

The proof of Goldbach's Conjecture

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Abstract.

Goldbach's Conjecture statement: "Every even integer greater than 2 can be expressed as the sum of two primes".

Initially, to prove this conjecture, we can form two arithmetic sequences (**A** and **B**) different for each even number, with all the natural numbers that can be primes, that can added, in pairs, result in the corresponding even number.

By analyzing the pairing process, in general, between all non-prime numbers of sequence **A** with terms of sequence **B**, or vice versa, to obtain the even number, we note that some pairs of primes are always formed. This allow us to develop a non-probabilistic formula to calculate the approximate number of pairs of primes that meet the conjecture for an even number x .

The result of this formula is always equal or greater than 1, and it tends to infinite when x tends to infinite, which allow us to confirm that Goldbach's Conjecture is true.

The prime numbers theorem by Carl Friedrich Gauss, the prime numbers theorem in arithmetic progressions and some axioms have been used to complete this investigation.

1. Prime numbers and composite numbers.

A *prime number* (or *prime*) is a natural number greater than 1 that has only two divisors, 1 and the number itself.

Examples of primes are: 2, 3, 5, 7, 11, 13, 17. The Greek mathematician Euclid proved that there are infinitely many primes, but they become more scarce as we move on the number line.

Except 2 and 3, all primes are of form $(6n + 1)$ or $(6n - 1)$ being n a natural number.

We can differentiate primes 2, 3 and 5 from the rest. The 2 is the first prime and the only one that is even, the 3 is the only one of form $(6n - 3)$ and the 5 is the only one finished in 5. All other primes are odd and its final digit will be 1, 3, 7 or 9.

In contrast to primes, a *composite number* (or *composite*) is a natural number that has more than two divisors.

Examples of composites are: 4 (divisors 1, 2, 4), 6 (1, 2, 3, 6), 15 (1, 3, 5, 15), 24 (1, 2, 3, 4, 6, 8, 12, 24).

Except 1, every natural number is prime or composite. By convention, number 1 is considered neither prime nor composite because it has only one divisor. Since it cannot meet the Goldbach conjecture, and for this demonstration, we will include the number 1 in the set of composite numbers. This question has no relevance in the development or in the final formula of this proof.

We can classify the set of primes (except 2, 3 and 5) in 8 groups depending of the situation of each of them with respect to multiples of 30, ($30 = 2 \cdot 3 \cdot 5$). Being: $n = 0, 1, 2, 3, 4, \dots, \infty$.

$$30n + 1 \quad 30n + 7 \quad 30n + 11 \quad 30n + 13 \quad 30n + 17 \quad 30n + 19 \quad 30n + 23 \quad 30n + 29$$

These expressions represent all arithmetic progressions of module 30, $(30n + b)$, such that $\text{gcd}(30, b) = 1$ being: $30 > b > 0$.

These 8 groups contain all primes (except 2, 3 and 5). They also include the number 1 and all composites that are multiples of primes greater than 5. As 30 and b are coprime, they cannot contain multiples of 2 or 3 or 5.

Logically, when n increases, decreases the primes proportion and increases the composites proportion that there are in each group.

Dirichlet's theorem statement^[1]: "An arithmetic progression $(an + b)$ such that $\text{gcd}(a, b) = 1$ contains infinitely many prime numbers". Applying this theorem for the 8 groups of primes, we can say that each of them contains infinitely many primes.

You can also apply the prime numbers theorem in arithmetic progressions. It states^[2]: "For every module a , the prime numbers tend to be distributed evenly among the different progressions $(an + b)$ such that $\text{gcd}(a, b) = 1$ ".

To verify the precision of this theorem, I used a programmable logic controller (PLC), like those that control automatic machines, having obtained the following data:

There are 50.847.531 primes lesser than 10^9 , (2, 3 and 5 not included), distributed as follows:

Group $(30n + 1)$	6.355.189 primes	12,49852033 %	$50.847.531 / 6.355.189 = 8,0009471$
Group $(30n + 7)$	6.356.475 primes	12,50104946 %	$50.847.531 / 6.356.475 = 7,999328401$
Group $(30n + 11)$	6.356.197 primes	12,50050273 %	$50.847.531 / 6.356.197 = 7,999678267$
Group $(30n + 13)$	6.356.062 primes	12,50023723 %	$50.847.531 / 6.356.062 = 7,999848176$
Group $(30n + 17)$	6.355.839 primes	12,49979866 %	$50.847.531 / 6.355.839 = 8,000128858$
Group $(30n + 19)$	6.354.987 primes	12,49812307 %	$50.847.531 / 6.354.987 = 8,001201419$
Group $(30n + 23)$	6.356.436 primes	12,50097276 %	$50.847.531 / 6.356.436 = 7,999377481$
Group $(30n + 29)$	6.356.346 primes	12,50079576 %	$50.847.531 / 6.356.346 = 7,999490745$

We can see that the maximum deviation for 10^9 , (between 6.354.987 and 6.355.941 average), is lesser than 0,01502 %.

I gather that, in compliance with this theorem, the maximum deviation tends to 0 % when larger numbers are analyzed.

2. Special cases of the conjecture.

As we have seen, numbers 2, 3 and 5 are different from all other primes and they are not included in the 8 described groups. We will study, as special cases, the even numbers 4, 6, 8, 10, 12 and 16 whose solutions, to meet the Goldbach conjecture, contain the primes 2, 3 or 5. We will write all possible pairs of terms for each of these even numbers, highlighting the primes in **bold**.

$$\begin{array}{ll}
 4 = 1 + \mathbf{3} = \mathbf{2} + \mathbf{2} & 4 = \mathbf{2} + \mathbf{2} \quad (\text{unique with prime } 2) \\
 6 = 1 + \mathbf{5} = \mathbf{2} + 4 = \mathbf{3} + \mathbf{3} & 6 = \mathbf{3} + \mathbf{3} \\
 8 = 1 + \mathbf{7} = \mathbf{2} + 6 = \mathbf{3} + \mathbf{5} = 4 + 4 & 8 = \mathbf{3} + \mathbf{5} \\
 10 = 1 + 9 = \mathbf{2} + 8 = \mathbf{3} + \mathbf{7} = 4 + 6 = \mathbf{5} + \mathbf{5} & 10 = \mathbf{3} + \mathbf{7} = \mathbf{5} + \mathbf{5} \\
 12 = 1 + \mathbf{11} = \mathbf{2} + 10 = \mathbf{3} + 9 = 4 + 8 = \mathbf{5} + \mathbf{7} = 6 + 6 & 12 = \mathbf{5} + \mathbf{7} \\
 16 = 1 + 15 = \mathbf{2} + 14 = \mathbf{3} + \mathbf{13} = 4 + 12 = \mathbf{5} + \mathbf{11} = 6 + 10 = \mathbf{7} + 9 = 8 + 8 & 16 = \mathbf{3} + \mathbf{13} = \mathbf{5} + \mathbf{11}
 \end{array}$$

We note that the even numbers 4, 6, 8, 10, 12 and 16 can be expressed as the sum of two primes. For all the other even numbers it must be proved or verified that they meet the conjecture with one or more prime pairs containing neither 3 nor 5.

3. Classification of even numbers.

Just as we did with primes, we can divide the set of even numbers (2, 4, 6, 8, 10, ...) in 15 groups depending of the situation of each of them with respect to multiples of 30. Being: $n = 0, 1, 2, 3, 4, \dots, \infty$.

$$\begin{array}{lll}
 30n + 2 & 30n + 12 & 30n + 22 \\
 30n + 4 & 30n + 14 & 30n + 24 \\
 30n + 6 & 30n + 16 & 30n + 26 \\
 30n + 8 & 30n + 18 & 30n + 28 \\
 30n + 10 & 30n + 20 & 30n + 30
 \end{array}$$

4. Combining groups of even numbers with groups of prime numbers.

Now, we will combine groups of even numbers with groups of primes to express the 36 possible combinations of Goldbach's conjecture. We can see that each group of even numbers has its own combinations that are different from the rest.

$$30n_1 + 2 = (30n_2 + 1) + (30n_3 + 1) = (30n_4 + 13) + (30n_5 + 19)$$

$$\text{Being: } n_1 = n_2 + n_3 = n_4 + n_5 + 1$$

We observe that for even numbers $(30n + 2)$ there are 2 different combinations using 3 groups of primes.

For number 2, it can only apply the combination $2 = 1 + 1$. This even number is excluded in Goldbach's conjecture statement.

$$30n_1 + 4 = (30n_2 + 11) + (30n_3 + 23) = (30n_4 + 17) + (30n_5 + 17)$$

$$\text{Being: } n_1 = n_2 + n_3 + 1 = n_4 + n_5 + 1$$

There are 2 different combinations using 3 groups of primes.

These combinations cannot be applied for number 4 so that, as I have indicated, it is considered a special case, $(4 = \mathbf{2} + \mathbf{2})$.

$$30n_1 + 6 = (30n_2 + 7) + (30n_3 + 29) = (30n_4 + 13) + (30n_5 + 23) = (30n_6 + 17) + (30n_7 + 19)$$

$$\text{Being: } n_1 = n_2 + n_3 + 1 = n_4 + n_5 + 1 = n_6 + n_7 + 1$$

There are 3 different combinations using 6 groups of primes.

These combinations cannot be applied for number 6, this is why it is considered a special case, $(6 = \mathbf{3} + \mathbf{3})$.

$$30n_1 + 8 = (30n_2 + 1) + (30n_3 + 7) = (30n_4 + 19) + (30n_5 + 19)$$

$$\text{Being: } n_1 = n_2 + n_3 = n_4 + n_5 + 1$$

There are 2 different combinations using 3 groups of primes.

For number 8, only the combination $8 = 1 + \mathbf{7}$ which does not meet the conjecture, this is why it is considered a special case, $(8 = \mathbf{3} + \mathbf{5})$.

$$30n_1 + 10 = (30n_2 + 11) + (30n_3 + 29) = (30n_4 + 17) + (30n_5 + 23)$$

$$\text{Being: } n_1 = n_2 + n_3 + 1 = n_4 + n_5 + 1$$

There are 2 different combinations using 4 groups of primes.

These combinations cannot be applied for number 10, this is why it is considered a special case, $(10 = \mathbf{3} + \mathbf{7} = \mathbf{5} + \mathbf{5})$.

$$30n_1 + 12 = (30n_2 + 1) + (30n_3 + 11) = (30n_4 + 13) + (30n_5 + 29) = (30n_6 + 19) + (30n_7 + 23)$$

$$\text{Being: } n_1 = n_2 + n_3 = n_4 + n_5 + 1 = n_6 + n_7 + 1$$

There are 3 different combinations using 6 groups of primes.

For number 12, only the combination $12 = 1 + \mathbf{11}$ which does not meet the conjecture, this is why it is considered a special case, $(12 = \mathbf{5} + \mathbf{7})$.

$$30n_1 + 14 = (30n_2 + 1) + (30n_3 + 13) = (30n_4 + 7) + (30n_5 + 7)$$

Being: $n_1 = n_2 + n_3 = n_4 + n_5$

There are 2 different combinations using 3 groups of primes.

$$30n_1 + 16 = (30n_2 + 17) + (30n_3 + 29) = (30n_4 + 23) + (30n_5 + 23)$$

Being: $n_1 = n_2 + n_3 + 1 = n_4 + n_5 + 1$

There are 2 different combinations using 3 groups of primes.

These combinations cannot be applied for number 16, this is why it is considered a special case, ($16 = 3 + 13 = 5 + 11$).

$$30n_1 + 18 = (30n_2 + 1) + (30n_3 + 17) = (30n_4 + 7) + (30n_5 + 11) = (30n_6 + 19) + (30n_7 + 29)$$

Being: $n_1 = n_2 + n_3 = n_4 + n_5 = n_6 + n_7 + 1$

There are 3 different combinations using 6 groups of primes.

For number 18, only the combinations $18 = 1 + 17 = 7 + 11$. (Also: $18 = 5 + 13$).

$$30n_1 + 20 = (30n_2 + 1) + (30n_3 + 19) = (30n_4 + 7) + (30n_5 + 13)$$

Being: $n_1 = n_2 + n_3 = n_4 + n_5$

There are 2 different combinations using 4 groups of primes.

$$30n_1 + 22 = (30n_2 + 11) + (30n_3 + 11) = (30n_4 + 23) + (30n_5 + 29)$$

Being: $n_1 = n_2 + n_3 = n_4 + n_5 + 1$

There are 2 different combinations using 3 groups of primes.

For number 22, only the combination $22 = 11 + 11$. (Also: $22 = 3 + 19 = 5 + 17$).

$$30n_1 + 24 = (30n_2 + 1) + (30n_3 + 23) = (30n_4 + 7) + (30n_5 + 17) = (30n_6 + 11) + (30n_7 + 13)$$

Being: $n_1 = n_2 + n_3 = n_4 + n_5 = n_6 + n_7$

There are 3 different combinations using 6 groups of primes.

$$30n_1 + 26 = (30n_2 + 7) + (30n_3 + 19) = (30n_4 + 13) + (30n_5 + 13)$$

Being: $n_1 = n_2 + n_3 = n_4 + n_5$

There are 2 different combinations using 3 groups of primes.

$$30n_1 + 28 = (30n_2 + 11) + (30n_3 + 17) = (30n_4 + 29) + (30n_5 + 29)$$

Being: $n_1 = n_2 + n_3 = n_4 + n_5 + 1$

There are 2 different combinations using 3 groups of primes.

For number 28, only the combination $28 = 11 + 17$. (Also: $28 = 5 + 23$).

$$30n_1 + 30 = (30n_2 + 1) + (30n_3 + 29) = (30n_4 + 7) + (30n_5 + 23) = (30n_6 + 11) + (30n_7 + 19) = (30n_8 + 13) + (30n_9 + 17)$$

Being: $n_1 = n_2 + n_3 = n_4 + n_5 = n_6 + n_7 = n_8 + n_9$

There are 4 different combinations using the 8 prime numbers groups available.

We observe that, for even numbers that are not multiples of 6 or 10, there are 2 different combinations using 3 groups of primes.

The multiples of 6, $(30n + 6)$, $(30n + 12)$, $(30n + 18)$ and $(30n + 24)$ have 3 different combinations using 6 groups of primes.

The multiples of 10, $(30n + 10)$ and $(30n + 20)$ have 2 different combinations using 4 groups of primes.

The multiples of 30, $(30n + 30)$, have 4 different combinations using the 8 prime numbers groups available.

It is reasonable to believe, that the number of prime pairs (also called Goldbach's partitions) that meet the conjecture depend on the number of groups of primes used by the even number (3, 4, 6 or 8). Examples with actual values:

3.600	125 partitions	8 groups	Multiple of 30	3.606	90 partitions	6 groups	Multiple of 6
3.602	48 partitions	3 groups		3.610	66 partitions	4 groups	Multiple of 10

Surprisingly, we can see that, due to the number of groups of primes used, some consecutive even numbers can have a noticeable difference in the partitions number. Example: 3.600 has 125 partitions and 3.602 has only 48.

5. Example.

The above concepts can be applied to number 784 serving as example for any of the 36 exposed combinations and for any even number x , even being a large number. We use the list of primes lesser than 1.000.

$$784 = 30 \cdot 26 + 4 = (30n_2 + 11) + (30n_3 + 23) = (30n_4 + 17) + (30n_5 + 17) \quad \text{Being: } 26 = n_2 + n_3 + 1 = n_4 + n_5 + 1$$

For the first combination, $784 = (30n_2 + 11) + (30n_3 + 23)$, we will write the sequence **A** of all numbers $(30n_2 + 11)$ from 0 to 784. Also, we will write the sequence **B** of all numbers $(30n_3 + 23)$ from 784 to 0. I highlight the primes in **bold**.

A 11 - 41 - 71 - 101-131-161-191-221-251-281-311-341-371-401-431-461-491-521-551-581-611-641-671-701-731-761
B 773-743-713-683-653-623-593-563-533-503-473-443-413-383-353-323-293-263-233-203-173-143-113 - 83 - 53 - 23

The second combination, $784 = (30n_4 + 17) + (30n_5 + 17)$, uses the same group of primes in the two sequences. We will write the sequence **A** of all numbers $(30n_4 + 17)$ from 0 to 392, ($\frac{1}{2}$ of 784). Also, we will write the sequence **B** of all numbers $(30n_5 + 17)$ from 784 to 392.

A 17 - 47 - 77 - 107-137-167-197-227-257-287-317-347-377
B 767-737-707-677-647-617-587-557-527-497-467-437-407

Sorted in this way, each term of each sequence **A** can be added with its partner in the corresponding sequence **B** to obtain 784. In the four above sequences, the 18 prime pairs that meet the Goldbach conjecture for number 784 are underlined.

The study of sequences **A** and **B**, individually and collectively, is the basis of this demonstration. I will analyze the complete sequences (**A** from 0 to x and **B** from x to 0). The halves sequences will be considered for the final formula.

To calculate the number of terms in each sequence **A** or **B** we must remember that these are arithmetic progressions of module 30.

$\frac{x}{30}$ Number of terms in each sequence **A** or **B** for an even number x . Obviously, it is equal to the number of pairs that are formed. (26 terms in each sequence and 26 pairs of terms that are formed to $x = 784$).

To analyze, in general, the above formula, would be:

Number of terms = number of pairs = formula result	if x is multiple of 30
Number of terms = number of pairs = integer part of result	if $n_1 = n_2 + n_3 + 1$
Number of terms = number of pairs = (integer part of result) + 1	if $n_1 = n_2 + n_3$

6. Applying the conjecture to small even numbers.

As we have seen, the composites present in the 8 groups of primes are multiples of primes greater than 5, (primes 7, 11, 13, 17, 19,...). The first composites that appear on them are:

$$49 = 7^2 \quad 77 = 7 \cdot 11 \quad 91 = 7 \cdot 13 \quad 119 = 7 \cdot 17 \quad 121 = 11^2 \quad 133 = 7 \cdot 19 \quad 143 = 11 \cdot 13 \quad 161 = 7 \cdot 23 \quad 169 = 13^2$$

And so on, forming products of two or more factors with primes greater than 5.

From the above we conclude that on even numbers lesser than 49 all terms of corresponding sequences **A-B** are primes (except 1) and all pairs meet the Goldbach conjecture (except those containing 1).

We will write all pairs between terms of sequences **A** and **B** of even numbers lesser than 49 (except special cases).

14 = 1 + 13 = 7 + 7	34 = 11 + 23 = 17 + 17
18 = 1 + 17 = 7 + 11	36 = 7 + 29 = 13 + 23 = 17 + 19
20 = 1 + 19 = 7 + 13	38 = 1 + 37 = 31 + 7 = 19 + 19
22 = 11 + 11	40 = 11 + 29 = 17 + 23
24 = 1 + 23 = 7 + 17 = 11 + 13	42 = 1 + 41 = 31 + 11 = 13 + 29 = 19 + 23
26 = 7 + 19 = 13 + 13	44 = 1 + 43 = 31 + 13 = 7 + 37
28 = 11 + 17	46 = 17 + 29 = 23 + 23
30 = 1 + 29 = 7 + 23 = 11 + 19 = 13 + 17	48 = 1 + 47 = 31 + 17 = 7 + 41 = 37 + 11 = 19 + 29
32 = 1 + 31 = 13 + 19	

Furthermore, we note that in the sequences **A-B** for number 784, used as an example, the prime numbers predominate (17 primes with 9 composites in the sequence **A** and 18 primes with 8 composites in the sequence **B**).

This occurs on the small even numbers (up to $x \approx 4.500$).

Therefore, for even numbers lesser than 4,500, is ensured the compliance with Goldbach's conjecture with the sequences **A** and **B** because, even in the event that all composites are paired with primes, there will always be, left over in the two sequences, some primes that will form pairs between them. Applying this reasoning to number 784 we would have:

$17 - 8 = 18 - 9 = 9$ prime pairs at least (in the previous chapter we can see that are 12 actual pairs).

7. Applying logical reasoning to the conjecture.

The sequences **A** and **B** are composed of terms that may be primes or composites that form pairs between them. To differentiate, I define as **free composite** the one which is not paired with another composite and having, as partner, a prime of the other sequence. Thus, the pairs between terms of sequences **A-B** will be formed by:

- (Composite of sequence **A**) + (Composite of sequence **B**) (CC pairs)
- (Free composite of sequence **A** or **B**) + (Prime of sequence **B** or **A**) (CP-PC pairs)
- (Prime of sequence **A**) + (Prime of sequence **B**) (PP pairs)

We will substitute the primes by a **P** and the composites by a **C** in the sequences **A-B** of number 784, that we use as example.

A P P P P P C P C **P** P **P** C C P P **P** P P C C C **P** C P C P
B P P C P P C P **P** C P C P C P P C P P **P** C **P** C **P** P P P

The number of pairs of primes (PP) that will be formed will depend on the free composites number of one of the sequences that are paired with primes of the another. In general, it can define the following axiom:

$$P_{PP} = (\text{Number of primes of A}) - (\text{number of free composites of B}) = (\text{Number of primes of B}) - (\text{number of free composites of A})$$

For number 784: $P_{PP} = 17 - 5 = 18 - 6 = 12$ prime pairs formed with the sequences **A-B**

I consider that this axiom is perfectly valid although being very simple and “obvious”. It will be used later in the proof of the conjecture.

Given this axiom, enough pairs of composites must be formed between the two sequences **A-B** because the number of free composites of sequence **A** cannot be greater than the number of primes of sequence **B**.

Conversely, the number of free composites of sequence **B** cannot be greater than the number of primes of sequence **A**.

This is particularly important for sequences **A-B** of very large even numbers in which the primes proportion is much lesser than the composites proportion.

Later, this question is analyzed in more detail when algebra is applied to the sequences **A-B**.

Let us consider, briefly, how are formed the different kinds of pairs between terms of sequences **A-B**.

A composite of sequence **A** will be paired with a composite of sequence **B** if both, as pair, meet some conditions that will depend on the characteristics of the even number **x** (if is a power of 2 or a multiple of a power of 3 or 5 or a multiple of one or more primes greater than 5, etc.). The composites of each sequence that fail to have a composite of the other, to meet the required conditions, will be paired with a prime (with this prime the conditions will be met). Finally, the remaining primes of the two sequences will form the pairs that will meet the Goldbach conjecture.

This question will be analyzed in more detail in the next chapter.

With what we have described, we can devise a logical reasoning to support the conclusion that Goldbach's conjecture is true.

Later, a general formula will be developed to calculate the approximate number of partitions.

As I indicated, compliance of the conjecture is secured for small even numbers (lesser than 4.500), since in corresponding sequences **A-B**, the primes predominate. Therefore, in these sequences we will find **PP** pairs and, if there are composites, **CC** and **CP-PC** pairs. If we verify increasingly larger numbers, we note that already predominate the composites and decreases the primes proportion.

Let us suppose that there is a sufficiently large even number that does not meet the Goldbach conjecture.

In this supposition, with all sequences **A-B** of this number (we remember, 2, 3 or 4 combinations of groups of primes depending on the even number) they could only be formed **CC** and **CP-PC** pairs, understanding that all free composites of each combination **A-B** would be paired with all primes of the same combination with extraordinary mathematical precision.

What happens, then, for even numbers greater than the one that, supposedly, does not meet the conjecture?

It can be assumed that the conjecture will not be achieved from the first even number that did not achieve it, but it is not possible because there will always be primes greater than it and that added to other primes will give us even numbers greater that will meet the conjecture.

Another question is that, as **x** increases, increases the composites proportion and decreases the primes proportion, so we could assume that, for very large numbers, some composites will not have a partner because there are not enough primes, which, obviously, is not possible because each term of sequence **A** has its corresponding partner in the sequence **B** and vice versa.

Not being possible both cases above, I conclude, although not serves as demonstration, that we will not find an even number that do not meet the conjecture. Therefore, I gather that Goldbach's conjecture is true.

Later, I will reinforce this deduction through the formula to calculate the approximate number of partitions for an even number **x**.

8. Studying how the pairs between terms of sequences A-B are formed.

We will analyze how the composite-composite pairs with the sequences **A** and **B** are formed. If the proportion of CC pairs is higher, there are less composites (free) that need a prime as a partner and, therefore, there will be more primes to form pairs.

The secret of Goldbach's conjecture is the number of composite-composite pairs formed with the sequences **A** and **B**.

For the following analysis, I consider m as the natural number that is not multiple of 2 or 3 or 5 and j as natural number (including 0).

Let us suppose that we applied Goldbach's conjecture to an even number that is a multiple of 7, ($x = 7q$, being q an even number).

In this supposition, any multiple of 7 ($7m_1$) of sequence **A** will be paired with a multiple of 7 ($7m_2$) of sequence **B** so that the sum of the two numbers will be a multiple of 7 ($7q$) as we have assumed for x .

Expanding this axiom, it can be confirmed that all multiples of 7 ($7m_1$) (including the prime 7, if it would be present) of sequence **A** will be paired with all multiples of 7 ($7m_2$) (including the prime 7, if it would be present) of sequence **B**.

$$x = 7q = 7m_1 + 7m_2 = 7(m_1 + m_2) \quad \text{being: } q = m_1 + m_2$$

We will apply this axiom to number $784 = 7 \cdot 112$. It serves as an example for any even number that is multiple of 7, even being a large number. We will write the corresponding sequences **A-B**. In them, the multiples of 7 are underlined.

A 11 - 41 - 71 - 101-131-161-191-221-251-281-311-341-371-401-431-461-491-521-551-581-611-641-671-701-731-761
B 773-743-713-683-653-623-593-563-533-503-473-443-413-383-353-323-293-263-233-203-173-143-113 - 83 - 53 - 23

Verifying: $784 = 7 \cdot 112 = 7m_1 + 7m_2 = 161 + 623 = 371 + 413 = 581 + 203$ being: $112 = 23 + 89 = 53 + 59 = 83 + 29$

We observe that the pairs of multiples of 7 can be “cross out” of the sequences **A-B** without affecting the prime pairs.

The same would happen if the even number would be multiple of any other prime greater than 5 and lesser than \sqrt{x} .

Mathematically, they are enough the primes lesser than \sqrt{x} to define all composites of sequences **A-B**.

If the even number would be multiple of several primes greater than 5, more pairs of composites would be formed being lesser the number of free composites which would give more possibilities to the primes to form pairs between them. For example, the number $2.002 = 2 \cdot 7 \cdot 11 \cdot 13$ has 44 partitions while the number $2.048 = 2^{11}$, that is greater, only 25. In the hypothetical event that an even number x were multiple of all primes lesser than \sqrt{x} , (of course, it is not possible), there would be no free composites and, therefore, all primes of sequences **A-B** would be paired between them (except those who are lesser than \sqrt{x}).

The worst case is when the even number x is not multiple of primes greater than 5. Example: $512 = 2^9$ $\sqrt{512} = 22,62$

For these even numbers, and following the order of primes (7, 11, 13, 17, 19, ..., previous to \sqrt{x}), we can write:

$x = 7r + a$	being: $r =$ natural number	$a =$ natural number < 7	$512 = 7 \cdot 73 + 1$	$r = 73$	$a = 1$
$x = 11s + b$	being: $s =$ natural number	$b =$ natural number < 11	$512 = 11 \cdot 46 + 6$	$s = 46$	$b = 6$
$x = 13t + c$	being: $t =$ natural number	$c =$ natural number < 13	$512 = 13 \cdot 39 + 5$	$t = 39$	$c = 5$
$x = 17u + d$	being: $u =$ natural number	$d =$ natural number < 17	$512 = 17 \cdot 30 + 2$	$u = 30$	$d = 2$
$x = 19v + e$	being: $v =$ natural number	$e =$ natural number < 19	$512 = 19 \cdot 26 + 18$	$v = 26$	$e = 18$

And so on to the prime previous to \sqrt{x} .

In this case, any multiple of 7 ($7m_{11}$) of sequence **A** will be paired with a term ($7j_{21} + a$) of sequence **B** so that the sum of the two numbers has the form ($7r + a$) as assumed for x .

Conversely, any term ($7j_{11} + a$) of sequence **A** will be paired with a multiple of 7 ($7m_{21}$) of sequence **B** so that, similarly, the sum of the two numbers has the form ($7r + a$).

Expanding this axiom, it can be confirmed that all multiples of 7 ($7m_{11}$) (including the prime 7, if it would be present) of sequence **A** will be paired with all terms ($7j_{21} + a$) of sequence **B**.

Conversely, all terms ($7j_{11} + a$) of sequence **A** will be paired with all multiples of 7 ($7m_{21}$) (including the prime 7, if it would be present) of sequence **B**.

Applying this axiom for all primes, from 7 to the previous to \sqrt{x} , it can be confirmed that all groups of multiples $7m_{11}, 11m_{12}, 13m_{13}, 17m_{14}, \dots$ (including the primes lesser than \sqrt{x} that are present) of sequence **A** will be paired, group to group, with all groups of terms ($7j_{21} + a$), ($11j_{22} + b$), ($13j_{23} + c$), ($17j_{24} + d$), ... of sequence **B**.

Conversely, all groups of terms ($7j_{11} + a$), ($11j_{12} + b$), ($13j_{13} + c$), ($17j_{14} + d$), ... of sequence **A** will be paired, group to group, with all groups of multiples $7m_{21}, 11m_{22}, 13m_{23}, 17m_{24}, \dots$ (including the primes lesser than \sqrt{x} that are present) of sequence **B**.

$$\begin{array}{ll}
x = 7r + a = 7m_{11} + (7j_{21} + a) = 7(m_{11} + j_{21}) + a & \text{being: } r = m_{11} + j_{21} \\
x = 11s + b = 11m_{12} + (11j_{22} + b) = 11(m_{12} + j_{22}) + b & \text{being: } s = m_{12} + j_{22} \\
x = 13t + c = 13m_{13} + (13j_{23} + c) = 13(m_{13} + j_{23}) + c & \text{being: } t = m_{13} + j_{23} \\
x = 17u + d = 17m_{14} + (17j_{24} + d) = 17(m_{14} + j_{24}) + d & \text{being: } u = m_{14} + j_{24} \\
\\
x = 7r + a = (7j_{11} + a) + 7m_{21} = 7(j_{11} + m_{21}) + a & \text{being: } r = j_{11} + m_{21} \\
x = 11s + b = (11j_{12} + b) + 11m_{22} = 11(j_{12} + m_{22}) + b & \text{being: } s = j_{12} + m_{22} \\
x = 13t + c = (13j_{13} + c) + 13m_{23} = 13(j_{13} + m_{23}) + c & \text{being: } t = j_{13} + m_{23} \\
x = 17u + d = (17j_{14} + d) + 17m_{24} = 17(j_{14} + m_{24}) + d & \text{being: } u = j_{14} + m_{24}
\end{array}$$

And so on to the prime previous to \sqrt{x} .

We will apply the above described, to even number $512 = 7 \cdot 73 + 1 = 11 \cdot 46 + 6 = 13 \cdot 39 + 5 = 17 \cdot 30 + 2 = 19 \cdot 26 + 18$.
It serves as an example for any even number x , even being a large number. We will write the corresponding sequences **A-B**.

$$\begin{array}{ll}
\mathbf{A} & \mathbf{13 - 43 - 73 - 103 - 133 - 163 - 193 - 223 - 253 - 283 - 313 - 343 - 373 - 403 - 433 - 463 - 493} \\
\mathbf{B} & \mathbf{499 - 469 - 439 - 409 - 379 - 349 - 319 - 289 - 259 - 229 - 199 - 169 - 139 - 109 - 79 - 49 - 19}
\end{array}$$

in **A**, the multiples $7m_{11}$ are underlined
in **B**, the terms $(7j_{21} + 1)$ are underlined

$$\text{Verifying: } 512 = 7 \cdot 73 + 1 = 7m_{11} + (7j_{21} + 1) = 133 + \mathbf{379} = 343 + 169 \qquad \text{being: } 73 = 19 + 54 = 49 + 24$$

$$\begin{array}{ll}
\mathbf{A} & \mathbf{13 - 43 - 73 - 103 - 133 - 163 - 193 - 223 - 253 - 283 - 313 - 343 - 373 - 403 - 433 - 463 - 493} \\
\mathbf{B} & \mathbf{499 - 469 - 439 - 409 - 379 - 349 - 319 - 289 - 259 - 229 - 199 - 169 - 139 - 109 - 79 - 49 - 19}
\end{array}$$

in **A**, the terms $(7j_{11} + 1)$ are underlined
in **B**, the multiples $7m_{21}$ are underlined

$$\text{Verifying: } 512 = 7 \cdot 73 + 1 = (7j_{11} + 1) + 7m_{21} = \mathbf{43} + 469 = 253 + 259 = \mathbf{463} + 49 \qquad \text{being: } 73 = 6 + 67 = 36 + 37 = 66 + 7$$

Applying the same procedure for the primes 11, 13, 17 and 19, it results:

$$\begin{array}{ll}
\text{For the prime 11:} & 512 = 11 \cdot 46 + 6 = 11m_{12} + (11j_{22} + 6) = 253 + 259 \qquad \text{being: } 46 = 23 + 23 \\
& 512 = 11 \cdot 46 + 6 = (11j_{12} + 6) + 11m_{22} = \mathbf{193} + 319 \qquad \text{being: } 46 = 17 + 29
\end{array}$$

$$\begin{array}{ll}
\text{For the prime 13:} & 512 = 13 \cdot 39 + 5 = 13m_{13} + (13j_{23} + 5) = \mathbf{13} + \mathbf{499} = 403 + \mathbf{109} \qquad \text{being: } 39 = 1 + 38 = 31 + 8 \\
& 512 = 13 \cdot 39 + 5 = (13j_{13} + 5) + 13m_{23} = 343 + 169 \qquad \text{being: } 39 = 26 + 13
\end{array}$$

$$\begin{array}{ll}
\text{For the prime 17:} & 512 = 17 \cdot 30 + 2 = 17m_{14} + (17j_{24} + 2) = 493 + \mathbf{19} \qquad \text{being: } 30 = 29 + 1 \\
& 512 = 17 \cdot 30 + 2 = (17j_{14} + 2) + 17m_{24} = \mathbf{223} + 289 \qquad \text{being: } 30 = 13 + 17
\end{array}$$

$$\begin{array}{ll}
\text{For the prime 19:} & 512 = 19 \cdot 26 + 18 = 19m_{15} + (19j_{25} + 18) = 133 + \mathbf{379} \qquad \text{being: } 26 = 7 + 19 \\
& 512 = 19 \cdot 26 + 18 = (19j_{15} + 18) + 19m_{25} = 493 + \mathbf{19} \qquad \text{being: } 26 = 25 + 1
\end{array}$$

The 7 pairs of terms of sequences **A-B** that do not appear in the above expressions are those that meet the Goldbach conjecture. In these pairs, the two terms are greater than $\sqrt{512}$.

We add the pair of primes (**13 + 499**) in which the first term is a multiple of 13 ($13m_{13}$) lesser than $\sqrt{512}$.

$$512 = \mathbf{73} + \mathbf{439} = \mathbf{103} + \mathbf{409} = \mathbf{163} + \mathbf{349} = \mathbf{283} + \mathbf{229} = \mathbf{313} + \mathbf{199} = \mathbf{373} + \mathbf{139} = \mathbf{433} + \mathbf{79} \qquad (\text{Also: } 512 = \mathbf{13} + \mathbf{499})$$

It can be seen that all multiples $7m, 11m, 13m, 17m, 19m, \dots$ of a sequence **A** or **B** are paired with multiples or primes of the other, to form multiple-multiple pairs, multiple-prime pairs and prime-multiple pairs, according to the defined axiom.

Finally, the remaining prime-prime pairs are those that meet the conjecture. In these pairs, the two primes are greater than \sqrt{x} .

Analyzing with detail the above axiom, we can say that the number of multiples (includes the composite numbers and the primes lesser than \sqrt{x} that are present) that there are in the terms $(7j_{11} + a), (11j_{12} + b), (13j_{13} + c), (17j_{14} + d), \dots$ of sequence **A** is always equal to the number of multiples that there are in the terms $(7j_{21} + a), (11j_{22} + b), (13j_{23} + c), (17j_{24} + d), \dots$ of sequence **B**, being the number of multiple-multiple pairs that are formed with the two sequences. I consider that this question is very important for this conjecture.

The above exposition helps us understand the relation between the terms of sequence **A** and the terms of sequence **B** of any even number x .

To numerically support the exposed axioms, I used a programmable controller to obtain data of sequences **A-B** corresponding to several even numbers (between 10^6 and 10^9) and that can be consulted from page 21.

They are the following data:

1. Number of multiples $7m, 11m, 13m, 17m, \dots$ in each sequence **A** or **B** (includes all composites and the primes lesser than \sqrt{x}).
2. Number of primes in each sequence **A** or **B** (only those who are greater than \sqrt{x}).
3. Number of multiples that there are in the terms $(7j + a), (11j + b), \dots$ in each sequence (all composites and the primes lesser than \sqrt{x}).
4. Number of primes that there are in the terms $(7j + a), (11j + b), (13j + c), \dots$ in each sequence (only those who are greater than \sqrt{x}).

9. Proving the conjecture.

For proving the conjecture, as a starting point, I will use the first part of the last axiom from the previous chapter:

All groups of multiples $7m_{11}, 11m_{12}, 13m_{13}, 17m_{14}, \dots$ (including the primes lesser than \sqrt{x} that are present) of sequence **A** will be paired, group to group, with all groups of terms $(7j_{21} + a), (11j_{22} + b), (13j_{23} + c), (17j_{24} + d), \dots$ of sequence **B**.

In this axiom, the concept of *multiple*, applied to the terms of each sequence **A** or **B**, includes all composites and the primes lesser than \sqrt{x} that are present. By this definition, all terms that are lesser than \sqrt{x} of each sequence **A** or **B** are *multiples*.

Simultaneously, and also in this axiom, the concept of *prime*, applied to the terms of each sequence **A** or **B**, refers only to primes greater than \sqrt{x} that are present in the corresponding sequence.

According to these concepts, each term of each sequence **A** or **B** will be *multiple* or *prime*. Thus, with the terms of the two sequences can be form multiple-multiple pairs, free multiple-prime pairs, prime-free multiple pairs and prime-prime pairs.

$\frac{x}{30}$ Number of terms in each sequence **A** or **B** for the even number x . (Page 4)

$\pi(x)$ Symbol^[3], normally used, to express the number of primes lesser or equal to x .

According to the prime numbers theorem^[3]: $\pi(x) \sim \frac{x}{\ln(x)}$ being: $\lim_{x \rightarrow \infty} \frac{\pi(x)}{\frac{x}{\ln(x)}} = 1$ $\ln(x)$ = natural logarithm of x

A better approach for this theorem is given by the offset logarithmic integral function **Li**(x): $\pi(x) \approx \mathbf{Li}(x) = \int_2^x \frac{dy}{\ln(y)}$

According to these formulas, for all $x \geq 5$ is true that $\pi(x) > \sqrt{x}$. This inequality becomes larger with increasing x .

$\pi(ax)$ Symbol to express the number of primes greater than \sqrt{x} in the sequence **A** for the even number x .

$\pi(bx)$ Symbol to express the number of primes greater than \sqrt{x} in the sequence **B** for the even number x .

For large values of x it can be accept that: $\pi(ax) \approx \pi(bx) \approx \frac{\pi(x)}{8}$ being 8 the number of groups of primes (page 1).

For $x = 10^9$, the maximum error of above approximation is 0,0215 % for group $(30n + 19)$.

$\frac{x}{30} - \pi(ax)$ Number of multiples of sequence **A** for the even number x . It includes the number 1 in the group $(30n + 1)$.

$\frac{x}{30} - \pi(bx)$ Number of multiples of sequence **B** for the even number x . It includes the number 1 in the group $(30n + 1)$.

We will define as a fraction $k(ax)$ of sequence **A**, or $k(bx)$ of sequence **B**, the ratio between the number of multiples and the total number of terms in the corresponding sequence. As the primes density decreases as we move on the number line, the $k(ax)$ and $k(bx)$ values gradually increase when increasing x and tend to 1 when x tends to infinite.

$$k(ax) = \frac{\frac{x}{30} - \pi(ax)}{\frac{x}{30}} = 1 - \frac{\pi(ax)}{\frac{x}{30}} \quad \text{For sequence A: } k(ax) = 1 - \frac{30\pi(ax)}{x} \quad \text{For sequence B: } k(bx) = 1 - \frac{30\pi(bx)}{x}$$

The central question of this chapter is to develop a general formula to calculate the number of multiples that there are in the terms $(7j_{21} + a), (11j_{22} + b), (13j_{23} + c), (17j_{24} + d), \dots$ of sequence **B** and that, complying the origin axiom, will be paired with an equal number of multiples $7m_{11}, 11m_{12}, 13m_{13}, 17m_{14}, \dots$ of sequence **A**. Known this data, we can calculate the number of free multiples of sequence **A** (and that will be paired with primes of sequence **B**). Finally, the remaining primes of sequence **B** will be paired with some primes of sequence **A** to determine the pairs number that meet the Goldbach conjecture.

We will study the terms $(7j + a), (11j + b), (13j + c), (17j + d), \dots$ of sequence **B**, in a general way.

The same procedure can be applied to the terms of sequence **A** if we use the second part of the axiom referred to in the above chapter.

We will analyze how the primes are distributed among terms $(7j + a), (11j + b), (13j + c), (17j + d), \dots$

For this purpose, we will see the relation between prime 7 and the 8 groups of primes, serving as example for any prime greater than 5.

We will analyze how are the groups of multiples of 7 ($7m$) and the groups $(7j + a)$ in generally, that's $(7j + 1), (7j + 2), (7j + 3), (7j + 4), (7j + 5)$ and $(7j + 6)$ of sequence **B**. Noting the fact that it is an axiom, I gather that they will be arithmetic progressions of module 210, $(210 = 7 \cdot 30)$.

In the following expressions, the 8 arithmetic progressions of module 210 correspond, respectively, with the 8 groups of primes of module 30. I highlight in **bold** the number that identifies each of these 8 groups. Being: $n = 0, 1, 2, 3, 4, \dots, \infty$.

$(210n + 90 + \mathbf{1})$, $(210n + \mathbf{7})$, $(210n + 150 + \mathbf{11})$, $(210n + 120 + \mathbf{13})$, $(210n + 60 + \mathbf{17})$, $(210n + 30 + \mathbf{19})$, $(210n + 180 + \mathbf{23})$ and $(210n + 90 + \mathbf{29})$ are multiples of 7 ($7m$). These groups do not contain primes, except the prime 7 in the group $(210n + \mathbf{7})$ for $n = 0$.

$(210n + \mathbf{1})$, $(210n + 120 + \mathbf{7})$, $(210n + 60 + \mathbf{11})$, $(210n + 30 + \mathbf{13})$, $(210n + 180 + \mathbf{17})$, $(210n + 150 + \mathbf{19})$, $(210n + 90 + \mathbf{23})$ and $(210n + \mathbf{29})$ are terms $(7j + 1)$.

$(210n + 120 + \mathbf{1})$, $(210n + 30 + \mathbf{7})$, $(210n + 180 + \mathbf{11})$, $(210n + 150 + \mathbf{13})$, $(210n + 90 + \mathbf{17})$, $(210n + 60 + \mathbf{19})$, $(210n + \mathbf{23})$ and $(210n + 120 + \mathbf{29})$ are terms $(7j + 2)$.

$(210n + 30 + \mathbf{1})$, $(210n + 150 + \mathbf{7})$, $(210n + 90 + \mathbf{11})$, $(210n + 60 + \mathbf{13})$, $(210n + \mathbf{17})$, $(210n + 180 + \mathbf{19})$, $(210n + 120 + \mathbf{23})$ and $(210n + 30 + \mathbf{29})$ are terms $(7j + 3)$.

$(210n + 150 + \mathbf{1})$, $(210n + 60 + \mathbf{7})$, $(210n + \mathbf{11})$, $(210n + 180 + \mathbf{13})$, $(210n + 120 + \mathbf{17})$, $(210n + 90 + \mathbf{19})$, $(210n + 30 + \mathbf{23})$ and $(210n + 150 + \mathbf{29})$ are terms $(7j + 4)$.

$(210n + 60 + \mathbf{1})$, $(210n + 180 + \mathbf{7})$, $(210n + 120 + \mathbf{11})$, $(210n + 90 + \mathbf{13})$, $(210n + 30 + \mathbf{17})$, $(210n + \mathbf{19})$, $(210n + 150 + \mathbf{23})$ and $(210n + 60 + \mathbf{29})$ are terms $(7j + 5)$.

$(210n + 180 + \mathbf{1})$, $(210n + 90 + \mathbf{7})$, $(210n + 30 + \mathbf{11})$, $(210n + \mathbf{13})$, $(210n + 150 + \mathbf{17})$, $(210n + 120 + \mathbf{19})$, $(210n + 60 + \mathbf{23})$ and $(210n + 180 + \mathbf{29})$ are terms $(7j + 6)$.

We can note that the groups of multiples of 7 ($7m$) of sequence **B** correspond to arithmetic progressions of module 210, $(210n + b)$, such that $\gcd(210, b) = 7$ being b lesser than 210, multiple of 7, and having 8 terms b , one of each group of primes.

Also, we can see that the groups of terms $(7j + 1)$, $(7j + 2)$, $(7j + 3)$, $(7j + 4)$, $(7j + 5)$ and $(7j + 6)$ of sequence **B** correspond to arithmetic progressions of module 210, $(210n + b)$, such that $\gcd(210, b) = 1$ being b lesser than 210, not multiple of 7, and having 48 terms b , 6 of each group of primes.

Finally, we can verify that the 56 terms b , $(8 + 48)$, are all those that appear in the 8 groups of primes and that are lesser than 210.

Applying the above axiom for all p (prime greater than 5 and lesser than \sqrt{x}) we can confirm that the groups of multiples of p (pm) of sequence **B** correspond to arithmetic progressions of module $30p$, $(30pn + b)$, such that $\gcd(30p, b) = p$ being b lesser than $30p$, multiple of p , and having 8 terms b , one of each group of primes.

Also, we can confirm that the groups of terms $(pj + 1)$, $(pj + 2)$, $(pj + 3), \dots, (pj + p - 2)$ and $(pj + p - 1)$ of sequence **B** correspond to arithmetic progressions of module $30p$, $(30pn + b)$, such that $\gcd(30p, b) = 1$ being b lesser than $30p$, not multiple of p , and having $8(p - 1)$ terms b , $(p - 1)$ of each group of primes.

Finally, we can confirm that the $8p$ terms b , $(8 + 8(p - 1))$, are all those that appear in the 8 groups of primes and that are lesser than $30p$.

On the other hand, an axiom that is met in the sequences **A** or **B** is that, in each set of p consecutive terms, there are one of each of the following groups: pm , $(pj + 1)$, $(pj + 2)$, $(pj + 3), \dots, (pj + p - 2)$ and $(pj + p - 1)$ (though not necessarily in this order). Example:

1	31	61	91	121	151	181	Terms $(30n + 1)$
$(7 \cdot 0 + 1)$	$(7 \cdot 4 + 3)$	$(7 \cdot 8 + 5)$	$7 \cdot 13$	$(7 \cdot 17 + 2)$	$(7 \cdot 21 + 4)$	$(7 \cdot 25 + 6)$	Terms $7m$ and $(7j + a)$

Therefore, and according to this axiom, $\frac{1}{p} \frac{x}{30}$ will be the number of multiples of p (pm) and, also, the number of terms that have each groups $(pj + 1)$, $(pj + 2)$, $(pj + 3), \dots, (pj + p - 2)$ and $(pj + p - 1)$ in each sequence **A** or **B**.

This same axiom allows us to say that these groups contain all terms of sequences **A** or **B** as follows:

1. Group pm : contains all multiples of p (including the prime p , if it would be present).
2. Groups $(pj + 1)$, $(pj + 2)$, $(pj + 3), \dots, (pj + p - 1)$: contain all multiples (except those of p) and the primes greater than \sqrt{x} .

As it has been described, the groups $(pj + 1)$, $(pj + 2)$, $(pj + 3), \dots, (pj + p - 2)$ and $(pj + p - 1)$ of sequence **B** (and similarly for sequence **A**) are arithmetic progressions of module $30p$, $(30pn + b)$, such that $\gcd(30p, b) = 1$.

Applying the prime numbers theorem in arithmetic progressions^[2], shown on page 1, to these groups we concluded that they all will have, approximately, the same amount of primes ($\approx \frac{\pi(bx)}{p-1}$ in the sequence **B**) and, as they all have the same number of terms, also they will have, approximately, the same number of multiples.

Similarly, we can apply this theorem to terms belonging to two or more groups. For example, the terms that are, at once, in the groups $(7j + a)$ and $(13j + c)$ correspond to arithmetic progressions of module 2730, $(2730 = 7 \cdot 13 \cdot 30)$. In this case, all groups of a sequence **A**

or **B** that contain these terms (72 groups that result of combining 6 *a* and 12 *c*) they will have, approximately, the same amount of primes and, as they all have the same number of terms, will also have, approximately, the same number of multiples.

As described, I gather that, of the $\frac{1}{7} \frac{x}{30}$ terms ($7j + a$) that there are in the sequence **B**, $\approx \frac{\pi(bx)}{6}$ will be primes. All other terms are multiples (of primes greater than 5, except the prime 7).

In general, I gather that, of the $\frac{1}{p} \frac{x}{30}$ terms ($pj + h$) that there are in the sequence **B**, $\approx \frac{\pi(bx)}{p-1}$ will be primes. All other terms are multiples (of primes greater than 5, except the prime *p*).

We will define as a fraction $k(7x)$ of sequence **B** the ratio between the number of multiples that there are in the group of terms ($7j + a$) and the total number of these.

Applying the above for all *p* (prime greater than 5 and lesser than \sqrt{x}) we will define as a fraction $k(px)$ of sequence **B** the ratio between the number of multiples that there are in the group of terms ($pj + h$) and the total number of these.

We can see the similarity between $k(bx)$ and the factors $k(7x)$, $k(11x)$, $k(13x)$, $k(17x)$, ..., $k(px)$, ... so their formulas will be similar. I will use \approx instead of = due to the imprecision in the number of primes that there are in each group ($7j + a$), ($11j + b$), ($13j + c$), ... Using the same procedure as for obtaining $k(bx)$:

$$k(px) \approx \frac{\frac{1}{p} \frac{x}{30} - \frac{\pi(bx)}{p-1}}{\frac{1}{p} \frac{x}{30}} = 1 - \frac{\frac{\pi(bx)}{p-1}}{\frac{1}{p} \frac{x}{30}} = 1 - \frac{30p\pi(bx)}{(p-1)x} \qquad k(px) \approx 1 - \frac{30\pi(bx)}{x} \frac{p}{p-1}$$

For the prime 7: $k(7x) \approx 1 - \frac{35\pi(bx)}{x}$ For the prime 11: $k(11x) \approx 1 - \frac{33\pi(bx)}{x}$ For the prime 31: $k(31x) \approx 1 - \frac{31\pi(bx)}{x}$

And so on to the prime previous to \sqrt{x} .

If we order these factors from lowest to highest value: $k(7x) < k(11x) < k(13x) < k(17x) < \dots < k(997x) < \dots < k(bx)$

In the formula to obtain $k(px)$ we have that: $\lim_{p \rightarrow \infty} \frac{p}{p-1} = 1$ so we can write: $\lim_{p \rightarrow \infty} k(px) = 1 - \frac{30\pi(bx)}{x} = k(bx)$

We can unify all factors $k(7x)$, $k(11x)$, $k(13x)$, ..., $k(px)$, ... into one, which we will call $k(jx)$, and that will group all of them together.

Applying the above, we will define as a fraction $k(jx)$ of sequence **B** the ratio between the number of multiples that there are in the set of all terms ($7j_{21} + a$), ($11j_{22} + b$), ($13j_{23} + c$), ($17j_{24} + d$), ... and the total number of these.

Logically, the $k(jx)$ value is determined by the $k(px)$ values corresponding to each primes from 7 to the one previous to \sqrt{x} .

Summarizing the exposed: a fraction $k(jx)$ of terms ($7j_{21} + a$), ($11j_{22} + b$), ($13j_{23} + c$), ($17j_{24} + d$), ... of sequence **B** will be multiples and, complying the origin axiom, will be paired with an equal fraction of multiples $7m_{11}$, $11m_{12}$, $13m_{13}$, $17m_{14}$, ... of sequence **A**.

To put it simply and in general:

A fraction $k(jx)$ of multiples of sequence **A** will have, as partner, a multiple of sequence **B**.

Recalling the axiom on page 5, and the formulas on page 8, we can record:

1. Number of multiple-multiple pairs = $k(jx)$ (Number of multiples of sequence **A**)
2. Number of free multiples in the sequence **A** = $(1 - k(jx))$ (Number of multiples of sequence **A**)
3. $P_{PP(x)} =$ Actual number of pairs of primes greater than \sqrt{x} that are formed with the sequences **A-B**
 $P_{PP(x)} =$ (Number of primes greater than \sqrt{x} of sequence **B**) – (Number of free multiples of sequence **A**)

Expressed algebraically: $P_{PP(x)} = \pi(bx) - (1 - k(jx))\left(\frac{x}{30} - \pi(ax)\right)$

Let us suppose that there may be a sufficiently large even number that does not meet the conjecture. In this case: $P_{PP(x)} = 0$. $P_{PP(x)}$ cannot be negative because the number of primes of sequence **B** cannot be lesser than the number of free multiples of sequence **A**.

We can define a factor, which I will call $k(0x)$ and that, replacing $k(jx)$ in the above formula, it results in $P_{PP(x)} = 0$.

As a concept, $k(0x)$ would be the minimum value of $k(jx)$ for which the conjecture would not be met.

$$0 = \pi(bx) - (1 - k(0x))\left(\frac{x}{30} - \pi(ax)\right) \qquad \pi(bx) = (1 - k(0x))\left(\frac{x}{30} - \pi(ax)\right)$$

Solving: $k(0x) = 1 - \frac{30\pi(bx)}{x - 30\pi(ax)}$

For the conjecture to be true, $k(jx)$ must be greater than $k(0x)$ for any x value.

Let us recall that the $k(jx)$ value is determined by values of the factors $k(7x), k(11x), k(13x), k(17x), \dots, k(px), \dots$

To analyze the relation between the factors $k(jx)$ and $k(0x)$, first, let us compare $k(0x)$ with the general factor $k(px)$.

$$k(0x) = 1 - \frac{30\pi(bx)}{x - 30\pi(ax)} = 1 - \frac{30\pi(bx)}{x} \frac{x}{x - 30\pi(ax)} = 1 - \frac{30\pi(bx)}{x} \frac{1}{1 - \frac{30\pi(ax)}{x}}$$

$$k(px) \approx 1 - \frac{30\pi(bx)}{x} \frac{p}{p-1} = 1 - \frac{30\pi(bx)}{x} \frac{1}{1 - \frac{1}{p}}$$

To compare $k(0x)$ with $k(px)$, simply compare $\frac{30\pi(ax)}{x}$ with $\frac{1}{p}$ which are the terms that differentiate the two formulas.

Let us recall, page 8, the prime numbers theorem: $\pi(x) \sim \frac{x}{\ln(x)}$ being $\pi(x)$ the number of primes lesser or equal to x .

As I have indicated, it can be accepted that: $\pi(ax) \approx \frac{\pi(x)}{8}$ being 8 the number of groups of primes.

Substituting $\pi(x)$ by its corresponding formula: $\pi(ax) \sim \frac{x}{8\ln(x)}$

The approximation of this formula does not affect the final result of the comparison between $k(0x)$ and $k(px)$ that we are analyzing.

Compare $\frac{30\pi(ax)}{x}$ with $\frac{1}{p}$ Substituting $\pi(ax)$ by its corresponding formula

Compare $\frac{30x}{8x\ln(x)}$ with $\frac{1}{p}$

Compare $\frac{3,75}{\ln(x)}$ with $\frac{3,75}{3,75p}$

Compare $\ln(x)$ with $3,75p$ Applying the natural logarithm concept

Compare x with $e^{3,75p}$ For powers of 10: $\ln 10 = 2,302585$ $3,75 / 2,302585 = 1,6286 \approx 1,63$

Compare x with $10^{1,63p}$

Comparison result: $k(0x)$ will be lesser than $k(px)$ if $x < 10^{1,63p}$ $k(0x)$ will be greater than $k(px)$ if $x > 10^{1,63p}$

In the following expressions, the exponents values are approximate. This does not affect the comparison result.

1. For the prime 7: $k(0x) < k(7x)$ if $x < 10^{11,4}$ $k(0x) > k(7x)$ if $x > 10^{11,4} \approx 4 \cdot 10^4$ primes lesser than $10^{5,7}$
2. For the prime 11: $k(0x) < k(11x)$ if $x < 10^{18}$ $k(0x) > k(11x)$ if $x > 10^{18} \approx 5,08 \cdot 10^7$ primes lesser than 10^9
3. For the prime 31: $k(0x) < k(31x)$ if $x < 10^{50}$ $k(0x) > k(31x)$ if $x > 10^{50} \approx 1,76 \cdot 10^{23}$ primes lesser than 10^{25}
4. For the prime 997: $k(0x) < k(997x)$ if $x < 10^{1620}$ $k(0x) > k(997x)$ if $x > 10^{1620} \approx 5,36 \cdot 10^{806}$ primes lesser than 10^{810}

By analyzing these data we can see that, for numbers lesser than $10^{11,4}$, $k(0x)$ is lesser than all factors $k(px)$ and, therefore, also will be lesser than $k(jx)$ which allows us to ensure that the Goldbach conjecture will be met, at least until $10^{11,4}$.

For the x values greater than $10^{11,4}$, we can see that $k(0x)$ overcomes gradually the factors $k(7x), k(11x), k(13x), k(17x), \dots, k(997x), \dots$

Looking in detail, we can note that if the p value, for which the comparison is applied, increases in geometric progression, the x value from which $k(0x)$ exceeds to $k(px)$ increases exponentially. Because of this, also increases exponentially (or slightly higher) the number of primes lesser than \sqrt{x} and whose factors $k(px)$ will determine the $k(jx)$ value.

Logically, if increases the number of primes lesser than \sqrt{x} , decreases the "relative weight" of each factor $k(px)$ in relation to the $k(jx)$ value. Thus, although from $10^{11,4}$ $k(7x)$ is lesser than $k(0x)$, the percentage of terms $(7j + a)$ which are not also in groups of primes greater than 7 will decrease and the factor $k(7x)$ will lose gradually influence on the $k(jx)$ value.

The same can be applied to the factors $k(11x), k(13x), k(17x), \dots$ that will lose gradually influence on the $k(jx)$ value with increasing x .

On the other hand, taking as an example the prime 997, we can note that, when $k(0x)$ exceeds $k(997x)$, there are already $\approx 5,36 \cdot 10^{806}$ primes whose factors $k(px)$ (which will be greater than $k(0x)$) added to factors $k(7x)$ to $k(997x)$ (165 factors that will be lesser than $k(0x)$) will determine the $k(jx)$ value. Note the large difference between 165 and $\approx 5,36 \cdot 10^{806}$.

These data allow us to intuit that $k(jx)$ will be greater than $k(0x)$ for any x value.

After these positive data, we continue developing the formula to calculate the approximate value of $k(jx)$.

Let us compare $k(bx)$ with $k(jx)$. Let us recall the definitions relating to these two factors.

$k(bx)$ = Ratio between the number of multiples and the total number of terms of sequence **B**.

<u>Sequence B</u>	$\frac{x}{30}$ terms	$\pi(bx)$ primes	$\frac{x}{30} - \pi(bx)$ multiples	$k(bx) = 1 - \frac{30\pi(bx)}{x}$
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Terms of sequence B 1/7 are multiples of 7, 1/11 are multiples of 11, 1/13 are multiples of 13, 1/17 are multiples of 17,...

And so on to the prime previous to \sqrt{x} .

$k(jx)$ = Ratio between the number of multiples that there are in the set of all terms $(7j_{21} + a)$, $(11j_{22} + b)$, $(13j_{23} + c)$, $(17j_{24} + d)$,... of sequence **B** and the total number of these. Its value is determined by the values of the factors $k(7x)$, $k(11x)$, $k(13x)$, $k(17x)$,...

As described when we applied the prime numbers theorem in arithmetic progressions, the actual number of primes that there are in each groups $(7j_{21} + a)$, $(11j_{22} + b)$, $(13j_{23} + c)$, $(17j_{24} + d)$,... will be, approximately, equal to the average value indicated.

In the case of an even number that would be multiple of any prime greater than 5, (for example 13), there would not be primes in the group $(13j_{23} + c)$ because $c = 0$, and its corresponding factor $k(13x)$ would equal 1, which, in the end, would slightly increase the $k(jx)$ value.

<u>Group (7j + a)</u>	$\frac{1}{7} \frac{x}{30}$ terms	$\approx \frac{\pi(bx)}{6}$ primes	$\approx \left(\frac{1}{7} \frac{x}{30} - \frac{\pi(bx)}{6} \right)$ multiples	$k(7x) \approx 1 - \frac{30\pi(bx)}{x} \frac{7}{6}$
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Terms (7j + a) No multiples of 7. 1/11 are multiples of 11, 1/13 are multiples of 13, 1/17 are multiples of 17,...

<u>Group (11j + b)</u>	$\frac{1}{11} \frac{x}{30}$ terms	$\approx \frac{\pi(bx)}{10}$ primes	$\approx \left(\frac{1}{11} \frac{x}{30} - \frac{\pi(bx)}{10} \right)$ multiples	$k(11x) \approx 1 - \frac{30\pi(bx)}{x} \frac{11}{10}$
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Terms (11j + b) 1/7 are multiples of 7, no multiples of 11. 1/13 are multiples of 13, 1/17 are multiples of 17,...

<u>Group (13j + c)</u>	$\frac{1}{13} \frac{x}{30}$ terms	$\approx \frac{\pi(bx)}{12}$ primes	$\approx \left(\frac{1}{13} \frac{x}{30} - \frac{\pi(bx)}{12} \right)$ multiples	$k(13x) \approx 1 - \frac{30\pi(bx)}{x} \frac{13}{12}$
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Terms (13j + c) 1/7 are multiples of 7, 1/11 are multiples of 11, no multiples of 13. 1/17 are multiples of 17,...

<u>Group (17j + d)</u>	$\frac{1}{17} \frac{x}{30}$ terms	$\approx \frac{\pi(bx)}{16}$ primes	$\approx \left(\frac{1}{17} \frac{x}{30} - \frac{\pi(bx)}{16} \right)$ multiples	$k(17x) \approx 1 - \frac{30\pi(bx)}{x} \frac{17}{16}$
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Terms (17j + d) 1/7 are multiples of 7, 1/11 are multiples of 11, 1/13 are multiples of 13, no multiples of 17....

And so on to the prime previous to \sqrt{x} .

It can be noted that, in compliance to the prime numbers theorem in arithmetic progressions, the groups $(7j_{21} + a)$, $(11j_{22} + b)$, $(13j_{23} + c)$, $(17j_{24} + d)$,... behave with some regularity, mathematically defined, for the number of terms, the number of primes and the number of multiples that contain, and that is maintained regardless of x value.

Continuing the study of these terms we can see some data, obtained with a programmable controller, that refers to the group $(30n + 29)$ (chosen as example) and the even numbers 10^6 , 10^7 , 10^8 and 10^9 .

Although for this analysis, any sequence of primes can be chosen, I will do it in ascending order (7, 11, 13, 17, 19, 23, ..., 307).

They are the following data, and are numbered as follows:

1. Total number of terms $(7j + a)$, $(11j + b)$, $(13j + c)$, $(17j + d)$,..., $(pj + h)$,...
2. Multiples that there are in the group $(7j + a)$: they are all included.
3. Multiples that there are in the group $(11j + b)$: not included those who are also $(7j + a)$.
4. Multiples that there are in the group $(13j + c)$: not included those who are also $(7j + a)$ or $(11j + b)$.
5. Multiples that there are in the group $(17j + d)$: not included those who are also $(7j + a)$ or $(11j + b)$ or $(13j + c)$.

And so on until the group of prime 307. These data can be consulted from page 21.

The indicated percentages are relative to the total number of terms $(7j + a)$, $(11j + b)$, $(13j + c)$, $(17j + d)$, ..., $(pj + h)$, ...

	10^6		10^7		10^8		10^9	
Terms $(7j + a)$, $(11j + b)$, ...	23.545		250.287		2.613.173		26.977.564	
Multiples $(7j + a)$ and %	3.140	13,34 %	33.750	13,48 %	356.077	13,63 %	3.702.786	13,73 %
Multiples $(11j + b)$ and %	1.767	7,5 %	19.045	7,61 %	199.519	7,63 %	2.067.520	7,66 %
Multiples $(13j + c)$ and %	1.374	5,84 %	14.714	5,88 %	154.831	5,92 %	1.600.628	5,93 %
Multiples $(17j + d)$ and %	1.010	4,29 %	10.549	4,21 %	110.081	4,21 %	1.137.457	4,22 %
Total multiples groups 7 to 307	15.008	63,74 %	156.956	62,71 %	1.642.061	62,84 %	17.014.540	63,07 %

These new data continue to confirm that the groups $(7j_{21} + a)$, $(11j_{22} + b)$, $(13j_{23} + c)$, $(17j_{24} + d)$, ... behave in a uniform manner, because the percentage of multiples that supply each is almost constant when x increases.

The regularity of these groups allows us to intuit that the approximate value of $k(jx)$ can be obtained by a general formula.

Considering the data of each group, and to develop the formula of $k(jx)$, we can think about adding, on one hand, the number of terms of all of them, on the other hand, the number of primes and finally the number of multiples and making the final calculations with the total of these sums. This method is not correct, since each term can be in several groups so they would be counted several times what would give us an unreliable result.

To resolve this question in a theoretical manner, but more accurate, each term $(7j_{21} + a)$, $(11j_{22} + b)$, $(13j_{23} + c)$, $(17j_{24} + d)$, ... should be analyzed individually and applying inclusion-exclusion principle, to define which are multiples and those who are primes.

After several attempts, I have found that this analytical method is quite complex, so that in the end, I rejected it.

In my opinion, the mathematician who solved this question in a rigorous way can use the approach outlined in this paper to demonstrate, in a definitive way, Goldbach's conjecture and the twin primes conjecture.

Given the difficulty of the mathematical analysis, I opted for an indirect method to obtain the formula for $k(jx)$.

Gathering information from the Internet of the latest demonstrations of mathematical conjectures, I have read that it has been accepted the use of computers to perform some of calculations or to verify the conjectures up to a certain number.

Given this information, I considered that I can use a programmable logic controller (PLC) to help me get the formula for $k(jx)$. To this purpose, I have developed the programs that the controller needs to perform this work.

I will begin by analyzing the exposed data from which it can be deduced:

1. The concepts of $k(jx)$ and $k(bx)$ are similar so, in principle, their formulas will use the same variables.
2. The parameters (number of terms, number of primes and number of multiples) involved in $k(jx)$ follow a certain "pattern".
3. The $k(jx)$ and $k(bx)$ values, and also those of $\pi(ax)$ and $\pi(bx)$, gradually increase with increasing x .
4. The $k(jx)$ value is lesser than the $k(bx)$ value (this may vary if the even number is a multiple of primes greater than 5).
5. The values of $k(jx)$ and $k(bx)$ will tend to equalize, in an asymptotically way, when x tends to infinite.

Here are some values, obtained by the controller, concerning to $k(bx)$, $k(jx)$ and the group $(30n + 29)$, (consult from page 21).

1. To 10^6	$k(bx) = 0,706447064$	$k(jx) = 0,698577192$	$k(jx) / k(bx) = 0,988859927$
2. To 10^7	$k(bx) = 0,751107751$	$k(jx) = 0,74568795$	$k(jx) / k(bx) = 0,992784256$
3. To 10^8	$k(bx) = 0,784035978$	$k(jx) = 0,780284734$	$k(jx) / k(bx) = 0,995215469$
4. To 10^9	$k(bx) = 0,809322428$	$k(jx) = 0,806541984$	$k(jx) / k(bx) = 0,996564479$
5. To $7 \cdot 10^6$, (multiple of 7)	$k(bx) = 0,744631063$	$k(jx) = 0,755471932$	$k(jx) / k(bx) = 1,014558712$

By analyzing these data, it can be seen that, as x increases, the $k(jx)$ value tends more rapidly to the $k(bx)$ value that the $k(bx)$ value with respect to 1.

Expressed numerically: To 10^6 : $(1 - 0,706447064) / (0,706447064 - 0,698577192) = 37,3$
 To 10^9 : $(1 - 0,809322428) / (0,809322428 - 0,806541984) = 68,57$

Then, based on the formulas for $k(bx)$ and $k(0x)$, I will propose a formula for $k(jx)$ with a constant. To calculate its value, I will use the programmable controller.

$$\text{Formula of } k(bx): \quad k(bx) = 1 - \frac{30\pi(bx)}{x} \qquad \text{Formula of } k(0x): \quad k(0x) = 1 - \frac{30\pi(bx)}{x - 30\pi(ax)}$$

$$\text{Proposed formula for } k(jx): \quad k(jx) = 1 - \frac{30\pi(bx)}{x - c(jx)\pi(ax)}$$

- Being: x = Even number for which the conjecture is applied and that defines the sequences **A-B**.
 $\pi(ax)$ = Number of primes greater than \sqrt{x} in the sequence **A** for x .
 $\pi(bx)$ = Number of primes greater than \sqrt{x} in the sequence **B** for x .
 $k(jx)$ = Factor in study. The data from the PLC allow calculate its value for several numbers x .
 $c(jx)$ = Constant that can be calculated if we know the values of $\pi(ax)$, $\pi(bx)$ and $k(jx)$ for each number x .

Let us recall that $k(jx)$ is lesser than $k(bx)$ so, comparing their corresponding formulas, it follows that $c(jx)$ would have a minimum value of 0. Also let us remember that, as a concept, $k(0x)$ would be the minimum value of $k(jx)$ for which the conjecture would not be met. According to this statement, and comparing their corresponding formulas, it follows that $c(jx)$ would have a maximum value of 30.

The program, which works in the programmable controller, is described below, in a simplified way:

1. It store the 3.398 primes that are lesser than $31.622 = 10^{4.5}$. With them, we can analyze the sequences **A-B** until number 10^9 .
2. It divides the even number x , ($\leq 10^9$), by the primes lesser than \sqrt{x} . The remains of these divisions are the values of a, b, c, d, \dots
3. It divides each term of each sequence **A** or **B** by the primes lesser than \sqrt{x} to define which are multiples and those who are primes.
4. In the same process, it determines the terms that are of form $(7j + a), (11j + b), (13j + c), (17j + d), \dots$ in each sequence **A** or **B**.
5. 8 counters are scheduled (4 in each sequence) to count the following data:
 6. Number of multiples that there are in each sequence **A** or **B** (it includes all composites and the primes who are lesser than \sqrt{x}).
 7. Number of primes that there are in each sequence **A** or **B** (only those who are greater than \sqrt{x}).
 8. Number of multiples that there are in the terms $(7j + a), (11j + b), (13j + c), (17j + d), \dots$ of each sequence **A** or **B** (as 6).
 9. Number of primes that there are in the terms $(7j + a), (11j + b), (13j + c), (17j + d), \dots$ of each sequence **A** or **B** (as 7).
10. With the final data of these counters, and using a calculator, the values of $k(ax), k(bx), k(jx), c(jx), \dots$ can be obtained.

Then, I indicate the calculated values of $c(jx)$ related to some even numbers, (between 10^6 and 10^9), and their corresponding groups of primes. The details of these calculations can be consulted in the numerical data presented from page 21.

	$\frac{(30n + 11) + (30n + 29)}{10^6}$	$\frac{(30n + 17) + (30n + 23)}{10^7}$							
	2,668	2,668	2,714	2,711					
	$\frac{(30n + 1) + (30n + 19)}{10^7}$	$\frac{(30n + 7) + (30n + 13)}{10^8}$							
	2,566	2,566	2,371	2,371					
	$\frac{(30n + 1) + (30n + 29)}{10^8}$	$\frac{(30n + 7) + (30n + 23)}{10^9}$							
	2,371	2,371	2,423	2,423					
	$\frac{(30n + 1) + (30n + 19)}{8 \cdot 10^6}$	$\frac{(30n + 7) + (30n + 13)}{8 \cdot 10^7}$							
	2,261	2,261	2,256	2,256					
	$\frac{(30n + 1) + (30n + 29)}{9 \cdot 10^6}$	$\frac{(30n + 7) + (30n + 23)}{9 \cdot 10^7}$	$\frac{(30n + 11) + (30n + 19)}{3 \cdot 10^8}$	$\frac{(30n + 13) + (30n + 17)}{3 \cdot 10^8}$					
	2,697	2,696	2,732	2,732	2,615	2,616	2,603	2,603	
	$\frac{(30n + 11) + (30n + 23)}{4.194.304 = 2^{22}}$	$\frac{(30n + 17) + (30n + 23)}{67.108.864 = 2^{26}}$	$\frac{(30n + 7) + (30n + 13)}{8.388.608 = 2^{23}}$	$\frac{(30n + 1) + (30n + 7)}{134.217.728 = 2^{27}}$			2,526	2,523	
	2,439	2,439	2,35	2,35			2,354	2,354	
	$\frac{(30n + 17) + (30n + 29)}{16.777.216 = 2^{24}}$	$\frac{(30n + 23) + (30n + 23)}{268.435.456 = 2^{28}}$	$\frac{(30n + 13) + (30n + 19)}{33.554.432 = 2^{25}}$	$\frac{(30n + 13) + (30n + 19)}{536.870.912 = 2^{29}}$			2,477	2,477	
	2,744	2,742	2,401	2,4	2,42	2,42	2,474	2,474	
	$\frac{(30n + 11) + (30n + 29)}{7 \cdot 10^6, \text{ (multiple of 7)}}$	$\frac{(30n + 17) + (30n + 23)}{7 \cdot 10^6, \text{ (multiple of 7)}}$	$\frac{(30n + 17) + (30n + 23)}{7 \cdot 10^6, \text{ (multiple of 7)}}$	$\frac{(30n + 17) + (30n + 23)}{7 \cdot 10^6, \text{ (multiple of 7)}}$			2,288	2,288	
	- 5,214	- 5,212	- 5,372	- 5,373			2,34	2,34	
		$\frac{(30n + 11) + (30n + 17)}{14.872.858 = 2 \cdot 7 \cdot 11 \cdot 13 \cdot 17 \cdot 19 \cdot 23}$	$\frac{(30n + 11) + (30n + 17)}{14.872.858 = 2 \cdot 7 \cdot 11 \cdot 13 \cdot 17 \cdot 19 \cdot 23}$	$\frac{(30n + 11) + (30n + 17)}{14.872.858 = 2 \cdot 7 \cdot 11 \cdot 13 \cdot 17 \cdot 19 \cdot 23}$			2,3	2,301	
		- 30,3	- 30,31	- 30,31			2,3	2,301	

To obtain the $c(jx)$ values for numbers greater than 10^9 , I have used actual data from MathWorld Web concerning to the Twin Primes Conjecture which says: "There are infinitely many primes p such that $(p + 2)$ is also prime."

We call *Twin Primes* the pair of consecutive primes that are separated only by an even number.

Applying to this conjecture a similar procedure that has been applied to Goldbach's conjecture it results in the next axiom:

All groups of multiples $7m_{11}, 11m_{12}, 13m_{13}, 17m_{14}, \dots$ (including the primes lesser than \sqrt{x} that are present) of sequence **A** are paired, group to group, with all groups of terms $(7m_{11} + 2), (11m_{12} + 2), (13m_{13} + 2), (17m_{14} + 2), \dots$ of sequence **B**.

Accordingly, the following average values of $c(jx)$ are referring to the terms $(7m_{11} + 2), (11m_{12} + 2), (13m_{13} + 2), (17m_{14} + 2), \dots$ of sequence **B**. For more details, consult the numerical data presented from page 32.

$\frac{10^{10}}{10^{11}} \approx 2,095$	$\frac{10^{12}}{10^{13}} \approx 2,058$	$\frac{10^{14}}{10^{15}} \approx 2,029$	$\frac{10^{16}}{10^{18}} \approx 2,005$
$\frac{10^{11}}{10^{12}} \approx 2,075$	$\frac{10^{13}}{10^{14}} \approx 2,042$	$\frac{10^{15}}{10^{16}} \approx 2,016$	$\frac{10^{17}}{10^{18}} \approx 1,987$

Consulting the numeric calculations presented from page 21 to 32, we can note that the axiom which has been used as a starting point at the beginning of this chapter is met:

1. The number of multiples $7m_{11}, 11m_{12}, \dots$ of sequence **A** is equal to the number of terms $(7j_{21} + a), (11j_{22} + b), \dots$ of sequence **B**.
2. The number of terms $(7j_{11} + a), (11j_{12} + b), \dots$ of sequence **A** is equal to the number of multiples $7m_{21}, 11m_{22}, \dots$ of sequence **B**.
3. The number of multiples that there are in the terms $(7j_{11} + a), (11j_{12} + b), \dots$ of sequence **A** is equal to the multiples in the terms $(7j_{21} + a), (11j_{22} + b), \dots$ of sequence **B**, being the number of multiple-multiple pairs that are formed with the two sequences.

Let's review the above data:

1. Lowest number analyzed: 10^6 .
2. Highest number analyzed with the programmable controller: 10^9 .
3. Highest number analyzed with data from MathWorld Web: 10^{18} .
4. Highest $c(jx)$ value: 2,744 for the even number $9 \cdot 10^6$ in the combination $(30n + 1) + (30n + 29)$.
5. Lowest $c(jx)$ value with the programmable controller: 2,223 for the number $9 \cdot 10^7$ in the combination $(30n + 7) + (30n + 23)$.
6. Lowest $c(jx)$ value with data from MathWorld Web: 1,98 for the number 10^{18} (average value referring to the twin primes conjecture).
7. Maximum number of terms analyzed by programmable controller in a sequence **A** or **B**: 33.333.333 for the number 10^9 .

In the analyzed numbers with PLC, 10^9 is 10^3 times greater than 10^6 . Using data from MathWorld Web, 10^{18} is 10^{12} times greater than 10^6 . It can be seen that, although there is a great difference between the values of the analyzed numbers, the $c(jx)$ values vary little (from 2,744 to 2,223 with PLC and up to 1,987 with data from MathWorld Web).

Looking in detail, it can be seen that for numbers greater than $16.777.216 = 2^{24}$, the $c(jx)$ value is lesser than 2,5. We also note that the average value of $c(jx)$ tends to decrease slightly when increasing x .

Finally, it can be intuited that, for large values of x , the average value of $c(jx)$ tends to an approximate value to 2,3.

I believe that this data is sufficiently representative to be applied in the proposed formula for $k(jx)$.

Given the above, we can define an approximate average value for $c(jx)$: $c(jx) \approx 2,5$ (for large numbers: $c(jx) \approx 2,3$)

With this average value of $c(jx)$, the final formula of $k(jx)$ can be written: $k(jx) \approx 1 - \frac{30\pi(bx)}{x - 2,5\pi(ax)}$

I consider that this formula is valid to prove the Goldbach conjecture although it has not been obtained through mathematical analysis. Also, I consider that it can be applied to large numbers because the regularity in the characteristics of terms $(7j_{21} + a), (11j_{22} + b), (13j_{23} + c), (17j_{24} + d), \dots$ is maintained, and I intuit that with more precision, when increasing x .

Likewise, I believe that this formula and the formula that can be obtained through a rigorous analytical method can be considered equivalent in purpose of validity to prove the conjecture although the respective numerical results may differ slightly.

Let us analyze the deviation that can affect the average value defined for $c(jx)$. Considering only the even numbers that are not multiples of primes greater than 5, $c(jx)$ it would have a minimum value greater than 0 because, in this case, $k(jx)$ is always lesser than $k(bx)$.

We can see that the maximum deviation decreasing is from 2,5 to 0 (or close to 0). I understand that, by symmetry, the maximum deviation increasing will be similar so that, in principle, the $c(jx)$ value always would be lesser than 5.

On the other hand, and as I have indicated, $c(jx)$ would have a maximum value of 30. Considering as valid the final formula proposed for $k(jx)$, considering that will be equivalent to analytical formula and comparing 30 with the calculated values of $c(jx)$, (between 2,744 and 2,223), it can be accepted that $c(jx) < 30$ always will be met.

At this point, let's make a summary of the exposed questions:

1. All multiples $7m_{11}, 11m_{12}, 13m_{13}, \dots$ of sequence **A** are paired with all terms $(7j_{21} + a), (11j_{22} + b), (13j_{23} + c), \dots$ of sequence **B**.
2. The groups $(7j_{21} + a), \dots$ follow a "pattern" for the number of terms, number of primes and number of multiples that contain.
3. We define as $k(jx)$ the fraction of terms $(7j_{21} + a), (11j_{22} + b), (13j_{23} + c), \dots$ of sequence **B** that are multiples.
4. The analysis of paragraph 2 allows us to intuit that the approximate value of $k(jx)$ can be obtained by a general formula.
5. Proposed formula for $k(jx)$: $k(jx) = 1 - \frac{30\pi(bx)}{x - c(jx)\pi(ax)}$. In the exposed calculations, the $c(jx)$ value has resulted to be lesser than 3.
6. Final formula for $k(jx)$: $k(jx) \approx 1 - \frac{30\pi(bx)}{x - 2,5\pi(ax)}$. I consider that will be equivalent to the formula obtained by mathematical analysis.
7. Considering valid the above formula and considering the calculated values of $c(jx)$, (< 3), it can be accepted that $c(jx) < 30$.
8. Applying $c(jx) < 30$ in the proposed formula for $k(jx)$: $k(jx) > 1 - \frac{30\pi(bx)}{x - 30\pi(ax)} = k(0x)$
9. Finally, for any x value: $k(jx) > k(0x)$. This statement must be rigorously demonstrated in the analytical formula.

Let us recall, page 10, the formula to calculate the pairs number of primes greater than \sqrt{x} that are formed with the sequences **A-B**.

$$P_{PP(x)} = \pi(bx) - (1 - k(jx))\left(\frac{x}{30} - \pi(ax)\right) \quad \text{Substituting } k(jx) \text{ for its formula: } k(jx) = 1 - \frac{30\pi(bx)}{x - c(jx)\pi(ax)}$$

$$P_{PP(x)} = \pi(bx) - \frac{30\pi(bx)}{x - c(jx)\pi(ax)}\left(\frac{x}{30} - \pi(ax)\right) = \pi(bx) - \frac{x\pi(bx) - 30\pi(ax)\pi(bx)}{x - c(jx)\pi(ax)} = \frac{x\pi(bx) - c(jx)\pi(ax)\pi(bx) - x\pi(bx) + 30\pi(ax)\pi(bx)}{x - c(jx)\pi(ax)}$$

$$P_{PP(x)} = \frac{(30 - c(jx))\pi(ax)\pi(bx)}{x - c(jx)\pi(ax)}$$

In this formula, we can replace $c(jx)$ by its already defined values:

$$c(jx) \approx 2,5 \quad P_{PP(x)} \approx \frac{(30 - 2,5)\pi(ax)\pi(bx)}{x - 2,5\pi(ax)} \quad P_{PP(x)} \approx \frac{27,5\pi(ax)\pi(bx)}{x - 2,5\pi(ax)} \quad \text{Number of pairs of primes greater than } \sqrt{x}$$

$$c(jx) < 30 \quad P_{PP(x)} > \frac{(30 - 30)\pi(ax)\pi(bx)}{x - 30\pi(ax)} \quad P_{PP(x)} > 0$$

This final expression indicates that $P_{PP(x)}$ is always greater than 0 and considering that by its nature, (prime pairs), cannot be a fractional number (must be greater than 0, cannot have a value between 0 and 1) I gather that $P_{PP(x)}$ will be a natural number equal to or greater than 1. Similarly, I conclude that the $P_{PP(x)}$ value will increase when increasing x because also increase $\pi(ax)$ and $\pi(bx)$.

We can record:

$$P_{PP(x)} \geq 1 \quad P_{PP(x)} \text{ will be a natural number and will increase when increasing } x$$

The above expression indicates that the pairs number of primes greater than \sqrt{x} that meet the conjecture for an even number x is always equal to or greater than 1.

With everything described, it can be confirmed that: **The Goldbach Conjecture is true.**

10. Final formula.

Considering that the conjecture has already been demonstrated, a formula can be defined to calculate the approximate number of partitions for an even number x .

According to the previous chapter, the number of these partitions formed with sequences **A-B** that are greater than \sqrt{x} is:

$$P_{PP(x)} \approx \frac{27,5\pi(ax)\pi(bx)}{x - 2,5\pi(ax)}$$

If no precision in the final formula is required, and for large values of x , the following can be considered:

1. On page 8 I have indicated that: $\pi(ax) \approx \pi(bx) \approx \frac{\pi(x)}{8}$ being $\pi(x)$ the number of primes lesser than or equal to x .
2. The term $2,5\pi(ax)$ can be neglected because it is be very small compared to x , (1,59 % of x for 10^9), (0,77 % of x for 10^{18}).
3. By applying the above, the value of the denominator will increase, so, to compensate, I will put in the numerator 28 instead of 27,5.
4. The exposed data allows us to intuit that, as x is larger, the average value of $c(jx)$ will decrease being lesser than 2,5.
5. The possible pairs of primes with one of them lesser than \sqrt{x} is very small compared to the total pairs number of primes.

With this in mind, the above formula can be slightly modified to make it more simple.

As a final concept, I consider that the numeric result of the obtained formula will be the approximate number of partitions which are formed with the sequences **A** and **B** and that meet Goldbach's conjecture for an even number x .

$$P_{PP(x)} \approx \frac{28 \frac{\pi(x)}{8} \frac{\pi(x)}{8}}{x} \quad P_{PP(x)} \approx \frac{7 \pi^2(x)}{16 x}$$

Now, let's analyze the halves sequences (page 4). In this case, the sequences **A** and **B** are formed with the same group of primes. Let us recall the number 784 used as example at the beginning.

$$784 = 30 \cdot 26 + 4 = (30n_4 + 17) + (30n_5 + 17) \quad \text{being: } 26 = n_4 + n_5 + 1$$

We will write the sequence **A** of all numbers $(30n_4 + 17)$ from 0 to 784. (We write the complete sequence in two halves).
Also we will write the sequence **B** of all numbers $(30n_5 + 17)$ from 784 to 0. (We write the complete sequence in two halves).

A 17 - 47 - 77 - 107-137-167-197-227-257-287-317-347-377 407-437-467-497-527-557-587-617-647-677-707-737-767
B 767-737-707-677-647-617-587-557-527-497-467-437-407 377-347-317-287-257-227-197-167-137-107 - 77 - 47 - 17

We can see that these two sequences have the same terms written in reverse order, so that the pairs of terms are repeated. If we apply the same procedure as used for the complete sequences we will obtain the same result. In this case, the number of different pairs of primes that will meet the Goldbach conjecture will be half of those in the complete sequences.

Then, and considering what has been described for the halves sequences, we will adjust the last formula (that uses 2 groups) to the number of groups of primes used (3, 6, 4 or 8) in each even number multiplying $\frac{7 \pi^2(x)}{16 x}$, respectively, by 3/2, 3, 2 or 4.

Performing the multiplications described and being $G(x), \dots$ the actual number of Goldbach's partitions for an even number x :

$$G(x) \approx \frac{21 \pi^2(x)}{32 x} \quad \text{Partitions number for the even number that is not multiple of 6 or 10.}$$

$$G_6(x) \approx \frac{21 \pi^2(x)}{16 x} \quad \text{Partitions number for the even number that is multiple of 6.}$$

$$G_{10}(x) \approx \frac{7 \pi^2(x)}{8 x} \quad \text{Partitions number for the even number that is multiple of 10.}$$

$$G_{30}(x) \approx \frac{7 \pi^2(x)}{4 x} \quad \text{Partitions number for the even number that is multiple of 30.}$$

Final formulas, being: $G(x), G_6(x), G_{10}(x), G_{30}(x)$ = Actual number of Goldbach's partitions for the even numbers x .
 x = Even number greater than 30.
 $\pi(x)$ = Number of primes lesser than or equal to x .

Although they could be obtained by the programmable controller, we will take the actual values of $\pi(x)$ from MathWorld Web to check the precision of formulas of $G_{10}(x)$ and $G(x)$. Although we will only check these two, I consider that the precision of the four above formulas will be similar.

	$\pi(x)$	$G_{10}(x)$ (PLC)	Formula result	Difference
1. To 10^6	78.498	5.382	5.392	+ 0,185 %
2. To 10^7	664.579	38.763	38.646	- 0,302 %
3. To 10^8	5.761.455	291.281	290.451	- 0,285 %
4. To 10^9	50.847.534	2.273.918	2.262.288	- 0,511 %
5. To $2^{28} = 268.435.456$	$\pi(x)$ (PLC) 14.630.810	$G(x)$ (PLC) 525.109	Formula result 523.319	Difference - 0,341 %

To express the final formulas as an x function, we will use the prime numbers theorem^[3], (page 8): $\pi(x) \sim \frac{x}{\ln(x)}$

Substituting $\pi(x)$ in the above formulas and simplifying:

$$G(x) \sim \frac{21 x}{32 \ln^2(x)} \quad \text{Partitions number for the even number that is not multiple of 6 or 10. To 16.384, formula: 114 partitions, actual: 151}$$

$$G_6(x) \sim \frac{21 x}{16 \ln^2(x)} \quad \text{Partitions number for the even number that is multiple of 6. To 13.122 formula: 191 partitions, actual: 245}$$

$$G_{10}(x) \sim \frac{7 x}{8 \ln^2(x)} \quad \text{Partitions number for the even number that is multiple of 10. To 31.250 formula: 255 partitions, actual: 326}$$

$$G_{30}(x) \sim \frac{7 x}{4 \ln^2(x)} \quad \text{Partitions number for the even number that is multiple of 30. To 21.870 formula: 383 partitions, actual: 483}$$

The sign \sim indicates that these formulas have an asymptotic behavior, giving results lesser than actual values when applied to small numbers but this difference gradually decreases as we analyze larger numbers.

If the even number is multiple of one or more primes greater than 5, and as seen on page 6, increases the pairs proportion of composites so that, ultimately, more pairs of primes will be formed.

Due to this, the difference between the actual number of partitions and the corresponding formula result will increase.

Examples:

To 16.016	Formula result: 112 partitions	Actual: 193 partitions	Multiple of 7, 11 and 13.
To 16.018	Formula result: 112 partitions	Actual: 152 partitions	

A better approach for this theorem is given by the offset logarithmic integral function^[3] $\text{Li}(x)$: $\pi(x) \approx \text{Li}(x) = \int_2^x \frac{dy}{\ln(y)}$

Substituting $\pi(x)$ again, we obtain four more precise formulas:

$G(x) \approx \frac{21}{32} \int_2^x \frac{dy}{\ln^2(y)}$ Partitions number for the even number that is not multiple of 6 or 10.

$G_6(x) \approx \frac{21}{16} \int_2^x \frac{dy}{\ln^2(y)}$ Partitions number for the even number that is multiple of 6.

$G_{10}(x) \approx \frac{7}{8} \int_2^x \frac{dy}{\ln^2(y)}$ Partitions number for the even number that is multiple of 10.

$G_{30}(x) \approx \frac{7}{4} \int_2^x \frac{dy}{\ln^2(y)}$ Partitions number for the even number that is multiple of 30.

11. Comparison with the Twin Primes Conjecture.

Twin Primes Conjecture statement^[4]: “There are infinitely many primes p such that $(p + 2)$ is also prime”.

We call *Twin Primes* the pair of consecutive primes that are separated only by an even number. Examples: (11, 13), (29, 31).

Goldbach's conjecture and the twin primes conjecture are similar in that both can be studied by combining two groups of primes to form pairs that add an even number, in the first, or pairs of twin primes in the second.

We will write the three combinations of groups of primes with which all pairs of twin primes greater than 7 will be formed:

$(30n_1 + 11)$ and $(30n_1 + 13)$ $(30n_2 + 17)$ and $(30n_2 + 19)$ $(30n_3 + 29)$ and $(30n_3 + 31)$

We will write the sequences **A** and **B** corresponding to number 780 and the combination $(30n_1 + 11)$ and $(30n_1 + 13)$, underlining the 11 twin prime pairs that are formed. We use the list of primes lesser than 1.000.

A 11-41-71-101-131-161-191-221-251-281-311-341-371-401-431-461-491-521-551-581-611-641-671-701-731-761
B 13-43-73-103-133-163-193-223-253-283-313-343-373-403-433-463-493-523-553-583-613-643-673-703-733-763

A first difference between these two conjectures refers to the order of the terms in the sequences **A-B**.

In Goldbach's conjecture, the terms are in reverse order (from lowest to highest in the sequence **A** and from highest to lowest in the sequence **B**) whilst in the twin primes conjecture, the terms of both sequences are in the same order (from lowest to highest).

Let us recall that the probability of a natural number being prime, decreases when increasing its value, so analyzing the sequences **A-B** of Goldbach's conjecture, we see that the two terms of each pairs that are formed have different probability to be primes because one of them has a value between 0 and $x/2$ and the other between $x/2$ and x .

On the other hand, analyzing the sequences **A-B** of the twin primes conjecture we see that the two terms of each pairs that are formed have virtually, the same probability to be primes since the difference between them is only two units.

Given the above, I conclude that in the Goldbach conjecture there is a greater “difficulty” to form pairs of primes.

As seen on page 6, this “difficulty” is lesser if the even number is a multiple of one or more primes greater than 5, because more pairs of composites are formed and, therefore, also more pairs of primes will be formed.

A second difference refers to the groups of terms analyzed in each demonstration that I have developed. In the proof of Goldbach's conjecture, we analyzed the groups of terms $(7j + a), (11j + b), (13j + c), (17j + d), \dots$ of sequence **B** (or **A**). In the proof of twin primes conjecture, we analyzed the groups of terms $(7m_{11} + 2), (11m_{12} + 2), (13m_{13} + 2), \dots$ of sequence **B**. In the latter, also we can analyze the groups of terms $(7m_{21} - 2), (11m_{22} - 2), (13m_{23} - 2), (17m_{24} - 2), \dots$ of sequence **A**.

Let us recall that the numbers a, b, c, d, \dots that appear in the proof of Goldbach's conjecture, are the remains of dividing the even number x by the primes from 7 to the prime previous to \sqrt{x} , so that a, b, c, d, \dots will have different values for each even number. If the even number is a multiple (for example, multiple of 7 and 11) we would have: $a = b = 0$.

In the case of the twin primes conjecture, we can say that $a = b = c = d = \dots = 2$ and, therefore, the terms analyzed always have the same configuration. Simply, it increases their number when x increases.

Because of this, it can be deduced that, when increasing x , the behavior of terms $(7m_{11} + 2), (11m_{12} + 2), \dots$ of the twin primes conjecture will be more regular, $(c(jx) \approx 2,2)$, than the behavior of terms $(7j + a), (11j + b), \dots$ of Goldbach's conjecture, $(c(jx) \approx 2,5)$.

A final difference would be related to the number of combinations of groups of primes and the number of these groups that are used. Let us recall this question for Goldbach's conjecture (page 3):

1. For the even number that is not multiple of 6 or 10: it results 2 different combinations using 3 groups of primes.
2. For the even number that is multiple of 6: it results 3 different combinations using 6 groups of primes.
3. For the even number that is multiple of 10: it results 2 different combinations using 4 groups of primes.
4. For the even number that is multiple of 30: it results 4 different combinations using the 8 prime numbers groups available.
5. For the twin primes conjecture: it results 3 different combinations using 6 groups of primes (always the same groups).

By analyzing the above data, we conclude that:

1. For the even number that is not multiple of 6 or 10: the partitions number is, approximately, $\frac{1}{2}$ of the number of twin prime pairs.
2. For the even number that is multiple of 6: the partitions number is, approximately, equal to the number of twin prime pairs.
3. For the even number that is multiple of 10: the partitions number is, approximately, $\frac{2}{3}$ of the number of twin prime pairs.
4. For the even number that is multiple of 30: the partitions number is, approximately, $\frac{4}{3}$ of the number of twin prime pairs.

As we have seen, the number of pairs of primes that add an even number x , (power of 2), is: $G(x) \approx \frac{21 \pi^2(x)}{32 x}$

For the even number that is multiple 10: $G_{10}(x) \approx \frac{7 \pi^2(x)}{8 x}$

According to the proof that I have developed, the number of twin prime pairs that are lesser than x is: $G_G(x) \approx \frac{21 \pi^2(x)}{16 x}$

As numerical support, and using the programmable controller, the following data has been obtained:

To $268.435.456 = 2^{28}$ 525.109 prime pairs that add 2^{28} , being both primes greater than 2^{14} .
 1.055.991 twin prime pairs that there are between 2^{14} and 2^{28} .
 $525.109 \approx \frac{1}{2}$ of 1.055.991

For the even number that is multiple of 10:

To 10^9 2.273.918 prime pairs that add 10^9 , being both primes greater than $10^{4.5}$.
 3.424.019 twin prime pairs that there are between $10^{4.5}$ and 10^9 .
 $2.273.918 \approx \frac{2}{3}$ of 3.424.019

Ternary Goldbach Conjecture

The conjecture that has been studied is called strong or binary because there is another, weak or ternary, Goldbach's also, which states [5]: "Every odd number greater than 7 can be written as the sum of three odd primes".

In May 2013, the Peruvian-born mathematician Harald Andrés Helfgott, a researcher at French CNRS at École Normale Supérieure in Paris, has published an article on web arXiv.org in which it is demonstrated that the ternary Goldbach conjecture is true for all odd numbers greater than 10^{29} . For the lesser odd numbers, and in collaboration with David Platt, computers have been used to verify that they also meet the conjecture.

Accepting that the binary Goldbach conjecture is proven, we can write down:

$$(\text{Even number} > 4) = (\text{odd prime}) + (\text{odd prime})$$

Adding the odd prime 3: $(\text{Even number} > 4) + 3 = (\text{odd prime}) + (\text{odd prime}) + 3$

Therefore, it follows: $(\text{Odd number} > 7) = (\text{odd prime}) + (\text{odd prime}) + (\text{odd prime})$

The article of Harald Helfgott and the above expression allows us to say that: **The Ternary Goldbach Conjecture is true.**

We will study now how to calculate the representations number of an odd number greater than 7 as the sum of three odd primes.

Hardy-Littlewood theorem statement [6]: "If General Riemann Hypothesis is true, then: $r_3(x) \sim \mathfrak{b}_3(x) \frac{x^2}{\ln^3(x)}$ ".

In this formula, $r_3(x)$ is the representations number of an odd number x greater than 7 as the sum of three odd primes and $\mathfrak{b}_3(x)$ a factor that depends of x being its value between two constants.

A simple mathematical reasoning can be done to obtain the formula of the previous theorem.

Let us recall that $\pi(x) \sim \frac{x}{\ln(x)}$ is the number of primes lesser than or equal to x . Being $p_1, p_2, p_3, p_4, \dots$ these primes (from first odd prime, $p_1 = 3$) and $N_1, N_2, N_3, N_4, \dots$ the even numbers remaining of each prime of the odd number x , we will have:

$$x = p_1 + N_1 = p_2 + N_2 = p_3 + N_3 = p_4 + N_4 = \dots$$

And so on until the last prime that is a distance greater than 4 of x .

Given the above, it can be deduced that the actual number of representations $r_3(x)$ will be equal to the sum of Goldbach's partitions $G_{(N_1)}, G_{(N_2)}, G_{(N_3)}, G_{(N_4)}, \dots$ of all the even numbers $N_1, N_2, N_3, N_4, \dots$

$$r_3(x) = G_{(N_1)} + G_{(N_2)} + G_{(N_3)} + G_{(N_4)} + \dots$$

In the above expression, the number of summands will be, approximately, equal to the number of primes lesser than x : $\pi(x) \sim \frac{x}{\ln(x)}$

Recalling the formulas to calculate the number of Goldbach's partitions, we will call $G_m(x) = \mathfrak{b}_3(x) \frac{x}{\ln^2(x)}$ the average value of $G_{(N_1)}, G_{(N_2)}, G_{(N_3)}, G_{(N_4)}, \dots$ being $\mathfrak{b}_3(x)$ a factor that depends of x . With this in mind, we can write:

$$r_3(x) \sim \frac{x}{\ln(x)} \mathfrak{b}_3(x) \frac{x}{\ln^2(x)} \qquad r_3(x) \sim \mathfrak{b}_3(x) \frac{x^2}{\ln^3(x)}$$

We can note that, after a "forced" reasoning, we obtain the same formula as the above theorem.

Getting data using a programmable controller

Let us recall: Multiples: include all composites and the primes lesser than \sqrt{x} .
Primes: only those who are greater than \sqrt{x} .

Sequence A

1. The four data highlighted in **bold** are those obtained by the programmable controller.
2. The sum of the number of multiples $7m, 11m, \dots$ and the number of primes is the total number of terms of the sequence.
It must match with the formula result: $\frac{x}{30}$ (page 4).
3. The sum of the number of multiples and the number of primes of form $(7j + a), (11j + b), \dots$ is the total number of these terms.
It must match with the number of multiples $7m, 11m, \dots$ of sequence **B** (page 6).
4. I used a calculator to obtain the following information:
 5. P_{PPx} = Number of prime pairs that add x being both greater than \sqrt{x} (with **A-B**). It must match with the P_{PPx} of sequence **B**.
 $P_{PPx} = (\text{Number of primes of sequence A}) - (\text{Number of primes of form } (7j + a), (11j + b), \dots \text{ of sequence A})$
 6. $k_{ax} =$ Number of multiples $7m, 11m, \dots$ divided by the total number of terms of sequence **A**.
 7. $k_{jx} =$ Number of multiples that there are in the terms $(7j + a), (11j + b), \dots$ divided by the total number of these.

Proposed formula for k_{jx} : $k(jx) = 1 - \frac{30\pi(ax)}{x - c(jx)\pi(bx)}$ (page 13).
 8. $c_{jx} =$ Constant of proposed formula for k_{jx} . Solving: $c(jx) = \frac{x - \frac{30\pi(ax)}{1 - k(jx)}}{\pi(bx)}$
 9. $k_{0x} =$ Minimum value of k_{jx} for which the conjecture would not be met: $k(0x) = 1 - \frac{30\pi(ax)}{x - 30\pi(bx)}$ (pages 10 and 11).

Sequence B

1. The four data highlighted in **bold** are those obtained by the programmable controller.
2. The sum of the number of multiples $7m, 11m, \dots$ and the number of primes is the total number of terms of the sequence.
It must match with the formula result: $\frac{x}{30}$ (page 4).
3. The sum of the number of multiples and the number of primes of form $(7j + a), (11j + b), \dots$ is the total number of these terms.
It must match with the number of multiples $7m, 11m, \dots$ of sequence **A** (page 6).
4. I used a calculator to obtain the following information:
 5. $P_{PPx} =$ Number of prime pairs that add x being both greater than \sqrt{x} (with **A-B**). It must match with the P_{PPx} of sequence **A**.
 $P_{PPx} = (\text{Number of primes of sequence B}) - (\text{Number of primes of form } (7j + a), (11j + b), \dots \text{ of sequence B})$
 6. $k_{bx} =$ Number of multiples $7m, 11m, \dots$ divided by the total number of terms of sequence **B**.
 7. $k_{jx} =$ Number of multiples that there are in the terms $(7j + a), (11j + b), \dots$ divided by the total number of these.

Proposed formula for k_{jx} : $k(jx) = 1 - \frac{30\pi(bx)}{x - c(jx)\pi(ax)}$ (page 13).
 8. $c_{jx} =$ Constant of proposed formula for k_{jx} . Solving: $c(jx) = \frac{x - \frac{30\pi(bx)}{1 - k(jx)}}{\pi(ax)}$
 9. $k_{0x} =$ Minimum value of k_{jx} for which the conjecture would not be met: $k(0x) = 1 - \frac{30\pi(bx)}{x - 30\pi(ax)}$ (pages 10 and 11).

Choosing the group $(30n + 29)$ as an example, we will count the number of multiples that there are in each of the groups $(7j + a), (11j + b), (13j + c), (17j + d), \dots$ until the group of prime 307. The obtained values are highlighted in **bold**. Although for this analysis, any sequence of primes can be chosen, and to count each term only once, we will do it in ascending order (7, 11, 13, 17, 19, 23, ..., 307).

1. Multiples that there are in the group $(7j + a)$: they are all included.
2. Multiples that there are in the group $(11j + b)$: not included those who are also $(7j + a)$.
3. Multiples that there are in the group $(13j + c)$: not included those who are also $(7j + a)$ or $(11j + b)$.

And so on until the group of prime 307.

The indicated percentages are relative to the total number of terms $(7j + a), (11j + b), (13j + c), (17j + d), \dots$

$$10^6 = (30n_1 + 10) = (30n_2 + 11) + (30n_3 + 29) \quad 33.333 \text{ pairs}$$

Highest prime to divide 997

Sequence A (30n₂ + 11)

Sequence B (30n₃ + 29)

Total number of terms	33.333
Multiples 7m, 11m,...	23.545
Primes greater than 10 ³	9.788
Number of terms (7j + a), (11j + b),...	23.548
Multiples (7j + a), (11j + b),...	16.448
Primes (7j + a), (11j + b),...	7.100

Total number of terms	33.333
Multiples 7m, 11m,...	23.548
Primes greater than 10 ³	9.785
Number of terms (7j + a), (11j + b),...	23.545
Multiples (7j + a), (11j + b),...	16.448
Primes (7j + a), (11j + b),...	7.097

P_{PPx} = Number of prime pairs being both greater than 10³ P_{PPx} = 9.788 – 7.100 = 9.785 – 7.097 = 2.688
 Not included the possible prime pairs in which one of them is lesser than 10³

k_{ax} = 0,70635...
 k_{jx} = 0,698488194 k_{jx} / k_{ax} = 0,988859927
 c_{jx} = 2,668143788
 k_{ox} = 0,584344256 k_{ox} / k_{ax} = 0,827264688

k_{bx} = 0,70644...
 k_{jx} = 0,698577192 k_{jx} / k_{bx} = 0,988859927
 c_{jx} = 2,668453186
 k_{ox} = 0,58441871 k_{ox} / k_{bx} = 0,827264687

Multiples (7j + a)	3.140	13,336 %	Multiples (31j + h)	451	1,915 %	Multiples (61j + i)	187	0,794 %
Multiples (11j + b)	1.767	7,504 %	Multiples (37j + i)	379	1,609 %	Multiples (67j + u)	178	0,756 %
Multiples (13j + c)	1.374	5,835 %	Multiples (41j + l)	327	1,389 %	Multiples (71j + v)	158	0,671 %
Multiples (17j + d)	1.010	4,289 %	Multiples (43j + o)	298	1,265 %	Multiples (73j + x)	157	0,666 %
Multiples (19j + e)	841	3,572 %	Multiples (47j + q)	271	1,151 %	Multiples (79j + y)	135	0,573 %
Multiples (23j + f)	659	2,799 %	Multiples (53j + r)	235	0,998 %	Multiples (83j + z)	126	0,535 %
Multiples (29j + g)	491	2,085 %	Multiples (59j + s)	208	0,883 %			
<hr/>								
Multiples group 89	121	0,514 %	Multiples group 163	69	0,293 %	Multiples group 239	49	0,208 %
Multiples group 97	108	0,459 %	Multiples group 167	70	0,297 %	Multiples group 241	54	0,229 %
Multiples group 101	108	0,459 %	Multiples group 173	71	0,301 %	Multiples group 251	47	0,2 %
Multiples group 103	111	0,471 %	Multiples group 179	62	0,263 %	Multiples group 257	48	0,204 %
Multiples group 107	96	0,408 %	Multiples group 181	61	0,259 %	Multiples group 263	43	0,183 %
Multiples group 109	103	0,437 %	Multiples group 191	59	0,251 %	Multiples group 269	41	0,174 %
Multiples group 113	92	0,391 %	Multiples group 193	61	0,259 %	Multiples group 271	43	0,183 %
Multiples group 127	84	0,357 %	Multiples group 197	55	0,234 %	Multiples group 277	43	0,183 %
Multiples group 131	83	0,352 %	Multiples group 199	57	0,242 %	Multiples group 281	40	0,17 %
Multiples group 137	80	0,34 %	Multiples group 211	59	0,251 %	Multiples group 283	38	0,161 %
Multiples group 139	78	0,331 %	Multiples group 223	47	0,2 %	Multiples group 293	42	0,178 %
Multiples group 149	67	0,284 %	Multiples group 227	52	0,221 %	Multiples group 307	35	0,149 %
Multiples group 151	76	0,323 %	Multiples group 229	43	0,183 %			
Multiples group 157	66	0,28 %	Multiples group 233	54	0,229 %	Total multiples in the groups 7 to 307	15.008	63,742 %

$$10^6 = (30n_1 + 10) = (30n_4 + 17) + (30n_5 + 23) \quad 33.333 \text{ pairs}$$

Highest prime to divide 997

Sequence A (30n₄ + 17)

Sequence B (30n₅ + 23)

Total number of terms	33.333
Multiples 7m, 11m,...	23.546
Primes greater than 10 ³	9.787
Number of terms (7j + a), (11j + b),...	23.514
Multiples (7j + a), (11j + b),...	16.421
Primes (7j + a), (11j + b),...	7.093

Total number of terms	33.333
Multiples 7m, 11m,...	23.514
Primes greater than 10 ³	9.819
Number of terms (7j + a), (11j + b),...	23.546
Multiples (7j + a), (11j + b),...	16.421
Primes (7j + a), (11j + b),...	7.125

P_{PPx} = Number of prime pairs being both greater than 10³ P_{PPx} = 9.787 – 7.093 = 9.819 – 7.125 = 2.694
 Not included the possible prime pairs in which one of them is lesser than 10³

k_{ax} = 0,70638...
 k_{jx} = 0,698349919 k_{jx} / k_{ax} = 0,98862218
 c_{jx} = 2,714499199
 k_{ox} = 0,583785776 k_{ox} / k_{ax} = 0,826438939

k_{bx} = 0,70542...
 k_{jx} = 0,697400832 k_{jx} / k_{bx} = 0,98862218
 c_{jx} = 2,711148007
 k_{ox} = 0,582992398 k_{ox} / k_{bx} = 0,826438955

$$10^7 = (30n_1 + 10) = (30n_2 + 11) + (30n_3 + 29) \quad 333.333 \text{ pairs}$$

Highest prime to divide 3.137

Sequence A (30n₂ + 11)

Sequence B (30n₃ + 29)

Total number of terms	333.333
Multiples 7m, 11m,...	250.287
Primes greater than 10 ^{3.5}	83.046
Number of terms (7j + a), (11j + b),...	250.369
Multiples (7j + a), (11j + b),...	186.636
Primes (7j + a), (11j + b),...	63.733

Total number of terms	333.333
Multiples 7m, 11m,...	250.369
Primes greater than 10 ^{3.5}	82.964
Number of terms (7j + a), (11j + b),...	250.287
Multiples (7j + a), (11j + b),...	186.636
Primes (7j + a), (11j + b),...	63.651

P_{PPx} = Number of prime pairs being both greater than 10^{3.5} P_{PPx} = 83.046 – 63.733 = 82.964 – 63.651 = 19.313
 Not included the possible prime pairs in which one of them is lesser than 10^{3.5}

$k_{ax} = 0,750861\dots$			$k_{bx} = 0,751107\dots$		
$k_{jx} = 0,745443725$	$k_{jx} / k_{ax} = 0,992784256$		$k_{jx} = 0,74568795$		$k_{jx} / k_{bx} = 0,992784256$
$c_{jx} = 2,565591768$			$c_{jx} = 2,56636034$		
$k_{0x} = 0,668306022$	$k_{0x} / k_{ax} = 0,890052025$		$k_{0x} = 0,668524975$		$k_{0x} / k_{bx} = 0,890052025$

Multiples $(7j + a)$	33.750	13,484 %	Multiples $(31j + h)$	4.843	1,935 %	Multiples $(61j + t)$	2.095	0,837 %
Multiples $(11j + b)$	19.045	7,609 %	Multiples $(37j + i)$	3.893	1,555 %	Multiples $(67j + u)$	1.862	0,744 %
Multiples $(13j + c)$	14.714	5,879 %	Multiples $(41j + l)$	3.441	1,375 %	Multiples $(71j + v)$	1.730	0,691 %
Multiples $(17j + d)$	10.549	4,215 %	Multiples $(43j + o)$	3.244	1,296 %	Multiples $(73j + x)$	1.680	0,671 %
Multiples $(19j + e)$	8.893	3,553 %	Multiples $(47j + q)$	2.876	1,149 %	Multiples $(79j + y)$	1.520	0,607 %
Multiples $(23j + f)$	6.985	2,791 %	Multiples $(53j + r)$	2.477	0,99 %	Multiples $(83j + z)$	1.425	0,569 %
Multiples $(29j + g)$	5.314	2,123 %	Multiples $(59j + s)$	2.192	0,876 %			

Multiples group 89	1.333	0,533 %	Multiples group 163	623	0,249 %	Multiples group 239	401	0,16 %
Multiples group 97	1.197	0,478 %	Multiples group 167	601	0,24 %	Multiples group 241	423	0,169 %
Multiples group 101	1.093	0,437 %	Multiples group 173	583	0,233 %	Multiples group 251	379	0,151 %
Multiples group 103	1.104	0,441 %	Multiples group 179	537	0,214 %	Multiples group 257	385	0,154 %
Multiples group 107	1.035	0,413 %	Multiples group 181	560	0,224 %	Multiples group 263	379	0,151 %
Multiples group 109	987	0,394 %	Multiples group 191	518	0,207 %	Multiples group 269	367	0,147 %
Multiples group 113	960	0,384 %	Multiples group 193	505	0,202 %	Multiples group 271	363	0,145 %
Multiples group 127	848	0,339 %	Multiples group 197	508	0,203 %	Multiples group 277	353	0,141 %
Multiples group 131	789	0,315 %	Multiples group 199	493	0,2 %	Multiples group 281	357	0,143 %
Multiples group 137	767	0,306 %	Multiples group 211	478	0,197 %	Multiples group 283	357	0,143 %
Multiples group 139	726	0,29 %	Multiples group 223	443	0,177 %	Multiples group 293	346	0,138 %
Multiples group 149	687	0,274 %	Multiples group 227	452	0,181 %	Multiples group 307	323	0,129 %
Multiples group 151	679	0,271 %	Multiples group 229	437	0,175 %			
Multiples group 157	636	0,254 %	Multiples group 233	416	0,166 %	Total multiples in the groups 7 to 307	156.956	62,71 %

$$10^7 = (30n_1 + 10) = (30n_4 + 17) + (30n_5 + 23) \quad 333.333 \text{ pairs}$$

Highest prime to divide 3.137

Sequence A $(30n_4 + 17)$

Sequence B $(30n_5 + 23)$

Total number of terms	333.333
Multiples $7m, 11m, \dots$	250.283
Primes greater than 3.162	83.050
Number of terms $(7j + a), (11j + b), \dots$	250.238
Multiples $(7j + a), (11j + b), \dots$	186.638
Primes $(7j + a), (11j + b), \dots$	63.600

Total number of terms	333.333
Multiples $7m, 11m, \dots$	250.238
Primes greater than 3.162	83.095
Number of terms $(7j + a), (11j + b), \dots$	250.283
Multiples $(7j + a), (11j + b), \dots$	186.638
Primes $(7j + a), (11j + b), \dots$	63.645

$P_{ppx} =$ Number of prime pairs being both greater than $10^{3.5}$

$$P_{ppx} = 83.050 - 63.600 = 83.095 - 63.645 = 19.450$$

Not included the possible prime pairs in which one of them is lesser than $10^{3.5}$

$k_{ax} = 0,750849\dots$			$k_{bx} = 0,750714\dots$		
$k_{jx} = 0,745841958$	$k_{jx} / k_{ax} = 0,9933305$		$k_{jx} = 0,745707858$		$k_{jx} / k_{bx} = 0,9933305$
$c_{jx} = 2,371314813$			$c_{jx} = 2,370922353$		
$k_{0x} = 0,668116395$	$k_{0x} / k_{ax} = 0,8898137$		$k_{0x} = 0,66799627$		$k_{0x} / k_{bx} = 0,8898137$

$$10^8 = (30n_1 + 10) = (30n_2 + 11) + (30n_3 + 29) \quad 3.333.333 \text{ pairs}$$

Highest prime to divide 9.973

Sequence A $(30n_2 + 11)$

Sequence B $(30n_3 + 29)$

Total number of terms	3.333.333
Multiples $7m, 11m, \dots$	2.613.173
Primes greater than 10^4	720.160
Number of terms $(7j + a), (11j + b), \dots$	2.613.453
Multiples $(7j + a), (11j + b), \dots$	2.039.019
Primes $(7j + a), (11j + b), \dots$	574.434

Total number of terms	3.333.333
Multiples $7m, 11m, \dots$	2.613.453
Primes greater than 10^4	719.880
Number of terms $(7j + a), (11j + b), \dots$	2.613.173
Multiples $(7j + a), (11j + b), \dots$	2.039.019
Primes $(7j + a), (11j + b), \dots$	574.154

$P_{ppx} =$ Number of prime pairs being both greater than 10^4

$$P_{ppx} = 720.160 - 574.434 = 719.880 - 574.154 = 145.726$$

Not included the possible prime pairs in which one of them is lesser than 10^4

$k_{ax} = 0,7839519\dots$			$k_{bx} = 0,7840359\dots$		
$k_{jx} = 0,780201136$	$k_{jx} / k_{ax} = 0,995215469$		$k_{jx} = 0,780284734$		$k_{jx} / k_{bx} = 0,995215469$
$c_{jx} = 2,370531694$			$c_{jx} = 2,370765683$		
$k_{0x} = 0,724441224$	$k_{0x} / k_{ax} = 0,924088775$		$k_{0x} = 0,724518848$		$k_{0x} / k_{bx} = 0,924088776$

Multiples $(7j + a)$	356.077	13,626 %	Multiples $(31j + h)$	50.268	1,924 %	Multiples $(61j + t)$	21.820	0,835 %
Multiples $(11j + b)$	199.519	7,635 %	Multiples $(37j + i)$	40.841	1,563 %	Multiples $(67j + u)$	19.527	0,747 %
Multiples $(13j + c)$	154.831	5,925 %	Multiples $(41j + l)$	35.803	1,37 %	Multiples $(71j + v)$	18.162	0,695 %
Multiples $(17j + d)$	110.081	4,212 %	Multiples $(43j + o)$	33.363	1,277 %	Multiples $(73j + x)$	17.423	0,667 %
Multiples $(19j + e)$	92.988	3,558 %	Multiples $(47j + q)$	29.791	1,14 %	Multiples $(79j + y)$	15.842	0,606 %
Multiples $(23j + f)$	73.084	2,797 %	Multiples $(53j + r)$	25.885	0,991 %	Multiples $(83j + z)$	14.889	0,57 %
Multiples $(29j + g)$	55.555	2,126 %	Multiples $(59j + s)$	22.922	0,877 %			

Multiples group 89	13.782	0,527 %	Multiples group 163	6.608	0,253 %	Multiples group 239	4.042	0,155 %
Multiples group 97	12.476	0,477 %	Multiples group 167	6.389	0,244 %	Multiples group 241	4.019	0,154 %
Multiples group 101	11.884	0,455 %	Multiples group 173	6.100	0,233 %	Multiples group 251	3.850	0,147 %
Multiples group 103	11.554	0,442 %	Multiples group 179	5.894	0,226 %	Multiples group 257	3.692	0,141 %
Multiples group 107	11.022	0,422 %	Multiples group 181	5.734	0,219 %	Multiples group 263	3.623	0,139 %
Multiples group 109	10.684	0,409 %	Multiples group 191	5.488	0,21 %	Multiples group 269	3.523	0,135 %
Multiples group 113	10.163	0,389 %	Multiples group 193	5.313	0,203 %	Multiples group 271	3.495	0,134 %
Multiples group 127	8.942	0,342 %	Multiples group 197	5.207	0,199 %	Multiples group 277	3.392	0,13 %
Multiples group 131	8.636	0,33 %	Multiples group 199	5.122	0,196 %	Multiples group 281	3.335	0,128 %
Multiples group 137	8.227	0,315 %	Multiples group 211	4.761	0,182 %	Multiples group 283	3.305	0,126 %
Multiples group 139	7.968	0,305 %	Multiples group 223	4.474	0,171 %	Multiples group 293	3.187	0,122 %
Multiples group 149	7.392	0,283 %	Multiples group 227	4.407	0,169 %	Multiples group 307	3.014	0,115 %
Multiples group 151	7.240	0,277 %	Multiples group 229	4.309	0,165 %			
Multiples group 157	6.897	0,264 %	Multiples group 233	4.240	0,162 %	Total multiples in the groups 7 to 307	1.642.061	62,838 %

$$10^8 = (30n_1 + 10) = (30n_4 + 17) + (30n_5 + 23) \quad 3.333.333 \text{ pairs}$$

Highest prime to divide 9.973

Sequence A $(30n_4 + 17)$

Sequence B $(30n_5 + 23)$

Total number of terms	3.333.333
Multiples $7m, 11m, \dots$	2.613.261
Primes greater than 10^4	720.072
Number of terms $(7j + a), (11j + b), \dots$	2.613.125
Multiples $(7j + a), (11j + b), \dots$	2.038.608
Primes $(7j + a), (11j + b), \dots$	574.517

Total number of terms	3.333.333
Multiples $7m, 11m, \dots$	2.613.125
Primes greater than 10^4	720.208
Number of terms $(7j + a), (11j + b), \dots$	2.613.261
Multiples $(7j + a), (11j + b), \dots$	2.038.608
Primes $(7j + a), (11j + b), \dots$	574.653

$P_{PPx} =$ Number of prime pairs being both greater than 10^4 $P_{PPx} = 720.072 - 574.517 = 720.208 - 574.653 = 145.555$
Not included the possible prime pairs in which one of them is lesser than 10^4

$k_{ax} = 0,7839783\dots$		$k_{bx} = 0,7839375\dots$	
$k_{jx} = 0,780141784$	$k_{jx} / k_{ax} = 0,99510625$	$k_{jx} = 0,780101183$	$k_{jx} / k_{bx} = 0,99510625$
$c_{jx} = 2,42296808$		$c_{jx} = 2,42285258$	
$k_{0x} = 0,724440312$	$k_{0x} / k_{ax} = 0,924056494$	$k_{0x} = 0,724402611$	$k_{0x} / k_{bx} = 0,924056495$

$$10^9 = (30n_1 + 10) = (30n_2 + 11) + (30n_3 + 29) \quad 33.333.333 \text{ pairs}$$

Highest prime to divide 31.607 square root 31.622 50.847.534 primes lesser than 10^9

Sequence A $(30n_2 + 11)$

Sequence B $(30n_3 + 29)$

Total number of terms	33.333.333
Multiples $7m, 11m, \dots$	26.977.564
Primes greater than $10^{4.5}$	6.355.769
Number of terms $(7j + a), (11j + b), \dots$	26.977.414
Multiples $(7j + a), (11j + b), \dots$	21.758.538
Primes $(7j + a), (11j + b), \dots$	5.218.876

Total number of terms	33.333.333
Multiples $7m, 11m, \dots$	26.977.414
Primes greater than $10^{4.5}$	6.355.919
Number of terms $(7j + a), (11j + b), \dots$	26.977.564
Multiples $(7j + a), (11j + b), \dots$	21.758.538
Primes $(7j + a), (11j + b), \dots$	5.219.026

$P_{PPx} =$ Number of prime pairs being both greater than $10^{4.5}$ $P_{PPx} = 6.355.769 - 5.218.876 = 6.355.919 - 5.219.026 = 1.136.893$
Not included the possible prime pairs in which one of them is lesser than $10^{4.5}$

$k_{ax} = 0,809326928$		$k_{bx} = 0,809322428$	
$k_{jx} = 0,806546468$	$k_{jx} / k_{ax} = 0,996564479$	$k_{jx} = 0,806541984$	$k_{jx} / k_{bx} = 0,996564479$
$c_{jx} = 2,261319599$		$c_{jx} = 2,26130754$	
$k_{0x} = 0,76440407$	$k_{0x} / k_{ax} = 0,944493559$	$k_{0x} = 0,76439982$	$k_{0x} / k_{bx} = 0,944493559$

Multiples $(7j + a)$	3.702.786	13,725 %	Multiples $(31j + h)$	517.983	1,92 %	Multiples $(61j + t)$	224.011	0,83 %
Multiples $(11j + b)$	2.067.520	7,664 %	Multiples $(37j + i)$	420.774	1,56 %	Multiples $(67j + u)$	200.865	0,745 %
Multiples $(13j + c)$	1.600.628	5,933 %	Multiples $(41j + l)$	369.785	1,371 %	Multiples $(71j + v)$	186.614	0,692 %
Multiples $(17j + d)$	1.137.457	4,216 %	Multiples $(43j + o)$	344.202	1,276 %	Multiples $(73j + x)$	178.908	0,663 %
Multiples $(19j + e)$	959.914	3,558 %	Multiples $(47j + q)$	307.696	1,141 %	Multiples $(79j + y)$	163.259	0,605 %
Multiples $(23j + f)$	753.704	2,794 %	Multiples $(53j + r)$	267.233	0,991 %	Multiples $(83j + z)$	153.399	0,569 %
Multiples $(29j + g)$	573.050	2,124 %	Multiples $(59j + s)$	235.662	0,874 %			

Multiples group 89	141.414	0,524 %	Multiples group 163	68.943	0,256 %	Multiples group 239	43.564	0,161 %
Multiples group 97	128.131	0,475 %	Multiples group 167	66.855	0,248 %	Multiples group 241	43.153	0,16 %
Multiples group 101	122.098	0,453 %	Multiples group 173	64.156	0,238 %	Multiples group 251	41.212	0,153 %
Multiples group 103	118.537	0,439 %	Multiples group 179	61.727	0,229 %	Multiples group 257	39.983	0,148 %
Multiples group 107	112.655	0,418 %	Multiples group 181	60.797	0,225 %	Multiples group 263	38.915	0,144 %
Multiples group 109	110.000	0,408 %	Multiples group 191	57.203	0,212 %	Multiples group 269	37.905	0,14 %
Multiples group 113	105.034	0,389 %	Multiples group 193	56.272	0,209 %	Multiples group 271	37.475	0,139 %
Multiples group 127	92.824	0,344 %	Multiples group 197	54.872	0,203 %	Multiples group 277	36.498	0,135 %
Multiples group 131	89.279	0,331 %	Multiples group 199	54.100	0,2 %	Multiples group 281	35.761	0,133 %
Multiples group 137	84.745	0,314 %	Multiples group 211	50.725	0,188 %	Multiples group 283	35.390	0,131 %
Multiples group 139	82.963	0,308 %	Multiples group 223	47.673	0,177 %	Multiples group 293	34.012	0,126 %
Multiples group 149	76.942	0,285 %	Multiples group 227	46.648	0,173 %	Multiples group 307	32.328	0,12 %
Multiples group 151	75.319	0,279 %	Multiples group 229	46.141	0,171 %			
Multiples group 157	71.971	0,267 %	Multiples group 233	44.870	0,166 %	Total multiples in the groups 7 to 307	17.014.540	63,069 %

$10^9 = (30n_1 + 10) = (30n_4 + 17) + (30n_5 + 23)$ 33.333.333 pairs Highest prime to divide 31.607 square root 31.622 50.847.534 primes lesser than 10^9

Sequence A $(30n_4 + 17)$

Total number of terms 33.333.333
 Multiples $7m, 11m, \dots$ **26.977.923**
 Primes greater than $10^{4.5}$ **6.355.410**
 Number of terms $(7j + a), (11j + b), \dots$ 26.977.320
 Multiples $(7j + a), (11j + b), \dots$ **21.758.935**
 Primes $(7j + a), (11j + b), \dots$ **5.218.385**

Sequence B $(30n_5 + 23)$

Total number of terms 33.333.333
 Multiples $7m, 11m, \dots$ **26.977.320**
 Primes greater than $10^{4.5}$ **6.356.013**
 Number of terms $(7j + a), (11j + b), \dots$ 26.977.923
 Multiples $(7j + a), (11j + b), \dots$ **21.758.935**
 Primes $(7j + a), (11j + b), \dots$ **5.218.988**

$P_{ppx} =$ Number of prime pairs being both greater than $10^{4.5}$ $P_{ppx} = 6.355.410 - 5.218.385 = 6.356.013 - 5.218.988 = 1.137.025$
 Not included the possible prime pairs in which one of them is lesser than $10^{4.5}$

$k_{ax} = 0,809337698$		$k_{bx} = 0,809319608$	
$k_{jx} = 0,806563995$	$k_{jx} / k_{ax} = 0,996572873$	$k_{jx} = 0,806545967$	$k_{jx} / k_{bx} = 0,996572873$
$c_{jx} = 2,255995237$		$c_{jx} = 2,255948601$	
$k_{0x} = 0,764416557$	$k_{0x} / k_{ax} = 0,944496418$	$k_{0x} = 0,764399471$	$k_{0x} / k_{bx} = 0,944496418$

$8.000.000 = 8 \cdot 10^6 = (30n_1 + 20) = (30n_2 + 1) + (30n_3 + 19)$ 266.667 pairs

Highest prime to divide 2.819

Sequence A $(30n_2 + 1)$ (number 1 is present)

Total number of terms 266.667
 Multiples $7m, 11m, \dots$ **199.312+1**
 Primes greater than 2.828 **67.354**
 Number of terms $(7j + a), (11j + b), \dots$ 199.314
 Multiples $(7j + a), (11j + b), \dots$ **147.802+1**
 Primes $(7j + a), (11j + b), \dots$ **51.511**

Sequence B $(30n_3 + 19)$

Total number of terms 266.667
 Multiples $7m, 11m, \dots$ **199.314**
 Primes greater than 2.828 **67.353**
 Number of terms $(7j + a), (11j + b), \dots$ 199.312
 Multiples $(7j + a), (11j + b), \dots$ **147.802**
 Primes $(7j + a), (11j + b), \dots$ **51.510**

$P_{ppx} =$ Number of prime pairs being both greater than 2.828 $P_{ppx} = 67.354 - 51.511 = 67.353 - 51.510 = 15.843$
 Not included the possible prime pairs in which one of them is lesser than 2.828

$k_{ax} = 0,747419065$		$k_{bx} = 0,747426565$	
$k_{jx} = 0,741553528$	$k_{jx} / k_{ax} = 0,992152277$	$k_{jx} = 0,741560969$	$k_{jx} / k_{bx} = 0,992152277$
$c_{jx} = 2,69727031$		$c_{jx} = 2,695611584$	
$k_{0x} = 0,662070338$	$k_{0x} / k_{ax} = 0,885808736$	$k_{0x} = 0,662073659$	$k_{0x} / k_{bx} = 0,88580429$

$8.000.000 = 8 \cdot 10^6 = (30n_1 + 20) = (30n_4 + 7) + (30n_5 + 13)$ 266.667 pairs

Highest prime to divide 2.819

Sequence A $(30n_4 + 7)$

Total number of terms 266.667
 Multiples $7m, 11m, \dots$ **199.224**
 Primes greater than 2.828 **67.443**
 Number of terms $(7j + a), (11j + b), \dots$ 199.241
 Multiples $(7j + a), (11j + b), \dots$ **147.663**
 Primes $(7j + a), (11j + b), \dots$ **51.578**

Sequence B $(30n_5 + 13)$

Total number of terms 266.667
 Multiples $7m, 11m, \dots$ **199.241**
 Primes greater than 2.828 **67.426**
 Number of terms $(7j + a), (11j + b), \dots$ 199.224
 Multiples $(7j + a), (11j + b), \dots$ **147.663**
 Primes $(7j + a), (11j + b), \dots$ **51.561**

$P_{ppx} =$ Number of prime pairs being both greater than 2.828 $P_{ppx} = 67.443 - 51.578 = 67.426 - 51.561 = 15.865$
 Not included the possible prime pairs in which one of them is lesser than 2.828

$k_{ax} = 0,747089066$		$k_{bx} = 0,747152816$	
$k_{jx} = 0,741127579$	$k_{jx} / k_{ax} = 0,992020379$	$k_{jx} = 0,74119082$	$k_{jx} / k_{bx} = 0,992020379$
$c_{jx} = 2,732174197$		$c_{jx} = 2,732386249$	
$k_{0x} = 0,661499827$	$k_{0x} / k_{ax} = 0,885436364$	$k_{0x} = 0,661556274$	$k_{0x} / k_{bx} = 0,885436365$

$80.000.000 = 8 \cdot 10^7 = (30n_1 + 20) = (30n_2 + 1) + (30n_3 + 19)$ 2.666.667 pairs

Highest prime to divide 8.941

Sequence A $(30n_2 + 1)$ (number 1 is present)

Total number of terms 2.666.667
 Multiples $7m, 11m, \dots$ **2.083.272+1**
 Primes greater than 8.944 **583.394**
 Number of terms $(7j + a), (11j + b), \dots$ 2.083.493
 Multiples $(7j + a), (11j + b), \dots$ **1.619.431+1**
 Primes $(7j + a), (11j + b), \dots$ **464.061**

Sequence B $(30n_3 + 19)$

Total number of terms 2.666.667
 Multiples $7m, 11m, \dots$ **2.083.493**
 Primes greater than 8.944 **583.174**
 Number of terms $(7j + a), (11j + b), \dots$ 2.083.272
 Multiples $(7j + a), (11j + b), \dots$ **1.619.431**
 Primes $(7j + a), (11j + b), \dots$ **463.841**

$P_{ppx} =$ Number of prime pairs being both greater than 8.944 $P_{ppx} = 583.394 - 464.061 = 583.174 - 463.841 = 119.333$
 Not included the possible prime pairs in which one of them is lesser than 8.944

$k_{ax} = 0,781226902$		$k_{bx} = 0,781309777$	
$k_{jx} = 0,77726731$	$k_{jx} / k_{ax} = 0,994931572$	$k_{jx} = 0,777349765$	$k_{jx} / k_{bx} = 0,994931572$
$c_{jx} = 2,438913946$		$c_{jx} = 2,438924811$	
$k_{0x} = 0,719992295$	$k_{0x} / k_{ax} = 0,921617385$	$k_{0x} = 0,720068328$	$k_{0x} / k_{bx} = 0,921616942$

$$80.000.000 = 8 \cdot 10^7 = (30n_1 + 20) = (30n_4 + 7) + (30n_5 + 13) \quad 2.666.667 \text{ pairs} \quad \text{Highest prime to divide } 8.941$$

Sequence A $(30n_4 + 7)$

Total number of terms	2.666.667
Multiples $7m, 11m, \dots$	2.083.064
Primes greater than 8.944	583.603
Number of terms $(7j + a), (11j + b), \dots$	2.083.030
Multiples $(7j + a), (11j + b), \dots$	1.619.203
Primes $(7j + a), (11j + b), \dots$	463.827

Sequence B $(30n_5 + 13)$

Total number of terms	2.666.667
Multiples $7m, 11m, \dots$	2.083.030
Primes greater than 8.944	583.637
Number of terms $(7j + a), (11j + b), \dots$	2.083.064
Multiples $(7j + a), (11j + b), \dots$	1.619.203
Primes $(7j + a), (11j + b), \dots$	463.861

$$P_{PPx} = \text{Number of prime pairs being both greater than } 8.944 \quad P_{PPx} = 583.603 - 463.827 = 583.637 - 463.861 = 119.776$$

Not included the possible prime pairs in which one of them is lesser than 8.944

$k_{ax} = 0,781148902$		$k_{bx} = 0,781136152$	
$k_{jx} = 0,777330619$	$k_{jx} / k_{ax} = 0,995111965$	$k_{jx} = 0,777317931$	$k_{jx} / k_{bx} = 0,995111965$
$c_{jx} = 2,350453759$		$c_{jx} = 2,350418595$	
$k_{0x} = 0,719829722$	$k_{0x} / k_{ax} = 0,921501291$	$k_{0x} = 0,719817973$	$k_{0x} / k_{bx} = 0,921501291$

$$9.000.000 = 9 \cdot 10^6 = (30n_1 + 30) = (30n_2 + 1) + (30n_3 + 29) \quad 300.000 \text{ pairs} \quad \text{Highest prime to divide } 2.999$$

Sequence A $(30n_2 + 1)$ (number 1 is present)

Total number of terms	300.000
Multiples $7m, 11m, \dots$	224.798+1
Primes greater than 3.000	75.201
Number of terms $(7j + a), (11j + b), \dots$	224.776
Multiples $(7j + a), (11j + b), \dots$	167.109+1
Primes $(7j + a), (11j + b), \dots$	57.666

Sequence B $(30n_3 + 29)$

Total number of terms	300.000
Multiples $7m, 11m, \dots$	224.776
Primes greater than 3.000	75.224
Number of terms $(7j + a), (11j + b), \dots$	224.798
Multiples $(7j + a), (11j + b), \dots$	167.109
Primes $(7j + a), (11j + b), \dots$	57.689

$$P_{PPx} = \text{Number of prime pairs being both greater than } 3.000 \quad P_{PPx} = 75.201 - 57.666 = 75.224 - 57.689 = 17.535$$

Not included the possible prime pairs in which one of them is lesser than 3.000

$k_{ax} = 0,749326666$		$k_{bx} = 0,749253333$	
$k_{jx} = 0,743446809$	$k_{jx} / k_{ax} = 0,992153145$	$k_{jx} = 0,743374051$	$k_{jx} / k_{bx} = 0,992153145$
$c_{jx} = 2,743605177$		$c_{jx} = 2,741843369$	
$k_{0x} = 0,66544026$	$k_{0x} / k_{ax} = 0,888050952$	$k_{0x} = 0,665372176$	$k_{0x} / k_{bx} = 0,888047001$

$$9.000.000 = 9 \cdot 10^6 = (30n_1 + 30) = (30n_4 + 7) + (30n_5 + 23) \quad 300.000 \text{ pairs} \quad \text{Highest prime to divide } 2.999$$

Sequence A $(30n_4 + 7)$

Total number of terms	300.000
Multiples $7m, 11m, \dots$	224.761
Primes greater than 3.000	75.239
Number of terms $(7j + a), (11j + b), \dots$	224.699
Multiples $(7j + a), (11j + b), \dots$	167.190
Primes $(7j + a), (11j + b), \dots$	57.509

Sequence B $(30n_5 + 23)$

Total number of terms	300.000
Multiples $7m, 11m, \dots$	224.699
Primes greater than 3.000	75.301
Number of terms $(7j + a), (11j + b), \dots$	224.761
Multiples $(7j + a), (11j + b), \dots$	167.190
Primes $(7j + a), (11j + b), \dots$	57.571

$$P_{PPx} = \text{Number of prime pairs being both greater than } 3.000 \quad P_{PPx} = 75.239 - 57.509 = 75.301 - 57.571 = 17.730$$

Not included the possible prime pairs in which one of them is lesser than 3.000

$k_{ax} = 0,749203333$		$k_{bx} = 0,748996666$	
$k_{jx} = 0,744062056$	$k_{jx} / k_{ax} = 0,993137674$	$k_{jx} = 0,743856807$	$k_{jx} / k_{bx} = 0,993137674$
$c_{jx} = 2,400922383$		$c_{jx} = 2,400313332$	
$k_{0x} = 0,665156498$	$k_{0x} / k_{ax} = 0,887818391$	$k_{0x} = 0,664973015$	$k_{0x} / k_{bx} = 0,88781839$

$$9.000.000 = 9 \cdot 10^6 = (30n_1 + 30) = (30n_6 + 11) + (30n_7 + 19) \quad 300.000 \text{ pairs} \quad \text{Highest prime to divide } 2.999$$

Sequence A $(30n_6 + 11)$

Total number of terms	300.000
Multiples $7m, 11m, \dots$	224.700
Primes greater than 3.000	75.300
Number of terms $(7j + a), (11j + b), \dots$	224.761
Multiples $(7j + a), (11j + b), \dots$	167.085
Primes $(7j + a), (11j + b), \dots$	57.676

Sequence B $(30n_7 + 19)$

Total number of terms	300.000
Multiples $7m, 11m, \dots$	224.761
Primes greater than 3.000	75.239
Number of terms $(7j + a), (11j + b), \dots$	224.700
Multiples $(7j + a), (11j + b), \dots$	167.085
Primes $(7j + a), (11j + b), \dots$	57.615

$$P_{PPx} = \text{Number of prime pairs being both greater than } 3.000 \quad P_{PPx} = 75.300 - 57.676 = 75.239 - 57.615 = 17.624$$

Not included the possible prime pairs in which one of them is lesser than 3.000

$k_{ax} = 0,749$		$k_{bx} = 0,749203333$	
$k_{jx} = 0,743389645$	$k_{jx} / k_{ax} = 0,992509539$	$k_{jx} = 0,743591455$	$k_{jx} / k_{bx} = 0,992509539$
$c_{jx} = 2,615264747$		$c_{jx} = 2,615912938$	
$k_{0x} = 0,664977465$	$k_{0x} / k_{ax} = 0,88782038$	$k_{0x} = 0,665157988$	$k_{0x} / k_{bx} = 0,887820379$

$$9.000.000 = 9 \cdot 10^6 = (30n_1 + 30) = (30n_8 + 13) + (30n_9 + 17) \quad 300.000 \text{ pairs} \quad \text{Highest prime to divide } 2.999$$

Sequence A $(30n_8 + 13)$

Total number of terms	300.000
Multiples $7m, 11m, \dots$	224.749
Primes greater than 3.000	75.251
Number of terms $(7j + a), (11j + b), \dots$	224.696
Multiples $(7j + a), (11j + b), \dots$	167.079
Primes $(7j + a), (11j + b), \dots$	57.617

Sequence B $(30n_9 + 17)$

Total number of terms	300.000
Multiples $7m, 11m, \dots$	224.696
Primes greater than 3.000	75.304
Number of terms $(7j + a), (11j + b), \dots$	224.749
Multiples $(7j + a), (11j + b), \dots$	167.079
Primes $(7j + a), (11j + b), \dots$	57.670

$$P_{PPx} = \text{Number of prime pairs being both greater than 3.000} \quad P_{PPx} = 75.251 - 57.617 = 75.304 - 57.670 = 17.634$$

Not included the possible prime pairs in which one of them is lesser than 3.000

$k_{ax} = 0,749163333$		$k_{bx} = 0,748986666$	
$k_{jx} = 0,743577989$	$k_{jx} / k_{ax} = 0,992544558$	$k_{jx} = 0,74340264$	$k_{jx} / k_{bx} = 0,992544558$
$c_{jx} = 2,603269404$		$c_{jx} = 2,602708519$	
$k_{0x} = 0,665098622$	$k_{0x} / k_{ax} = 0,88778854$	$k_{0x} = 0,664941779$	$k_{0x} / k_{bx} = 0,887788539$

$$90.000.000 = 9 \cdot 10^7 = (30n_1 + 30) = (30n_2 + 1) + (30n_3 + 29) \quad 3.000.000 \text{ pairs} \quad \text{Highest prime to divide } 9.479$$

Sequence A $(30n_2 + 1)$ (number 1 is present)

Total number of terms	3.000.000
Multiples $7m, 11m, \dots$	2.348.173+1
Primes greater than 9.486	651.827-1
Number of terms $(7j + a), (11j + b), \dots$	2.347.976
Multiples $(7j + a), (11j + b), \dots$	1.829.138
Primes $(7j + a), (11j + b), \dots$	518.838

Sequence B $(30n_3 + 29)$

Total number of terms	3.000.000
Multiples $7m, 11m, \dots$	2.347.976
Primes greater than 9.486	652.024
Number of terms $(7j + a), (11j + b), \dots$	2.348.173
Multiples $(7j + a), (11j + b), \dots$	1.829.138
Primes $(7j + a), (11j + b), \dots$	519.035

$$P_{PPx} = \text{Number of prime pairs being both greater than 9.486} \quad P_{PPx} = 651.827 - 518.838 = 652.024 - 519.035 = 132.989 \quad (-1)$$

Not included the possible prime pairs in which one of them is lesser than 9.486

$k_{ax} = 0,782724333$		$k_{bx} = 0,782658666$	
$k_{jx} = 0,779027553$	$k_{jx} / k_{ax} = 0,995277035$	$k_{jx} = 0,778962197$	$k_{jx} / k_{bx} = 0,995277035$
$c_{jx} = 2,309215414$		$c_{jx} = 2,309036271$	
$k_{0x} = 0,722387707$	$k_{0x} / k_{ax} = 0,922914589$	$k_{0x} = 0,722327102$	$k_{0x} / k_{bx} = 0,922914589$

$$90.000.000 = 9 \cdot 10^7 = (30n_1 + 30) = (30n_4 + 7) + (30n_5 + 23) \quad 3.000.000 \text{ pairs} \quad \text{Highest prime to divide } 9.479$$

Sequence A $(30n_4 + 7)$

Total number of terms	3.000.000
Multiples $7m, 11m, \dots$	2.348.028
Primes greater than 9.486	651.972
Number of terms $(7j + a), (11j + b), \dots$	2.347.948
Multiples $(7j + a), (11j + b), \dots$	1.829.330
Primes $(7j + a), (11j + b), \dots$	518.618

Sequence B $(30n_5 + 23)$

Total number of terms	3.000.000
Multiples $7m, 11m, \dots$	2.347.948
Primes greater than 9.486	652.052
Number of terms $(7j + a), (11j + b), \dots$	2.348.028
Multiples $(7j + a), (11j + b), \dots$	1.829.330
Primes $(7j + a), (11j + b), \dots$	518.698

$$P_{PPx} = \text{Number of prime pairs being both greater than 9.486} \quad P_{PPx} = 651.972 - 518.618 = 652.052 - 518.698 = 133.354$$

Not included the possible prime pairs in which one of them is lesser than 9.486

$k_{ax} = 0,782676$		$k_{bx} = 0,782649333$	
$k_{jx} = 0,779118617$	$k_{jx} / k_{ax} = 0,995454846$	$k_{jx} = 0,779092072$	$k_{jx} / k_{bx} = 0,995454846$
$c_{jx} = 2,222960729$		$c_{jx} = 2,222890318$	
$k_{0x} = 0,72232264$	$k_{0x} / k_{ax} = 0,922888449$	$k_{0x} = 0,72229803$	$k_{0x} / k_{bx} = 0,922888449$

$$90.000.000 = 9 \cdot 10^7 = (30n_1 + 30) = (30n_6 + 11) + (30n_7 + 19) \quad 3.000.000 \text{ pairs} \quad \text{Highest prime to divide } 9.479$$

Sequence A $(30n_6 + 11)$

Total number of terms	3.000.000
Multiples $7m, 11m, \dots$	2.347.956
Primes greater than 9.486	652.044
Number of terms $(7j + a), (11j + b), \dots$	2.348.253
Multiples $(7j + a), (11j + b), \dots$	1.828.760
Primes $(7j + a), (11j + b), \dots$	519.493

Sequence B $(30n_7 + 19)$

Total number of terms	3.000.000
Multiples $7m, 11m, \dots$	2.348.253
Primes greater than 9.486	651.747
Number of terms $(7j + a), (11j + b), \dots$	2.347.956
Multiples $(7j + a), (11j + b), \dots$	1.828.760
Primes $(7j + a), (11j + b), \dots$	519.196

$$P_{PPx} = \text{Number of prime pairs being both greater than 9.486} \quad P_{PPx} = 652.044 - 519.493 = 651.747 - 519.196 = 132.551$$

Not included the possible prime pairs in which one of them is lesser than 9.486

$k_{ax} = 0,782652$		$k_{bx} = 0,782751$	
$k_{jx} = 0,778774689$	$k_{jx} / k_{ax} = 0,995045932$	$k_{jx} = 0,778873198$	$k_{jx} / k_{bx} = 0,995045932$
$c_{jx} = 2,420244777$		$c_{jx} = 2,420526556$	
$k_{0x} = 0,722328045$	$k_{0x} / k_{ax} = 0,922923655$	$k_{0x} = 0,722419415$	$k_{0x} / k_{bx} = 0,922923656$

$$90.000.000 = 9 \cdot 10^7 = (30n_1 + 30) = (30n_8 + 13) + (30n_9 + 17) \quad 3.000.000 \text{ pairs} \quad \text{Highest prime to divide } 9.479$$

Sequence A $(30n_8 + 13)$

Total number of terms	3.000.000
Multiples $7m, 11m, \dots$	2.347.924
Primes greater than 9.486	652.076
Number of terms $(7j + a), (11j + b), \dots$	2.347.962
Multiples $(7j + a), (11j + b), \dots$	1.828.299
Primes $(7j + a), (11j + b), \dots$	519.663

Sequence B $(30n_9 + 17)$

Total number of terms	3.000.000
Multiples $7m, 11m, \dots$	2.347.962
Primes greater than 9.486	652.038
Number of terms $(7j + a), (11j + b), \dots$	2.347.924
Multiples $(7j + a), (11j + b), \dots$	1.828.299
Primes $(7j + a), (11j + b), \dots$	519.625

$$P_{PPx} = \text{Number of prime pairs being both greater than 9.486} \quad P_{PPx} = 652.076 - 519.663 = 652.038 - 519.625 = 132.413$$

Not included the possible prime pairs in which one of them is lesser than 9.486

$k_{ax} = 0,782641333$		$k_{bx} = 0,782654$	
$k_{jx} = 0,778674867$	$k_{jx} / k_{ax} = 0,994931949$	$k_{jx} = 0,77868747$	$k_{jx} / k_{bx} = 0,994931949$
$c_{jx} = 2,473675008$		$c_{jx} = 2,473711418$	
$k_{0x} = 0,722280002$	$k_{0x} / k_{ax} = 0,922874848$	$k_{0x} = 0,722291692$	$k_{0x} / k_{bx} = 0,922874848$

$$300.000.000 = 3 \cdot 10^8 = (30n_1 + 30) = (30n_2 + 1) + (30n_3 + 29) \quad 10^7 \text{ pairs} \quad \text{Highest prime to divide } 17.317$$

Sequence A $(30n_2 + 1)$ (number 1 is present)

Total number of terms	10.000.000
Multiples $7m, 11m, \dots$	7.969.053+1
Primes greater than 17.320	2.030.946
Number of terms $(7j + a), (11j + b), \dots$	7.968.588
Multiples $(7j + a), (11j + b), \dots$	6.324.515+1
Primes $(7j + a), (11j + b), \dots$	1.644.072

Sequence B $(30n_3 + 29)$

Total number of terms	10.000.000
Multiples $7m, 11m, \dots$	7.968.588
Primes greater than 17.320	2.031.412
Number of terms $(7j + a), (11j + b), \dots$	7.969.053
Multiples $(7j + a), (11j + b), \dots$	6.324.515
Primes $(7j + a), (11j + b), \dots$	1.644.538

$$P_{PPx} = \text{Number of prime pairs being both greater than 17.320} \quad P_{PPx} = 2.030.946 - 1.644.072 = 2.031.412 - 1.644.538 = 386.874$$

Not included the possible prime pairs in which one of them is lesser than 17.320

$k_{ax} = 0,7969053$		$k_{bx} = 0,7968588$	
$k_{jx} = 0,793680762$	$k_{jx} / k_{ax} = 0,995953675$	$k_{jx} = 0,79363445$	$k_{jx} / k_{bx} = 0,995953675$
$c_{jx} = 2,308152416$		$c_{jx} = 2,307957765$	
$k_{0x} = 0,745131006$	$k_{0x} / k_{ax} = 0,935030807$	$k_{0x} = 0,745087434$	$k_{0x} / k_{bx} = 0,93503069$

$$300.000.000 = 3 \cdot 10^8 = (30n_1 + 30) = (30n_4 + 7) + (30n_5 + 23) \quad 10^7 \text{ pairs} \quad \text{Highest prime to divide } 17.317$$

Sequence A $(30n_4 + 7)$

Total number of terms	10.000.000
Multiples $7m, 11m, \dots$	7.968.657
Primes greater than 17.320	2.031.343
Number of terms $(7j + a), (11j + b), \dots$	7.968.695
Multiples $(7j + a), (11j + b), \dots$	6.323.804
Primes $(7j + a), (11j + b), \dots$	1.644.891

Sequence B $(30n_5 + 23)$

Total number of terms	10.000.000
Multiples $7m, 11m, \dots$	7.968.695
Primes greater than 17.320	2.031.305
Number of terms $(7j + a), (11j + b), \dots$	7.968.657
Multiples $(7j + a), (11j + b), \dots$	6.323.804
Primes $(7j + a), (11j + b), \dots$	1.644.853

$$P_{PPx} = \text{Number of prime pairs being both greater than 17.320} \quad P_{PPx} = 2.031.343 - 1.644.891 = 2.031.305 - 1.644.853 = 386.452$$

Not included the possible prime pairs in which one of them is lesser than 17.320

$k_{ax} = 0,7968657$		$k_{bx} = 0,7968695$	
$k_{jx} = 0,793580881$	$k_{jx} / k_{ax} = 0,995877826$	$k_{jx} = 0,793584665$	$k_{jx} / k_{bx} = 0,995877826$
$c_{jx} = 2,350215256$		$c_{jx} = 2,350225822$	
$k_{0x} = 0,745084609$	$k_{0x} / k_{ax} = 0,935019049$	$k_{0x} = 0,745088162$	$k_{0x} / k_{bx} = 0,935019048$

$$300.000.000 = 3 \cdot 10^8 = (30n_1 + 30) = (30n_6 + 11) + (30n_7 + 19) \quad 10^7 \text{ pairs} \quad \text{Highest prime to divide } 17.317$$

Sequence A $(30n_6 + 11)$

Total number of terms	10.000.000
Multiples $7m, 11m, \dots$	7.968.686
Primes greater than 17.320	2.031.314
Number of terms $(7j + a), (11j + b), \dots$	7.968.872
Multiples $(7j + a), (11j + b), \dots$	6.324.084
Primes $(7j + a), (11j + b), \dots$	1.644.788

Sequence B $(30n_7 + 19)$

Total number of terms	10.000.000
Multiples $7m, 11m, \dots$	7.968.872
Primes greater than 17.320	2.031.128
Number of terms $(7j + a), (11j + b), \dots$	7.968.686
Multiples $(7j + a), (11j + b), \dots$	6.324.084
Primes $(7j + a), (11j + b), \dots$	1.644.602

$$P_{PPx} = \text{Number of prime pairs being both greater than 17.320} \quad P_{PPx} = 2.031.314 - 1.644.788 = 2.031.128 - 1.644.602 = 386.526$$

Not included the possible prime pairs in which one of them is lesser than 17.320

$k_{ax} = 0,7968686$		$k_{bx} = 0,7968872$	
$k_{jx} = 0,793598391$	$k_{jx} / k_{ax} = 0,995896175$	$k_{jx} = 0,793616915$	$k_{jx} / k_{bx} = 0,995896175$
$c_{jx} = 2,34016453$		$c_{jx} = 2,340214659$	
$k_{0x} = 0,74509391$	$k_{0x} / k_{ax} = 0,935027318$	$k_{0x} = 0,745111301$	$k_{0x} / k_{bx} = 0,935027317$

$$300.000.000 = 3 \cdot 10^8 = (30n_1 + 30) = (30n_8 + 13) + (30n_9 + 17) \quad 10^7 \text{ pairs} \quad \text{Highest prime to divide } 17.317$$

Sequence A $(30n_8 + 13)$

Total number of terms	10.000.000
Multiples $7m, 11m, \dots$	7.968.552
Primes greater than 17.320	2.031.448
Number of terms $(7j + a), (11j + b), \dots$	7.968.561
Multiples $(7j + a), (11j + b), \dots$	6.324.315
Primes $(7j + a), (11j + b), \dots$	1.644.246

$$P_{PPx} = \text{Number of prime pairs being both greater than 17.320} \quad P_{PPx} = 2.031.448 - 1.644.246 = 2.031.439 - 1.644.237 = 387.202$$

Not included the possible prime pairs in which one of them is lesser than 17.320

$$\begin{aligned} k_{ax} &= 0,7968552 \\ k_{jx} &= 0,793658353 & k_{jx} / k_{ax} &= 0,99598817 \\ c_{jx} &= 2,287981074 \\ k_{0x} &= 0,745067145 & k_{0x} / k_{ax} &= 0,935009453 \end{aligned}$$

Sequence B $(30n_9 + 17)$

Total number of terms	10.000.000
Multiples $7m, 11m, \dots$	7.968.561
Primes greater than 17.320	2.031.439
Number of terms $(7j + a), (11j + b), \dots$	7.968.552
Multiples $(7j + a), (11j + b), \dots$	6.324.315
Primes $(7j + a), (11j + b), \dots$	1.644.237

$$\begin{aligned} k_{bx} &= 0,7968561 \\ k_{jx} &= 0,793659249 & k_{jx} / k_{bx} &= 0,99598817 \\ c_{jx} &= 2,287983736 \\ k_{0x} &= 0,745067987 & k_{0x} / k_{bx} &= 0,935009454 \end{aligned}$$

$$2^{22} = 4.194.304 = (30n_1 + 4) = (30n_2 + 11) + (30n_3 + 23) \quad 139.810 \text{ pairs} \quad \text{Highest prime to divide } 2.039$$

Sequence A $(30n_2 + 11)$

Total number of terms	139.810
Multiples $7m, 11m, \dots$	102.838
Primes greater than 2.048	36.972
Number of terms $(7j + a), (11j + b), \dots$	102.841
Multiples $(7j + a), (11j + b), \dots$	74.981
Primes $(7j + a), (11j + b), \dots$	27.860

$$P_{PPx} = \text{Number of prime pairs being both greater than 2.048} \quad P_{PPx} = 36.972 - 27.860 = 36.969 - 27.857 = 9.112$$

Not included the possible prime pairs in which one of them is lesser than 2.048

$$\begin{aligned} k_{ax} &= 0,735555396 \\ k_{jx} &= 0,729096372 & k_{jx} / k_{ax} &= 0,991218846 \\ c_{jx} &= 2,70514953 \\ k_{0x} &= 0,640494043 & k_{0x} / k_{ax} &= 0,870762482 \end{aligned}$$

Sequence B $(30n_3 + 23)$

Total number of terms	139.810
Multiples $7m, 11m, \dots$	102.841
Primes greater than 2.048	36.969
Number of terms $(7j + a), (11j + b), \dots$	102.838
Multiples $(7j + a), (11j + b), \dots$	74.981
Primes $(7j + a), (11j + b), \dots$	27.857

$$\begin{aligned} k_{bx} &= 0,735576854 \\ k_{jx} &= 0,729117641 & k_{jx} / k_{bx} &= 0,991218846 \\ c_{jx} &= 2,705221428 \\ k_{0x} &= 0,640512728 & k_{0x} / k_{bx} &= 0,870762482 \end{aligned}$$

$$2^{23} = 8.388.608 = (30n_1 + 8) = (30n_2 + 1) + (30n_3 + 7) \quad 279.621 \text{ pairs} \quad \text{Highest prime to divide } 2.887$$

Sequence A $(30n_2 + 1)$ (number 1 is present)

Total number of terms	279.621
Multiples $7m, 11m, \dots$	209.240+1
Primes greater than 2.896	70.380
Number of terms $(7j + a), (11j + b), \dots$	209.137
Multiples $(7j + a), (11j + b), \dots$	155.356+1
Primes $(7j + a), (11j + b), \dots$	53.780

$$P_{PPx} = \text{Number of prime pairs being both greater than 2.896} \quad P_{PPx} = 70.380 - 53.780 = 70.484 - 53.884 = 16.600$$

Not included the possible prime pairs in which one of them is lesser than 2.896

$$\begin{aligned} k_{ax} &= 0,748298589 \\ k_{jx} &= 0,742843208 & k_{jx} / k_{ax} &= 0,992709619 \\ c_{jx} &= 2,526146914 \\ k_{0x} &= 0,663473002 & k_{0x} / k_{ax} &= 0,886642058 \end{aligned}$$

Sequence B $(30n_3 + 7)$

Total number of terms	279.621
Multiples $7m, 11m, \dots$	209.137
Primes greater than 2.896	70.484
Number of terms $(7j + a), (11j + b), \dots$	209.240
Multiples $(7j + a), (11j + b), \dots$	155.356
Primes $(7j + a), (11j + b), \dots$	53.884

$$\begin{aligned} k_{bx} &= 0,747930234 \\ k_{jx} &= 0,742477537 & k_{jx} / k_{bx} &= 0,992709619 \\ c_{jx} &= 2,5233892 \\ k_{0x} &= 0,663143231 & k_{0x} / k_{bx} &= 0,886637818 \end{aligned}$$

$$2^{24} = 16.777.216 = (30n_1 + 16) = (30n_2 + 17) + (30n_3 + 29) \quad 559.240 \text{ pairs} \quad \text{Highest prime to divide } 4.093$$

Sequence A $(30n_2 + 17)$

Total number of terms	559.240
Multiples $7m, 11m, \dots$	424.629
Primes greater than 4.096	134.611
Number of terms $(7j + a), (11j + b), \dots$	424.501
Multiples $(7j + a), (11j + b), \dots$	320.414
Primes $(7j + a), (11j + b), \dots$	104.087

$$P_{PPx} = \text{Number of prime pairs being both greater than 4.096} \quad P_{PPx} = 134.611 - 104.087 = 134.739 - 104.215 = 30.524$$

Not included the possible prime pairs in which one of them is lesser than 4.096

$$\begin{aligned} k_{ax} &= 0,759296545 \\ k_{jx} &= 0,754801519 & k_{jx} / k_{ax} &= 0,994080013 \\ c_{jx} &= 2,282775574 \\ k_{0x} &= 0,682896316 & k_{0x} / k_{ax} &= 0,899380249 \end{aligned}$$

Sequence B $(30n_3 + 29)$

Total number of terms	559.240
Multiples $7m, 11m, \dots$	424.501
Primes greater than 4.096	134.739
Number of terms $(7j + a), (11j + b), \dots$	424.629
Multiples $(7j + a), (11j + b), \dots$	320.414
Primes $(7j + a), (11j + b), \dots$	104.215

$$\begin{aligned} k_{bx} &= 0,759067663 \\ k_{jx} &= 0,754573992 & k_{jx} / k_{bx} &= 0,994080013 \\ c_{jx} &= 2,282139908 \\ k_{0x} &= 0,682690464 & k_{0x} / k_{bx} &= 0,899380249 \end{aligned}$$

$$2^{25} = 33.554.432 = (30n_1 + 2) = (30n_2 + 13) + (30n_3 + 19) \quad 1.118.481 \text{ pairs} \quad \text{Highest prime to divide } 5.791$$

Sequence A $(30n_2 + 13)$

Total number of terms	1.118.481
Multiples $7m, 11m, \dots$	860.580
Primes greater than 5.792	257.901
Number of terms $(7j + a), (11j + b), \dots$	860.726
Multiples $(7j + a), (11j + b), \dots$	658.409
Primes $(7j + a), (11j + b), \dots$	202.317

$$P_{PPx} = \text{Number of prime pairs being both greater than } 5.792 \quad P_{PPx} = 257.901 - 202.317 = 257.755 - 202.171 = 55.584$$

Not included the possible prime pairs in which one of them is lesser than 5.792

$$\begin{aligned} k_{ax} &= 0,769418523 \\ k_{jx} &= 0,764946103 & k_{jx} / k_{ax} &= 0,994187272 \\ c_{jx} &= 2,476961963 \\ k_{0x} &= 0,700368084 & k_{0x} / k_{ax} &= 0,910256332 \end{aligned}$$

Sequence B $(30n_3 + 19)$

Total number of terms	1.118.481
Multiples $7m, 11m, \dots$	860.726
Primes greater than 5.792	257.755
Number of terms $(7j + a), (11j + b), \dots$	860.580
Multiples $(7j + a), (11j + b), \dots$	658.409
Primes $(7j + a), (11j + b), \dots$	202.171

$$\begin{aligned} k_{bx} &= 0,769549058 \\ k_{jx} &= 0,765075879 & k_{jx} / k_{bx} &= 0,994187272 \\ c_{jx} &= 2,477347165 \\ k_{0x} &= 0,700486904 & k_{0x} / k_{bx} &= 0,910256332 \end{aligned}$$

$$2^{26} = 67.108.864 = (30n_1 + 4) = (30n_2 + 11) + (30n_3 + 23) \quad 2.236.962 \text{ pairs} \quad \text{Highest prime to divide } 8.191$$

Sequence A $(30n_2 + 11)$

Total number of terms	2.236.962
Multiples $7m, 11m, \dots$	1.742.437
Primes greater than 8.192	494.525
Number of terms $(7j + a), (11j + b), \dots$	1.742.039
Multiples $(7j + a), (11j + b), \dots$	1.350.052
Primes $(7j + a), (11j + b), \dots$	391.987

$$P_{PPx} = \text{Number of prime pairs being both greater than } 8.192 \quad P_{PPx} = 494.525 - 391.987 = 494.923 - 392.385 = 102.538$$

Not included the possible prime pairs in which one of them is lesser than 8.192

$$\begin{aligned} k_{ax} &= 0,778930084 \\ k_{jx} &= 0,774983797 & k_{jx} / k_{ax} &= 0,994933708 \\ c_{jx} &= 2,378037211 \\ k_{0x} &= 0,716122909 & k_{0x} / k_{ax} &= 0,919367377 \end{aligned}$$

Sequence B $(30n_3 + 23)$

Total number of terms	2.236.962
Multiples $7m, 11m, \dots$	1.742.039
Primes greater than 8.192	494.923
Number of terms $(7j + a), (11j + b), \dots$	1.742.437
Multiples $(7j + a), (11j + b), \dots$	1.350.052
Primes $(7j + a), (11j + b), \dots$	392.385

$$\begin{aligned} k_{bx} &= 0,778752164 \\ k_{jx} &= 0,774806779 & k_{jx} / k_{bx} &= 0,994933708 \\ c_{jx} &= 2,377536801 \\ k_{0x} &= 0,715959336 & k_{0x} / k_{bx} &= 0,919367378 \end{aligned}$$

$$2^{27} = 134.217.728 = (30n_1 + 8) = (30n_2 + 1) + (30n_3 + 7) \quad 4.473.925 \text{ pairs} \quad \text{Highest prime to divide } 11.579$$

Sequence A $(30n_2 + 1)$ (number 1 is present)

Total number of terms	4.473.925
Multiples $7m, 11m, \dots$	3.523.978+1
Primes greater than 11.585	949.946
Number of terms $(7j + a), (11j + b), \dots$	3.523.523
Multiples $(7j + a), (11j + b), \dots$	2.762.690+1
Primes $(7j + a), (11j + b), \dots$	760.832

$$P_{PPx} = \text{Number of prime pairs being both greater than } 11.585 \quad P_{PPx} = 949.946 - 760.832 = 950.402 - 761.288 = 189.114$$

Not included the possible prime pairs in which one of them is lesser than 11.585

$$\begin{aligned} k_{ax} &= 0,787670334 \\ k_{jx} &= 0,784070375 & k_{jx} / k_{ax} &= 0,995429611 \\ c_{jx} &= 2,354564853 \\ k_{0x} &= 0,730398752 & k_{0x} / k_{ax} &= 0,927289908 \end{aligned}$$

Sequence B $(30n_3 + 7)$

Total number of terms	4.473.925
Multiples $7m, 11m, \dots$	3.523.523
Primes greater than 11.585	950.402
Number of terms $(7j + a), (11j + b), \dots$	3.523.978
Multiples $(7j + a), (11j + b), \dots$	2.762.690
Primes $(7j + a), (11j + b), \dots$	761.288

$$\begin{aligned} k_{bx} &= 0,787568633 \\ k_{jx} &= 0,783969139 & k_{jx} / k_{bx} &= 0,995429611 \\ c_{jx} &= 2,354141254 \\ k_{0x} &= 0,730304239 & k_{0x} / k_{bx} &= 0,927289645 \end{aligned}$$

$$2^{28} = 268.435.456 = (30n_1 + 16) = (30n_2 + 17) + (30n_3 + 29) \quad 8.947.848 \text{ pairs} \quad \text{Highest prime to divide } 16.381$$

Sequence A $(30n_2 + 17)$

Total number of terms	8.947.848
Multiples $7m, 11m, \dots$	7.119.164
Primes greater than 16.384	1.828.684
Number of terms $(7j + a), (11j + b), \dots$	7.119.276
Multiples $(7j + a), (11j + b), \dots$	5.640.478
Primes $(7j + a), (11j + b), \dots$	1.478.798

$$P_{PPx} = \text{Number of prime pairs being both greater than } 16.384 \quad P_{PPx} = 1.828.684 - 1.478.798 = 1.828.572 - 1.478.686 = 349.886$$

Not included the possible prime pairs in which one of them is lesser than 16.384

$$\begin{aligned} k_{ax} &= 0,795628624 \\ k_{jx} &= 0,792282529 & k_{jx} / k_{ax} &= 0,9957944 \\ c_{jx} &= 2,36480182 \\ k_{0x} &= 0,743136259 & k_{0x} / k_{ax} &= 0,934024035 \end{aligned}$$

Sequence B $(30n_3 + 29)$

Total number of terms	8.947.848
Multiples $7m, 11m, \dots$	7.119.276
Primes greater than 16.384	1.828.572
Number of terms $(7j + a), (11j + b), \dots$	7.119.164
Multiples $(7j + a), (11j + b), \dots$	5.640.478
Primes $(7j + a), (11j + b), \dots$	1.478.686

$$\begin{aligned} k_{bx} &= 0,795641141 \\ k_{jx} &= 0,792294994 & k_{jx} / k_{bx} &= 0,9957944 \\ c_{jx} &= 2,364835628 \\ k_{0x} &= 0,74314795 & k_{0x} / k_{bx} &= 0,934024035 \end{aligned}$$

$$2^{28} = 268.435.456 = (30n_1 + 16) = (30n_4 + 23) + (30n_5 + 23) \quad 4.473.924 \text{ pairs} \quad \text{Highest prime to divide } 16.381$$

Sequence A $(30n_4 + 23)$ from 0 to 134.217.728

Total number of terms	4.473.924
Multiples $7m, 11m, \dots$	3.523.743
Primes greater than 16.384	950.181
Number of terms $(7j + a), (11j + b), \dots$	3.595.452
Multiples $(7j + a), (11j + b), \dots$	2.820.494
Primes $(7j + a), (11j + b), \dots$	774.958

Sequence B $(30n_5 + 23)$ from 134.217.728 to 268.435.456

Total number of terms	4.473.924
Multiples $7m, 11m, \dots$	3.595.452
Primes greater than 16.384	878.472
Number of terms $(7j + a), (11j + b), \dots$	3.523.743
Multiples $(7j + a), (11j + b), \dots$	2.820.494
Primes $(7j + a), (11j + b), \dots$	703.249

$P_{PPx} =$ Number of prime pairs being both greater than 16.384 $P_{PPx} = 950.181 - 774.958 = 878.472 - 703.249 = 175.223$
 Not included the possible prime pairs in which one of them is lesser than 16.384

$k_{ax} = 0,787617983$		$k_{bx} = 0,803646195$	
$k_{jx} = 0,784461592$	$k_{jx} / k_{ax} = 0,995992484$	$k_{jx} = 0,80042557$	$k_{jx} / k_{bx} = 0,995992484$
$c_{jx} = 2,237432384$		$c_{jx} = 2,279504226$	
$k_{0x} = 0,735726988$	$k_{0x} / k_{ax} = 0,934116542$	$k_{0x} = 0,750699204$	$k_{0x} / k_{bx} = 0,934116541$

$$2^{29} = 536.870.912 = (30n_1 + 2) = (30n_2 + 13) + (30n_3 + 19) \quad 17.895.697 \text{ pairs} \quad \text{Highest prime to divide } 23.167$$

Sequence A $(30n_2 + 13)$

Total number of terms	17.895.697
Multiples $7m, 11m, \dots$	14.371.638
Primes greater than 23.170	3.524.059
Number of terms $(7j + a), (11j + b), \dots$	14.372.432
Multiples $(7j + a), (11j + b), \dots$	11.498.803
Primes $(7j + a), (11j + b), \dots$	2.873.629

Sequence B $(30n_3 + 19)$

Total number of terms	17.895.697
Multiples $7m, 11m, \dots$	14.372.432
Primes greater than 23.170	3.523.265
Number of terms $(7j + a), (11j + b), \dots$	14.371.638
Multiples $(7j + a), (11j + b), \dots$	11.498.803
Primes $(7j + a), (11j + b), \dots$	2.872.835

$P_{PPx} =$ Number of prime pairs being both greater than 23.170 $P_{PPx} = 3.524.059 - 2.873.629 = 3.523.265 - 2.872.835 = 650.430$
 Not included the possible prime pairs in which one of them is lesser than 23.170

$k_{ax} = 0,803077857$		$k_{bx} = 0,803122225$	
$k_{jx} = 0,800059655$	$k_{jx} / k_{ax} = 0,996241707$	$k_{jx} = 0,800103857$	$k_{jx} / k_{bx} = 0,996241707$
$c_{jx} = 2,300236897$		$c_{jx} = 2,300353829$	
$k_{0x} = 0,754804268$	$k_{0x} / k_{ax} = 0,939889278$	$k_{0x} = 0,754845969$	$k_{0x} / k_{bx} = 0,939889278$

$$7.000.000 = 7 \cdot 10^6 = (30n_1 + 10) = (30n_2 + 11) + (30n_3 + 29) \quad 233.333 \text{ pairs} \quad \text{Highest prime to divide } 2.633$$

In this case: $a = 0$

Sequence A $(30n_2 + 11)$

Total number of terms	233.333
Multiples $7m, 11m, \dots$	173.796
Primes greater than 2.645	59.537
Number of terms $(7j + a), (11j + b), \dots$	173.747
Multiples $(7j + a), (11j + b), \dots$	131.298
Primes $(7j + a), (11j + b), \dots$	42.449

Sequence B $(30n_3 + 29)$

Total number of terms	233.333
Multiples $7m, 11m, \dots$	173.747
Primes greater than 2.645	59.586
Number of terms $(7j + a), (11j + b), \dots$	173.796
Multiples $(7j + a), (11j + b), \dots$	131.298
Primes $(7j + a), (11j + b), \dots$	42.498

$P_{PPx} =$ Number of prime pairs being both greater than 2.645 $P_{PPx} = 59.537 - 42.449 = 59.586 - 42.498 = 17.088$
 Not included the possible prime pairs in which one of them is lesser than 2.645

$k_{ax} = 0,744841064$		$k_{bx} = 0,744631063$	
$k_{jx} = 0,75568499$	$k_{jx} / k_{ax} = 1,014558712$	$k_{jx} = 0,755471932$	$k_{jx} / k_{bx} = 1,014558712$
$c_{jx} = -5,214054931$		$c_{jx} = -5,212329083$	
$k_{0x} = 0,657335748$	$k_{0x} / k_{ax} = 0,882518136$	$k_{0x} = 0,657150419$	$k_{0x} / k_{bx} = 0,882518136$

$$7.000.000 = 7 \cdot 10^6 = (30n_1 + 10) = (30n_4 + 17) + (30n_5 + 23) \quad 233.333 \text{ pairs} \quad \text{Highest prime to divide } 2.633$$

In this case: $a = 0$

Sequence A $(30n_4 + 17)$

Total number of terms	233.333
Multiples $7m, 11m, \dots$	173.764
Primes greater than 2.645	59.569
Number of terms $(7j + a), (11j + b), \dots$	173.783
Multiples $(7j + a), (11j + b), \dots$	131.356
Primes $(7j + a), (11j + b), \dots$	42.427

Sequence B $(30n_5 + 23)$

Total number of terms	233.333
Multiples $7m, 11m, \dots$	173.783
Primes greater than 2.645	59.550
Number of terms $(7j + a), (11j + b), \dots$	173.764
Multiples $(7j + a), (11j + b), \dots$	131.356
Primes $(7j + a), (11j + b), \dots$	42.408

$P_{PPx} =$ Number of prime pairs being both greater than 2.645 $P_{PPx} = 59.569 - 42.427 = 59.550 - 42.408 = 17.142$
 Not included the possible prime pairs in which one of them is lesser than 2.645

$k_{ax} = 0,744703921$		$k_{bx} = 0,744785349$	
$k_{jx} = 0,755862196$	$k_{jx} / k_{ax} = 1,014983505$	$k_{jx} = 0,755944844$	$k_{jx} / k_{bx} = 1,014983505$
$c_{jx} = -5,372347613$		$c_{jx} = -5,373039944$	
$k_{0x} = 0,657222595$	$k_{0x} / k_{ax} = 0,882528715$	$k_{0x} = 0,657294458$	$k_{0x} / k_{bx} = 0,882528715$

$$14.872.858 = 2 \cdot 7 \cdot 11 \cdot 13 \cdot 17 \cdot 19 \cdot 23 = (30n_1 + 28) = (30n_2 + 11) + (30n_3 + 17) \quad 495.762 \text{ pairs} \quad \text{Highest prime to divide } 3.853$$

In this case: $a = b = c = d = e = f = 0$

Sequence A $(30n_2 + 11)$

Total number of terms	495.762
Multiples $7m, 11m, \dots$	375.402
Primes greater than 3.856	120.360
Number of terms $(7j + a), (11j + b), \dots$	375.451
Multiples $(7j + a), (11j + b), \dots$	302.245
Primes $(7j + a), (11j + b), \dots$	73.206

Sequence B $(30n_3 + 17)$

Total number of terms	495.762
Multiples $7m, 11m, \dots$	375.451
Primes greater than 3.856	120.311
Number of terms $(7j + a), (11j + b), \dots$	375.402
Multiples $(7j + a), (11j + b), \dots$	302.245
Primes $(7j + a), (11j + b), \dots$	73.157

$P_{ppx} =$ Number of prime pairs being both greater than 3.856 $P_{ppx} = 120.360 - 73.206 = 120.311 - 73.157 = 47.154$
 Not included the possible prime pairs in which one of them is lesser than 3.856

$k_{ax} = 0,757222215$		$k_{bx} = 0,757321053$	
$k_{jx} = 0,805018497$	$k_{jx} / k_{ax} = 1,063120549$	$k_{jx} = 0,805123574$	$k_{jx} / k_{bx} = 1,063120549$
$c_{jx} = -30,30331135$		$c_{jx} = -30,31126361$	
$k_{0x} = 0,679425487$	$k_{0x} / k_{ax} = 0,897260372$	$k_{0x} = 0,67951417$	$k_{0x} / k_{bx} = 0,897260372$

The programmable controller used is very slow to perform calculations with numbers greater than 10^9 .

To know the approximate values of c_{jx} for higher numbers, we will use data from MathWorld Web concerning to the Twin Primes Conjecture.

Twin Primes Conjecture statement: "There are infinitely many primes p such that $(p + 2)$ is also prime."

We call *Twin Primes* the pair of consecutive primes that are separated only by an even number. To form twin prime pairs, the following combinations of groups of primes are used:

$$(30n_1 + 11) \text{ and } (30n_1 + 13), \quad (30n_2 + 17) \text{ and } (30n_2 + 19), \quad (30n_3 + 29) \text{ and } (30n_3 + 31)$$

Goldbach's conjecture and the twin primes conjecture are similar in that both can be studied by combining two groups of primes to form pairs of primes that add an even number, in the first, or pairs of twin primes in the second. With this in mind, we can use data (*) from MathWorld Web concerning to the number of primes and to the number of twin prime pairs that are lesser than a given number (from 10^{10} to 10^{18}).

$$10^{10} \quad 455.052.511^* \text{ primes} \quad 27.412.679^* \text{ twin prime pairs}$$

Number of terms in each sequence **A** or **B**: $10^{10} / 30 = 333.333.333$
 Approximate number of primes in each sequence **A** or **B**: $455.052.511 / 8 = 56.881.563$ (1)
 Approximate number of twin prime pairs in the sequences **A-B** (1 combination of 3): $27.412.679 / 3 = 9.137.559$
 Approximate number of multiples $7m, 11m, \dots$ $333.333.333 - 56.881.563 = 276.451.770$ (2)
 Number of terms $(7m + 2), (11m + 2), \dots$ is, approximately, equal to the number of multiples $7m, 11m, \dots$
 Approximate number of primes $(7m + 2), (11m + 2), \dots$ $56.881.563 - 9.137.559 = 47.744.004$ (3)
 Approximate number of multiples $(7m + 2), (11m + 2), \dots$ $276.451.770 - 47.744.004 = 228.707.766$ (4)

Total number of terms in the sequence B	333.333.333		
Multiples $7m, 11m, \dots$	$\approx 276.451.770$ (2)	$k_{bx} \approx 0,82935531$	
Primes greater than 10^5	$\approx 56.881.563$ (1)	$k_{jx} \approx 0,827297166$	$k_{jx} / k_{bx} \approx 0,99751838$
Number of terms $(7m + 2), (11m + 2), \dots$	$\approx 276.451.770$ (2)	$c_{jx} \approx 2,095100568$	
Multiples $(7m + 2), (11m + 2), \dots$	$\approx 228.707.766$ (4)	$k_{0x} \approx 0,794244171$	$k_{0x} / k_{bx} \approx 0,957664539$
Primes $(7m + 2), (11m + 2), \dots$	$\approx 47.744.004$ (3)	$k_{7x} \approx 0,800914529$	

$$10^{11} \quad 4.118.054.813^* \text{ primes} \quad 224.376.048^* \text{ twin prime pairs}$$

Number of terms in each sequence **A** or **B**: $10^{11} / 30 = 3.333.333.333$
 Approximate number of primes in each sequence **A** or **B**: $4.118.054.813 / 8 = 514.756.851$ (1)
 Approximate number of twin prime pairs in the sequences **A-B** (1 combination of 3): $224.376.048 / 3 = 74.792.016$
 Approximate number of multiples $7m, 11m, \dots$ $3.333.333.333 - 514.756.851 = 2.818.576.482$ (2)
 Number of terms $(7m + 2), (11m + 2), \dots$ is, approximately, equal to the number of multiples $7m, 11m, \dots$
 Approximate number of primes $(7m + 2), (11m + 2), \dots$ $514.756.851 - 74.792.016 = 439.964.835$ (3)
 Approximate number of multiples $(7m + 2), (11m + 2), \dots$ $2.818.576.482 - 439.964.835 = 2.378.611.647$ (4)

Total number of terms in the sequence B	3.333.333.333		
Multiples $7m, 11m, \dots$	$\approx 2.818.576.482$ (2)	$k_{bx} \approx 0,845572944$	
Primes greater than $10^{5,5}$	$\approx 514.756.851$ (1)	$k_{jx} \approx 0,843905305$	$k_{jx} / k_{bx} \approx 0,998027799$
Number of terms $(7m + 2), (11m + 2), \dots$	$\approx 2.818.576.482$ (2)	$c_{jx} \approx 2,075447865$	
Multiples $(7m + 2), (11m + 2), \dots$	$\approx 2.378.611.647$ (4)	$k_{0x} \approx 0,817369919$	$k_{0x} / k_{bx} \approx 0,966646254$
Primes $(7m + 2), (11m + 2), \dots$	$\approx 439.964.835$ (3)	$k_{7x} \approx 0,819835102$	$k_{0x} < k_{7x}$ (page 11)

10^{12} 37.607.912.018* primes 1.870.585.220* twin prime pairs

Number of terms in each sequence **A** or **B**: $10^{12} / 30 = 33.333.333.333$
 Approximate number of primes in each sequence **A** or **B**: $37.607.912.018 / 8 = 4.700.989.002$ (1)
 Approximate number of twin prime pairs in the sequences **A-B** (1 combination of 3): $1.870.585.220 / 3 = 623.528.406$
 Approximate number of multiples $7m, 11m, \dots$ $33.333.333.333 - 4.700.989.002 = 28.632.344.331$ (2)
 Number of terms $(7m + 2), (11m + 2), \dots$ is, approximately, equal to the number of multiples $7m, 11m, \dots$
 Approximate number of primes $(7m + 2), (11m + 2), \dots$ $4.700.989.002 - 623.528.406 = 4.077.460.596$ (3)
 Approximate number of multiples $(7m + 2), (11m + 2), \dots$ $28.632.344.331 - 4.077.460.596 = 24.554.883.735$ (4)

Total number of terms in the sequence B	33.333.333.333		$k_{bx} \approx 0,85897033$	
Multiples $7m, 11m, \dots$	$\approx 28.632.344.331$	(2)	$k_{jx} \approx 0,857592499$	$k_{jx} / k_{bx} \approx 0,99839595$
Primes greater than 10^6	$\approx 4.700.989.002$	(1)	$c_{jx} \approx 2,058134681$	
Number of terms $(7m + 2), (11m + 2), \dots$	$\approx 28.632.344.331$	(2)	$k_{0x} \approx 0,835815434$	$k_{0x} / k_{bx} \approx 0,973043427$
Multiples $(7m + 2), (11m + 2), \dots$	$\approx 24.554.883.735$	(4)	$k_{7x} \approx 0,835465384$	$k_{0x} > k_{7x}$ (page 11)
Primes $(7m + 2), (11m + 2), \dots$	$\approx 4.077.460.596$	(3)	$k_{11x} \approx 0,844867362$	$k_{0x} < k_{11x}$ (page 11)

10^{13} 346.065.536.839* primes 15.834.664.872* twin prime pairs

Number of terms in each sequence **A** or **B**: $10^{13} / 30 = 333.333.333.333$
 Approximate number of primes in each sequence **A** or **B**: $346.065.536.839 / 8 = 43.258.192.105$ (1)
 Approximate number of twin prime pairs in the sequences **A-B** (1 combination of 3): $15.834.664.872 / 3 = 5.278.221.624$
 Approximate number of multiples $7m, 11m, \dots$ $333.333.333.333 - 43.258.192.105 = 290.075.141.228$ (2)
 Number of terms $(7m + 2), (11m + 2), \dots$ is, approximately, equal to the number of multiples $7m, 11m, \dots$
 Approximate number of primes $(7m + 2), (11m + 2), \dots$ $43.258.192.105 - 5.278.221.624 = 37.979.970.481$ (3)
 Approximate number of multiples $(7m + 2), (11m + 2), \dots$ $290.075.141.228 - 37.979.970.481 = 252.095.170.747$ (4)

Total number of terms in the sequence B	333.333.333.333		$k_{bx} \approx 0,870225423$	
Multiples $7m, 11m, \dots$	$\approx 290.075.141.228$	(2)	$k_{jx} \approx 0,869068509$	$k_{jx} / k_{bx} \approx 0,998670559$
Primes greater than $10^{6.5}$	$\approx 43.258.192.105$	(1)	$c_{jx} \approx 2,042626025$	
Number of terms $(7m + 2), (11m + 2), \dots$	$\approx 290.075.141.228$	(2)	$k_{0x} \approx 0,85087246$	$k_{0x} / k_{bx} \approx 0,977760977$
Multiples $(7m + 2), (11m + 2), \dots$	$\approx 252.095.170.747$	(4)		
Primes $(7m + 2), (11m + 2), \dots$	$\approx 37.979.970.481$	(3)		

10^{14} 3.204.941.750.802* primes 135.780.321.665* twin prime pairs

Number of terms in each sequence **A** or **B**: $10^{14} / 30 = 3.333.333.333.333$
 Approximate number of primes in each sequence **A** or **B**: $3.204.941.750.802 / 8 = 400.617.718.850$ (1)
 Approximate number of twin prime pairs in the sequences **A-B** (1 combination of 3): $135.780.321.665 / 3 = 45.260.107.221$
 Approximate number of multiples $7m, 11m, \dots$ $3.333.333.333.333 - 400.617.718.850 = 2.932.715.614.483$ (2)
 Number of terms $(7m + 2), (11m + 2), \dots$ is, approximately, equal to the number of multiples $7m, 11m, \dots$
 Approximate number of primes $(7m + 2), (11m + 2), \dots$ $400.617.718.850 - 45.260.107.221 = 355.357.611.629$ (3)
 Approximate number of multiples $(7m + 2), (11m + 2), \dots$ $2.932.715.614.483 - 355.357.611.629 = 2.577.358.002.854$ (4)

Total number of terms in the sequence B	3.333.333.333.333		$k_{bx} \approx 0,879814684$	
Multiples $7m, 11m, \dots$	$\approx 2.932.715.614.483$	(2)	$k_{jx} \approx 0,878829842$	$k_{jx} / k_{bx} \approx 0,998880626$
Primes greater than 10^7	$\approx 400.617.718.850$	(1)	$c_{jx} \approx 2,028807737$	
Number of terms $(7m + 2), (11m + 2), \dots$	$\approx 2.932.715.614.483$	(2)	$k_{0x} \approx 0,863397011$	$k_{0x} / k_{bx} \approx 0,981339623$
Multiples $(7m + 2), (11m + 2), \dots$	$\approx 2.577.358.002.854$	(4)		
Primes $(7m + 2), (11m + 2), \dots$	$\approx 355.357.611.629$	(3)		

10^{15} 29.844.570.422.669* primes 1.177.209.242.304* twin prime pairs

Number of terms in each sequence **A** or **B**: $10^{15} / 30 = 33.333.333.333.333$
 Approximate number of primes in each sequence **A** or **B**: $29.844.570.422.669 / 8 = 3.730.571.302.833$ (1)
 Approximate number of twin prime pairs in the sequences **A-B** (1 combination of 3): $1.177.209.242.304 / 3 = 392.403.080.768$
 Approximate number of multiples $7m, 11m, \dots$ $33.333.333.333.333 - 3.730.571.302.833 = 29.602.762.030.500$ (2)
 Number of terms $(7m + 2), (11m + 2), \dots$ is, approximately, equal to the number of multiples $7m, 11m, \dots$
 Approximate number of primes $(7m + 2), (11m + 2), \dots$ $3.730.571.302.833 - 392.403.080.768 = 3.338.168.221.065$ (3)
 Approximate number of multiples $(7m + 2), (11m + 2), \dots$ $29.602.762.030.500 - 3.338.168.221.065 = 26.264.593.809.435$ (4)

Total number of terms in the sequence B	33.333.333.333.333		$k_{bx} \approx 0,888082861$	
Multiples $7m, 11m, \dots$	$\approx 29.602.762.030.500$	(2)	$k_{jx} \approx 0,887234568$	$k_{jx} / k_{bx} \approx 0,999044804$
Primes greater than $10^{7.5}$	$\approx 3.730.571.302.833$	(1)	$c_{jx} \approx 2,016482789$	
Number of terms $(7m + 2), (11m + 2), \dots$	$\approx 29.602.762.030.500$	(2)	$k_{0x} \approx 0,873978945$	$k_{0x} / k_{bx} \approx 0,984118693$
Multiples $(7m + 2), (11m + 2), \dots$	$\approx 26.264.593.809.435$	(4)		
Primes $(7m + 2), (11m + 2), \dots$	$\approx 3.338.168.221.065$	(3)		

10^{16}

279.238.341.033.925* primes

10.304.195.697.298* twin prime pairs

Number of terms in each sequence **A** or **B**: $10^{16} / 30 = 333.333.333.333.333$

Approximate number of primes in each sequence **A** or **B**: $279.238.341.033.925 / 8 = 34.904.792.629.240$ (1)

Approximate number of twin prime pairs in the sequences **A-B** (1 combination of 3): $10.304.195.697.298 / 3 = 3.434.731.897.432$

Approximate number of multiples $7m, 11m, \dots$ $333.333.333.333.333 - 34.904.792.629.240 = 298.428.540.704.093$ (2)

Number of terms $(7m + 2), (11m + 2), \dots$ is, approximately, equal to the number of multiples $7m, 11m, \dots$

Approximate number of primes $(7m + 2), (11m + 2), \dots$ $34.904.792.629.240 - 3.434.731.897.432 = 31.470.060.721.808$ (3)

Approximate number of multiples $(7m + 2), (11m + 2), \dots$ $298.428.540.704.093 - 31.470.060.721.808 = 266.958.479.982.285$ (4)

Total number of terms in the sequence B	333.333.333.333.333		$k_{bx} \approx 0,895285622$	
Multiples $7m, 11m, \dots$	$\approx 298.428.540.704.093$	(2)	$k_{jx} \approx 0,894547415$	$k_{jx} / k_{bx} \approx 0,999175451$
Primes greater than 10^8	$\approx 34.904.792.629.240$	(1)	$c_{jx} \approx 2,005561339$	
Number of terms $(7m + 2), (11m + 2), \dots$	$\approx 298.428.540.704.093$	(2)	$k_{ox} \approx 0,883038021$	$k_{ox} / k_{bx} \approx 0,986319895$
Multiples $(7m + 2), (11m + 2), \dots$	$\approx 266.958.479.982.285$	(4)	$k_{7x} \approx 0,877833225$	$k_{ox} > k_{7x}$ (page 11)
Primes $(7m + 2), (11m + 2), \dots$	$\approx 31.470.060.721.808$	(3)	$k_{11x} \approx 0,884814184$	$k_{ox} < k_{11x}$ (page 11)
			$k_{13x} \approx 0,886559424$	$k_{ox} < k_{13x}$ (page 11)

10^{18}

24.739.954.287.740.860* primes

808.675.888.577.436* twin prime pairs

Number of terms in each sequence **A** or **B**: $10^{18} / 30 = 33.333.333.333.333.333$

Approximate number of primes in each sequence **A** or **B**: $24.739.954.287.740.860 / 8 = 3.092.494.285.967.607$ (1)

Approximate number of twin prime pairs in the sequences **A-B** (1 combination of 3): $808.675.888.577.436 / 3 = 269.558.629.525.812$

Approximate number of multiples $7m, 11m, \dots$ $33.333.333.333.333.333 - 3.092.494.285.967.607 = 30.240.839.047.365.726$ (2)

Number of terms $(7m + 2), (11m + 2), \dots$ is, approximately, equal to the number of multiples $7m, 11m, \dots$

Approximate number of primes $(7m + 2), (11m + 2), \dots$ $3.092.494.285.967.607 - 269.558.629.525.812 = 2.822.935.656.441.795$ (3)

Approximate number of multiples $(7m + 2), (11m + 2), \dots$ $30.240.839.047.365.726 - 2.822.935.656.441.795 = 27.417.903.390.923.931$ (4)

Total number of terms in the sequence B	33.333.333.333.333.333		$k_{bx} \approx 0,907225171$	
Multiples $7m, 11m, \dots$	$\approx 30.240.839.047.365.726$	(2)	$k_{jx} \approx 0,906651543$	$k_{jx} / k_{bx} \approx 0,999367712$
Primes greater than 10^9	$\approx 3.092.494.285.967.607$	(1)	$c_{jx} \approx 1,987076711$	
Number of terms $(7m + 2), (11m + 2), \dots$	$\approx 30.240.839.047.365.726$	(2)	$k_{ox} \approx 0,897737814$	$k_{ox} / k_{bx} \approx 0,989542445$
Multiples $(7m + 2), (11m + 2), \dots$	$\approx 27.417.903.390.923.931$	(4)	$k_{7x} \approx 0,8917627$	$k_{ox} > k_{7x}$ (page 11)
Primes $(7m + 2), (11m + 2), \dots$	$\approx 2.822.935.656.441.795$	(3)	$k_{11x} \approx 0,897947688$	$k_{ox} \approx k_{11x}$ (page 11)
			$k_{13x} \approx 0,899493935$	$k_{ox} < k_{13x}$ (page 11)

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