

# The Bipolar Linear *Momentum* transported by the Electromagnetic Waves: Origin of the Gravitational Interaction

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Besides energy, the electromagnetic waves transport linear *momentum*. Then, if this momentum is absorbed by a surface, pressure is exerted on the surface. This is the so-called Radiation Pressure. Here we show that this pressure has a *negative* component (opposite to the direction of propagation of the radiation) due to the existence of the negative linear *momentum* transported by the electromagnetic waves. This fact leads to an important theoretical discovery: the velocity of the electromagnetic waves in free space is not a constant. In addition, a *generalized equation* of the Newton's law of Gravitation, is deduced starting from the concept of *negative* radiation pressure applied on the Gravitational Interaction.

**Key words:** Gravity, Gravitation, Electromagnetic Waves, Radiation Pressure.

## 1. Introduction

Electromagnetic waves transport energy as well as linear *momentum*. Then, if this *momentum* is absorbed by a surface, pressure is exerted on the surface. Maxwell showed that, if the incident energy  $U$  is totally absorbed by the surface during a time  $t$ , then the total *momentum*  $q$  transferred to the surface is  $q = U/v$  [1]. Then, the pressure,  $p$  (defined as force  $F$  per unit area  $A$ ), exerted on the surface, is given by

$$p = \frac{F}{A} = \frac{1}{A} \frac{dq}{dt} = \frac{1}{A} \frac{d}{dt} \left( \frac{U}{v} \right) = \frac{1}{v} \frac{(dU/dt)}{A} \quad (1)$$

whence we recognize the term  $(dU/dt)/A$  as the radiation *power density*,  $D$ , (in *watts/m<sup>2</sup>*) arriving at the surface<sup>1</sup>. Thus, if  $v = c$  the *radiation pressure* exerted on the surface is

$$p = \frac{D}{c} \quad (2)$$

Here we show that this pressure has a *negative* component (*opposite to the direction of propagation of the photons*) due to the existence of the negative linear *momentum* transported by the photons. This fact leads to an important theoretical discovery: the velocity of the electromagnetic waves in free space is not a constant. In addition, a *generalized equation* of the Newton's law of Gravitation is deduced starting from the concept of *negative* radiation pressure applied on the Gravitational Interaction.

## 2. Theory

The energy of a harmonic oscillator is quantized in multiples of  $hf$ , and given by

$$E_n = \left(n + \frac{1}{2}\right)hf \quad n = 0, 1, 2, \dots \quad (3)$$

where  $f$  is the classical frequency of oscillation, and  $h$  is the Planck's constant [2]. When  $n = 0$ , Eq. (3) shows that  $E_0 = \frac{1}{2}hf$ . This value is called *energy of the zero point*. Thus, the energy of the harmonic oscillator, at equilibrium with the surrounding medium, does not tend to zero when temperature approaches to absolute zero, but stays equal to  $E_0$ .

In the particular case of *massless oscillators* (photons, for example),  $E_0$  does not correspond to the lowest value of the energy, which the oscillator can have, because, when temperature approaches to absolute zero the oscillator frequency becomes dependent of the temperature  $T$ , as show the well-known expression of the *thermal De Broglie wavelength* ( $\Lambda$ ) for *massless particles* [3, 4], which is given by

$$\Lambda = \frac{ch}{2\pi^{\frac{1}{3}}kT} \quad \Rightarrow \quad \frac{1}{2}hf = \pi^{\frac{1}{3}}kT \quad (4)$$

Then, *the lowest value of the energy*  $\frac{1}{2}hf$ , in the case of the *photon*, for example, will be a *fraction* of the value  $\frac{1}{2}hf$  correspondent to a critical temperature  $T_c$  very close to absolute zero. The mentioned *fraction* must be only related to the frequency  $f$ , and a frequency limit,  $f_g$ , whose value must be extremely large. Just a simple algebraic form, the quotient  $f/f_g$ , can express satisfactorily the mentioned fraction. Thus, according to Eq. (4), we can write that

<sup>1</sup> This value is also called of *Poynting vector*.

$$\frac{1}{2}hf\left(\frac{f}{f_g}\right) = \pi^{\frac{1}{3}}kT \quad (5)$$

Above  $T_c$ , the photon absorbs energy from the surrounding medium<sup>2</sup> [5], and its energy becomes equal to  $hf$ . Therefore, the energy absorbed by the photon is

$$U = hf - \frac{1}{2}hf\left(\frac{f}{f_g}\right) \quad (6)$$

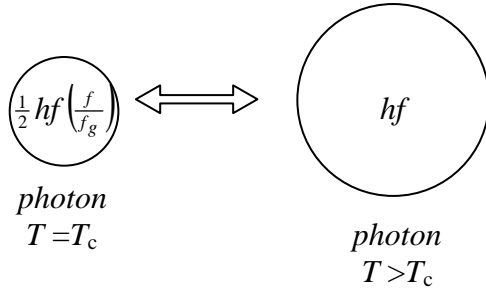


Fig. 1 – Above  $T_c$ , the photon absorbs energy from the surrounding medium, and its energy becomes equal to  $hf$ .

The absorbed energy is that thrust the photon and gives to it its velocity  $\vec{v}$ . Consequently, the momentum  $q$  transported by the photon with velocity  $\vec{v}$  will be expressed by

$$\begin{aligned} \vec{q} = \frac{U}{\vec{v}} &= \frac{hf - \frac{1}{2}hf\left(\frac{f}{f_g}\right)}{\vec{v}} = \left(1 - \frac{1}{2}\frac{f}{f_g}\right)\frac{hf}{\vec{v}} = \\ &= \left(1 - \frac{1}{2}\frac{f}{f_g}\right)\frac{hf}{\vec{v}}\left(\frac{c}{c}\right) = \left(1 - \frac{1}{2}\frac{f}{f_g}\right)\frac{hf}{c}\left(\frac{c}{\vec{v}}\right) = \\ &= \left(1 - \frac{1}{2}\frac{f}{f_g}\right)\frac{hf}{c}\vec{n}_r \end{aligned} \quad (7)$$

Equation above shows the existence of a bipolar linear momentum transported by the electromagnetic waves. For  $f < 2f_g$  the resultant momentum transported by the photon is positive, i.e., If this momentum is absorbed by a surface, pressure is exerted on the surface, in the same direction of propagation of the photon. These photons are

well-known. However, Eq. (7) point to a new type of photons when  $f = 2f_g$ . In this case  $q = 0$ , i.e., this type of photon does not exert pressure when it incides on a surface. What means that it does not interact with matter. Obviously, this corresponds to a special type of photon, which we will call of neutral photon. Finally, if  $f > 2f_g$  the resultant momentum transported by the photon is negative. If this momentum is absorbed by a surface, pressure is exerted on the surface, in the opposite direction of propagation of the photon. This special type of photon will be denominated of attractive photon.

The quantization of gravity shows that the gravitational mass  $m_g$  and inertial mass  $m_i$  are correlated by means of the following factor [6]:

$$\chi = \frac{m_g}{m_{i0}} = \left\{ 1 - 2 \left[ \sqrt{1 + \left(\frac{\Delta p}{m_{i0}c}\right)^2} - 1 \right] \right\} \quad (8)$$

where  $m_{i0}$  is the rest inertial mass of the particle and  $\Delta p$  is the variation in the particle's kinetic momentum.

Another important equation obtained in the quantization theory of gravity is the new expression for the momentum  $q$  of a particle with gravitational mass  $M_g$  and velocity  $v$ , which is given by

$$\vec{q} = M_g \vec{v} \quad (9)$$

where  $M_g = m_g / \sqrt{1 - v^2/c^2}$ ;  $m_g$  is given by Eq.(8), i.e.,  $m_g = \chi m_i$ .

By comparing Eq. (9) with (7) we obtain

$$v = c \sqrt{\frac{hf}{M_g c^2} \left(1 - \frac{f}{2f_g}\right)} \quad (10)$$

Mass–energy equivalence principle states that anything having mass has an equivalent amount of energy and vice versa. In the particular case of photons, the energy of the photons,  $E = hf$ , has a corresponding equivalent mass,  $M_g$ , given by its energy  $E$  divided by the speed of light squared  $c^2$ , i.e.,

$$M_g \equiv E/c^2 \quad (11)$$

or

<sup>2</sup> In order to provide the equilibrium the harmonic oscillator absorbs energy from the surrounding medium.

$$M_g c^2 \equiv hf \quad (12)$$

Considering this expression, Eq. (10) can be rewritten as follows

$$v = c \sqrt{\left(1 - \frac{f}{2f_g}\right)} \quad (13)$$

Equation (13) shows that the speed of the electromagnetic waves in free space is *not a constant*. Thus, also the speed of light is not a constant.

Theories proposing a varying speed of light have recently been widely proposed under the claim that they offer a solution to cosmological puzzles [7, 8].

It is known that the interactions are communicated by means of the changing of “virtual” *quanta*. The maximum velocity of these *quanta* is a *constant* called maximum velocity of propagation of the interactions. Currently, it is assumed that this velocity is equal to the velocity of the electromagnetic waves in free space ( $c$ ). This is the reason of the constant  $c$  to appear in the relativistic factor (Eq. (10)). However, Eq. (13) shows that the velocity of the electromagnetic waves in free space is not a constant. In addition, Eq. (13) shows that for  $f > 2f_g$  the velocity of the photon is *imaginary*. This means that the *attractive* photons are **virtual photons**.

Now we will apply the concept of *negative radiation pressure*, here developed, to the Gravitational Interaction.

According to Eq. (7), the resultant *momentum* transported by the photons with frequency  $f > 2f_g$  is *negative*. If this *momentum* is absorbed by a surface, pressure is exerted on the surface, in the *opposite direction of propagation of the photon*.

Now consider two particles  $A$  and  $B$  with *gravitational masses*  $m_A$  and  $m_B$ , respectively. If both particles emit *attractive radiation* ( $f > 2f_g$ ), then the powers  $P_A$  and  $P_B$  emitted from  $A$  and  $B$ , according to Eq. (6), are respectively given by

$$P_A = N_A hf \left(1 - \frac{f}{2f_g}\right) 2f \quad (14)$$

$$P_B = N_B hf \left(1 - \frac{f}{2f_g}\right) 2f \quad (15)$$

where  $N_A$  and  $N_B$  are respectively, the number of attractive photons emitted from  $A$  and  $B$ , during the time interval  $\cong 1/2f$ .

Equations (14) and (15) can be rewritten as follows

$$P_A \cong \left[2N_A hf \left(1 - \frac{f}{2f_g}\right)\right] f = E_{g(A)} f = \left(\frac{1}{\sqrt{1 - \frac{V_A^2}{c^2}}}\right) M_{g(A)} c^2 f \quad (16)$$

$$P_B \cong \left[2N_B hf \left(1 - \frac{f}{2f_g}\right)\right] f = E_{g(B)} f = \left(\frac{1}{\sqrt{1 - \frac{V_B^2}{c^2}}}\right) M_{g(B)} c^2 f \quad (17)$$

where  $E_{g(A)}$  and  $E_{g(B)}$  are respectively the *total energies* of the particles  $A$  and  $B$ , given respectively by:  $E_{g(A)} = M_{g(A)} c^2 = m_A c^2 / \sqrt{1 - V_A^2/c^2}$  and  $E_{g(B)} = M_{g(B)} c^2 = m_B c^2 / \sqrt{1 - V_B^2/c^2}$  [6];  $V_A$  is the velocity of the particle  $A$  in respect to the particle  $B$  and  $V_B$  is the velocity of the particle  $B$  in respect to the particle  $A$ . Obviously,  $V_A = V_B$ .

Thus, if  $r$  is the distance between the mentioned particles, then the power densities of the *attractive radiation* in  $A$  and  $B$  are respectively, given by

$$D_A = \frac{P_A}{4\pi r^2} = \left(\frac{1}{\sqrt{1 - \frac{V^2}{c^2}}}\right) \frac{M_{g(A)} c^2 f}{4\pi r^2} = \left(\frac{1}{\sqrt{1 - \frac{V^2}{c^2}}}\right) \xi_0 \frac{M_{g(A)}}{r^2} \quad (18)$$

$$D_B = \frac{P_B}{4\pi r^2} = \left(\frac{1}{\sqrt{1 - \frac{V^2}{c^2}}}\right) \frac{M_{g(B)} c^2 f}{4\pi r^2} = \left(\frac{1}{\sqrt{1 - \frac{V^2}{c^2}}}\right) \xi_0 \frac{M_{g(B)}}{r^2} \quad (19)$$

where  $V = V_A = V_B$ ;  $\xi_0$  is expressed by

$$\xi_0 = \frac{c^2 f}{4\pi} \quad (20)$$

Let us now show that the *gravitational attraction* between two particles  $A$  and  $B$  is generated by the interchange of *attractive*

photons (*virtual photons with  $f > 2f_g$* ), emitted reciprocally by the two particles.

It is known that the *electric force* on an electric charge A, due to another electric charge B is related to the product of the charge of A ( $q_A$ ) by the flux density on A, due to the charge of B, ( $D_B = q_B/4\pi r^2$ ), i.e.,  $\vec{F}_{BA} \propto \vec{D}_B q_A$ . By analogy, the charge A exerts an opposite electric force on the charge B, which is related to the product of the charge of B ( $q_B$ ) by the flux density on B, due to the charge of A, ( $D_A = q_A/4\pi r^2$ ), i.e.,  $\vec{F}_{AB} = -\vec{F}_{BA}$ ,  $\vec{F}_{AB} \propto \vec{D}_A q_B$ . These proportionalities are usually written by means of the following equations:

$$\vec{F}_{AB} = -\vec{F}_{BA} = \frac{\vec{D}_A q_B}{\epsilon_0} = -\frac{\vec{D}_B q_A}{\epsilon_0} = \frac{q_A q_B}{4\pi \epsilon_0 r^2} \hat{\mu} \quad (21)$$

where  $\epsilon_0$  is the so-called *permissivity constant* for free space ( $\epsilon_0 = 8.854 \times 10^{-12} F/m$ ).

Similarly, the *magnetic force* that a magnetic pole B exerts on another magnetic pole A is related to the product of the pole intensity  $p_A$  of the pole A by the flux density on A, due to the pole intensity  $p_B$  of the pole B, ( $D_B = p_B/4\pi r^2$ ), i.e.,  $\vec{F}_{BA} \propto \vec{D}_B p_A$ . By analogy, the pole A exerts an opposite magnetic force on the pole B, which is related to the product of the pole intensity  $p_B$  of the pole B by the flux density on B, due to the pole intensity  $p_A$  of the pole A, ( $D_A = p_A/4\pi r^2$ ), i.e.,  $\vec{F}_{AB} = -\vec{F}_{BA}$ ,  $\vec{F}_{AB} \propto \vec{D}_A p_B$ . Usually these proportionalities are expressed by means of the following equations:

$$\vec{F}_{AB} = -\vec{F}_{BA} = \frac{\vec{D}_A p_B}{\mu_0} = -\frac{\vec{D}_B p_A}{\mu_0} = \frac{p_A p_B}{4\pi \mu_0 r^2} \hat{\mu} \quad (22)$$

where  $\mu_0$  is the so-called *permeability constant* for free space ( $\mu_0 = 4\pi \times 10^{-7} H/m$ ).

In the case of the forces produced by the action of *attractive photons* emitted reciprocally from the particles A and B, their expressions can be deduced by using the same argument previously shown in order to obtain the expressions of the electric forces and magnetic forces. That is, the force exerted on the particle A (whose gravitational mass is  $M_{g(A)}$ ), by another particle B (whose gravitational mass is

$M_{g(B)}$ ) is related to the product of  $M_{g(A)}$  by the flux density (*power density*) on A, due to the mass  $M_{g(B)}$ , ( $D_B = P_B/4\pi r^2$ . See Eq.(19)), i.e.,  $\vec{F}_{BA} \propto \vec{D}_B M_{g(A)}$ . By analogy, the particle A exerts an opposite force on the particle B, which is related to the product of the mass  $M_{g(A)}$  by the flux density (*power density*) on B, due to the mass  $M_{g(B)}$  of the particle B, ( $D_A = P_A/4\pi r^2$ . See Eq.(18)), i.e.,  $\vec{F}_{AB} = -\vec{F}_{BA}$ ,  $\vec{F}_{AB} \propto \vec{D}_A M_{g(B)}$ . Thus, we can write that

$$\vec{F}_{AB} = -\vec{F}_{BA} = \frac{\vec{D}_A M_{g(A)}}{k_0} \hat{\mu} = -\frac{\vec{D}_B M_{g(B)}}{k_0} \hat{\mu} \quad (23)$$

Substitution of Eqs. (18) and (19) into Eq. (23) we get

$$\vec{F}_{AB} = -\vec{F}_{BA} = \left( \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \right) \left( \frac{\xi_0}{k_0} \right) \frac{M_{g(A)} M_{g(B)}}{r^2} \hat{\mu} \quad (24)$$

For  $V = 0$  Eq. (24) reduces to

$$\begin{aligned} \vec{F}_{AB} = -\vec{F}_{BA} &= \left( \frac{\xi_0}{k_0} \right) \frac{m_A m_B}{r^2} \hat{\mu} = \\ &= G \frac{m_A m_B}{r^2} \hat{\mu} \end{aligned} \quad (25)$$

where  $G = 6.67 \times 10^{-11} N.m^2.kg^{-2}$  is the Universal constant of Gravitation.

Equation (25) tells us that

$$\xi_0/k_0 = G \quad (26)$$

By substituting  $\xi_0$  given by Eq. (20) into this expression, we obtain

$$k_0 = \frac{\xi_0}{G} = \frac{c^2 f}{4\pi G} \quad (27)$$

From the above exposed, we can then conclude that the *gravitational interaction* is caused by the interchange of *virtual photons* with frequencies  $f > 2f_g$  (attractive photons). In this way, the called *graviton* must have spin 1 and not 2. Consequently, the gravitational forces are also *gauge forces* because they are yielded by the exchange of *virtual quanta* of spin 1, such as the electromagnetic forces and the weak and strong nuclear forces.

Now consider the emission of  $N$  attractive photons with frequency  $f > 2f_g$  (*gravitons*)

from a particle with mass  $m_{i0}$ . According to Eq. (8), and considering that  $q = U/v$ , we get

$$\chi = \frac{m_g}{m_{i0}} = \left\{ 1 - 2 \left[ \sqrt{1 + \left( \frac{U}{m_{i0}cv} \right)^2} - 1 \right] \right\} \quad (28)$$

Substitution of Eq. (6) and (13) into Eq. (28) gives

$$\frac{m_g}{m_{i0}} = \left\{ 1 - 2 \left[ \sqrt{1 + \left( \frac{Nhf}{m_{i0}c^2} \sqrt{1 - \frac{f}{2f_g}} \right)^2} - 1 \right] \right\} \quad (29)$$

In case of **protons**, for example, the number of attractive photons emitted from a proton,  $N_p$ , can be expressed by means of the following relation:  $N_p = N (m_{p0}/m_{i0})$ , where  $N$  is the number of attractive photons emitted from the particle with mass  $m_{i0}$ ;  $m_{p0}$  is the rest inertial mass of the proton. In this case, the equation (29) will be rewritten as follows:

$$\begin{aligned} \frac{m_{gp}}{m_{p0}} &= \left\{ 1 - 2 \left[ \sqrt{1 + \left( \frac{N_p hf}{m_{p0}c^2} \sqrt{1 - \frac{f}{2f_g}} \right)^2} - 1 \right] \right\} = \\ &= \left\{ 1 - 2 \left[ \sqrt{1 + \left( \frac{N hf}{m_{i0}c^2} \sqrt{1 - \frac{f}{2f_g}} \right)^2} - 1 \right] \right\} \quad (30) \end{aligned}$$

In the case of, **electrons**, for example, the number of attractive photons emitted from an electron  $N_e$ , can be expressed by means of the following relation:  $N_e = N (m_{e0}/m_{i0})$ , where  $m_{e0}$  is the rest inertial mass of the electron. In this case, the equation (29) will be rewritten as

$$\begin{aligned} \frac{m_{ge}}{m_{e0}} &= \left\{ 1 - 2 \left[ \sqrt{1 + \left( \frac{N_e hf}{m_{e0}c^2} \sqrt{1 - \frac{f}{2f_g}} \right)^2} - 1 \right] \right\} = \\ &= \left\{ 1 - 2 \left[ \sqrt{1 + \left( \frac{N hf}{m_{i0}c^2} \sqrt{1 - \frac{f}{2f_g}} \right)^2} - 1 \right] \right\} \quad (31) \end{aligned}$$

This is *exactly the same expression* for the proton (Eq. (30)).

In the case of, **neutrinos**, for example, the number of attractive photons,  $N_n$ , emitted from a neutrino can be expressed by means of the following relation:  $N_n = N (m_{n0}/m_{i0})$ , where  $m_{n0}$  is the rest inertial mass of the neutrino. In

this case, the equation (29) will be rewritten as follows:

$$\begin{aligned} \frac{m_{gn}}{m_{n0}} &= \left\{ 1 - 2 \left[ \sqrt{1 + \left( \frac{N_n hf}{m_{n0}c^2} \sqrt{1 - \frac{f}{2f_g}} \right)^2} - 1 \right] \right\} = \\ &= \left\{ 1 - 2 \left[ \sqrt{1 + \left( \frac{N hf}{m_{i0}c^2} \sqrt{1 - \frac{f}{2f_g}} \right)^2} - 1 \right] \right\} \quad (32) \end{aligned}$$

Also, in this case, the obtained expression is *exactly the same expression* for the proton and the electron (the term  $N/m_{i0}$  is a constant). In short, the result is the same for any particle with non-null mass.

By solving equation below

$$\begin{aligned} \left\{ 1 - 2 \left[ \sqrt{1 + \left( \frac{N hf}{m_{i0}c^2} \sqrt{1 - \frac{f}{2f_g}} \right)^2} - 1 \right] \right\} &= \\ \left\{ 1 - 2 \left[ \sqrt{1 + \underbrace{\left( \frac{N h}{m_{i0}c^2} \right)^2}_{\text{constant}} f^2 \left( 1 - \frac{f}{2f_g} \right)} - 1 \right] \right\} &= \chi \quad (33) \end{aligned}$$

we get:

$$f^2 \left( 1 - \frac{f}{2f_g} \right) = \frac{m_{i0}^2 c^4}{N^2 h^2} \left( \frac{\chi^2 - 6\chi + 5}{4} \right) \quad (34)$$

Note that, for  $N = N_{\max}$ , the value of  $f$  in Eq. (34) becomes equal to  $f_{\min}$ . But, the minimum frequency of the gravific photons is very close to  $2f_g$ , then we can write that  $f = f_{\min} \cong 2f_g$ . Under these circumstances, Eq. (33) shows that we have that  $\chi = \chi_{\min} \cong 1$ . Consequently, both terms of the Eq. (34) become approximately equal to zero. On the other hand, for  $N = N_{\min} = 1$  (one gravific photon) the value of  $f$  in Eq. (34) becomes equal to  $f_{\max}$  (the maximum frequency of the gravific photons) and Eq. (33) shows that, in this case,  $\chi = \chi_{\max}$ , i.e.,

$$\left\{ 1 - 2 \left[ \sqrt{1 + \underbrace{\left( \frac{N h}{m_{i0}c^2} \right)^2}_{\text{constant}} f_{\max}^2 \left( 1 - \frac{f_{\max}}{2f_g} \right)} - 1 \right] \right\} = \chi_{\max} \quad (35)$$

By solving this equation, we obtain:

$$f_{\max}^2 \left(1 - \frac{f_{\max}}{2f_g}\right) = \frac{m_{i0}^2 c^4}{h^2} \left(\frac{\chi_{\max}^2 - 6\chi_{\max} + 5}{4}\right) \quad (36)$$

The energy density  $D$  at a distance  $r$  from the mentioned particle can be expressed by:  $D = (U/\Delta t)/S$ . Assuming that  $\Delta t = 1/f$  and considering Eq. (6), then we can write that

$$D = \frac{U/\Delta t}{S} = \frac{hf \left(1 - f/2f_g\right) (f)}{4\pi r^2} \quad (37)$$

Substitution of Eq.(36) into Eq. (37) yields

$$D = \left(\frac{m_{i0} c^4}{4\pi h}\right) \left(\frac{\chi_{\max}^2 - 6\chi_{\max} + 5}{4}\right) \frac{m_{i0}}{r^2} \quad (38)$$

By comparing Eq. (38) with equations (18) and (19), we can conclude that

$$\left(\frac{m_{i0} c^4}{4\pi h}\right) \left(\frac{\chi_{\max}^2 - 6\chi_{\max} + 5}{4}\right) = \xi_0 \quad (39)$$

By comparing this equation with Eq.(20), we obtain

$$f_{\max} = \left(\frac{\chi_{\max}^2 - 6\chi_{\max} + 5}{4}\right) \frac{m_{i0} c^2}{h} \quad (40)$$

Now, taking Eq. (8), where the term  $\Delta p/m_{i0}c$  is putted in the following form:

$\Delta p/m_{i0}c = (v/c) \left(1 - v^2/c^2\right)^{\frac{1}{2}}$  [6], we get

$$\chi = \frac{m_g}{m_{i0}} = \left\{1 - 2 \left[ \sqrt{1 + \left(\frac{v/c}{\sqrt{1 - v^2/c^2}}\right)^2} - 1 \right]\right\} \quad (41)$$

In practice, how close  $c$  the velocity  $v$  can approach? At the *Large Hadron Collider* (LHC) the protons each have energy of 6.5 TeV, giving total collision energy of 13 TeV. At this energy the protons move with velocity  $v = 0.999999990c$ . Possibly this value will can be increased up to  $v = 0.999999999999c$ , in the next experiments at the LHC. In this case, Eq. (41) gives  $\chi \cong -10^7$ . Since  $\chi_{\max}$  is obviously, *very greater than* this value, then we can conclude that the term  $(\chi_{\max}^2 - 6\chi_{\max} + 5/4)$  in Eq. (40) is *very greater than*  $10^{14}$ ; showing, therefore, that the maximum frequency  $f$  of the *gravific photons* (See Eq. (40)) is *very greater than*  $10^{14} (m_{i0} c^2/h) \cong 10^{64} m_{i0}$ . Thus, we can define the *frequency spectrum of the gravific photons* by means of the following expression:

$$f_{\min} \cong 2f_g \leq f \leq f_{\max} \gg 10^{64} m_{i0} \quad (42)$$

This expression shows then that *the frequency spectrum of the gravific photons must be above the spectrum of the gamma rays (neutrino mass:  $m_{i0} \cong 10^{-37} \text{ kg}$  [9]).* Thus, considering that the *highest energy of gamma ray detected is approximately  $3 \times 10^{13} \text{ eV}$  [10]*<sup>3</sup>, in terms of frequency  $f_{\gamma \max} \approx 10^{28} \text{ Hz}$ , then we can assume that *the characteristic value,  $2f_g$* , in the Eq.(34), in spite to be greater than  $f_{\gamma \max}$ , it should be very close it, because the spectrum of the attractive photons should make limit with the gamma rays spectrum (See Fig.2). Thus, we can write that

$$2f_g \gtrsim f_{\gamma \max} \approx 10^{28} \text{ Hz} \quad (43)$$

It is very unlikely that there are gamma rays in the Nature with frequency much greater than the aforementioned value, but if they exist, they would only show that the value of  $2f_g$  would be situated above the value indicated by Eq (43).

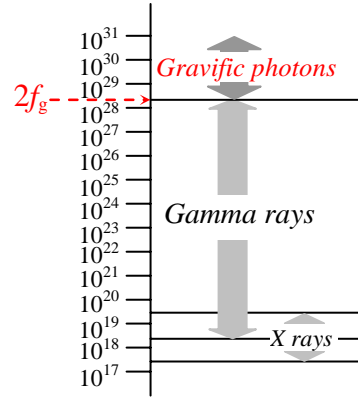


Fig. 2 – The *Gravific Photons* Spectrum (above  $2f_g \approx 10^{28} \text{ Hz}$ ).

<sup>3</sup> The largest air shower detected is from a particle of around  $4 \times 10^{20} \text{ eV}$  but this is thought to be from a cosmic ray *particle* rather than photon.

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