

# Fuzzy Pairwise L-Open Sets and Fuzzy Pairwise L-Continuous Functions

M. E. Abd El-Monsef<sup>1,\*</sup>, A. Kozac<sup>2</sup>, A. A. Salama<sup>1</sup>, H. M. Elagamy<sup>2</sup>

<sup>1</sup>Department of Mathematics and Computer Sciences, Faculty of Sciences, Port Said University, Egypt

<sup>2</sup>Department of Mathematics, Faculty of Science, Tanta University, Egypt

**Abstract** The aim of this paper is to introduce and study some new fuzzy pairwise notion in fuzzy bitopological ideals spaces. We also generalize the notion of fuzzy L-open sets due to Abd El-Monsef et al[1]. In addition to generalize the concept of fuzzy L-closed sets, fuzzy L-continuity and L-open functions due to Abd El-Monsef et al[1]. Relationships between the above new fuzzy pairwise notions and there other relevant classes are investigated. Recently, we define and study two different types of fuzzy pairwise functions.

**Keywords** Fuzzy Ideals, Fuzzy Bitopological Spaces, Fuzzy L-open Sets, Fuzzy L-continuous Functions

## 1. Introduction

The concept of fuzzy sets was first introduced by Zadeh[8]. Subsequently, Chang defined the notion of fuzzy topology[4]. Since then various aspects of general topology were investigated and carried out in fuzzy since by several authors of this field. The notions of fuzzy ideal and fuzzy pairwise local function introduced and studied in[2]. Nough[6] initiated the study of fuzzy bitopological spaces. A fuzzy set equipped with two fuzzy topologies is called a fuzzy bitopological space. Concepts of fuzzy ideals and fuzzy local function were introduced by Sarkar[7]. The purpose of this paper is to introduce and study some new pairwise fuzzy notion in fuzzy bitopological ideals spaces. We also generalize the notion of fuzzy L-open sets due to Abd El-Monsef et al[1]. In addition to generalize the concept of L-closed sets, L-continuity and L-open functions due to Abd El-Monsef et al[1].

## 2. Preliminaries

Throughout this paper, by  $(X, \tau_1, \tau_2)$ , we mean a fuzzy bitopological space (fbts in short) in Nough's[6] sense. A fuzzy point in X with support  $x \in X$  and value  $\varepsilon$   $0 < \varepsilon \leq 1$  is denoted by  $x_\varepsilon$  in[3]. A fuzzy point  $x_\varepsilon$  is said to be contained in a fuzzy set  $\mu$  in  $I^X$  iff  $\varepsilon \leq \mu$  and this will be denoted by  $x_\varepsilon \in \mu$  [3]. For a fuzzy set  $\mu$  in a fbts

$(X, \tau_1, \tau_2)$ ,  $\tau_i - cl(\mu)$ ,  $\tau_i - Int(\mu)$ ,  $i \in \{1, 2\}$ , and  $\mu^c$  will respectively denote closure, interior and complement of  $\mu$ . The constant fuzzy sets taking values 0 and 1 on X are denoted by  $0_x$ ,  $1_x$  respectively. A fuzzy set  $\mu$  in fts is said to be quasi-coincident[5] with a fuzzy set  $\eta$ , denoted by  $\mu q \eta$ , if there exists  $x \in X$  such that  $\mu(x) + \eta(x) > 1$ . A fuzzy set  $\nu$  in a fts  $(X, \tau)$ , is called a q-nbd[3,5] of a fuzzy point  $x_\varepsilon$  iff there exists a fuzzy open set  $\mu$  such that  $x_\varepsilon q \mu \subseteq \nu$ , we will denote the set of all q-nbd of  $x_\varepsilon$  in  $(X, \tau)$  by  $N(x_\varepsilon)$ . A nonempty collection of fuzzy sets L of a set X is called fuzzy ideal[5] on X iff i)  $\mu \in L$  and  $\eta \subseteq \mu \Rightarrow \eta \in L$  (heredity), (ii)  $\mu \in L$  and  $\eta \in L \Rightarrow \mu \vee \eta \in L$  (finite additivity). The fuzzy local function[7]  $\mu^*(L, \tau)$  of a fuzzy set  $\mu$  is the union of all fuzzy points  $x_\varepsilon$  such that  $\nu \in N(x_\varepsilon)$  and  $\rho \in L$  then there is at least one  $r \in X$  for which  $\nu(r) + \mu(r) - 1 > \rho(r)$ . For a fts  $(X, \tau)$  with fuzzy ideal L,  $cl^*(\mu) = \mu \vee \mu^*$  [7] for any fuzzy set  $\mu$  of X and  $\tau^*(L)$  be the fuzzy topology generated by  $cl^*$ [7].

**Definition.2.1.[2].** A fuzzy set  $\mu$  in a fbts  $(X, \tau_1, \tau_2)$  is called pairwise quasi-coincident with a fuzzy set  $\eta$  denoted by  $P(\mu q \eta)$ , if there exists  $x \in X$  such that  $\mu(x) + \eta(x) > 1$ . Obviously, for any two fuzzy sets  $\mu$  and  $\eta$ ,  $P(\mu q \eta)$  will imply  $P(\mu q \eta)$ .

**Definition.2.2.[2].** A fuzzy set  $\mu$  in a fbts) is  $(X, \tau_1, \tau_2)$  called Pairwise quasi-neighborhood of point  $x_\varepsilon$  if and only if there exists a fuzzy  $\tau_i - open$ ,  $i \in \{1, 2\}$  set  $\rho$

\* Corresponding author:

drsalama44@gmail.com (M. E. Abd El-Monsef)

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such that  $x_\varepsilon \in \rho \subseteq \mu$  we will denote the set of all pairwise q-nbd of  $x_\varepsilon$  in  $(X, \tau_1, \tau_2)$  by  $PN(x_\varepsilon, \tau_i), i \in \{1, 2\}$

**Definition 2.3.[2].** Let  $(X, \tau_1, \tau_2)$  be a fbts with fuzzy ideal  $L$  on  $X$ , and  $\mu \in I^X$ . Then the fuzzy pairwise local function  $P\mu^*(L, \tau_i), i \in \{1, 2\}$  of  $\mu$  is the union of all fuzzy points  $x_\varepsilon$  such that for  $\rho \in PN(x_\varepsilon, \tau_i)$  and  $\ell \in L$  then there is at least one  $r \in X$  for which  $\rho(r) + \mu(r) - 1 > \ell(r)$ , where  $PN(x_\varepsilon, \tau_i)$  is the set of all q-nbd of  $x_\varepsilon$ . Therefore, any  $x_\varepsilon \notin P\mu^*(L, \tau_i), i \in \{1, 2\}$  (for any  $x_\varepsilon \notin \mu$  (any fuzzy set) implies hereafter,  $x_\varepsilon$  is not contained in the fuzzy set  $\mu$ , i.e.  $\varepsilon > \mu(x)$  implies there is at least one  $\rho \in PN(x_\varepsilon, \tau_i)$  such that for every  $r \in X$  for which  $\rho(r) + \mu(r) - 1 > \ell(r)$  for some  $\ell \in L$ .

We will occasionally write  $P\mu^*$  or  $P\mu^*(L)$  for  $P\mu^*(L, \tau_i)$ . We define  $P^*$ -fuzzy closure operator, denoted by  $pcl^*$  for fuzzy topology  $\tau_i^*(L)$  finer than  $\tau_i$  as follows:

$Pcl^*(\mu) = \mu \vee p\mu^*$  for every fuzzy set  $\mu$  on  $X$ . When there is no ambiguity, we will simply write for  $P\mu^*$  and  $\tau_i^*$  for  $P\mu^*(L, \tau_i)$  and  $\tau_i^*(L)$ , respectively.

**Definition 2.4.[2].** Let  $(X, \tau_1, \tau_2)$  be a fbts with fuzzy ideal  $L$  on  $X$ , a fuzzy pairwise local function  $P\mu^*(L, \tau_1 \vee \tau_2), i \in \{1, 2\}$  of  $\mu \in I^X$  is the union of all fuzzy points  $x_\varepsilon$  such that for and  $\ell \in L$  then there is at least one  $r \in X$  for which  $\rho(r) + \mu(r) - 1 > \ell(r)$ , where  $PN(x_\varepsilon, \tau_i)$  is the set of all q-nbd of  $x_\varepsilon$  in  $\tau_1 \vee \tau_2$  (where  $\tau_1 \vee \tau_2$  is fuzzy topology generated by  $\tau_1, \tau_2$ ).

**Example: 2.1.[2].** One may easily verify that

If  $L = \{0_x\}$  then  $P\mu^*(L, \tau_i) = \tau_i - cl(\mu)$  for any  $\mu \in I^X, i \in \{1, 2\}$ .

$L = I^X$ , then  $P\mu^*(L, \tau_i) = 0_x$ , for any  $\mu \in I^X, i \in \{1, 2\}$ .

**Theorem 2.1.[2].** Let  $(X, \tau_1, \tau_2)$  be a fbts with fuzzy ideal  $L$  on  $X$ ,  $\mu, \eta \in I^X$  and  $\sigma = \tau_1 \vee \tau_2$ . Then we have:

- i)  $P\mu^*(L, \sigma) \subseteq P\mu^*(L, \tau_i); i \in \{1, 2\}$ .
- ii) If  $\mu \subseteq \eta$  then  $P\mu^*(L, \sigma) \subseteq P\eta^*(L, \tau_i); i \in \{1, 2\}$ .
- iii)  $P\mu^*(L, \sigma) \subseteq \sigma - cl(\mu) \subseteq \tau_i - cl(\mu)$ .
- iv)  $P\mu^{**}(L, \sigma) \subseteq P\mu^*(L, \tau_i); i \in \{1, 2\}$ .

**Theorem 2.2.[2].** Let  $(X, \tau_1, \tau_2)$  be a fbts with fuzzy ideal  $L$  on  $X$ ,  $\mu \in I^X$ , If  $\tau_1 \subseteq \tau_2$ , then

- i)  $P\mu^*(L, \tau_2) \subseteq P\mu^*(L, \tau_1)$ , for every  $\mu \in I^X$ ,
- ii)  $\tau_1^* \subseteq \tau_2^*$ .

iii) Clearly  $\tau_1^* \subseteq \tau_2^*$  as  $P\mu^*(L, \tau_2) \subseteq P\mu^*(L, \tau_1)$

**Theorem 2.3.[2].** Let  $(X, \tau_1, \tau_2)$  be a fbts and  $L, J$  be two fuzzy ideals on  $X$ . Then for any fuzzy sets  $\mu, \rho \in I^X$

- i.)  $\mu \subseteq \rho \Rightarrow P\mu^*(L, \tau_i) \subseteq P\rho^*(L, \tau_i), i \in \{1, 2\}$ .
- ii.)  $L \subseteq J \Rightarrow P\mu^*(J, \tau_i) \subseteq P\mu^*(L, \tau_i), i \in \{1, 2\}$
- iii)  $P\mu^* = \tau_i - cl(P\mu^*) \subseteq \tau_i - cl(\mu), i \in \{1, 2\}$ .
- vi)  $P\mu^{**}(L, \tau_i) \subseteq P\mu^*(L, \tau_i), i \in \{1, 2\}$
- v)  $P(\mu \cup \rho)^*(L, \tau_i) = P\mu^*(L, \tau_i) \cup P\rho^*(L, \tau_i)$ .
- iv)  $\rho \in L \Rightarrow P(\mu \cup \rho)^*(L, \tau_i) = P\mu^*(L, \tau_i)$

### 3. On Fuzzy Pairwise Local Function

**Definition 3.1.** Given  $(X, \tau_1, \tau_2)$  be a fbts with fuzzy ideal  $L$  on  $X$ ,  $\mu \in I^X$ . Then  $\mu$  is said to be:

- i) Fuzzy pairwise  $\tau_i^*$ -closed,  $i \in \{1, 2\}$  (or PF $^*$ -closed) if  $P\mu^* \leq \mu$
- ii) Fuzzy pairwise PL-dense – in – itself (or PF $^*$ -dense- in – itself) if  $\mu \leq P\mu^*$ .
- (iii) Fuzzy pairwise  $^*$ -perfect if  $\mu$  is PF $^*$ -closed and PF $^*$ -dense – in itself.

**Theorem 3.1.** Given  $(X, \tau_i), i \in \{1, 2\}$  be a fbts with fuzzy ideal  $L$  on  $X$ ,  $\mu \in I^X$  then  $\mu$  is

- i) PF $^*$ - closed iff  $cl^*(\mu) = \mu$
- (ii) PF $^*$ - dense – in – itself iff  $cl^*(\mu) = p\mu^*$ .
- (iii) PF $^*$ - perfect iff  $cl^*(\mu) = p\mu^* = \mu$

**Proof:** Follows directly from the fuzzy pairwise closure operator  $cl^*$  for a fuzzy bitopological  $\tau_i^*(L), i \in \{1, 2\}$  in [2] and Definition 3.1

**Remark 3.1.** One can deduce that

- (i) Every PF $^*$ -dense- in – itself is fuzzy pairwise dense set.
- (ii) Every fuzzy pairwise closed (resp. fuzzy pairwise open) set is PF $^*$ -closed (resp. PF $^*$  $\tau_i^*$  – open,  $i \in \{1, 2\}$ ).

**Corollary 3.1.** Given  $(X, \tau_i), i \in \{1, 2\}$  be a fbts with fuzzy ideal  $L$  on  $X$ ,  $\mu \in \tau_i$  then we have :

- (i) If  $\mu$  is PF $^*$ -closed then  $p\mu^* \leq \text{int}(\mu) \leq cl(\mu)$ .
- (ii) If  $\mu$  is PF $^*$ -dense- itself then  $\text{int}(\mu) \leq p\mu^*$ .
- (iii) If  $\mu$  is PF $^*$ - perfect then  $\text{int}(\mu) = cl(\mu) = p\mu^*$ .

**Proof:** Obvious.

**Theorem 3.1.** Given  $(X, \tau_i), i \in \{1, 2\}$  be a fbts with fuzzy ideal  $L_n$  on  $X$ ,  $\mu \in I^X$  then we have the following:

- (i.)  $\mu$  is fuzzy pairwise  $\alpha$  - closed iff  $\mu$  is PF $^*$ - closed.
- (ii.)  $\mu$  is fuzzy pairwise  $\beta$  – open iff  $\mu$  is PF $^*$ - dense in itself.
- (iii.)  $\mu$  is fuzzy regular pairwise closed iff  $\mu$  is PF $^*$ - perfect.

**Proof:** It's clear.

**Corollary 3.2.** For an fbts  $(X, \tau_i)$ ,  $i \in \{1,2\}$  with fuzzy ideal  $L$  on  $X$ ,  $\mu \in I^X$  the following holds:

- (i) If  $\mu \in PFC(X)$  then  $\mu$  is  $PF^*$ -closed.
- (ii) If  $\mu \in PFBC(X)$  then  $\text{int}(\text{int}(P\mu^*)) \leq \mu$ .
- (iii) If  $\mu \in PFSC(X)$  then  $\text{int}(P\mu^*) \leq \mu$ .

**Proof:** Obvious.

### 4. Fuzzy Pairwise L-open and Fuzzy Pairwise L- closed Sets

**Definition.4.1.** Given  $(X, \tau_i)$ ,  $i \in \{1,2\}$  be a fbts with fuzzy ideal  $L$  on  $X$ ,  $\mu \in I^X$  and  $\mu$  is called a fuzzy pairwise  $L$ -open set iff there exists  $\zeta \in \tau_i$ ,  $i \in \{1,2\}$  such that  $\mu \subseteq \zeta \subseteq P\mu^*(L, \tau_i)$   $i \in \{1,2\}$ .

We will denote the family of all fuzzy pairwise

$L$ -open on  $(X, \tau_i) = \{\mu \in \tau_1 - \text{int}(P\mu^*(L, \tau_1))\}$  and  $\mu \in \tau_2 - \text{int}(P\mu^*(L, \tau_2))$ ,  $i \in \{1,2\}$ . (simplify  $FPLO(X)$ ).

When there is no chance for confusion.

**Theorem.4.1.** let  $(X, \tau_i)$ ,  $i \in \{1,2\}$  be a fbts with fuzzy ideal  $L$  then  $\mu \in FPOL(X)$  iff  $\mu \in \tau_i - \text{int}(P\mu^*(L, \tau_i))$  for  $i \in \{1,2\}$

**Proof.** Assume that  $\mu \in FPOL(X)$  then Definition.4.1. there exists  $\zeta \in \tau_i$ , such that  $\mu \subseteq \zeta \subseteq P\mu^*(L, \tau_i)$ ,  $i \in \{1,2\}$ . But  $\text{int}(P\mu^*) \subseteq P\mu^*$ , put  $\zeta = \text{int}(P\mu^*)$ . Hence  $\mu \subseteq \text{int}(P\mu^*) = P\mu^*$ . Conversely  $\mu \subseteq \text{int}(P\mu^*) \subseteq P\mu^*$ . Then there exists  $\zeta \subseteq \text{int}(P\mu^*) \in \tau_i$ . Hence  $\mu \in FPOL(X)$ .

**Definition.4.2.** The largest  $\tau_i - FPL - \text{open}$  (simply  $\tau_i - FPLO(X)$ ) set contained in  $\mu$  is called a  $\tau_i - FPL - \text{interior}$  of  $\mu$ . The complement of fuzzy pairwise L-open subset of  $X$  is a fuzzy pairwise L-closed subset of  $X$  (simply  $FPLC(X)$ ). We denoted by  $FPL - \text{int}(\mu)$ .

**Theorem.4.2.** Let  $(X, \tau_i)$ ,  $i \in \{1,2\}$  be a fbts with fuzzy ideal  $L$ ,  $\mu \in I^X$  and  $J$  is an arbitrary set then

- i) The union of fuzzy pairwise L-open subsets is fuzzy pairwise L-open.
- ii) If  $\nu$  is fuzzy pairwise open and  $\mu$  is fuzzy pairwise L-open subset of  $X$ . Then  $\mu \cap \nu$  pairwise L-open subset.

**Proof.**

i) Let  $\{\mu_i : i \in J\}$  be a family of  $FPLO(X)$ . Then for each  $j \in J$ ,  $\mu_j \subseteq \tau_i - \text{int}(P\mu_j^*)$  and so  $Y_i \mu_j \subseteq Y_i \tau_i - \text{int}(P\mu_j^*) \subseteq \tau_i - \text{int}(Y_i \mu_j^*)$ .

ii) Assume that  $\nu$  is fuzzy pairwise open and  $\mu$  is fuzzy pairwise L-open subsets of  $X$ . Then  $\mu \cap \nu \subseteq \nu \cap (\tau_i - \text{int}(P\mu^*)) \subseteq \tau_i - \text{int}(\nu \cap P\mu^*) \subseteq \tau_i - \text{int}(\nu \cap \mu)^*$ .

**Definition.4.3.** Let  $(X, \tau_i), i \in \{1,2\}$  be a fbts with fuzzy ideal  $L$  on  $X, \mu \in I^X$ . Then  $\mu$  is said to be

- i) fuzzy  $\tau_i^*$ -closed iff  $\tau_i^* - \text{cl}^*(\mu) = \mu$ .
- ii) fuzzy  $\tau_i^*$ -dense-in-itself if  $\mu \subseteq P\mu^*(L, \tau_i)$ .
- iii) fuzzy  $\tau_i^*$ -perfect if  $\mu$  is  $\tau_i^*$ -closed and  $\tau_i^*$ -dense in itself.

**Theorem.4.3.** Given  $(X, \tau_i), i \in \{1,2\}$  a fbts with fuzzy ideal  $L$  on  $X, \mu \in I^X$ , then the following holds:

- (i) If  $\mu$  is both fuzzy pairwise L-open and  $\tau_i^*$ -perfect then  $\mu$  is fuzzy pairwise open.
- (ii) If  $\mu$  is both fuzzy pairwise open and  $\tau_i^*$ -dense-in-itself then  $\mu$  is fuzzy pairwise L-open.

**Proof.** Follows directly from the fuzzy closure operator for  $\tau_i^*$  and Definition.4.1

**Corollary.4.1.** For a fuzzy subset  $\mu$  of a fbts  $(X, \tau_i)$ ,  $i \in \{1,2\}$  with fuzzy ideal  $L$  on  $X$ , we have:

If  $\mu$  is  $\tau_1^*$ -closed and  $FPL - \text{open}$  then  $\text{int}(\mu) = \text{int}(P\mu^*)$ .

If  $\mu$  is  $\tau_1^*$ -perfect and  $FPL - \text{open}$  set then  $\text{int}(\mu) = \text{int}(P\mu^*)$ .

**Theorem.4.4.** If  $(X, \tau_i), i \in \{1,2\}$  a fbts with fuzzy ideal  $L$  and  $\mu \in I^X$  then

- (i)  $\mu \cap \text{int}(P\mu^*)$  is fuzzy L-open set.
- (ii)  $FPL \tau_i - \text{int}(\mu) = 0_X$  iff  $\text{int}(P\mu^*) = 0_X$ .

**Proof.**

(i) Since  $\text{int}(P\mu^*) = P\mu^* \cap \text{int}(P\mu^*)$  then  $\text{int}(P\mu^*) = P\mu^* \cap \text{int}(P\mu^*) \subseteq P(\mu \cap \mu^*)$ . Thus  $\mu \cap P\mu^* \subseteq (\mu \cap (\mu \cap \text{int}(P\mu^*)))^* \subseteq \text{int}(P(\mu \cap \text{int}(P\mu^*)))^*$ . Hence  $\mu \cap \text{int}(P\mu^*) \in FPLO(X)$ .

(ii) Let  $FPL \tau_i - \text{int}(\mu) = 0_X$ . Then  $\mu \cap (P\mu^*) = 0_X$ , implies  $\text{cl}(\mu \cap \text{int}(P\mu^*)) = 0_X$  and so  $\mu \cap \text{int}(P\mu^*) = 0_X$  conversely assume that  $\text{int}(P\mu^*) = 0_X$  then  $\mu \cap \text{int}(P\mu^*) = 0_X$ . Hence  $FPL \tau_i - \text{int}(\mu) = 0_X$ .

**Theorem.4.5.** If  $(X, \tau_i)$ ,  $i \in \{1,2\}$  a fbts with fuzzy ideal  $L$  on  $X$ ,  $\mu \in I^X$  then  $FPL \tau_i - \text{int}(\mu) = \mu \wedge \text{int}(P\mu^*)$ .

**Proof.** Clear

**Definition.4.4:** Given  $(X, \tau_i), i \in \{1,2\}$  a fbts with fuzzy ideal  $L$  and  $\zeta \in I^X$ ,  $\zeta$  called fuzzy pairwise L-closed set if its complement is fuzzy L-open set. We will denote the family of fuzzy L-closed sets by  $FPLC(X)$ .

**Theorem.4.6.** Given  $(X, \tau_i), i \in \{1,2\}$  a fbts with fuzzy ideal  $L$  and  $\zeta \in I^X$  is fuzzy pairwise L-closed set, then  $P(\text{int} \zeta)^* \leq \zeta$ .

Proof. It's clear.

**Theorem 4.7.** Given  $(X, \tau_i), i \in \{1, 2\}$  be a fbts with fuzzy ideal  $L$  on  $X$  and  $\zeta \in I^X$  such that  $P(\text{int } \zeta)^{*c} = \text{int } P(\zeta^c)$ . then  $\zeta \in FPLC(X)$  iff  $P(\text{int } \zeta)^* \leq \zeta$

**Proof.** (Necessity). Follows immedially from the above theorem. (Sufficiency). Let  $P(\text{int } \zeta)^* \leq \zeta$  then  $\zeta^c \leq (P \text{int } \zeta)^{*c} = \text{int } \zeta^{c*}$  from the hypothesis. Hence  $\zeta^c \in FPLO(X)$ . Thus  $\zeta \in FPLC(X)$ .

**Corollary 4.2.** For a fbts  $(X, \tau_i), i \in \{1, 2\}$  with fuzzy ideal  $L$  on  $X$ , then the union of fuzzy pairwise L-closed sets is fuzzy pairwise L-closed set.

## 5. Fuzzy Pairwise L-Continuous Functions

By utilizing the notion of pairwise L-open sets, we establish in this article a class of fuzzy pairwise L-continuous function. Each of fuzzy pairwise L-continuous and fuzzy pairwise continuous function are independent concepts. Many characterizations and properties of this concept are investigated.

**Definition 5.1.** A fuzzy pairwise function  $f : (X, \tau_i) \rightarrow (Y, \sigma), i \in \{1, 2\}$  with fuzzy ideal  $L$  on  $X$  is said to be fuzzy pairwise L-continuous if for every  $\zeta \in \sigma, f^{-1}(\zeta) \in FPLO(X)$ .

**Remark 5.1:** Every fuzzy pairwise L-continuity is fuzzy pairwise precontinuity but the converse is not true in general as seen by the following example.

**Example 5.1:** Let  $X = Y = \{x\}$ , fuzzy pairwise indiscrete bitopological,  $\sigma$  is fuzzy pairwise discrete bitopological and  $L = \{0_x, \mu\} \vee \{x_\varepsilon : \varepsilon \leq 0.3\} \mu(x) = 0.3$ . The fuzzy pairwise identity function  $f : (X, \tau_i) \rightarrow (Y, \sigma), i \in \{1, 2\}$  is fuzzy pairwise precontinuous but not fuzzy pairwise L-continuous, since  $\mu \in \sigma$  while  $f^{-1}(\mu) \notin FPLO(X)$ .

**Theorem 5.1:** For a function  $f : (X, \tau_i) \rightarrow (Y, \sigma), i \in \{1, 2\}$  with fuzzy ideal  $L$  on  $X$  the following are equivalent:

- (i.)  $f$  is fuzzy pairwise L-continuous.
- (ii.) For  $x_\varepsilon$  in  $X$  and each  $\zeta \in \sigma$  containing  $f(x_\varepsilon)$  there exists  $\mu \in FPLO(X)$  containing  $x_\varepsilon$  such that  $f(\mu) \leq \zeta$ .
- (iii.) For each fuzzy pairwise point  $x_\varepsilon$  in  $X$  and  $\zeta \in \sigma$  containing  $f(x_\varepsilon)$ ,  $(f^{-1}(\zeta))^*$  is fuzzy pairwise nbd of  $x_\varepsilon$ .
- (iv.) The inverse image of each fuzzy pairwise closed set in  $Y$  is fuzzy pairwise L-closed.

**Proof:** (i.)  $\rightarrow$  (ii.). Since  $\zeta \in \sigma$  containing,  $f(x_\varepsilon)$  then by (i),  $f^{-1}(\zeta) \in FPLO(X)$ , by putting  $\mu = f^{-1}(\zeta)$  which containing  $x_\varepsilon$ , we have  $f(\mu) \leq \zeta$  (ii.)  $\rightarrow$  (iii.). Let  $\zeta \in \sigma$  containing  $f(x_\varepsilon)$ . Then by (ii) there exists  $\mu \in PLO(X)$  containing  $x_\varepsilon$  such that  $f(\mu) \leq \zeta$  so  $x_\varepsilon \in \mu \leq \text{int } \mu^* \leq \text{int}(f^{-1}(\zeta))^* \leq (f^{-1}(\zeta))^*$ . Hence  $(f^{-1}(\zeta))^*$  is fuzzy npbd of  $x_\varepsilon$ .

(iii.)  $\rightarrow$  (i.) Let  $\zeta \in \sigma$ , since  $(f^{-1}(\zeta))$  is fuzzy pairwise npbd of any point  $f^{-1}(\zeta)$ , every point  $x_\varepsilon \in (f^{-1}(\zeta))^*$  is a fuzzy pairwise interior point of  $f^{-1}(\zeta)^*$ . Then  $f^{-1}(\zeta) \leq \text{int}(f^{-1}(\zeta))^*$  and hence  $f$  is fuzzy pairwise L-continuous.

(i.)  $\rightarrow$  (iv.) Let  $\zeta \in \sigma$  be a fuzzy pairwise closed set. Then  $\zeta^c$  is fuzzy pairwise open set, by  $f^{-1}(\zeta^c) = f^{-1}(\zeta)^c \in FPLO(X)$ . Thus  $f^{-1}(\zeta)$  is fuzzy PL-closed set.

The following theorem establish the relationship between fuzzy pairwise L-continuous and fuzzy pairwise continuous by using the previous fuzzy pairwise notions.

**Theorem 5.2.** Given  $f : (X, \tau_i) \rightarrow (Y, \sigma), i \in \{1, 2\}$  is a function with ideal  $L$  on  $X$  then we have. If  $f$  is fuzzy pairwise L-continuous of each fuzzy pairwise\*-perfect set in  $X$ , then  $f$  is fuzzy pairwise continuous.

**Proof:** Obvious.

**Corollary 5.1.** Given a function  $f : (X, \tau_i) \rightarrow (Y, \sigma), i \in \{1, 2\}$  and each member of  $X$  is fuzzy pairwise\*-dense-in-itself.

Then we have:

- (i.) Every fuzzy pairwise continuous function is fuzzy pairwise L-continuous.
- (ii.) Each of fuzzy pairwise precontinuous function and fuzzy pairwise L-continuous are equivalent.

**Proof:** It's clear.

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