Schrödinger Equation and the Quantization of Celestial Systems

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In the present article, we argue that it is possible to generalize Schrödinger equation to describe quantization of celestial systems. While this hypothesis has been described by some authors, including Nottale, here we argue that such a macroquantization was formed by topological superfluid vortice. We also provide derivation of Schrödinger equation from Gross-Pitaevskii-Ginzburg equation, which supports this superfluid dynamics interpretation.

1 Introduction

In the present article, we argue that it is possible to generalize Schrödinger equation to describe quantization of celestial systems, based on logarithmic nature of Schrödinger equation, and also its exact mapping to Navier-Stokes equations [1].

While this notion of macro-quantization is not widely accepted yet, as we will see the logarithmic nature of Schrödinger equation could be viewed as a support of its applicability to larger systems. After all, the use of Schrödinger equation has proved itself to help in finding new objects known as extrasolar planets [2, 3]. And we could be sure that new extrasolar planets are to be found in the near future. As an alternative, we will also discuss an outline for how to derive Schrödinger equation from simplification of Ginzburg-Landau equation. It is known that Ginzburg-Landau equation exhibits fractal character, which implies that quantization could happen at *any scale*, supporting topological interpretation of quantized vortices [4].

First, let us rewrite Schrödinger equation in its common form [5]

$$\left[i\,\frac{\partial}{\partial t} + \frac{\bar{\nabla}^2}{2m} - U(x) \right] \psi = 0 \tag{1}$$

or

$$i\,\frac{\partial \psi}{\partial t} = H\,\psi\,. \tag{2}$$

Now, it is worth noting here that Englman and Yahalom [5] argues that this equation exhibits logarithmic character

$$\ln \psi(x,t) = \ln \big(|\psi(x,t)|\big) + i \arg\big(\psi(x,t)\big)\,. \tag{3}$$

Schrödinger already knew this expression in 1926, which then he used it to propose his equation called “eigentliche Wellengleichung” [5]. Therefore equation (1) can be re-written as follows

$$2m\frac{\partial\big(\ln|\psi|\big)}{\partial t} + 2\bar{\nabla}\ln|\psi|\,\bar{\nabla}\arg\big[\psi\big] + \bar{\nabla}\bar{\nabla}\arg\big[\psi\big] = 0\,. \tag{4}$$

Interestingly, Nottale’s scale-relativistic method [2, 3] was also based on generalization of Schrödinger equation to describe quantization of celestial systems. It is known that Nottale-Schumacher’s method [6] could predict new exoplanets in good agreement with observed data. Nottale’s scale-relativistic method is essentially based on the use of first-order scale-differentiation method defined as follows [2]

$$\frac{\partial V}{\partial(\ln \delta t)} = \beta(V) = a + b\,V + \dots\,. \tag{5}$$

Now it seems clear that the natural-logarithmic derivation, which is essential in Nottale’s scale-relativity approach, also has been described properly in Schrödinger’s original equation [5]. In other words, its logarithmic form ensures applicability of Schrödinger equation to describe macro-quantization of celestial systems. [7, 8]

2 Quantization of celestial systems and topological quantized vortices

In order to emphasize this assertion of the possibility to describe quantization of celestial systems, let us quote Fischer’s description [4] of relativistic momentum from superfluid dynamics. Fischer [4] argues that the circulation is in the relativistic dense superfluid, defined as the integral of the momentum

$$\gamma_s = \oint p_\mu\, dx^\mu = 2\pi N_v\,\hbar\,, \tag{6}$$

and is quantized into multiples of Planck’s quantum of action. This equation is the covariant Bohr-Sommerfeld quantization of γ_s. And then Fischer [4] concludes that the Maxwell equations of ordinary electromagnetism can be written in the form of conservation equations of relativistic perfect fluid hydrodynamics [9]. Furthermore, the topological character of equation (6) corresponds to the notion of topological electronic liquid, where compressible electronic liquid represents superfluidity [25]. For the plausible linkage between superfluid dynamics and cosmological phenomena, see [16–24].

It is worth noting here, because vortices could be defined as elementary objects in the form of stable topological excitations [4], then equation (6) could be interpreted as Bohr-Sommerfeld-type quantization from *topological quantized vortices*. Fischer [4] also remarks that equation (6) is quite interesting for the study of superfluid rotation in the context of gravitation. Interestingly, application of Bohr-Sommerfeld quantization for celestial systems is known in literature [7, 8], which here in the context of Fischer's arguments it has special meaning, i. e. it suggests that *quantization of celestial systems actually corresponds to superfluid-quantized vortices at large-scale* [4]. In our opinion, this result supports known experiments suggesting neat correspondence between condensed matter physics and various cosmology phenomena [16–24].

To make the conclusion that quantization of celestial systems actually corresponds to superfluid-quantized vortices at large-scale a bit conceivable, let us consider the problem of quantization of celestial orbits in solar system.

In order to obtain planetary orbit prediction from this hypothesis we could begin with the Bohr-Sommerfeld's conjecture of quantization of angular momentum. This conjecture may originate from the fact that according to BCS theory, superconductivity can exhibit macroquantum phenomena [26, 27]. In principle, this hypothesis starts with observation that in quantum fluid systems like superfluidity [28]; it is known that such vortexes are subject to quantization condition of integer multiples of 2π, or $\oint v_s\,dl = 2\pi n\hbar/m$. As we know, for the wavefunction to be well defined and unique, the momenta must satisfy Bohr-Sommerfeld's quantization condition [28]

$$\oint_\Gamma p\,dx = 2\pi n\hbar \tag{6a}$$

for any closed classical orbit Γ. For the free particle of unit mass on the unit sphere the left-hand side is [28]

$$\int_0^T v^2 d\tau = \omega^2 T = 2\pi\omega\,, \tag{7}$$

where $T = 2\pi/\omega$ is the period of the orbit. Hence the quantization rule amounts to quantization of the rotation frequency (the angular momentum): $\omega = n\hbar$. Then we can write the force balance relation of Newton's equation of motion [28]

$$\frac{GMm}{r^2} = \frac{mv^2}{r}\,. \tag{8}$$

Using Bohr-Sommerfeld's hypothesis of quantization of angular momentum, a new constant g was introduced [28]

$$mvr = \frac{ng}{2\pi}\,. \tag{9}$$

Just like in the elementary Bohr theory (before Schrödinger), this pair of equations yields a known simple solution for the orbit radius for any quantum number of the form [28]

$$r = \frac{n^2 g^2}{4\pi^2 GMm^2}\,, \tag{10}$$

which can be rewritten in the known form of gravitational Bohr-type radius [2, 7, 8]

$$r = \frac{n^2 GM}{v_0^2}\,, \tag{11}$$

where r, n, G, M, v_0 represents orbit radii, quantum number ($n = 1, 2, 3, \dots$), Newton gravitation constant, and mass of the nucleus of orbit, and specific velocity, respectively. In this equation (11), we denote [28]

$$v_0 = \frac{2\pi}{g}\,GMm\,. \tag{12}$$

The value of m is an adjustable parameter (similar to g) [7, 8]. In accordance with Nottale, we assert that the specific velocity v_0 is 144 km/sec for planetary systems. By noting that m is meant to be mass of celestial body in question, then we could find g parameter (see also [28] and references cited therein).

Using this equation (11), we could predict quantization of celestial orbits in the solar system, where for Jovian planets we use least-square method and use M in terms of reduced mass $\mu = \frac{(M_1+M_2)}{M_1 M_2}$. From this viewpoint the result is shown in Table 1 below [28].

For comparison purpose, we also include some recent observation by Brown-Trujillo team from Caltech [29–32]. It is known that Brown et al. have reported not less than four new planetoids in the outer side of Pluto orbit, including 2003EL61 (at 52 AU), 2005FY9 (at 52 AU), 2003VB12 (at 76 AU, dubbed as Sedna). And recently Brown-Trujillo team reported a new planetoid finding, called 2003UB31 (97 AU). This is not to include their previous finding, Quaoar (42 AU), which has orbit distance more or less near Pluto (39.5 AU), therefore this object is excluded from our discussion. It is interesting to remark here that all of those new "planetoids" are within 8% bound from our prediction of celestial quantization based on the above Bohr-Sommerfeld quantization hypothesis (Table 1). While this prediction is not so precise compared to the observed data, one could argue that the 8% bound limit also corresponds to the remaining planets, including inner planets. Therefore this 8% uncertainty could be attributed to macroquantum uncertainty and other local factors.

While our previous prediction only limits new planet finding until $n = 9$ of Jovian planets (outer solar system), it seems that there are sufficient reasons to suppose that more planetoids in the Oort Cloud will be found in the near future. Therefore it is recommended to extend further the same quantization method to larger n values. For prediction purpose, we include in Table 1 new expected orbits based

Object	No.	Titius	Nottale	CSV	Observ.	Δ, %
	1		0.4	0.43		
	2		1.7	1.71		
Mercury	3	4	3.9	3.85	3.87	0.52
Venus	4	7	6.8	6.84	7.32	6.50
Earth	5	10	10.7	10.70	10.00	−6.95
Mars	6	16	15.4	15.4	15.24	−1.05
Hungarias	7		21.0	20.96	20.99	0.14
Asteroid	8		27.4	27.38	27.0	1.40
Camilla	9		34.7	34.6	31.5	−10.00
Jupiter	2	52		45.52	52.03	12.51
Saturn	3	100		102.4	95.39	−7.38
Uranus	4	196		182.1	191.9	5.11
Neptune	5			284.5	301	5.48
Pluto	6	388		409.7	395	−3.72
2003EL61	7			557.7	520	−7.24
Sedna	8	722		728.4	760	4.16
2003UB31	9			921.8	970	4.96
Unobserv.	10			1138.1		
Unobserv.	11			1377.1		

Table 1: Comparison of prediction and observed orbit distance of planets in Solar system (in 0.1AU unit) [28].

on the same quantization procedure we outlined before. For Jovian planets corresponding to quantum number $n = 10$ and $n = 11$, our method suggests that it is likely to find new orbits around 113.81 AU and 137.71 AU, respectively. It is recommended therefore, to find new planetoids around these predicted orbits.

As an interesting alternative method supporting this proposition of quantization from superfluid-quantized vortices (6), it is worth noting here that Kiehn has argued in favor of re-interpreting the square of the wavefunction of Schrödinger equation as the vorticity distribution (including topological vorticity defects) in the fluid [1]. From this viewpoint, Kiehn suggests that there is *exact mapping* from Schrödinger equation to Navier-Stokes equation, using the notion of quantum vorticity [1]. Interestingly, de Andrade and Sivaram [33] also suggest that there exists formal analogy between Schrödinger equation and the Navier-Stokes viscous dissipation equation:

$$\frac{\partial V}{\partial t} = \nu\, \nabla^2 V\,, \tag{13}$$

where ν is the kinematic viscosity. Their argument was based on propagation torsion model for quantized vortices [23]. While Kiehn's argument was intended for ordinary fluid, nonetheless the neat linkage between Navier-Stokes equation and superfluid turbulence is known in literature [34, 24].

At this point, it seems worth noting that some criticism arises concerning the use of quantization method for describing the motion of celestial systems. These criticism proponents usually argue that quantization method (wave mechanics) is oversimplifying the problem, and therefore cannot explain other phenomena, for instance planetary migration etc. While we recognize that there are phenomena which do not correspond to quantum mechanical process, at least we can argue further as follows:

1. Using quantization method like Nottale-Schumacher did, one can expect to predict new exoplanets (extra-solar planets) with remarkable result [2, 3];
2. The "conventional" theories explaining planetary migration normally use fluid theory involving *diffusion* process;
3. Alternatively, it has been shown by Gibson et al. [35] that these migration phenomena could be described via Navier-Stokes approach;
4. As we have shown above, Kiehn's argument was based on *exact-mapping* between Schrödinger equation and Navier-Stokes equations [1];
5. Based on Kiehn's vorticity interpretation one these authors published prediction of some new planets in 2004 [28]; which seems to be in good agreement with Brown-Trujillo's finding (March 2004, July 2005) of planetoids in the Kuiper belt;
6. To conclude: while our method as described herein may be interpreted as an oversimplification of the real planetary migration process which took place sometime in the past, at least it could provide us with useful tool for prediction;
7. Now we also provide new prediction of other planetoids which are likely to be observed in the near future (around 113.8 AU and 137.7 AU). It is recommended to use this prediction as guide to finding new objects (in the inner Oort Cloud);
8. There are of course other theories which have been developed to explain planetoids and exoplanets [36]. Therefore quantization method could be seen as merely a "plausible" theory between others.

All in all, what we would like to emphasize here is that the quantization method does not have to be the *true* description of reality with regards to celestial phenomena. As always this method could explain some phenomena, while perhaps lacks explanation for other phenomena. But at least it can be used to predict something quantitatively, i. e. measurable (exoplanets, and new planetoids in the outer solar system etc.).

In the meantime, it seems also interesting here to consider a plausible generalization of Schrödinger equation in particular in the context of viscous dissipation method [1]. First, we could write Schrödinger equation for a charged particle interacting with an external electromagnetic field [1] in the form of Ulrych's unified wave equation [14]

$$\begin{aligned}\big[(-i\hbar\nabla - qA)_\mu(-i\hbar\nabla - qA)^\mu\psi\big] &=\\ &= \left[-i2m\,\frac{\partial}{\partial t} + 2mU(x)\right]\psi\,.\end{aligned} \tag{14}$$

In the presence of electromagnetic potential, one could include another term into the LHS of equation (14)

$$\begin{aligned}\big[(-i\hbar\nabla - qA)_\mu(-i\hbar\nabla - qA)^\mu + eA_0\big]\psi &=\\ &= 2m\left[-i\,\frac{\partial}{\partial t} + U(x)\right]\psi\,.\end{aligned} \tag{15}$$

This equation has the physical meaning of Schrödinger equation for a charged particle interacting with an external electromagnetic field, which takes into consideration Aharonov effect [37]. Topological phase shift becomes its immediate implication, as already considered by Kiehn [1].

As described above, one could also derived equation (11) from scale-relativistic Schrödinger equation [2, 3]. It should be noted here, however, that Nottale's method [2, 3] differs appreciably from the viscous dissipative Navier-Stokes approach of Kiehn [1], because Nottale only considers his equation in the Euler-Newton limit [3]. Nonetheless, it shall be noted here that in his recent papers (2004 and up), Nottale has managed to show that his scale relativistic approach has linkage with Navier-Stokes equations.

3 Schrödinger equation derived from Ginzburg-Landau equation

Alternatively, in the context of the aforementioned superfluid dynamics interpretation [4], one could also derive Schrödinger equation from simplification of Ginzburg-Landau equation. This method will be discussed subsequently. It is known that Ginzburg-Landau equation can be used to explain various aspects of superfluid dynamics [16, 17]. For alternative approach to describe superfluid dynamics from Schrödinger-type equation, see [38, 39].

According to Gross, Pitaevskii, Ginzburg, wavefunction of N bosons of a reduced mass m^* can be described as [40]

$$-\left(\frac{\hbar^2}{2m^*}\right)\nabla^2\psi + \kappa\,|\psi|^2\,\psi = i\hbar\,\frac{\partial\psi}{\partial t}\,. \tag{16}$$

For some conditions, it is possible to replace the potential energy term in equation (16) with Hulthen potential. This substitution yields

$$-\left(\frac{\hbar^2}{2m^*}\right)\nabla^2\psi + V_{\text{Hulthen}}\psi = i\hbar\,\frac{\partial\psi}{\partial t}\,, \tag{17}$$

where

$$V_{\text{Hulthen}} = -Ze^2\,\frac{\delta\,e^{-\delta r}}{1 - e^{-\delta r}}\,. \tag{18}$$

This equation (18) has a pair of exact solutions. It could be shown that for small values of δ, the Hulthen potential (18) approximates the effective Coulomb potential, in particular for large radius

$$V^{\text{eff}}_{\text{Coulomb}} = -\frac{e^2}{r} + \frac{\ell(\ell+1)\,\hbar^2}{2mr^2}\,. \tag{19}$$

By inserting (19), equation (17) could be rewritten as

$$-\left(\frac{\hbar^2}{2m^*}\right)\nabla^2\psi + \left[-\frac{e^2}{r} + \frac{\ell(\ell+1)\hbar^2}{2mr^2}\right]\psi = i\hbar\,\frac{\partial\psi}{\partial t}\,. \tag{20}$$

For large radii, second term in the square bracket of LHS of equation (20) reduces to zero [41],

$$\frac{\ell(\ell+1)\,\hbar^2}{2mr^2} \to 0\,, \tag{21}$$

so we can write equation (20) as

$$\left[-\left(\frac{\hbar^2}{2m^*}\right)\nabla^2 + U(x)\right]\psi = i\hbar\,\frac{\partial\psi}{\partial t}\,, \tag{22}$$

where Coulomb potential can be written as

$$U(x) = -\frac{e^2}{r}\,. \tag{22a}$$

This equation (22) is nothing but Schrödinger equation (1), except for the mass term now we get mass of Cooper pairs. In other words, we conclude that it is possible to re-derive Schrödinger equation from simplification of (Gross-Pitaevskii) Ginzburg-Landau equation for superfluid dynamics [40], in the limit of small screening parameter, δ. Calculation shows that introducing this Hulthen effect (18) into equation (17) will yield essentially similar result to (1), in particular for small screening parameter. Therefore, we conclude that for most celestial quantization problems the result of TDGL-Hulthen (20) is *essentially* the same with the result derived from equation (1). Now, to derive gravitational Bohr-type radius equation (11) from Schrödinger equation, one could use Nottale's scale-relativistic method [2, 3].

4 Concluding remarks

What we would emphasize here is that this derivation of Schrödinger equation from (Gross-Pitaevskii) Ginzburg-Landau equation is in good agreement with our previous conjecture that equation (6) implies macroquantization corresponding to superfluid-quantized vortices. This conclusion is the main result of this paper. Furthermore, because Ginzburg-Landau equation represents superfluid dynamics at low-temperature [40], the fact that we can derive quantization of celestial systems from this equation seems to support the idea of Bose-Einstein condensate cosmology [42, 43]. Nonetheless, this hypothesis of Bose-Einstein condensate cosmology deserves discussion in another paper.

Above results are part of our book *Multi-Valued Logic, Neutrosophy, and Schrödinger Equation* that is in print.

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