Connections between Extension Logic and Refined Neutrosophic Logic

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Abstract.

The aim of this presentation is to connect Extension Logic with new fields of research, i.e. fuzzy logic and neutrosophic logic.

We show herein:

- How Extension Logic is connected to the 3-Valued Neutrosophic Logic,
- How Extension Logic is connected to the 4-Valued Neutrosophic Logic,
- How Extension Logic is connected to the n-Valued Neutrosophic Logic,

when contradictions occurs. As extension transformation one uses the normalization of the neutrosophic logic components.

Introduction.

In this paper we present a short history of logics: from particular cases of *2*-symbol or numerical valued logic to the general case of *n*-symbol or numerical valued logic, and the way they are connected to Prof. Cai Wen's Extension Logic Theory (1983). We show generalizations of *2*-valued Boolean logic to fuzzy logic, also from the Kleene's and Lukasiewicz' *3*-symbol valued logics or Belnap's *4*-symbol valued logic to the most general *n-symbol or numerical valued refined neutrosophic logic*. Two classes of neutrosophic norm (*n-norm*) and neutrosophic conorm (*n-conorm*) are defined. Examples of applications of neutrosophic logic to physics are listed in the last section.
Similar generalizations can be done for *n-Valued Refined Neutrosophic Set*, and respectively *n-Valued Refined Neutrosopjhic Probability* in connections with Extension Logic.

The essential difference between extension logic and neutrosophic logic is that the sum of the components in the extension logic is greater than 1. And the relationship between extension logic and refined neutrosophic logic is that both of them can be normalized (by dividing each logical component by the sum of all components), thus using an extension transformation.

1. Two-Valued Logic

a) The <u>Two Symbol-Valued Logic.</u>

It is the Chinese philosophy: *Yin* and *Yang* (or Femininity and Masculinity) as contraries:

Fig 1. Ying and Yang

It is also the Classical or *Boolean Logic*, which has two symbol-values: truth *T* and falsity *F*.

b) The <u>Two Numerical-Valued Logic.</u>

It is also the Classical or *Boolean Logic*, which has two numerical-values: truth *1* and falsity *0*.

More general it is the *Fuzzy Logic*, where the truth (*T*) and the falsity (*F*) can be any numbers in *[0,1]* such that $T + F = 1$.

Even more general, *T* and *F* can be subsets of *[0, 1]*.

2. Three-Valued Logic

a) The <u>Three Symbol-Valued Logics:</u>

i) *Łukasiewicz 's Logic*: True, False, and Possible.
ii) *Kleene's Logic*: True, False, Unknown (or Undefined).
iii) Chinese philosophy extended to: *Yin, Yang,* and *Neuter* (or Femininity, Masculinity, and Neutrality) – as in Neutrosophy.

Neutrosophy philosophy was born from neutrality between various philosophies. Connected with <u>Extension Logic</u> (Prof. Cai Wen, 1983), and Paradoxism (F. Smarandache, 1980).

<u>Neutrosophy</u> is a new branch of philosophy that studies the origin, nature, and scope of neutralities, as well as their interactions with different ideational spectra.

This theory considers every notion or idea <A> together with its opposite or negation <antiA> and with their spectrum of neutralities <neutA> in between them (i.e. notions or ideas supporting neither <A> nor <antiA>).

The <neutA> and <antiA> ideas together are referred to as <nonA>.

Neutrosophy is a generalization of Hegel's dialectics (the last one is based on <A> and <antiA> only).

According to this theory every idea <A> tends to be neutralized and balanced by <antiA> and <nonA> ideas - as a state of equilibrium.

In a classical way <A>, <neutA>, <antiA> are disjoint two by two. But, since in many cases the borders between notions are vague, imprecise, Sorites, it is possible that <A>, <neutA>, <antiA> (and <nonA> of course) have common parts two by two, or even all three of them as well. Such contradictions involves Extension Logic.

Neutrosophy is the base of all neutrosophics and it is used in engineering applications (especially for software and information fusion), medicine, military, airspace, cybernetics, physics.

b) The Three Numerical-Valued Logic:

i) Kleene's Logic: True (*1*), False (*0*), Unknown (or Undefined) (*1/2*), and uses "min" for $\wedge$, "max" for $\vee$, and "1-" for negation.

ii) More general is the *Neutrosophic Logic* [Smarandache, 1995], where the truth (*T*) and the falsity (*F*) and the indeterminacy (*I*) can be any numbers in *[0, 1],* then $0 \leq T + I + F \leq 3$. More general: Truth (*T*), Falsity (*F*), and Indeterminacy (*I*) are standard or nonstandard subsets of the nonstandard interval *]-0, 1+[.*

When t + f > 1 we have conflict, hence Extension Logic.

3. Four-Valued Logic

a) The Four Symbol-Valued Logic

i) It is *Belnap's Logic:* True (*T*), False (*F*), Unknown (*U*), and Contradiction (*C*), where *T, F, U, C* are symbols, not numbers.

Now we have Extension Logic, thanks to C = contradiction.

Below is the Belnap's conjunction operator table:

∩	F	U	C	T
F	F	F	F	F
U	F	U	F	U
C	F	F	C	C
T	F	U	C	T

Table 1.

Restricted to *T, F, U,* and to *T, F, C,* the Belnap connectives coincide with the connectives in Kleene's logic.

ii) Let *G* = Ignorance. We can also propose the following two 4-Symbol Valued Logics: *(T, F, U, G),* and *(T, F, C, G).*

iii) *Absolute-Relative 2-, 3-, 4-, 5-, or 6-Symbol Valued Logics* [Smarandache, 1995].

Let T_A be truth in all possible worlds (according to Leibniz's definition);

T_R be truth in at last one world but not in all worlds;

and similarly let I_A be indeterminacy in all possible worlds;

I_R be indeterminacy in at last one world but not in all worlds;

also let F_A be falsity in all possible worlds;

F_R be falsity in at last one world but not in all worlds;

Then we can form several Absolute-Relative 2-, 3-, 4-, 5-, or 6-Symbol Valued Logics just taking combinations of the symbols T_A, T_R, I_A, I_R, F_A, and F_R.

As particular cases, very interesting would be to study the Absolute-Relative 4-Symbol Valued Logic (T_A, T_R, F_A, F_R), as well as the Absolute-Relative *6*-Symbol Valued Logic $(T_A, T_R, I_A, I_R, F_A, F_R)$.

b) *Four Numerical-Valued Neutrosophic Logic:* Indeterminacy I is refined (split) as U =

Unknown, and C = contradiction.
T, F, U, C are subsets of [0, 1], instead of symbols;
This logic generalizes Belnap's logic since one gets a degree of truth, a degree of falsity, a degree of unknown, and a degree of contradiction.
Since C = T/\F, this logic involves the Extension Logic.

4. **Five-Valued Logic**

a) *Five Symbol-Valued Neutrosophic Logic* [Smarandache, 1995]:
Indeterminacy *I* is refined (split) as *U* = Unknown, *C* = contradiction, and *G* = ignorance; where the symbols represent:
T = truth;
F = falsity;
U = neither *T* nor *F* (undefined);
C = *T/\F,* which involves the Extension Logic;
G = *T\/F.*

b) If *T, F, U, C, G* are subsets of *[0, 1]* then we get: a *Five Numerical-Valued Neutrosophic Logic*.

5. **Seven-Valued Logic**

a) *Seven Symbol-Valued Neutrosophic Logic* [Smarandache, 1995]:
I is refined (split) as *U, C, G,* but *T* also is refined as T_A = absolute truth and T_R = relative truth, and *F* is refined as F_A = absolute falsity and F_R = relative falsity. Where:
U = neither (T_A or T_R) nor (F_A or F_R) (i.e. undefined);
C = (T_A or T_R) /\ (F_A or F_R) (i.e. Contradiction), which involves the Extension Logic;
G = (T_A or T_R) \/ (F_A or F_R) (i.e. Ignorance).
All are symbols.

b) But if T_A, T_R, F_A, F_R, *U, C, G* are subsets of *[0, 1]*, then we get a *Seven Numerical-Valued Neutrosophic Logic*.

6. **n-Valued Logic**

a) *The n-Symbol-Valued Refined Neutrosophic Logic* [Smarandache, 1995].
In general:
T can be split into many types of truths: $T_1, T_2, \ldots, T_p$, and *I* into many types of indeterminacies: $I_1, I_2, \ldots, I_r$, and *F* into many types of falsities: $F_1, F_2, \ldots, F_s$,, where all $p, r, s \geq 1$ are integers, and $p + r + s = n$.

All subcomponents T_j, I_k, F_l are symbols for $j\in \{1,2,\ldots,p\}$, $k\in \{1,2,\ldots,r\}$, and $l\in \{1,2,\ldots,s\}$.
If at least one $I_k = T_j /\backslash\ F_l$ = contradiction, we get again the Extension Logic.

b) *The n-Numerical-Valued Refined Neutrosophic Logic.*

In the same way, but all subcomponents T_j, I_k, F_l are not symbols, but subsets of *[0,1],* for all

$j \in \{1,2,\ldots,p\}$, all $k \in \{1,2,\ldots,r\}$, and all $l \in \{1,2,\ldots,s\}$.

If all sources of information that separately provide neutrosophic values for a specific subcomponent are independent sources, then in the general case we consider that each of the subcomponents T_j, I_k, F_l is independent with respect to the others and it is in the non-standard set $]^-0, 1^+[$. Therefore per total we have for crisp neutrosophic value subcomponents T_j, I_k, F_l that:

$$^-0 \le \sum_{j=1}^{p} T_j + \sum_{k=1}^{r} I_k + \sum_{l=1}^{s} F_l \le n^+ \qquad (1)$$

where of course $n = p + r + s$ as above.

If there are some dependent sources (or respectively some dependent subcomponents), we can treat those dependent subcomponents together. For example, if T_2 and I_3 are dependent, we put them together as $^-0 \le T_2 + I_3 \le 1^+$.

The non-standard unit interval $]^-0, 1^+[$, used to make a distinction between *absolute* and *relative* truth/indeterminacy/falsehood in philosophical applications, is replace for simplicity with the standard (classical) unit interval *[0, 1]* for technical applications.

For at least one $I_k = T_j \wedge F_l$ = contradiction, we get again the Extension Logic.

7. Neutrosophic Cube and its Extension Logic Part

The most important distinction between IFS and NS is showed in the below **Neutrosophic Cube** A'B'C'D'E'F'G'H' introduced by J. Dezert in 2002.

Because in technical applications only the classical interval is used as range for the neutrosophic parameters , we call the cube the **technical neutrosophic cube** and its extension the **neutrosophic cube** (or **absolute neutrosophic cube**), used in the fields where we need to differentiate between *absolute* and *relative* (as in philosophy) notions.

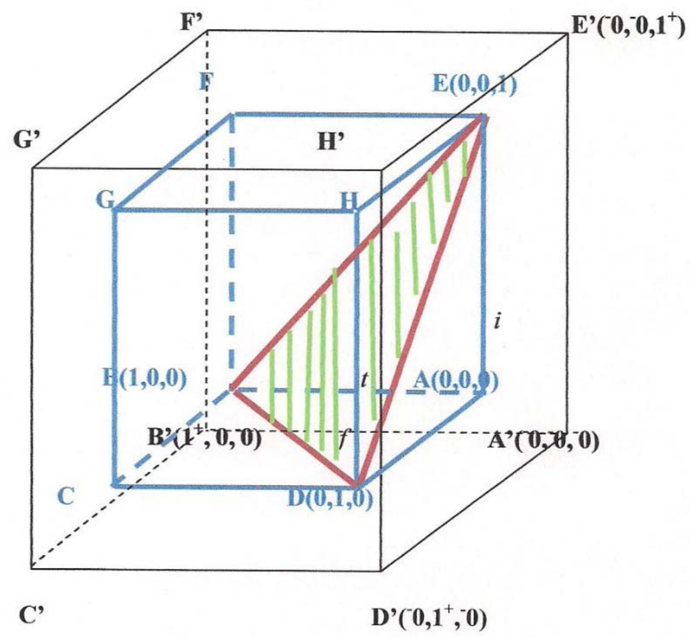

Fig. 2. Neutrosophic Cube

Let's consider a 3D-Cartesian system of coordinates, where t is the truth axis with value range in $]^-0,1^+[$, i is the false axis with value range in $]^-0,1^+[$, and similarly f is the indeterminate axis with value range in $]^-0,1^+[$.

We now divide the technical neutrosophic cube ABCDEFGH into three disjoint regions:

1) The equilateral triangle BDE, whose sides are equal to √(2) which represents the geometrical locus of the points whose sum of the coordinates is 1.

   If a point Q is situated on the sides of the triangle BDE or inside of it, then tQ+iQ+fQ=1 as in Atanassov-intuitionistic fuzzy set (A-IFS).
2) The pyramid EABD {situated in the right side of the triangle EBD, including its faces triangle ABD(base), triangle EBA, and triangle EDA (lateral faces), but excluding its face: triangle BDE } is the locus of the points whose sum of coordinates is less than 1 (Incomplete Logic).

3) In the left side of triangle BDE in the cube there is the solid EFGCDEBD (excluding triangle BDE) which is the locus of points whose sum of their coordinates is greater than 1 as in the paraconsistent logic. This is the **Extension Logic** part.

It is possible to get the **sum of coordinates strictly less than 1** (in Incomplete information), or **strictly greater than 1 (in contradictory** Extension Logic**)**. For example:

We have a source which is capable to find only the degree of membership of an element; but it is unable to find the degree of non-membership;

Another source which is capable to find only the degree of non-membership of an element;

Or a source which only computes the indeterminacy.

Thus, when we put the results together of these sources, it is possible that their sum is not 1, but smaller (Incomplete) or greater (Extension Logic).

8. Example of Extension Logic in 3-Valued Neutrosophic Logic

About a proposition P, the first source of information provides the truth-value T=0.8.

Second source of information provides the false-value F=0.7.

Third source of information provides the indeterminacy-value I=0.2.

Hence NL3(P) = (0.8, 0.2, 0.7).

Got Extension Logic, since Contradiction: T + F = 0.8 + 0.7 > 1.

Can remove Contradiction by normalization:

NL(P) = (0.47, 0.12, 0.41); now T+F ≤ 1.

9. Example of Extension Logic in 4-Valued Neutrosophic Logic

About a proposition Q, the first source of information provides the truth-value T=0.4, second source provides the false-value F=0.3, third source provides the undefined-value U=0.1, fourth source provides the contradiction-value C=0.2.

Hence NL4(Q) = (0.4, 0.1, 0.2, 0.3).

Got Extension Logic, since Contradiction C = 0.2 > 0.

Since C =T/\F, we can remove it by transferring its value 0.2 to T and F (since T and F were involved in the conflict) proportionally w.r.t. their values 0. 4,0.3:

xT/0.4 = yF/0.3 = 0.2/(0.4+0.3), whence xT=0.11, yF=0.09.

Thus T=0.4+0.11=0.51, F=0.3+0.09=0.39, U=0.1, C=0.

Conclusion

Many types of logics have been presented above related with Extension Logic. Examples of Neutrosophic Cube and its Extension Logic part, and Extension Logic in 3-Valued and 4-Valued Neutrosophic Logics are given.

Similar generalizations are done for **n-Valued Refined Neutrosophic Set**, and respectively **n-Valued Refined Neutrosopjhic Probability** in connections with Extension Logic.

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