DARK MATTER MODEL BY QUANTUM VACUUM

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ABSTRAT

Title: Dark Matter Model by Quantum Vacuum

The idea of the model is quite simple:
The physical vacuum is a quantum system with a minimum energy when it is in his ground state. When the space has a gravitational field the space state increase his energy levels and the mass of the space begin to increase as well. The mass of the space would be the dark matter (D.M.).

Model refers to the same conception about 'vacuum' as the Quantum Electrodynamics (QED), which consider the space full of virtual electrons and virtual positrons.
Therefore it is easy to think that there are virtual particles which feel the gravitational forces named virtual gravitons. It is logical to think that the gravitational field excites the vacuum states in a way that "the vacuum would be heavier, the more intense gravitational field". The hypothesis of this DM model is that virtual gravitons are Dark Matter.

The paper is organised in nine parts:

The first one shows graphs about measures of spin speed stars on Andromeda galaxy and Milky Way.
The second one shows a simple mathematical model for flat galactic disks. Also it is got the density formula of total mass.

The third one studies the NFW model (Navarro, Frenk & White. 1996) for Dark Matter in bulge, disk and halo on galaxies.

The fourth one explains the model theoretically in a simple way because in my opinion the ultimate theory of DM will be explain by the Quantum Gravity, which there is not exist yet.

In the fifth one it is proved mathematically there is a function $\varphi_{MO-NFW}(E) = g \circ f^{-1}(E)$ which connects density φ_{MO-NFW} with gravitational field.
The sixth one explains some consequences of Dark Matter Model on galactic haloes, such as radius and spherical or elliptical shape.

In the seventh, the Dark Matter model is extended to galactic clusters. Through the model it is justified cluster haloes in a similar way to galactic haloes. Also it is explained the Virial method and gravitational lensing method to measure cluster masses in order to check the model predictions.

In the eighth it is shown that there is coherence between DM model and experimental evidences known about DM in Bullet Cluster and other clusters.

The ninth one proposes five experimental tests to check the model through astronomical measures.

1. INTRODUCTION

The idea of the model is quite simple:

The physical vacuum is a quantum system with a minimum energy when it is in his ground state. When the space has a gravitational field the space state increase his energy levels and the mass of the space begin to increase as well. The mass of the space would be the dark matter (D.M.).

Model refers to the same conception about 'vacuum' as the Quantum Electrodynamics' theory (QED). Readers with knowledge about QED know that virtual particles are able to exist during a short period of time only if $\Delta E \bullet \Delta t \leq \frac{\hbar}{2}$. So according to QED virtual particles could break the conservation energy principle for a tiny period of time.

Similarly Quantum Chromo Dynamic (QCD) considers the space full of particles named virtual gluons which interact with protons, neutrons and other particles.
It is right to think that space is full of particles which are concerned by gravitational field. The amount of this particles (virtual gravitons) would be bigger as the gravitational field is bigger.

In this paper, the author try to explain the basis of his theory in a qualitative way, because the final theory will be the Quantum gravity (QG) which there is not exist yet.
Theoretical physicist have been trying to develop QG for decades but they have failed for now, although there have been remarkable contributions to this theory.

The paper is organised in nine parts:

The first one shows graphs about measures of spin speed stars on Andromeda galaxy and Milky Way.
The second one shows a simple mathematical model for flat galactic disks. Also it is got the density formula $\rho(r)_{Total\ Mass} = \frac{K_G}{4\pi r^2}$
The third one studies the NFW model (Navarro, Frenk & White. 1996) for Dark Matter in bulge, disk and halo on galaxies.

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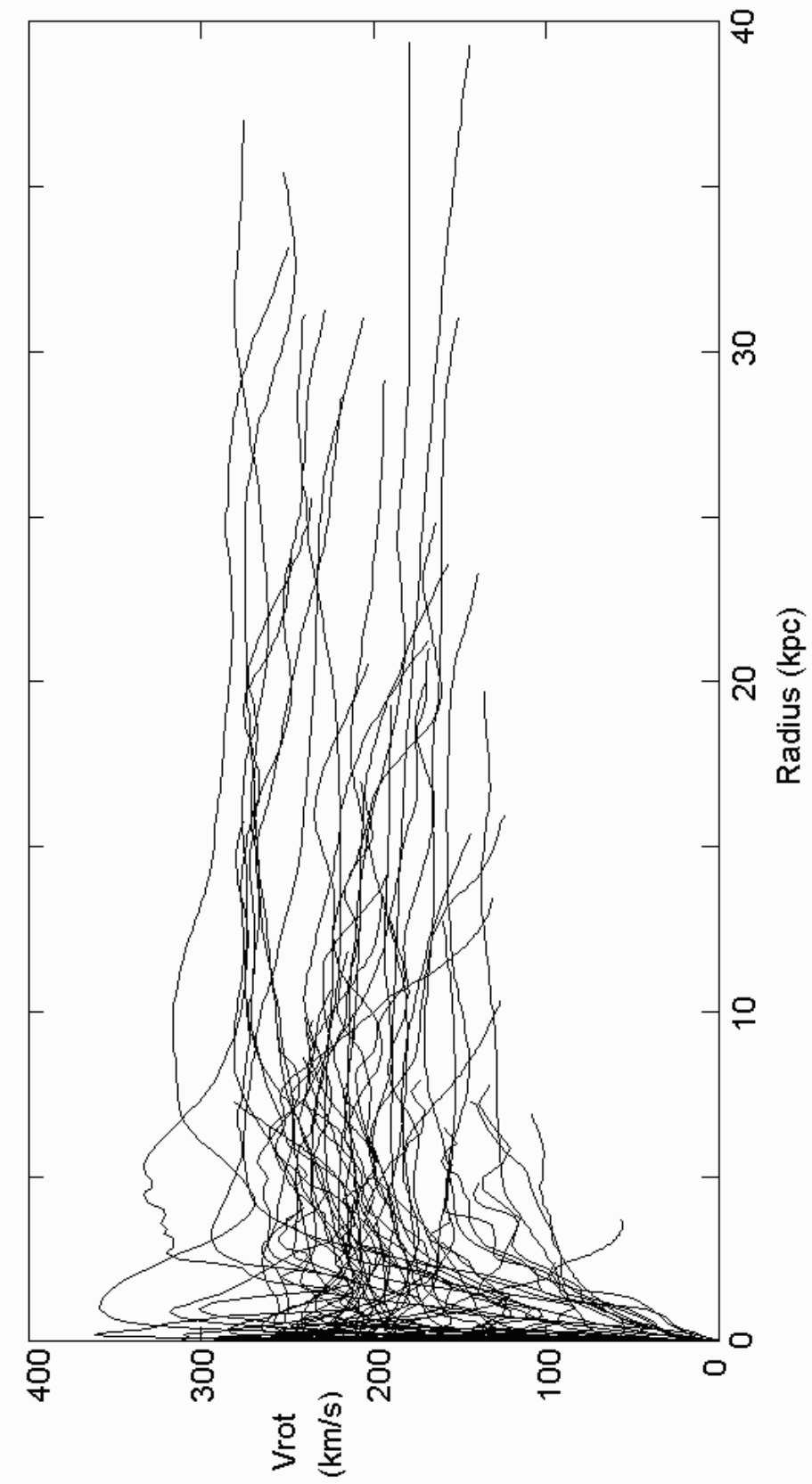
The ninth one proposes seven experimental tests to check the model through astronomical measures.

2. REAL SPIN SPEED GRAPHS

In the picture are shown real spin speed of stars in lots of galaxies, in their bulge and disk. It is remarkable the fact that curves are almost flat in disk region. Therefore is a good model to consider a curve flat in disk region.



2.1 SPIN SPEED GRAPHIC IN DISK OF ANDROMEDA

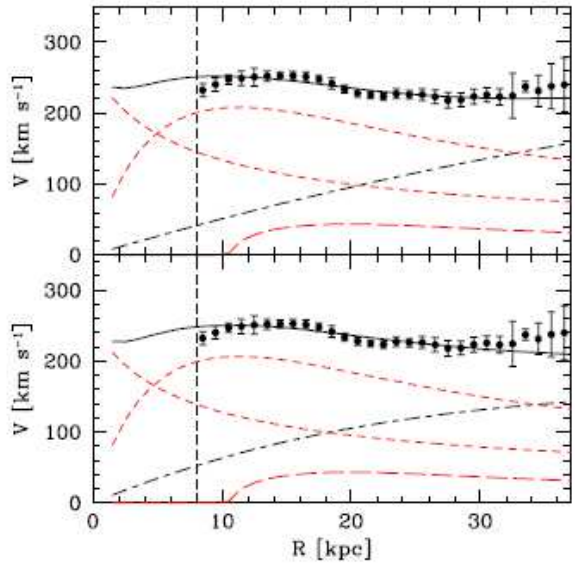


Fig. 12. The M31 rotation curve (points) and the best-fitting mass models (solid line) using a Burkert dark halo profile with h_d = 5.1 kpc, h_b = 2 kpc and n=4. Also shown are the dark halo contribution (dot-dashed line), the stellar disk and bulge (short-dashed line) and the gas contribution (long-dashed line). In the top panel, we show the best fit mass model (χ^2 = 0.81) with $(M/L)_b$ = 4.5 $M_\odot/L_\odot$, $(M/L)_d$ = 8.0 $M_\odot/L_\odot$ and R_B = 77 kpc. The case shown in the bottom panel refers to a fixed, lower value of the core radius, namely R_B = 28 kpc. For this case the best fitting values of the mass-to-light ratios are $(M/L)_b$ = 4.9 $M_\odot/L_\odot$ and $(M/L)_d$ = 7.4 $M_\odot/L_\odot$ (χ^2 = 1.17).

As Andromeda it is the nearest galaxy, it has lot of astrophysical measures.

As it is shown, a flat curve at disk is an acceptable model.

At disk there are baryonic mass and Dark matter.

Both kind of mass are the origin of flat curve of spin speed of stars.

Bibliography [4]: A wide-field H I mosaic of Messier 31(Andrómeda)
II. The disk warp, rotation and the dark matter halo.
Authors: Edvige Corbelli, Silvio Lorenzoni, Rene Walterbos, Robert Braun, and David Thilker
Published in arXiv:0912.4133v1 [astro-ph.CO] 21 Dec 2009

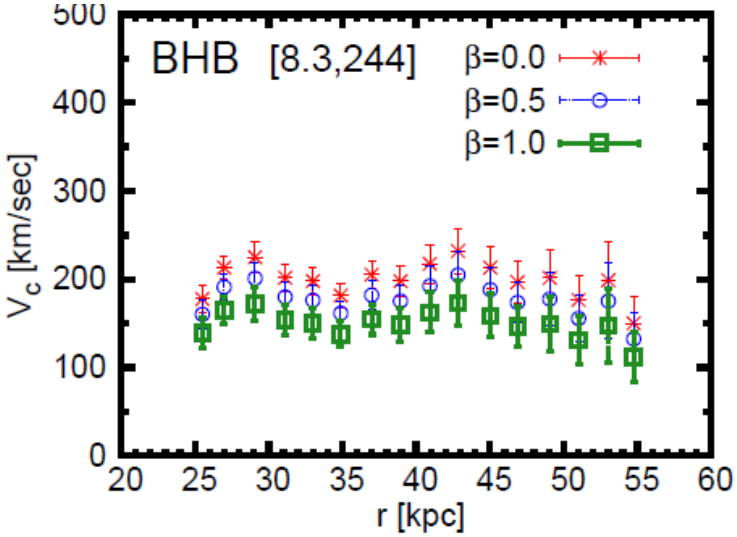
2.2 SPIN SPEED IN MILKY WAY HALO

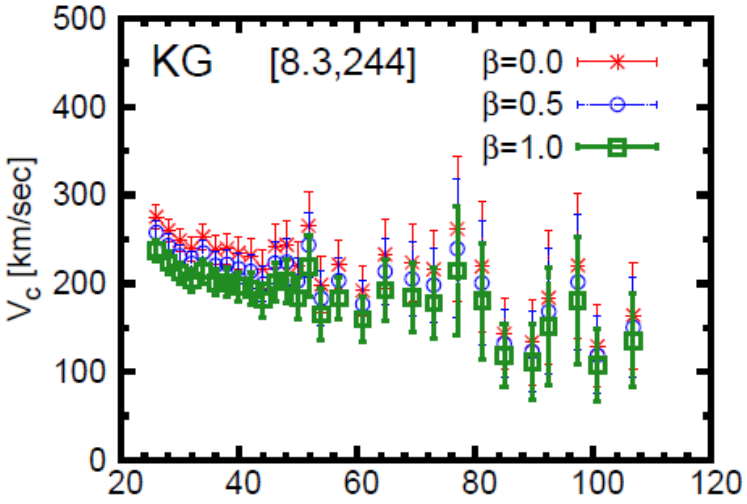
It is accepted that halo in Milky Way begin at 20 kpc and finish at 300 kpc approximately. According experimental results, it is remarkable that spin speed of stars decrease clearly for distances bigger than 60 kpc

Authors have calculated speed function as $v \sim r^{-0,3}$

It is known that Kepler behaviour is $v \sim r^{-0,5}$

So, it is needed Dark Matter hypothesis to explain a spin speed of star in halo so high.





Bibliography [6]: ROTATION CURVE OF THE MILKY WAY OUT TO 200 KPC
Authors: Pijushpani Bhattacharjee, Soumini Chaudhury, and Susmita Kundu
Published in arXiv:1310.2659v3 [astro-ph.GA] 26 Feb 2014

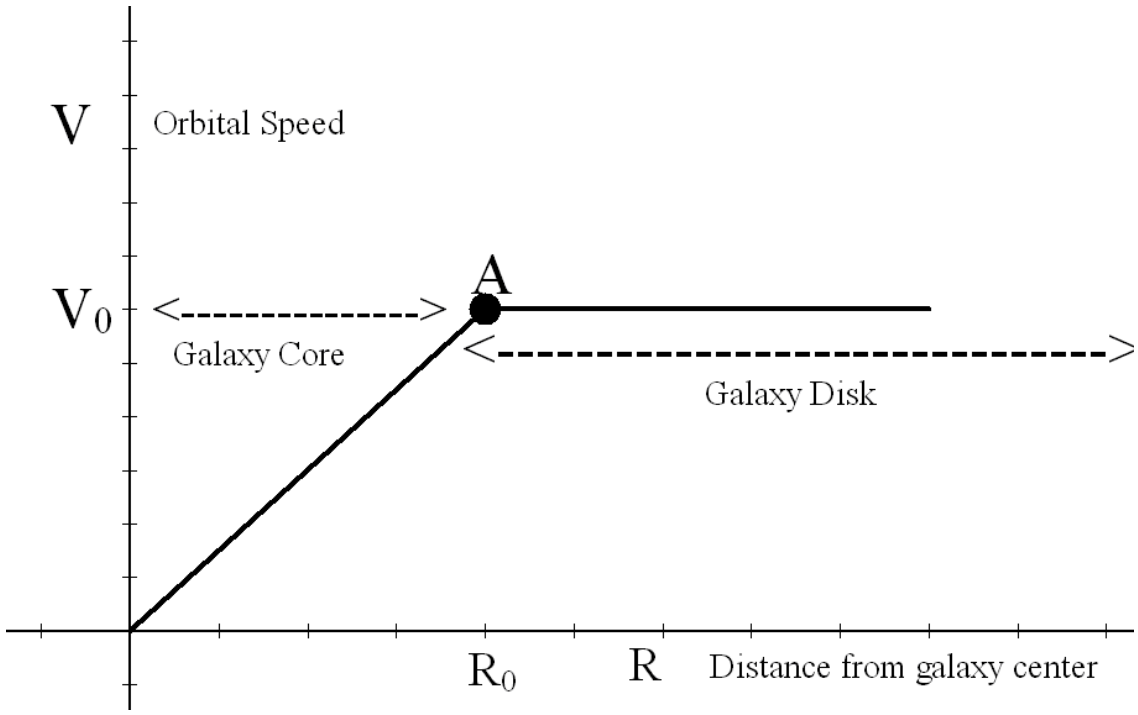
3. A MODEL FOR ROTATIONAL SPEED CURVES OF STARS IN GALAXIES

3.1 INTRODUCTION

According to experimental evidences, in the bulge, the angular velocity of its stars is almost constant. However in the galactic disk, the rotational speed of its stars is perceptibly constant and unexpectedly high in terms of ordinary matter (O.M.)

The rotational curve of star speed can be modelled in a easy way by the function that you can see in the picture.

Rotational speed curve model of stars around the galaxy centre in bulge and disk region.



As it is shown in the graphic, there are two regions clearly bounded:
Core or Bulge and Disk.

Core or Bulge Up to R_0 the angular speed is constant.
R_0 is the radius of galactic core.
Disk From R_0 the speed is constant and the curve is almost horizontal until the furthest stars of galaxy.

R represents the spin radius of a star around the galactic core, and M (< r) represents the total mass (ordinary and dark) contained by a sphere with radius R.

Galactic Bulge

As in this region ω = constant, from the third Keplero´s law written as $\omega^2 = \frac{GM}{R^3} = \frac{4\pi\rho G}{3} =$ constant, it is deduced that in this region, there is a density of mass constant.
Experimental evidences show that galactic core has a density of ordinary matter approximately constant with spherical symmetry and a mass density much bigger than the galactic disk. Therefore the constant angular speed can be reasonably explained by the amount of ordinary matter observed.

Galactic Disk

Disk is a large part of a galaxy which begins near the bulge, extending itself to the most external stars. Experimental data says that stars which are in this region have constant linear velocity, which means that its rotational speed is independent of the distance to the galactic core.
The spin speed of stars (from 200 km/s to 300 km/s) in this area is inexplicable through observable matter measured by astronomical methods. In all the galaxies whose mass and rotation curve have been measured, it has been checked that there is a large mass defect, which in some galaxies is 90% dark matter (DM) compared with 10% of ordinary matter or baryonic matter.

3.2 TOTAL MASS FORMULA IN THE GALACTIC DISK

If it is considered that in this region spin speed of stars is constant, the Virial theorem confirms that $v^2 = \frac{GM}{R} = \text{constant}$. From graphic point **A** it is deduced that $\frac{M_0}{R_0} = \frac{M}{R} = \frac{V_0^{\,2}}{G} = \text{Constant}=K_{GALAXY}$, where M_0 represents the total mass contained by the galactic bulge, R_0 represents its radius, V_0 represents the spin speed of stars in the galactic disk, R is the spin radius of a star placed in the galactic disk and M represents the total mass enclosed by a sphere with radius R. Therefore K_G is a constant which depend on each galaxy.

In conclusion, in the galactic disk $M_{TOTAL} = K_G \cdot R$ where M_{TOTAL} represents the total mass enclosed by a sphere with radius R.
For example in the Milky Way, taking into account $V_{SUN} = 220$ Km/s = V_0 . It is got

$$K_{M\text{-}W}=7{,}25 \cdot 10^{20}\ Kg/m.$$

3.3 MASS DENSITY FORMULA IN THE GALACTIC DISK

Taking into account $\begin{aligned} M_{TOTAL} &= K_G \bullet R --> dM_T = K_G dr \\ M_{TOTAL} &= \rho \bullet V --> dM_T = \rho 4\pi r^2 dr \end{aligned}$ it is concluded $\rho(r)_{TOTAL\,MASS} = \frac{K_G}{4\pi r^2}$, where $\rho(r)$ is the density function of total mass (ordinary + dark) in the galactic disk.

3.4 MASS IN MILKY WAY DISK

By integration of $\rho(r)_{MASA\ TOTAL}$ it is got

$M_{TOTAL}(< r)=M_{BULGE} + K_G \cdot (r- R_{BULGE})$ where r € Disco.

Supposing R_{BULGE}= 5 Kpc and $M_{BULGE} = 1,6 \cdot 10^{10}\ M_\odot$

$$M_{TOTAL}(< r) = 1.6 \cdot 10^{10}\ M_\odot + (r - 5Kpc) \cdot 1.1 \cdot 10^{10}\ M_\odot \quad r\ (Kpc) \in Disk$$

For example, mass in the whole disk would be

$$M_{TOTAL}(< 20\ Kpc) = 1,6 \cdot 10^{10}\ M_\odot + (20 - 5) \cdot 1.1 \cdot 10^{10}\ M_\odot \simeq 1,8 \cdot 10^{11}\ M_\odot$$

Which is an acceptable result.

4.NFW PROFILE FOR DARK MATTER IN GALAXIES

$\varphi_{DM-NFW}(R)$ it is considered as the best profile for dark matter in galaxies by the scientific community.

$\varphi_{DM-NFW}(R) = \frac{\varphi_0}{X(1+X)^2}$ where R € bulge, disk or halo and X = R / h

φ_0 and h depend on each galaxy.

In my opinion the reason why DM can be calculate by a simple law as $\varphi_{MO-NFW}(R)$ is that DM generation mechanism is universal.

It happens a similar thing in the Solar System, where the planets periods can be calculate by the gravitation law which depend on Solar mass as a parameter.

4.1 NFW PROFILE FOR MILKY WAY

In Milky Way it is accepted.

$\varphi_0 = (1.06\pm0.14)\times10^{-2} M_\odot pc^{-3}$ *y* h = 12.53±0.88 kpc

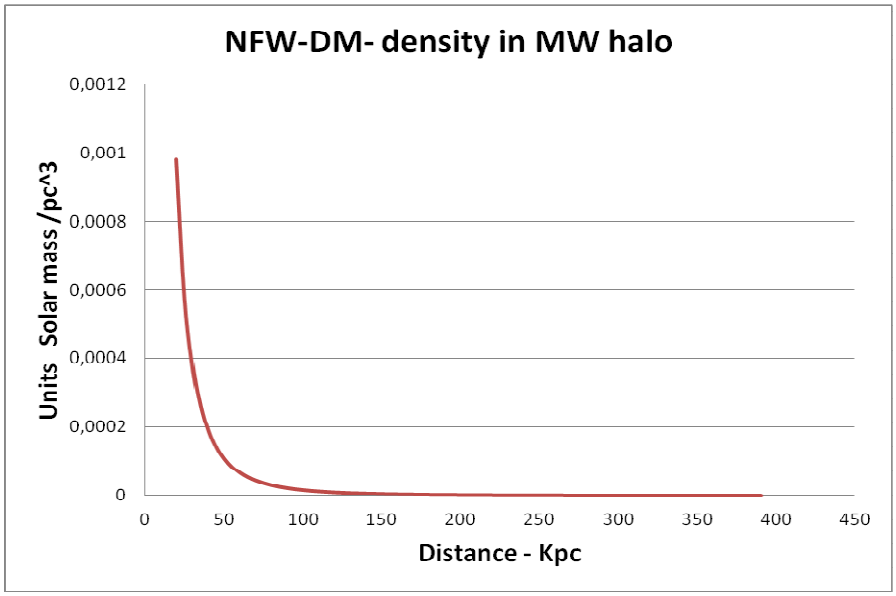
See bibliography [10] Yoshiaki Sofue-2013.

Approximately $5\,Kpc \le Disk \le 20\,Kpc$ and Halo < 385 Kpc so X= R /h < 31.

In chapter seven will be explained the reason why 385 Kpc is Milky Way radius halo.

Below is organised and plotted density function, units $M_\odot pc^{-3}$, depending on distance (kpc).

R Kpc	X	Densidad
20,048	1,6	0,00098003
25,06	2	0,000588889
50,12	4	0,000106
75,18	6	3,60544E-05
100,24	8	1,6358E-05
125,3	10	8,76033E-06
150,36	12	5,22682E-06
175,42	14	3,36508E-06
200,48	16	2,29239E-06
225,54	18	1,63127E-06
250,6	20	1,20181E-06
275,66	22	9,10809E-07
300,72	24	7,06667E-07
325,78	26	5,59249E-07
350,84	28	4,50144E-07
375,9	30	3,67673E-07



4.2 DARK MATTER IN MILKY WAY

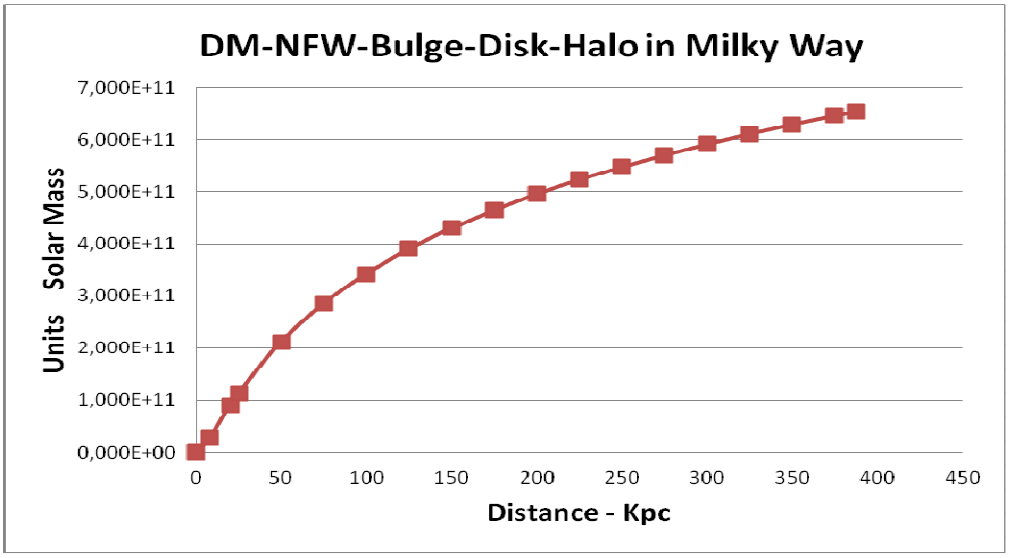
Density DM profile $\varphi_{DM-NFW}(R)$ works in bulge, disk and halo, so by integration it is got

$$M_{MO}(<R) = 4\pi\varphi_0 h^3\left[\ln(1+x) - \frac{x}{1+x}\right] \quad x = R/h \quad x \in Bu\lg e, Disk\ and\ \ halo\ y\ \ \varphi_0 h^3 = 2.085\cdot 10^{10} M_\odot$$

where 0 < X < 31 because 0 < R < 385 Kpc. $M_{DM}(<R)$ Represents DM inside a sphere with R as radius. Particularly $M_{DM}(<R=h) = 2.4272\,\varphi_0 h^3$ because X=1

Below is organised and plotted DM each 25 kpc in bulge, disk and halo. Second column is for X= R / h and third column is mass in Solar masses units ($M_\odot$)

R Kpc		
0	0	0,000E+00
8= $R_\odot$	0,64	2,737E+10
20=R_{DISk}	1,6	8,913E+10
25	2	1,132E+11
50	4	2,121E+11
75	6	2,853E+11
100	8	3,428E+11
125	10	3,901E+11
150	12	4,302E+11
175	14	4,650E+11
200	16	4,958E+11
225	18	5,233E+11
250	20	5,482E+11
275	22	5,710E+11
300	24	5,919E+11
325	26	6,113E+11
350	28	6,294E+11
375	30	6,463E+11
387,5	31	6,543E+11



SOME DATA ABOUT MASSES IN MILKY WAY

Materia Oscura en la Via Lactea

a) $M_{DM}(R < R_{\odot} = 8kpc) = 2.7 \cdot 10^{10} M_{\odot}$

b) $M_{DM}(R < galactic\ disk = 20kpc) = 8.8 \cdot 10^{10} M_{\odot}$

c) $M_{Total\ DM\ Milky\ Way}(R < Halo = 385\,kpc) = 6.5 \cdot 10^{11} M_{\odot}$

It is remarkable that total DM is 7.4 times DM inside galactic disk.

Another data [10] Yoshiaki Sofue-2013

Solar mass $M_{\odot} \approx 2 \cdot 10^{30} Kg$

Barionic mass $M_{Bulge} = 1,6 \cdot 10^{10} M_{\odot}$ $M_{Disk} = 3.4 \cdot 10^{10} M_{\odot}$ $M_{Bulge+Disk} = 5 \cdot 10^{10} M_{\odot}$

Total mass $M_{Barionic+DM} = 5 \cdot 10^{10} M_{\odot} + 6.5 \cdot 10^{11} M_{\odot} = 7 \cdot 10^{11} M_{\odot}$

Barionic mass fraction $M_{Barionic} / M_{TOTAL} = \dfrac{5 \cdot 10^{10} M_{\odot}}{7 \cdot 10^{11} M_{\odot}} = 0.07 --> 7\%$

4.3 TOTAL MASS IN MILKY WAY HALO DEPENDING ON RADIUS

As barionic mass in Milky Way is estimated in $5 \cdot 10^{10} M_{\odot}$. It is acceptable for halo

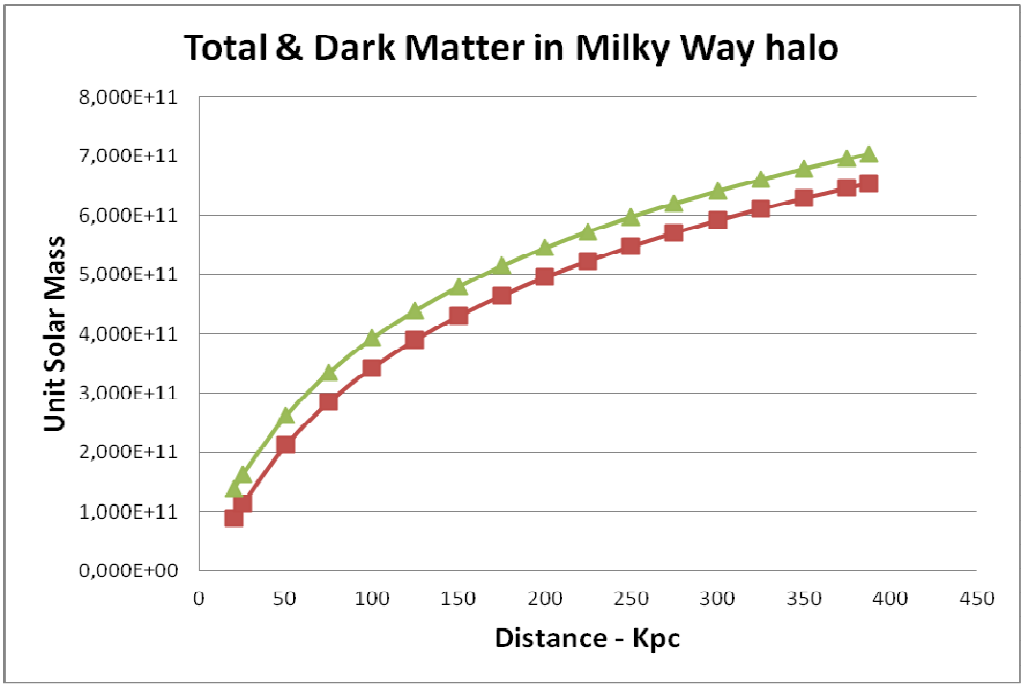
$$M_{TOTAL}(< R) = M_{BARIONIC} + M_{DM}(< R) = 5 \cdot 10^{10} M_{\odot} + 4\pi\varphi_0 h^3 \left[\ln(1+x) - \frac{x}{1+x} \right]$$

where $x = R/h$ $x \in halo$ $1.64 < x < 31$ *y* $\varphi_0 h^3 = 2.085 \cdot 10^{10} M_{\odot}$

because baryonic mass in halo is negligible.

Below are organised and plotted total mass and DM each 25 kpc.

R Kpc	X	M_{DARK}(<R)	M tot(<R)
20	1,6	8,908E+10	1,391E+11
25	2	1,131E+11	1,631E+11
50	4	2,120E+11	2,620E+11
75	6	2,852E+11	3,352E+11
100	8	3,427E+11	3,927E+11
125	10	3,899E+11	4,399E+11
150	12	4,300E+11	4,800E+11
175	14	4,648E+11	5,148E+11
200	16	4,955E+11	5,455E+11
225	18	5,230E+11	5,730E+11
250	20	5,479E+11	5,979E+11
275	22	5,707E+11	6,207E+11
300	24	5,916E+11	6,416E+11
325	26	6,110E+11	6,610E+11
350	28	6,290E+11	6,790E+11
375	30	6,459E+11	6,959E+11
387,5	31	6,540E+11	7,040E+11

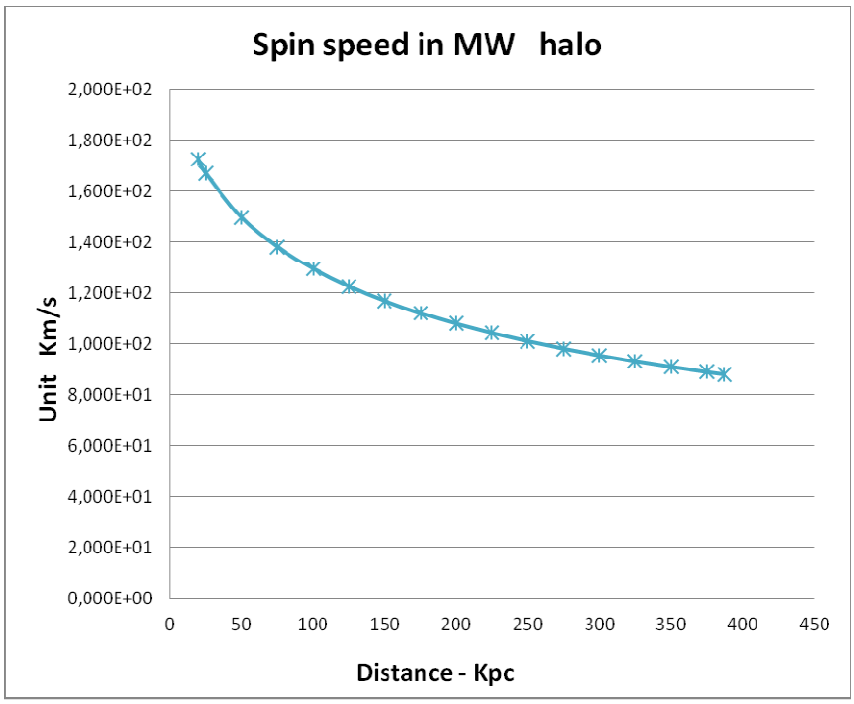


4.4 SPIN SPEED IN MILKY WAY HALO

According Virial Theorem , formula for speed is $v^2 = \frac{GM(<r)}{r^2}$.

Below are organised and plotted magnitudes each 25Kpc.

R Kpc	X	DM(<R)	M tot(<R)	V^2	V Km/s	Exponent
20	1,6	8,908E+10	1,391E+11	2,971E+10	1,724E+02	-0,14
25	2	1,131E+11	1,631E+11	2,788E+10	1,670E+02	
50	4	2,120E+11	2,620E+11	2,239E+10	1,496E+02	-0,195
75	6	2,852E+11	3,352E+11	1,909E+10	1,382E+02	
100	8	3,427E+11	3,927E+11	1,678E+10	1,295E+02	-0,245
125	10	3,899E+11	4,399E+11	1,504E+10	1,226E+02	
150	12	4,300E+11	4,800E+11	1,367E+10	1,169E+02	-0,271
175	14	4,648E+11	5,148E+11	1,257E+10	1,121E+02	
200	16	4,955E+11	5,455E+11	1,165E+10	1,080E+02	-0,295
225	18	5,230E+11	5,730E+11	1,088E+10	1,043E+02	
250	20	5,479E+11	5,979E+11	1,022E+10	1,011E+02	-0,305
275	22	5,707E+11	6,207E+11	9,643E+09	9,820E+01	
300	24	5,916E+11	6,416E+11	9,138E+09	9,559E+01	-0,313
325	26	6,110E+11	6,610E+11	8,690E+09	9,322E+01	
350	28	6,290E+11	6,790E+11	8,289E+09	9,105E+01	-0,323
375	30	6,459E+11	6,959E+11	7,929E+09	8,904E+01	
387,5	31	6,540E+11	7,040E+11	7,762E+09	8,810E+01	-0,323



In the last column in table is shown the exponent of formula $V(R) = KR^{\exp}$ where V represents velocity and R distance. It is remarkable that exp > -0.5.
It is known that exponent = -0.5 is Kepler decreasing.

5.DARK MATTER MODEL BY QUANTUM VACUUM

It is known that QED theory consider the space as a place plenty of virtual particles which feel the electro-weak forces. Readers with knowledge about QED know that virtual particles are able to exist during a short period of time only if $\Delta E \bullet \Delta t \leq \frac{\hbar}{2}$. So according to QED virtual particles could break the conservation energy principle for a tiny period of time.

Similarly the Quantum Chromo Dynamics consider the space full of virtual particles, called gluons. Therefore it is easy to think that there is virtual particles which feel the gravitational forces named virtual gravitons.

It is logical to think that the gravitational field excites the vacuum states in a way that **"the vacuum would be heavier, the more intense gravitational field is".** This one would be the main hypothesis of Dark Matter Model by Quantum Vacuum.

In a further chapter it will be got a formula to justify this weird hypothesis about DM nature.

To explain this interaction between gravity and vacuum, we are going to see the well-known case of the relationship between physical electron and nude electron according to Quantum Electrodynamics (QED).

The distinction between physical-electron and nude electron could be proved when particle accelerators had enough energy to penetrate in the physical electron. In other words, when high energy electrons are shot against the electrons, they suffer a dispersion that can not be explained by a Coulomb type potential because when the electron break through the physical electron, the net charge that is "noticed" by the projectile electron is higher than the physical electron charge. However, the physical electron is very small because the virtual positrons shield the nude electron in a very small region, so that the electric force become a Coulomb´s force in an extremely small distance around the nude electron. Readers can find lot of books about QED, for instance, my source about nude electron and physical electron is Harald Fritzsch. Los quarks, la materia prima de nuestro Universo. 1982. Alianza Editorial.

There are two main differences between electric and gravitational force

a) Intensity of gravitational force is much lower than electric force.
b) The shielding of the nude electron happens in a extremely small area around the nude electron since the virtual positrons are an opposite sign and they decrease the total charge of the physical electron. Because of the virtual positron charge is opposite to the charge of the electron, the physical electron net charge is lower than the naked electron one.

However, the gravitational force is always attractive, therefore, virtual gravitons are attractive too and because of this the total mass (usual mass + dark mass) increases as we consider a growing volume of space around ordinary matter.

For these two reasons, in a Solar System scale, the net dark matter is negligible versus ordinary matter. However, on a galaxy scale, the dark matter is dominant against ordinary matter because of the huge size of space into a galaxy.

5.1 THE ULTIMATE DIFFERENCE BETWEEN DARK MATTER AND ORDINARY MATTER

According this model DM is generated by ordinary matter (O.M.). In other words, DM can not exist independently from ordinary matter.

By now I do not know any experimental evidence of an astronomical region with pure DM. In my opinion this fact is surprising because it is accepted that DM is 90% in the Universe versus 10% of ordinary matter. In addition the difference between big galaxies and small galaxies is very wide. However the proportion DM versus OM is not very different all over galaxies according to the experimental measures. In my view, this fact could be explained because DM is generated by OM, so there is a functional dependence between both kind of matters.

I am going to explain the difference between DM and OM using as example the physic and nude electron from QED.

To begin I would say that virtual particles in the quantum vacuum exist because it is possible to create baryonic particles if you give the energy to that virtual particles, in other words: virtual particles + energy = baryonic particles.

Now we consider a nude electron which produces a big instability in the space surrounding it, so a virtual positrons cloud go around the nude electron. I would say that virtual positron exist because thanks to them the total charge of physic electron is lower that the charge of nude electron.

I am going to translate this ideas to try to explain the DM nature:

a) If we consider a galaxy, its ordinary matter produces a gravitational field which excite the virtual gravitons in the surrounding space, so this way the space has a bigger mass.
b) Inside a big cosmic void the gravitational field is very weak, so the space has his virtual gravitons in their ground state and as a consequence the mass of the space is minimum. I think that in this situation, it is not possible for pure DM to create central gravitational fields. In the 9 epigraph, it will be proposed a experimental proof based on gravitational lenses to check this theoretical prediction.

According this model. in what sense is real DM? DM is real because it is responsible of 90 % of total mass in a galaxy, although DM is composed by virtual particles.

This model explain in a simple way the impossibility to have a region with pure DM, also this model explain easily the proportion DM versus OM because DM depend on ordinary matter.

The ultimate explanation of the dark matter in a galaxy and of the rotation curve of the stars will only be possible when a complete theory of quantum gravity be available, which, unfortunately there is not exist nowadays despite the fact that the most brilliant theoretical physicists have been working on this theory for decades.

5.2 DARK MATTER MODEL IS COHERENT WITH NFW PROFILE

It is known that NFW profile $\varphi_{MO-NFW}(X) = \frac{\varphi_0}{X(1+X)^2}$ can explain DM in galaxies. The only thing is needed is a couple of parameters φ_0 y h to adapt the profile each galaxy.
In my opinion, this fact is a reason to think that DM is generated by a universal mechanism.

The main hypothesis this work is to state that mechanism has a quantum gravitational nature, which depend on gravitational field intensity.
In conclusion the model is coherent with NFW profile because it is based on a universal mechanism as well.
A direct consequence of universal mechanism of DM generation is that it works at different scales: stars, galaxies and galaxy clusters.

In my opinion, that would explain rightly the DM subhaloes property inside galactic halo or inside cluster halo.

So in general it is possible to state the total DM mass for galaxies.
$M_{MO}(<R) = 4\pi\varphi_{G0}h_G^{\,3}\left[\ln(1+x) - \frac{x}{1+x}\right]$ $x = R/h_G$ $x \in Bu\lg e, Disk, halo.$ Where φ_{G0} and h_G depend on each galaxy.

6. NFW DM DENSITY AS A FUNCTION OF GRAVITATIONAL FIELD INTENSITY

The main goal this chapter is to demonstrate that it exist a mathematical function for DM density depending on gravitational field intensity E.

6.1 GRAVITATIONAL FIELD E IN HALO DEPENDING ON DISTANCE TO THE GALACTIC CENTRE

In order to do some calculus it is considered Milky Way data

As $E = \frac{GM_{TOTAL}(<R)}{R^2}$, where

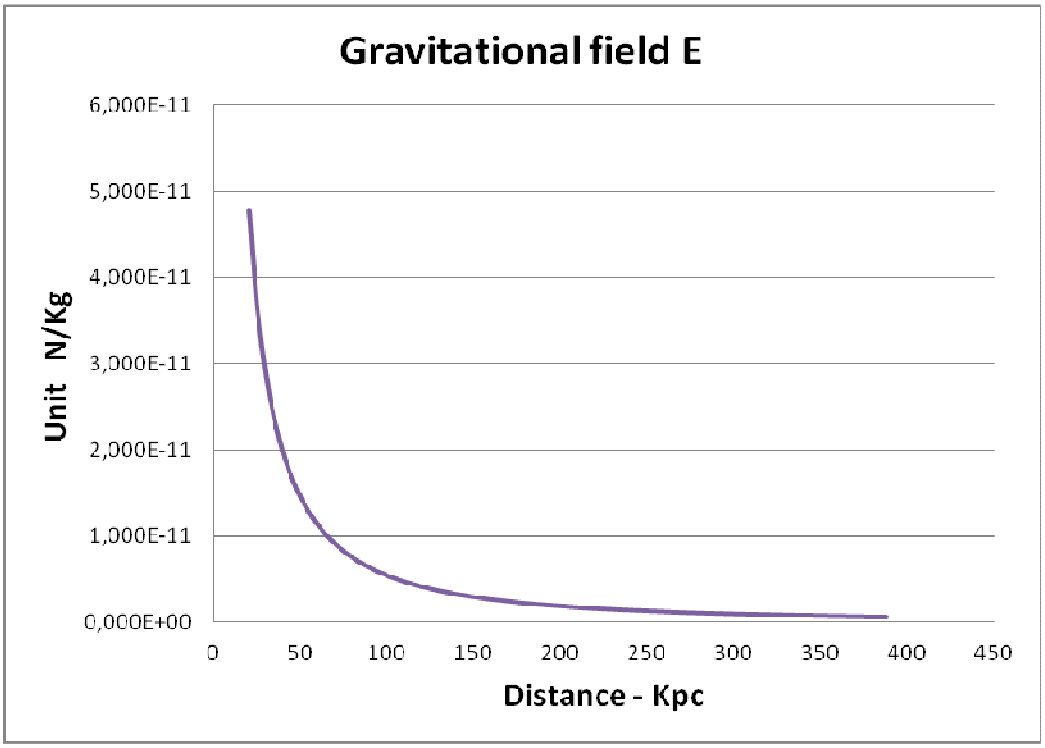
$$M_{TOTAL}(<R) = M_{Barionic} + 4\pi\varphi_0 h^3\left[\ln(1+x) - \frac{x}{1+x}\right]$$

$$where\ x = \frac{R}{h} \in halo 1.64 < X < 31\ and\ M_{Barionic} = 5{\cdot}10^{10} M_{\odot}$$

$$E = \frac{GM_{TOTAL}(<R)}{R^2} = \frac{GM_{TOTAL}(<R)}{h^2 X^2} = \frac{G\left\langle M_{B-VIA\ LACTEA} + 4\pi\varphi_0 h^3\left[\ln(1+x) - \frac{x}{1+x}\right]\right\rangle}{h^2 X^2}$$

This formula shows how E depend on distance R. E = f(x) where x=R/h
It is easy to see that E is decreasing with distance. However below is organised and plotted, which shows this property rightly.

		$M_{TOTAL}(M_{\odot})$	Field E
R Kpc	X	M tot(<R)	N/Kg
20	1,6	1,391E+11	4,781E-11
25	2	1,631E+11	3,589E-11
50	4	2,620E+11	1,441E-11
75	6	3,352E+11	8,193E-12
100	8	3,927E+11	5,399E-12
125	10	4,399E+11	3,871E-12
150	12	4,800E+11	2,933E-12
175	14	5,148E+11	2,311E-12
200	16	5,455E+11	1,875E-12
225	18	5,730E+11	1,556E-12
250	20	5,979E+11	1,315E-12
275	22	6,207E+11	1,129E-12
300	24	6,416E+11	9,802E-13
325	26	6,610E+11	8,605E-13
350	28	6,790E+11	7,622E-13
375	30	6,959E+11	6,805E-13
387,5	31	7,040E+11	6,446E-13



It is known a mathematical theorem which states that there is inverse function for decreasing functions, therefore if E = f(x) where 1,6 < x < 31 e $\Rightarrow$ Exits x= $f^{-1}(E)$ where 6,446E-13 < E < 4,781E-11
It is obvious that formula for $f^{-1}(E)$ is very complicated, but its existence it the only thing important for us.

6.2 NFW PROFILE DM DENSITY DEPENDING ON GRAVITATIONAL FIELD E IN HALO

It will be considered Milky Way data as an example, but it is a general results.

$$\varphi_{MO-NFW}(X)=\frac{\varphi_0}{X(1+X)^2}$$ where r € halo 20 Kpc < R< 385 Kpc → 1,6 < x < 31

In previous epigraph it was demonstrated that x= $f^{-1}(E)$ where 6,446E-13 < E < 4,781E-11 and E= f (x) is the right function, which is shown below.

$$E=\frac{G\left\langle M_{B-VIA\ LACTEA}+4\pi\varphi_0 h^3\left[\ln(1+x)-\frac{x}{1+x}\right]\right\rangle}{h^2X^2}$$

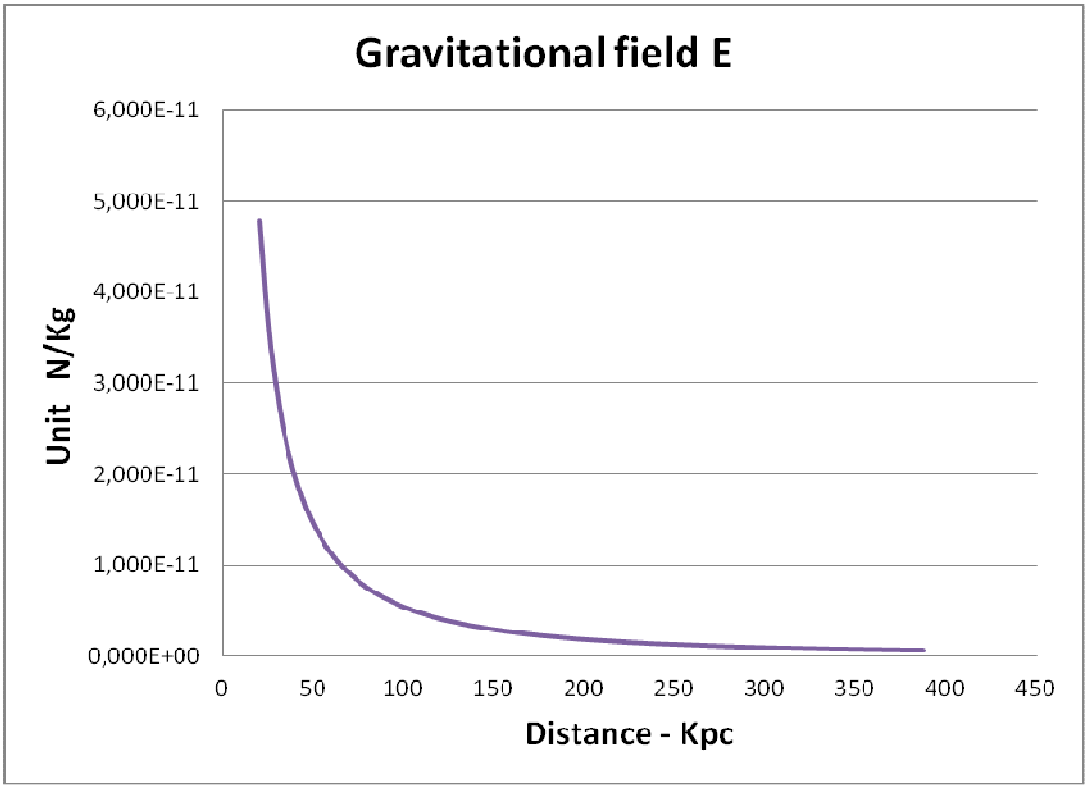
If $\varphi_{MO}(x)=g(x)=\frac{\varphi_0}{X(1+X)^2}$ replacing x = $f^{-1}(E)$ inside g(x) it is got

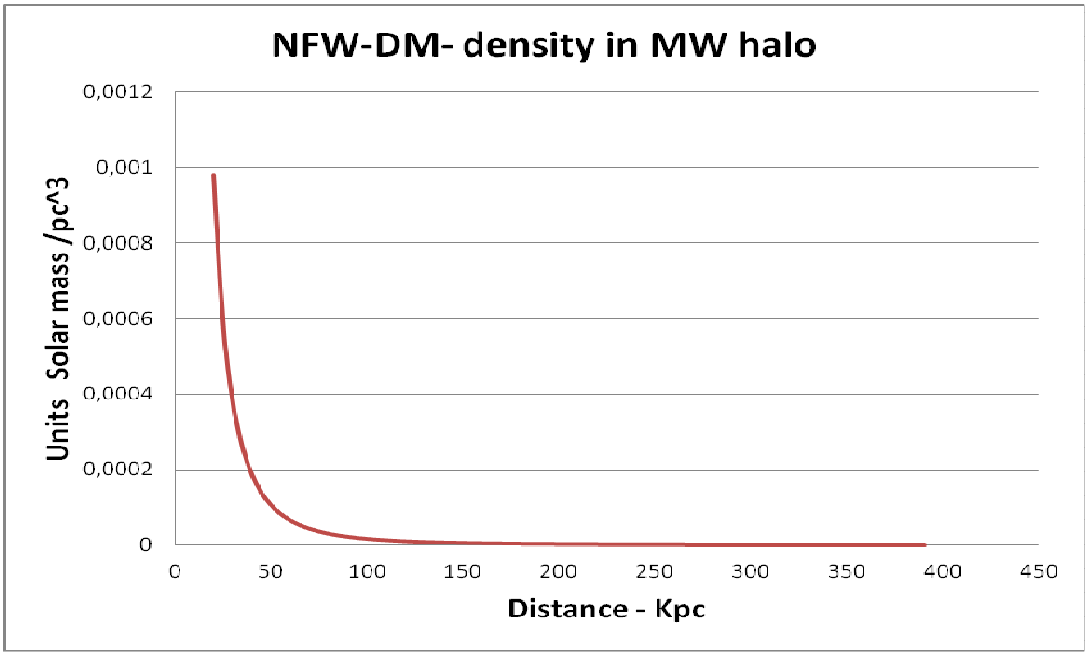
$$\varphi_{MO}(E)=g\circ f^{-1}(E)=\frac{\varphi_0}{\left[f^{-1}(E)\right](1+\left[f^{-1}(E)\right])^2}$$ where 6,446E-13 < E < 4,781E-11

Through this formula $\varphi_{MO-NFW}(E)=g\circ f^{-1}(E)$ has been demonstrated that it exists a mathematical dependence between DM density and gravitational field.

The main goal this paper is to show that there are evidences about DM density depend on gravitational field physically, not only mathematically.

Below there are joined both graphs



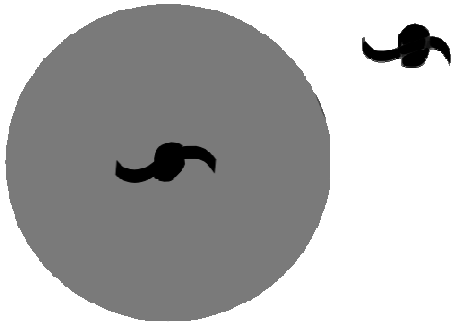


7. DM MODEL CONSEQUENCES ON GALACTIC HALOES

It has been shown $\varphi_{MO-NFW}(E) = g \circ f^{-1}(E)$. As it is known E is a vectorial field, which will have important consequences on haloes.

7.1 GALACTIC HALO

According the model, DM is generated by gravitational field,
So it is right to think that galactic halo is the region where galactic gravitational field dominates over galactic field of his neighbour galaxies.
This kind of halo may be called ***Gravitational halo.***

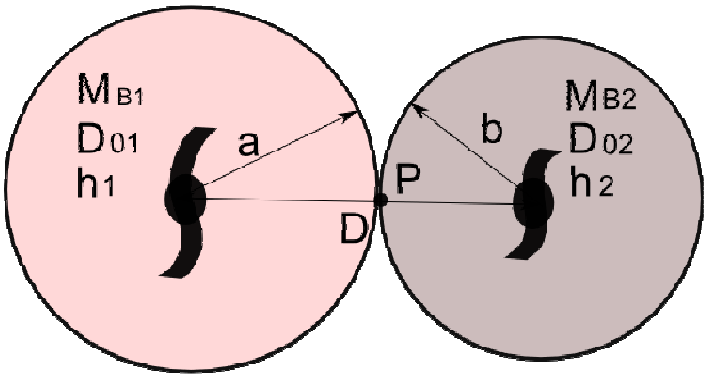


For example, Andromeda and Milky Way are two similar galaxies whose distance is 770 kpc, so it is right to think that a half distance both gravitational field add zero.
It is clear that this halo definition is totally different from Virial radius R_{200} , which is widely used by scientific community.

7.2 GALACTIC HALO RADIUS DEPEND ON GALAXY NEIGHBOUR MASS AS WELL

To calculate radius, it is considered point P, where total field is zero.

Picture shows two galaxies G1 and G2



Its baryonic masses are Mb1 y Mb2
$D_{01} = \rho_{01}$, $D_{02} = \rho_{02}$, h_1 *y* h_2 are
Its NFW parameters.
$x=R/h_1$ for G1
$y=R/h_2$ for G2.
Distance D, and radii a and b. So
$a+b=D \dashrightarrow b=D-a$.
At point P it is got

$$x=\frac{a}{h_1} \dashrightarrow \quad b=D-xh_1 \qquad y=\frac{b}{h_2} \dashrightarrow \quad y=\frac{D-xh_1}{h_2}$$

According the formula for field

$$E_1=\frac{G\left\langle M_{B1}+4\pi\varphi_{01}h_1^{\,3}\left[\ln(1+x)-\frac{x}{1+x}\right]\right\rangle}{h_1^{\,2}x^2} \text{ y } E_2=\frac{G\left\langle M_{B2}+4\pi\varphi_{02}h_2^{\,3}\left[\ln(1+y)-\frac{y}{1+y}\right]\right\rangle}{h_2^{\,2}y^2}$$

Since at P point both field are equals it is got

$$\frac{G\left\langle M_{B1}+4\pi\varphi_{01}h_1^{\,3}\left[\ln(1+x)-\frac{x}{1+x}\right]\right\rangle}{h_1^{\,2}x^2}=\frac{G\left\langle M_{B2}+4\pi\varphi_{02}h_2^{\,3}\left[\ln(1+y)-\frac{y}{1+y}\right]\right\rangle}{h_2^{\,2}y^2}$$

Plus equation $y = \frac{D - xh_1}{h_2}$ made a system with two equation and two unknown quantities

The system is very hard to solve, but it is possible.
Particularly, for twins galaxies x=y → a=b → a=D/2
This is precisely what happen in Milky Way and Andromeda.

7.3 TOTAL DM FOR A GALAXY DEPEND ON GALACTIC NEIGHBOUR MASS AS WELL

The formula for total DM is

$M_{TOTAL\ DM}(< R_{Halo}) = 4\pi\varphi_{G0}h_G^{\ 3}\left[\ln(1+X_{Halo}) - \frac{X_{Halo}}{1+X_{Halo}}\right]$ where $4\pi\varphi_{G0}h_G^{\ 3}$ depend on each galxy R_{Halo} is radius and $X_{Halo} = R_{Halo} / h$.

As R_{Halo} depend on radius and mass of galaxy neighbour it is concluded that $M_{TOTAL\ DM}$ depend on radius and mass of galactic neighbour as well.

Since total DM depend on halo radius through $f(X) = \ln(1+X) - \frac{X}{1+X}$

as $X_{Halo} = R_{Halo} / h >> 1$ then $f(X) \simeq \ln(1+X) - 1$

This fact means that total DM for R= 385 Kpc (***gravitational halo*** for Milky Way) it is not so different from total DM for R_{200}=305 kpc (Virial radius of Milky Way).

In conclusion, although the gravitational halo defined in the model is very different from Virial radius for halo, the amount of DM inside halo with Virial halo is similar to DM calculated with the gravitational halo.

7.4 HALO SYMMETRY DEPEND ON GRAVITATIONAL FIELD SYMMETRY

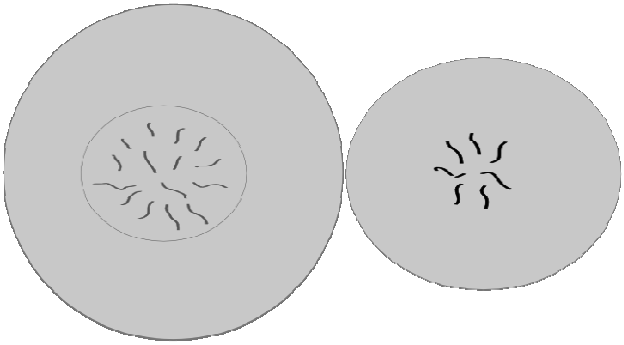
According the model is right this property. In general haloes are not spherical because gravitational field has not spherical symmetry.

Recently it has been proved that Milky Way halo is elliptical and the main axis are in the plane of galactic disk. This fact may be explained by the model because the gravitational field in galactic disk directions is more strong than perpendicular direction.

8. DARK MATTER MODEL FOR GALAXY CLUSTERS

8.1 CLUSTER HALO

According the model is right to define cluster halo as the region where own gravitational field dominates over the neighbour ones. That is, according the model, cluster would have gravitational halo.

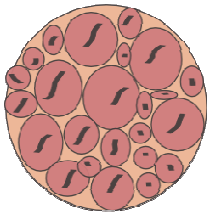


In similar way to galaxies, for twins cluster whose typical distance is 10 Mpc, its radius is 5 Mpc.

According the model, galaxies not only generate DM inside their own haloes, but also in the cluster halo.

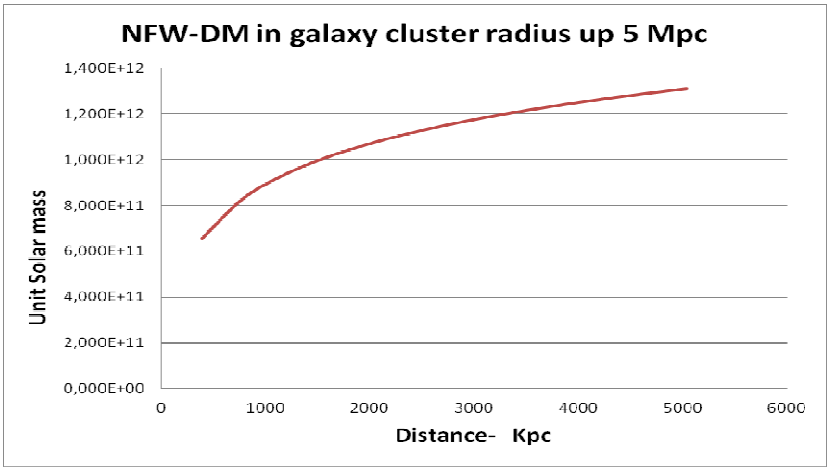
Below is organised and plotted data from the same Milky Way model used in chapter 4, for a halo extended to 5 Mpc.
First date belong to total DM in MW and the last one date belong to a halo with radius 5 Mpc, which has two times more mass, although the radius is 13 times bigger.

CLUSTER HALO



DM Milky Way model for R= 5 Mpc

R Kpc	X	M(<X)
0	0	0,000E+00
387,5	31	6,543E+11
775	62	8,278E+11
1162,5	93	9,313E+11
1550	124	1,005E+12
1937,5	155	1,063E+12
2325	186	1,110E+12
2712,5	217	1,150E+12
3100	248	1,185E+12
3487,5	279	1,215E+12
3875	310	1,243E+12
4262,5	341	1,268E+12
4650	372	1,290E+12
5037,5	403	1,311E+12



8.2 NFW PROFILE FOR DM IN GALAXY CLUSTER

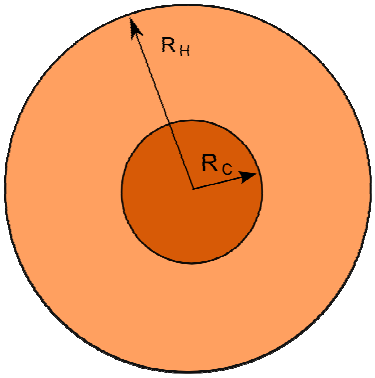
As according model, mechanism of DM generation is universal then NFW profile density DM should be right for clusters. $\varphi_{DM-NFW-CLUSTER}(R)=\frac{\varphi_{0C}}{X(1+X)^2}$ *where* $X=\frac{R}{h_C}$

And φ_{0C} *y* h_C depend on each cluster.

φ_{0C} *and* h_C may be calculated through two DM measures on cluster.

The first one would be over the sphere enclose all galaxies, which is represented in picture with radius R_C. The method in this case would be the Virial theorem.

The second one would be over a sphere including cluster halo, which has radius R_H. The method would be gravitational lensing because is able to measure the total mass spread through a region so wide as a cluster halo.



Both methods measure total mass, so it is needed subtract baryonic mass, which should be calculated with a different method.

So $DM_{\text{INSIDE CLUSTER}} = M_{\text{TOTAL- VIRIAL}} - M_B$ would be DM inside a sphere with R_C radius.

$DM_{\text{TOTAL CLUSTER}} = M_{\text{TOTAL- GRAVITATIONAL LENSING}} - M_B$ would be total DM included DM from halo cluster.

The two masses measured allow to calculate φ_{0C} *y* h_C through equations (1) y (2)

$$M_{DM\ INSIDE\ CLUSTER}(<R_{CLUSTER})=4\pi\varphi_{0C}h_C^{\,3}\left[\ln(1+X_C)-\frac{X_C}{1+X_C}\right] \quad (1)$$

where $X_C=\frac{R_{CLUSTER}}{h_C}$ *and* R_C *is cluster radio.*

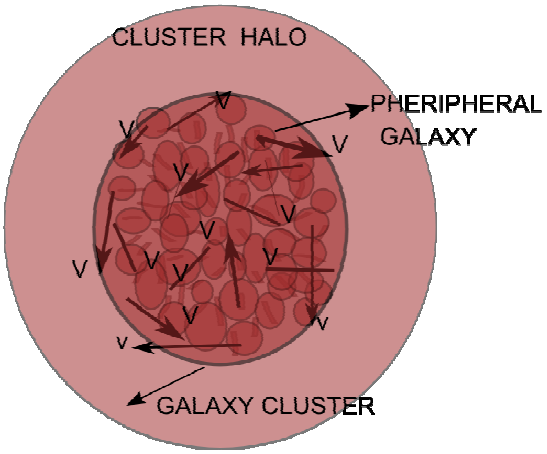
$$M_{TOTAL\ DM}(<R_H)=4\pi\varphi_{0C}h_C^{\,3}\left[\ln(1+X_H)-\frac{X_H}{1+X_H}\right] \quad (2)$$

where $X_H=\frac{R_{HALO}}{h_C}$ *and* R_H *is halo radius.*

Then it is possible to write NFW DM density $\varphi_{DM\ NFW\ CLUSTER}(R)=\frac{\varphi_{0C}}{X(1+X)^2}$ *where* $X=\frac{R}{h_C}$

8.3 VIRIAL THEOREM IN GALAXY CLUSTER

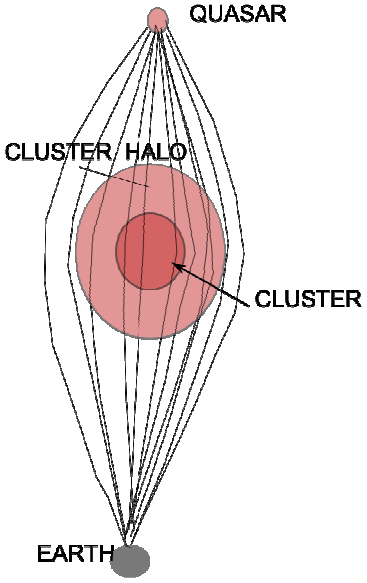
If it is considered gravitational equilibrium then speed of a pheripheral galaxy in cluster, allow to calculate total mass enclosed by galaxy cluster.
Total mass would be $M = R \cdot V^2 / G$.
Such mass is renamed as
Mass $_{VIRIAL\ TOTAL} = R \cdot V^2 / G$
Therefore it is possible to calculate DM by subtraction of baryonic mass M_B
$DM_{INSIDE\ CLUSTER} = Mass_{VIRIAL\ TOTAL} - M_B$



8.4 GRAVITATIONAL LENSING TO MEASURE TOTAL MASS

Gravitational lensing is a good method to calculate total mass in galaxy cluster = $M_{TOTAL\ GRAVITATIONAL\ LENSING}$.
So Total DM would be
$DM_{TOTAL} = M_{TOTAL\ GRAVITATIONAL\ LENSING} - M_B$
Where M_B is baryonic mass.

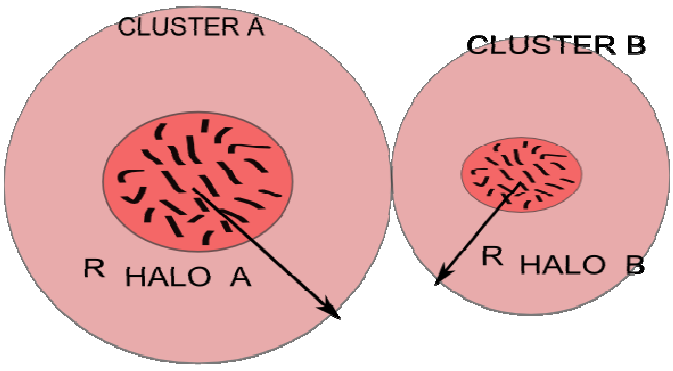
Some experimental results about DM measures in galaxy cluster have shown that DM fraction is bigger than DM fraction in galaxies.
This evidence may be explained thanks to DM belonging to cluster halo.



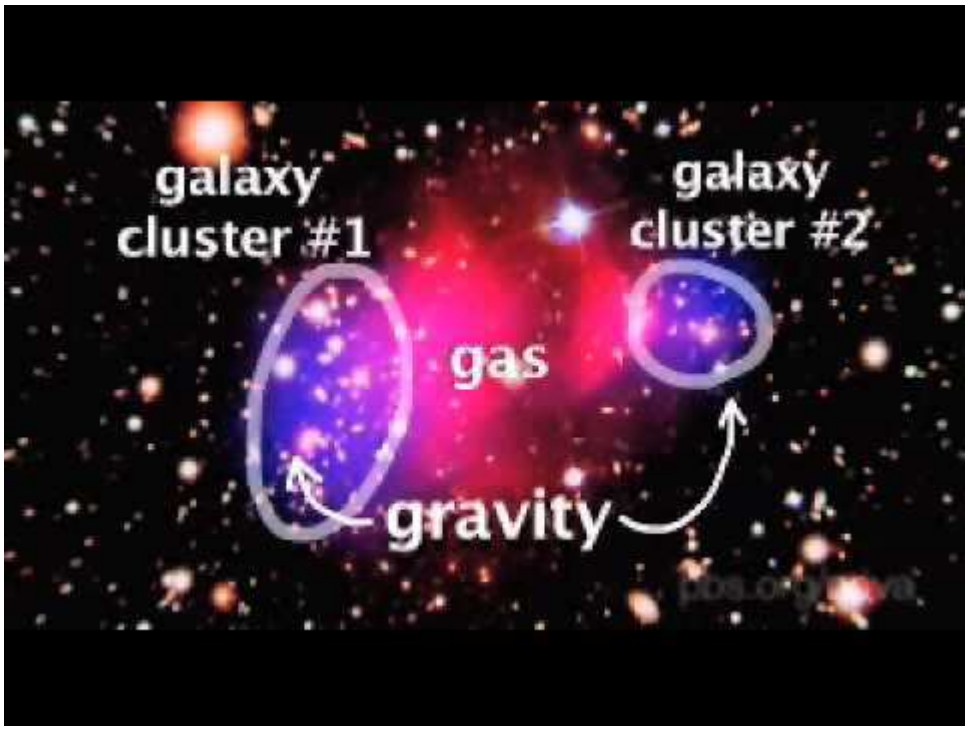
8.5 RADIUS HALO CLUSTER DEPEND ON NEIGHBOUR CLUSTER MASS AND DISTANCE

Since DM model is universal, what happens in galactic haloes happens in cluster haloes. So it is not necessary to explain the same idea for clusters.

In general, if neighbour cluster have similar mass, its halo radius is a half of distance between them.



9. DARK MATTER MODEL IN BULLET CLUSTER



The **Bullet Cluster** consists in two colliding clusters of galaxies. Strictly speaking, the name *Bullet Cluster* refers to the smaller sub cluster (2), moving away from the larger one (1). They move away each other at 10 million km/h. As a result their collision the gas, in red, has 70 million K of temperature and it emits a X-ray radiation.

Both clusters collided 150 millions years ago. Stars did not collided, stars simply changed a bit their trajectory. However gas interacted electromagnetically, so gas decelerated strongly. As a consequence the two gas clouds remain a bit joined at high temperature emitting X-ray radiation. In addition, there is no evidences about DM collision.

It has been estimated experimentally that gas, in red, is two times more massive than star masses, in blue. However by gravitational lens methods it has been checked that the blue area is much more massive than red area.
This fact has been the ultimate evidence to accept the DM hypothesis and refuse the MOND theory (Modified Newton Dynamic theory).
The previous information was discovered some years ago.

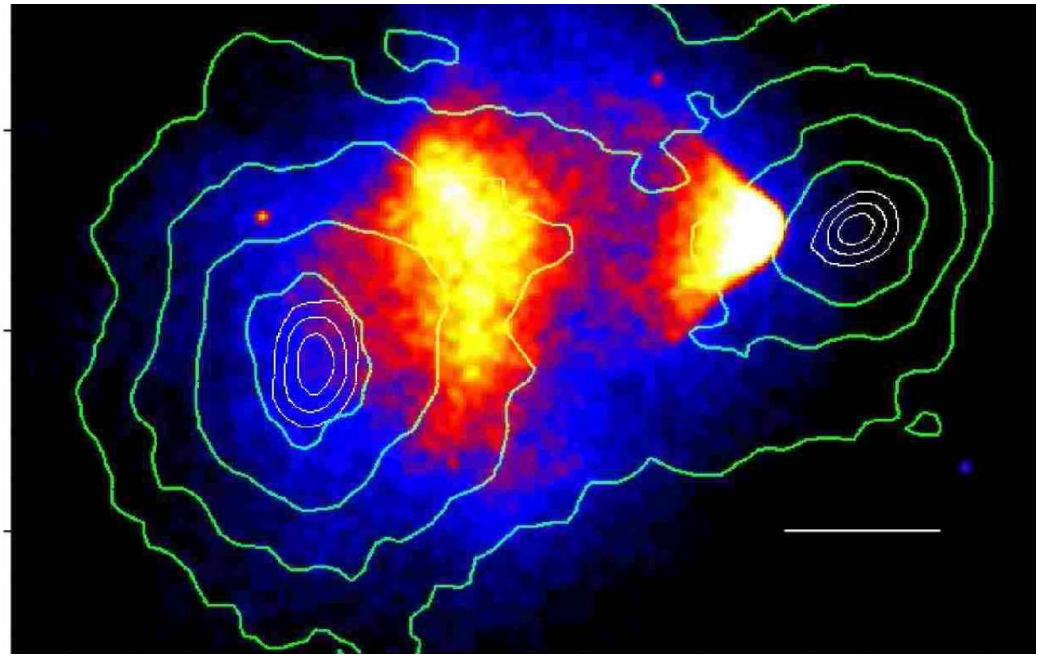
Now I would like to think about two weird evidences.

- DM is a substance which is concerned only by gravitation, moreover DM does not collided despite the fact DM is a light substance widely spread through the space.

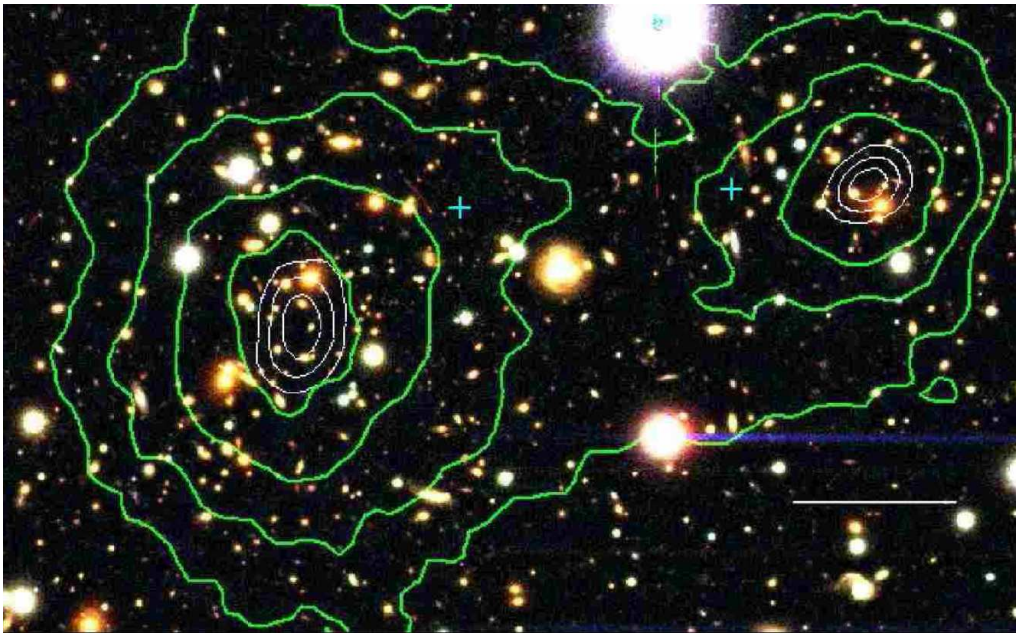
- DM remains joined mainly to galaxy cluster although a fraction is distributed through the gas region.

  Both properties are shown in the pictures below.

The Bullet Cluster is a pair of colliding galaxy clusters (Clowe et al. 2006)



Cloud in white yellow red and blue shows the X-ray radiation from gas as a result their collision.



Curves show density mass distribution which has a great spherical symmetry except in the inner region between clusters. This one means two things:

a) DM does not collide at all.
b) DM remain near galaxy cluster mainly, although there is a fraction of DM between clusters, where DM makes filaments.

Both evidences can be explained by the DM model.

9.1 DM MODEL EXPLAIN DM PUZZLE IN BULLET CLUSTER

Since hypothesis of DM model is based on a universal mechanism DM generation it is posible to suppose $\varphi_{DM\,CLUSTER}(E) = g \circ f^{-1}(E)$
The formula shows that DM depends on gravitational field, therefore DM will be a conservative field because E is a conservative field. According the model, this is the reason why DM is not able to collide whereas gas clouds in bullet cluster collided strongly.
In addition, if DM does not collide then DM will keep the spherical symmetry in the clusters.

At the beginning of epigraph it was said that gas cloud has more than two times the cluster mass, so according the model apparently they have to produce much more DM than cluster galaxies.

In the following epigraph it will be explained how the model could explain the fact that DM in gas clouds is considerably less than DM in clusters. DM in gas clouds would made filaments, which have a great deal lower DM than clusters.

Chapter bibliography
The remarkable pictures about Bullet Cluster has been taken from the paper:
Schneider, Peter (2012). Dark matter in clusters and large- scale structure.
Published in XXIV Canary Islands winter school of astrophysics.

10. DM FILAMENTS CONNECTING CLUSTERS

The model states that DM is generated by gravitational field E, which is generated by ordinary matter, stars and gas clouds.
However there is a big difference between both types of baryonic substances. Stars are substances billion of billions more dense than gas clouds.

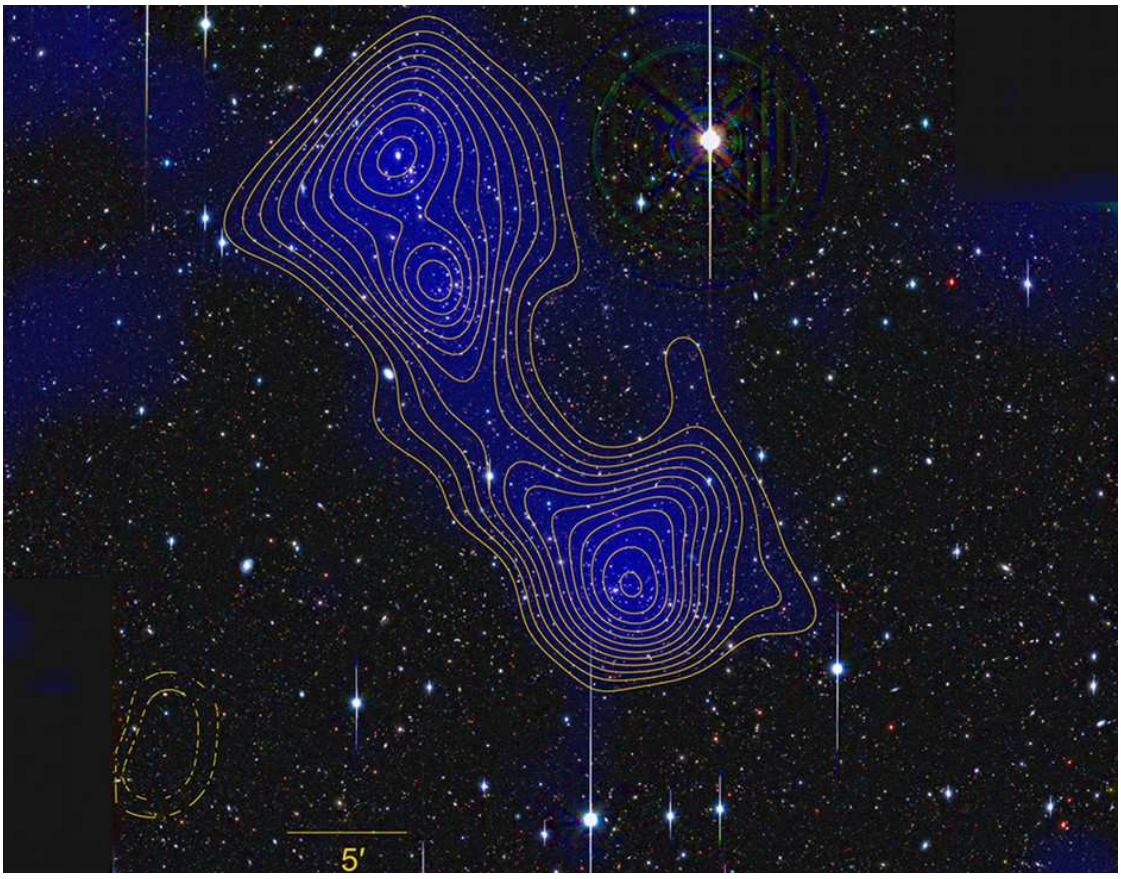
In chapter 6 was developed mathematical function between DM density and gravitational field E $\varphi_{DM\,CLUSTER}(E) = g \circ f^{-1}(E)$ .This function is very complicated. However it is observed in chapter 6 graphics that density decreases strongly as E decreases. Therefore density of DM generated by gas cloud will be considerably less than density generated by galaxies, because gravitational field in gas cloud is very weak.

So the total amount of DM got by integration of $\rho(r)_{D.M.}$ all over the gas cloud is lower than DM generated by galaxies in cluster region.
This reason could explain the lower amount of DM measured in gas clouds region in Bullet Cluster compared with the amount of DM inside the clusters.
The amount of DM placed between clusters would be the filament of DM.

In the picture below it is possible to see the level of DM through the curves.
As you can see in the region between clusters there is DM filaments, which have a density of DM lower than cluster region.
In conclusion, despite the fact there is a big amount of baryonic matter as a gas cloud between clusters, the DM produced by this gas is lower that DM produced by galaxies inside the clusters.

As it has been shown, the DM model can explain easily the weird experimental evidences about DM in Bullet Cluster. Also can explain easily the filaments of DM generated by gas clouds

The double cluster A222/A223 (Dietrich et al. 2012)

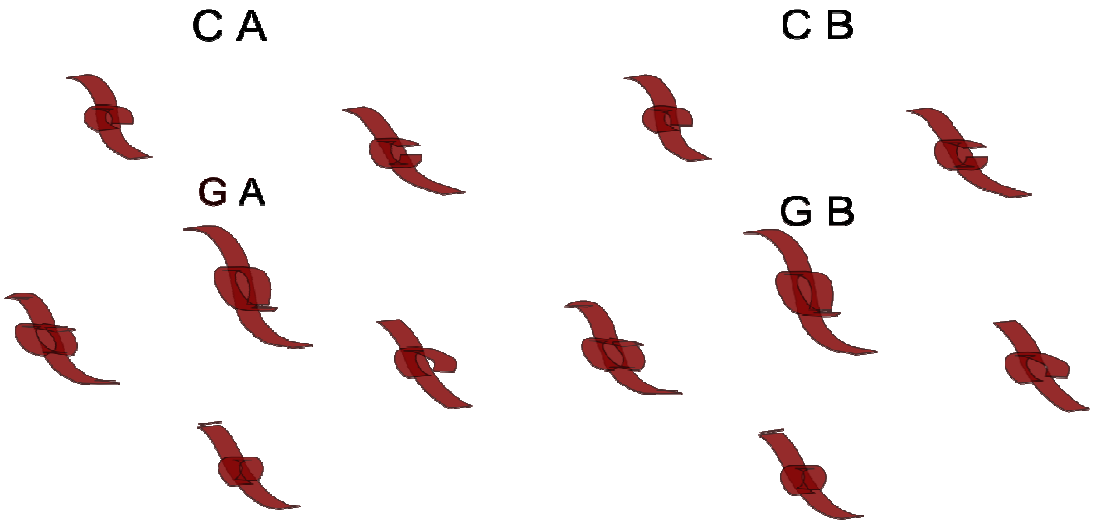


11. EXPERIMENTAL TESTS TO CHECK THE DM MODEL PREDICTIONS

Through the paper has been reiterated, that model is based on a causal relation between gravitational field and DM and as a result there are mathematical functions to calculate DM density: $\varphi_{DM\ NFW}(R)$ and $\varphi_{MO-NFW}(E) = g \circ f^{-1}(E)$

As a consequence the previous idea, the experimental tests proposed try to check that generation mechanism of DM is universal.
In others words, if there are two region A and B with similar gravitational field then DM density generated should be similar an total DM generated should be similar.
The regions may be around a star , a galaxy or a galaxy cluster because it does not matter scale.

11.1 SIMILAR GALAXIES BELONGING TO DIFFERENT CLUSTERS



An essential test would be that the dark matter in two similar galaxies belonging two different cluster, which are surrounded by similar galaxies should have the same DM.

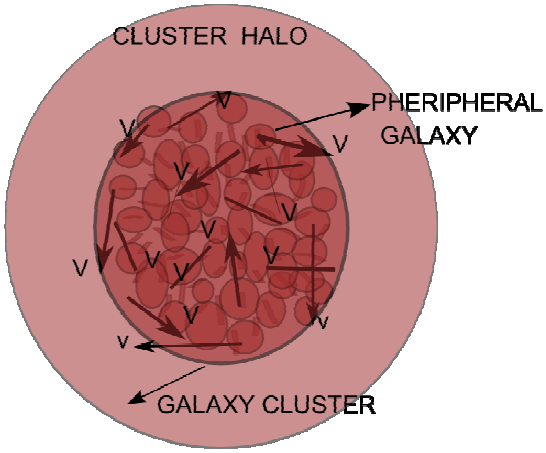
The reason is simple. Since the generation mechanism of DM is universal, if G_A and G_B are galaxies with similar structure, similar baryonic mass and similar radius haloes then the total DM generated in both galaxies should be similar.

It is evident that if in two similar galaxies their dark matters are quite different, the model have to be rejected.

11.2 DM IN GALAXY CLUSTER MEASURED TROUGH VIRIAL THEOREM

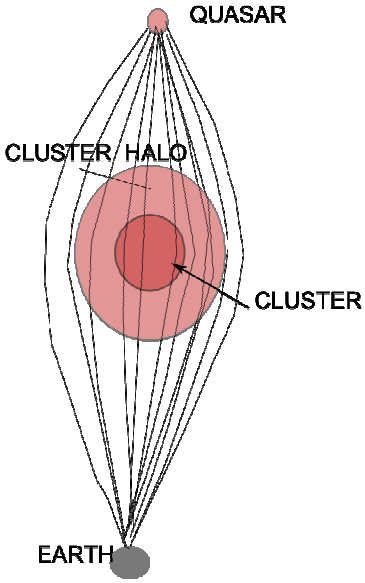
If are chosen two galaxy clusters, C_A and C_B with similar baryonic mass, similar baryonic cloud mass, similar radius and similar number of galaxies then according the model, total DM generated inside the clusters should be similar.

Measuring total mass with Virial theorem method and subtracting baryonic mass, might be calculated total DM in both clusters.



11.3 GALAXY CLUSTERS WITH SIMILAR BARYONIC MASS THROUGH GRAVITATIONAL LENSING

As it was explained in chapter 9, gravitational lensing is a good method to measure total mass of galaxy clusters, included DM belonging to cluster halo.
According the model if two clusters have similar baryonic mass, similar cloud mass, similar number of galaxies and similar halo sizes then they should have similar total mass as well.
The reason is that according the model, mechanism of DM generation is universal.



11.4 COSMIC VOIDS

The Hubble telescope took a picture of a gravitational lens composed by the Abel galaxy cluster 2218, which is placed to 2000 millions light years far away. The lens show a distorted images of a galaxy which is placed 10000 million light years far away.

Excellent picture and superb spectacle¡

It is known that a gravitational lens needs only a central gravitational field to work. So if the viewer, the lens and the object are in the same line then the viewer wil be able to see the image if he or she has a good telescope¡

We are going to propose a fourth experimental proof to check the model through the gravitational lens effect because it is obvious that the only way to look for DM pure is that effect.

According the model it is not possible that DM pure exist. If pure DM could exist the best places to look for it would be the cosmic voids. The pure DM would create central gravitational field which would be a gravitational lens. Until this moment there is not any experimental evidence of gravitational lens of pure DM. Perhaps in the future it would have been found it. However this model predicts the impossibility to find it.

By now there is no experimental evidence of pure DM, despite the fact that the sky is monitored by hundreds of telescopes with the highest technology.

In my opinion if pure DM existed it would have been already discovered.

11.5 SUBHALO STRUCTURE

Subhalo structure may be explained rightly through the model, because gravitational field in Universe has a behaviour similar to fractal structures.
If there is a distribution mass with spherical symmetry, the field has spherical symmetry. When it is considered a small scale, the field is generated mainly by the bigger and closer mass. In conclusion the subhalo structure arises when is measured DM generated by masses at lower scale.
So subhalo structure may be a good way to check the model, because according the model the only cause of subhalo structure is gravitational field.
According the model, even a single star produces gravitational field which generates DM.
The problem is be able to measure the little amount of DM generated by a single star and its planets.

CONCLUSION

The main model hypothesis is that DM density is generated by gravitational field, through a gravitational quantum effect, which is unknown for now. Moreover, this effect is universal. In others words, it works similarly everywhere and every scale, stars, galaxies and clusters.
Taking into account this hypothesis the paper has shown the following results:

a) *DM model is coherent with DM density NFW, which is based on a universal formula for DM as well.*

b) *Starting with the formula of gravitational field depending on distance*

$$E = \frac{GM_{TOTAL}(<R)}{R^2} = \frac{GM_{TOTAL}(<R)}{h_G{}^2 X^2} = \frac{G\left\langle M_B + 4\pi\varphi_{0G} h_G{}^3 \left[\ln(1+x) - \frac{x}{1+x} \right] \right\rangle}{h_G{}^2 X^2}$$

It has been proved mathematically, it exists a function between DM density NFW and gravitational field $\varphi_{MO-NFW}(E) = g \circ f^{-1}(E)$.

c) *The halo radius of a galaxy measures the border of the región where the own gravitational field dominates over the neighbours ones. In the border the gravitational field is cancelated because of vectorial nature of gravitational field. As a consequence the halo radius of a galaxy depend on not only their own mass but also the neighbour galaxy mass and distance between them.*

d) *If mechanism DM generation is universal it is right to suppose that it works in a similar way for a bigger scale in galaxy cluster, particularly it has been supposed:*
   - *DM density function follows a NFW function in galaxy clusters.*
   - *Cluster halo is the same concept as galactic halo that is: Cluster halo is the region where the gravitational field dominates over the neighbours ones.*

e) *DM model may explain easily weird properties that DM has in Bullet Cluster and DM filaments between clusters.*

f) *It has been proposed five experimental test to try to check that generation mechanism of DM is universal and consequently DM depend on gravitational field.*
   The reason is right:
   If in everywhere galaxies exits the same density function, such as NFW, the reason has to be the only thing in common: the gravitational field.

If someone of these experimental proofs contradicted model predictions it would have to discard the model.
However if all the five tests supported the model it would have plenty of reasons to consider the model as a serious candidate to explain the DM nature.

The unsuccessful search for particles responsible of the dark matter for the whole international scientific community during several decades, using the most sophisticated technology could be an indication that dark matter has another nature.
In addition, it is accepted by the scientific community the fact that Dark Energy is a Quantum Gravity phenomenon, so for symmetry reasons DM should be produced by Quantum Gravity as well.

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