THE TWIN PRIMES CONJECTURE

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ABSTRACT

Euclid's proof of the infinitude of the primes has generally been regarded as elegant. It is a proof by contradiction, or, *reductio ad absurdum*, and it relies on an algorithm which will always bring in larger and larger primes, an infinite number of them. However, the proof is also subtle and has been misinterpreted by some with one well-known mathematician even remarking that the algorithm might not work for extremely large numbers. This paper, which is a revision/expansion of the author's earlier paper published in an international mathematics journal in 2003, presents a strong argument which supports the validity of the twin primes conjecture, using reasoning similar to that of Euclid's proof of the infinity of the primes.

MSC: 11-XX (Number Theory)

INTRODUCTION

In 1919, Viggo Brun (1885 - 1978) proved that the sum of the reciprocals of the twin primes converges to Brun's constant:

$1/3 + 1/5 + 1/7 + 1/11 + 1/13 + 1/17 + 1/19 + \ldots = 1.9021605 \ldots$

It is evident that the twin primes thin out as infinity is approached. The problem of whether there is an infinitude of twin primes is an inherently difficult one to solve, as infinity (normally symbolised by: ∞) is a difficult concept and is against common sense. It is impossible to count, calculate or live to infinity, perhaps with the exception of God. Infinity is a nebulous idea and appears to be only an abstraction devoid of any actual practical meaning. How do we quantify infinity? How big is infinity? We could either attempt to prove that the twin primes are finite, or, infinite. If the twin primes were finite, how could we prove that a particular pair of twin primes is the largest existing pair of twin primes, and, if they were infinite, how could we prove that there are always larger and larger pairs of them? It is evidently difficult to prove either, with the former appearing more difficult to prove as the odds seem against it. This paper provides proof of the latter, i.e., the infinitude of the twin primes.

Keywords: indivisible, impossibility, new primes/twin primes

Theorem:- The twin primes are infinite.

Proof:-

Let 3, 5, 7, 11, 13, 17, 19, …….., n - 2, n be the list of consecutive primes, wherein n & n - 2 are assumed to be the largest existing twin primes pair, within the infinite list of the primes.

Let 3 x 5 x 7 x 11 x 13 x 17 x 19 = a .

Lemma: (a x …….. x n - 2 x n) - 2, &, (a x …….. x n - 2 x n) - 4 will never be divisible by any of the consecutive primes in the list: 3, 5, 7, 11, 13, 17, 19, …….., n - 2, n, whether they are prime or composite. (See Appendix 1.)

This implies that:

If (a x …….. x n - 2 x n) - 2 &/V (a x …….. x n - 2 x n) - 4 are prime, then:

(a x …….. x n - 2 x n) - 2 > (a x …….. x n - 2 x n) - 4 > n > n - 2

If (a x …….. x n - 2 x n) - 2 &/V (a x …….. x n - 2 x n) - 4 are non-prime/composite, then:

(a) each prime factor, e.g., y below, of (a x …….. x n - 2 x n) - 2 > n > n - 2
(b) each prime factor, e.g., z below, of (a x …….. x n - 2 x n) - 4 > n > n - 2

(a x …….. x n - 2 x n) - 2 = prime V composite (1)

(a x …….. x n - 2 x n) - 4 = prime V composite (2)

(1) & (2) = twin primes, if both (1) & (2) are prime

(1) & (2) > n & n - 2

Let Y represent the prime factors of (a x …….. x n - 2 x n) - 2 if (a x …….. x n - 2 x n) - 2 is not prime (i.e., it is composite), each prime factor may pair up with another prime which differs from it by 2 to form twin primes. Let y = prime factor in Y .

y & y +/- 2 = twin primes, if y +/- 2 is prime

y & y +/- 2 > n & n - 2

Let Z represent the prime factors of (a x …….. x n - 2 x n) - 4 if (a x …….. x n - 2 x n) - 4 is not prime (i.e., it is composite), each prime factor may pair up with another prime which differs from it by 2 to form twin primes. Let z = prime factor in Z .

z & z +/- 2 = twin primes, if z +/- 2 is prime z & z +/- 2 > n & n - 2

Therefore: (a x …….. x n - 2 x n) - 2 > (a x …….. x n - 2 x n) - 4 > y V y +/- 2 V z V z +/- 2 > n > n - 2

By the above, the following, which implies that n & n - 2 are the largest existing twin primes pair, is an impossibility:

n > n - 2 > (a x …….. x n - 2 x n) - 2 > (a x …….. x n - 2 x n) - 4 > y V y +/- 2 V z V z +/- 2

It is hence clear that no n & n - 2 in any list of consecutive primes can ever possibly be the largest existing twin primes pair and larger twin primes than them can always be found by applying the same mathematical logic (as is described in Appendix 1), e.g., by utilising the evidently effective Algorithm 1, or, Algorithm 2 described in Appendix 3. That is, a largest existing twin primes pair is an impossibility, which implies that the twin primes are infinite. It is possible to find larger twin primes than n & n - 2 no matter how large n & n - 2 are, with the following formulae involving the list of consecutive primes: (a x …….. x n) - 2 & (ax …….. x n) - 4, which by the nature of their composition are capable of generating new primes/twin primes which will always be larger than n & n - 2 (see Appendix 1); this operation is part of Algorithm 1 described in Appendix 3. This is an indirect proof or proof by contradiction (reductio ad absurdum) of the infinity of the twin primes, for our assumption of n & n - 2 as the largest existing twin primes pair will be contradicted by the discovery of larger twin primes, implying the infinity of the twin primes. Again, by applying the same mathematical logic (described in Appendix 1), by way of this evidently effective Algorithm 1 in Appendix 3, and going one step further, we can find that many twin odd integers found between n and (a x …….. x n) - 2, which differ from one another by 2 and are not divisible by any of the primes in the list of consecutive primes: 3, 5, 7, 11, 13, 17,19, …….., n, will be twin primes larger than n & n - 2, our assumed largest existing twin primes pair, which is a contradiction of this assumption, thus implying or proving the infinitude of the twin primes. In this manner, i.e., by resorting to Algorithm 1 in Appendix 3, by continually adding more and more consecutive primes to the list of consecutive primes: 3, 5, 7, 11, 13, 17, 19, …….., n, i.e., continually extending the value of n, and utilising the formula: (a x …….. x n) - 2, as well as the formula: (ax …….. x n) - 4, to perform the computations a la Algorithm 1 in Appendix 3, many larger and larger twin primes can be found, all the way to infinity, in parallel with the infinitude of the list of consecutive primes: 3, 5, 7, 11, 13, 17, 19, …….. of which the twin primes are a part together with other primes pairs, wherein the twin primes are not likely to be finite (as is evident from Appendix 2) and can be expected to be infinite. (Algorithm 2 in Appendix 3 may also be utilised for this purpose but it is evidently a longer and less efficient method.) A largest existing twin primes pair is indeed an impossibility. The twin primes are infinite.

CONCLUDING REMARKS

There are 376 pairs of twin primes (752 primes) found within the 2,500 consecutive primes from 2 to 22,307 - this means that 30.08%, which is sizeable, of the 2,500, not a small quantity, consecutive primes are twin primes. 3, 5 & 7 are the only "triple" primes found. There is no regularity in pattern in the appearance of the twin primes, except that the intervals between consecutive twin primes vary greatly by from 4 integers to 370 integers - the intervals between the consecutive twin primes increase and decrease, and, then increase and decrease again, by turns, giving rise to a graph that is characterised by many peaks, i.e., the curve is rough and nonlinear, making its description (hence, forecast of the twin primes) by differential equations practically impossible.

The argument used here to prove the twin primes' infinity is the indirect (reductio ad absurdum) method, which had been used by Euclid and other mathematicians after him. Logically, 1 or 2 examples of "contradiction" should be sufficient proof of infinity, for it does not make sense to have a need for an infinite number of cases of "contradiction", as our proof would then have to be infinitely and impossibly long, an absurdity. This method of proof is "proof by implication" as a result of "contradiction" - which is a "short-cut" and smart way in proving infinity, instead of "proving infinity by counting to infinity", which is ludicrous, and, impossible. Hence, 1 or 2 cases of "contradiction" should be sufficient for implying that there would be an infinitude of twin primes, which of course also tacitly implies that there would be an infinitude of the number of cases of such "contradiction". (Euclid evidently had this logical point in mind when he formulated the indirect (reductio ad absurdum) proof of the infinity of the primes.) This method of proof had been cleverly used by a number of mathematicians, not the least by the great German mathematician, David Hilbert. For example, Hilbert had used an indirect method (the "reductio ad absurdum" proof) to prove Gordan's Theorem without having to show an actual "construction", a proof which had been accepted by his peers.

The paper presents 2 algorithms for generating or sieving all the twin primes in any range of odd numbers - by utilising any of these 2 algorithms (preferably the evidently more efficient Algorithm 1), we will be able to find many twin primes which are all larger than those in any chosen list of consecutive primes, i.e., we will be able to generate many larger and larger twin primes. This is indeed significant. There is evidently some deep meaning in the ease with which the twin primes turn up, as is shown in this paper. It is thus evident that the twin primes are an inherent characteristic of the infinite prime numbers (as well as odd numbers), a characteristic which could be regarded as "self-similar" or "fractal". A twin primes pair is in effect any pair of odd numbers which differ from one another by 2 and are indivisible by any number except itself, the negative of itself, +1 and -1 (i.e., the pair of odd numbers are prime numbers). Any consecutive odd numbers or odd numbers that differ from one another by 2 are therefore potential prime numbers, as well as potential twin primes, and, the likelihood of them being prime is infinite (vide Euclid's proof and Dirichlet's Theorem), i.e., the primes will always be found amongst them and will be there all the way to infinity (the primes being evidently the "atoms" or building-blocks of all the whole numbers or integers, i.e., all the odd numbers and even numbers - every odd number or integer is either a prime number or composite of prime numbers (i.e., the integer has prime factors), and, every even number is the sum of two prime numbers (vide the Goldbach conjecture which, it appears, practically all mathematicians believe to be true), as well as the product of prime numbers (composite)); hence, the likelihood of them being twin primes is infinite as well (the twin primes being an inherent property of the infinite prime numbers - as well as odd numbers - the twin primes can in fact be likened to next-door neighbours, which are a common, expected thing).

So far, there has not been any indication or confirmation that the number of twin primes is finite and the so-called largest existing pair of twin primes has not been found and confirmed (which of course would be impossible to find and confirm if the twin primes were infinite). On the other hand, practically everyone could intuit that the number of twin primes is infinite.

Due to the evident effectiveness of the 2 algorithms described in Appendix 3 in bringing in larger and larger twin primes, the above proof of the infinitude of the twin primes is not only an indirect proof or proof by contradiction (reductio ad absurdum), importantly, it is also a constructive proof. It should be noted that the characteristic of a mountain or infinite volume of sand is reflected in the characteristic of some grains of sand found there so that studying the characteristic of some grains of sand found there is enough for deducing the characteristic of the mountain or infinite volume of sand, to ascertain the quality of a batch of products it is only necessary to inspect some carefully selected samples from that batch of products and not every one of the products and to carry out a population census, i.e., find out the characteristics of a population, it is only necessary to carry out a survey on some carefully selected respondents and not the whole population. With any of these 2 algorithms (preferably the evidently more efficient Algorithm 1), in like manner, by the same principle, we could carry out a study of a carefully selected list of integers and their associated primes and twin primes and deduce by induction whether the twin primes would always turn up, appear infinitely, in the list which is itself infinite - this act is rather like extrapolation.

APPENDIX 1

Note: The (only) even prime 2 is omitted from the list of consecutive primes: 3, 5, 7, 11, 13, 17, 19, …….., n - 2, n stated in the paper, wherein n & n - 2 are assumed to be the largest existing twin primes pair.

The list of newly created primes, and, twin primes for n = 5, 7, 11, 13, 17, 19, …….. (n = 19 being the maximum limit achievable with a hand-held calculator) is as follows:-

1] For n = 5, we get the following new primes/new twin primes:

$(3 \times 5) - 2 = 13$ (α)
$(3 \times 5) - 4 = 11$ (β)

2] For n = 7, we get the following new primes/new twin primes:

(3 x 5 x 7) - 2 =103 (α)
(3 x 5 x 7) - 4 = 101 (β)

3] For n = 11, we get the following new primes/new twin primes:

(3 x 5 x 7 x 11) - 2 = 1,153 (α)
(3 x 5 x 7 x 11) - 4 = 1,151 (β)

4] For n = 13, we get the following new prime and composite number with its prime factors:

(3 x 5 x 7 x 11 x 13) - 2 = 15,013 (α) - Prime Number
(3 x 5 x 7 x 11 x 13) - 4 = 15,011 (β) - Composite Number (= 17 x 883, with 17 pairing with 19 to form a twin primes pair and 883 pairing with 881 to form another twin primes pair)

5] For n = 17, we get the following new primes/new twin primes:

(3 x 5 x 7 x 11 x 13 x 17) - 2 = 255,253 (α)
(3 x 5 x 7 x 11 x 13 x 17) - 4 = 255,251 (β)

6] For n = 19, we get the following new prime and composite number with its prime factors:

(3 x 5 x 7 x 11 x 13 x 17 x 19) - 2 = 4,849,843 (α) - Prime Number
(3 x 5 x 7 x 11 x 13 x 17 x 19) - 4 = 4,849,841 (β) - Composite Number (= 43 x 112,787, with 43 pairing with 41 to form a twin primes pair while 112,787 is a stand-alone prime)

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Results Of α And β Above

1) α above generates 6 new primes (13; 103; 1,153; 15,013; 255,253; 4,849,843), nil composite numbers.
2) β above generates 4 new primes (11; 101; 1,151; 255,251), 2 composite numbers (15,011 = 17 x 883; 4,849,841 = 43 x 112,787).
3) α and β above together produce 4 pairs of new twin primes (13 & 11; 103 & 101; 1,153 & 1,151; 255,253& 255,251).
4) The prime factors of α and β above form 3 pairs of new twin primes with prime partners which differ from them by 2 (19 & 17; 43 & 41; 883 & 881).

5) All the new twin primes in (3) and (4) above are larger than n & n - 2, the assumed largest existing twin primes pair, which is indirect proof of the infinitude of the twin primes.

Why It Is Impossible For Any n & n - 2 To Be The Largest Existing Twin Primes Pair

α = (3 x 5 x 7 x 11 x 13 x 17 x 19 x .... x n) - 2, and, β = (3 x 5 x 7 x 11 x 13 x 17 x 19 x .... x n) - 4 will never be divisible by any of the consecutive prime numbers in the list: 3, 5, 7, 11, 13, 17, 19, ..., n, whether they are prime or composite (non-prime and divisible by prime numbers or prime factors). This means that none of the consecutive prime numbers in the list: 3, 5, 7, 11, 13, 17, 19, ..., n can ever be factors of α and β, and, α and β must be new primes/twin primes larger than all the consecutive prime numbers in the list: 3, 5, 7, 11, 13, 17, 19, ..., n, or, if they were composite (non-prime and divisible by prime numbers or prime factors), their prime factors (and "twin prime" partners which differ from them by 2) must be larger than all the consecutive prime numbers in the list: 3, 5, 7, 11, 13, 17, 19, ..., n. This is a very important mathematical logic, which needs to be grasped in order to understand the proof.

This all implies that no n & n - 2 (if n - 2 were also a prime number) in any list of consecutive prime numbers can ever possibly be the largest existing twin primes pair, since all the new primes/twin primes produced or generated by α and β will always be larger than n & n - 2. That is, a largest existing twin primes pair is an impossibility, which implies the infinitude of the list of the primes/twin primes.

In other words, by the mathematical logic stated above, which explains why all the new primes/twin primes, which α and β by the nature of their composition are capable of producing or generating, will always be larger than n & n - 2, no n & n - 2 in any list of consecutive prime numbers: 3, 5, 7, 11, 13, 17, 19, ..., n can ever possibly be the largest existing twin primes pair, i.e., a largest existing twin primes pair is indeed an impossibility, thus implying the infinitude of the list of the twin primes. This is a very important inference.

Regardless of how long the list of the twin primes pairs is, it is possible to find some new twin primes pairs which will always be larger than n & n - 2, our assumed largest existing twin primes pair - the largest twin primes pair in our assumed finite list of the twin primes pairs, with α and β, which is indirect proof of the infinity of the twin primes. In fact, by the same principle, many twin odd integers found between n and α, which differ from one another by 2 and are not divisible by any of the primes in the list of consecutive primes: 3, 5, 7, 11, 13, 17, 19, ……., n, will be twin primes pairs larger than n & n - 2, our assumed largest existing twin primes pair, which is a contradiction of this assumption, hence implying or proving the infinitude of the twin primes. (Refer to Algorithm 1, as well as Algorithm 2, in Appendix 3.)

APPENDIX 2

Anecdotal Evidence Of The Infinity Of The Twin Primes

TOP TWIN PRIMES IN 2000, 2001, 2007 & 2009

In the year 2000, 4648619711505 x $2^{60000} \pm 1$ (18,075 digits) had been the top twin primes pair which had been discovered. In the year 2001, it only ranked eighth in the list of top 20 twin primes pairs, with $318032361 \cdot 2^{107001} \pm 1$ (32,220 digits) topping the list. In the year 2007, in the list of top 20 twin primes pairs, $318032361 \cdot 2^{107001} \pm 1$ (32,220 digits) ranked eighth, while 4648619711505 x $2^{60000} \pm 1$ (18,075 digits) was nowhere to be seen; 2003663613*2^195000-1 and 2003663613*2^195000+1 (58,711 digits), which was discovered on January 15, 2007, by Eric Vautier (from France) of the Twin Prime Search (TPS) project in collaboration with PrimeGrid (BOINC platform), was at the top of the list. As at August 2009, $65516468355 \cdot 2^{333333}-1$ and $65516468355 \cdot 2^{333333}+1$ (100,355 digits) is at the top of the list of top 20 twin primes pairs, while $318032361 \cdot 2^{107001} \pm 1$ (32,220 digits) ranks $11^{th.}$, and, 2003663613*2^195000-1 and 2003663613*2^195000+1 (58,711 digits) ranks second in this list.

We can expect larger twin primes than these extremely large twin primes, much larger ones, infinitely larger ones, to be discovered in due course.

LIST OF PRIMES PAIRS FOR THE FIRST 2,500 CONSECUTIVE PRIMES, 2 TO 22,307, RANKED ACCORDING TO THEIR FREQUENCIES OF APPEARANCE

S. No.	Ranking	Prime Pairs	No. Of Pairs	Percentage
(1)	1	primes pair separated by 6 integers	482	19.29 %
(2)	2	primes pair separated by 4 integers	378	15.13 %

(3)	3	primes pair separated by 2 integers (t. p.)	376	15.05 %
(4)	4	primes pair separated by 12 integers	267	10.68 %
(5)	5	primes pair separated by 10 integers	255	10.20 %
(6)	6	primes pair separated by 8 integers	229	9.16 %
(7)	7	primes pair separated by 14 integers	138	5.52 %
(8)	8	primes pair separated by 18 integers	111	4.44 %
(9)	9	primes pair separated by 16 integers	80	3.20 %
(10)	10	primes pair separated by 20 integers	47	1.88 %
(11)	11	primes pair separated by 22 integers	46	1.84 %
(12)	12	primes pair separated by 30 integers	24	0.96 %
(13)	13	primes pair separated by 28 integers	19	0.76 %
(14)	14	primes pair separated by 24 integers	16	0.64 %
(15)	15	primes pair separated by 26 integers	10	0.40 %
(16)	16	primes pair separated by 34 integers	9	0.36 %
(17)	17	primes pair separated by 36 integers	5	0.20 %
(18)	18	primes pair separated by 32 integers	2	0.08 %
(19)	18	primes pair separated by 40 integers	2	0.08 %
(20)	19	primes pair separated by 42 integers	1	0.04 %
(21)	19	primes pair separated by 52 integers	1	0.04 %

Total No. Of Primes Pairs In List: 2,498

It is evident in the above list that the primes pairs separated by 6 integers, 4 integers and 2 integers (twin primes), among the 21 classifications of primes pairs separated by from 2 integers to 52 integers (primes pairs separated by 38 integers, 44 integers, 46 integers, 48 integers & 50 integers are not among them, but, they are expected to appear further down in the infinite list of the primes), are the most dominant, important. There is a long list of other primes pairs, besides those shown in the above list, which also play a part as the building-blocks of the infinite list of the integers.

The list of the integers is infinite. The list of the primes is also infinite. The infinite primes are the building-blocks of the infinite integers - the infinite odd integers are all either primes or composites of primes, and, the infinite even integers, except for 2 which is a prime, are all also composites of primes. Therefore, all the primes pairs separated by the integers of various magnitudes, as described above, can never all be finite. If there is any possibility at all for any of these primes pairs to be finite, there is only the possibility that a number of these primes pairs are finite (but never all of them). However, will it have to be the primes pairs separated by 2 integers or twin primes (which are the subject of our investigation here), which are the only primes pair, or, one among a number of primes pairs, which are finite? Why question only the infinity of the primes pairs separated by 2 integers, the twin primes? Are not the infinities of the primes pairs separated by 8 integers and more, whose frequencies of appearance are lower, as compared to those of the primes pairs which are separated by 6, 4 and 2 integers respectively, in the above list of primes pairs, more questionable? Why single out only the twin primes? (There are at least 18 other primes pairs, separated by from 8 integers to 52 integers, whose respective infinities should be more suspect, as is evident from the above list of primes pairs, if any infinities should be doubted. Evidently, the primes pairs separated by 2 integers (twin primes) are not that likely to be finite.)

The above represents anecdotal evidence that the twin primes are infinite, which is a ratification of the actual proof given earlier.

APPENDIX 3

The following algorithms will be able to generate or sieve all the twin primes in any range of odd numbers which are all larger than those in the list of known consecutive primes/twin primes; these 2 important algorithms will provide plenty of numerical evidence that the twin primes are infinite:-

Algorithm 1

We would provide an example with Items (1) to (3) from the following list of products of consecutive primes/twin primes, which should be sufficient for our purpose here:-

1) 3 x 5 = 15
2) 3 x 5 x 7 = 105
3) 3 x 5 x 7 x 11 = 1,155
4) 3 x 5 x 7 x 11 x 13 = 15, 015
5) 3 x 5 x 7 x 11 x 13 x 17 = 255,255
6) 3 x 5 x 7 x 11 x 13 x 17 x 19 = 4,849,845

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The example is as follows:-

1) For 3 x 5 = 15, we would find all the consecutive pairs of odd numbers between 5 & 15 which differ from one another by 2 and are not divisible by any of the consecutive primes/twin primes 3 & 5 in the list of consecutive primes/twin primes 3 x 5 whose product is 15.

   There is only 1 pair of odd numbers between 5 & 15 which differ from one another by 2 and are not divisible by the consecutive primes/twin primes 3 & 5 in the list of consecutive primes/twin primes 3 x 5 - they are the twin primes 11 & 13.

2) Similarly, for 3 x 5 x 7 = 105, we would find all the consecutive pairs of odd numbers between 7 & 105 which differ from one another by 2 and are not divisible by any of the consecutive primes/twin primes 3, 5 & 7 in the list of consecutive primes/twin primes 3 x 5 x 7 whose product is 105.

   The consecutive pairs of odd numbers between 7 & 105 which differ from one another by 2 and are not divisible by the consecutive primes/twin primes 3, 5 & 7 are the following consecutive twin primes:

(a) 11 & 13
(b) 17 & 19
(c) 29 & 31
(d) 41 & 43
(e) 59 & 61
(f) 71 & 73
(g) 101 & 103

3) Similarly, in this final case, for 3 x 5 x 7 x 11 = 1,155, we would find all the consecutive pairs of odd numbers between 11 & 1,155 which differ from one another by 2 and are not divisible by any of the consecutive primes/twin primes 3, 5, 7 & 11 in the list of consecutive primes/twin primes 3 x 5 x 7 x 11 whose product is 1,155.

   Many of the consecutive pairs of odd numbers between 11 & 1,155 which differ from one another by 2 and are not divisible by the consecutive primes/twin primes 3, 5, 7 & 11 are twin primes (while the rest are primes larger than 3, 5, 7 & 11 and/or composite numbers whose prime factors are each larger than 3, 5, 7 & 11), some of which are as follows:

   (a) 17 & 19
   (b) 29 & 31
   (c) 41 & 43
   (d) 59 & 61
   (e) 71 & 73
   (f) 101 & 103
   (g) 107 & 109
   (h) 137 & 139
   (i) 149 & 151
   (j) 179 & 181
   (k) Etc. to 1,151 & 1,153

In this way, we would also be able to achieve the following:-

1) For 3 x 5 x 7 x 11 x 13 = 15,015, find all the consecutive twin primes between 13 and 15,015.
2) For 3 x 5 x 7 x 11 x 13 x 17 = 255,255, find all the consecutive twin primes between 17 and 255,255.
3) For 3 x 5 x 7 x 11 x 13 x 17 x 19 = 4,849,845, find all the consecutive twin primes between 19 and 4,849,845.

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Algorithm 2

We would, similar to Algorithm 1 above, also provide an example with Items (1) to (3) from the following list of products of consecutive primes/twin primes, which should be sufficient for our purpose here:-

1) 3 x 5 = 15
2) 3 x 5 x 7 = 105
3) 3 x 5 x 7 x 11 = 1,155
4) 3 x 5 x 7 x 11 x 13 = 15, 015
5) 3 x 5 x 7 x 11 x 13 x 17 = 255,255
6) 3 x 5 x 7 x 11 x 13 x 17 x 19 = 4,849,845

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The example is as follows:-

1) For 3 x 5 = 15, we would first find all the consecutive pairs of even numbers between 5 & 15 which differ from one another by 2 and are not divisible by any of the consecutive primes/twin primes 3 & 5 in the list of consecutive primes/twin primes 3 x 5. Then we deduct each of these consecutive pairs of even numbers which are not divisible by any of the consecutive primes/twin primes 3 & 5 from the product of these consecutive primes/twin primes 3 x 5 which is 15. The results would each be 1 pair of twin primes, 1 prime & 1 composite of primes, or, 2 composites of primes. In this way, we would be able to find all the consecutive twin primes between 5 & 15.

   There is only 1 pair of even numbers between 5 & 15 which differ from one another by 2 and are not divisible by any of the consecutive primes/twin primes 3 & 5 in the list of consecutive primes/twin primes 3 x 5 - they are the pair 2 & 4.

   The following is the result after we deduct this pair of even numbers 2 & 4 which are not divisible by any of the consecutive primes/twin primes 3 & 5 from the product of these consecutive primes/twin primes 3 x 5 which is 15:

   (a) 15 - 2 & 15 - 4: 13 & 11 (twin primes)

2) Similarly, for 3 x 5 x 7 = 105, we would first find all the consecutive pairs of even numbers between 7 & 105 which differ from one another by 2 and are not divisible by any of the consecutive primes/twin primes 3, 5 & 7 in the list of consecutive primes/twin primes 3 x 5 x 7, which are as follows:

   (a) 2 & 4
   (b) 32 & 34
   (c) 44 & 46
   (d) 62 & 64

(e) 74 & 76
(f) 86 & 88
(g) 92 & 94

Then we deduct each of these consecutive pairs of even numbers which are not divisible by any of the consecutive primes/twin primes 3, 5 & 7 from the product of these consecutive primes/twin primes 3 x 5 x 7 which is 105. The results would each be 1 pair of twin primes, 1 prime & 1 composite of primes, or, 2 composites of primes. In this way, we would be able to find all the consecutive twin primes between 7 & 105, which are as follows:

(a) 105 - 2 & 105 - 4: 103 & 101 (twin primes)
(b) 105 - 32 & 105 - 34: 73 & 71 (twin primes)
(c) 105 - 44 & 105 - 46: 61 & 59 (twin primes)
(d) 105 - 62 & 105 - 64: 43 & 41 (twin primes)
(e) 105 - 74 & 105 - 76: 31 & 29 (twin primes)
(f) 105 - 86 & 105 - 88: 19 & 17 (twin primes)
(g) 105 - 92 & 105 - 94: 13 & 11 (twin primes)

3) Similarly, in this final case, for 3 x 5 x 7 x 11 = 1,155, we would first find all the consecutive pairs of even numbers between 11 & 1,155 which differ from one another by 2 and are not divisible by any of the consecutive primes/twin primes 3, 5, 7 & 11 in the list of consecutive primes/twin primes 3 x 5 x 7 x 11, some of which are as follows:

(a) 2 & 4
(b) 32 & 34
(c) 62 & 64
(d) 74 & 76
(e) 92 & 94
(f) 116 & 118
(g) 122 & 124
(h) 134 & 136
(i) Etc. to 1,136 & 1,138

Next we deduct each of these consecutive pairs of even numbers which are not divisible by any of the consecutive primes/twin primes 3, 5, 7 & 11 from the product of these consecutive primes/twin primes 3 x 5 x 7 x 11 which is 1,155. The results would each be 1 pair of twin primes, 1 prime & 1 composite of primes, or, 2 composites of primes. In this way, we would be able to find all the consecutive twin primes between 11 & 1,155, some of which are as follows:

(a) 1,155 - 2 & 1,155 - 4: 1,153 & 1,151 (twin primes)
(b) 1,155 - 32 & 1,155 - 34: 1,123 (prime) & 1,121 (composite of primes which are each larger than 3, 5, 7 & 11 = 19 x 59)
(c) 1,155 - 62 & 1,155 - 64: 1,093 & 1,091 (twin primes)
(d) 1,155 - 74 & 1,155 - 76: 1,081 & 1,079

(composite of primes which are each larger than 3, 5, 7 & 11 = 23 x 47) (composite of primes which are each larger than 3, 5, 7 & 11 = 13 x 83)

(e) 1,155 - 92 & 1,155 - 94: 1,063 & 1,061 (twin primes)
(f) 1,155 - 116 & 1,155 - 118: 1,039 (prime) & 1,037 (composite of primes which are each larger than 3, 5, 7 & 11 = 17 x 61)
(g) 1,155 - 122 & 1,155 - 124: 1,033 & 1,031 (twin primes)
(h) 1,155 - 134 & 1,155 - 136: 1,021 & 1,019 (twin primes)
(i) Etc. to 1,155 - 1,136 & 1,155 - 1,138: 19 & 17 (twin primes)

In like manner, we would also be able to achieve the following:-

1) For 3 x 5 x 7 x 11 x 13 = 15,015, find all the consecutive twin primes between 13 and 15,015.
2) For 3 x 5 x 7 x 11 x 13 x 17 = 255,255, find all the consecutive twin primes between 17 and 255,255.
3) For 3 x 5 x 7 x 11 x 13 x 17 x 19 = 4,849,845, find all the consecutive twin primes between 19 and 4,849,845.

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By utilising any of the above algorithms (preferably the evidently more efficient Algorithm 1), we will be able to find many twin primes which are all larger than those in any chosen list of consecutive primes/twin primes, i.e., we will be able to generate many larger and larger twin primes with these algorithms.

It would evidently be difficult to accept a proof of the twin primes conjecture without having to confirm or check the validity of the logic by computing a sufficiently long list of twin primes, even to the extent of looking out for counter-examples. Hence, the great importance of the above algorithms.

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