Humboldtian model as a third generation of universities, Yuko Harayama emphasizes that within it the situation of the academic subjects is a situation of constant discovery. This means that "the teaching and learning process" occurs through "research activities" (Harayama Y., 1997, p. 13). In other words, the discoveries occur in university; possibly even in the teaching process. To reach this stage, the university has gone through, Yuko Harayama asserts, two models.

The first of university system emerges in the eleventh century and the twelfth century. Its elements are the teachers and students. The function of the system is one of knowledge transfer (knowledge is validated and scientific information is consecrated and preserved). The teachers do not create, do not innovate, do not discover. They take knowledge and new knowledge elements and they teach them. The new elements of knowledge are generated outside academia. The function of this university is one of "teaching."

A second generation of universities, according to Professor Yuko Harayama, keeps the non-investigative character and guides the teaching act only toward the elites of the religious and political spectrum. We would say that this model is focused on “teaching” too, its characteristic being the limitation induced by the religious or political pressures.

The third model, introduced by Wilhelm von Humboldt, is bi-functional: “teaching and research.”

Today the university model is based on the Humboldtian model. The technological university process is essentially a “teaching-learning process.” Over time this process has always been the focus of academic management in order to increase its efficiency and effectiveness. On the other hand, he was doubled at a time by the research process. The opinion of Professor Constantin Brătianu is similar: “The fundamental competences of a generic university are: teaching, learning and research. All of these are knowledge dynamic processes”(2009, p. 69). These two key functions have been multiplied in the policies developed in universities. Thus the universities are no longer limited today to the two functions. As Howard Newby argues “Today's universities are expected to engage in lifelong learning (not just teaching), research, knowledge transfer, social inclusion (via widening participation or access for non-traditional students), local and regional economic development, citizenship training and much more”(2008, pp. 57-58).

4.2. The third function: utility and social engagement

During the early twentieth century, the external environment required that universities have a stronger orientation toward utility. University transfer generates a system of high education that should acquire a more remarkable social, economic, financial and moral utility. He who brings in this practical necessity is John Henry Cardinal Newman. In his “The Idea of University,” he considers theology as a “branch of knowledge” (1999, p. 19) and militates for “useful knowledge” and for “usefulness” (1999, pp. 102-109). Through the knowledge provided, the university must exercise a function of utility and social involvement, locally, regionally or nationally. The transferred knowledge is required to acquire utility and practical involvement.

4.3. Entrepreneurial function. Entrepreneurial Paradigm

The functional development of the university has as its main purpose the performance and the competitiveness. Modern and post-modern universities are financed either by public funds or private funds and sometimes have a double funding. Private universities were the first who raised the question of self-financing. Related, the research function included an economic efficiency criterion. Therefore, having at least this double causality, the commercial, and economic entrepreneurial function has enforced in the set of functions. This remodeled the principal functions too, the ones of “teaching, learning and researching.” High education institutions have also assumed the entrepreneurial task function. In 1983, in the article “Entrepreneurial Scientists and Entrepreneurial Universities in American Academic Science,” Henry Etzkowitz launched the concept of “entrepreneurial university.” He argued that Thorstein Veblen had admitted at the beginning of the twentieth-century the possibility “that American universities would increasingly take on commercial characteristics.” Then, Henry Etzkowitz noted that “universities... are considering the possibilities of new sources of funds to come from patenting the discoveries made by holding academic appointments from the sale of knowledge gained by research done under the contract with commercial firms, and from entry into partnerships with private business enterprises” (1983, p. 198). A university exerting such an entrepreneurial function is an entrepreneurial university. In 2000, Henry Etzkowitz and his colleagues would find that “entrepreneurial university is a global phenomenon” and understand that it was “the triple helix model of academic-industry-government relations.” They speak, in this case, of the “entrepreneurial paradigm” (H. Etzkowitz, A. Webster, C. Gebhardt, Cantisano, Terra BR, 2000, p. 313). The concept of “entrepreneurial university” was considered lucrative and was developed so that, in 2007, David Woollard, Oswald Jones and Michael Zhang realized that this feature (generally accepted as a function) is, along with “teaching and researching the third mission” (2007, p. 1), meaning “commercialization of science .”

However, the concept also keeps a dose of lack of understanding and a dose of misunderstanding (Stanciu. Şt., 2008, pp. 130-134). However, in Romania the concern for an entrepreneurial university is already solid. Since 1998, professor Panaite Nica has taken scientifically into account the entrepreneurial function. Subsequently, Professor Valentin Mureşan (2002) brought in convergence opinions of university entrepreneurial specialists from France, England and Romania. For now, the concept of “Entrepreneurial University is still fuzzy and culturally dependent” (Brătianu C., Stanciu Şt., 2010, p. 133).

V. Collaborative-Communications Paradigm, the fifth function: function of communication, collaboration-integration

The functions of the university system are related to the mending demands required by the internal environment and by the needs to adapt to the external environment. These functions are initially mission assumed by the management structure. Once proven, the practical validity and the mission effectiveness, for a longer period and in several universities, it becomes a function of the global university system.

Functions are ways of permanent structural changing-transforming of the university system in relation to the internal requirements and external needs. As specified by Andrei Marga, university functions in society and fulfills “functions which develop along with the changes around them” (2009, p. 152). Following the same line of ideas, Andrei Marga takes into account “the multiple functions of university” (2004, p. 13). In exercising these functions, the university is presented “as a powerful scientific research center… for acquiring and applying knowledge,” and “as a source of technological innovation, as an intellectual authority in critically examining situations; as a space for commitment to civil rights, social justice and reforms“ (Marga A., 2004, p. 13).

Functions are, in general, “institutionalized” by the laws that give the university the character of institution. Thus, for example, social utility missions or entrepreneurial plans that were undertaken by some universities 25 years ago are now a function of the university system in general. Moreover, supranational authorities currently allow future university functions.

“The Bologna Declaration” (1999) mentions many of the functions of the university, teaching, research and a predicted communication-dissemination function. “The University functions in the societies having differing organization being the consequence of different geographical and historical conditions, and represents an institute that critically interprets and disseminates culture by the way of research and teaching.”

Nowadays, the environment university develops is one it has contributed to. This environment is not one in which the university decides. It must adapt to it.

The globalization of economic, financial, social phenomena is, on the one hand, the result of knowledge development, of creativity and innovation, and on the other, of their putting into practice. The world is in the Information Age. There has been a digital revolution that has succeeded everywhere. Interaction, networking, connectivity that is always the engine of society acquires new values in the new context. Social relations are digitally imprinted. Some of them even develop completely or partially, as mediated by computers. Many social relations have a virtual component.

The Information Age began after 1970 with the first personal computers, expanded after 1990 with the introduction of the Internet and strengthened after 2000 with the generalization of the Internet, with its use widely and globally.

In his trilogy, Information Age (1996, 1997, 1998, second edition 2000, 2001, 2004), Manuel Castells states: “Toward the end of second millenium of the christian era several events of historical significance transformed the social landscape of human life. A technological revolution, centered around information technologies, began to reshape, at accelerated peace, the material basis of society.

People increasingly organize their meaning not around what they do but on the basis of what they are. Meanwhile, on the other hand, global networks of instrumental exchanges selectively switch on and off individuals, groups, regions and even countries. “Our societies are increasingly structured around a bipolar opposition between the Net and the Self” (Castells M ., 1996, p. 1 p. 2 and p. 3). Taking ideas expressed in the late 1980s, Manuel Castells formulates and sets in trilogy the concept of the “Information Age.” “Prologue: the Net and the Self” opens the first volume “The Rise of the Network Society.” Here with the idea of the Information Age, two more ideas are displayed, that of the “network society” and that of the opposition between “Net” and “Self.” Later, in his book, Communication Power (2009), Manuel Castells will talk about the Information Age as the “digital age” or “network age.” The Information Age is the era of information society, information economy, information policy, etc. It is not a change of vision, but a transformation of substance, a historic turning point transformation. There is the digitization, globalization and putting in interaction to the components of the global social system.

Illustrating for the practical impact of digitization is the banks case. The globalization and interdependence brought by digitization went beyond any boundaries. They induced significant changes, major changes, namely functional changes. Banks, like all other operators, actors, and factors of the social, economic, and political systems, found themselves confronted with their own limits: some uncontrollable limits. In this respect, Lloyd Darlington points out: “For the first time in 300 years, the very nature of banking has changed. We still handle money, but information, not money, is now the lifeblood of our industry. From what was essentially a transaction-based business, where customers come to you (or didn’t), banking has to make the leap into what is essentially a sale-and-marketing culture” (1998, p. 115).

The Information era has induced significant changes in the internal environment and external environment of the university system. It has generated changes in the way the system should respond to the challenges and opportunities generated by the digital revolution, the technological revolution. The university system must adapt to external processes. To the external environmental changes, the university management must respond adaptively. The technological revolution has brought not only the transformation of the external environment, but it has also brought new tools for the university system to adapt. The challenge is primarily one of the university system functioning as a management coordinate and, secondly, in its “production” coordinate. The vision, missions and academic values are going through changes. In their content, strategic management includes adaptive tasks to respond to exogenous factors induced by digitization: extended or sometimes generalized computing and Internet communication, as well as rapid globalization of knowledge, discoveries, innovations, etc.

University is becoming more and more a place for creative knowledge. In visions, missions and values, functional commitments begin to transpire. In other words, on their own some universities assume new functions. In time, through their inter-university resonance, similar commitments in visions, mission and values go national. They are institutionalized and become functions of any university system.

For example, in his strategic document, Oxford Brooks University mentions the traditional, modern and postmodern functions and it involves performing activities we think will become functions specific to the Information Age. In "Our strategy for 2020,"Oxford Brooks University stated: "Oxford Brooks University occupies a strong position in UK higher education. We have a sound and growing international reputation for the quality of our teaching, learning and research and we are a vital part of and contributor to the local and national economy and society."

Remain fundamental nuclear functions of the university: "teaching, learning and researching."

Public interest and entrepreneurial functions were institutionalized: "we are a vital part of and contributor to the local and national economy and society." The strategy states: "We also need to ensure that our organizational structures support staff and students in their activities, that they facilitate the integration of research and teaching and promote inter-disciplinarity and diversity. We are international in our orientation: in our curriculum, our staff, our student body and our increasingly interdependent world partnership in an increasingly interdependent world. We aspire to be a university which makes a commitment to an educational culture where mentorship is valued and teaching is integrated with both research and cutting-edge practice from the professions."

In the space it exists, the university must place itself as the main generator and supplier of knowledge. The relevant context of the current university system is structured mainly by the action of three factors. These factors-buoys of the context are:

a) Computing, technology, rapid innovation (prefigured by and currently under development by Gordon Moore's law: "the computing power of microchips doubles every 18 months");

b) Accelerated extension of the information-communication systems, (categories of users increase, diversify and amplify their importance: according to Robert Metcalfe's postulate: "a network's value grows proportionally with the numbers of users" and according to George Gilder's law "the total bandwidth of communication systems triple every 12 months");

c) Development and accreditation of a collaborative and disseminating academic environment (the transition from unilateral projects to international and multilateral projects, the application of the principle of "shared knowledge," the liberalization of flows of knowledge and the setting of new dissemination channels).

The fundamental phenomena taking place in the internal environment are a permissive-adaptive and intelligent replication of those from the external environment: tech-digitization, globalization and interdependence. They have a direct impact on the activities carried out in the university and indirectly (mediated by management) on the functions of the university system.

According to the strategy Oxford - 2020, management assures ("ensure") in connection with the involvement in reforming the functions of "teaching" and "research": "facilitate the integration of research and reaching" and "commitment to"... "teaching integrated with both research and cutting-edge practice."

Related, we mention a commitment to "promote inter-disciplinarity and diversity." A direction with a functional touch is the decision that the university should be "international in our orientation: in our curriculum, our staff, our student body and our partnership." If at first already accredited four functions are mentioned, this latter functional commitment is specific to the Information Age world: "an increasingly interdependent world."

Manuel Castells considers "globalisation and digitization" as "the two most profound social and economic trends of our age" (2009, p. 70). The main feature of globalization is reflected in the fulminant emergence of networks. A "Global Network Society" emerges. "Network society is to the Information Age," Castells states, "what the industrial society was to the Industrial Age" (2009, p. 12). In the "Global Network Society" image, universities are characterized as academic institutions with a recognizable profile. They "are at the cutting edge of research and teaching on the global network society." Keeping in mind two of the functions of the university "teaching" and "research," we may notice the acceptance of a commitment project: "project of situation the university within the technological and intellectual conditions of the Information Age" (Castells M., 2009, p. 3). Manuel Castells is not concerned with how the university should develop in the Information Age.

Our thesis is that in the context of the "Digital Age," the university system must assume new functions adaptively. These functions are not surprising occurrences. They have been preliminarily mentioned in the university strategies, either incidentally as vision, mission and values or as precise missions. In the context of separation of functions the university system had to institutionalize, we mention Professor Andrei Marga's point of view. He has argued that the twenty-first century university is forced to face many challenges, listing ten: "the implementation of the Bologna Declaration (1999), globalization, the sustainability and the identity of a university, the autonomy, the quality assurance, the

Phenomenon of "brain drain," the issue of multiculturalism of leadership, the climate of change, the overcoming of relativism, and the recuperation of the vision based on knowledge "(Marga A., 2008).

Smart organizations are characterized, among other things, by flexibility, learning and a high potential for change. As the most important pole of knowledge and as a decisive development pole, the university is among the most intelligent organizations. Therefore, we anticipate that university systems will even take on new functions according to the Digital Age opportunities. They will not expect that from opportunities, the challenges should become necessities. The new paradigm of a pure specificity for the Information Age will be a collaborative-communicational paradigm.

We predict that the current university system will connect into a single network under a title like "Universities Global Network." It is already mentioned, as Professor Adrian Ghicov does, about the "matching network" for an "efficient learning" (2008, p. 29) and about the "idea of integration and completeness" (Callo T., 2005, p. 49). Following the same line of ideas, Bogdan Danciu, Margaret Dinca and Valeria Savu consider communication and collaboration as concepts of adaptation in the "academic field" (2010, p. 87).

University collaborative platforms will be open in areas and disciplines. Yuko Harayama and René Carraz count on "scientific collaboration," a feature found in the Japanese university system; see Harayama Y., R. Carraz, 2008.) Thus, "teaching" and "researching" could be carried out in the network. In this respect, Ilie Bădescu, Radu Baltasiu and Cristian Bădescu talk about "research networks" (2011, p. 248). IT infrastructure will enable the exchange of lectures held by teachers, live, interactively, in the videoconferencing system. Teachers specialize in certain subjects or who have important contributions on specific topics will be able to teach, using computer highways, the students from other universities in different regions or even other continents. As Ana Maria Marhan argues, cognitive players have not only become users of information technology, but they have mentally adjusted with the computer tools for learning, research, knowledge: a lucrative relationship between man and computer has been established (2007, pp. 12-14). Moreover, the teaching-learning in the network will capitalize improving the effect of "social facilitation" discovered by Robert B. Zajonc; "the mere presence of others" improves performance (1965, p. 274). The presence of students and teachers from other universities in videoconferencing will enhance the performance of teaching-learning knowledge and information. Students, as stated by Gheorghe Iosif, Ştefan Trăuşan-Matu, Ana-Maria Marhan, Ion Juvină şi Gheorghe Marius (2001), will be involved in designing cooperatively, with teachers, educational objectives; the training-educational process will be accomplished in relation to the "learning needs" and the "learning tasks," using computer technology, especially the Internet.

The integration of university research will start by regional, national projects and will expand globally. Collaborative platforms will allow the dissemination and unification of knowledge in areas and disciplines. In this manner, a knowledge base will arise for each discipline to avoid knowledge, research, parallel investigation or discovery in some places of old discoveries made in other units of knowledge. On the platforms, virtual research teams may rise which can synthesize all relevant knowledge on a specific subject and to continue research on behalf of the entire community of specialists. Researchers from different universities will work on joint projects in virtual teams in collaboration platforms. Interdependence of the world will be so fully visible regarding the interdependence of research and learning too. Research will be better and more equitable and professional and student performance indicators will gain a unique and relevant basis for reporting and evaluation. At this moment it has already achieved the digitization of some of the activities induced by the use and occurrence in university of the traditional university-canonical function. Decisive steps were taken to implement computer strategies concerning the "learning-teaching" function. Well-known Australian specialist, Som Naidu, notes that today student should learn in a new context, one "of e-learning; open, distant, and flexible learning environments" (2003, p. 362). Naidu says that "In the midst of all this interest in the proliferation of e-learning, there is a great deal of variability in the quality of e-learning and teaching." (2003, p. 354). On this basis and related, the professor at the University of Melbourne develops a guide of principles and procedures. The study requires the idea of digitization by "e-learning and teaching" and other processes undertaken by the university system (S. Naidu, 2003).

We value and fight for strengthening and developing the communicative-collaborative-integrative functions of the global university system. If the Digital Age brings, however, globalization and interdependence, we should not expect that they be imposed, but we should welcome them. It is good to settle all opportunities from challenges. It would be a beneficial and wonderful feed-forward response. In fact, some steps toward this emerging fifth function have already been taken.

Finally, it is arguable that it is about a global e-university in a global e-system and that e-communication and collaboration function applies not only to universities, but to all institutions, and even to individuals entering the electronic global communication system.

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Análisis de textos de José Martí utilizando mapas cognitivos neutrosóficos

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Florentin Smarandache

Yo vengo de todas partes,

Y hacia todas partes voy:

Arte soy entre las artes,

Y en los montes, montes soy.

José Martí

Palabras clave: José Martí, mapas cognitivos neutrosóficos, causalidad

1. Introducción

Martí, el más universal de los cubanos, fue un autor preocupado y con fe en la utilidad de la virtud. En este aspecto merece especial atención su artículo Maestros Ambulantes publicado en Nueva York, mayo de 1884 [1]. En el presente trabajo se presenta una propuesta para facilitar el análisis de su obra haciendo uso de los mapas cognitivos neutrosóficos (MCN) [2].

El trabajo está motivado por la importancia de interpretar las relaciones causales en los textos escritos por José Martí. La causalidad es una elemento fundamental para el entendimiento del mundo[3], y juega un papel fundamental en el aprendizaje especialmente en niños [4].

El artículo continúa de la siguiente forma. En la Sección 2 se aborda la tema tica de los Mapas Cognitivos Difusos/Neutrosóficos, En la Sección 3 se desarrolla la propuesta y en la Sección 4 se presentan las conclusiones y recomendaciones de trabajos futuros.

2. Mapas cognitivos difusos/neutrosóficos.

La causalidad es un tipo de relación entre dos entidades, causa y efecto. Es un proceso directo cuando A causa B y B es el efecto directo de A, o indirecto cuando A causa C a través de B y C es un efecto indirecto de A [3]. A pesar de la dificultad de desarrollar una definición de la causalidad los humanos poseen una comprensión de esta que permite elaborar modelos mentales de la interacción entre los fenómenos existentes a su alrededor [5].

En el mundo cotidiano los enlaces entre causa y efecto son frecuentemente imprecisos o imperfectos por naturaleza [6]. Este tipo de causalidad, es denominada causalidad imperfecta, desempeña un papel importante en el análisis de textos [7].

Los mapas cognitivos difusos (MCD) fueron introducidos por Kosko [8] como una mejora de los mapas cognitivos [9]. Los MCD extienden los mapas cognitivos describiendo la fortaleza de la relación mediante el empleo de valores difusos en el intervalo [-1,1]. Los nodos son conceptos causales y pueden representar distintos elementos [14].

Constituyen una estructura de grafo difuso con retroalimentación empleados para representar causalidad. Los MCD ofrecen un marco de trabajo flexible para representar el conocimiento humano y para el razonamiento automático [10].

Los MCN constituyen una extensión de los MCD basado en la lógica neutrosófica. La lógica neutrosófica es una extensión de la lógica difusa que permite representar la indeterminación en las relaciones causales [2].

3. Desarrollo

El texto martiano Maestros Ambulantes [1] se encuentra cargado de expresiones simbólicas. A continuación se analizan un fragmento y se buscan posibles relaciones causales.

“Ser bueno es el único modo de ser dichoso. Ser culto es el único modo de ser libre. Pero, en lo común de la naturaleza humana, se necesita ser próspero para ser bueno.”

Como posibles nodos son los siguientes: bondad (N1), dicha (N2), cultura (N3), libertad (N4), prosperidad (N5). A continuación se representa las relaciones causales existentes en el texto (Figura 1).

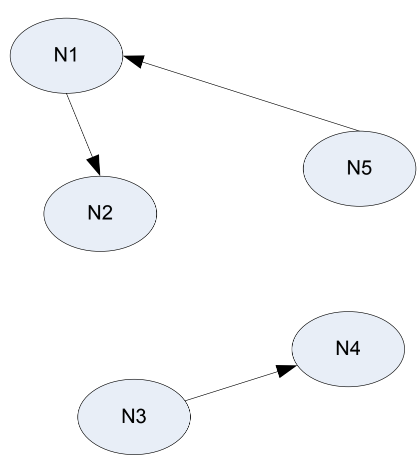

Figura 1. Mapa Cognitivo Difuso.

A continuación se muestra la matriz de adyacencia obtenida (Figura 2).

$$E = \begin{matrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{matrix}$$

Figura 2. Matriz de adyacencia del MCD

Se puede introducir el concepto de indeterminación entre los nodos y obtener a partir de expertos la interpretación en este caso la relación libertad (N4) y prosperidad (N5) (Figura 3).

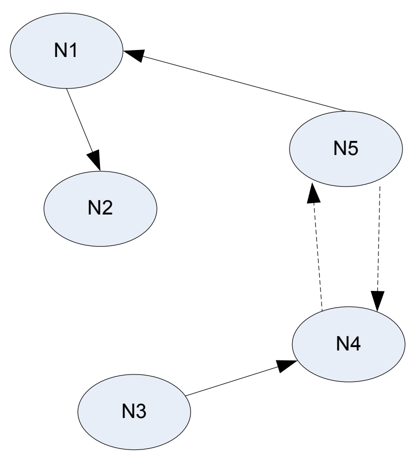

Figura 3. Mapa Cognitivo Neutrosófico.

A continuación se muestra la matriz de adyacencia obtenida mostrado la relación de indeterminación entre N5 y N4 (figura 4).

$$E = \begin{matrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & I \\ 1 & 0 & 0 & I & 0 \end{matrix}$$

Figura 4. Matriz de adyacencia del MCN

Este modelo puede ser utilizado en la enseñanza contribuyendo a la interpretación de los textos. Sobre el modelo se puede desarrollar análisis estático y dinámico.

4. Conclusiones

En el trabajo se presenta la posibilidad de interpretar los textos martianos a partir de mapas cognitivos neutrosóficos y su posibilidad de ser empleado en la enseñanza de su obra. Como trabajos futuros se encuentran el desarrollo de procedimientos semiautomáticos para el análisis de oraciones causales en textos[3]. Otra área de futuros trabajos es desarrollar la creación de un repositorio de modelos causales.

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Neutrosofia, o Nouă Ramură a Filosofiei

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Abstract:
În această lucrare este prezentată o nouă ramură a filosofiei, numită neutrosofie, care studiază originea, natura, şi scopul neutralităţilor, precum şi interacţiunile lor cu diferite spectre de ideatic. Teza fundamentală: Orice idee *<A>* este *T%* adevărată, *I%* nedeterminată şi *F%* falsă, unde *T, I, F* sunt submulţimi standard sau non-standard incluse în intervalul non-standard $]^-0, ^+1[$.
Teoria fundamentală:
Fiecare idee *<A>* tinde să fie neutralizată, diminuată, echilibrată de idei *<Non-A>* (nu numai *<Anti-A>*, cum a susţinut Hegel) - ca o stare de echilibru.
Neutrosofia stă la baza logicii neutrosofice, o logică cu valoare multiplă care generalizează logica fuzzy, la baza mulţimii neutrosofice care generalizează mulţimea fuzzy, la baza probabilităţii neutrosfice şi a statisticilor neutrosofice, care generalizează probabilitatea clasică şi imprecisă şi respectiv statisticile.

Cuvinte cheie şi expresii: analiza non-standard, număr hiper-real, infinitezimal, monadă, interval unitar real non-standard, operaţiuni cu mulţimi.

1991 MSC: 00A30, 03-02, 03B50

1.1. **Cuvînt înainte**.

Pentru că lumea este plină de nedeterminare, o imprecizie mai precisă este necesară. De aceea, în acest studiu este introdus un nou punct de vedere în filosofie, care ajută la generalizarea 'teoriei probabilităţilor', 'mulţimii fuzzy', şi 'logicii fuzzy' la < probabilitate neutrosofică>, <mulţime neutrosofică> , şi respectiv <logică neutrosofică>. Ele sunt utile în domeniul inteligenţei artificiale, reţelelor neuronale, programării evoluţionare, sistemelor neutrosofice dinamice, şi mecanicii cuantice.

În special în teoria cuantică există o incertitudine cu privire la energie şi la momentul particulelor, deoarece particulele nu au poziţii exacte în lumea subatomică, vom calcula mai bine probabilităţile lor neutrosofice la unele puncte (adică implicând un procentaj de incertitudine şi nedeterminare - în spatele procentajelor de adevăr şi respectiv de falsitate) decât probabilităţile lor clasice.

În afară de matematică şi de filosofia inter-relaţională, se caută Matematica în conexiune cu Psihologia, Sociologia, Economia, şi Literatura.

Acesta este un studiu de bază al filosofiei neutrosofice, deoarece consider că un întreg colectiv de cercetători ar trebui să treacă prin toate şcolile / mişcările/ tezele / ideile filozofice şi să extragă caracteristici pozitive, negative, şi neutre.
Filosofia este supusă interpretării.
Prezentăm o propedeutică şi o primă încercare de astfel de tratat.
[O filozofie neutrosofică exhaustivă (dacă aşa ceva este posibil) ar trebui să fie o sinteză a tuturor filozofiilor dintr-un sistem neutrosofic.]

Acest articol se compune dintr-o colecţie de fragmente concise, scurte observaţii, diverse citate, aforisme, unele dintre ele într-o formă poetică. (Referinţele principale sunt enumerate după mai multe fragmente individuale.) De asemenea, articolul introduce şi explorează noi termeni în cadrul avangardei şi al metodelor filozofice experimentale sub diverse logici valorice.

1.2. Neutrosofie, o nouă ramură de Filosofie

A) Etimologie:
Neutru-sofie [Din francezul *neutre* <latinul *neuter*, neutru, şi grecescul *sophia*, calificare / înţelepciune] înseamnă cunoaştere a gândirii neutre.

B) Definiţie:
Neutrosofia este o nouă ramură a filosofiei, care studiază originea, natura, şi scopul neutralităţilor, precum şi interacţiunile lor cu diferite spectre ideatice.

C) Caracteristici:
Acest mod de gândire:
- propune noi teze filozofice, principii, legi, metode, formule, mişcări;
- arată că lumea este plină de nedeterminare;
- interpretează neinterpretabilul;
- tratează din unghiuri diferite concepte şi sisteme vechi,
aratând că o idee, care este adevărată într-un sistem de referinţă dat, poate fi falsă în altul -- şi invers;
- încearcă să atenueze războiul de idei, şi să se războiască cu ideile paşnice;
- măsoară stabilitatea sistemelor instabile, şi instabilitatea sistemelor stabile.

D) Metode de Studiu Neutrosofic:
matematizare (logica neutrosofică, probabilitatea neutrosofică şi statisticile neutrosofice, dualitate), generalizare, complementaritate, contradicţie, paradox, tautologie, analogie, reinterpretare, asociere, interferenţă, aforistic, lingvistic, transdisciplinaritate.

E) Formalizarea:
Să notăm cu <*A*> o idee, sau propunere, teorie, eveniment, concept, entitate, cu <*Non-A*> ceea ce nu este <*A*>, şi cu <*Anti-A*> opusul <*A*>. De asemenea, <*Neut-A*> simbolizează ceea ce nu este nici <*A*> nici <*Anti-A*> şi anume neutralitatea dintre cele două extreme. Şi <*A'*> o versiune a <*A*>.

<*Non-A*> este diferit de <*Anti-A*>.

De exemplu:

Dacă <*A*> = alb, apoi <*Anti-A*> = negru (antonim),
dar <*Non-A*> = verde, roşu, albastru, galben, negru, etc. (orice culoare, mai putin albul),
în timp ce <*Neut-A*> = verde, roşu, albastru, galben, etc. (orice culoare, cu excepţia albului şi negrului),
şi <*A'*> = alb închis, etc. (orice nuanţă de alb).

Într-un mod clasic:
<Neut-A> $\equiv$ <Neut-(Anti-A)>, adică neutralităţile lui <*A*> sunt identice cu neutralităţile lui <*Anti-A*>.

<*Non-A*> $\supset$ <*Anti-A*> şi <*Non-A*> $\supset$ <*Neut-A*> precum şi,

<*A*> ∩ <*Anti*-*A*> = ϕ,

<*A*> ∩ <*Non*-*A*> = ϕ,

ori <*A*>, <*Neut*-*A*> şi <*Anti*-*A*> sunt disjuncte două câte două.
<*Non*-*A*> este completarea lui <*A*> cu privire la mulţimea universală.

Dar pentru că în multe cazuri frontierele dintre noţiuni sunt vagi, imprecise, este posibil ca <*A*>, <*Neut*-*A*>, <*Anti*-*A*> (şi bineinţeles <*Non*-*A*>) să aibă părţi comune, două cîte două.

F) Principiul fundamental:

Între o idee <*A*> şi opusul ei <*Anti*-*A*>, există un spectru continuu de putere a neutralităţilor <*Neut*-*A*>.

G) Teza fundamentală:

Orice idee <*A*> este *T%* adevărată, *I%* nedeterminată, şi *F%* falsă, unde *T*, *I*, *F* $\subset$ $]^{-}0, 1^{+}[$.

H) Legi principale:

Să luăm < α > ca atribut şi *(T, I, F) în* $]^{-}0, 1^{+}[^{3}$. Atunci:
- Există o propunere <*P*> şi un sistem de referinţă *{R}*, astfel că <*P*> este *T%* < α >, *I%* nedeterminat sau <*Neut*- α >, şi *F%* <*Anti*- α >.
- Pentru orice propunere <*P*>, există un sistem de referinţă *{R}*, astfel că <*P*> este *T%* < α >, *I%* nedeterminat sau <*Neut*- α > şi *F%* <*Anti*- α >.
- < α > este într-o anumită măsură <*Anti*- α >, în timp ce <*Anti*- α > este într-o anumită măsură < α >.

Prin urmare:

Pentru fiecare propoziţie <*P*> există sisteme de referinţă $\{R_1\}, \{R_2\}, \ldots$, astfel că <*P*> arată diferit în fiecare dintre ele - obţinand toate stările posibile de la <*P*> la <*Neut*-*P*> până la <*Anti*-*P*>.
Şi ca o consecinţă, pentru oricare două propoziţii <*M*> şi <*N*>, există două sisteme de referinţă $\{R_M\}$ şi respectiv $\{R_N\}$, astfel că <*M*> şi <*N*> arată la fel.
Sistemele de referinţă sunt ca nişte oglinzi de curburi diferite care reflectă propoziţiile.

I) Motto-uri:

- Totul este posibil chiar şi imposibilul!
- Nimic nu este perfect, nici chiar perfecţiunea!

J) Teorie Fundamentală:

Fiecare <*A*> idee tinde să fie neutralizată, diminuată, echilibrată de idei <*Non*-*A*> (nu numai <*Anti*-*A*>, cum a sustinut Hegel) - ca o stare de echilibru. Între <*A*> şi <*Anti*-*A*> există infinit de multe idei <*Neut*-*A*>, care pot echilibra <*A*> fără a fi necesare versiuni <*Anti*-*A*>.
Pentru a neutraliza o idee trebuiesc descoperite toate cele trei laturi ale sale: de sens (adevărul), de nonsens (falsitate), şi de imprecizie (nedeterminare) – apoi trebuiesc inversate / combinate. Ulterior, ideea va fi clasificată ca neutralitate.

K) Delimitarea de alte concepte şi teorii filozofice:

1. Neutrosofia se bazează nu numai pe analiza de propuneri opuse, aşa cum face dialectica, ci de asemenea pe analiza neutralităţilor dintre ele.

2. În timp ce *epistemologia* studiază limitele cunoaşterii şi ale raţionamentului, neutrosofia trece de aceste limite şi ia sub lupă nu numai caracteristicile definitorii şi condiţiile de fond ale unei entităţi <*E*> - dar şi tot spectrul <*E'*> în legătură cu <*Neut-E*> .
Epistemologia studiază contrariile filozofice, de exemplu <*E*> versus <*Anti-E*>, neutrosofia studiaza <*Neut-E*> versus <*E*> şi versus <*Anti-E*> ceea ce înseamnă logică bazată pe neutralităţi.
3-4. *Monismul neutru* afirmă că realitatea ultimă nu este nici fizică nici mentală. Neutrosofia constă într-un punct de vedere pluralist: o infinitudine de nuanţe separate şi ultime conturează lumea.
5. *Hermeneutica* este arta sau ştiinţa interpretarii, în timp ce neutrosofia creează idei noi şi analizează o gamă largă de câmp ideatic prin echilibrarea sistemelor instabile şi dezechilibrarea sistemelor stabile.
6. *Filozofia Perennis* spune adevărul comun al punctelor de vedere contradictorii, neutrosofia combină cu adevărul de asemenea şi neutralele.
7. *Falibilismul* atribuie incertitudine fiecărei clase de convingeri sau propuneri, în timp ce neutrosofia acceptă afirmaţii *100%* adevărate precum şi afirmaţii *100%* false, - în plus, verifică în care sisteme de referinţă procentajul incertitudinii se apropie de zero sau de *100*.

L) Limitele filozofiei:

Întreaga filozofie este un *tautologism*: adevărat în virtutea de formă, pentru că orice idee lansată pentru prima oară este dovedită ca adevărată de către iniţiatorul(ii) său(i). Prin urmare, filozofia este goală sau dezinformativă,
şi reprezintă a priori cunoaşterea.
Se poate afirma: Totul este adevărat, chiar si falsul!

Şi totuşi, întreaga filozofie este un *nihilism*: pentru ca orice idee, odată dovedită adevarată, este mai târziu dovedită ca falsă de către urmaşi. Este o contradicţie: fals în virtute de formă. Prin urmare, filozofia este supra-informativă şi o cunoaştere a posteriori.
Atfel, se poate afirma: Totul este fals, chiar şi adevărul!

Toate ideile filozofice care nu au fost încă contrazise vor fi mai devreme sau mai târziu contrazise deoarece fiecare filozof încearcă să găsească o breşă în sistemele vechi. Chiar şi această nouă teorie (care sunt sigur că nu este sigură!) va fi inversată ... Şi mai târziu alţii o vor instala inapoi ...

Prin urmare, filozofia este logic necesară şi logic imposibilă. Agostoni Steuco din Gubbio a avut dreptate, diferenţele dintre filozofi sunt de nediferenţiat.
Expresia lui Leibniz <adevarat în orice lume posibilă> este de prisos, peiorativă, întrucat mintea noastră poate construi de asemenea o lume imposibilă, care devine posibilă în imaginaţia noastră.
(F. Smarandache, "Sisteme de axiome inconsistente", 1995.)
- În această teorie nu se poate dovedi nimic!
- În această teorie nu se poate nega nimic!
Filosofism = Tautologism + nihilism.

M) Clasificarea de idei:

a) acceptate cu uşurinţă, uitate repede;
b) acceptate cu uşurinţă, uitate greu;

c) acceptate greu, uitate repede;
d) acceptate greu, uitate greu.

Şi versiuni diferite între orice două categorii.

N) Evoluţia unui idei *<A>* în lume nu este ciclică (dupa cum a afirmat Marx), dar discontinuă, înnodată, fără margini:
<Neut-A> = fond ideatic existent, înainte de apariţia lui *<A>;*
<Pre-A> = o pre-idee, un precursor al lui *<A>;*
<Pre-A'> = Spectru de versiuni *<Pre-A>;*
<A> = Ideea în sine, care dă naştere implicit la:
<Non-A> = ceea ce este în afara lui *<A>;*
<A'> = Spectru de versiuni *<A>* după interpretări / înţelegeri (greşite) de către persoane, şcoli, culture diferite;
<A/Neut-A> = Spectru de derivate /deviaţii <A>, deoarece <A> se amestecă parţial mai întâi cu idei neutre;
<Anti-A> = Opusul direct al *<A>,* dezvoltat în interiorul lui *<Non-A>;*
<Anti-A'> = Spectru de versiuni *<Anti-A>* după interpretări / întelegeri (greşite) de către persoane, şcoli, culture diferite;
<Anti-A/Neut-A> = Spectru de derivate /deviaţii *<Anti-A>,* ceea ce înseamnă *<Anti-A>* parţial şi *<Neut-A>* partial combinate în procentaje diferite;
<A'/Anti-A'> = Spectru de derivate /deviaţii după amestecarea spectrelor *<A'>* şi *<Anti-A'>;*
<Post-A> = După *<A>,* o post-idee, o concluzie;
<Post-A'> = Spectru de versiuni *<Post-A>;*
<Neo-A> = *<A>* reluat într-un mod nou, la un alt nivel, în condiţii noi, ca într-o curbă iregulată cu puncte inflexiune, în perioade evolute şi evolvente, într-un mod de recurent, viaţa lui *<A>* re-începe.

"Spirala" evoluţiei, a lui Marx, este înlocuita cu o curbă diferenţială complexă, cu urcuşurile şi coborâşurile sale, cu noduri - pentru că evoluţia înseamnă şi cicluri de involuţie.
Aceasta este *dinafilozofia* = studiul drumului infinit al unei idei.
<Neo-A> are o sferă mai largă (incluzînd, în afară de părţi vechi ale *<A>* şi părţi ale *<Neut-A>* rezultate din combinaţii anterioare), mai multe caracteristici, este mai eterogen (după combinaţii cu diferite idei *<Non-A>*). Dar, *<Neo-A>*, ca un întreg în sine, are tendinţa de a-şi omogeniza conţinutul şi apoi de a dezomogeniza prin alaturarea cu alte idei.
Şi aşa mai departe, până când *<A>*-ul anterior ajunge la un punct în care încorporează în mod paradoxal întregul *<Non-A>,* fiind nedesluşit de ansamblu. Şi acesta este punctul în care ideea moare, nu poate fi distinctă de altele. Întregul se destramă, pentru că mişcarea îi este caracteristică într-o pluralitate de idei noi (unele dintre ele conţinând părţi din orginalul *<A>*), care îşi încep viaţa lor într-un mod similar. Drept un imperiu multinaţional.
Nu este posibil să se treacă de la o idee la opusul său, fără a trece peste un spectru de versiuni ale ideii, de abateri sau idei neutre între cele două.
Astfel, în timp, *<A>* ajunge să se amestece cu *<Neut-A>* şi *<Anti-A>*.

Nu am spune că "opusele se atrag", dar <*A*> şi <*Non-A*> (adică interiorul, exteriorul şi neutrul ideii).

Prin urmare, rezumatul lui Hegel a fost incomplet: o teză este înlocuită de alta, numită anti-teză; contradicţia dintre teză şi anti-teză este depăşită şi astfel rezolvată printr-o sinteză. Deci Socrate la început, sau Marx şi Engels (materialismul dialectic).
Nu este un sistem triadic:
- Teză, antiteză, sinteză (Hegelieni);

sau
- Afirmaţie, negare, negaţie a negaţiei (Marxişti);

ci un sistem piramidal pluradic, aşa cum se vede mai sus.

Antiteza <*Anti-T*> a lui Hegel şi Marx nu rezultă pur şi simplu din teza <*T*>.
<*T*> apare pe un fond de idei preexistente, şi se amestecă cu ele în evoluţia sa.
<*Anti-T*> este construit pe un fond ideatic similar, nu pe un câmp gol, şi foloseşte în construcţia sa, nu numai elemente opuse <*T*>, dar şi elementele de <*Neut-T*>, precum şi elemente de <*T* >.
Căci o teză <*T*> este înlocuită nu numai de către o antiteză <*Anti-T*>, dar şi de diferite versiuni ale neutralităţilor <*Neut-T*>.
Am putea rezuma astfel: teză-neutră (fond ideatic înainte de teză), pre-teză, teză, pro-teză, non-teză (diferită, dar nu opusă), anti-teză, post-teză, neo-teză.
Sistemul lui Hegel a fost purist, teoretic, idealist. A fost necesară generalizarea. De la simplism la organicism.

O) Formule filozofice:

De ce există atât de multe şcoli filozofice distincte (chiar contrare)?
De ce, concomitent cu introducerea unei noţiuni <*A*>, rezulta inversul ei <*Non-A*>?

Acum, sunt prezentate formule filozofice numai pentru că în domeniul spiritual este foarte dificil să obţii formule (exacte).

a) Legea Echilibrului:

Cu cît <*A*> creşte mai mult, cu atat scade <*Anti-A*>. Relatia este urmatoarea:

$$<A> \times <\mathit{Anti}\text{-}A> = k \times <\mathit{Neut}\text{-}A>,$$

unde *k* este o constantă care depinde de <*A*> şi <*Neut-A*> este un punct de sprijin pentru echilibrarea celor două extreme.

În cazul în care punctul de sprijin este centrul de greutate al neutralităţilor, atunci formula de mai sus este simplificată:

$$<A> \times <\mathit{Anti}\text{-}A> = k,$$

unde *k* este o constantă care depinde de <*A*>.

Cazuri particulare interesante:

Industrializare × *Spiritualizare* = *constant*, pentru orice societate.
Cu cît o societate este mai industrializată, cu atât scade nivelul spiritual al cetăţenilor săi.

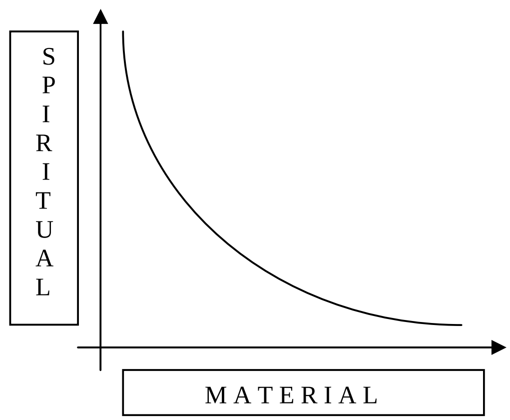
Ştiinţa × Religie = constant.

Alb × Negru = constant.

Plus × *Minus* = *constant.*
Împingînd limitele, în alte cuvinte, calculând în spaţiul absolut, se obţine:
Totul × *Nimic* = *universal constan.*

Ne îndreptăm către o matematizare a filozofiei, dar nu în sens Platonian.

Graficul 5. O.a.1:
Materialism ×*Idealism* = *constant*, pentru orice societate.



Axele carteziene verticale şi orizontale sunt asimptote pentru curba $M \times I = k$.

b) Legea Anti-reflexivitate:
<*A*> în oglindă cu <*A*> dispare treptat.
Sau <*A*>-ul lui <*A*> se poate transforma într-un <*A*> distorsionat.

Exemple:
Căsătoria între rude dă naştere la descendenţi anoşti (de multe ori cu handicap).
De aceea, amestecând specii de plante (şi uneori, rase de animale şi oameni), obţinem hibrizi cu calităţi şi / sau cantităţi mai bune. Teoria biologică a amestecării speciilor.
De aceea, emigrarea este benignă întrucat aduce sânge proaspat într-o populaţie statică.
Nihilismul, propovăduit după romanul "Părinţi şi copii" de Turgeniev în 1862 drept o negare absolută, neagă totul, prin urmare, se neagă pe sine!
Dadaismul dadaismului dispare.

c) Legea de Complementaritate:
<*A*> simte nevoia să se completeze prin <*Non-A*> cu scopul de a forma un întreg.

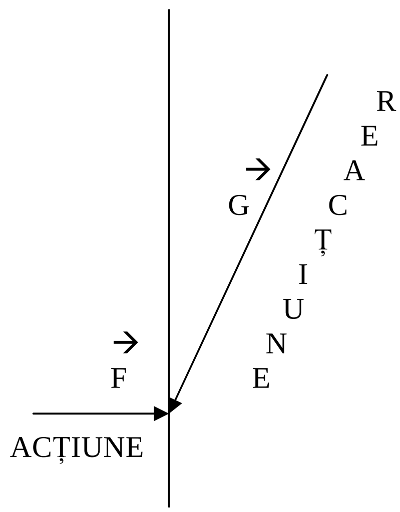
Exemple:
Persoanele diferite simt nevoia să se completeze reciproc şi să se asocieze. (Barbatul cu femeia.)
Culorile complementare (care, combinate la intensităţile potrivite, produc albul).

d) Legea Efectului Invers:

Atunci când este încercată convertirea cuiva la o idee, credinţă, sau religie prin repetiţii plictisitoare sau prin forţă, acea persoana ajunge să o urască.

Exemple:
Cu cât rogi pe cineva să facă ceva, cu atât persoana vrea mai puţin să o facă.
Dublând regula, ajungi la înjumătăţire.
Ce e mult, nu e bine ...
(invers proporţional).
Când esti sigur, nu fii!

Atunci când forţăm pe cineva să facă ceva, persoana va avea o reacţie diferită (nu necesar opusă, precum afirma axioma legii a treia a mişcării a lui Newton):



e) Legea Identificării Intoarse:
<*Non-A*> este un <*A*> mai bun decât <*A*>.

Exemplu:
Poezia este mai filozofică decât filozofia.

f) Legea Întreruperii Conectate:
<*A*> şi <*Non-A*> au elemente în comun.

Exemple:
Există o distincţie mică între "bine" şi "rău".
Raţionalul şi iraţionalul funcţionează împreună inseparate.
Conştiinţa şi inconştienţa în mod similar.

"Vino, sufletul mi-a spus, haide să scriem poezii pentru corpul meu, căci suntem Unul" (Walt Whitman).

Finitul este infinit [vezi microinfinitatea].

g) Legea Întreruperii de Identităţi:

Lupta permanentă între <*A*> şi <*A'*> (unde <*A'*> sunt diferite nuanţe de <*A*>).

Exemple:

Lupta permanentă între adevărul absolut şi adevărul relativ.

Distincţia dintre falsul clar şi falsul neutrosofic (cea de doua noţiune reprezintă o combinaţie de grade de falsitate, nedeterminare, şi adevăr).

h) Legea Compensaţiei:

Dacă acum <*A*>, atunci mai tîrziu <*Non-A*>.

Exemple:

Orice pierdere are un câstig.
[ceea ce înseamnă că mai târziu va fi mai bine, pentru că ai învăţat ceva din pagubă].

Nu există niciun succes fără eşec
[aveţi răbdare!].

i) Legea Stării Stabilite:

Nu pot fi depăşite limitele proprii.
(Ne învârtim în cercul propriu.)

j) Legea Gravitaţiei Ideaţionale:

Fiecare idee <*A*> atrage şi respinge altă idee <*B*> cu o forţă direct proporţională cu produsul măsurilor lor neutrosofice şi exponenţialul distanţei lor.

(În opoziţie cu reafirmarea modernă a Legeii lui Newton cu privire la gravitaţia particulelor de materie, distanţa influenţează direct proporţional - nu indirect: cu cât sunt ideile mai opuse (distanţate), cu atât se atrag mai puternic.)

k) Legea Gravitaţiei Universale Ideaţionale:

<*A*> tinde către <*Non-A*> (nu către <*Anti-A*> cum a spus Hegel) şi reciproc. Există forţe care acţionează asupra lui <*A*>, orientând-o către <*Non-A*>, până când un punct critic este atins, iar apoi <*A*> se întoarce.
<*A*> şi <*Non-A*> sunt în continuă mişcare, iar limitele lor se schimbă în consecinţă.

Exemple:

Perfecţiunea duce la imperfecţiune.

Ignoranţa este mulţumitoare.

Caz particular:

Fiecare persoană tinde să se apropie de nivelul său de… incompetenţă!
Aceasta nu este o glumă, ci purul adevăr:
Să spunem că *X* obţine un loc de muncă la nivelul *L1*;
dacă este bun, este promovat la nivelul *L2*;
daca este bun în noua sa poziţie, este promovat mai departe la *L3*;

şi aşa mai departe ... până când el nu mai este bun şi prin urmare, nu mai este promovat;
astfel, el a ajuns la nivelul său de incompetenţă.
<*A*> tinde catre <*Non-A*>.
Prin urmare, idealul fiecăruia este de a tinde către ceea ce nu poate face.

Dar mişcarea este neliniară.
<*Non-A*> dispune de o gamă largă (continuum de putere), de versiuni care "nu sunt <*A*>"   (<*A*> exterior), să le indexăm în mulţimea $\{<Non\text{-}A>_i\}_i$.

(Toate versiunile $\{<Anti\text{-}A>_i\}_i$ sunt incluse în <*Non-A*>.)
Prin urmare, infinit de multe versiuni $<Non\text{-}A>_i$ gravitează, precum planetele în jurul unui atru, pe orbitele lui <*A*>. Şi între fiecare versiune $<Non\text{-}A>_i$ şi centrul de greutate al "astrului" <*A*>, există forţe de atracţie şi de respingere. Se apropie una de alta până când se ajunge la anumite limite critice minime: $P_{m(i)}$ pentru <*A*> şi $Q_{m(i)}$ pentru $<Non\text{-}A>_i$ şi apoi se departează una de cealaltă până la atingerea anumitor limite maxime: $P_{M(i)}$ pentru <*A*> şi $Q_{M(i)}$ pentru $<Non\text{-}A>_i$.
Prin ecuaţii diferenţiale putem calcula distanţele minime şi maxime (spirituale) dintre <*A*> şi $<Non\text{-}A>_i$, coordonatele carteziene ale punctelor critice şi *status quo*-ul fiecărei versiuni.
Am putea spune că <*A*> şi o versiune $<Non\text{-}A>_i$ se întâlnesc într-un punct absolut / infinit.
Când toate versiunile $<Non\text{-}A>_i$ cad sub categoria <*A*> avem o catastrofă!

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Logica modernă se învecinează cu...nevoia de nelogică

Funcţii, Secvenţe, Paradoxuri "Smarandache", Logica "Smarandache" – cîteva cărţi de vizită în plus pentru recunoaşterea internaţională a şcolii româneşti de matematică.

Florentin Smarandache, actualmente profesor la Universitatea New Mexico, a absolvit Facultatea de Matematică a Universităţii din Craiova. Este cunoscut în lumea ştiinţifică internaţională pentru contribuţiile originale în domeniul matematicilor moderne. Înainte de a părăsi litoralul, a fost invitatul redacţiei noastre, pentru a ne vorbi despre principala sa pasiune.

Reporter: Enumeraţi pentru cititorii ziarului "Telegraf", poate pasionaţi matematicieni, cîteva dintre realizările personale în acest domeniu.

Florentin Smarandache:În matematică există cîteva noţiuni ca: Funcţii, Secvenţe, Constante, Paradoxuri, care poartă numele meu. Cea mai cunoscută este funcţia din Teoria Numerelor, Funcţia "Smarandache", care poate fi definită: *S(n)* este cel mai mic număr întreg astfel încît *S(n)!* este divizibil cu n. În logică, am generalizat logicile "Fuzzy" (ale propoziţiilor nedeterminate) la aşa numita logică neutrosofică (numită şi logica "Smarandache"). În teoria probabilităţilor, am propus generalizarea probabilităţii clasice şi a probabilităţii neutrosifice (definită ca un vector tridimensional, ale cărei componenţi sînt submulţimi reale). Am fost atras de domeniul geometriei, mai exact al geometriilor neeuclidiene, ale lui Rieman şi Lobacevski, pe care le-am combinat într-o concepţie proprie.

R.: La ce nivel sînt recunoscute aceste realizări teoretice?

F.S.: Noţiunile din Teoria Numerelor sînt amintite în enciclopediile americane, iar elementele care defines logica neutrosofică pot fi regăsite şi în Dictionary of Computing, editat în Anglia. Există oameni de ştiinţă care sunt la curent cu activitatea mea, cum ar fi în Italia Felice Russo, în Suedia Henry Ibsted, în SUA Charles Asabacher; în România, la catedra de matematică a universităţii din Craiova, s-a constituit un grup de cercetare asupra funcţiei din Teoria Numerelor pe care am descoperit-o, alcătuit din profesorii Vasile Seleacu şi Constantin Dumitrescu. În Japonia, lucrările mele sînt studiate si de Kenikiro Kashihara.

R.: Cum estimaţi şcoala românească de matematică şi dacă consideraţi că emigrarea v-a răsplătit?

F.S.: Este ştiut că există o şcoală puternică de matematică în România. La universităţile străine, din Occident şi din America, există numeroşi doctoranzi români în matematici. Aceştia nu se mai întorc. Venirea în America mi-a permis o cunoaştere mai largă, din diferite unghiuri ale existenţei. Am cunoscut un nou mod de viaţă şi o deschidere spre lume care m-a avantajat în contactele cu matematicieni, scriitori. Am avut ocazia unor întîlniri şi dialoguri prin participarea la conferinţe, schimburi de lucrări. Ca şi accesul la o tehnologic mai avansată, care favorizează comunicarea şi o mass media mai deschisă, mai democratică.

R.: Aţi fost atras de logica modernă, care se pare, tinde să sondeze dincolo de realitatea concretă, lumea necunoscutului, a misterelor.

F.S.: Da, este adevărat. Logica modernă operează cu noţiuni ca neprecisul, inexactul, contradictoriul, vagul, neconturatul, care sînt, totuşi noţiuni bine delimitate. În matematică se mai fac studii care analizează fenomenul creaţiei artistice; în urma acestora se pot crea algoritmi pe baza cărora calculatorul poate scrie literatură sau să picteze.

La un moment dat, în unul din volumele dedicate exilului, medita la faptul că logica te limitează, că ai nevoie şi de o doză de religiozitate, de percepţie metafizică şi ... multă gîndire nelogică.

De zece ani, se tipăreşte oa revistă de matematică, "Smarandache Notions Journal". Apare anual, într-un tiraj de 800 de exemplare, între 225-325 pagini. Este tipărită în America, dar şi pe Internet, la http://www.gallup.unm.edu/-smarandache/. Are colaboratori şi abonaţi de pe mapamond, materialele spre publicare putînd fi trimise pe adresa de mai sus. Ocazional sînt inserate şi articole de fizică sau filosofie.

Magdalena Vlădilă, "Telegraf", Constanţa, Anul VII, Nr. 180(2507), p.21, Miercuri, 2 August 2000.

Neutrosofia, un concept original în filosofie, creat de un român

Florentin Smarandache a părăsit România în 1988 (avea pe atunci 33 de ani), petrecînd doi ani într-un lagăr de transfugi din Turcia, aşteptînd cu tenacitate viza pentru Statele Unite. Pînă în acel an între 1982-1984, predase matematicile în Maroc.

Absolvent al Facultăţii de Matematică a Universităţii Craiova, se distinsese în domeniul ştiinţific, descoperind funcţii si secvente din Teoria Numerelor care îi poartă numele.

Dezamăgit de regimul communist, deopotrivă persecutat, interzicîndu-i-se să publice în domeniul matematicii şi al literaturii, cere azil politic, convins că pe această cale poate intra mai uşor în America. Motivul: interzicerea participării la congrese, interzicerea doctoratului şi a publicării lucrărilor.

Pînă la vîrsta emigrării scrisese foarte mult, fără a publica aproape nimic. La plecare a îngropat manuscrisele în via casei părinteşti din Vîlcea, ascunse în cutii de aluminiu in care se vindea laptele praf.

Personalitate neobişnuit de creativă, Florin Smarandache s-a distins deopotrivă în domeniul matematicii, poeziei, filosofiei. Actualul professor al Universităţii New Mexico din SUA a fost invitatul redacţiei noastre, fiind rugat să ne spună cîteva cuvinte despre contribuţia sa originală în domeniul filozofiei.

Reporter:Pentru cei mai puţin familiarizaţi cu termenul de neutrosofie, concept original, care vă aparţine, vă rugăm să ne spuneţi cum aţi ajuns la constituirea sa?

Florin Smarandache:*Am pornit de la logica clasică, care afirmă că o propoziţie poate avea valoarea adevărat sau fals. În logica modernă se admite că valoarea de adevăr a unei propoziţii poate include posibilităţile: adevărat, fals, nedeterminat.*

R.: Se afirmă că este o generalizare a dialecticii lui Hegel. Ce componentă a dialecticii sale aţi exploatat spre a vă configura propriul concept?

F.S.: *Hegel în dialectica susţine că pentru orice propoziţie atîta vreme cît există şi poate fi afirmată, apare automat şi opusul ei. Aceeaşi dialectică a aplicat-o şi Marx în analiza socială, iar Hegel a folosit-o la nivelul ideilor.*

R.: Neutrosofia este un termen care vă aparţine. Care este justificarea, sensul său? Şi care este conţinutul său?

FS.: *Pornind de la faptul că există propoziţii (= paradoxurile) concomitent adevărate şi false, am semnalat că acest caz nu era prins în logica fuzzy, a propoziţiilor vagi.*

Creatorii logicii fuzzy, L Zadeh afirmă că o propoziţie are două componente: să fie adevărată de pildă 30% şi restul 70% falsă. Părţii de nedeterminare, care se găseşte între adevăr şi fals împreună cu părţile de adevăr şi fals le-am extins limita pînă la 300% pe total, fiecare component $\in\ ]-0{,}100^{+}[$. *Astfel încît extremele adevărat – fals au fost împinse în aleatoriu.*

Acest interval virtual al aleatorului aparţine unor parametri ascunşi care, pe parcursul experimentărilor practice sau a operaţiilor teoretice se pot releva.

Am numit "neutru" acest interval, în care o dimensiune se află în starea de a nu putea primi valoarea "adevărat" sau "fals". Această ştiinţă se dinamizează din confruntarea cu realitatea. Aplicaţiile sale sînt în inteligenţa artificială, care lucrează cu noţiuni vagi, imprecise, contradictorii, sau cu sferă nedeterminată.

Magdalena Vlădilă, "Telegraf", Constanţa, p. 29, Nr. 169(2496) /20 iulie 2000

Neutrosophic Theory means Neutrosophy applied in many fields in order to solve problems related to indeterminacy.

Neutrosophy considers every entity <A> together with its opposite or negation <antiA>, and with their spectrum of neutralities <neutA> in between them (i.e. entities supporting neither <A> nor <antiA>). Where <neutA>, which of course depends on <A>, can be indeterminacy, neutrality, tie (game), unknown, vagueness, contradiction, ignorance, incompleteness, imprecision, etc.

Hence, in one hand, the Neutrosophic Theory is based on the triad <A>, <neutA>, and <antiA>.

In the other hand, Neutrosophic Theory studies the indeterminacy in general, labelled as I, with $I^n = I$ for $n \geq 1$, and $mI + nI = (m+n)I$, in neutrosophic structures developed in algebra, geometry, topology etc.

This volume contains 45 papers, written by the author alone or in collaboration with the following co-authors: Mumtaz Ali, Said Broumi, Sukanto Bhattacharya, Mamoni Dhar, Irfan Deli, Mincong Deng, Alexandru Gal, Valeri Kroumov, Pabitra Kumar Maji, Maikel Leyva-Vazquez, Feng Liu, Pinaki Majumdar, Munazza Naz, Karina Perez-Teruel, Rıdvan Sahin, A. A. Salama, Muhammad Shabir, Rajshekhar Sunderraman, Luige Vladareanu, Magdalena Vladila, Stefan Vladutescu, Haibin Wang, Hongnian Yu, Yan-Qing Zhang.

ISBN 978-1-59973-320-3

9 781599 733203 >

87.50 €