IS A MATHEMATICAL PROOF A TAUTOLOGY?

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ABSTRACT

This article addresses the issue of mathematical proof.

Research mathematics evidently concerns the proving of theorems, e.g., the infinitude of the non-trivial zeros of the zeta function of the Riemann hypothesis, the infinity of the solutions for the Birch-Swinnerton-Dyer conjecture, etc.. A proof might just require several lines of text and symbols, or, more, and, in the more extreme case, even hundreds or more pages of text and symbols. But is a proof of a theorem in fact a tautology, i.e., just another way of expressing or stating a theorem so that the theorem is clearly understood and accepted as valid in its logic? That is, is a proof of a theorem in fact the following tautological statement?:

Theorem A = confirmed (or proven) Theorem A

A proof has to be rigourous, water-tight, so the mathematician says, to have no gaps or occasions for doubt. However, when pressed to explain what being rigourous means the mathematician is bound to be tongue-tied. Even the meaning of "rigour" found in a standard dictionary is vague, e.g., stating the meaning as "strictness, severity, or harshness", whatever that really means.

A proof involves reasoning with axioms and/or lemmas. For example, a mathematician might use axioms and/or lemmas to prove Theorem A. After a possibly long chain of reasoning, he might (or might not, sometimes) confirm that Theorem A is really a theorem. The chain of mathematical reasoning in the proof is there to convince its reader of the truth or validity of Theorem A. The mathematical reader however might not be convinced due to lack of understanding or intelligence, bias (e.g., cultural bias which is a common phenomenon), prejudice and what-not; he might even stubbornly not be convinced, e.g., if he did not think highly of the mathematical reasoner (of course, if he thought highly of the mathematical reasoner, he could be more easily convinced). Reasoning or logic should be objective and clear-cut, one might think, but sadly it often appears not which possibly explains all the different schools of thought.

On the other hand, an ultra-intelligent person, possibly an extra-terrestrial, might know that Theorem A is really Theorem A without needing to be convinced by the above-said chain of reasoning, possibly having his own way of ascertaining it, and even possibly by pure, powerful intuition.

Could a proof be non-tautological? One might wonder. A proof would not be tautological if, e.g., the chain of reasoning leads from Theorem A to Theorem B, or, more other theorems:

Theorem A ⟶ Theorem B (etc.)

Of course this also happens when smaller theorems or lemmas help the mathematician to arrive at the main theorem (the so-called tautology).

This kind of reasoning is known as deduction, the kind of reasoning the iconic fictional private detective Sherlock Holmes had famously used. Just from some clues Sherlock Holmes was able to deduce who the criminal or murderer was:

clues ⟶ criminal/murderer

Should and could mathematics not also make use of such non-tautological reasoning? In other words, shouldn't mathematical reasoning also be non-tautological, which could result in more theorems being deduced or discovered, as is stated above?

REFERENCES

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