The proof of Goldbach's Conjecture

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Abstract.

Goldbach's Conjecture statement: "Every even integer greater than 2 can be expressed as the sum of two primes".

Initially, to prove this conjecture, we can form two arithmetic sequences (**A** and **B**) different for each even number, with all the natural numbers that can be primes, that can added, in pairs, result in the corresponding even number.
By analyzing the pairing process, in general, between all non-prime numbers of sequence **A**, with terms of sequence **B**, or vice versa, to obtain the even number, we note that some pairs of primes are always formed. This allow us to develop a non-probabilistic formula, to calculate the approximate number of pairs of primes that meet the conjecture for an even number x.
The result of this formula is always equal or greater than 1, and it tends to infinite when x tends to infinite, which allow us to confirm that Goldbach's Conjecture is true.
The prime numbers theorem by Carl Friedrich Gauss, the prime numbers theorem in arithmetic progressions and some axioms have been used to complete this investigation.

1. Prime numbers and composite numbers.

A *prime number* (or *prime*) is a natural number greater than 1 that has only two divisors, 1 and the number itself.
Examples of primes are: 2, 3, 5, 7, 11, 13, 17. The Greek mathematician Euclid proved that there are infinitely many primes, but they become more scarce as we move on the number line.
Except 2 and 3, all primes are of form $(6n + 1)$ or $(6n – 1)$ being n a natural number.
We can differentiate primes 2, 3 and 5 from the rest. The 2 is the first prime and the only one that is even, the 3 is the only one of form $(6n – 3)$ and 5 is the only one finished in 5. All other primes are odd and its final digit will be 1, 3, 7 or 9.

In contrast to primes, a *composite number* (or *composite*) is a natural number that has more than two divisors.
Examples of composites are: 4 (divisors 1, 2, 4), 6 (1, 2, 3, 6), 15 (1, 3, 5, 15), 24 (1, 2, 3, 4, 6, 8, 12, 24).

Except 1, every natural number is prime or composite. By convention, number 1 is considered neither prime nor composite because it has only one divisor. Since it cannot meet the conjecture, and for this demonstration, we will include the number 1 in the composite set. This question has no relevance in the development or in the final formula of this proof.
We can classify the set of primes (except 2, 3 and 5) in 8 groups depending of the situation of each of them with respect to multiples of 30, $(30 = 2\cdot3\cdot5)$. Being: $n = 0, 1, 2, 3, 4,\ldots, \infty$.

$30n + 1$ $30n + 7$ $30n + 11$ $30n + 13$ $30n + 17$ $30n + 19$ $30n + 23$ $30n + 29$

These expressions represent all arithmetic progressions of module 30, $(30n + b)$, such that $\gcd(30, b) = 1$ being: $30 > b > 0$.
These 8 groups contain all primes (except 2, 3 and 5). They also include the number 1 and all composites that are multiples of primes greater than 5. As 30 and b are coprime, they cannot contain multiples of 2 or 3 or 5.
Logically, when n increases, decreases the primes proportion and increases the composites proportion that there are in each group.

Dirichlet's theorem statement[1]: "An arithmetic progression $(an + b)$ such that $\gcd(a, b) = 1$ contains infinitely many prime numbers". Applying this theorem for the 8 groups of primes, we can say that each of them contains infinitely many primes.

You can also apply the prime numbers theorem in arithmetic progressions. It states[2]: "For every module a, the prime numbers tend to be distributed evenly among the different progressions $(an + b)$ such that $\gcd(a, b) = 1$".
To verify the precision of this theorem, I used a programmable logic controller (PLC), like those that control automatic machines, having obtained the following data:
There are 50.847.531 primes lesser than 10^9, (2, 3 and 5 not included), distributed as follows:

Group $(30n + 1)$	6.355.189 primes	12,49852033 %	50.847.531 / 6.355.189 = 8,0009471
Group $(30n + 7)$	6.356.475 primes	12,50104946 %	50.847.531 / 6.356.475 = 7,999328401
Group $(30n + 11)$	6.356.197 primes	12,50050273 %	50.847.531 / 6.356.197 = 7,999678267
Group $(30n + 13)$	6.356.062 primes	12,50023723 %	50.847.531 / 6.356.062 = 7,999848176
Group $(30n + 17)$	6.355.839 primes	12,49979866 %	50.847.531 / 6.355.839 = 8,000128858
Group $(30n + 19)$	6.354.987 primes	12,49812307 %	50.847.531 / 6.354.987 = 8,001201419
Group $(30n + 23)$	6.356.436 primes	12,50097276 %	50.847.531 / 6.356.436 = 7,999377481
Group $(30n + 29)$	6.356.346 primes	12,50079576 %	50.847.531 / 6.356.346 = 7,999490745

We can see that the maximum deviation for 10^9, (between 6.354.987 and 6.355.941 average), is lesser than 0,01502 %.
I gather that, in compliance with this theorem, the maximum deviation tends to 0 % when larger numbers are analyzed.

2. Special cases of the conjecture.

As we have seen, numbers 2, 3 and 5 are different from all other primes and they are not included in the 8 groups described.
We will study, as special cases, the even numbers 4, 6, 8, 10, 12 and 16 whose solutions, to meet the conjecture, contain the primes 2, 3 or 5. We will write all possible pairs of terms for each of these even numbers, highlighting the primes in **bold**.

4 = 1 + **3** = **2** + **2**	4 = **2** + **2** (unique with prime 2)
6 = 1 + **5** = **2** + 4 = **3** + **3**	6 = **3** + **3**
8 = 1 + **7** = **2** + 6 = **3** + **5** = 4 + 4	8 = **3** + **5**
10 = 1 + 9 = **2** + 8 = **3** + **7** = 4 + 6 = **5** + **5**	10 = **3** + **7** = **5** + **5**
12 = 1 + **11** = **2** + 10 = **3** + 9 = 4 + 8 = **5** + **7** = 6 + 6	12 = **5** + **7**
16 = 1 + 15 = **2** + 14 = **3** + **13** = 4 + 12 = **5** + **11** = 6 + 10 = **7** + 9 = 8 + 8	16 = **3** + **13** = **5** + **11**

We note that the even numbers 4, 6, 8, 10, 12 and 16 can be expressed as the sum of two primes. For all the other even numbers, it must be proved or verified that they meet the conjecture with one or more prime pairs containing neither 3 nor 5.

3. Classification of even numbers.

Just as we did with primes, we can divide the set of even numbers (2, 4, 6, 8, 10,…) in 15 groups depending of the situation of each of them with respect to multiples of 30. Being: $n = 0, 1, 2, 3, 4, \ldots, \infty$.

$30n + 2$	$30n + 12$	$30n + 22$
$30n + 4$	$30n + 14$	$30n + 24$
$30n + 6$	$30n + 16$	$30n + 26$
$30n + 8$	$30n + 18$	$30n + 28$
$30n + 10$	$30n + 20$	$30n + 30$

4. Combining groups of even numbers with groups of prime numbers.

Now, we will combine groups of even numbers with groups of primes to express the 36 possible combinations of Goldbach's conjecture. We can see that each group of even numbers has its own combinations that are different from the rest.

$30n_1 + 2 = (30n_2 + 1) + (30n_3 + 1) = (30n_4 + 13) + (30n_5 + 19)$
Being: $n_1 = n_2 + n_3 = n_4 + n_5 + 1$
We observe that for even numbers $(30n + 2)$ there are 2 different combinations using 3 groups of primes.
For number 2, it can only apply the combination 2 = 1 + 1. This even number is excluded in Goldbach's conjecture statement.

$30n_1 + 4 = (30n_2 + 11) + (30n_3 + 23) = (30n_4 + 17) + (30n_5 + 17)$
Being: $n_1 = n_2 + n_3 + 1 = n_4 + n_5 + 1$
There are 2 different combinations using 3 groups of primes.
These combinations cannot be applied for number 4 so that, as I have indicated, it is considered a special case, (4 = **2** + **2**).

$30n_1 + 6 = (30n_2 + 7) + (30n_3 + 29) = (30n_4 + 13) + (30n_5 + 23) = (30n_6 + 17) + (30n_7 + 19)$
Being: $n_1 = n_2 + n_3 + 1 = n_4 + n_5 + 1 = n_6 + n_7 + 1$
There are 3 different combinations using 6 groups of primes.
These combinations cannot be applied for number 6, this is why it is considered a special case, (6 = **3** + **3**).

$30n_1 + 8 = (30n_2 + 1) + (30n_3 + 7) = (30n_4 + 19) + (30n_5 + 19)$
Being: $n_1 = n_2 + n_3 = n_4 + n_5 + 1$
There are 2 different combinations using 3 groups of primes.
For number 8, only the combination 8 = 1 + **7** which does not meet the conjecture, this is why it is considered a special case, (8 = **3** + **5**).

$30n_1 + 10 = (30n_2 + 11) + (30n_3 + 29) = (30n_4 + 17) + (30n_5 + 23)$
Being: $n_1 = n_2 + n_3 + 1 = n_4 + n_5 + 1$
There are 2 different combinations using 4 groups of primes.
These combinations cannot be applied for number 10, this is why it is considered a special case, (10 = **3** + **7** = **5** + **5**).

$30n_1 + 12 = (30n_2 + 1) + (30n_3 + 11) = (30n_4 + 13) + (30n_5 + 29) = (30n_6 + 19) + (30n_7 + 23)$
Being: $n_1 = n_2 + n_3 = n_4 + n_5 + 1 = n_6 + n_7 + 1$
There are 3 different combinations using 6 groups of primes.
For number 12, only the combination 12 = 1 + **11** which does not meet the conjecture, this is why it is considered a special case, (12 = **5** + **7**).

$30n_1 + 14 = (30n_2 + 1) + (30n_3 + 13) = (30n_4 + 7) + (30n_5 + 7)$
Being: $n_1 = n_2 + n_3 = n_4 + n_5$
There are 2 different combinations using 3 groups of primes.

$30n_1 + 16 = (30n_2 + 17) + (30n_3 + 29) = (30n_4 + 23) + (30n_5 + 23)$
Being: $n_1 = n_2 + n_3 + 1 = n_4 + n_5 + 1$
There are 2 different combinations using 3 groups of primes.
These combinations cannot be applied for number 16, this is why it is considered a special case, (16 = **3** + **13** = **5** + **11**).

$30n_1 + 18 = (30n_2 + 1) + (30n_3 + 17) = (30n_4 + 7) + (30n_5 + 11) = (30n_6 + 19) + (30n_7 + 29)$
Being: $n_1 = n_2 + n_3 = n_4 + n_5 = n_6 + n_7 + 1$
There are 3 different combinations using 6 groups of primes.
For number 18, only the combinations 18 = 1 + **17** = **7** + **11**. (Also: 18 = **5** + **13**).

$30n_1 + 20 = (30n_2 +1) + (30n_3 + 19) = (30n_4 + 7) + (30n_5 + 13)$
Being: $n_1 = n_2 + n_3 = n_4 + n_5$
There are 2 different combinations using 4 groups of primes.

$30n_1 + 22 = (30n_2 + 11) + (30n_3 + 11) = (30n_4 + 23) + (30n_5 + 29)$
Being: $n_1 = n_2 + n_3 = n_4 + n_5 + 1$
There are 2 different combinations using 3 groups of primes.
For number 22, only the combination 22 = **11** + **11**. (Also: 22 = **3** + **19** = **5** + **17**).

$30n_1 + 24 = (30n_2 + 1) + (30n_3 + 23) = (30n_4 + 7) + (30n_5 + 17) = (30n_6 + 11) + (30n_7 + 13)$
Being: $n_1 = n_2 + n_3 = n_4 + n_5 = n_6 + n_7$
There are 3 different combinations using 6 groups of primes.

$30n_1 + 26 = (30n_2 + 7) + (30n_3 + 19) = (30n_4 + 13) + (30n_5 + 13)$
Being: $n_1 = n_2 + n_3 = n_4 + n_5$
There are 2 different combinations using 3 groups of primes.

$30n_1 + 28 = (30n_2 + 11) + (30n_3 + 17) = (30n_4 + 29) + (30n_5 + 29)$
Being: $n_1 = n_2 + n_3 = n_4 + n_5 + 1$
There are 2 different combinations using 3 groups of primes.
For number 28, only the combination 28 = **11** + **17**. (Also: 28 = **5** + **23**).

$30n_1 + 30 = (30n_2 + 1) + (30n_3 + 29) = (30n_4 + 7) + (30n_5 + 23) = (30n_6 + 11) + (30n_7 + 19) = (30n_8 + 13) + (30n_9 + 17)$
Being: $n_1 = n_2 + n_3 = n_4 + n_5 = n_6 + n_7 = n_8 + n_9$
There are 4 different combinations using the 8 prime numbers groups available.

We observe that, for even numbers that are not multiples of 6 or 10, there are 2 different combinations using 3 groups of primes.
The multiples of 6, $(30n + 6)$, $(30n + 12)$, $(30n + 18)$ and $(30n + 24)$ have 3 different combinations using 6 groups of primes.
The multiples of 10, $(30n + 10)$ and $(30n + 20)$ have 2 different combinations using 4 groups of primes.
The multiples of 30, $(30n + 30)$, have 4 different combinations using the 8 prime numbers groups available.

It is reasonable to believe, that the number of prime pairs (also called Goldbach's partitions) that meet the conjecture depend on the number of groups of primes used by the even number (3, 4, 6 or 8). Examples with actual values:

3.600	125 partitions.	8 groups.	Multiple of 30.	3.606	90 partitions.	6 groups.	Multiple of 6.
3.602	48 partitions.	3 groups.		3.610	66 partitions.	4 groups.	Multiple of 10.

Surprisingly, we can see that, due to the number of groups of primes used, consecutive even numbers have a noticeable difference in the partitions number. Example: 3.600 has 125 partitions and 3.602 has only 48.

5. Example.

The concepts described can be applied to number 784 serving as example for any of the 36 exposed combinations and for any even number $\boldsymbol{x}$, even being a large number. We use the list of primes lesser than 1.000.

$784 = 30\cdot26 + 4 = (30n_2 + 11) + (30n_3 + 23) = (30n_4 + 17) + (30n_5 + 17)$ Being: $26 = n_2 + n_3 + 1 = n_4 + n_5 + 1$

For the first combination, $784 = (30n_2 + 11) + (30n_3 + 23)$, we will write the sequence **A** of all numbers $(30n_2 + 11)$ from 0 to 784.
Also, we will write the sequence **B** of all numbers $(30n_3 + 23)$ from 784 to 0. I highlight the primes in **bold**.

A **11** **-** **41** **- 71 -101-131**-161-**191**-221-**251-281-311**-341-371-**401-431-461-491-521**-551-581-611-**641**-671-**701**-731-**761**
B **773-743**-713-**683-653**-623-**593-563**-533-**503**-473-**443**-413-**383-353**-323-**293-263-233**-203-**173**-143-**113 - 83 - 53 – 23**

The second combination, 784 = ($30n_4$ + 17) + ($30n_5$ + 17), uses the same group of primes in the two sequences.
We will write the sequence **A** of all numbers ($30n_4$ + 17) from 0 to 392, (½ of 784).
Also, we will write the sequence **B** of all numbers ($30n_5$ + 17) from 784 to 392.

A **17 - 47 -** 77- **107-137-167-197-227-257**-287-**317-347**-377
B 767-737-707-**677-647-617-587-557**-527-497-**467**-437-407

Sorted in this way, each term in a sequence **A** can be added with its partner in the corresponding sequence **B** to obtain 784.
In the above 4 sequences, the 18 prime pairs that meet the conjecture for number 784 are underlined.

The study of sequences **A** and **B**, individually and collectively, is the basis of this demonstration.
I will analyze the complete sequences (**A** from 0 to x and **B** from x to 0). The halves sequences will be considered for the final formula.

To calculate the number of terms in each sequence **A** or **B** we must remember that these are arithmetic progressions of module 30.

$\frac{x}{30}$ Number of terms in each sequence **A** or **B** for an even number x. Obviously, it is equal to the number of pairs that are formed. (26 terms in each sequence and 26 pairs of terms that are formed to x = 784).

To analyze, in general, the above formula, would be:

Number of terms = number of pairs = formula result	if x is multiple of 30
Number of terms = number of pairs = integer part of result	if $n_1 = n_2 + n_3 + 1$
Number of terms = number of pairs = (integer part of result) + 1	if $n_1 = n_2 + n_3$

6. Applying the conjecture to small even numbers.

As we have seen, the composites present in the 8 groups of primes are multiples of primes greater than 5 (primes 7, 11, 13, 17, 19,…). The first composites that appear on them are:

49 = $\mathbf{7^2}$ 77 = **7·11** 91 = **7·13** 119 = **7·17** 121 = $\mathbf{11^2}$ 133 = **7·19** 143 = **11·13** 161 = **7·23** 169 = $\mathbf{13^2}$

And so on, forming products of two or more factors with primes greater than 5.

From the above, we conclude that on even numbers lesser than 49, all terms of corresponding sequences **A-B** are primes (except 1) and all pairs meet the conjecture (except those containing 1).
We will write all pairs between terms of sequences **A** and **B** of even numbers lesser than 49 (except special cases).

14 = 1 + **13** = **7** + **7**
18 = 1 + **17** = **7** + **11**
20 = 1 + **19** = **7** + **13**
22 = **11** + **11**
24 = 1 + **23** = **7** + **17** = **11** + **13**
26 = **7** + **19** = **13** + **13**
28 = **11** + **17**
30 = 1 + **29** = **7** + **23** = **11** + **19** = **13** + **17**
32 = 1 + **31** = **13** + **19**
34 = **11** + **23** = **17** + **17**
36 = **7** + **29** = **13** + **23** = **17** + **19**
38 = 1 + **37** = **31** + **7** = **19** + **19**
40 = **11** + **29** = **17** + **23**
42 = 1 + **41** = **31** + **11** = **13** + **29** = **19** + **23**
44 = 1 + **43** = **31** + **13** = **7** + **37**
46 = **17** + **29** = **23** + **23**
48 = 1 + **47** = **31** + **17** = **7** + **41** = **37** + **11** = **19** + **29**

Furthermore, we note that in the complete sequences **A-B** for number 784, used as an example, the primes predominate (17 primes with 9 composites in sequence **A** and 18 primes with 8 composites in sequence **B**).
This occurs on small even numbers (up to $x \approx$ 4.500).
Therefore, for even numbers lesser than 4.500, is ensured the compliance with Goldbach's conjecture with the sequences **A** and **B** because, even in the event that all composites are paired with primes, there will always be, left over in the two sequences, some primes that will form pairs between them. Applying this reasoning to number 784 we would have:
17 – 8 = 18 – 9 = 9 prime pairs at least (in the previous chapter we can see that are 12 actual pairs).

7. Applying logical reasoning to the conjecture.

The sequences **A** and **B** are composed of terms that may be primes or composites that form pairs between them. To differentiate, I define as **free composite** the one which is not paired with another composite and having, as partner, a prime of the other sequence. Thus, the pairs between terms of sequences **A-B** will be formed by:

(Composite of sequence **A**) + (Composite of sequence **B**)	(CC pairs)
(Free composite of sequence **A** or **B**) + (Prime of sequence **B** or **A**)	(C**P**-**P**C pairs)
(Prime of sequence **A**) + (Prime of sequence **B**)	(**PP** pairs)

We will substitute the primes by a **P** and the composites by a C in the sequences **A-B** of number 784, that we use as example.

A	**P**	**P**	**P**	**P**	**P**	C	**P**	C	**P**	**P**	**P**	C	C	**P**	**P**	**P**	**P**	**P**	C	C	C	**P**	C	**P**	C	**P**
B	**P**	**P**	C	**P**	**P**	C	**P**	**P**	C	**P**	C	**P**	C	**P**	**P**	C	**P**	**P**	**P**	C	**P**	C	**P**	**P**	**P**	**P**

The number of pairs of primes (P_{PP}) that will be formed will depend on the free composites number of one sequence that are paired with primes of the another sequence. In general, it can define the following axiom:

P_{PP} = (Number of primes of **A**) – (number of free composites of **B**) = (Number of primes of **B**) – (number of free composites of **A**)

For number 784: $P_{PP} = 17 - 5 = 18 - 6 = 12$ prime pairs in the sequences **A-B**.

I consider that this axiom is perfectly valid although being very simple and "obvious". It will be used later in the proof of the conjecture.

Given this axiom, enough pairs of composites must be formed between the two sequences **A-B** because the number of free composites of sequence **A** cannot be greater than the number of primes of sequence **B**.
Conversely, the number of free composites of sequence **B** cannot be greater than the number of primes of sequence **A**.
This is particularly important for sequences **A-B** of very large even numbers in which the primes proportion is much lesser than the composites proportion.
Later, this question is analyzed in more detail when algebra is applied to the sequences **A-B**.

Let us consider, briefly, how are formed the different kinds of pairs between terms of sequences **A-B**.

A composite of sequence **A** will be paired with a composite of sequence **B** if both, as pair, meet some conditions that will depend on the characteristics of the even number x (if is a power of 2 or a multiple of a power of 3 or 5 or a multiple of one or more primes greater than 5, etc.). The composites of each sequence that fail to have a composite of the other, to meet the required conditions, will be paired with a prime (with this prime the conditions will be met). Finally, the remaining primes of the two sequences will form the pairs that will meet the conjecture.
This question will be analyzed in more detail in the next chapter.

With what we have described, we can devise a logical reasoning to support the conclusion that Goldbach's conjecture is true.
Later, a general formula will be developed to calculate the approximate number of partitions.

As I indicated, compliance of the conjecture is secured for small even numbers (lesser than 4.500), since in corresponding sequences **A** and **B**, the primes predominate. Therefore, in these sequences we will find **PP** pairs and, if there are composites, CC and C**P**-**P**C pairs. If we verify increasingly larger numbers, we note that already predominate the composites and decreases the primes proportion.

Let us suppose that there is a sufficiently large even number that does not meet the conjecture. In this supposition, with all sequences **A-B** of this number (we remember, 2, 3 or 4 combinations of groups of primes depending on the even number) they could only be formed CC and C**P**-**P**C pairs, understanding that all free composites of each combination **A-B** would be paired with all primes of the same combination with extraordinary mathematical precision.

What happens, then, for even numbers greater than the one that, supposedly, does not meet the conjecture?

It can be assumed that the conjecture will not be achieved from the first even number that did not achieve it, but it is not possible because there will always be primes greater than it and that added to other primes will give us even numbers greater that will meet the conjecture.
Another question is that, as x increases, increases the composites proportion and decreases the primes proportion, so we could assume that, for very large numbers, some composites will not have a partner because there are not enough primes, which, obviously, is not possible because each term of sequence **A** has its corresponding partner in sequence **B** and vice versa.

Not being possible both cases above, I conclude, although not serve as demonstration, that we will not find an even number that do not meet the conjecture. Therefore, I gather that Goldbach's conjecture is true.
Later, I will reinforce this deduction through the formula to calculate the approximate number of partitions for an even number x.

8. Studying how the pairs between terms of sequences A-B are formed.

We will analyze how the composite-composite pairs with the sequences **A** and **B** are formed. If the proportion of CC pairs is higher, there are less composites (free) that need a prime as a partner and, therefore, there will be more primes to form pairs.

The secret of Goldbach's conjecture is the number of composite-composite pairs formed with the sequences **A** and **B**.

For this analysis, I consider *m* as the natural number that is not multiple of 2 or 3 or 5 and *j* as natural number (including 0).

Let us suppose that we applied Goldbach's conjecture to an even number that is a multiple of 7, ($\boldsymbol{x} = 7q$, being q an even number).

In this supposition, any multiple of 7 ($7m_1$) of sequence **A** will be paired with a multiple of 7 ($7m_2$) of sequence **B** so that the sum of the two numbers will be a multiple of 7 ($7q$) as we have assumed for $\boldsymbol{x}$.
Expanding this axiom, it can be confirmed that all multiples of 7 ($7m_1$) (including prime 7, if it would be present) of sequence **A** will be paired with all multiples of 7 ($7m_2$) (including prime 7, if it would be present) of sequence **B**.

$$\boldsymbol{x} = 7q = 7m_1 + 7m_2 = 7(m_1 + m_2) \qquad \text{being: } q = m_1 + m_2$$

We will apply this axiom to number 784 = 7·112. It serves as an example for any even number that is multiple of 7, even being a large number. We will write the corresponding sequences **A-B**. In them, the multiples of 7 are underlined.

A **11 - 41 - 71 -101-131-**161**-191-**221**-251-281-311-**341-371**-401-431-461-491-521-**551-581-611**-641-**671**-701-**731**-761**
B **773-743-**713**-683-653-**623**-593-563-**533**-503-**473**-443-**413**-383-353-**323**-293-263-233-**203**-173-**143**-113 - 83 - 53 - 23**

Verifying: 784 = 7·112 = $7m_1 + 7m_2$ = 161 + 623 = 371 + 413 = 581 + 203 being: 112 = 23 + 89 = 53 + 59 = 83 + 29

We observe that the pairs of multiples of 7 can be "cross out" of the sequences **A-B** without affecting the prime pairs.
The same would happen if the even number would be multiple of any other prime greater than 5 and lesser than $\sqrt{\boldsymbol{x}}$.
Mathematically, they are enough primes lesser than $\sqrt{\boldsymbol{x}}$ to define all composites of sequences **A-B**.

If the even number would be multiple of several primes greater than 5, more pairs of composites would be formed being lesser the number of free composites which would give more possibilities to the primes to form pairs between them. For example, the number 2.002 = 2·7·11·13 has 44 partitions while the number 2.048 = 2^{11}, that is greater, only 25. In the hypothetical event that an even number $\boldsymbol{x}$ were multiple of all primes lesser than $\sqrt{\boldsymbol{x}}$, (of course, it is not possible), there would be no free composites and, therefore, all primes of sequences **A-B** would be paired between them (except lesser than $\sqrt{\boldsymbol{x}}$).

The worst case is when the even number $\boldsymbol{x}$ is not multiple of primes greater than 5. Example: 512 = 2^9 $\sqrt{\mathbf{512}}$ = 22,62
For these even numbers, and following the order of primes (7, 11, 13, 17, 19,…, previous to $\sqrt{\boldsymbol{x}}$), we can write:

$\boldsymbol{x} = 7r + a$	being:	r = natural number	a = natural number < 7	512 = 7·73 + 1	$a = 1$
$\boldsymbol{x} = 11s + b$	being:	s = natural number	b = natural number < 11	512 = 11·46 + 6	$b = 6$
$\boldsymbol{x} = 13t + c$	being:	t = natural number	c = natural number < 13	512 = 13·39 + 5	$c = 5$
$\boldsymbol{x} = 17u + d$	being:	u = natural number	d = natural number < 17	512 = 17·30 + 2	$d = 2$
$\boldsymbol{x} = 19v + e$	being:	v = natural number	e = natural number < 19	512 = 19·26 + 18	$e = 18$

And so on to the prime previous to $\sqrt{\boldsymbol{x}}$.

In this case, any multiple of 7 ($7m_{11}$) of sequence **A** will be paired with a term ($7j_{21} + a$) of sequence **B** so that the sum of the two numbers has the form ($7r + a$) as assumed for $\boldsymbol{x}$.
Conversely, any term ($7j_{11} + a$) of sequence **A** will be paired with a multiple of 7 ($7m_{21}$) of sequence **B** so that, similarly, the sum of the two numbers has the form ($7r + a$).

Expanding this axiom, it can be confirmed that all multiples of 7 ($7m_{11}$) (including the prime 7, if it would be present) of sequence **A** will be paired with all terms ($7j_{21} + a$) of sequence **B**.
Conversely, all terms ($7j_{11} + a$) of sequence **A** will be paired with all multiples of 7 ($7m_{21}$) (including the prime 7, if it would be present) of sequence **B**.

Applying this axiom for all primes, from 7 to the previous to $\sqrt{\boldsymbol{x}}$, it can be confirmed that all groups of multiples $7m_{11}$, $11m_{12}$, $13m_{13}$, $17m_{14}$,… (including the primes lesser than $\sqrt{\boldsymbol{x}}$ that are present) of sequence **A** will be paired, group to group, with all groups of terms ($7j_{21} + a$), ($11j_{22} + b$), ($13j_{23} + c$), ($17j_{24} + d$),… of sequence **B**.
Conversely, all groups of terms ($7j_{11} + a$), ($11j_{12} + b$), ($13j_{13} + c$), ($17j_{14} + d$),… of sequence **A** will be paired, group to group, with all groups of multiples $7m_{21}$, $11m_{22}$, $13m_{23}$, $17m_{24}$,… (including the primes lesser than $\sqrt{\boldsymbol{x}}$ that are present) of sequence **B**.

$\boldsymbol{x} = 7r + a = 7m_{11} + (7j_{21} + a) = 7(m_{11} + j_{21}) + a$ being: $r = m_{11} + j_{21}$
$\boldsymbol{x} = 11s + b = 11m_{12} + (11j_{22} + b) = 11(m_{12} + j_{22}) + b$ being: $s = m_{12} + j_{22}$
$\boldsymbol{x} = 13t + c = 13m_{13} + (13j_{23} + c) = 13(m_{13} + j_{23}) + c$ being: $t = m_{13} + j_{23}$
$\boldsymbol{x} = 17u + d = 17m_{14} + (17j_{24} + d) = 17(m_{14} + j_{24}) + d$ being: $u = m_{14} + j_{24}$

$\boldsymbol{x} = 7r + a = (7j_{11} + a) + 7m_{21} = 7(j_{11} + m_{21}) + a$ being: $r = j_{11} + m_{21}$
$\boldsymbol{x} = 11s + b = (11j_{12} + b) + 11m_{22} = 11(j_{12} + m_{22}) + b$ being: $s = j_{12} + m_{22}$
$\boldsymbol{x} = 13t + c = (13j_{13} + c) + 13m_{23} = 13(j_{13} + m_{23}) + c$ being: $t = j_{13} + m_{23}$
$\boldsymbol{x} = 17u + d = (17j_{14} + d) + 17m_{24} = 17(j_{14} + m_{24}) + d$ being: $u = j_{14} + m_{24}$

And so on to the prime previous to $\sqrt{\boldsymbol{x}}$.

We will apply the above described, to even number 512 = 7•73 + 1 = 11•46 + 6 = 13•39 + 5 = 17•30 + 2 = 19•26 + 18.
It serves as an example for any even number $\boldsymbol{x}$, even being a large number. We will write the corresponding sequences **A-B**.

A **13 - 43 - 73- 103-**133**-163-193-223-**253**-283-313-**343**-373-**403**-433-463-**493 — in **A**, multiples $7m_{11}$ are underlined
B **499-**469**-439-409-379-349-**319-289-259**-229-199-**169**-139-109 - 79 -** 49 **- 19** — in **B**, terms $(7j_{21} + 1)$ are underlined

Verifying: 512 = 7•73 + 1 = $7m_{11} + (7j_{21} + 1)$ = 133 + **379** = 343 + 169 — being: 73 = 19 + 54 = 49 + 24

A **13 - 43 - 73- 103-**133**-163-193-223-**253**-283-313-**343**-373-**403**-433-463-**493 — in **A**, terms $(7j_{11} + 1)$ are underlined
B **499-**469**-439-409-379-349-**319-289-259**-229-199-**169**-139-109 - 79 -** 49 **- 19** — in **B**, multiples $7m_{21}$ are underlined

Verifying: 512 = 7•73 + 1 = $(7j_{11} + 1) + 7m_{21}$ = **43** + 469 = 253 + 259 = **463** + 49 — being: 73 = 6 + 67 = 36 + 37 = 66 + 7

Applying the same procedure for the primes 11, 13, 17 and 19, it results:

For the prime 11: 512 = 11•46 + 6 = $11m_{12} + (11j_{22} + 6)$ = 253 + 259 — being: 46 = 23 + 23
512 = 11•46 + 6 = $(11j_{12} + 6) + 11m_{22}$ = **193** + 319 — being: 46 = 17 + 29

For the prime 13: 512 = 13•39 + 5 = $13m_{13} + (13j_{23} + 5)$ = **13 + 499** = 403 + **109** — being: 39 = 1 + 38 = 31 + 8
512 = 13•39 + 5 = $(13j_{13} + 5) + 13m_{23}$ = 343 + 169 — being: 39 = 26 + 13

For the prime 17: 512 = 17•30 + 2 = $17m_{14} + (17j_{24} + 2)$ = 493 + **19** — being: 30 = 29 + 1
512 = 17•30 + 2 = $(17j_{14} + 2) + 17m_{24}$ = **223** + 289 — being: 30 = 13 + 17

For the prime 19: 512 = 19•26 + 18 = $19m_{15} + (19j_{25} + 18)$ = 133 + **379** — being: 26 = 7 + 19
512 = 19•26 + 18 = $(19j_{15} + 18) + 19m_{25}$ = 493 + **19** — being: 26 = 25 + 1

The 7 pairs of terms of sequences **A-B** that do not appear in the above expressions are those that meet the conjecture. In these pairs, the two terms are greater than $\sqrt{\mathbf{512}}$ and lesser than 512.
We add the pair of primes (**13** + **499**) in which the first term is a multiple of 13 ($13m_{13}$) lesser than $\sqrt{\mathbf{512}}$.

512 = **73** + **439** = **103** + **409** = **163** + **349** = **283** + **229** = **313** + **199** = **373** + **139** = **433** + **79** (Also: 512 = **13** + **499**)

It can be seen that all multiples $7m$, $11m$, $13m$, $17m$, $19m$,… of a sequence **A** or **B** are paired with multiples or primes of the other, to form multiple-multiple pairs, multiple-prime pairs and prime-multiple pairs, according to the axiom defined.
Finally, the remaining prime-prime pairs are those that meet the conjecture. In these pairs, the two primes are greater than $\sqrt{\boldsymbol{x}}$.

The above exposition helps us understand the relation between terms of sequence **A** and terms of sequence **B** of any even number $\boldsymbol{x}$.

To numerically support the axioms exposed, I used a programmable controller to obtain data of sequences **A-B** corresponding to several even numbers (between 10^6 and 10^9) and that can be consulted from page 21.
They are the following data:

1. Number of multiples $7m$, $11m$, $13m$, $17m$,… in each sequence **A** or **B**, (includes all composites and the primes lesser than $\sqrt{\boldsymbol{x}}$).
2. Number of primes in each sequence **A** or **B**, (only those who are greater than $\sqrt{\boldsymbol{x}}$).
3. Number of multiples that there are in terms $(7j + a)$, $(11j + b)$,… in each sequence, (all composites and the primes lesser than $\sqrt{\boldsymbol{x}}$).
4. Number of primes that there are in terms $(7j + a)$, $(11j + b)$, $(13j + c)$,… in each sequence, (only those who are greater than $\sqrt{\boldsymbol{x}}$).

9. Proving the conjecture.

For proving the conjecture, as a starting point, I will use the first part of the last axiom from the previous chapter:
All multiples $7m_{11}$, $11m_{12}$, $13m_{13}$, $17m_{14}$,… (including the primes lesser than $\sqrt{x}$ that are present) of sequence **A** will be paired, respectively, with all terms $(7j_{21} + a)$, $(11j_{22} + b)$, $(13j_{23} + c)$, $(17j_{24} + d)$,… of sequence **B**.

In this axiom, the concept of *multiple*, applied to the terms of each sequence **A** or **B**, includes all composites and the primes lesser than $\sqrt{x}$ that are present. By this definition, all terms that are lesser than $\sqrt{x}$ of each sequence **A** or **B** are *multiples.*
Simultaneously, and also in this axiom, the concept of *prime*, applied to the terms of each sequence **A** or **B**, refers only to primes greater than $\sqrt{x}$ that are present in the corresponding sequence.
According to these concepts, each term of sequences **A** or **B** will be *multiple* or *prime*. Thus, with the terms of the two sequences can be form multiple-multiple pairs, free multiple-prime pairs, prime-free multiple pairs and prime-prime pairs.

$\frac{x}{30}$ — Number of terms in each sequence **A** or **B** for the even number x. (Page 4)

$\pi(x)$ — Symbol[3], normally used, to express the number of primes lesser or equal to x.

According to the prime numbers theorem[3]: $\pi(x) \sim \frac{x}{\ln(x)}$ being: $\lim_{x\to\infty} \frac{\pi(x)}{\frac{x}{\ln(x)}} = 1$ ln(x) = natural logarithm of x

A better approach for this theorem is given by the offset logarithmic integral function **Li**(x): $\pi(x) \approx \mathrm{Li}(x) = \int_2^x \frac{dy}{\ln(y)}$

According to these formulas, for all $x \geq 5$ is true that $\pi(x) > \sqrt{x}$. This inequality becomes larger with increasing x.

$\pi(ax)$ — Symbol to express the number of primes greater than $\sqrt{x}$ in sequence **A** for the even number x.

$\pi(bx)$ — Symbol to express the number of primes greater than $\sqrt{x}$ in sequence **B** for the even number x.

For large values of x it can be accept that: $\pi(ax) \approx \pi(bx) \approx \frac{\pi(x)}{8}$ being 8 the number of groups of primes (page 1).

For $x = 10^9$, the maximum error of above approximation is 0,0215 % for group $(30n + 19)$.

$\frac{x}{30} - \pi(ax)$ — Number of multiples of sequence **A** for the even number x. It includes the number 1 in the group $(30n + 1)$.

$\frac{x}{30} - \pi(bx)$ — Number of multiples of sequence **B** for the even number x. It includes the number 1 in the group $(30n + 1)$.

We will define as a fraction $k(ax)$ of sequence **A**, or $k(bx)$ of sequence **B**, the ratio between the number of multiples and the total number of terms in the corresponding sequence. As the primes density decreases as we move on the number line, the $k(ax)$ and $k(bx)$ values gradually increase when increasing x and tend to 1 when x tends to infinite.

$$k(ax) = \frac{\frac{x}{30} - \pi(ax)}{\frac{x}{30}} = 1 - \frac{\pi(ax)}{\frac{x}{30}}$$

For sequence **A**: $k(ax) = 1 - \frac{30\pi(ax)}{x}$ For sequence **B**: $k(bx) = 1 - \frac{30\pi(bx)}{x}$

The central question of this chapter is to develop a general formula to calculate the number of multiples that there are in terms $(7j_{21} + a)$, $(11j_{22} + b)$, $(13j_{23} + c)$, $(17j_{24} + d)$,… of sequence **B** and that, complying the origin axiom, will be paired with an equal number of multiples $7m_{11}$, $11m_{12}$, $13m_{13}$, $17m_{14}$,… of sequence **A**. Known this data, we can calculate the number of free multiples of sequence **A** (and that will be paired with primes of sequence **B**). Finally, the remaining primes of sequence **B** will be paired with some primes of sequence **A** to determine the pairs number that meet the conjecture.

We will study the terms $(7j + a)$, $(11j + b)$, $(13j + c)$, $(17j + d)$,… of sequence **B**, in a general way.
The same procedure can be applied to the terms of sequence **A** if we use the second part of the axiom referred to in the above chapter.

We will analyze how primes are distributed among terms $(7j + a)$, $(11j + b)$, $(13j + c)$, $(17j + d)$,…
For this purpose, we will see the relation between prime 7 and the 8 groups of primes, serving as example for any prime greater than 5.

We will analyze how are the groups of multiples of 7 ($7m$) and the groups $(7j + a)$ in generally, that's $(7j + 1)$, $(7j + 2)$, $(7j + 3)$, $(7j + 4)$, $(7j + 5)$ and $(7j + 6)$. Noting the fact that it is an axiom, I gather that they will be arithmetic progressions of module 210, $(210 = 7 \cdot 30)$.

In the following expressions, the 8 arithmetic progressions of module 210 correspond, respectively, with the 8 groups of primes of module 30. I highlight in **bold** the number that identifies each of these 8 groups. Being: $n = 0, 1, 2, 3, 4, \ldots, \infty$.

$(210n + 90 + \mathbf{1})$, $(210n + \mathbf{7})$, $(210n + 150 + \mathbf{11})$, $(210n + 120 + \mathbf{13})$, $(210n + 60 + \mathbf{17})$, $(210n + 30 + \mathbf{19})$, $(210n + 180 + \mathbf{23})$ and $(210n + 90 + \mathbf{29})$ are multiples of 7 ($7m$). These groups do not contain primes, except the prime 7 in the group $(210n + \mathbf{7})$ for $n = 0$.

$(210n + \mathbf{1})$, $(210n + 120 + \mathbf{7})$, $(210n + 60 + \mathbf{11})$, $(210n + 30 + \mathbf{13})$, $(210n + 180 + \mathbf{17})$, $(210n + 150 + \mathbf{19})$, $(210n + 90 + \mathbf{23})$ and $(210n + \mathbf{29})$ are terms $(7j + 1)$.

$(210n + 120 + \mathbf{1})$, $(210n + 30 + \mathbf{7})$, $(210n + 180 + \mathbf{11})$, $(210n + 150 + \mathbf{13})$, $(210n + 90 + \mathbf{17})$, $(210n + 60 + \mathbf{19})$, $(210n + \mathbf{23})$ and $(210n + 120 + \mathbf{29})$ are terms $(7j + 2)$.

$(210n + 30 + \mathbf{1})$, $(210n + 150 + \mathbf{7})$, $(210n + 90 + \mathbf{11})$, $(210n + 60 + \mathbf{13})$, $(210n + \mathbf{17})$, $(210n + 180 + \mathbf{19})$, $(210n + 120 + \mathbf{23})$ and $(210n + 30 + \mathbf{29})$ are terms $(7j + 3)$.

$(210n + 150 + \mathbf{1})$, $(210n + 60 + \mathbf{7})$, $(210n + \mathbf{11})$, $(210n + 180 + \mathbf{13})$, $(210n + 120 + \mathbf{17})$, $(210n + 90 + \mathbf{19})$, $(210n + 30 + \mathbf{23})$ and $(210n + 150 + \mathbf{29})$ are terms $(7j + 4)$.

$(210n + 60 + \mathbf{1})$, $(210n + 180 + \mathbf{7})$, $(210n + 120 + \mathbf{11})$, $(210n + 90 + \mathbf{13})$, $(210n + 30 + \mathbf{17})$, $(210n + \mathbf{19})$, $(210n + 150 + \mathbf{23})$ and $(210n + 60 + \mathbf{29})$ are terms $(7j + 5)$.

$(210n + 180 + \mathbf{1})$, $(210n + 90 + \mathbf{7})$, $(210n + 30 + \mathbf{11})$, $(210n + \mathbf{13})$, $(210n + 150 + \mathbf{17})$, $(210n + 120 + \mathbf{19})$, $(210n + 60 + \mathbf{23})$ and $(210n + 180 + \mathbf{29})$ are terms $(7j + 6)$.

We can note that the groups of multiples of 7 ($7m$) correspond to arithmetic progressions of module 210, $(210n + b)$, such that $\gcd(210, b) = 7$ being b lesser than 210, multiple of 7, and having 8 terms b, one of each group of primes.

Also, we can see that the groups of terms $(7j + 1)$, $(7j + 2)$, $(7j + 3)$, $(7j + 4)$, $(7j + 5)$ and $(7j + 6)$ correspond to arithmetic progressions of module 210, $(210n + b)$, such that $\gcd(210, b) = 1$ being b lesser than 210, not multiple of 7, and having 48 terms b, 6 of each group of primes.

Finally, we can verify that the 56 terms b, $(8 + 48)$, are all those that appear in the 8 groups of primes and that are lesser than 210.

Applying the above axiom for all p (prime greater than 5 and lesser than $\sqrt{\boldsymbol{x}}$) we can confirm that the groups of multiples of p (pm) correspond to arithmetic progressions of module $30p$, $(30pn + b)$, such that $\gcd(30p, b) = p$ being b lesser than $30p$, multiple of p, and having 8 terms b, one for each group of primes.

Also, we can confirm that the groups of terms $(pj + 1)$, $(pj + 2)$, $(pj + 3)$,…, $(pj + p – 2)$ and $(pj + p – 1)$ correspond to arithmetic progressions of module $30p$, $(30pn + b)$, such that $\gcd(30p, b) = 1$ being b lesser than $30p$, not multiple of p, and having $8(p – 1)$ terms b, $(p – 1)$ of each group of primes.

Finally, we can confirm that the $8p$ terms b, $(8 + 8(p – 1))$, are all those that appear in the 8 groups of primes and that are lesser than $30p$.

On the other hand, an axiom that is met in the sequences **A** or **B** is that, in each set of p consecutive terms, there are one of each of the following groups: pm, $(pj + 1)$, $(pj + 2)$, $(pj + 3)$,…, $(pj + p – 2)$ and $(pj + p – 1)$ (though not necessarily in this order). Example:

1	**31**	**61**	91	121	**151**	**181**	Terms $(30n + 1)$
$(7\cdot 0 + 1)$	$(7\cdot 4 + 3)$	$(7\cdot 8 + 5)$	$7\cdot 13$	$(7\cdot 17 + 2)$	$(7\cdot 21 + 4)$	$(7\cdot 25 + 6)$	Terms $7m$ and $(7j + a)$

Therefore, and according to this axiom, $\boldsymbol{\frac{1}{p}\frac{x}{30}}$ will be the number of multiples of p (pm) (including p, if it would be present) and, also, the number of terms that have each groups $(pj + 1)$, $(pj + 2)$, $(pj + 3)$,…, $(pj + p – 2)$ and $(pj + p – 1)$ in each sequence **A** or **B**.

This same axiom allows us to say that these groups contain all terms of sequences **A** or **B** as follows:

1. Group pm: contains all multiples of p.
2. Groups $(pj + 1)$, $(pj + 2)$, $(pj + 3)$,…, $(pj + p – 1)$: contain all multiples (except those of p) and the primes greater than $\sqrt{\boldsymbol{x}}$.

As it has been described, the groups $(pj + 1)$, $(pj + 2)$, $(pj + 3)$,…, $(pj + p – 2)$ and $(pj + p – 1)$ are arithmetic progressions of module $30p$, $(30pn + b)$, such that $\gcd(30p, b) = 1$.
Applying the prime numbers theorem in arithmetic progressions [2], shown on page 1, to these groups we concluded that they all will have, approximately, the same amount of primes ($\approx \boldsymbol{\frac{\pi(bx)}{p-1}}$ in sequence **B**) and, as they all have the same number of terms, also they will have, approximately, the same number of multiples.
Similarly, we can apply this theorem to terms belonging to two or more groups. For example, the terms that are, at once, in groups $(7j + a)$ and $(13j + c)$ correspond to arithmetic progressions of module 2730, $(2730 = 7\cdot 13\cdot 30)$. In this case, all groups of a sequence **A**

or **B** that contain these terms (72 groups that result of combining 6 a and 12 c) they will have, approximately, the same amount of primes and, as they all have the same number of terms, will also have, approximately, the same number of multiples.

As described, I gather that, of the $\frac{1}{7}\frac{x}{30}$ terms ($7j + a$) that there are in sequence **B**, $\approx \frac{\pi(bx)}{6}$ will be primes.
All other terms are multiples (of primes greater than 5, except the prime 7).

In general, I gather that, of the $\frac{1}{p}\frac{x}{30}$ terms ($pj + h$) that there are in sequence **B**, $\approx \frac{\pi(bx)}{p-1}$ will be primes.
All other terms are multiples (of primes greater than 5, except the prime p).

We will define as a fraction $k(7x)$ of sequence **B** the ratio between the number of multiples that there are in the group of terms ($7j + a$) and the total number of these.
Applying the above for all p (prime greater than 5 and lesser than $\sqrt{x}$) we will define as a fraction $k(px)$ of sequence **B** the ratio between the number of multiples that there are in the group of terms ($pj + h$) and the total number of these.

We can see the similarity between $k(bx)$ and factors $k(7x)$, $k(11x)$, $k(13x)$, $k(17x)$,…, $k(px)$,… so their formulas will be similar.
I will use ≈ instead of = due to the imprecision in the number of primes that there are in each group.
Using the same procedure as for obtaining $k(bx)$:

$$k(px) \approx \frac{\frac{1}{p}\frac{x}{30} - \frac{\pi(bx)}{p-1}}{\frac{1}{p}\frac{x}{30}} = 1 - \frac{\frac{\pi(bx)}{p-1}}{\frac{1}{p}\frac{x}{30}} = 1 - \frac{30p\pi(bx)}{(p-1)x} \qquad k(px) \approx 1 - \frac{30\pi(bx)}{x}\frac{p}{p-1}$$

For the prime 7: $k(7x) \approx 1 - \frac{35\pi(bx)}{x}$ For the prime 11: $k(11x) \approx 1 - \frac{33\pi(bx)}{x}$ For the prime 31: $k(31x) \approx 1 - \frac{31\pi(bx)}{x}$

And so on to the prime previous to $\sqrt{x}$.

If we order these factors from lowest to highest value: $k(7x) < k(11x) < k(13x) < k(17x) <\ldots < k(997x) <\ldots < k(bx)$

In the formula to obtain $k(px)$ we have that: $\lim_{p\to\infty} \frac{p}{p-1} = 1$ so we can write: $\lim_{p\to\infty} k(px) = 1 - \frac{30\pi(bx)}{x} = k(bx)$

We can unify all factors $k(7x)$, $k(11x)$, $k(13x)$,…, $k(px)$,… into one, which we will call $k(jx)$, and that will group all of them together.

Applying the above, we will define as a fraction $k(jx)$ of sequence **B** the ratio between the number of multiples that there are in the set of all terms ($7j_{21} + a$), ($11j_{22} + b$), ($13j_{23} + c$), ($17j_{24} + d$),… and the total number of these.
Logically, the $k(jx)$ value is determined by the $k(px)$ values corresponding to each primes from 7 to the one previous to $\sqrt{x}$.

Summarizing the exposed: a fraction $k(jx)$ of terms ($7j_{21} + a$), ($11j_{22} + b$), ($13j_{23} + c$), ($17j_{24} + d$),… of sequence **B** will be multiples and that, complying the origin axiom, will be paired with an equal fraction of multiples $7m_{11}$, $11m_{12}$, $13m_{13}$, $17m_{14}$,… of sequence **A**.

To put it simply and in general:
A fraction $k(jx)$ of multiples of sequence **A** will have, as partner, a multiple of sequence **B**.

Recalling the axiom on page 5, and the formulas on page 8, we can record:

1. Number of multiple-multiple pairs = $k(jx)$ (Number of multiples of sequence **A**)
2. Number of free multiples in sequence **A** = ($1 - k(jx)$) (Number of multiples of sequence **A**)
3. $P_{PP}(x)$ = actual number of pairs of primes greater than $\sqrt{x}$
   $P_{PP}(x)$ = (Number of primes greater than $\sqrt{x}$ of sequence **B**) – (Number of free multiples of sequence **A**)

Expressed algebraically: $P_{PP}(x) = \pi(bx) - (1 - k(jx))\left(\frac{x}{30} - \pi(ax)\right)$

Let us suppose that there may be a sufficiently large even number that does not meet the conjecture. In this case: $P_{PP}(x) = 0$. $P_{PP}(x)$ cannot be negative because the number of primes of sequence **B** cannot be lesser than the number of free multiples of sequence **A**.
We can define a factor, which I will call $k(0x)$ and that, replacing $k(jx)$ in the above formula, it results in $P_{PP}(x) = 0$.
As a concept, $k(0x)$ would be the minimum value of $k(jx)$ for which the conjecture would not be met.

$$0 = \pi(bx) - (1 - k(0x))\left(\frac{x}{30} - \pi(ax)\right) \qquad \pi(bx) = (1 - k(0x))\left(\frac{x}{30} - \pi(ax)\right)$$

Solving: $$k(0x) = 1 - \frac{30\pi(bx)}{x - 30\pi(ax)}$$

For the conjecture to be true, $k(jx)$ must be greater than $k(0x)$ for any x value.
Let us recall that the $k(jx)$ value is determined by values of the factors $k(7x)$, $k(11x)$, $k(13x)$, $k(17x)$,…, $k(px)$,…
To analyze the relation between the factors $k(jx)$ and $k(0x)$, first, let us compare $k(0x)$ with the general factor $k(px)$.

$$k(0x) = 1 - \frac{30\pi(bx)}{x - 30\pi(ax)} = 1 - \frac{30\pi(bx)}{x}\frac{x}{x - 30\pi(ax)} = 1 - \frac{30\pi(bx)}{x}\frac{1}{1 - \frac{30\pi(ax)}{x}}$$

$$k(px) \approx 1 - \frac{30\pi(bx)}{x}\frac{p}{p-1} = 1 - \frac{30\pi(bx)}{x}\frac{1}{1-\frac{1}{p}}$$

To compare $k(0x)$ with $k(px)$, simply compare $\frac{30\pi(ax)}{x}$ with $\frac{1}{p}$ which are the terms that differentiate the two formulas.

Let us recall, page 8, the prime numbers theorem: $\pi(x) \sim \frac{x}{\ln(x)}$ being $\pi(x)$ the number of primes lesser or equal to x.

As I have indicated, it can be accepted that: $\pi(ax) \approx \frac{\pi(x)}{8}$ being 8 the number of groups of primes.

Substituting $\pi(x)$ by its corresponding formula: $\pi(ax) \sim \frac{x}{8\ln(x)}$

The approximation of this formula does not affect the final result of the comparison between $k(0x)$ and $k(px)$ that we are analyzing.

Compare	$\frac{30\pi(ax)}{x}$	with	$\frac{1}{p}$	Substituting $\pi(ax)$ by its corresponding formula
Compare	$\frac{30x}{8x\ln(x)}$	with	$\frac{1}{p}$	
Compare	$\frac{3,75}{\ln(x)}$	with	$\frac{3,75}{3,75p}$	
Compare	$\ln(x)$	with	$3,75p$	Applying the natural logarithm concept
Compare	x	with	$e^{3,75p}$	For powers of 10: ln10 = 2,302585 3,75 / 2,302585 = 1,6286 ≈ 1,63
Compare	x	with	$10^{1,63p}$	

Comparison result: $k(0x)$ will be lesser than $k(px)$ if $x < 10^{1,63p}$ $k(0x)$ will be greater than $k(px)$ if $x > 10^{1,63p}$

In the following expressions, the exponents values are approximate. This does not affect the comparison result.

1. For the prime 7: $k(0x) < k(7x)$ if $x < 10^{11,4}$ $k(0x) > k(7x)$ if $x > 10^{11,4}$ $\approx 4 \cdot 10^4$ primes lesser than $10^{5,7}$
2. For the prime 11: $k(0x) < k(11x)$ if $x < 10^{18}$ $k(0x) > k(11x)$ if $x > 10^{18}$ $\approx 5,08 \cdot 10^7$ primes lesser than 10^9
3. For the prime 31: $k(0x) < k(31x)$ if $x < 10^{50}$ $k(0x) > k(31x)$ if $x > 10^{50}$ $\approx 1,76 \cdot 10^{23}$ primes lesser than 10^{25}
4. For the prime 997: $k(0x) < k(997x)$ if $x < 10^{1620}$ $k(0x) > k(997x)$ if $x > 10^{1620}$ $\approx 5,36 \cdot 10^{806}$ primes lesser than 10^{810}

By analyzing these data we can see that, for numbers lesser than $10^{11,4}$, $k(0x)$ is lesser than all factors $k(px)$ and, therefore, also will be lesser than $k(jx)$ which allows us to ensure that the Goldbach conjecture will be met, at least until $10^{11,4}$.
For the x values greater than $10^{11,4}$, we can see that $k(0x)$ overcomes gradually the factors $k(px)$ ($k(7x)$, $k(11x)$, $k(13x)$, $k(17x)$,…, $k(997x)$,…).
Looking in detail, we can note that if the p value, for which the comparison is applied, increases in geometric progression, the x value from which $k(0x)$ exceeds to $k(px)$ increases __ exponentially. Because of this, also increases __ exponentially (or slightly higher) the number of primes lesser than $\sqrt{x}$ and whose factors $k(px)$ will determine the $k(jx)$ value.
Logically, if increases the number of primes lesser than $\sqrt{x}$, decreases the "relative weight" of each factor $k(px)$ in relation to the $k(jx)$ value. Thus, although from $10^{11,4}$ $k(7x)$ is lesser than $k(0x)$, the percentage of terms $(7j + a)$ which are not in upper groups will decrease and the factor $k(7x)$ will lose gradually influence on the $k(jx)$ value.
The same can be applied to the factors $k(11x)$, $k(13x)$, $k(17x)$,… that will lose gradually influence on the $k(jx)$ value with increasing x.

On the other hand, taking as an example the prime 997, we can note that, when $k(0x)$ exceeds $k(997x)$, there are already $\approx 5{,}36 \cdot 10^{806}$ primes whose factors $k(px)$ (which will be greater than $k(0x)$) added to factors $k(7x)$ to $k(997x)$ (165 factors that will be lesser than $k(0x)$) will determine the $k(jx)$ value. Note the large difference between 165 and $\approx 5{,}36 \cdot 10^{806}$.

These data allow us to intuit that $k(jx)$ will be greater than $k(0x)$ for any x value.

After these positive data, we continue developing the formula to calculate the approximate value of $k(jx)$.

Let us compare $k(bx)$ with $k(jx)$. Let us recall the definitions relating to these two factors.

$k(bx)$ = ratio between the number of multiples and the total number of terms of sequence **B**.

Sequence **B**	$\frac{x}{30}$ terms	$\pi(bx)$ primes	$\frac{x}{30} - \pi(bx)$ multiples	$k(bx) = 1 - \frac{30\pi(bx)}{x}$
Terms of sequence **B**	1/7 are multiples of 7,	1/11 are multiples of 11,	1/13 are multiples of 13,	1/17 are multiples of 17,…

And so on to the prime previous to $\sqrt{x}$.

$k(jx)$ = ratio between the number of multiples that there are in the set of all terms $(7j_{21} + a)$, $(11j_{22} + b)$, $(13j_{23} + c)$, $(17j_{24} + d)$,… of sequence **B** and the total number of these. Its value is determined by the values of the factors $k(7x)$, $k(11x)$, $k(13x)$, $k(17x)$,…

As described when we applied the prime numbers theorem in arithmetic progressions, the actual number of primes that there are in each groups $(7j_{21} + a)$, $(11j_{22} + b)$, $(13j_{23} + c)$, $(17j_{24} + d)$,… will be, approximately, equal to the average value indicated.
In the case of an even number that would be multiple of any prime greater than 5 (for example 13), there would not be primes in the group $(13j_{23} + c)$ because $c = 0$, and its corresponding factor $k(13x)$ would equal 1, which, in the end, would slightly increase the $k(jx)$ value.

Group $(7j + a)$	$\frac{1}{7}\frac{x}{30}$ terms	$\approx \frac{\pi(bx)}{6}$ primes	$\approx \left(\frac{1}{7}\frac{x}{30} - \frac{\pi(bx)}{6}\right)$ multiples	$k(7x) \approx 1 - \frac{30\pi(bx)}{x}\frac{7}{6}$
Terms $(7j + a)$	No multiples of 7,	1/11 are multiples of 11,	1/13 are multiples of 13,	1/17 are multiples of 17,…
Group $(11j + b)$	$\frac{1}{11}\frac{x}{30}$ terms	$\approx \frac{\pi(bx)}{10}$ primes	$\approx \left(\frac{1}{11}\frac{x}{30} - \frac{\pi(bx)}{10}\right)$ multiples	$k(11x) \approx 1 - \frac{30\pi(bx)}{x}\frac{11}{10}$
Terms $(11j + b)$	1/7 are multiples of 7,	no multiples of 11,	1/13 are multiples of 13,	1/17 are multiples of 17,…
Group $(13j + c)$	$\frac{1}{13}\frac{x}{30}$ terms	$\approx \frac{\pi(bx)}{12}$ primes	$\approx \left(\frac{1}{13}\frac{x}{30} - \frac{\pi(bx)}{12}\right)$ multiples	$k(13x) \approx 1 - \frac{30\pi(bx)}{x}\frac{13}{12}$
Terms $(13j + c)$	1/7 are multiples of 7,	1/11 are multiples of 11,	no multiples of 13,	1/17 are multiples of 17,…
Group $(17j + d)$	$\frac{1}{17}\frac{x}{30}$ terms	$\approx \frac{\pi(bx)}{16}$ primes	$\approx \left(\frac{1}{17}\frac{x}{30} - \frac{\pi(bx)}{16}\right)$ multiples	$k(17x) \approx 1 - \frac{30\pi(bx)}{x}\frac{17}{16}$
Terms $(17j + d)$	1/7 are multiples of 7,	1/11 are multiples of 11,	1/13 are multiples of 13,	no multiples of 17,…

And so on to the prime previous to $\sqrt{x}$.

It can be noted that, in compliance to the prime numbers theorem in arithmetic progressions, the groups $(7j_{21} + a)$, $(11j_{22} + b)$, $(13j_{23} + c)$, $(17j_{24} + d)$,… behave with some regularity, mathematically defined, for the number of terms, the number of primes and the number of multiples that contain, and that is maintained regardless of x value.

Continuing the study of these terms we can see some data, obtained with a programmable controller, that refers to the group $(30n + 29)$ (chosen as example) and the even numbers 10^6, 10^7, 10^8 and 10^9.
Although for this analysis, any sequence of primes can be chosen, I will do it in ascending order (7, 11, 13, 17, 19, 23,…, 307).
They are the following data, and are numbered as follows:

1. Total number of terms $(7j + a)$, $(11j + b)$, $(13j + c)$, $(17j + d)$,…, $(pj + h)$,…
2. Multiples that there are in the group $(7j + a)$: they are all included.
3. Multiples that there are in the group $(11j + b)$: not included those who are also $(7j + a)$.
4. Multiples that there are in the group $(13j + c)$: not included those who are also $(7j + a)$ or $(11j + b)$.
5. Multiples that there are in the group $(17j + d)$: not included those who are also $(7j + a)$ or $(11j + b)$ or $(13j + c)$.

And so on until the group of prime 307. These data can be consulted from page 21.
The percentages indicated are relative to the total number of terms $(7j + a)$, $(11j + b)$, $(13j + c)$, $(17j + d)$,…, $(pj + h)$,…

	10^6		10^7		10^8		10^9	
Terms $(7j + a)$, $(11j + b)$,…	23.545		250.287		2.613.173		26.977.564	
Multiples $(7j + a)$ and %	3.140	13,34 %	33.750	13,48 %	356.077	13,63 %	3.702.786	13,73 %
Multiples $(11j + b)$ and %	1.767	7,5 %	19.045	7,61 %	199.519	7,63 %	2.067.520	7,66 %
Multiples $(13j + c)$ and %	1.374	5,84 %	14.714	5,88 %	154.831	5,92 %	1.600.628	5,93 %
Multiples $(17j + d)$ and %	1.010	4,29 %	10.549	4,21 %	110.081	4,21 %	1.137.457	4,22 %
Total multiples groups 7 to 307	15.008	63,74 %	156.956	62,71 %	1.642.061	62,84 %	17.014.540	63,07 %

These new data continue to confirm that the groups $(7j_{21} + a)$, $(11j_{22} + b)$, $(13j_{23} + c)$, $(17j_{24} + d)$,… behave in a uniform manner, because the percentage of multiples that supply each is almost constant when $\boldsymbol{x}$ increases.

The regularity of these groups allows us to intuit that the approximate value of $k(jx)$ can be obtained by a general formula.

Considering the data of each group, and to develop the formula of $k(jx)$, we can think about adding, on one hand, the number of terms of all of them, on the other hand, the number of primes and finally the number of multiples and making the final calculations with the total of these sums. This method is not correct, since each term can be in several groups so they would be counted several times what would give us an unreliable result.
To resolve this question in a theoretical manner, but more accurate, each term $(7j_{21} + a)$, $(11j_{22} + b)$, $(13j_{23} + c)$, $(17j_{24} + d)$,… should be analyzed individually and applying inclusion-exclusion principle, to define which are multiples and those who are primes.
After several attempts, I have found that this analytical method is quite complex, so that in the end, I rejected it.
I hope that any mathematician interested in this topic may resolve this question in a rigorous way.
Given the difficulty of the mathematical analysis, I opted for an indirect method to obtain the formula for $k(jx)$.

Gathering information from the Internet of the latest demonstrations of mathematical conjectures, I have read that it has been accepted the use of computers to perform some of calculations or to verify the conjectures up to a certain number.
Given this information, I considered that I can use a programmable logic controller (PLC) to help me get the formula for $k(jx)$. To this purpose, I have developed the programs that the controller needs to perform this work.
I will begin by analyzing the exposed data from which it can be deduced:

1. The concepts of $k(jx)$ and $k(bx)$ are similar so, in principle, their formulas will use the same variables.
2. The parameters (number of terms, number of primes and number of multiples) involved in $k(jx)$ follow a certain "pattern".
3. The $k(jx)$ and $k(bx)$ values, and also those of $\boldsymbol{\pi(ax)}$ and $\boldsymbol{\pi(bx)}$, gradually increase with increasing $\boldsymbol{x}$.
4. The $k(jx)$ value is lesser than the $k(bx)$ value (this may vary if the even number is a multiple of primes greater than 5).
5. The values of $k(jx)$ and $k(bx)$ will tend to equalize, in an asymptotically way, when $\boldsymbol{x}$ tends to infinite.

Here are some values, obtained by the controller, concerning to $k(bx)$, $k(jx)$ and the group $(30n + 29)$, (consult from page 21).

1. To 10^6	$k(bx) = 0{,}706447064$	$k(jx) = 0{,}698577192$	$k(jx) / k(bx) = 0{,}988859927$
2. To 10^7	$k(bx) = 0{,}751107751$	$k(jx) = 0{,}74568795$	$k(jx) / k(bx) = 0{,}992784256$
3. To 10^8	$k(bx) = 0{,}784035978$	$k(jx) = 0{,}780284734$	$k(jx) / k(bx) = 0{,}995215469$
4. To 10^9	$k(bx) = 0{,}809322428$	$k(jx) = 0{,}806541984$	$k(jx) / k(bx) = 0{,}996564479$
5. To $7{\cdot}10^6$, (multiple of 7)	$k(bx) = 0{,}744631063$	$k(jx) = 0{,}755471932$	$k(jx) / k(bx) = 1{,}014558712$

By analyzing these data, it can be seen that, as $\boldsymbol{x}$ increases, the $k(jx)$ value tends more rapidly to the $k(bx)$ value that the $k(bx)$ value with respect to 1.
Expressed numerically: To 10^6: $(1 – 0{,}706447064) / (0{,}706447064 – 0{,}698577192) = 37{,}3$
To 10^9: $(1 – 0{,}809322428) / (0{,}809322428 – 0{,}806541984) = 68{,}57$

Then, based on the formulas for $k(bx)$ and $k(0x)$, I will propose a formula for $k(jx)$ with a constant. To calculate its value, I will use the programmable controller.

Formula of $k(bx)$: $\boldsymbol{k(bx)} = \boldsymbol{1} - \dfrac{\boldsymbol{30\pi(bx)}}{\boldsymbol{x}}$ Formula of $k(0x)$: $\boldsymbol{k(0x)} = \boldsymbol{1} - \dfrac{\boldsymbol{30\pi(bx)}}{\boldsymbol{x} - \boldsymbol{30\pi(ax)}}$

Proposed formula for $k(jx)$: $\boldsymbol{k(jx)} = \boldsymbol{1} - \dfrac{\boldsymbol{30\pi(bx)}}{\boldsymbol{x} - \boldsymbol{C(jx)\pi(ax)}}$

Being: $\boldsymbol{x}$ = Even number for which the conjecture is applied and that defines the sequences **A-B**.
$\boldsymbol{\pi(ax)}$ = Number of primes greater than $\sqrt{\boldsymbol{x}}$ in sequence **A** for $\boldsymbol{x}$.
$\boldsymbol{\pi(bx)}$ = Number of primes greater than $\sqrt{\boldsymbol{x}}$ in sequence **B** for $\boldsymbol{x}$.
$\boldsymbol{k(jx)}$ = Factor in study. The data from the PLC allow calculate its value for various numbers $\boldsymbol{x}$.
$\boldsymbol{C(jx)}$ = Constant that can be calculated if we know the values of $\boldsymbol{\pi(ax)}$, $\boldsymbol{\pi(bx)}$ and $\boldsymbol{k(jx)}$ for each number $\boldsymbol{x}$.

Let us recall that $k(jx)$ is lesser than $k(bx)$ so, comparing their corresponding formulas, it follows that $c(jx)$ would have a minimum value of 0. Also let us remember that, as a concept, $k(0x)$ would be the minimum value of $k(jx)$ for which the conjecture would not be met. According to this statement, and comparing their corresponding formulas, it follows that $c(jx)$ would have a maximum value of 30.

The program, which works in the programmable controller, is described below, in a simplified way:

1. It store the 3.398 primes that are lesser than 31.622. With them, we can analyze the sequences **A-B** until number 10^9.
2. It divides the even number $\boldsymbol{x}$ ($\leq 10^9$) by the primes lesser than $\sqrt{\boldsymbol{x}}$. The remains of these divisions are the values of a, b, c, d,…
3. It divides each term of each sequence **A** or **B**, by the primes lesser than $\sqrt{\boldsymbol{x}}$, to define which are multiples and those who are primes.
4. In the same process, it determines the terms that are of form $(7j + a)$, $(11j + b)$, $(13j + c)$, $(17j + d)$,… in each sequence **A** or **B**.
5. 8 counters are scheduled (4 in each sequence) to count the following data:
   6. Number of multiples that there are in each sequence **A** or **B** (it includes all composites and the primes who are lesser than $\sqrt{\boldsymbol{x}}$).
   7. Number of primes that there are in each sequence **A** or **B** (only which are greater than $\sqrt{\boldsymbol{x}}$).
   8. Number of multiples that there are in the terms $(7j + a)$, $(11j + b)$, $(13j + c)$, $(17j + d)$,… of each sequence **A** or **B** (as 6).
   9. Number of primes that there are in the terms $(7j + a)$, $(11j + b)$, $(13j + c)$, $(17j + d)$,… of each sequence **A** or **B** (as 7).
10. With the final data of these counters, and using a calculator, the values of $k(ax)$, $k(bx)$, $k(jx)$, $c(jx)$,…can be obtained.

Then, I indicate the calculated values of $c(jx)$ related to some even numbers (between 10^6 and 10^9) and their corresponding groups of primes. The details of these calculations can be consulted in the numerical data presented from page 21.

	(30*n* + 11) + (30*n* + 29)		(30*n* + 17) + (30*n* + 23)	
10^6	2,668	2,668	2,714	2,711
10^7	2,566	2,566	2,371	2,371
10^8	2,371	2,371	2,423	2,423
10^9	2,261	2,261	2,256	2,256

	(30*n* + 1) + (30*n* + 19)		(30*n* + 7) + (30*n* + 13)	
$8 \cdot 10^6$	2,697	2,696	2,732	2,732
$8 \cdot 10^7$	2,439	2,439	2,35	2,35

	(30*n* + 1) + (30*n* + 29)		(30*n* + 7) + (30*n* + 23)		(30*n* + 11) + (30*n* + 19)		(30*n* + 13) + (30*n* + 17)	
$9 \cdot 10^6$	2,744	2,742	2,401	2,4	2,615	2,616	2,603	2,603
$9 \cdot 10^7$	2,309	2,309	2,223	2,223	2,42	2,42	2,474	2,474
$3 \cdot 10^8$	2,308	2,308	2,35	2,35	2,34	2,34	2,288	2,288

	(30*n* + 11) + (30*n* + 23)	
$4.194.304 = 2^{22}$	2,705	2,705
$67.108.864 = 2^{26}$	2,378	2,378

	(30*n* + 1) + (30*n* + 7)	
$8.388.608 = 2^{23}$	2,526	2,523
$134.217.728 = 2^{27}$	2,354	2,354

	(30*n* + 17) + (30*n* + 29)		(30*n* + 23) + (30*n* + 23)	
$16.777.216 = 2^{24}$	2,283	2,282		
$268.435.456 = 2^{28}$	2,365	2,365	2,237	2,279

	(30*n* + 13) + (30*n* + 19)	
$33.554.432 = 2^{25}$	2,477	2,477
$536.870.912 = 2^{29}$	2,3	2,301

	(30*n* + 11) + (30*n* + 29)		(30*n* + 17) + (30*n* + 23)	
$7 \cdot 10^6$, (multiple of 7)	– 5,214	– 5,212	– 5,372	– 5,373

	(30*n* + 11) + (30*n* + 17)	
$14.872.858 = 2 \cdot 7 \cdot 11 \cdot 13 \cdot 17 \cdot 19 \cdot 23$	– 30,3	– 30,31

To obtain the $c(jx)$ values for numbers greater than 10^9, I have used actual data from Wikipedia concerning to the Twin Primes Conjecture which says: “There are infinitely many primes p such that $(p + 2)$ is also prime.”
We call *Twin Primes* the pair of consecutive primes that are separated only by an even number.
Applying to this conjecture, a similar procedure that has been applied to Goldbach's conjecture, it results in the next axiom:
All multiples $7m_{11}$, $11m_{12}$, $13m_{13}$, $17m_{14}$,… (including the primes lesser than $\sqrt{\boldsymbol{x}}$ that are present) of sequence **A** are paired, respectively, with all terms $(7m_{11} + 2)$, $(11m_{12} + 2)$, $(13m_{13} + 2)$, $(17m_{14} + 2)$,… of sequence **B**.
Accordingly, the following average values of $c(jx)$ are referring to the terms $(7m_{11} + 2)$, $(11m_{12} + 2)$, $(13m_{13} + 2)$, $(17m_{14} + 2)$,… of sequence **B**. For more details, consult the numerical data presented from page 32.

10^{10}	≈ 2,095	10^{12}	≈ 2,058	10^{14}	≈ 2,029	10^{16}	≈ 2,005
10^{11}	≈ 2,075	10^{13}	≈ 2,042	10^{15}	≈ 2,016	10^{18}	≈ 1,987

Consulting the numeric calculations presented from page 21 to 32, we can note that the axiom which has been used as a starting point at the beginning of this chapter is met:

1. The number of multiples $7m_{11}$, $11m_{12}$,… of sequence **A** is equal to the number of terms $(7j_{21} + a)$, $(11j_{22} + b)$,… of sequence **B**.
2. The number of terms $(7j_{11} + a)$, $(11j_{12} + b)$,… of sequence **A** is equal to the number of multiples $7m_{21}$, $11m_{22}$,… of sequence **B**.
3. The number of multiples that there are in the terms $(7j_{11} + a)$, $(11j_{12} + b)$,… of sequence **A** is equal to the multiples in the terms $(7j_{21} + a)$, $(11j_{22} + b)$,… of sequence **B**, being the number of multiple-multiple pairs that are formed with the two sequences.

Let's review the above data:

1. Lowest number analyzed: 10^6.
2. Highest number analyzed with the programmable controller: 10^9.
3. Highest number analyzed with data from Wikipedia: 10^{18}.
4. Highest $c_{(jx)}$ value: 2,744 for the even number $9 \cdot 10^6$ in the combination $(30n + 1) + (30n + 29)$.
5. Lowest $c_{(jx)}$ value with the programmable controller: 2,223 for the number $9 \cdot 10^7$ in the combination $(30n + 7) + (30n + 23)$.
6. Lowest $c_{(jx)}$ value with data from Wikipedia: 1,987 for the number 10^{18} (average value) (referring to the twin primes conjecture).
7. Maximum number of terms analyzed by programmable controller in a sequence **A** or **B**: 33.333.333 for the number 10^9.

In the analyzed numbers with PLC, 10^9 is 10^3 times greater than 10^6. Using data from Wikipedia, 10^{18} is 10^{12} times greater than 10^6.
It can be seen that, although there is a great difference between the values of the analyzed numbers, the $c_{(jx)}$ values vary little (from 2,744 to 2,223 with PLC and up to 1,987 with data from Wikipedia).
Looking in detail, it can be seen that for numbers greater than $16.777.216 = 2^{24}$, the $c_{(jx)}$ value is lesser than 2,5. We also note that the average value of $c_{(jx)}$ tends to decrease slightly when increasing $\boldsymbol{x}$.
Finally, it can be intuited that, for large values of $\boldsymbol{x}$, the average value of $c_{(jx)}$ tends to an approximate value to 2,3.
I believe that this data is sufficiently representative to be applied in the proposed formula for $k_{(jx)}$.

Given the above, we can define an approximate average value for $c_{(jx)}$: $c_{(jx)} \approx 2{,}5$ (for large numbers: $c_{(jx)} \approx 2{,}3$)

With this average value of $c_{(jx)}$, the final formula of $k_{(jx)}$ can be written: $\boldsymbol{k_{(jx)} \approx 1 - \dfrac{30\pi_{(bx)}}{x - 2{,}5\pi_{(ax)}}}$

I consider that this formula is valid to prove the conjecture although it has not been obtained through mathematical analysis.
Also, I consider that it can be applied to large numbers because the regularity in the characteristics of terms $(7j_{21} + a)$, $(11j_{22} + b)$, $(13j_{23} + c)$, $(17j_{24} + d)$,… is maintained, and I intuit that with more precision, when increasing $\boldsymbol{x}$.
Likewise, I believe that this formula and the formula that can be obtained through a rigorous analytical method can be considered equivalent in purpose of validity to prove the conjecture although the respective numerical results may differ slightly.

Let us analyze the deviation that can affect the average value defined for $c_{(jx)}$. Considering only the even numbers that are not multiples of primes greater than 5, $c_{(jx)}$ it would have a minimum value greater than 0 because, in this case, $k_{(jx)}$ is always lesser than $k_{(bx)}$.
We can see that the maximum deviation decreasing is from 2,5 to 0 (or close to 0). I understand that, by symmetry, the maximum deviation increasing will be similar so that, in principle, the $c_{(jx)}$ value would always be lesser than 5.

On the other hand, and as I have indicated, $c_{(jx)}$ would have a maximum value of 30. Considering as valid the final formula proposed for $k_{(jx)}$, considering that will be equivalent to analytical formula and comparing 30 with the calculated values of $c_{(jx)}$, (between 2,744 and 2,223), it can be accepted that $c_{(jx)} < 30$ will always be met.

At this point, let's make a summary of the exposed questions:

1. All multiples $7m_{11}$, $11m_{12}$, $13m_{13}$,… of sequence **A** are paired with all terms $(7j_{21} + a)$, $(11j_{22} + b)$, $(13j_{23} + c)$,… of sequence **B**.
2. The groups $(7j_{21} + a)$,… follow a "pattern" for the number of terms, number of primes and number of multiples that contain.
3. We define as $k_{(jx)}$ the fraction of terms $(7j_{21} + a)$, $(11j_{22} + b)$, $(13j_{23} + c)$,… of sequence **B** that are multiples.
4. The analysis of paragraph 2 allows us to intuit that the approximate value of $k_{(jx)}$ can be obtained by a general formula.
5. Proposed formula for $k_{(jx)}$: $\boldsymbol{k_{(jx)} = 1 - \dfrac{30\pi_{(bx)}}{x - c_{(jx)}\pi_{(ax)}}}$. In the exposed calculations, the $c_{(jx)}$ value has resulted to be lesser than 3.
6. Final formula for $k_{(jx)}$: $\boldsymbol{k_{(jx)} \approx 1 - \dfrac{30\pi_{(bx)}}{x - 2{,}5\pi_{(ax)}}}$. I consider that will be equivalent to the formula obtained by mathematical analysis.
7. Considering valid the above formula and considering the calculated values of $c_{(jx)}$, (< 3), it can be accepted that $c_{(jx)} < 30$.
8. Applying $c_{(jx)} < 30$ in the proposed formula for $k_{(jx)}$: $\boldsymbol{k_{(jx)} > 1 - \dfrac{30\pi_{(bx)}}{x - 30\pi_{(ax)}} = k_{(0x)}}$
9. Finally, for any $\boldsymbol{x}$ value: $\boldsymbol{k_{(jx)} > k_{(0x)}}$. This statement must be rigorously demonstrated in the analytical formula.

Let us recall, page 10, the formula to calculate the pairs number of primes greater than $\sqrt{x}$ that are formed with the sequences **A-B**.

$$P_{PP(x)} = \pi_{(bx)} - (1 - k_{(jx)})\left(\frac{x}{30} - \pi_{(ax)}\right) \qquad \text{Substituting } k_{(jx)} \text{ for its formula:} \qquad k_{(jx)} = 1 - \frac{30\pi_{(bx)}}{x - c_{(jx)}\pi_{(ax)}}$$

$$P_{PP(x)} = \pi_{(bx)} - \frac{30\pi_{(bx)}}{x - c_{(jx)}\pi_{(ax)}}\left(\frac{x}{30} - \pi_{(ax)}\right) = \pi_{(bx)} - \frac{x\pi_{(bx)} - 30\pi_{(ax)}\pi_{(bx)}}{x - c_{(jx)}\pi_{(ax)}} = \frac{x\pi_{(bx)} - c_{(jx)}\pi_{(ax)}\pi_{(bx)} - x\pi_{(bx)} + 30\pi_{(ax)}\pi_{(bx)}}{x - c_{(jx)}\pi_{(ax)}}$$

$$P_{PP(x)} = \frac{(30 - c_{(jx)})\pi_{(ax)}\pi_{(bx)}}{x - c_{(jx)}\pi_{(ax)}}$$

In this formula, we can replace $c_{(jx)}$ by its already defined values:

$c_{(jx)} \approx 2{,}5$ $\qquad P_{PP(x)} \approx \frac{(30 - 2{,}5)\pi_{(ax)}\pi_{(bx)}}{x - 2{,}5\pi_{(ax)}}$ $\qquad P_{PP(x)} \approx \frac{27{,}5\pi_{(ax)}\pi_{(bx)}}{x - 2{,}5\pi_{(ax)}}$

$c_{(jx)} < 30$ $\qquad P_{PP(x)} > \frac{(30 - 30)\pi_{(ax)}\pi_{(bx)}}{x - 30\pi_{(ax)}}$ $\qquad P_{PP(x)} > 0$

This final expression indicates that $P_{PP(x)}$ is always greater than 0 and considering that by its nature, (prime pairs), cannot be a fractional number (must be greater than 0, cannot have a value between 0 and 1) I gather that $P_{PP(x)}$ will be a natural number equal to or greater than 1. Similarly, I conclude that the $P_{PP(x)}$ value will increase when increasing x because also increase $\pi_{(ax)}$ and $\pi_{(bx)}$.
We can record:

$P_{PP(x)} \geq 1$ $\qquad$ $P_{PP(x)}$ will be a natural number and will increase when increasing x

The above expression indicates that the pairs number of primes greater than $\sqrt{x}$ that meet the conjecture for an even number x is always equal to or greater than 1.

With everything described, it can be confirmed that: **<u>The Goldbach Conjecture is true.</u>**

10. Final formula.

Considering that the conjecture has already been demonstrated, a formula can be defined to calculate the approximate number of partitions for an even number x.
According to the previous chapter, the number of these partitions, formed with sequences **A-B**, greater than $\sqrt{x}$ and lesser than x is:

$$P_{PP(x)} \approx \frac{27{,}5\pi_{(ax)}\pi_{(bx)}}{x - 2{,}5\pi_{(ax)}}$$

If no precision in the final formula is required, and for large values of x, the following can be considered:

1. On page 8 I have indicated that: $\pi_{(ax)} \approx \pi_{(bx)} \approx \frac{\pi_{(x)}}{8}$ being $\pi_{(x)}$ the number of primes lesser than or equal to x.
2. The term $2{,}5\pi_{(ax)}$ can be neglected because it is be very small compared to x, (1,59 % of x for 10^9), (0,77 % of x for 10^{18}).
3. By applying the above, the value of the denominator will increase, so, to compensate, I will put in the numerator 28 instead of 27,5.
4. The exposed data allows us to intuit that, as x is larger, the average value of $c_{(jx)}$ will decrease being lesser than 2,5.
5. The possible pairs of primes with one of them lesser than $\sqrt{x}$ is very small compared to the total pairs number of primes.

With this in mind, the above formula can be slightly modified to make it more simple.
As a final concept, I consider that the numeric result of the obtained formula will be the approximate number of partitions which are formed with the sequences **A** and **B** and that meet Goldbach's conjecture for an even number x.

$$P_{PP(x)} \approx \frac{28\frac{\pi_{(x)}}{8}\frac{\pi_{(x)}}{8}}{x} \qquad P_{PP(x)} \approx \frac{7}{16}\frac{\pi^2_{(x)}}{x}$$

Now, let's analyze the halves sequences (page 4). In this case, the sequences **A** and **B** are formed with the same group of primes.
Let us recall the number 784, used as example at the beginning.

$784 = 30 \cdot 26 + 4 = (30n_4 + 17) + (30n_5 + 17)$ being: $26 = n_4 + n_5 + 1$

We will write the sequence **A** of all numbers $(30n_4 + 17)$ from 0 to 784. (We write the complete sequence in two halves).
Also we will write the sequence **B** of all numbers $(30n_5 + 17)$ from 784 to 0. (We write the complete sequence in two halves).

A	**17 - 47 -** 77- **<u>107</u>-<u>137</u>-<u>167</u>-<u>197</u>-<u>227</u>-257-**287-**<u>317</u>-347-**377	407-437-**<u>467</u>**-497-527-**<u>557</u>-<u>587</u>-<u>617</u>-<u>647</u>-<u>677</u>**-707-737-767
B	767-737-707-**<u>677</u>-<u>647</u>-<u>617</u>-<u>587</u>-<u>557</u>**-527-497-**<u>467</u>**-437-407	377-**347-<u>317</u>**-287-**257-<u>227</u>-<u>197</u>-<u>167</u>-<u>137</u>-<u>107</u>** - 77 - 47 **- 17**

We can see that these two sequences have the same terms written in reverse order, so that the pairs of terms are repeated. If we apply the same procedure as used for the complete sequences, we will obtain the same result. In this case, the number of different pairs of primes that will meet the conjecture will be half of those in the complete sequence.

Then, and considering what has been described for the halves sequences, we will adjust the last formula (that uses 2 groups) to the number of groups of primes used (3, 6, 4 or 8) in each even number multiplying $\frac{7}{16}\frac{\pi^2(x)}{x}$, respectively, by 3/2, 3, 2 or 4.

Performing the multiplications described and being **G**(x),… the actual number of Goldbach's partitions for an even number x:

$$G(x) \approx \frac{21}{32}\frac{\pi^2(x)}{x}$$ Partitions number for the even number that is not multiple of 6 or 10.

$$G_6(x) \approx \frac{21}{16}\frac{\pi^2(x)}{x}$$ Partitions number for the even number that is multiple of 6.

$$G_{10}(x) \approx \frac{7}{8}\frac{\pi^2(x)}{x}$$ Partitions number for the even number that is multiple of 10.

$$G_{30}(x) \approx \frac{7}{4}\frac{\pi^2(x)}{x}$$ Partitions number for the even number that is multiple of 30.

Final formulas, being: $G(x)$, $G_6(x)$, $G_{10}(x)$, $G_{30}(x)$ = Actual number of Goldbach's partitions for the even numbers x.
x = Even number greater than 30.
$\pi(x)$ = Number of primes lesser than or equal to x.

Although they could be obtained by the programmable controller, we will take the actual values of $\pi(x)$ from Wikipedia to check the precision of formulas of $G_{10}(x)$ and $G(x)$. Although we will only check these two, I consider that the precision of the four above formulas will be similar.

	$\pi(x)$	$G_{10}(x)$ (PLC)	Formula result	Difference
1. To 10^6	78.498	5.382	5.392	+ 0,185 %
2. To 10^7	664.579	38.763	38.646	– 0,302 %
3. To 10^8	5.761.455	291.281	290.451	– 0,285 %
4. To 10^9	50.847.534	2.273.918	2.262.288	– 0,511 %

	$\pi(x)$ (PLC)	$G(x)$ (PLC)	Formula result	Difference
5. To 2^{28} = 268.435.456	14.630.810	525.109	523.319	– 0,341 %

To express the final formulas as an x function, we will use the prime numbers theorem[3], (page 8): $\pi(x) \sim \frac{x}{\ln(x)}$
Substituting $\pi(x)$ in the above formulas and simplifying:

$G(x) \sim \frac{21}{32}\frac{x}{\ln^2(x)}$ Partitions number for the even number that is not multiple of 6 or 10. To 16.384, formula: 114 partitions, actual: 151

$G_6(x) \sim \frac{21}{16}\frac{x}{\ln^2(x)}$ Partitions number for the even number that is multiple of 6. To 13.122 formula: 191 partitions, actual: 245

$G_{10}(x) \sim \frac{7}{8}\frac{x}{\ln^2(x)}$ Partitions number for the even number that is multiple of 10. To 31.250 formula: 255 partitions, actual: 326

$G_{30}(x) \sim \frac{7}{4}\frac{x}{\ln^2(x)}$ Partitions number for the even number that is multiple of 30. To 21.870 formula: 383 partitions, actual: 483

The sign ~ indicates that these formulas have an asymptotic behavior, giving results lesser than actual values when applied to small numbers but this difference gradually decreases as we analyze larger numbers.

If the even number is multiple of one or more primes greater than 5, and as seen on page 6, increases the pairs proportion of composites so that, ultimately, more pairs of primes will be formed.
Due to this, the difference between the actual number of partitions and the corresponding formula result will increase.
Examples:

To 16.016	Formula result:	112 partitions.	Actual: 193 partitions.	Multiple of 7, 11 and 13.
To 16.018	Formula result:	112 partitions.	Actual: 152 partitions.	

A better approach for this theorem is given by the offset logarithmic integral function[3] **Li**(x): $\boldsymbol{\pi}(x) \approx \mathbf{Li}(x) = \int_2^x \frac{dy}{\ln(y)}$

Substituting $\boldsymbol{\pi}(x)$ again, we obtain four more precise formulas:

$\mathbf{G}(x) \approx \frac{21}{32}\int_2^x \frac{dy}{\ln^2(y)}$ Partitions number for the even number that is not multiple of 6 or 10.

$\mathbf{G_6}(x) \approx \frac{21}{16}\int_2^x \frac{dy}{\ln^2(y)}$ Partitions number for the even number that is multiple of 6.

$\mathbf{G_{10}}(x) \approx \frac{7}{8}\int_2^x \frac{dy}{\ln^2(y)}$ Partitions number for the even number that is multiple of 10.

$\mathbf{G_{30}}(x) \approx \frac{7}{4}\int_2^x \frac{dy}{\ln^2(y)}$ Partitions number for the even number that is multiple of 30.

11. Comparison with the Twin Primes Conjecture.

Twin Primes Conjecture statement[4]: "There are infinitely many primes p such that p + 2 is also prime".
We call *Twin Primes* the pair of consecutive primes that are separated only by an even number. Examples: (**11**, **13**), (**29**, **31**).

Goldbach's conjecture and the twin primes conjecture are similar in that both can be studied by combining two groups of primes to form pairs that add an even number, in the first, or pairs of twin primes in the second.

We will write the three combinations of groups of primes with which all pairs of twin primes greater than 7 will be formed:

$(30n_1 + 11)$ and $(30n_1 + 13)$ $(30n_2 + 17)$ and $(30n_2 + 19)$ $(30n_3 + 29)$ and $(30n_3 + 31)$

We will write the sequences **A** and **B** corresponding to number 780 and the combination $(30n_1 + 11)$ and $(30n_1 + 13)$, underlining the 11 twin prime pairs that are formed. We use list of primes lesser than 1.000.

A <u>**11**</u>-<u>**41**</u>-<u>**71**</u>-<u>**101**</u>-**131**-161-<u>**191**</u>-221-**251**-<u>**281**</u>-<u>**311**</u>-341-371-**401**-<u>**431**</u>-<u>**461**</u>-**491**-<u>**521**</u>-551-581-611-<u>**641**</u>-671-**701**-731-**761**
B <u>**13**</u>-<u>**43**</u>-<u>**73**</u>-<u>**103**</u>-133-**163**-<u>**193**</u>-**223**-253-<u>**283**</u>-<u>**313**</u>-343-**373**-403-<u>**433**</u>-<u>**463**</u>-493-<u>**523**</u>-553-583-**613**-<u>**643**</u>-**673**-703-**733**-763

A first difference between these two conjectures refers to the order of the terms in sequences **A-B**.
In Goldbach's conjecture, the terms are in reverse order (from lowest to highest in sequence **A** and from highest to lowest in sequence **B**) whilst in the twin primes conjecture, the terms of both sequences are in the same order (from lowest to highest).

Let us recall that the probability of a natural number being prime, decreases when increasing its value, so analyzing the sequences **A-B** of Goldbach's conjecture, we see that the two terms of each pairs that are formed have different probability to be primes because one of them has a value between 0 and $\boldsymbol{x}$/2 and the other between $\boldsymbol{x}$/2 and $\boldsymbol{x}$.
On the other hand, analyzing the sequences **A-B** of the twin primes conjecture we see that the two terms of each pairs that are formed have virtually, the same probability to be primes since the difference between them is only two units.

Given the above, I conclude that in the Goldbach conjecture there is a greater "difficulty" to form pairs of primes.
As seen on page 6, this "difficulty" is lesser if the even number is a multiple of one or more primes greater than 5, because more pairs of composites are formed and, therefore, also more pairs of primes will be formed.

A second difference refers to the groups of terms analyzed in each demonstration that I have developed.
In the proof of Goldbach's conjecture, we analyzed the groups of terms ($7j + a$), ($11j + b$), ($13j + c$), ($17j + d$),… of sequence **B** (or **A**).
In the proof of twin primes conjecture, we analyzed the groups of terms ($7m + 2$), ($11m + 2$), ($13m + 2$), ($17m + 2$),… of sequence **B**.
In the latter, also we can analyze the groups of terms ($7m – 2$), ($11m – 2$), ($13m – 2$), ($17m – 2$),… of sequence **A**.

Let us recall that the numbers a, b, c, d,… that appear in the proof of Goldbach's conjecture, are the remains of dividing the even number $\boldsymbol{x}$ by the primes from 7 to the prime previous to $\sqrt{\boldsymbol{x}}$, so that a, b, c, d,… will have different values for each even number.
If the even number is a multiple (for example, multiple of 7 and 11) we would have: $a = b = 0$.

In the case of the twin primes conjecture, we can say that $a = b = c = d = … = 2$ and, therefore, the terms analyzed always have the same configuration. Simply, it increases their number when $\boldsymbol{x}$ increases.
Because of this, it can be deduced that, when increasing $\boldsymbol{x}$, the behavior of terms ($7m + 2$), ($11m + 2$),… of the twin primes conjecture will be more regular, ($c(j\boldsymbol{x}) \approx 2{,}2$), than the behavior of terms ($7j + a$), ($11j + b$),… of Goldbach's conjecture, ($c(j\boldsymbol{x}) \approx 2{,}5$).

A final difference would be related to the number of combinations of groups of primes and the number of these groups that are used.
Let us recall this question for Goldbach's conjecture (page 3):

1. For the even number that is not multiple of 6 or 10: it results 2 different combinations using 3 groups of primes.
2. For the even number that is multiple of 6: it results 3 different combinations using 6 groups of primes.
3. For the even number that is multiple of 10: it results 2 different combinations using 4 groups of primes.
4. For the even number that is multiple of 30: it results 4 different combinations using the 8 prime numbers groups available.

5. For the twin primes conjecture: it results 3 different combinations using 6 groups of primes (always the same groups).

By analyzing the above data, we conclude that:

1. For the even number that is not multiple of 6 or 10: the partitions number is, approximately, ½ of the number of twin prime pairs.
2. For the even number that is multiple of 6: the partitions number is, approximately, equal to the number of twin prime pairs.
3. For the even number that is multiple of 10: the partitions number is, approximately, 2/3 of the number of twin prime pairs.
4. For the even number that is multiple of 30: the partitions number is, approximately, 4/3 of the number of twin prime pairs.

As we have seen, the number of pairs of primes that add an even number $\boldsymbol{x}$ (power of 2) is: $\mathbf{G}(x) \approx \dfrac{\mathbf{21}}{\mathbf{32}} \dfrac{\boldsymbol{\pi}^{\mathbf{2}}(x)}{\boldsymbol{x}}$

For the even number that is multiple 10: $\mathbf{G}_{\mathbf{10}}(x) \approx \dfrac{\mathbf{7}}{\mathbf{8}} \dfrac{\boldsymbol{\pi}^{\mathbf{2}}(x)}{\boldsymbol{x}}$

According to the proof that I have developed, the number of twin prime pairs that are lesser than $\boldsymbol{x}$ is: $\mathbf{G}_{\mathbf{G}}(x) \approx \dfrac{\mathbf{21}}{\mathbf{16}} \dfrac{\boldsymbol{\pi}^{\mathbf{2}}(x)}{\boldsymbol{x}}$

As numerical support, and using the programmable controller, the following data has been obtained:

To 268.435.456 = 2^{28}	525.109	prime pairs that add 2^{28}, being both primes greater than 2^{14}.
	1.055.991	twin prime pairs that are greater than 2^{14} and lesser than 2^{28}.

For the even number that is multiple of 10:

To 10^9	2.273.918	prime pairs that add 10^9, being both primes greater than $10^{4,5}$.
	3.424.019	twin prime pairs that are greater than $10^{4,5}$ and lesser than 10^9.

Ternary Goldbach Conjecture

The conjecture that has been studied is called strong or binary because there is another, weak or ternary, Goldbach's also, which states [5]: "All odd number greater than 7 can be written as a sum of three odd primes".

In May 2013, the Peruvian-born mathematician Harald Andrés Helfgott, a researcher at French CNRS at École Normale Supérieure in Paris, has published an article on web arXiv.org in which it is demonstrated that the ternary Goldbach conjecture is true for all odd numbers greater than 10^{29}. For the lesser odd numbers, and in collaboration with David Platt, computers have been used to verify that they also meet the conjecture.

Accepting that the binary Goldbach conjecture is proven, we can write down:

(Even number > 4) = (odd prime) + (odd prime)

Adding the odd prime 3: (Even number > 4) + 3 = (odd prime) + (odd prime) + 3

Therefore, it follows: (Odd number > 7) = (odd prime) + (odd prime) + (odd prime)

The article of Harald Helfgott and the above expression allows us to say that: **The Ternary Goldbach Conjecture is true.**

We will study now how to calculate the representations number of an odd number greater than 7 as a sum of three odd primes.

Hardy-Littlewood theorem statement [6]: "If General Riemann Hypothesis is true, then: $\boldsymbol{r}_3(x) \sim \mathfrak{б}_3(x)\,\dfrac{x^2}{\ln^3(x)}$ ".

In this formula, $r_3(x)$ is the representations number of an odd number x greater than 7 as the sum of three odd primes and $\mathfrak{б}_3(x)$ a factor that depends of x being its value between two constants.

A simple mathematical reasoning can be done to obtain the formula of the previous theorem.

Let us recall that $\pi(x) \sim \dfrac{x}{\ln(x)}$ is the number of primes lesser than or equal to x. Being p_1, p_2, p_3, p_4,… these primes (from first odd prime, $p_1 = 3$) and N_1, N_2, N_3, N_4,… the even numbers remaining of each prime of the odd number x, we will have:

$$x = p_1 + N_1 = p_2 + N_2 = p_3 + N_3 = p_4 + N_4 = \ldots$$

And so on until the last prime that is a distance greater than 4 of x.

Given the above, it can be deduced that the actual number of representations $r_3(x)$ will be equal to the sum of Goldbach's partitions $G_{(N1)}$, $G_{(N2)}$, $G_{(N3)}$, $G_{(N4)}$,… of all the even numbers N_1, N_2, N_3, N_4,…

$$r_3(x) = G_{(N1)} + G_{(N2)} + G_{(N3)} + G_{(N4)} + \ldots$$

In the above expression, the number of summands will be, approximately, equal to the number of primes lesser than x.

Recalling the formulas to calculate the number of Goldbach's partitions, we will call $\mathbf{G_m}(x) = \mathfrak{б}_3(x)\,\dfrac{x}{\ln^2(x)}$ the average value of $G_{(N1)}$, $G_{(N2)}$, $G_{(N3)}$, $G_{(N4)}$,… being $\mathfrak{б}_3(x)$ a factor that depends of x. With this in mind, we can write:

$$\boldsymbol{r}_3(x) \sim \frac{x}{\ln(x)}\,\mathfrak{б}_3(x)\,\frac{x}{\ln^2(x)} \qquad\qquad \boldsymbol{r}_3(x) \sim \mathfrak{б}_3(x)\,\frac{x^2}{\ln^3(x)}$$

We can note that, after a "forced" reasoning, we obtain the same formula as the above theorem.

Getting data using a programmable controller

Let us recall: Multiples: include all composites and the primes lesser than $\sqrt{x}$.
Primes: only those that are greater than $\sqrt{x}$.

Sequence A

1. The four data highlighted in **bold** are those obtained by the programmable controller.
2. The sum of the number of multiples 7*m*, 11*m*,… and the number of primes is the total number of terms of the sequence.
   It must match with the formula result: $\frac{x}{30}$ (page 4).
3. The sum of the number of multiples and the number of primes of form (7*j* + *a*), (11*j* + *b*),… is the total number of these terms.
   It must match with the number of multiples 7*m*, 11*m*,… of sequence **B** (page 6).
4. I used a calculator to obtain the following information:
5. P_{PPx} = Number of prime pairs being both greater than $\sqrt{x}$. It must match with the P_{PPx} of sequence **B**.
   P_{PPx} = (Number of primes of sequence **A**) – (Number of primes of form (7*j* + *a*), (11*j* + *b*),… of sequence **A**)
6. k_{ax} = Number of multiples 7*m*, 11*m*,… divided by the total number of terms of sequence **A**.
7. k_{jx} = Number of multiples that there are in the terms (7*j* + *a*), (11*j* + *b*),… divided by the total number of these.

   Proposed formula for k_{jx}: $k(jx) = 1 - \frac{30\pi(ax)}{x - c(jx)\pi(bx)}$ (page 13).
8. c_{jx} = Constant of proposed formula for k_{jx}. Solving: $c(jx) = \frac{x - \frac{30\pi(ax)}{1 - k(jx)}}{\pi(bx)}$
9. k_{0x} = Minimum value of k_{jx} for which the conjecture would not be met: $k(0x) = 1 - \frac{30\pi(ax)}{x - 30\pi(bx)}$ (pages 10 and 11).

Sequence B

1. The four data highlighted in **bold** are those obtained by the programmable controller.
2. The sum of the number of multiples 7*m*, 11*m*,… and the number of primes is the total number of terms of the sequence.
   It must match with the formula result: $\frac{x}{30}$ (page 4).
3. The sum of the number of multiples and the number of primes of form (7*j* + *a*), (11*j* + *b*),… is the total number of these terms.
   It must match with the number of multiples 7*m*, 11*m*,… of sequence **A** (page 6).
4. I used a calculator to obtain the following information:
5. P_{PPx} = Number of prime pairs being both greater than $\sqrt{x}$. It must match with the P_{PPx} of sequence **A**.
   P_{PPx} = (Number of primes of sequence **B**) – (Number of primes of form (7*j* + *a*), (11*j* + *b*),… of sequence **B**)
6. k_{bx} = Number of multiples 7*m*, 11*m*,… divided by the total number of terms of sequence **B**.
7. k_{jx} = Number of multiples that there are in the terms (7*j* + *a*), (11*j* + *b*),… divided by the total number of these.

   Proposed formula for k_{jx}: $k(jx) = 1 - \frac{30\pi(bx)}{x - c(jx)\pi(ax)}$ (page 13).
8. c_{jx} = Constant of proposed formula for k_{jx}. Solving: $c(jx) = \frac{x - \frac{30\pi(bx)}{1 - k(jx)}}{\pi(ax)}$
9. k_{0x} = Minimum value of k_{jx} for which the conjecture would not be met: $k(0x) = 1 - \frac{30\pi(bx)}{x - 30\pi(ax)}$ (pages 10 and 11).

Choosing the group (30*n* + 29) as an example, we will count the number of multiples that there are in each of the groups (7*j* + *a*), (11*j* + *b*), (13*j* + *c*), (17*j* + *d*),… until the group of prime 307. The obtained values are highlighted in **bold**.
Although for this analysis, any sequence of primes can be chosen, and to count each term only once, we will do it in ascending order (7, 11, 13, 17, 19, 23,…, 307).

1. Multiples that there are in the group (7*j* + *a*): they are all included.
2. Multiples that there are in the group (11*j* + *b*): not included those who are also (7*j* + *a*).
3. Multiples that there are in the group (13*j* + *c*): not included those who are also (7*j* + *a*) or (11*j* + *b*).

And so on until the group of prime 307.
The percentages indicated are relative to the total number of terms (7*j* + *a*), (11*j* + *b*), (13*j* + *c*), (17*j* + *d*),…

$\underline{10^6} = (30n_1 + 10) = (30n_2 + 11) + (30n_3 + 29)$ 33.333 pairs

Highest prime to divide 997

Sequence **A** $(30n_2 + 11)$		Sequence **B** $(30n_3 + 29)$	
Total number of terms	33.333	Total number of terms	33.333
Multiples $7m$, $11m$,…	**23.545**	Multiples $7m$, $11m$,…	**23.548**
Primes greater than 10^3	**9.788**	Primes greater than 10^3	**9.785**
Number of terms $(7j + a)$, $(11j + b)$,…	23.548	Number of terms $(7j + a)$, $(11j + b)$,…	23.545
Multiples $(7j + a)$, $(11j + b)$,…	**16.448**	Multiples $(7j + a)$, $(11j + b)$,…	**16.448**
Primes $(7j + a)$, $(11j + b)$,…	**7.100**	Primes $(7j + a)$, $(11j + b)$,…	**7.097**

P_{PPx} = Number of prime pairs being both greater than 10^3 $P_{PPx} = 9.788 - 7.100 = 9.785 - 7.097 = 2.688$
Not included the possible prime pairs in which one of them is lesser than 10^3

$k_{ax} = 0{,}70635…$		$k_{bx} = 0{,}70644…$	
$k_{jx} = 0{,}698488194$	$k_{jx} / k_{ax} = 0{,}988859927$	$k_{jx} = 0{,}698577192$	$k_{jx} / k_{bx} = 0{,}988859927$
$c_{jx} = 2{,}668143788$		$c_{jx} = 2{,}668453186$	
$k_{0x} = 0{,}584344256$	$k_{0x} / k_{ax} = 0{,}827264688$	$k_{0x} = 0{,}58441871$	$k_{0x} / k_{bx} = 0{,}827264687$

Multiples $(7j + a)$	**3.140**	13,336 %	Multiples $(31j + h)$	**451**	1,915 %	Multiples $(61j + t)$	**187**	0,794 %
Multiples $(11j + b)$	**1.767**	7,504 %	Multiples $(37j + i)$	**379**	1,609 %	Multiples $(67j + u)$	**178**	0,756 %
Multiples $(13j + c)$	**1.374**	5,835 %	Multiples $(41j + l)$	**327**	1,389 %	Multiples $(71j + v)$	**158**	0,671 %
Multiples $(17j + d)$	**1.010**	4,289 %	Multiples $(43j + o)$	**298**	1,265 %	Multiples $(73j + x)$	**157**	0,666 %
Multiples $(19j + e)$	**841**	3,572 %	Multiples $(47j + q)$	**271**	1,151 %	Multiples $(79j + y)$	**135**	0,573 %
Multiples $(23j + f)$	**659**	2,799 %	Multiples $(53j + r)$	**235**	0,998 %	Multiples $(83j + z)$	**126**	0,535 %
Multiples $(29j + g)$	**491**	2,085 %	Multiples $(59j + s)$	**208**	0,883 %			

Multiples group 89	**121**	0,514 %	Multiples group 163	**69**	0,293 %	Multiples group 239	**49**	0,208 %
Multiples group 97	**108**	0,459 %	Multiples group 167	**70**	0,297 %	Multiples group 241	**54**	0,229 %
Multiples group 101	**108**	0,459 %	Multiples group 173	**71**	0,301 %	Multiples group 251	**47**	0,2 %
Multiples group 103	**111**	0,471 %	Multiples group 179	**62**	0,263 %	Multiples group 257	**48**	0,204 %
Multiples group 107	**96**	0,408 %	Multiples group 181	**61**	0,259 %	Multiples group 263	**43**	0,183 %
Multiples group 109	**103**	0,437 %	Multiples group 191	**59**	0,251 %	Multiples group 269	**41**	0,174 %
Multiples group 113	**92**	0,391 %	Multiples group 193	**61**	0,259 %	Multiples group 271	**43**	0,183 %
Multiples group 127	**84**	0,357 %	Multiples group 197	**55**	0,234 %	Multiples group 277	**43**	0,183 %
Multiples group 131	**83**	0,352 %	Multiples group 199	**57**	0,242 %	Multiples group 281	**40**	0,17 %
Multiples group 137	**80**	0,34 %	Multiples group 211	**59**	0,251 %	Multiples group 283	**38**	0,161 %
Multiples group 139	**78**	0,331 %	Multiples group 223	**47**	0,2 %	Multiples group 293	**42**	0,178 %
Multiples group 149	**67**	0,284 %	Multiples group 227	**52**	0,221 %	Multiples group 307	**35**	0,149 %
Multiples group 151	**76**	0,323 %	Multiples group 229	**43**	0,183 %			
Multiples group 157	**66**	0,28 %	Multiples group 233	**54**	0,229 %	Total multiples in the groups 7 to 307	15.008	63,742 %

$\underline{10^6} = (30n_1 + 10) = (30n_2 + 17) + (30n_3 + 23)$ 33.333 pairs

Highest prime to divide 997

Sequence **A** $(30n_2 + 17)$		Sequence **B** $(30n_3 + 23)$	
Total number of terms	33.333	Total number of terms	33.333
Multiples $7m$, $11m$,…	**23.546**	Multiples $7m$, $11m$,…	**23.514**
Primes greater than 10^3	**9.787**	Primes greater than 10^3	**9.819**
Number of terms $(7j + a)$, $(11j + b)$,…	23.514	Number of terms $(7j + a)$, $(11j + b)$,…	23.546
Multiples $(7j + a)$, $(11j + b)$,…	**16.421**	Multiples $(7j + a)$, $(11j + b)$,…	**16.421**
Primes $(7j + a)$, $(11j + b)$,…	**7.093**	Primes $(7j + a)$, $(11j + b)$,…	**7.125**

P_{PPx} = Number of prime pairs being both greater than 10^3 $P_{PPx} = 9.787 - 7.093 = 9.819 - 7.125 = 2.694$
Not included the possible prime pairs in which one of them is lesser than 10^3

$k_{ax} = 0{,}70638…$		$k_{bx} = 0{,}70542…$	
$k_{jx} = 0{,}698349919$	$k_{jx} / k_{ax} = 0{,}98862218$	$k_{jx} = 0{,}697400832$	$k_{jx} / k_{bx} = 0{,}98862218$
$c_{jx} = 2{,}714499199$		$c_{jx} = 2{,}711148007$	
$k_{0x} = 0{,}583785776$	$k_{0x} / k_{ax} = 0{,}826438939$	$k_{0x} = 0{,}582992398$	$k_{0x} / k_{bx} = 0{,}826438955$

$\underline{10^7} = (30n_1 + 10) = (30n_2 + 11) + (30n_3 + 29)$ 333.333 pairs

Highest prime to divide 3.137

Sequence **A** $(30n_2 + 11)$		Sequence **B** $(30n_3 + 29)$	
Total number of terms	333.333	Total number of terms	333.333
Multiples $7m$, $11m$,…	**250.287**	Multiples $7m$, $11m$,…	**250.369**
Primes greater than $10^{3.5}$	**83.046**	Primes greater than $10^{3.5}$	**82.964**
Number of terms $(7j + a)$, $(11j + b)$,…	250.369	Number of terms $(7j + a)$, $(11j + b)$,…	250.287
Multiples $(7j + a)$, $(11j + b)$,…	**186.636**	Multiples $(7j + a)$, $(11j + b)$,…	**186.636**
Primes $(7j + a)$, $(11j + b)$,…	**63.733**	Primes $(7j + a)$, $(11j + b)$,…	**63.651**

P_{PPx} = Number of prime pairs being both greater than $10^{3.5}$ $P_{PPx} = 83.046 - 63.733 = 82.964 - 63.651 = 19.313$
Not included the possible prime pairs in which one of them is lesser than $10^{3.5}$

k_{ax} = 0,75086175
k_{jx} = 0,745443725 k_{jx} / k_{ax} = 0,992784256
c_{jx} = 2,565591768
k_{0x} = 0,668306022 k_{0x} / k_{ax} = 0,890052025

k_{bx} = 0,751107…
k_{jx} = 0,74568795 k_{jx} / k_{bx} = 0,992784256
c_{jx} = 2,56636034
k_{0x} = 0,668524975 k_{0x} / k_{bx} = 0,890052025

Multiples (7*j* + *a*)	**33.750**	13,484 %	Multiples (31*j* + *h*)	**4.843**	1,935 %	Multiples (61*j* + *t*)	**2.095**	0,837 %
Multiples (11*j* + *b*)	**19.045**	7,609 %	Multiples (37*j* + *i*)	**3.893**	1,555 %	Multiples (67*j* + *u*)	**1.862**	0,744 %
Multiples (13*j* + *c*)	**14.714**	5,879 %	Multiples (41*j* + *l*)	**3.441**	1,375 %	Multiples (71*j* + *v*)	**1.730**	0,691 %
Multiples (17*j* + *d*)	**10.549**	4,215 %	Multiples (43*j* + *o*)	**3.244**	1,296 %	Multiples (73*j* + *x*)	**1.680**	0,671 %
Multiples (19*j* + *e*)	**8.893**	3,553 %	Multiples (47*j* + *q*)	**2.876**	1,149 %	Multiples (79*j* + *y*)	**1.520**	0,607 %
Multiples (23*j* + *f*)	**6.985**	2,791 %	Multiples (53*j* + *r*)	**2.477**	0,99 %	Multiples (83*j* + *z*)	**1.425**	0,569 %
Multiples (29*j* + *g*)	**5.314**	2,123 %	Multiples (59*j* + *s*)	**2.192**	0,876 %			

Multiples group 89	**1.333**	0,533 %	Multiples group 163	**623**	0,249 %	Multiples group 239	**401**	0,16 %
Multiples group 97	**1.197**	0,478 %	Multiples group 167	**601**	0,24 %	Multiples group 241	**423**	0,169 %
Multiples group 101	**1.093**	0,437 %	Multiples group 173	**583**	0,233 %	Multiples group 251	**379**	0,151 %
Multiples group 103	**1.104**	0,441 %	Multiples group 179	**537**	0,214 %	Multiples group 257	**385**	0,154 %
Multiples group 107	**1.035**	0,413 %	Multiples group 181	**560**	0,224 %	Multiples group 263	**379**	0,151 %
Multiples group 109	**987**	0,394 %	Multiples group 191	**518**	0,207 %	Multiples group 269	**367**	0,147 %
Multiples group 113	**960**	0,384 %	Multiples group 193	**505**	0,202 %	Multiples group 271	**363**	0,145 %
Multiples group 127	**848**	0,339 %	Multiples group 197	**508**	0,203 %	Multiples group 277	**353**	0,141 %
Multiples group 131	**789**	0,315 %	Multiples group 199	**493**	0,2 %	Multiples group 281	**357**	0,143 %
Multiples group 137	**767**	0,306 %	Multiples group 211	**478**	0,197 %	Multiples group 283	**357**	0,143 %
Multiples group 139	**726**	0,29 %	Multiples group 223	**443**	0,177 %	Multiples group 293	**346**	0,138 %
Multiples group 149	**687**	0,274 %	Multiples group 227	**452**	0,181 %	Multiples group 307	**323**	0,129 %
Multiples group 151	**679**	0,271 %	Multiples group 229	**437**	0,175 %			
Multiples group 157	**636**	0,254 %	Multiples group 233	**416**	0,166 %	Total multiples in the groups 7 to 307	156.956	62,71 %

<u>10^7</u> = (30n_1 + 10) = (30n_2 + 17) + (30n_3 + 23) 333.333 pairs

Highest prime to divide 3.137

Sequence **A** (30n_2 + 17)		Sequence **B** (30n_3 + 23)	
Total number of terms	333.333	Total number of terms	333.333
Multiples 7*m*, 11*m*,…	**250.283**	Multiples 7*m*, 11*m*,…	**250.238**
Primes greater than 3.162	**83.050**	Primes greater than 3.162	**83.095**
Number of terms (7*j* + *a*), (11*j* + *b*),…	250.238	Number of terms (7*j* + *a*), (11*j* + *b*),…	250.283
Multiples (7*j* + *a*), (11*j* + *b*),…	**186.638**	Multiples (7*j* + *a*), (11*j* + *b*),…	**186.638**
Primes (7*j* + *a*), (11*j* + *b*),…	**63.600**	Primes (7*j* + *a*), (11*j* + *b*),…	**63.645**

P_{PPx} = Number of prime pairs being both greater than $10^{3.5}$ P_{PPx} = 83.050 – 63.600 = 83.095 – 63.645 = 19.450
Not included the possible prime pairs in which one of them is lesser than $10^{3.5}$

k_{ax} = 0,75084975
k_{jx} = 0,745841958 k_{jx} / k_{ax} = 0,9933305
c_{jx} = 2,371314813
k_{0x} = 0,668116395 k_{0x} / k_{ax} = 0,8898137

k_{bx} = 0,75071475
k_{jx} = 0,745707858 k_{jx} / k_{bx} = 0,9933305
c_{jx} = 2,370922353
k_{0x} = 0,66799627 k_{0x} / k_{bx} = 0,8898137

<u>10^8</u> = (30n_1 + 10) = (30n_2 + 11) + (30n_3 + 29) 3.333.333 pairs

Highest prime to divide 9.973

Sequence **A** (30n_2 + 11)		Sequence **B** (30n_3 + 29)	
Total number of terms	3.333.333	Total number of terms	3.333.333
Multiples 7*m*, 11*m*,…	**2.613.173**	Multiples 7*m*, 11*m*,…	**2.613.453**
Primes greater than 10^4	**720.160**	Primes greater than 10^4	**719.880**
Number of terms (7*j* + *a*), (11*j* + *b*),…	2.613.453	Number of terms (7*j* + *a*), (11*j* + *b*),…	2.613.173
Multiples (7*j* + *a*), (11*j* + *b*),…	**2.039.019**	Multiples (7*j* + *a*), (11*j* + *b*),…	**2.039.019**
Primes (7*j* + *a*), (11*j* + *b*),…	**574.434**	Primes (7*j* + *a*), (11*j* + *b*),…	**574.154**

P_{PPx} = Number of prime pairs being both greater than 10^4 P_{PPx} = 720.160 – 574.434 = 719.880 – 574.154 = 145.726
Not included the possible prime pairs in which one of them is lesser than 10^4

k_{ax} = 0,7839519…
k_{jx} = 0,780201136 k_{jx} / k_{ax} = 0,995215469
c_{jx} = 2,370531694
k_{0x} = 0,724441224 k_{0x} / k_{ax} = 0,924088775

k_{bx} = 0,7840359…
k_{jx} = 0,780284734 k_{jx} / k_{bx} = 0,995215469
c_{jx} = 2,370765683
k_{0x} = 0,724518848 k_{0x} / k_{bx} = 0,924088776

Multiples (7*j* + *a*)	**356.077**	13,626 %	Multiples (31*j* + *h*)	**50.268**	1,924 %	Multiples (61*j* + *t*)	**21.820**	0,835 %
Multiples (11*j* + *b*)	**199.519**	7,635 %	Multiples (37*j* + *i*)	**40.841**	1,563 %	Multiples (67*j* + *u*)	**19.527**	0,747 %
Multiples (13*j* + *c*)	**154.831**	5,925 %	Multiples (41*j* + *l*)	**35.803**	1,37 %	Multiples (71*j* + *v*)	**18.162**	0,695 %
Multiples (17*j* + *d*)	**110.081**	4,212 %	Multiples (43*j* + *o*)	**33.363**	1,277 %	Multiples (73*j* + *x*)	**17.423**	0,667 %
Multiples (19*j* + *e*)	**92.988**	3,558 %	Multiples (47*j* + *q*)	**29.791**	1,14 %	Multiples (79*j* + *y*)	**15.842**	0,606 %
Multiples (23*j* + *f*)	**73.084**	2,797 %	Multiples (53*j* + *r*)	**25.885**	0,991 %	Multiples (83*j* + *z*)	**14.889**	0,57 %
Multiples (29*j* + *g*)	**55.555**	2,126 %	Multiples (59*j* + *s*)	**22.922**	0,877 %			

Multiples group 89	**13.782**	0,527 %	Multiples group 163	**6.608**	0,253 %	Multiples group 239	**4.042**	0,155 %
Multiples group 97	**12.476**	0,477 %	Multiples group 167	**6.389**	0,244 %	Multiples group 241	**4.019**	0,154 %
Multiples group 101	**11.884**	0,455 %	Multiples group 173	**6.100**	0,233 %	Multiples group 251	**3.850**	0,147 %
Multiples group 103	**11.554**	0,442 %	Multiples group 179	**5.894**	0,226 %	Multiples group 257	**3.692**	0,141 %
Multiples group 107	**11.022**	0,422 %	Multiples group 181	**5.734**	0,219 %	Multiples group 263	**3.623**	0,139 %
Multiples group 109	**10.684**	0,409 %	Multiples group 191	**5.488**	0,21 %	Multiples group 269	**3.523**	0,135 %
Multiples group 113	**10.163**	0,389 %	Multiples group 193	**5.313**	0,203 %	Multiples group 271	**3.495**	0,134 %
Multiples group 127	**8.942**	0,342 %	Multiples group 197	**5.207**	0,199 %	Multiples group 277	**3.392**	0,13 %
Multiples group 131	**8.636**	0,33 %	Multiples group 199	**5.122**	0,196 %	Multiples group 281	**3.335**	0,128 %
Multiples group 137	**8.227**	0,315 %	Multiples group 211	**4.761**	0,182 %	Multiples group 283	**3.305**	0,126 %
Multiples group 139	**7.968**	0,305 %	Multiples group 223	**4.474**	0,171 %	Multiples group 293	**3.187**	0,122 %
Multiples group 149	**7.392**	0,283 %	Multiples group 227	**4.407**	0,169 %	Multiples group 307	**3.014**	0,115 %
Multiples group 151	**7.240**	0,277 %	Multiples group 229	**4.309**	0,165 %			
Multiples group 157	**6.897**	0,264 %	Multiples group 233	**4.240**	0,162 %			

Total multiples in the groups 7 to 307   1.642.061   62,838 %

<u>10^8</u> = ($30n_1$ + 10) = ($30n_2$ + 17) + ($30n_3$ + 23)   3.333.333 pairs   Highest prime to divide 9.973

Sequence **A** ($30n_2$ + 17)		Sequence **B** ($30n_3$ + 23)	
Total number of terms	3.333.333	Total number of terms	3.333.333
Multiples 7*m*, 11*m*,…	**2.613.261**	Multiples 7*m*, 11*m*,…	**2.613.125**
Primes greater than 10^4	**720.072**	Primes greater than 10^4	**720.208**
Number of terms (7*j* + *a*), (11*j* + *b*),…	2.613.125	Number of terms (7*j* + *a*), (11*j* + *b*),…	2.613.261
Multiples (7*j* + *a*), (11*j* + *b*),…	**2.038.608**	Multiples (7*j* + *a*), (11*j* + *b*),…	**2.038.608**
Primes (7*j* + *a*), (11*j* + *b*),…	**574.517**	Primes (7*j* + *a*), (11*j* + *b*),…	**574.653**

P_{PPx} = Number of prime pairs being both greater than 10^4 P_{PPx} = 720.072 – 574.517 = 720.208 – 574.653 = 145.555
Not included the possible prime pairs in which one of them is lesser than 10^4

k_{ax} = 0,7839783…		k_{bx} = 0,7839375…	
k_{jx} = 0,780141784	k_{jx} / k_{ax} = 0,99510625	k_{jx} = 0,780101183	k_{jx} / k_{bx} = 0,99510625
c_{jx} = 2,42296808		c_{jx} = 2,42285258	
k_{0x} = 0,724440312	k_{0x} / k_{ax} = 0,924056494	k_{0x} = 0,724402611	k_{0x} / k_{bx} = 0,924056495

<u>10^9</u> = ($30n_1$ + 10) = ($30n_2$ + 11) + ($30n_3$ + 29)   33.333.333 pairs   Highest prime to divide 31.607   square root 31.622   50.847.534 primes lesser than 10^9

Sequence **A** ($30n_2$ + 11)		Sequence **B** ($30n_3$ + 29)	
Total number of terms	33.333.333	Total number of terms	33.333.333
Multiples 7*m*, 11*m*,…	**26.977.564**	Multiples 7*m*, 11*m*,…	**26.977.414**
Primes greater than $10^{4.5}$	**6.355.769**	Primes greater than $10^{4.5}$	**6.355.919**
Number of terms (7*j* + *a*), (11*j* + *b*),…	26.977.414	Number of terms (7*j* + *a*), (11*j* + *b*),…	26.977.564
Multiples (7*j* + *a*), (11*j* + *b*),…	**21.758.538**	Multiples (7*j* + *a*), (11*j* + *b*),…	**21.758.538**
Primes (7*j* + *a*), (11*j* + *b*),…	**5.218.876**	Primes (7*j* + *a*), (11*j* + *b*),…	**5.219.026**

P_{PPx} = Number of prime pairs being both greater than $10^{4.5}$ P_{PPx} = 6.355.769 – 5.218.876 = 6.355.919 – 5.219.026 = 1.136.893
Not included the possible prime pairs in which one of them is lesser than $10^{4.5}$

k_{ax} = 0,809326928		k_{bx} = 0,809322428	
k_{jx} = 0,806546468	k_{jx} / k_{ax} = 0,996564479	k_{jx} = 0,806541984	k_{jx} / k_{bx} = 0,996564479
c_{jx} = 2,261319599		c_{jx} = 2,26130754	
k_{0x} = 0,76440407	k_{0x} / k_{ax} = 0,944493559	k_{0x} = 0,76439982	k_{0x} / k_{bx} = 0,944493559

Multiples (7*j* + *a*)	**3.702.786**	13,725 %	Multiples (31*j* + *h*)	**517.983**	1,92 %	Multiples (61*j* + *t*)	**224.011**	0,83 %
Multiples (11*j* + *b*)	**2.067.520**	7,664 %	Multiples (37*j* + *i*)	**420.774**	1,56 %	Multiples (67*j* + *u*)	**200.865**	0,745 %
Multiples (13*j* + *c*)	**1.600.628**	5,933 %	Multiples (41*j* + *l*)	**369.785**	1,371 %	Multiples (71*j* + *v*)	**186.614**	0,692 %
Multiples (17*j* + *d*)	**1.137.457**	4,216 %	Multiples (43*j* + *o*)	**344.202**	1,276 %	Multiples (73*j* + *x*)	**178.908**	0,663 %
Multiples (19*j* + *e*)	**959.914**	3,558 %	Multiples (47*j* + *q*)	**307.696**	1,141 %	Multiples (79*j* + *y*)	**163.259**	0,605 %
Multiples (23*j* + *f*)	**753.704**	2,794 %	Multiples (53*j* + *r*)	**267.233**	0,991 %	Multiples (83*j* + *z*)	**153.399**	0,569 %
Multiples (29*j* + *g*)	**573.050**	2,124 %	Multiples (59*j* + *s*)	**235.662**	0,874 %			

Multiples group 89	**141.414**	0,524 %	Multiples group 163	**68.943**	0,256 %	Multiples group 239	**43.564**	0,161 %
Multiples group 97	**128.131**	0,475 %	Multiples group 167	**66.855**	0,248 %	Multiples group 241	**43.153**	0,16 %
Multiples group 101	**122.098**	0,453 %	Multiples group 173	**64.156**	0,238 %	Multiples group 251	**41.212**	0,153 %
Multiples group 103	**118.537**	0,439 %	Multiples group 179	**61.727**	0,229 %	Multiples group 257	**39.983**	0,148 %
Multiples group 107	**112.655**	0,418 %	Multiples group 181	**60.797**	0,225 %	Multiples group 263	**38.915**	0,144 %
Multiples group 109	**110.000**	0,408 %	Multiples group 191	**57.203**	0,212 %	Multiples group 269	**37.905**	0,14 %
Multiples group 113	**105.034**	0,389 %	Multiples group 193	**56.272**	0,209 %	Multiples group 271	**37.475**	0,139 %
Multiples group 127	**92.824**	0,344 %	Multiples group 197	**54.872**	0,203 %	Multiples group 277	**36.498**	0,135 %
Multiples group 131	**89.279**	0,331 %	Multiples group 199	**54.100**	0,2 %	Multiples group 281	**35.761**	0,133 %
Multiples group 137	**84.745**	0,314 %	Multiples group 211	**50.725**	0,188 %	Multiples group 283	**35.390**	0,131 %
Multiples group 139	**82.963**	0,308 %	Multiples group 223	**47.673**	0,177 %	Multiples group 293	**34.012**	0,126 %
Multiples group 149	**76.942**	0,285 %	Multiples group 227	**46.648**	0,173 %	Multiples group 307	**32.328**	0,12 %
Multiples group 151	**75.319**	0,279 %	Multiples group 229	**46.141**	0,171 %			
Multiples group 157	**71.971**	0,267 %	Multiples group 233	**44.870**	0,166 %			

Total multiples in the groups 7 to 307   17.014.540   63,069 %

$\underline{10^9} = (30n_1 + 10) = (30n_2 + 17) + (30n_3 + 23)$ 33.333.333 pairs Highest prime to divide 31.607 square root 31.622 50.847.534 primes lesser than 10^9

	Sequence **A** $(30n_2 + 17)$	Sequence **B** $(30n_3 + 23)$
Total number of terms	33.333.333	33.333.333
Multiples $7m$, $11m$,…	**26.977.923**	**26.977.320**
Primes greater than $10^{4.5}$	**6.355.410**	**6.356.013**
Number of terms $(7j + a)$, $(11j + b)$,…	26.977.320	26.977.923
Multiples $(7j + a)$, $(11j + b)$,…	**21.758.935**	**21.758.935**
Primes $(7j + a)$, $(11j + b)$,…	**5.218.385**	**5.218.988**

P_{PPx} = Number of prime pairs being both greater than $10^{4.5}$ P_{PPx} = 6.355.410 – 5.218.385 = 6.356.013 – 5.218.988 = 1.137.025
Not included the possible prime pairs in which one of them is lesser than $10^{4.5}$

k_{ax} = 0,809337698
k_{jx} = 0,806563995 k_{jx} / k_{ax} = 0,996572873
c_{jx} = 2,255995237
k_{0x} = 0,764416557 k_{0x} / k_{ax} = 0,944496418

k_{bx} = 0,809319608
k_{jx} = 0,806545967 k_{jx} / k_{bx} = 0,996572873
c_{jx} = 2,255948601
k_{0x} = 0,764399471 k_{0x} / k_{bx} = 0,944496418

$\underline{8.000.000 = 8\cdot 10^6} = (30n_1 + 20) = (30n_2 + 1) + (30n_3 + 19)$ 266.667 pairs Highest prime to divide 2.819

	Sequence **A** $(30n_2 + 1)$	Sequence **B** $(30n_3 + 19)$
Total number of terms	266.667	266.667
Multiples $7m$, $11m$,…	**199.312+1**	**199.314**
Primes greater than 2.828	**67.354**	**67.353**
Number of terms $(7j + a)$, $(11j + b)$,…	199.314	199.312
Multiples $(7j + a)$, $(11j + b)$,…	**147.802+1**	**147.802**
Primes $(7j + a)$, $(11j + b)$,…	**51.511**	**51.510**

P_{PPx} = Number of prime pairs being both greater than 2.828 P_{PPx} = 67.354 – 51.511 = 67.353 – 51.510 = 15.843
Not included the possible prime pairs in which one of them is lesser than 2.828

k_{ax} = 0,747419065
k_{jx} = 0,741553528 k_{jx} / k_{ax} = 0,992152277
c_{jx} = 2,69727031
k_{0x} = 0,662070338 k_{0x} / k_{ax} = 0,885808736

k_{bx} = 0,747426565
k_{jx} = 0,741560969 k_{jx} / k_{bx} = 0,992152277
c_{jx} = 2,695611584
k_{0x} = 0,662073659 k_{0x} / k_{bx} = 0,88580429

$\underline{8.000.000 = 8\cdot 10^6} = (30n_1 + 20) = (30n_2 + 7) + (30n_3 + 13)$ 266.667 pairs Highest prime to divide 2.819

	Sequence **A** $(30n_2 + 7)$	Sequence **B** $(30n_3 + 13)$
Total number of terms	266.667	266.667
Multiples $7m$, $11m$,…	**199.224**	**199.241**
Primes greater than 2.828	**67.443**	**67.426**
Number of terms $(7j + a)$, $(11j + b)$,…	199.241	199.224
Multiples $(7j + a)$, $(11j + b)$,…	**147.663**	**147.663**
Primes $(7j + a)$, $(11j + b)$,…	**51.578**	**51.561**

P_{PPx} = Number of prime pairs being both greater than 2.828 P_{PPx} = 67.443 – 51.578 = 67.426 – 51.561 = 15.865
Not included the possible prime pairs in which one of them is lesser than 2.828

k_{ax} = 0,747089066
k_{jx} = 0,741127579 k_{jx} / k_{ax} = 0,992020379
c_{jx} = 2,732174197
k_{0x} = 0,661499827 k_{0x} / k_{ax} = 0,885436364

k_{bx} = 0,747152816
k_{jx} = 0,74119082 k_{jx} / k_{bx} = 0,992020379
c_{jx} = 2,732386249
k_{0x} = 0,661556274 k_{0x} / k_{bx} = 0,885436365

$\underline{80.000.000 = 8\cdot 10^7} = (30n_1 + 20) = (30n_2 + 1) + (30n_3 + 19)$ 2.666.667 pairs Highest prime to divide 8.941

	Sequence **A** $(30n_2 + 1)$	Sequence **B** $(30n_3 + 19)$
Total number of terms	2.666.667	2.666.667
Multiples $7m$, $11m$,…	**2.083.272+1**	**2.083.493**
Primes greater than 8.944	**583.394**	**583.174**
Number of terms $(7j + a)$, $(11j + b)$,…	2.083.493	2.083.272
Multiples $(7j + a)$, $(11j + b)$,…	**1.619.431+1**	**1.619.431**
Primes $(7j + a)$, $(11j + b)$,…	**464.061**	**463.841**

P_{PPx} = Number of prime pairs being both greater than 8.944 P_{PPx} = 583.394 – 464.061 = 583.174 – 463.841 = 119.333
Not included the possible prime pairs in which one of them is lesser than 8.944

k_{ax} = 0,781226902
k_{jx} = 0,77726731 k_{jx} / k_{ax} = 0,994931572
c_{jx} = 2,438913946
k_{0x} = 0,719992295 k_{0x} / k_{ax} = 0,921617385

k_{bx} = 0,781309777
k_{jx} = 0,777349765 k_{jx} / k_{bx} = 0,994931572
c_{jx} = 2,438924811
k_{0x} = 0,720068328 k_{0x} / k_{bx} = 0,921616942

$\underline{80.000.000 = 8 \cdot 10^7} = (30n_1 + 20) = (30n_2 + 7) + (30n_3 + 13)$ 2.666.667 pairs Highest prime to divide 8.941

Sequence **A** $(30n_2 + 7)$		Sequence **B** $(30n_3 + 13)$	
Total number of terms	2.666.667	Total number of terms	2.666.667
Multiples $7m$, $11m$,…	**2.083.064**	Multiples $7m$, $11m$,…	**2.083.030**
Primes greater than 8.944	**583.603**	Primes greater than 8.944	**583.637**
Number of terms $(7j + a)$, $(11j + b)$,…	2.083.030	Number of terms $(7j + a)$, $(11j + b)$,…	2.083.064
Multiples $(7j + a)$, $(11j + b)$,…	**1.619.203**	Multiples $(7j + a)$, $(11j + b)$,…	**1.619.203**
Primes $(7j + a)$, $(11j + b)$,…	**463.827**	Primes $(7j + a)$, $(11j + b)$,…	**463.861**

P_{PPx} = Number of prime pairs being both greater than 8.944 P_{PPx} = 583.603 – 463.827 = 583.637 – 463.861 = 119.776
Not included the possible prime pairs in which one of them is lesser than 8.944

k_{ax} = 0,781148902		k_{bx} = 0,781136152	
k_{jx} = 0,777330619	k_{jx} / k_{ax} = 0,995111965	k_{jx} = 0,777317931	k_{jx} / k_{bx} = 0,995111965
c_{jx} = 2,350453759		c_{jx} = 2,350418595	
k_{0x} = 0,719829722	k_{0x} / k_{ax} = 0,921501291	k_{0x} = 0,719817973	k_{0x} / k_{bx} = 0,921501291

$\underline{9.000.000 = 9 \cdot 10^6} = (30n_1 + 30) = (30n_2 + 1) + (30n_3 + 29)$ 300.000 pairs Highest prime to divide 2.999

Sequence **A** $(30n_2 + 1)$		Sequence **B** $(30n_3 + 29)$	
Total number of terms	300.000	Total number of terms	300.000
Multiples $7m$, $11m$,…	**224.798+1**	Multiples $7m$, $11m$,…	**224.776**
Primes greater than 3.000	**75.201**	Primes greater than 3.000	**75.224**
Number of terms $(7j + a)$, $(11j + b)$,…	224.776	Number of terms $(7j + a)$, $(11j + b)$,…	224.798
Multiples $(7j + a)$, $(11j + b)$,…	**167.109+1**	Multiples $(7j + a)$, $(11j + b)$,…	**167.109**
Primes $(7j + a)$, $(11j + b)$,…	**57.666**	Primes $(7j + a)$, $(11j + b)$,…	**57.689**

P_{PPx} = Number of prime pairs being both greater than 3.000 P_{PPx} = 75.201 – 57.666 = 75.224 – 57.689 = 17.535
Not included the possible prime pairs in which one of them is lesser than 3.000

k_{ax} = 0,749326666		k_{bx} = 0,749253333	
k_{jx} = 0,743446809	k_{jx} / k_{ax} = 0,992153145	k_{jx} = 0,743374051	k_{jx} / k_{bx} = 0,992153145
c_{jx} = 2,743605177		c_{jx} = 2,741843369	
k_{0x} = 0,66544026	k_{0x} / k_{ax} = 0,888050952	k_{0x} = 0,665372176	k_{0x} / k_{bx} = 0,888047001

$\underline{9.000.000 = 9 \cdot 10^6} = (30n_1 + 30) = (30n_2 + 7) + (30n_3 + 23)$ 300.000 pairs Highest prime to divide 2.999

Sequence **A** $(30n_2 + 7)$		Sequence **B** $(30n_3 + 23)$	
Total number of terms	300.000	Total number of terms	300.000
Multiples $7m$, $11m$,…	**224.761**	Multiples $7m$, $11m$,…	**224.699**
Primes greater than 3.000	**75.239**	Primes greater than 3.000	**75.301**
Number of terms $(7j + a)$, $(11j + b)$,…	224.699	Number of terms $(7j + a)$, $(11j + b)$,…	224.761
Multiples $(7j + a)$, $(11j + b)$,…	**167.190**	Multiples $(7j + a)$, $(11j + b)$,…	**167.190**
Primes $(7j + a)$, $(11j + b)$,…	**57.509**	Primes $(7j + a)$, $(11j + b)$,…	**57.571**

P_{PPx} = Number of prime pairs being both greater than 3.000 P_{PPx} = 75.239 – 57.509 = 75.301 – 57.571 = 17.730
Not included the possible prime pairs in which one of them is lesser than 3.000

k_{ax} = 0,749203333		k_{bx} = 0,748996666	
k_{jx} = 0,744062056	k_{jx} / k_{ax} = 0,993137674	k_{jx} = 0,743856807	k_{jx} / k_{bx} = 0,993137674
c_{jx} = 2,400922383		c_{jx} = 2,400313332	
k_{0x} = 0,665156498	k_{0x} / k_{ax} = 0,887818391	k_{0x} = 0,664973015	k_{0x} / k_{bx} = 0,88781839

$\underline{9.000.000 = 9 \cdot 10^6} = (30n_1 + 30) = (30n_2 + 11) + (30n_3 + 19)$ 300.000 pairs Highest prime to divide 2.999

Sequence **A** $(30n_2 + 11)$		Sequence **B** $(30n_3 + 19)$	
Total number of terms	300.000	Total number of terms	300.000
Multiples $7m$, $11m$,…	**224.700**	Multiples $7m$, $11m$,…	**224.761**
Primes greater than 3.000	**75.300**	Primes greater than 3.000	**75.239**
Number of terms $(7j + a)$, $(11j + b)$,…	224.761	Number of terms $(7j + a)$, $(11j + b)$,…	224.700
Multiples $(7j + a)$, $(11j + b)$,…	**167.085**	Multiples $(7j + a)$, $(11j + b)$,…	**167.085**
Primes $(7j + a)$, $(11j + b)$,…	**57.676**	Primes $(7j + a)$, $(11j + b)$,…	**57.615**

P_{PPx} = Number of prime pairs being both greater than 3.000 P_{PPx} = 75.300 – 57.676 = 75.239 – 57.615 = 17.624
Not included the possible prime pairs in which one of them is lesser than 3.000

k_{ax} = 0,749		k_{ax} = 0,749203333	
k_{jx} = 0,743389645	k_{jx} / k_{ax} = 0,992509539	k_{jx} = 0,743591455	k_{jx} / k_{bx} = 0,992509539
c_{jx} = 2,615264747		c_{jx} = 2,615912938	
k_{0x} = 0,664977465	k_{0x} / k_{ax} = 0,88782038	k_{0x} = 0,665157988	k_{0x} / k_{bx} = 0,887820379

$\underline{9.000.000 = 9\cdot 10^6}$ = $(30n_1 + 30) = (30n_2 + 13) + (30n_3 + 17)$ 300.000 pairs Highest prime to divide 2.999

Sequence **A** $(30n_2 + 13)$		Sequence **B** $(30n_3 + 17)$	
Total number of terms	300.000	Total number of terms	300.000
Multiples $7m$, $11m$,…	**224.749**	Multiples $7m$, $11m$,…	**224.696**
Primes greater than 3.000	**75.251**	Primes greater than 3.000	**75.304**
Number of terms $(7j + a)$, $(11j + b)$,…	224.696	Number of terms $(7j + a)$, $(11j + b)$,…	224.749
Multiples $(7j + a)$, $(11j + b)$,…	**167.079**	Multiples $(7j + a)$, $(11j + b)$,…	**167.079**
Primes $(7j + a)$, $(11j + b)$,…	**57.617**	Primes $(7j + a)$, $(11j + b)$,…	**57.670**

P_{PPx} = Number of prime pairs being both greater than 3.000 P_{PPx} = 75.251 – 57.617 = 75.304 – 57.670 = 17.634
Not included the possible prime pairs in which one of them is lesser than 3.000

k_{ax} = 0,749163333
k_{jx} = 0,743577989 k_{jx} / k_{ax} = 0.992544558
c_{jx} = 2,603269404
k_{0x} = 0,665098622 k_{0x} / k_{ax} = 0,88778854

k_{bx} = 0,748986666
k_{jx} = 0,74340264 k_{jx} / k_{bx} = 0,992544558
c_{jx} = 2,602708519
k_{0x} = 0,664941779 k_{0x} / k_{bx} = 0,887788539

$\underline{90.000.000 = 9\cdot 10^7}$ = $(30n_1 + 30) = (30n_2 + 1) + (30n_3 + 29)$ 3.000.000 pairs Highest prime to divide 9.479

Sequence **A** $(30n_2 + 1)$		Sequence **B** $(30n_3 + 29)$	
Total number of terms	3.000.000	Total number of terms	3.000.000
Multiples $7m$, $11m$,…	**2.348.173+1**	Multiples $7m$, $11m$,…	**2.347.976**
Primes greater than 9.486	**651.827–1**	Primes greater than 9.486	**652.024**
Number of terms $(7j + a)$, $(11j + b)$,…	2.347.976	Number of terms $(7j + a)$, $(11j + b)$,…	2.348.173
Multiples $(7j + a)$, $(11j + b)$,…	**1.829.138**	Multiples $(7j + a)$, $(11j + b)$,…	**1.829.138**
Primes $(7j + a)$, $(11j + b)$,…	**518.838**	Primes $(7j + a)$, $(11j + b)$,…	**519.035**

P_{PPx} = Number of prime pairs being both greater than 9.486 P_{PPx} = 651.827 – 518.838 = 652.024 – 519.035 = 132.989 (–1)
Not included the possible prime pairs in which one of them is lesser than 9.486

k_{ax} = 0,782724333
k_{jx} = 0,779027553 k_{jx} / k_{ax} = 0,995277035
c_{jx} = 2,309215414
k_{0x} = 0,722387707 k_{0x} / k_{ax} = 0,922914589

k_{bx} = 0,782658666
k_{jx} = 0,778962197 k_{jx} / k_{bx} = 0,995277035
c_{jx} = 2,309036271
k_{0x} = 0,722327102 k_{0x} / k_{bx} = 0,922914589

$\underline{90.000.000 = 9\cdot 10^7}$ = $(30n_1 + 30) = (30n_2 + 7) + (30n_3 + 23)$ 3.000.000 pairs Highest prime to divide 9.479

Sequence **A** $(30n_2 + 7)$		Sequence **B** $(30n_3 + 23)$	
Total number of terms	3.000.000	Total number of terms	3.000.000
Multiples $7m$, $11m$,…	**2.348.028**	Multiples $7m$, $11m$,…	**2.347.948**
Primes greater than 9.486	**651.972**	Primes greater than 9.486	**652.052**
Number of terms $(7j + a)$, $(11j + b)$,…	2.347.948	Number of terms $(7j + a)$, $(11j + b)$,…	2.348.028
Multiples $(7j + a)$, $(11j + b)$,…	**1.829.330**	Multiples $(7j + a)$, $(11j + b)$,…	**1.829.330**
Primes $(7j + a)$, $(11j + b)$,…	**518.618**	Primes $(7j + a)$, $(11j + b)$,…	**518.698**

P_{PPx} = Number of prime pairs being both greater than 9.486 P_{PPx} = 651.972 – 518.618 = 652.052 – 518.698 = 133.354
Not included the possible prime pairs in which one of them is lesser than 9.486

k_{ax} = 0,782676
k_{jx} = 0,779118617 k_{jx} / k_{ax} = 0,995454846
c_{jx} = 2,222960729
k_{0x} = 0,72232264 k_{0x} / k_{ax} = 0,922888449

k_{bx} = 0,782649333
k_{jx} = 0,779092072 k_{jx} / k_{bx} = 0,995454846
c_{jx} = 2,222890318
k_{0x} = 0,72229803 k_{0x} / k_{bx} = 0,922888449

$\underline{90.000.000 = 9\cdot 10^7}$ = $(30n_1 + 30) = (30n_2 + 11) + (30n_3 + 19)$ 3.000.000 pairs Highest prime to divide 9.479

Sequence **A** $(30n_2 + 11)$		Sequence **B** $(30n_3 + 19)$	
Total number of terms	3.000.000	Total number of terms	3.000.000
Multiples $7m$, $11m$,…	**2.347.956**	Multiples $7m$, $11m$,…	**2.348.253**
Primes greater than 9.486	**652.044**	Primes greater than 9.486	**651.747**
Number of terms $(7j + a)$, $(11j + b)$,…	2.348.253	Number of terms $(7j + a)$, $(11j + b)$,…	2.347.956
Multiples $(7j + a)$, $(11j + b)$,…	**1.828.760**	Multiples $(7j + a)$, $(11j + b)$,…	**1.828.760**
Primes $(7j + a)$, $(11j + b)$,…	**519.493**	Primes $(7j + a)$, $(11j + b)$,…	**519.196**

P_{PPx} = Number of prime pairs being both greater than 9.486 P_{PPx} = 652.044 – 519.493 = 651.747 – 519.196 = 132.551
Not included the possible prime pairs in which one of them is lesser than 9.486

k_{ax} = 0,782652
k_{jx} = 0,778774689 k_{jx} / k_{ax} = 0,995045932
c_{jx} = 2,420244777
k_{0x} = 0,722328045 k_{0x} / k_{ax} = 0,922923655

k_{bx} = 0,782751
k_{jx} = 0,778873198 k_{jx} / k_{bx} = 0,995045932
c_{jx} = 2,420526556
k_{0x} = 0,722419415 k_{0x} / k_{bx} = 0,922923656

$90.000.000 = 9\cdot10^7 = (30n_1 + 30) = (30n_2 + 13) + (30n_3 + 17)$ 3.000.000 pairs Highest prime to divide 9.479

Sequence **A** $(30n_2 + 13)$

Total number of terms	3.000.000
Multiples $7m$, $11m$,…	**2.347.924**
Primes greater than 9.486	**652.076**
Number of terms $(7j + a)$, $(11j + b)$,…	2.347.962
Multiples $(7j + a)$, $(11j + b)$,…	**1.828.299**
Primes $(7j + a)$, $(11j + b)$,…	**519.663**

Sequence **B** $(30n_3 + 17)$

Total number of terms	3.000.000
Multiples $7m$, $11m$,…	**2.347.962**
Primes greater than 9.486	**652.038**
Number of terms $(7j + a)$, $(11j + b)$,…	2.347.924
Multiples $(7j + a)$, $(11j + b)$,…	**1.828.299**
Primes $(7j + a)$, $(11j + b)$,…	**519.625**

P_{PPx} = Number of prime pairs being both greater than 9.486 P_{PPx} = 652.076 – 519.663 = 652.038 – 519.625 = 132.413
Not included the possible prime pairs in which one of them is lesser than 9.486

k_{ax} = 0,782641333
k_{jx} = 0,778674867 k_{jx} / k_{ax} = 0,994931949
c_{jx} = 2,473675008
k_{0x} = 0,722280002 k_{0x} / k_{ax} = 0,922874848

k_{bx} = 0,782654
k_{jx} = 0,77868747 k_{jx} / k_{bx} = 0,994931949
c_{jx} = 2,473711418
k_{0x} = 0,722291692 k_{0x} / k_{bx} = 0,922874848

$300.000.000 = 3\cdot10^8 = (30n_1 + 30) = (30n_2 + 1) + (30n_3 + 29)$ 10^7 pairs Highest prime to divide 17.317

Sequence **A** $(30n_2 + 1)$

Total number of terms	10.000.000
Multiples $7m$, $11m$,…	**7.969.053+1**
Primes greater than 17.320	**2.030.946**
Number of terms $(7j + a)$, $(11j + b)$,…	7.968.588
Multiples $(7j + a)$, $(11j + b)$,…	**6.324.515+1**
Primes $(7j + a)$, $(11j + b)$,…	**1.644.072**

Sequence **B** $(30n_3 + 29)$

Total number of terms	10.000.000
Multiples $7m$, $11m$,…	**7.968.588**
Primes greater than 17.320	**2.031.412**
Number of terms $(7j + a)$, $(11j + b)$,…	7.969.053
Multiples $(7j + a)$, $(11j + b)$,…	**6.324.515**
Primes $(7j + a)$, $(11j + b)$,…	**1.644.538**

P_{PPx} = Number of prime pairs being both greater than 17.320 P_{PPx} = 2.030.946 – 1.644.072 = 2.031.412 – 1.644.538 = 386.874
Not included the possible prime pairs in which one of them is lesser than 17.320

k_{ax} = 0,7969053
k_{jx} = 0,793680762 k_{jx} / k_{ax} = 0,995953675
c_{jx} = 2,308152416
k_{0x} = 0,745131006 k_{0x} / k_{ax} = 0,935030807

k_{bx} = 0,7968588
k_{jx} = 0,79363445 k_{jx} / k_{bx} = 0,995953675
c_{jx} = 2.307957765
k_{0x} = 0,745087434 k_{0x} / k_{bx} = 0,93503069

$300.000.000 = 3\cdot10^8 = (30n_1 + 30) = (30n_2 + 7) + (30n_3 + 23)$ 10^7 pairs Highest prime to divide 17.317

Sequence **A** $(30n_2 + 7)$

Total number of terms	10.000.000
Multiples $7m$, $11m$,…	**7.968.657**
Primes greater than 17.320	**2.031.343**
Number of terms $(7j + a)$, $(11j + b)$,…	7.968.695
Multiples $(7j + a)$, $(11j + b)$,…	**6.323.804**
Primes $(7j + a)$, $(11j + b)$,…	**1.644.891**

Sequence **B** $(30n_3 + 23)$

Total number of terms	10.000.000
Multiples $7m$, $11m$,…	**7.968.695**
Primes greater than 17.320	**2.031.305**
Number of terms $(7j + a)$, $(11j + b)$,…	7.968.657
Multiples $(7j + a)$, $(11j + b)$,…	**6.323.804**
Primes $(7j + a)$, $(11j + b)$,…	**1.644.853**

P_{PPx} = Number of prime pairs being both greater than 17.320 P_{PPx} = 2.031.343 – 1.644.891 = 2.031.305 – 1.644.853 = 386.452
Not included the possible prime pairs in which one of them is lesser than 17.320

k_{ax} = 0,7968657
k_{jx} = 0,793580881 k_{jx} / k_{ax} = 0,995877826
c_{jx} = 2,350215256
k_{0x} = 0,745084609 k_{0x} / k_{ax} = 0,935019049

k_{bx} = 0,7968695
k_{jx} = 0,793584665 k_{jx} / k_{bx} = 0,995877826
c_{jx} = 2,350225822
k_{0x} = 0,745088162 k_{0x} / k_{bx} = 0,935019048

$300.000.000 = 3\cdot10^8 = (30n_1 + 30) = (30n_2 + 11) + (30n_3 + 19)$ 10^7 pairs Highest prime to divide 17.317

Sequence **A** $(30n_2 + 11)$

Total number of terms	10.000.000
Multiples $7m$, $11m$,…	**7.968.686**
Primes greater than 17.320	**2.031.314**
Number of terms $(7j + a)$, $(11j + b)$,…	7.968.872
Multiples $(7j + a)$, $(11j + b)$,…	**6.324.084**
Primes $(7j + a)$, $(11j + b)$,…	**1.644.788**

Sequence **B** $(30n_3 + 19)$

Total number of terms	10.000.000
Multiples $7m$, $11m$,…	**7.968.872**
Primes greater than 17.320	**2.031.128**
Number of terms $(7j + a)$, $(11j + b)$,…	7.968.686
Multiples $(7j + a)$, $(11j + b)$,…	**6.324.084**
Primes $(7j + a)$, $(11j + b)$,…	**1.644.602**

P_{PPx} = Number of prime pairs being both greater than 17.320 P_{PPx} = 2.031.314 – 1.644.788 = 2.031.128 – 1.644.602 = 386.526
Not included the possible prime pairs in which one of them is lesser than 17.320

k_{ax} = 0,7968686
k_{jx} = 0,793598391 k_{jx} / k_{ax} = 0,995896175
c_{jx} = 2,34016453
k_{0x} = 0,74509391 k_{0x} / k_{ax} = 0,935027318

k_{bx} = 0,7968872
k_{jx} = 0,793616915 k_{jx} / k_{bx} = 0,995896175
c_{jx} = 2,340214659
k_{0x} = 0,745111301 k_{0x} / k_{bx} = 0,935027317

$\underline{300.000.000 = 3\cdot10^8}$ = $(30n_1 + 30) = (30n_2 + 13) + (30n_3 + 17)$ 10^7 pairs Highest prime to divide 17.317

Sequence **A** $(30n_2 + 13)$		Sequence **B** $(30n_3 + 17)$	
Total number of terms	10.000.000	Total number of terms	10.000.000
Multiples $7m$, $11m$,…	**7.968.552**	Multiples $7m$, $11m$,…	**7.968.561**
Primes greater than 17.320	**2.031.448**	Primes greater than 17.320	**2.031.439**
Number of terms $(7j + a)$, $(11j + b)$,…	7.968.561	Number of terms $(7j + a)$, $(11j + b)$,…	7.968.552
Multiples $(7j + a)$, $(11j + b)$,…	**6.324.315**	Multiples $(7j + a)$, $(11j + b)$,…	**6.324.315**
Primes $(7j + a)$, $(11j + b)$,…	**1.644.246**	Primes $(7j + a)$, $(11j + b)$,…	**1.644.237**

P_{PPx} = Number of prime pairs being both greater than 17.320 P_{PPx} = 2.031.448 – 1.644.246 = 2.031.439 – 1.644.237 = 387.202
Not included the possible prime pairs in which one of them is lesser than 17.320

k_{ax} = 0,7968552		k_{bx} = 0,7968561	
k_{jx} = 0,793658353	k_{jx} / k_{ax} = 0,99598817	k_{jx} = 0,793659249	k_{jx} / k_{bx} = 0,99598817
c_{jx} = 2,287981074		c_{jx} = 2,287983736	
k_{0x} = 0,745067145	k_{0x} / k_{ax} = 0,935009453	k_{0x} = 0,745067987	k_{0x} / k_{bx} = 0,935009454

$\underline{2^{22} = 4.194.304}$ = $(30n_1 + 4) = (30n_2 + 11) + (30n_3 + 23)$ 139.810 pairs Highest prime to divide 2.039

Sequence **A** $(30n_2 + 11)$		Sequence **B** $(30n_3 + 23)$	
Total number of terms	139.810	Total number of terms	139.810
Multiples $7m$, $11m$,…	**102.838**	Multiples $7m$, $11m$,…	**102.841**
Primes greater than 2.048	**36.972**	Primes greater than 2.048	**36.969**
Number of terms $(7j + a)$, $(11j + b)$,…	102.841	Number of terms $(7j + a)$, $(11j + b)$,…	102.838
Multiples $(7j + a)$, $(11j + b)$,…	**74.981**	Multiples $(7j + a)$, $(11j + b)$,…	**74.981**
Primes $(7j + a)$, $(11j + b)$,…	**27.860**	Primes $(7j + a)$, $(11j + b)$,…	**27.857**

P_{PPx} = Number of prime pairs being both greater than 2.048 P_{PPx} = 36.972 – 27.860 = 36.969 – 27.857 = 9.112
Not included the possible prime pairs in which one of them is lesser than 2.048

k_{ax} = 0,735555396		k_{bx} = 0,735576854	
k_{jx} = 0,729096372	k_{jx} / k_{ax} = 0,991218846	k_{jx} = 0,729117641	k_{jx} / k_{bx} = 0,991218846
c_{jx} = 2,70514953		c_{jx} = 2,705221428	
k_{0x} = 0,640494043	k_{0x} / k_{ax} = 0,870762482	k_{0x} = 0,640512728	k_{0x} / k_{bx} = 0,870762482

$\underline{2^{23} = 8.388.608}$ = $(30n_1 + 8) = (30n_2 + 1) + (30n_3 + 7)$ 279.621 pairs Highest prime to divide 2.887

Sequence **A** $(30n_2 + 1)$		Sequence **B** $(30n_3 + 7)$	
Total number of terms	279.621	Total number of terms	279.621
Multiples $7m$, $11m$,…	**209.240+1**	Multiples $7m$, $11m$,…	**209.137**
Primes greater than 2.896	**70.380**	Primes greater than 2.896	**70.484**
Number of terms $(7j + a)$, $(11j + b)$,…	209.137	Number of terms $(7j + a)$, $(11j + b)$,…	209.240
Multiples $(7j + a)$, $(11j + b)$,…	**155.356+1**	Multiples $(7j + a)$, $(11j + b)$,…	**155.356**
Primes $(7j + a)$, $(11j + b)$,…	**53.780**	Primes $(7j + a)$, $(11j + b)$,…	**53.884**

P_{PPx} = Number of prime pairs being both greater than 2.896 P_{PPx} = 70.380 – 53.780 = 70.484 – 53.884 = 16.600
Not included the possible prime pairs in which one of them is lesser than 2.896

k_{ax} = 0,748298589		k_{bx} = 0,747930234	
k_{jx} = 0,742843208	k_{jx} / k_{ax} = 0,992709619	k_{jx} = 0,742477537	k_{jx} / k_{bx} = 0,992709619
c_{jx} = 2,526146914		c_{jx} = 2,5233892	
k_{0x} = 0,663473002	k_{0x} / k_{ax} = 0,886642058	k_{0x} = 0,663143231	k_{0x} / k_{bx} = 0,886637818

$\underline{2^{24} = 16.777.216}$ = $(30n_1 + 16) = (30n_2 + 17) + (30n_3 + 29)$ 559.240 pairs Highest prime to divide 4.093

Sequence **A** $(30n_2 + 17)$		Sequence **B** $(30n_3 + 29)$	
Total number of terms	559.240	Total number of terms	559.240
Multiples $7m$, $11m$,…	**424.629**	Multiples $7m$, $11m$,…	**424.501**
Primes greater than 4.096	**134.611**	Primes greater than 4.096	**134.739**
Number of terms $(7j + a)$, $(11j + b)$,…	424.501	Number of terms $(7j + a)$, $(11j + b)$,…	424.629
Multiples $(7j + a)$, $(11j + b)$,…	**320.414**	Multiples $(7j + a)$, $(11j + b)$,…	**320.414**
Primes $(7j + a)$, $(11j + b)$,…	**104.087**	Primes $(7j + a)$, $(11j + b)$,…	**104.215**

P_{PPx} = Number of prime pairs being both greater than 4.096 P_{PPx} = 134.611 – 104.087 = 134.739 – 104.215 = 30.524
Not included the possible prime pairs in which one of them is lesser than 4.096

k_{ax} = 0,759296545		k_{bx} = 0,759067663	
k_{jx} = 0,754801519	k_{jx} / k_{ax} = 0,994080013	k_{jx} = 0,754573992	k_{jx} / k_{bx} = 0,994080013
c_{jx} = 2,282775574		c_{jx} = 2,282139908	
k_{0x} = 0,682896316	k_{0x} / k_{ax} = 0,899380249	k_{0x} = 0,682690464	k_{0x} / k_{bx} = 0,899380249

2^{25} = 33.554.432 = ($30n_1 + 2$) = ($30n_2 + 13$) + ($30n_3 + 19$)   1.118.481 pairs   Highest prime to divide 5.791

Sequence **A** ($30n_2 + 13$)		Sequence **B** ($30n_3 + 19$)	
Total number of terms	1.118.481	Total number of terms	1.118.481
Multiples 7*m*, 11*m*,…	**860.580**	Multiples 7*m*, 11*m*,…	**860.726**
Primes greater than 5.792	**257.901**	Primes greater than 5.792	**257.755**
Number of terms (7*j* + *a*), (11*j* + *b*),…	860.726	Number of terms (7*j* + *a*), (11*j* + *b*),…	860.580
Multiples (7*j* + *a*), (11*j* + *b*),…	**658.409**	Multiples (7*j* + *a*), (11*j* + *b*),…	**658.409**
Primes (7*j* + *a*), (11*j* + *b*),…	**202.317**	Primes (7*j* + *a*), (11*j* + *b*),…	**202.171**

P_{PPx} = Number of prime pairs being both greater than 5.792 P_{PPx} = 257.901 – 202.317 = 257.755 – 202.171 = 55.584
Not included the possible prime pairs in which one of them is lesser than 5.792

k_{ax} = 0,769418523		k_{bx} = 0,769549058	
k_{jx} = 0,764946103	k_{jx} / k_{ax} = 0,994187272	k_{jx} = 0,765075879	k_{jx} / k_{bx} = 0,994187272
c_{jx} = 2,476961963		c_{jx} = 2,477347165	
k_{0x} = 0,700368084	k_{0x} / k_{ax} = 0,910256332	k_{0x} = 0,700486904	k_{0x} / k_{bx} = 0,910256333

2^{26} = 67.108.864 = ($30n_1 + 4$) = ($30n_2 + 11$) + ($30n_3 + 23$)   2.236.962 pairs   Highest prime to divide 8.191

Sequence **A** ($30n_2 + 11$)		Sequence **B** ($30n_3 + 23$)	
Total number of terms	2.236.962	Total number of terms	2.236.962
Multiples 7*m*, 11*m*,…	**1.742.437**	Multiples 7*m*, 11*m*,…	**1.742.039**
Primes greater than 8.192	**494.525**	Primes greater than 8.192	**494.923**
Number of terms (7*j* + *a*), (11*j* + *b*),…	1.742.039	Number of terms (7*j* + *a*), (11*j* + *b*),…	1.742.437
Multiples (7*j* + *a*), (11*j* + *b*),…	**1.350.052**	Multiples (7*j* + *a*), (11*j* + *b*),…	**1.350.052**
Primes (7*j* + *a*), (11*j* + *b*),…	**391.987**	Primes (7*j* + *a*), (11*j* + *b*),…	**392.385**

P_{PPx} = Number of prime pairs being both greater than 8.192 P_{PPx} = 494.525 – 391.987 = 494.923 – 392.385 = 102.538
Not included the possible prime pairs in which one of them is lesser than 8.192

k_{ax} = 0,778930084		k_{bx} = 0,778752164	
k_{jx} = 0,774983797	k_{jx} / k_{ax} = 0,994933708	k_{jx} = 0,774806779	k_{jx} / k_{bx} = 0,994933708
c_{jx} = 2,378037211		c_{jx} = 2,377536801	
k_{0x} = 0,716122909	k_{0x} / k_{ax} = 0,919367377	k_{0x} = 0,715959336	k_{0x} / k_{bx} = 0,919367378

2^{27} = 134.217.728 = ($30n_1 + 8$) = ($30n_2 + 1$) + ($30n_3 + 7$)   4.473.925 pairs   Highest prime to divide 11.579

Sequence **A** ($30n_2 + 1$)		Sequence **B** ($30n_3 + 7$)	
Total number of terms	4.473.925	Total number of terms	4.473.925
Multiples 7*m*, 11*m*,…	**3.523.978+1**	Multiples 7*m*, 11*m*,…	**3.523.523**
Primes greater than 11.585	**949.946**	Primes greater than 11.585	**950.402**
Number of terms (7*j* + *a*), (11*j* + *b*),…	3.523.523	Number of terms (7*j* + *a*), (11*j* + *b*),…	3.523.978
Multiples (7*j* + *a*), (11*j* + *b*),…	**2.762.690+1**	Multiples (7*j* + *a*), (11*j* + *b*),…	**2.762.690**
Primes (7*j* + *a*), (11*j* + *b*),…	**760.832**	Primes (7*j* + *a*), (11*j* + *b*),…	**761.288**

P_{PPx} = Number of prime pairs being both greater than 11.585 P_{PPx} = 949.946 – 760.832 = 950.402 – 761.288 = 189.114
Not included the possible prime pairs in which one of them is lesser than 11.585

k_{ax} = 0,787670334		k_{bx} = 0,787568633	
k_{jx} = 0,784070375	k_{jx} / k_{ax} = 0,995429611	k_{jx} = 0,783969139	k_{jx} / k_{bx} = 0,995429611
c_{jx} = 2,354564853		c_{jx} = 2,354141254	
k_{0x} = 0,730398752	k_{0x} / k_{ax} = 0,927289908	k_{0x} = 0,730304239	k_{0x} / k_{bx} = 0,927289645

2^{28} = 268.435.456 = ($30n_1 + 16$) = ($30n_2 + 17$) + ($30n_3 + 29$)   8.947.848 pairs   Highest prime to divide 16.381

Sequence **A** ($30n_2 + 17$)		Sequence **B** ($30n_3 + 29$)	
Total number of terms	8.947.848	Total number of terms	8.947.848
Multiples 7*m*, 11*m*,…	**7.119.164**	Multiples 7*m*, 11*m*,…	**7.119.276**
Primes greater than 16.384	**1.828.684**	Primes greater than 16.384	**1.828.572**
Number of terms (7*j* + *a*), (11*j* + *b*),…	7.119.276	Number of terms (7*j* + *a*), (11*j* + *b*),…	7.119.164
Multiples (7*j* + *a*), (11*j* + *b*),…	**5.640.478**	Multiples (7*j* + *a*), (11*j* + *b*),…	**5.640.478**
Primes (7*j* + *a*), (11*j* + *b*),…	**1.478.798**	Primes (7*j* + *a*), (11*j* + *b*),…	**1.478.686**

P_{PPx} = Number of prime pairs being both greater than 16.384 P_{PPx} = 1.828.684 – 1.478.798 = 1.828.572 – 1.478.686 = 349.886
Not included the possible prime pairs in which one of them is lesser than 16.384

k_{ax} = 0,795628624		k_{bx} = 0,795641141	
k_{jx} = 0,792282529	k_{jx} / k_{ax} = 0,9957944	k_{jx} = 0,792294994	k_{jx} / k_{bx} = 0,9957944
c_{jx} = 2,36480182		c_{jx} = 2,364835628	
k_{0x} = 0,743136259	k_{0x} / k_{ax} = 0,934024035	k_{0x} = 0,74314795	k_{0x} / k_{bx} = 0,934024035

2^{28} = 268.435.456 = $(30n_1 + 16) = (30n_4 + 23) + (30n_5 + 23)$ 4.473.924 pairs Highest prime to divide 16.381

Sequence **A** $(30n_4 + 23)$ from 0 to 134.217.728		Sequence **B** $(30n_5 + 23)$ from 134.217.728 to 268.435.456	
Total number of terms	4.473.924	Total number of terms	4.473.924
Multiples $7m$, $11m$,…	**3.523.743**	Multiples $7m$, $11m$,…	**3.595.452**
Primes greater than 16.384	**950.181**	Primes greater than 16.384	**878.472**
Number of terms $(7j + a)$, $(11j + b)$,…	3.595.452	Number of terms $(7j + a)$, $(11j + b)$,…	3.523.743
Multiples $(7j + a)$, $(11j + b)$,…	**2.820.494**	Multiples $(7j + a)$, $(11j + b)$,…	**2.820.494**
Primes $(7j + a)$, $(11j + b)$,…	**774.958**	Primes $(7j + a)$, $(11j + b)$,…	**703.249**

P_{PPx} = Number of prime pairs being both greater than 16.384 P_{PPx} = 950.181 – 774.958 = 878.472 – 703.249 = 175.223
Not included the possible prime pairs in which one of them is lesser than 16.384

k_{ax} = 0,787617983
k_{jx} = 0,784461592 k_{jx} / k_{ax} = 0,995992484
c_{jx} = 2,237432384
k_{0x} = 0,735726988 k_{0x} / k_{ax} = 0,934116542

k_{bx} = 0,803646195
k_{jx} = 0,80042557 k_{jx} / k_{bx} = 0,995992484
c_{jx} = 2,279504226
k_{0x} = 0,750699204 k_{0x} / k_{bx} = 0,934116541

2^{29} = 536.870.912 = $(30n_1 + 2) = (30n_2 + 13) + (30n_3 + 19)$ 17.895.697 pairs Highest prime to divide 23.167

Sequence **A** $(30n_2 + 13)$		Sequence **B** $(30n_3 + 19)$	
Total number of terms	17.895.697	Total number of terms	17.895.697
Multiples $7m$, $11m$,…	**14.371.638**	Multiples $7m$, $11m$,…	**14.372.432**
Primes greater than 23.170	**3.524.059**	Primes greater than 23.170	**3.523.265**
Number of terms $(7j + a)$, $(11j + b)$,…	14.372.432	Number of terms $(7j + a)$, $(11j + b)$,…	14.371.638
Multiples $(7j + a)$, $(11j + b)$,…	**11.498.803**	Multiples $(7j + a)$, $(11j + b)$,…	**11.498.803**
Primes $(7j + a)$, $(11j + b)$,…	**2.873.629**	Primes $(7j + a)$, $(11j + b)$,…	**2.872.835**

P_{PPx} = Number of prime pairs being both greater than 23.170 P_{PPx} = 3.524.059 – 2.873.629 = 3.523.265 – 2.872.835 = 650.430
Not included the possible prime pairs in which one of them is lesser than 23.170

k_{ax} = 0,803077857
k_{jx} = 0,800059655 k_{jx} / k_{ax} = 0,996241707
c_{jx} = 2,300236897
k_{0x} = 0,754804268 k_{0x} / k_{ax} = 0,939889278

k_{bx} = 0,803122225
k_{jx} = 0,800103857 k_{jx} / k_{bx} = 0,996241707
c_{jx} = 2,300353829
k_{0x} = 0,754845969 k_{0x} / k_{bx} = 0,939889278

7.000.000 = $7 \cdot 10^6$ = $(30n_1 + 10) = (30n_2 + 11) + (30n_3 + 29)$ 233.333 pairs Highest prime to divide 2.633

In this case: $a = 0$

Sequence **A** $(30n_2 + 11)$		Sequence **B** $(30n_3 + 29)$	
Total number of terms	233.333	Total number of terms	233.333
Multiples $7m$, $11m$,…	**173.796**	Multiples $7m$, $11m$,…	**173.747**
Primes greater than 2.645	**59.537**	Primes greater than 2.645	**59.586**
Number of terms $(7j + a)$, $(11j + b)$,…	173.747	Number of terms $(7j + a)$, $(11j + b)$,…	173.796
Multiples $(7j + a)$, $(11j + b)$,…	**131.298**	Multiples $(7j + a)$, $(11j + b)$,…	**131.298**
Primes $(7j + a)$, $(11j + b)$,…	**42.449**	Primes $(7j + a)$, $(11j + b)$,…	**42.498**

P_{PPx} = Number of prime pairs being both greater than 2.645 P_{PPx} = 59.537 – 42.449 = 59.586 – 42.498 = 17.088
Not included the possible prime pairs in which one of them is lesser than 2.645

k_{ax} = 0,744841064
k_{jx} = 0,75568499 k_{jx} / k_{ax} = 1,014558712
c_{jx} = – 5,214054931
k_{0x} = 0,657335748 k_{0x} / k_{ax} = 0,882518136

k_{bx} = 0,744631063
k_{jx} = 0,755471932 k_{jx} / k_{bx} = 1,014558712
c_{jx} = – 5,212329083
k_{0x} = 0,657150419 k_{0x} / k_{bx} = 0,882518136

7.000.000 = $7 \cdot 10^6$ = $(30n_1 + 10) = (30n_2 + 17) + (30n_3 + 23)$ 233.333 pairs Highest prime to divide 2.633

In this case: $a = 0$

Sequence **A** $(30n_2 + 17)$		Sequence **B** $(30n_3 + 23)$	
Total number of terms	233.333	Total number of terms	233.333
Multiples $7m$, $11m$,…	**173.764**	Multiples $7m$, $11m$,…	**173.783**
Primes greater than 2.645	**59.569**	Primes greater than 2.645	**59.550**
Number of terms $(7j + a)$, $(11j + b)$,…	173.783	Number of terms $(7j + a)$, $(11j + b)$,…	173.764
Multiples $(7j + a)$, $(11j + b)$,…	**131.356**	Multiples $(7j + a)$, $(11j + b)$,…	**131.356**
Primes $(7j + a)$, $(11j + b)$,…	**42.427**	Primes $(7j + a)$, $(11j + b)$,…	**42.408**

P_{PPx} = Number of prime pairs being both greater than 2.645 P_{PPx} = 59.569 – 42.427 = 59.550 – 42.408 = 17.142
Not included the possible prime pairs in which one of them is lesser than 2.645

k_{ax} = 0,744703921		k_{bx} = 0,744785349	
k_{jx} = 0,755862196	k_{jx} / k_{ax} = 1,014983505	k_{jx} = 0,755944844	k_{jx} / k_{bx} = 1,014983505
c_{jx} = – 5,372347613		c_{jx} = – 5,373039944	
k_{0x} = 0,657222595	k_{0x} / k_{ax} = 0,882528715	k_{0x} = 0,657294458	k_{0x} / k_{bx} = 0,882528715

<u>14.872.858 = 2·7·11·13·17·19·23</u> = ($30n_1$ + 28) = ($30n_2$ + 11) + ($30n_3$ + 17)   495.762 pairs   Highest prime to divide 3.853

In this case: $a = b = c = d = e = f = 0$

Sequence **A** ($30n_2$ + 11)		Sequence **B** ($30n_3$ + 17)	
Total number of terms	495.762	Total number of terms	495.762
Multiples 7*m*, 11*m*,…	**375.402**	Multiples 7*m*, 11*m*,…	**375.451**
Primes greater than 3.856	**120.360**	Primes greater than 3.856	**120.311**
Number of terms (7*j* + *a*), (11*j* + *b*),…	375.451	Number of terms (7*j* + *a*), (11*j* + *b*),…	375.402
Multiples (7*j* + *a*), (11*j* + *b*),…	**302.245**	Multiples (7*j* + *a*), (11*j* + *b*),…	**302.245**
Primes (7*j* + *a*), (11*j* + *b*),…	**73.206**	Primes (7*j* + *a*), (11*j* + *b*),…	**73.157**

P_{PPx} = Number of prime pairs being both greater than 3.856 P_{PPx} = 120.360 – 73.206 = 120.311 – 73.157 = 47.154
Not included the possible prime pairs in which one of them is lesser than 3.856

k_{ax} = 0,757222215		k_{bx} = 0,757321053	
k_{jx} = 0,805018497	k_{jx} / k_{ax} = 1,063120549	k_{jx} = 0,805123574	k_{jx} / k_{bx} = 1,063120549
c_{jx} = – 30,30331135		c_{jx} = – 30,31126361	
k_{0x} = 0,679425487	k_{0x} / k_{ax} = 0,897260372	k_{0x} = 0,67951417	k_{0x} / k_{bx} = 0,897260372

The programmable controller used is very slow to perform calculations with numbers greater than 10^9.

To know the approximate values of c_{jx} for higher numbers, we will use data from Wikipedia concerning to the Twin Primes Conjecture.
Twin Primes Conjecture statement: "There are infinitely many primes *p* such that *p* + 2 is also prime."
We call *Twin Primes* the pair of consecutive primes that are separated only by an even number. To form twin prime pairs, the following combinations of groups of primes are used:

($30n_1$ + 11) and ($30n_1$ + 13),   ($30n_2$ + 17) and ($30n_2$ + 19),   ($30n_3$ + 29) and ($30n_3$ + 31)

Goldbach's conjecture and the twin primes conjecture are similar in that both can be studied by combining two groups of primes to form pairs of primes that add an even number, in the first, or pairs of twin primes in the second. With this in mind, we can use data (*) from Wikipedia concerning to the number of primes and to the number of twin prime pairs lesser than a given number (from 10^{10} to 10^{18}).

<u>10^{10}</u>   455.052.511* primes   27.412.679* twin prime pairs

Number of terms in each sequence **A** or **B**: 10^{10} / 30 = 333.333.333
Approximate number of primes in each sequence **A** or **B**: 455.052.511 / 8 = 56.881.563 (1)
Approximate number of twin prime pairs in the sequences **A-B** (1 combination of 3): 27.412.679 / 3 = 9.137.559
Approximate number of multiples 7*m*, 11*m*,… 333.333.333 – 56.881.563 = 276.451.770 (2)
Number of terms (7*m* + 2), (11*m* + 2),… is, approximately, equal to the number of multiples 7*m*, 11*m*,…
Approximate number of primes (7*m* + 2), (11*m* + 2),… 56.881.563 – 9.137.559 = 47.744.004 (3)
Approximate number of multiples (7*m* + 2), (11*m* + 2),… 276.451.770 – 47.744.004 = 228.707.766 (4)

Total number of terms in sequence **B**	333.333.333			
Multiples 7*m*, 11*m*,…	≈ 276.451.770 (2)	k_{bx} ≈ 0,82935531		
Primes greater than 10^5	≈ 56.881.563 (1)	k_{jx} ≈ 0,827297166	k_{jx} / k_{bx} ≈ 0,99751838	
Number of terms (7*m* + 2), (11*m* + 2),…	≈ 276.451.770 (2)	c_{jx} ≈ 2,095100568		
Multiples (7*m* + 2), (11*m* + 2),…	≈ 228.707.766 (4)	k_{0x} ≈ 0,794244171	k_{0x} / k_{bx} ≈ 0,957664539	
Primes (7*m* + 2), (11*m* + 2),…	≈ 47.744.004 (3)	k_{7x} ≈ 0,800914529		

<u>10^{11}</u>   4.118.054.813* primes   224.376.048* twin prime pairs

Number of terms in each sequence **A** or **B**: 10^{11} / 30 = 3.333.333.333
Approximate number of primes in each sequence **A** or **B**: 4.118.054.813 / 8 = 514.756.851 (1)
Approximate number of twin prime pairs in the sequences **A-B** (1 combination of 3): 224.376.048 / 3 = 74.792.016
Approximate number of multiples 7*m*, 11*m*,… 3.333.333.333 – 514.756.851 = 2.818.576.482 (2)
Number of terms (7*m* + 2), (11*m* + 2),… is, approximately, equal to the number of multiples 7*m*, 11*m*,…
Approximate number of primes (7*m* + 2), (11*m* + 2),… 514.756.851 – 74.792.016 = 439.964.835 (3)
Approximate number of multiples (7*m* + 2), (11*m* + 2),… 2.818.576.482 – 439.964.835 = 2.378.611.647 (4)

Total number of terms in sequence **B**	3.333.333.333		
Multiples 7*m*, 11*m*,…	≈ 2.818.576.482 (2)	k_{bx} ≈ 0,845572944	
Primes greater than $10^{5,5}$	≈ 514.756.851 (1)	k_{jx} ≈ 0,843905305	k_{jx} / k_{bx} ≈ 0,998027799
Number of terms (7*m* + 2), (11*m* + 2),…	≈ 2.818.576.482 (2)	c_{jx} ≈ 2,075447865	
Multiples (7*m* + 2), (11*m* + 2),…	≈ 2.378.611.647 (4)	k_{0x} ≈ 0,817369919	k_{0x} / k_{bx} ≈ 0,966646254
Primes (7*m* + 2), (11*m* + 2),…	≈ 439.964.835 (3)	k_{7x} ≈ 0,819835102	

$\underline{10^{12}}$ 37.607.912.018* primes 1.870.585.220* twin prime pairs

Number of terms in each sequence **A** or **B**: 10^{12} / 30 = 33.333.333.333
Approximate number of primes in each sequence **A** or **B**: 37.607.912.018 / 8 = 4.700.989.002 (1)
Approximate number of twin prime pairs in the sequences **A-B** (1 combination of 3): 1.870.585.220 / 3 = 623.528.406
Approximate number of multiples 7*m*, 11*m*,… 33.333.333.333 – 4.700.989.002 = 28.632.344.331 (2)
Number of terms (7*m* + 2), (11*m* + 2),… is, approximately, equal to the number of multiples 7*m*, 11*m*,…
Approximate number of primes (7*m* + 2), (11*m* + 2),… 4.700.989.002 – 623.528.406 = 4.077.460.596 (3)
Approximate number of multiples (7*m* + 2), (11*m* + 2),… 28.632.344.331 – 4.077.460.596 = 24.554.883.735 (4)

Total number of terms in sequence **B**	33.333.333.333		$k_{bx} \approx 0{,}85897033$	
Multiples 7*m*, 11*m*,…	≈ 28.632.344.331	(2)	$k_{jx} \approx 0{,}857592499$	$k_{jx} / k_{bx} \approx 0{,}99839595$
Primes greater than 10^6	≈ 4.700.989.002	(1)	$c_{jx} \approx 2{,}058134681$	
Number of terms (7*m* + 2), (11*m* + 2),…	≈ 28.632.344.331	(2)	$k_{0x} \approx 0{,}835815434$	$k_{0x} / k_{bx} \approx 0{,}973043427$
Multiples (7*m* + 2), (11*m* + 2),…	≈ 24.554.883.735	(4)	$k_{7x} \approx 0{,}835465384$	
Primes (7*m* + 2), (11*m* + 2),…	≈ 4.077.460.596	(3)	$k_{11x} \approx 0{,}844867362$	

$\underline{10^{13}}$ 346.065.536.839* primes 15.834.664.872* twin prime pairs

Number of terms in each sequence **A** or **B**: 10^{13} / 30 = 333.333.333.333
Approximate number of primes in each sequence **A** or **B**: 346.065.536.839 / 8 = 43.258.192.105 (1)
Approximate number of twin prime pairs in the sequences **A-B** (1 combination of 3): 15.834.664.872 / 3 = 5.278.221.624
Approximate number of multiples 7*m*, 11*m*,… 333.333.333.333 – 43.258.192.105 = 290.075.141.228 (2)
Number of terms (7*m* + 2), (11*m* + 2),… is, approximately, equal to the number of multiples 7*m*, 11*m*,…
Approximate number of primes (7*m* + 2), (11*m* + 2),… 43.258.192.105 – 5.278.221.624 = 37.979.970.481 (3)
Approximate number of multiples (7*m* + 2), (11*m* + 2),… 290.075.141.228 – 37.979.970.481 = 252.095.170.747 (4)

Total number of terms in sequence **B**	333.333.333.333			
Multiples 7*m*, 11*m*,…	≈ 290.075.141.228	(2)	$k_{bx} \approx 0{,}870225423$	
Primes greater than $10^{6,5}$	≈ 43.258.192.105	(1)	$k_{jx} \approx 0{,}869068509$	$k_{jx} / k_{bx} \approx 0{,}998670559$
Number of terms (7*m* + 2), (11*m* + 2),…	≈ 290.075.141.228	(2)	$c_{jx} \approx 2{,}042626025$	
Multiples (7*m* + 2), (11*m* + 2),…	≈ 252.095.170.747	(4)	$k_{0x} \approx 0{,}85087246$	$k_{0x} / k_{bx} \approx 0{,}977760977$
Primes (7*m* + 2), (11*m* + 2),…	≈ 37.979.970.481	(3)		

$\underline{10^{14}}$ 3.204.941.750.802* primes 135.780.321.665* twin prime pairs

Number of terms in each sequence **A** or **B**: 10^{14} / 30 = 3.333.333.333.333
Approximate number of primes in each sequence **A** or **B**: 3.204.941.750.802 / 8 = 400.617.718.850 (1)
Approximate number of twin prime pairs in the sequences **A-B** (1 combination of 3): 135.780.321.665 / 3 = 45.260.107.221
Approximate number of multiples 7*m*, 11*m*,… 3.333.333.333.333 – 400.617.718.850 = 2.932.715.614.483 (2)
Number of terms (7*m* + 2), (11*m* + 2),… is, approximately, equal to the number of multiples 7*m*, 11*m*,…
Approximate number of primes (7*m* + 2), (11*m* + 2),… 400.617.718.850 – 45.260.107.221 = 355.357.611.629 (3)
Approximate number of multiples (7*m* + 2), (11*m* + 2),… 2.932.715.614.483 – 355.357.611.629 = 2.577.358.002.854 (4)

Total number of terms in sequence **B**	3.333.333.333.333			
Multiples 7*m*, 11*m*,…	≈ 2.932.715.614.483	(2)	$k_{bx} \approx 0{,}879814684$	
Primes greater than 10^7	≈ 400.617.718.850	(1)	$k_{jx} \approx 0{,}878829842$	$k_{jx} / k_{bx} \approx 0{,}998880626$
Number of terms (7*m* + 2), (11*m* + 2),…	≈ 2.932.715.614.483	(2)	$c_{jx} \approx 2{,}028807737$	
Multiples (7*m* + 2), (11*m* + 2),…	≈ 2.577.358.002.854	(4)	$k_{0x} \approx 0{,}863397011$	$k_{0x} / k_{bx} \approx 0{,}981339623$
Primes (7*m* + 2), (11*m* + 2),…	≈ 355.357.611.629	(3)		

$\underline{10^{15}}$ 29.844.570.422.669* primes 1.177.209.242.304* twin prime pairs

Number of terms in each sequence **A** or **B**: 10^{15} / 30 = 33.333.333.333.333
Approximate number of primes in each sequence **A** or **B**: 29.844.570.422.669 / 8 = 3.730.571.302.833 (1)
Approximate number of twin prime pairs in the sequences **A-B** (1 combination of 3): 1.177.209.242.304 / 3 = 392.403.080.768
Approximate number of multiples 7*m*, 11*m*,… 33.333.333.333.333 – 3.730.571.302.833 = 29.602.762.030.500 (2)
Number of terms (7*m* + 2), (11*m* + 2),… is, approximately, equal to the number of multiples 7*m*, 11*m*,…
Approximate number of primes (7*m* + 2), (11*m* + 2),… 3.730.571.302.833 – 392.403.080.768 = 3.338.168.221.065 (3)
Approximate number of multiples (7*m* + 2), (11*m* + 2),… 29.602.762.030.500 – 3.338.168.221.065 = 26.264.593.809.435 (4)

Total number of terms in sequence **B**	33.333.333.333.333			
Multiples 7*m*, 11*m*,…	≈ 29.602.762.030.500	(2)	$k_{bx} \approx 0{,}888082861$	
Primes greater than $10^{7,5}$	≈ 3.730.571.302.833	(1)	$k_{jx} \approx 0{,}887234568$	$k_{jx} / k_{bx} \approx 0{,}999044804$
Number of terms (7*m* + 2), (11*m* + 2),…	≈ 29.602.762.030.500	(2)	$c_{jx} \approx 2{,}016482789$	
Multiples (7*m* + 2), (11*m* + 2),…	≈ 26.264.593.809.435	(4)	$k_{0x} \approx 0{,}873978945$	$k_{0x} / k_{bx} \approx 0{,}984118693$
Primes (7*m* + 2), (11*m* + 2),…	≈ 3.338.168.221.065	(3)		

10^{16} 279.238.341.033.925* primes 10.304.195.697.298* twin prime pairs

Number of terms in each sequence **A** or **B**: 10^{16} / 30 = 333.333.333.333.333
Approximate number of primes in each sequence **A** or **B**: 279.238.341.033.925 / 8 = 34.904.792.629.240 (1)
Approximate number of twin prime pairs in the sequences **A**-**B** (1 combination of 3): 10.304.195.697.298 / 3 = 3.434.731.897.432
Approximate number of multiples 7*m*, 11*m*,… 333.333.333.333.333 – 34.904.792.629.240 = 298.428.540.704.093 (2)
Number of terms (7*m* + 2), (11*m* + 2),… is, approximately, equal to the number of multiples 7*m*, 11*m*,…
Approximate number of primes (7*m* + 2), (11*m* + 2),… 34.904.792.629.240 – 3.434.731.897.432 = 31.470.060.721.808 (3)
Approximate number of multiples (7*m* + 2), (11*m* + 2),… 298.428.540.704.093 – 31.470.060.721.808 = 266.958.479.982.285 (4)

			$k_{bx} \approx 0{,}895285622$	
Total number of terms in sequence **B**	333.333.333.333.333		$k_{jx} \approx 0{,}894547415$	$k_{jx} / k_{bx} \approx 0{,}999175451$
Multiples 7*m*, 11*m*,…	≈ 298.428.540.704.093	(2)	$c_{jx} \approx 2{,}005561339$	
Primes greater than 10^8	≈ 34.904.792.629.240	(1)	$k_{0x} \approx 0{,}883038021$	$k_{0x} / k_{bx} \approx 0{,}986319895$
Number of terms (7*m* + 2), (11*m* + 2),…	≈ 298.428.540.704.093	(2)	$k_{7x} \approx 0{,}877833225$	
Multiples (7*m* + 2), (11*m* + 2),…	≈ 266.958.479.982.285	(4)	$k_{11x} \approx 0{,}884814184$	
Primes (7*m* + 2), (11*m* + 2),…	≈ 31.470.060.721.808	(3)	$k_{13x} \approx 0{,}886559424$	

10^{18} 24.739.954.287.740.860* primes 808.675.888.577.436* twin prime pairs

Number of terms in each sequence **A** or **B**: 10^{18} / 30 = 33.333.333.333.333.333
Approximate number of primes in each sequence **A** or **B**: 24.739.954.287.740.860 / 8 = 3.092.494.285.967.607 (1)
Approximate number of twin prime pairs in the sequences **A**-**B** (1 combination of 3): 808.675.888.577.436 / 3 = 269.558.629.525.812
Approximate number of multiples 7*m*, 11*m*,… 33.333.333.333.333.333 – 3.092.494.285.967.607 = 30.240.839.047.365.726 (2)
Number of terms (7*m* + 2), (11*m* + 2),… is, approximately, equal to the number of multiples 7*m*, 11*m*,…
Approximate number of primes (7*m* + 2), (11*m* + 2),… 3.092.494.285.967.607 – 269.558.629.525.812 = 2.822.935.656.441.795 (3)
Approximate number of multiples (7*m* + 2), (11*m* + 2),… 30.240.839.047.365.726 – 2.822.935.656.441.795 = 27.417.903.390.923.931 (4)

			$k_{bx} \approx 0{,}907225171$	
Total number of terms in sequence **B**	33.333.333.333.333.333		$k_{jx} \approx 0{,}906651543$	$k_{jx} / k_{bx} \approx 0{,}999367712$
Multiples 7*m*, 11*m*,…	≈ 30.240.839.047.365.726	(2)	$c_{jx} \approx 1{,}987076711$	
Primes greater than 10^9	≈ 3.092.494.285.967.607	(1)	$k_{0x} \approx 0{,}897737814$	$k_{0x} / k_{bx} \approx 0{,}989542445$
Number of terms (7*m* + 2), (11*m* + 2),…	≈ 30.240.839.047.365.726	(2)	$k_{7x} \approx 0{,}8917627$	
Multiples (7*m* + 2), (11*m* + 2),…	≈ 27.417.903.390.923.931	(4)	$k_{11x} \approx 0{,}897947688$	
Primes (7*m* + 2), (11*m* + 2),…	≈ 2.822.935.656.441.795	(3)	$k_{13x} \approx 0{,}899493935$	

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