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Gravity cannot be quantized

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Abstract Besides the common difficulties for gravity quantization, this work shows that the gravitational fermion associated to the gravitational field does not exist. Consequently, the graviton does not exist either. Therefore, gravity cannot be quantized.

Keywords Quantum gravity, graviton.

1. Introduction

All nongravitational forces are described in terms of quantum mechanics (QM), that is, in terms of quantum field theory (QFT). There are some justifications for the necessity of a quantum formulation of gravity. One of them, for example, is directly linked to the Big Bang theory and has to do with the description of matter in the beginning of the universe.

Although for classical weak gravitational fields Newtonian gravity (NG) is the most used theory, the most general theory of gravity is general relativity (GR). However, a quantum mechanical formulation of gravity should yield both NG and GR in their appropriate limits.

Some relevant literature on quantum gravity (QG) is presented below within the discussion of the difficulties for obtaining QG.

This paper is an updated version of the preprint [1].

2. The Spacetime Background Dependence of Quantum Gravity

Both NG and QM, and also relativistic quantum mechanics (RQM), have fixed spacetime backgrounds. In the case of RQM Minkowski spacetime is its fixed spacetime background, and as for NG and QM the fixed spacetime background is Euclidean spacetime. On the other hand GR has no fixed spacetime background. Thus, a quantum formulation of gravity has to solve this difficult conundrum. An important theoretical effort in this direction is loop quantum gravity [2,3,4,5] which is a spacetime background independent theory. The reader can find a detailed discussion on the above mentioned conundrum in the introduction of reference [3].

The basis of loop quantum gravity is the assumption that spacetime has an elementary quantum granular structure at the Planck scale. However, let us recall that quantum states are states of fermions, and the mediation between any two of them is carried on by bosons. For example, nuclear states in nuclei are quantum states of nucleons or quantum states in solids, liquids and gases are states of electrons (either directly or indirectly). Therefore, the quantum states of spacetime have to be associated to fermionic states of one or more fermionic gravitational charge carriers. An effort towards towards this direction is the work by Morales-Técoti and Rovelli [6]. Some difficult problems of this paperin my view are: i) the definition of a scalar field as a clock for measuring the physical time for studying the evolution of the fermion-gravity system; ii) lack of identification of a gravitational charge associated to the fermions; iii) the action of the Einstein-Weyl system contains the vector field t^a with affine parameter t without any physically plausible justification for such a field.

3. Nonrenormalizability of Quantum Gravity

It is well known that when gravity is treated as a particle field it is not renormalizable [7]. That is, the infinite

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quantities due to the interactions of the gravitational particles cannot be removed.

Contrary to quantum electrodynamics, QG needs an infinite number of independent parameters, such as charge, mass etc. A way out of this is string theory in which new symmetry principles reduce the number of parameters to a finite set, but string theory has many drawbacks, among which is the assumption of mysterious extra dimensions for spacetime.

4. The Gravitational Fermionic Charge

It is well known that the curvature of spacetime around a massive body depends strongly on the body's mass. In NG the gravitational field produced by a massive body around it depends also on its mass. Thus, the gravitational charge has to be proportional to mass (rest mass). Of course this is already a drawback since rest mass is not always conserved.

On the other hand the quantization of gravity has to be valid in either curved spacetime or flat spacetime. For example, it is expected that immediately after a supernova explosion we should observe gravitational fermions propagating through space, carrying gravitational charges.

5. The Gravitational Fermion

Despite the drawback of the nonconservation of rest mass, let us admit that the gravitational fermion exists, and let us call it *masson*. Admitting that it is a 1/2 spin fermion, in flat spacetime a free *masson* has to satisfy Dirac equation

$$\left(i\hbar\gamma^{\nu}\partial_{\nu}-mc\right)\varphi=0 \tag{1}$$

where m is its rest mass. Associated to the *masson* there should exist a mass current whose expression should depend on the nature of the gravitational field.

In the considerations below for the different types of currents, symmetric tensors are out, of course, and as it was discussed above it is expected that the currents are proportional to mass.

5.1 Vectorial mass current

In this case

$$j^{\mu}=mc\bar{\varphi}\gamma^{\mu}\varphi \tag{2}$$

but from Dirac equation we have

$$i\hbar\gamma^{\nu}\partial_{\nu}\varphi=mc\varphi$$

which multiplied from the left by γ^{μ} yields

$$i\hbar\gamma^{\mu}\gamma^{\nu}\partial_{\nu}\varphi=mc\gamma^{\mu}\varphi \tag{3}$$

Since $\gamma^{\mu}\varphi$ is also a solution of Dirac equation we obtain

$$i\hbar\gamma^{\nu}\partial_{\nu}\left(\gamma^{\mu}\varphi\right)=mc\left(\gamma^{\mu}\varphi\right) \tag{4}$$

and

$$i\hbar\gamma^{\nu}\gamma^{\mu}\partial_{\nu}\varphi=mc\left(\gamma^{\mu}\varphi\right). \tag{5}$$

Summing up equations (3) and (5), and using

$$\gamma^{\nu}\gamma^{\mu}+\gamma^{\mu}\gamma^{\nu}=2g^{\nu\mu}$$

we obtain

$$i\hbar\bar{\varphi}g^{\nu\mu}\partial_{\nu}\varphi=mc\bar{\varphi}\left(\gamma^{\mu}\varphi\right) \tag{6}$$

where

$$g^{\nu\mu}=\begin{pmatrix}1&0&0&0\\0&-1&0&0\\0&0&-1&0\\0&0&0&-1\end{pmatrix}$$

Therefore, we obtain the mass current

$$j^{\mu}=i\hbar\bar{\varphi}g^{\nu\mu}\partial_{\nu}\varphi=i\hbar\bar{\varphi}g^{\nu\mu}\partial_{\nu}\varphi \tag{7}$$

which depends on the metric $g^{\nu\mu}$.

In curved spacetime we can always choose a small region where spacetime is approximately flat. Hence, we can extend the meaning of $g^{\nu\mu}$ to include curved spacetime. We take the orthogonal metric such as

$$g^{\nu\mu}=\begin{pmatrix}g^{00}&0&0&0\\0&g^{11}&0&0\\0&0&g^{22}&0\\0&0&0&g^{33}\end{pmatrix} \tag{8}$$

and consider that we are in a very small region of curved spacetime, that is, without large curvature. Thus, we can make $g^{00} \approx 1+f^{00}$, $g^{11} \approx -1+f^{11}$, $g^{22} \approx -1+f^{22}$, and $g^{33} \approx -1+f^{33}$, and thus we obtain for small f^{ij}

$$\delta j^{\mu} \approx i\hbar\bar{\varphi}\Delta^{\mu a} e_a^{\nu} D_{\nu}\varphi \tag{9}$$

where

$$\Delta^{\nu\mu} = \begin{pmatrix} f^{00} & 0 & 0 & 0 \\ 0 & f^{11} & 0 & 0 \\ 0 & 0 & f^{22} & 0 \\ 0 & 0 & 0 & f^{33} \end{pmatrix}$$

and thus δj^{μ} is a mass current directly acquired from curvature. The change from $\partial_{\nu} \to e_a^{\nu} D_{\nu}$ was done because in curved spacetime Dirac equation is given by [8,9]

$$i\hbar\gamma^a e_a^{\mu} D_{\nu}\varphi - mc\varphi = 0$$

where e_a^{ν} is the vierbein and D_{μ} is the covariant derivative defined as

$$D_{\mu} = \partial_{\mu} - \frac{i}{4}\omega_{\mu}^{ab}\sigma_{ab}$$

in which σ_{ab} is the commutator of Dirac matrices

$$\sigma_{ab} = \frac{i}{2}[\gamma_a, \gamma_b]$$

and ω_{μ}^{ab} are the spin connection components.

5.2 Pseudovectorial mass current

Now we have the current

$$j^{\mu} = mc\bar{\varphi}\gamma^{\mu}\gamma^5\varphi \tag{10}$$

and from Dirac equation we have

$$i\hbar\gamma^{\nu}\partial_{\nu}\varphi = mc\varphi \tag{11}$$

from which we obtain

$$i\hbar\gamma^{\mu}\gamma^5\gamma^{\nu}\partial_{\nu}\varphi = mc\gamma^{\mu}\gamma^5\varphi$$

and

$$-i\hbar\gamma^5\gamma^{\mu}\gamma^{\nu}\partial_{\nu}\varphi = mc\gamma^{\mu}\gamma^5\varphi \tag{12}$$

As $\gamma^{\mu}\varphi$ is also a solution of Dirac equation we obtain

$$i\hbar\gamma^{\nu}\partial_{\nu}\left(\gamma^{\mu}\varphi\right) = mc\left(\gamma^{\mu}\varphi\right)$$

and thus

$$i\hbar\gamma^{\nu}\gamma^{\mu}\partial_{\nu}\varphi = mc\gamma^{\mu}\varphi$$

which multiplied by γ^5 becomes

$$i\hbar\gamma^5\gamma^{\nu}\gamma^{\mu}\partial_{\nu}\varphi = mc\gamma^5\gamma^{\mu}\varphi$$

from which we obtain

$$-i\hbar\gamma^5\gamma^{\nu}\gamma^{\mu}\partial_{\nu}\varphi = mc\gamma^{\mu}\gamma^5\varphi\,. \tag{13}$$

Summing up equations (12) and (13), and using

$$\gamma^{\nu}\gamma^{\mu} + \gamma^{\mu}\gamma^{\nu} = 2g^{\nu\mu}$$

we obtain

$$-i\hbar\gamma^5 g^{\nu\mu}\partial_{\nu}\varphi = mc\gamma^{\mu}\gamma^5\varphi \tag{14}$$

and the mass current

$$j^{\mu} = -i\hbar\bar{\varphi}\gamma^5 g^{\nu\mu}\partial_{\nu}\varphi$$

which depends on the metric $g^{\nu\mu}$, and thus, there is the acquisition from curvature of the mass current

$$\delta j^{\mu} \approx -i\hbar\bar{\varphi}\gamma^5\Delta^{\mu a} e_a^{\nu} D_{\nu}\varphi \tag{15}$$

5.3 Scalar mass current

In the case of a scalar mass current

$$j = mc\bar{\varphi}\varphi \tag{16}$$

and from Dirac equation we obtain

$$i\hbar\gamma^{\nu}\partial_{\nu}\varphi = mc\varphi\,. \tag{17}$$

As $\gamma^{\mu}\varphi$ is also a solution of Dirac equation we obtain

$$i\hbar\gamma^{\nu}\partial_{\nu}\left(\gamma^{\mu}\varphi\right) = mc\left(\gamma^{\mu}\varphi\right).$$

Multiplying this equation from the left by γ^{μ} and taking into account that $\gamma_\mu \gamma^\mu = 4$ we have

$$i\hbar \gamma_\mu \gamma^\nu \gamma^\mu \partial_\nu \varphi = mc\gamma_\mu \gamma^\mu \varphi = 4mc\varphi . \quad (18)$$

And multiplying Eq. (17) by $\gamma_\mu \gamma^\mu$ we obtain

$$i\hbar \gamma_\mu \gamma^\mu \gamma^\nu \partial_\nu \varphi = mc\gamma_\mu \gamma^\mu \varphi = 4mc\varphi . \quad (19)$$

Summing up equations (18) and (19) yields

$$i\hbar \gamma_\mu \left(\gamma^\nu \gamma^\mu + \gamma^\mu \gamma^\nu \right) \partial_\nu \varphi = 8mc\varphi \quad (20)$$

but as $\gamma^\nu \gamma^\mu + \gamma^\mu \gamma^\nu = 2g^{\nu\mu}$ Eq. (20) becomes

$$i\hbar \gamma_\mu g^{\nu\mu} \partial_\nu \varphi = 4mc\varphi$$

and thus the mass current

$$j = \frac{1}{4} i\hbar \bar{\varphi} \gamma_\mu g^{\nu\mu} \partial_\nu \varphi \quad (21)$$

which also depends on the metric. And thus curved spacetime yields the additional mass current

$$j = \frac{1}{4} i\hbar \bar{\varphi} \gamma_\mu g^{a\mu} e_a^\nu D_\nu \varphi .$$

5.4 Pseudoscalar mass current

In such a case we have the mass current

$$j = mc\bar{\varphi}\gamma^5 \varphi . \quad (22)$$

The derivation is similar to those above and yields the mass current

$$j = -\frac{1}{4} i\hbar \bar{\varphi} \gamma_\mu \gamma^5 g^{\nu\mu} \partial_\nu \varphi \quad (23)$$

which also depends on the metric. Thus in curved spacetime curvature produces the additional mass current

$$\delta j = -\frac{1}{4} i\hbar \bar{\varphi} \gamma_\mu \gamma^5 g^{a\mu} e_a^\nu D_\nu \varphi . \quad (24)$$

5.5 Antisymmetric tensorial mass current

For this case the current is given by

$$j^{\mu\nu} = mc\bar{\varphi}\sigma^{\mu\nu} \varphi \quad (25)$$

where $\sigma^{\mu\nu} = \frac{i}{2}\left(\gamma^\mu \gamma^\nu - \gamma^\nu \gamma^\mu \right)$.

From Dirac equation we have

$$i\hbar \gamma^\nu \partial_\nu \varphi = mc\varphi .$$

Multiplying this equation by $\gamma^\mu \gamma^\nu - \gamma^\nu \gamma^\mu$ we obtain

$$i\hbar \left(\gamma^\mu \gamma^\nu - \gamma^\nu \gamma^\mu \right) \gamma^\nu \partial_\nu \varphi = mc\left(\gamma^\mu \gamma^\nu - \gamma^\nu \gamma^\mu \right) \varphi \quad (26)$$

that is,

$$i\hbar \sigma^{\mu\nu} \gamma^\nu \partial_\nu \varphi = mc\sigma^{\mu\nu} \varphi$$

and thus the current

$$j^{\mu\nu} = i\hbar \bar{\varphi} \sigma^{\mu\nu} \gamma^\nu \partial_\nu \varphi \quad (27)$$

which *does not depend on the metric*. In curved spacetime Eq. (27) becomes

$$j^{\mu\nu} = i\hbar \bar{\varphi} \sigma^{\mu\nu} \gamma^a e_a^\nu D_\nu \varphi .$$

It is worth having in mind that $\mathrm{I}_4, \gamma^5, \gamma^\mu, \gamma^\mu \gamma^5$ and $\sigma^{\mu\nu}$ form a basis for the space of all 4×4 matrices, and thus, any other tensor $\chi^{\mu\nu}$ can be written in terms of a linear combination of these 16 matrices.

As it was shown above the only possibility for the gravitational fermion is to be associated to an antisymmetric tensorial field. This result is not a surprise and agrees well with GR. Misner, Thorne and Wheeler [10] who have proven that the *classical* gravitational field is an antisymmetric tensorial field.

Unfortunately, as it is shown below, such a 1/2 spin fermion is incompatible with the graviton.

5.6 The spin prohibition

Bosons and fermions related to the same fundamental interaction work together. As it is shown below any fermion is incompatible with the graviton which is supposed to be a spin 2 massless boson.

Since it is massless its spin projections are -2, -1, 1, 2. Each one of these projections should correspond to differences between two spin projections of the corresponding fermionic states. For example, we should have

$$-2 = m_{s2} - m_{s1} \quad (28)$$

where m_{s1} and m_{s2} are the spin projections of the initial and final fermionic states. Of course, m_{s1} and m_{s2} of the fermions are the fractional numbers $\pm 1/2, \pm 3/2, \pm 5/2, ...$ Thus, for a hypothetical 5/2 spin fermion we would have, for example, the results for allowed transitions (for $S_z = -2$ of the graviton)

$$-2 = -3/2 - (+1/2)$$
$$-2 = -1/2 - (+3/2)$$
$$-2 = -5/2 - (-1/2)$$
$$-2 = -1/2 - (-5/2)$$

but, for example, the fermionic transitions yielding the results

$$-3/2 - (+3/2) = -3$$
$$+3/2 - (-3/2) = +3$$
$$-1/2 - (+1/2) = -1$$
$$+1/2 - (-1/2) = +1$$

would be mysteriously forbidden. And for a 1/2 spin fermion we would have only the two possibilities below for allowed transitions

$$-1 = -1/2 - (+1/2)$$
$$+1 = +1/2 - (-1/2)$$

and thus, no fermionic transitions would produce the graviton spin projections -2 and +2.

5.7 The prohibition due to the constitution of matter

Any known ordinary matter is not constituted by such gravitational fermions and all masses involved in ordinary matter have been accounted for.

5.10 The photonic prohibition

It is well known that, according to GR, the motion of a photon in the gravitational field of a massive body is described by the geodesic equation

$$\frac{dp^{\alpha}}{d\lambda^{*}} + \Gamma^{\alpha}_{\beta\gamma} p^{\beta} p^{\gamma} = 0 \qquad (29)$$

where p is the four-momentum of the photon, λ is an affine parameter and $\Gamma^{\alpha}_{\beta\gamma}$ is the Christoffel symbol of the second kind. Eq. (29) has been experimentally proven for a photon in the gravitational field of the Sun. In this case a photon suffers a deflection given by [11]

$$\Delta\phi = 1''.75(\mathrm{R}_S / b)$$

where R_S is the Sun's radio and b is the impact parameter. How to obtain the same result with a quantum mechanical formulation?

From a quantum mechanical point of view this would only be possible if the photon would interact with a virtual *masson*, and thus there would exist a virtual sea of such fermions and their corresponding antifermions. And, of course, we would also have the real counterparts of such virtual particles in the world. But such particles have not been seen anywhere.

Besides this above drawback, an interaction between a photon and *massons* continuously yielding a photon in the final state (for following a geodesic) would only be possible if the *masson* had spin equal to 1/2, but as it was shown above, this is not possible.

6. Conclusion

Besides other difficulties faced by QG, we have shown above that the gravitational fermion does not exist. And as fermions and bosons of a fundamental interaction work together, the graviton does not exist either.

Therefore, gravity cannot be quantized.

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