Massless electroweak field propagator predicts mass gap

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23 August 2014

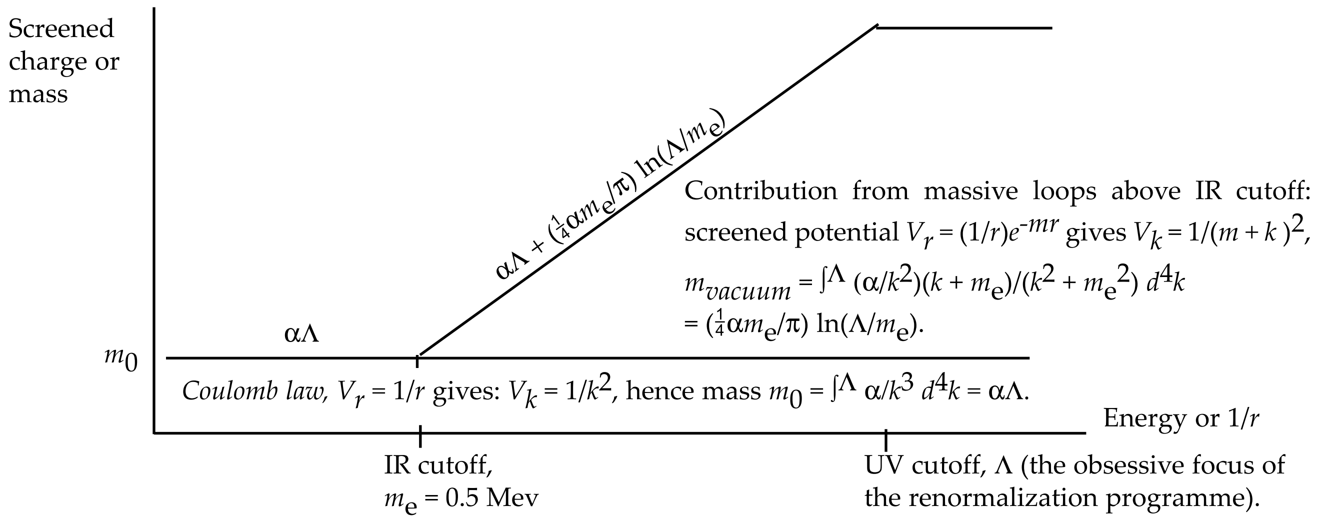
ABSTRACT

Conventionally, the Coulomb field potential $V_r = 1/r$ is replaced by the exponentially screened potential $V_r = (1/r)e^{-mr}$ to ensure that a Laplace transform of the potential gives a propagator with a mass term, whose integral gives the usual logarithmic running with energy. Sidney Coleman suggested the trick of treating the force-mediating propagator photon as having a mass during the calculation, and then setting the mass equal to zero at the end. The QFT textbooks such as Zee's follow Coleman's prescription. The Laplace transform converts the Coulomb potential as a function of radial distance, *r*, into a propagator V_k which is a function of momentum, *k*. $V_k = \int V_r\, e^{-kr} d^3r$. Because we're integrating over spherical volume, d^3r, we can remember that the volume of a sphere is the integral of volume of a spherical shell of surface area $4\pi r^2$ and thickness dr, so that $d^3r \rightarrow 4\pi r^2\, dr$, giving $V_k = \int V_r\, e^{-kr}(4\pi r^2)\, dr$. The substitution of $V_r = (1/r)e^{-mr}$ then gives $V_k = \int [(1/r)e^{-mr}]\, e^{-kr}(4\pi r^2)\, dr = 4\pi \int r e^{-r(m+k)}\, dr = 1/(m+k)^2$, the propagator. Integrating the propagator by Feynman's rules in momentum space gives logarithmic running, $m_{vacuum} = \int^{\Lambda} (\alpha/k^2)(k + m_e)/(k^2 + m_e^2)\, d^4k = (\frac{1}{4}\alpha m_e/\pi) \ln(\Lambda/m_e)$. In renormalization, this running mass term is added on to the observed electron mass, m_0.

But as demonstrated by Figure 1, there are two contributions to the propagator, one from massless field quanta which generate the bare mass that doesn't have a running coupling, and one from massive field quanta which generate the logarithmic running contribution to mass which is utilized in renormalization procedures. In other words, the observed electron mass at low energy (which remains constant at all energies below 1 Mev, where no pair production or vacuum polarization occurs), is the integral of the *massless* Coulomb field potential. The Laplace transform using the Coulomb field potential $V_r = 1/r$ (which is experimentally valid below the IR cutoff of about 1 Mev), gives a propagator which doesn't contain a mass term: $V_k = \int (1/r)\, e^{-kr}(4\pi r^2)\, dr = 4\pi \int r e^{-rk}\, dr = 1/k^2$. Using this in Feynman's rules: $m = \int^{\Lambda} (\alpha/k^3)\, d^4k = \alpha\Lambda$.

So the change in the propagator from $k^2 + m_e^2 \rightarrow 1/k^2$ results in a physically meaningful non-running so-called "bare" mass m_0 due to the *Coulomb* low-energy propagator valid below the 1 Mev IR cutoff and this Coulomb portion of the field contributes a simple interaction-current mass term, $\alpha\Lambda$: $\int^{\Lambda} \alpha(k + m_e)/(k^2 + m_e^2)\, d^4k \rightarrow \int^{\Lambda} \alpha/k^3\, d^4k = \alpha\Lambda$.

Therefore, we argue that the conventional (Schwinger) formula $m_e = m_0 + m_{\text{vacuum}}$ should be revised to represent the bare mass m_0 as being the non-running mass contributed by the massless Coulomb propagator, i.e. $m_0 = \int^{\Lambda} \alpha/k^3\, d^4k$.



Note that although Coulomb potential predominates below 0.5 Mev, it still contributes mass up to the UV cutoff, Λ.

Figure 1: writing the mass of a particle as the sum of contributions from the Coulomb law propagator (representing the non-running mass of a particle below the IR cutoff of 1 Mev) and the logarithmic running contribution due to the vacuum polarization screen ed potential, which results from a potential of the Klein-Gordon type, giving rise to a propagator containing a mass term.

For a *single* one-loop: $m_e = m_0 + m_{vacuum} = \int^{\Lambda} \alpha/k^3\, d^4k + \int^{\Lambda} (\alpha/k^2)(k + m_e)/(k^2 + m_e^2)\, d^4k = \alpha\Lambda + (\frac{1}{4}\alpha m_e/\pi)\ln(\Lambda/m_e)$

A electron-sized mass gap in quantum field theory is given when written using a IR cutoff theory for the Lagrangian obtained by integrating a *two* one-loop Feynman diagrams with massless electroweak force propagator over spacetime up to an electroweak cutoff scale which is defined as the charged weak boson mass, $\Lambda = m_W$. For *two* one-loops:

$$\int^{\Lambda} \alpha^2(k^2 + m_e^2)/(k^2 + m_e^2)^2]\, d^4k \rightarrow \int^{\Lambda} (\alpha^2/k^3)\, d^4k = \alpha^2\Lambda.$$

However, the integral for the Feynman diagram for mass acquisition is weighted by the ratio of the neutral Z-boson mass to the Higgs mass, on the basis of the mechanism that when a particle core is physically accelerated, its acceleration is impeded by its interaction with Z-bosons in its own field, i.e. neutral (weak) currents. The Higgs field gives mass to weak bosons, which then mire particles, causing mass (inertia). If $\Lambda = m_Z = 91.19$:

$$m_e = \int^{\Lambda} [(\alpha/\pi)^2/k^3]\, d^4k$$
$$= m_Z(\alpha/\pi)^2$$

The masses of the unstable charged leptons, and also of all hadrons (mesons and baryons) can be represented by a simpler Feynman diagram propagator, containing only a first power of alpha, based on the 1-loop mass renormalization propagator with the mass term dropped to eliminate the standard solution, i.e. the logarithmic running of mass (which is only present at energies exceeding an IR cutoff of 1 Mev for pair production and vacuum polarization):

$$\int^{\Lambda} [(\alpha/k^2)(k + m_e)/(k^2 + m_e^2)]\, d^4k \rightarrow \int^{\Lambda} (\alpha/k^3)\, d^4k, \text{ where: } \int^{\Lambda} (\alpha/k^3)\, d^4k = \alpha\Lambda.$$

Ref: see page 8 of the book *Supersymmetry Demystified,* equations 1.2 and 1.3, where the non-logarithmic running solutions for zero mass are mentioned in passing but are however dismissed as being "naive power counting," and it is stated that a two one-loop Feynman diagram contributes to mass for Yang-Mills fields, while only a single one-loop contributes for Abelian fields. Since the standard electroweak theory is in fact a mixture of linked Abelian U(1) hypercharge and Yang-Mills SU(2), and since mass results from this mixed symmetry breaking, it is clear that both a single and a double one-loop mass acquisition mechanism must contribute to particle masses in the standard electroweak theory. Hence,

$$m_{n,N} = \int^{\Lambda} n(N + 1)m_Z\, \alpha/(2\pi\, k^3)\ \ d^4k$$
$$= n(N + 1)m_Z\, \alpha/(2\pi)$$
$$= 35.0n(N + 1) \text{ Mev},$$

where n is the number of onshell particles (1 for leptons, 2 quarks for mesons, 3 quarks for baryons) and N is the total number of shell-structured "effective onshell" particles created by pair production and polarization in the field field around each onshell particle's core. See Table 1 for a comparison of predictions with observations.

The muon and tauon correspond respectively with nucleon shell theory magic numbers $N = 2$ and 50, respectively, while nucleons correspond to the nucleon shell theory's magic number of $N = 8$. Thus, we can use the Pauli exclusion principle to predict relatively stable particle masses, which gives a solid basis for objectively picking out observed quantized masses instead of using *ad hoc* landscape speculations, such as occur using string theory (often claimed to be the best hope for masses). There is a simple, understandable, predictive model for masses, based on vacuum fields:

1. Virtual fermions are radially polarized (driven further apart) by the electric field in which they formed.

2. This polarization supplies the virtual fermions energy, at the expense of electric field, which is thus partly "screened."

3. The energy supplied to virtual fermions by their radial polarization extends their lifetime beyond Heisenberg's $\hbar/E$.

4. This supply of extra energy moves "virtual" fermions towards the real mass shell, so they briefly obey Pauli's principle.

5. As a result of this, the "virtual" fermions become structured like electron orbits, thereby contributing quantized mass. Different isomers are possible which allow various weak decay routes, thereby predicting the CKM matrix mechanism.

6. Because neutrinos only have weak fields, they have little pair production, accounting for their small masses.

Neutrino masses suggest that flavor oscillations are differences in virtual particle structuring. Interactions by particles with a vacuum Higgs field are postulated in the Standard Model to produce fermion masses. A photon is slowed and refracted in a block of glass by the "loading" interactions of its electromagnetic field with the fields of the electrons in the crystal lattice of glass. Similarly, a photon passing near the sun is slowed and refracted by the interactions of its field with the gravitation field of the sun. In effect, the photon is "loaded" by interactions with the field quanta through which it passes, be this an electromagnetic or gravitational field, slowing the photon and therefore providing a quantum field theory mechanism and explanation for the phenomena of relativistic time-dilation. Vacuum polarization is also the basis for the logarithmic runnings of force coupling strengths, i.e. effective charges and masses, with interaction energy. The running of mass with energy has been a standard part of QED's renormalization procedure since 1949. There is nothing speculative about these observations. This paper merely sorts out the confused muddle created by a fog of obfuscation, it does not speculate (see reference in further reading section for further discussion of the mechanism and hard data).

TABLE 1

COMPARISON OF OBSERVED WITH PREDICTED MASSES OF ELEMENTARY PARTICLES*

N (number of black hole, gravitationally trapped Z-bosons associated with each core)	n (number of fundamental particles per observable particle core)		
	1	2 (i.e., 2 quarks)	3 (i.e., 3 quarks)
	Leptons	Mesons	Baryons
1	*Electron (stable, 0.511 Mev measured):* $M_Z\alpha^2/(1.5*2\pi)$ = 0.51 MeV	*Pions (139.57, 134.96 Mev measured):* $M_e n(N + 1)/(2\alpha) = 35n(N+1)$ = 140 MeV	'… I do feel strongly that this [string theory] is nonsense! … I think all this superstring stuff is crazy and is in the wrong direction. … I don't like it that they're not calculating anything. … why are the masses of the various particles such as quarks what they are? All these numbers … have no explanations in these string theories - absolutely none! …' – Richard P. Feynman in Davies & Brown, *Superstrings* 1988, at pages 194-195
2* *Magic number of nuclear physics => High stability	*Muon (Most stable particle after neutron, 105.66 Mev measured):* $M_e n(N + 1)/(2\alpha) = 35n(N+1)$ = 105 Mev		
6		*Kaons (493.67, 497.67 Mev measured):* $M_e n(N + 1)/(2\alpha) = 35n(N+1)$ = 490 Mev	
7		*Eta (548.8 Mev measured):* $M_e n(N + 1)/(2\alpha) = 35n(N+1)$ = 560 Mev	
8* *Magic number of nuclear physics => High stability			*Nucleons (VERY STABLE, 938.28, 939.57 Mev measured):* $M_e n(N + 1)/(2\alpha) = 35n(N+1) \sim 945$ Mev
10			*Lambda & Sigmas (1115.6, 1189.36, 1192.46, 1197.34 Mev measured):* $M_e n(N + 1)/(2\alpha) = 35n(N+1)$ = 1155 Mev
12			*Xi (1314.9, 1321.3 Mev measured):* $M_e n(N + 1)/(2\alpha) = 35n(N+1)$ = 1365 Mev
15			*Omega (1672.5 Mev measured):* $M_e n(N + 1)/(2\alpha) = 35n(N+1)$ = 1680 Mev
50* *Magic number of nuclear physics => High stability	*Tauon (1784.2 MeV measured):* $M_e n(N + 1)/(2\alpha) = 35n(N+1)$ = 1785 Mev		

*Only particles with lifetimes above 10^{-23} s are included above. The blank spaces predict other particles. The integer formula is very close, as statistical tests show. (Notice that the periodic table of chemistry did not explain discrepancies from integer masses until mass defect due to binding energy, isotopic composition, and other factors were discovered long after the periodic table was widely accepted. Doubtless there is analogous 'noise' in the measurements due to field interactions.)

Further reading:

"Mechanism of renormalization can predict particle masses," N. B. Cook, vixra, 24 July 2014.