On Crittenden and Vanden Eynden's Conjecture

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It is possible to cover all (positive) integers with n geometrical progressions of integers?

Find a necessary and sufficient condition for a general class of positive integer sequences such that, for a fixed n, there are n (distinct) sequences of this class which cover all integers.

Comments:

a) No. Let $a_1,...,a_n$ be respectively the first terms of each geometrical progression, and $q_1,...,q_n$ respectively their ratios. Let p be a prime number different from $a_1,...,a_n, q_1,...,q_n$. Then p does not belong to the union of these n geometrical progressions.

b) For example, the class of progressions $A_f = \{\{a_n\}_{n\geq 1} : a_n = f(a_{n-1},...,a_{n-i})$ for $n \geq i+1$, and $i, a_1, a_2,.. \in N^*\}$ with the property $\exists y \in N^*,\ \forall (x_1,...,x_i) \in N^{*i} : f(x_1,...,x_i) \neq y$. Does it cover all integers?

But, if $\forall y \in N^*,\ \exists (x_1,...,x_i) \in N^{*i} : f(x_1,...,x_i) = y$?

(Generally no.)

This (solved and unsolved) problem remembers Crittenden and Vanden Eynden's conjecture.

References:

[1] R.B. Crittenden and C. L. Vanden Eynden, *Any n arithmetic progressions covering the first 2^n integers covers all integers*, Proc. Amer. Math. Soc. 24 (1970) 475-481.

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[3] R. K. Guy, *Unsolved Problem in Number Theory*, Springer-Verlag, NewYork, Heidelberg, Berlin, 1981, Problem E23, p.136.