Speakable and unspeakable in special relativity. I. Synchronization and clock rhythms.

Rodrigo de Abreu and Vasco Guerra

Departamento de Física, Instituto Superior Técnico, Universidade Técnica de Lisboa, 1049-001 Lisboa, Portugal

E-mail: `vguerra@ist.utl.pt`

Abstract. The traditional presentation of special relativity is made from a rupture with previous ideas, such as the notion of absolute motion, emphasizing the antagonism of the Lorentz-Poincaré's views and Einstein's ideas. However, a weaker formulation of the postulates allows to recover all the mathematical results from Einstein's special relativity and reveals that both viewpoints are merely different perspectives of one and the same theory. The apparent contradiction simply stems from different procedures for clock "synchronization," associated with different choices of the *coordinates* used to *describe* the physical world. Even very fundamental claims, such as the constancy of the speed of light, relativity of simultaneity and relativity of time dilation, are seen to be no more than a consequence of a misleading language adopted in the description of the physical reality, which confuses clock *rhythms* with clock *time readings*. Indeed, the latter depend on the "synchronization" adopted, whereas the former do *not*. As such, these supposedly fundamental claims are *not* essential aspects of the theory, as reality is not altered by a mere change of coordinates. The relation between the rhythms of clocks in relative motion is derived with generality. This relation, which is not the standard textbook expression, markedly exposes the indeterminacy of special relativity, connected with the lack of knowledge of the value of the one-way speed of light. Moreover, the theory does not collapse and remains valid if some day the one-way speed of light is truly measured and the indeterminacy is removed. It is further shown that the slow transport method of "synchronization" cannot be seen as distinct from Einstein's procedure.

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1. Introduction

In previous works we undertook a reflection on the foundations of special relativity [1–5]. An inspiring source in this journey was John Bell's book "Speakable and unspeakable in quantum mechanics" [6]. More precisely, the book includes a chapter entitled "How to teach special relativity," in which Bell recommends the use of a *Lorentzian pedagogy*, *i.e.*, that special relativity should be taught starting from the idea of a preferred frame:

> "I have for long thought that if I had the opportunity to teach this subject, I would emphasize the continuity with earlier ideas. Usually it is the discontinuity which is stressed, the radical break with more primitive notions of space and time. Often the result is to destroy completely the confidence of the student in perfectly sound and useful concepts already acquired."

John Bell's idea goes much deeper than the questions of "continuity" and "confidence." Indeed, he continues by adding and stressing an ingredient of special relativity still somewhat unnoticed, namely that all the results from special relativity can be derived either by following the ideas of Lorentz and Poincaré of the existence of a "preferred reference frame" or Einstein's "equivalence of all inertial frames." He acknowledges a difference of philosophy – and a difference of style – on two approaches describing the same physics:

> The difference of philosophy is this. Since it is experimentally impossible to say which uniformly moving system is *really* at rest, Einstein declares the notions 'really resting' and 'really moving' as meaningless. For him only the *relative* motion of two or more uniformly moving objects is real. Lorentz, on the other hand, preferred the view that there is indeed a state of *real* rest, defined by the 'aether', even though the laws of physics conspire to prevent us identifying it experimentally. The facts of physics do not oblige us to accept one philosophy than the other.

The last quoted assertion, although well-known by specialists, still startles most physicists. Nevertheless, it is simply the quite obvious affirmation that the study of relative motion can be made without any reference to absolute motion, but is not incompatible with it. A fact well-known by Galileo, Newton, Lorentz and Poincaré and deeply connected with the principle of relativity, as carefully debated in [4], that most textbooks tend to forget.

In his outstanding 1905 relativity paper [7], Einstein considers the reference to absolute motion as "superfluous." Modern physics lead to a widespread acceptance of a strict operational view of physics, making it easy to identify the word "superfluous" with "meaningless." This constitutes the essence of the "difference in phylosophy." However, the bases of Einstein's special relativity are much less solid than it is generally accepted. In short, both postulates from special relativity are too strong and can be formulated in weaker forms [4], while keeping fully compatible with all available observations and experimental results [1–5]. This more general formulation of special relativity, briefly reviewed in sections 2 and 4, strikingly evinces that there is an indeterminacy in the theory [4], since there are quantities which eventually cannot be measured, such as the *one-way* speed of light, as noted early by Reichenbach [8, 9] and discussed by many authors [2, 10–19]. As a consequence, a deadlock arises in practical terms – although not in fundamental ones – and some additional assumptions have to be required to cut this Gordian knot. Einstein's theory solves the problem in an extremely simple and elegant way, with his methodology for "synchronization" of distant clocks, providing a straightforward and effective *operational* procedure to study physics [4]. Still, other approaches to the problem are possible, fully compatible with Einsteins relativity in practice, but leading to very different assertions in fundamental and philosophical terms.

It seems reasonable to conceed that when additional restrictions are included on top of those implied by the physical reality, then it is likely we are describing only part of it. The difficulty in transposing this somewhat evident statement into the context of Bell's two philosophies lays in a misleading interpretation of the "symbols" employed in the mathematical formalism, with t and v on the first line. In fact, a negligent use of language, associated with the unclear separation of scientific results and "philosophical" or "ideological" statements (defined here as statements that are dependent on an arbitrary convention or on an interpretation relying on additional assumptions not imposed by experiment), has led to a terminological confusion and *apparent* contradictions.

The difficulty in accepting Bell's point is more on the *speech* or *discourse* surrounding special relativity, not so much on the calculations actually performed. As a simple example, "relativity of simultaneity," one of the trademarks of special relativity, presented almost always in the very beginning of any text or media content about relativity, is one debatable "philosophical" statement. As a matter of fact, it depends on the choice of *coordinates* (*cf.* section 3) and, therefore, by no means is an intrinsic feature of the theory [12]. The same is true regarding "relativity of time dilation" (*cf.* section 5). Despite the correctness of the underlying calculations, these affirmations are repeatedly given an abusive semantics they do not possess, as detailed in the body of this paper, as they mix the notions of clock *rhythms* (or clock tick rates) and *time readings* (or time coordinates) displayed by clocks. Indeed, the former are *independent* of any "synchronization" procedure, whereas the latter are *not*.

Following [12], we note that emphasis should be given to the properties that do *not* depend on the choice of coordinates, or, equivalently, on the "synchronization" procedure adopted. To avoid the problem of coordinate-dependent quantities, Oziewicz [20,21] and Ivezic [22] have developed "coordinate-free" approaches to special relativity.

The existence of alternative formalisms and broader views of special relativity following the general lines presented above is rather consensual. As illustrations, we can name the works of Edwards [11], Mansouri and Sexl [10], Leubner *et al* [12] or Selleri [15]. However, the *speech* surrounding special relativity has gained a strong ideological charge, nearly dogmatic. Thus, if these unconventional theories and the corresponding calculations are widely accepted, their implications remain "unspeakable." As a consequence, the establishment of a broader and more general view of special relativity was hindered up to now and a minimal interpretation prevails.

The aim of this work is to accentuate the need for a general formulation of special relativity, by reconciling two apparently contradictory discourses. Hence, one should not speak about two philosophies, as they are different aspects of one and the same theory. In particular, one should not say that the results from special relativity can be derived *either* by following the ideas of Lorentz and Poincaré of the existence of a "preferred reference frame" *or* Einstein's "equivalence of all inertial frames," but rather use the word *both*. For instance, Lorentz's view is usually associated with the sentence "the speed of light in vacuum is c only in one reference frame," whereas Einstein's view with the seemingly contradictory sentence "the speed of light in vacuum is c in all inertial frames." These statements induce to think of a severe incongruity, that could be depicted schematically as in figure 1a). The conflict can be easily elucidated with the simultaneous use of different procedures for clock "synchronization," to which are associated different choices of the time *coordinates* used to *describe* physical events [2,4]. A key concept is the notion of "Einstein-speed" previously introduced in [2] and reviewed in section 6. Within the proposed formulation of special relativity, the former sentences have to be rephrased to "the one-way speed of light in vacuum is c in one reference frame; the two-way speed of light in vacuum is c in all inertial frames" and "the one-way Einstein-speed of light in vacuum is c in all inertial frames," which could be represented as in figure 1b). One explicit case to exemplify this assertion can be found in section 5 from [2]. It shows that special relativity was developed under the shadow of a false dichotomy and that with a precise language all conflicts disappear at the onset.

The structure of this paper is the following. In the next section we present the weak formulation of the postulates. Synchronization procedures and the definition of simultaneity are discussed in section 3. The mathematical formalism is introduced in section 4, with the presentation of the IST transformation and its relation with the Lorentz transformation. Section 5 contains a debate on the difference between clock rhythms and clock time readings, in a short and simple subsection, which nevertheless is a cornerstone of this article. The concept of Einstein speed is reviewed in section 6, where the full compatibility of Bell's two philosophies and the picture of figure 1b) is definitely established. The bridge between the two philosophies is completed in section 7, where the role of the Lorentz transformation is further discussed. In section 8 we obtain the relation of rhythms between two clocks in relative motion and its correlation with the usual time dilation expressions, in another central section of this work. The slow transport method of "synchronization" is presented in section 9 and shown to be equivalent to Einstein's procedure. Finally, section 10 summarizes our main findings.

2. The weak statement of special relativity postulates

We stand for a "weak" form of the postulates, in which the number of assumptions is kept to a minimum and no additional restrictions to those required by experiment are

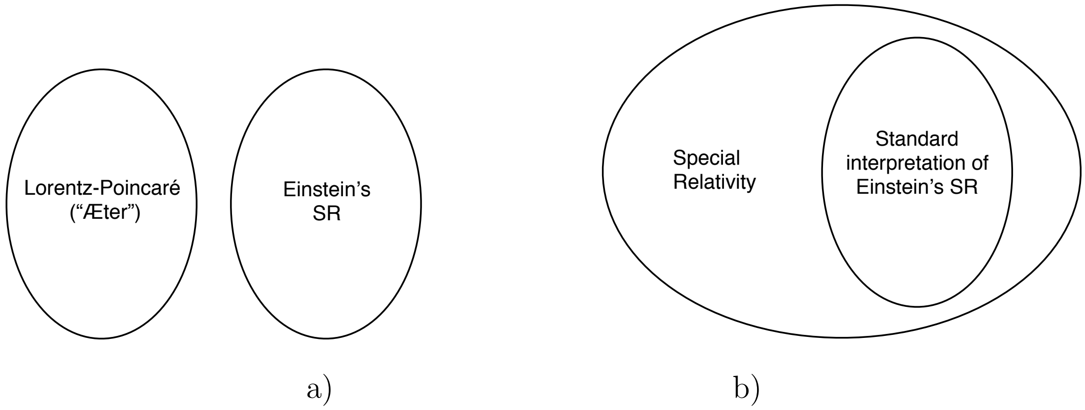

Figure 1. a) Usual view: Lorentz-Poincaré's and Einstein's philosophies seen as irreconciliable; b) Current formulation: Lorentz-Poincaré's and Einstein's views focus different aspects of the same theory.

imposed. In this view, we start, exactly as it has been done by Einstein in his 1905 paper [7], with the definition of the "rest system." Einstein defined it as "a system of co-ordinates in which the equations of Newtonian mechanics hold good (*i.e.* to first approximation)." We define it as a system in which the *one-way* speed of light in empty space is c in any direction, independently of the velocity of the source emitting the light. As the name indicates, the one-way speed of light is the speed of light in a path in just one direction.

One may argue that it may be impossible to know which is the rest system. The answer to this remark is somewhat disconcerting, as the issue is of no relevance for the point we are trying to make. If we accept that when a photon travels between two points in space, it does so with a certain speed, regardless of our knowledge of its value, then there is no conceptual difficulty at this stage, although there may exist a practical one. How to deal with this impossibility has been already discussed [2, 4] and the question is readdressed in section 7.

The postulate of the constancy of the speed of light is then stated as follows [4]:

- the *two-way* speed of light in empty space is c in any inertial frame, independently of the velocity of the source emitting the light.

Here, an inertial frame is any frame moving at constant velocity in relation to the rest system, and the two-way speed of light is its average speed on a round-trip. It is worth to emphasize that what one learns from the Michelson-Morley experiment is the constancy of the two-way speed of light in vacuum and no information can be obtained regarding its one-way value. The definition of the rest system is important to give a starting reference point to the theory. In addition, it is interesting to note that the existence of the rest system can be deduced from the constancy of the two-way speed of light and the assumption of homogeneity of space [23].

No further assumptions in this postulate are required to develop special relativity, in particular no claims have to be made regarding the detectability and/or uniqueness of the rest system nor to the value of the one-way speed of light in inertial frames. However, although for operational reasons any inertial frame can be treated *as if* it were the rest system, the rest system is indeed unique [2,4], as shown in section 4. How this affirmation is compatible with Einstein's view is clarified in section 6. In passing, let us refer that the constancy of the two-way speed of light can also be derived from a "conceptualization of time" grounded on very fundamental assumptions [1].

A general formulation of the principle of relativity is more subtle than it may look at first sight. We have made a comprehensive analysis of the principle of relativity in [4], where we defend its introduction on a late stage of the presentation of the theory and suggest a formulation close to the one proposed in Feynman's "Lectures on Physics" [24]:

- all the experiments performed in a closed cabin in any moving inertial frame will appear the same as if performed in the rest system, provided, of course, that one does not look outside.

The importance of not "looking outside" is stressed by Feynman and was carefully discussed before [4]. One critical implication of not looking outside is the need to perform an *internal synchronization* of clocks (see section 3), such as the one proposed by Einstein [7]. With Einstein's "synchronization" procedure the space-time *coordinates* of events in different frames become related by the Lorentz transformation, so that an alternative way to express the principle of relativity is "all laws of physics, when written with Lorentzian coordinates, keep the same form in all inertial frames, the same as in the rest system" [4]. The privileged role of the Lorentz transformation in special relativity was examined in our former publications [2, 4] and further appreciation is made in section 7.

3. Synchronization and simultaneity

Synchronization of distant clocks is a key issue in special relativity, debated since the very beginning of the theory and continuing nowadays [2,7–19,25]. One important trend in the discussion of this topic is the so-called "conventionality of simultaneity thesis." A good overview of this thesis was presented by Marco Mamone Capria [16] and was quickly reviewed in [4].

In this work we follow our former analysis [2], based to a big extent on the work of Mansouri and Sexl [10]. We start with the synchronization of clocks in the rest system. In this system there is no need for any "stipulation" nor any element of convention. Since the value of the one-way speed of light is known in this frame and it is the same in the to-and-fro paths, we can use Einstein's procedure: a photon is emitted from a point A at time t_A^0 measured by a punctual clock in point A, it is reflected at a point B at a time t_B to be defined, and arrives back at point A at time t_A^1; the time t_B of a punctual clock in point B is set as $t_B = (t_A^0 + t_A^1)/2$. All clocks in the rest system can be synchronized in this way. Alternatively, if the distance between points A and B is L, $t_B = t_A^0 + L/c$.

We can now try to synchronize clocks in any inertial frame. The simplest method is to perform an "external synchronization" [2,4,10], in which we "look outside" and the synchronized clocks from the rest system are used as a reference. For instance, observers in the moving frame set their clocks to 0 when they fly past by a clock in the rest system marking 0 as well. We denote clocks synchronized in this way by *synchronized clocks* and their time readings by *synchronized times* or simply "times." An alternative way is to perform an "internal synchronization" [2, 4, 10], where the moving observers do not look outside and use the same procedure employed in the rest system. This is of course Einstein's synchronization. We call clocks "synchronized" with this scheme by *Lorentzian clocks*, as their time readings, that we name *Lorentzian times*, are related by the Lorentz transformation. In this case the moving observers have proceeded *as if* they were in the rest system: despite not knowing the value of the one-way speed of light in their frame, they proceed as if it were c. A Lorentzian clock has a constant offset of $-vx'/c^2$ in relation to a co-punctual synchronized clock [2, 4].

One very intuitive idea to set the initial adjustment of distant clocks is the *slow transport method* of clock "synchronization." It consists of setting all the clocks at the same location and then to move them slowly until they reach their final positions. Hence, the slow transport procedure does not involve "looking outside." As anticipated in [4], this method is equivalent to the "internal synchronization" scheme [10, 26–30], where we proceed as if the value of the one-way speed of light were c, in accordance with the current formulation of the principle of relativity. Hence, slow transport can be used to synchronize clocks in the rest system, but we must be fully aware it leads to Lorentzian clocks if used in a moving inertial frame. The equivalence between slow transport and internal synchronization is deduced from the present formulation of special relativity in section 9.

A first step towards the clarification on how to reconcile Bell's two philosophies can already be made. It consists in having both synchronized and Lorentzian clocks in one inertial frame. An example of this configuration is shown in figure 2. In this figure S and S' denote the rest system and the inertial frame, respectively, t is the time in S, t' and t'_L are the time readings of synchronized and Lorentzian clocks in S', respectively, the speed of S' is $v = 0.6c$, and the distances $\overline{AB}$ and $\overline{BC}$ are the same and equal to 1800 km (in S). Clocks D and E are co-punctual with clocks B and A, respectively, at $t = 0$. We will consider similar setups repeatedly.

Figure 2 acutely exposes the difficulties with the concept of *simultaneity.* Are two events, both occurring at $t = 0$, one at point A and the other at point B, simultaneous in the moving frame as well? The difficulty is that while the synchronized clocks display the same time readings for the two events, the Lorentzian clocks display different ones, so that the comparison of clock time readings by itself does not provide an answer. However, on the one hand, the Lorentzian clocks have been adjusted with one additional assumption, namely, a stipulated value for the one-way speed of light in the moving frame used for operational purposes. On the other hand, a photon emitted from an observer in the inertial frame S' propagates in the rest system with a one-way speed of light c,

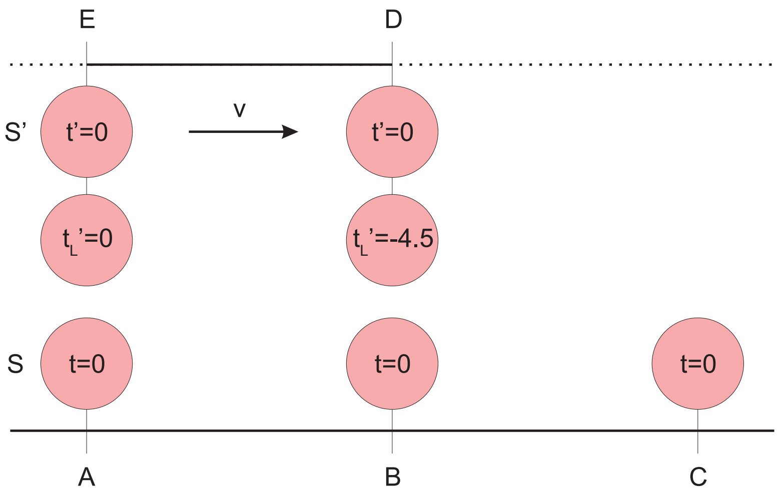

Figure 2. Lorentzian (t'_L) and synchronized (t') clocks in an inertial frame S', obtained from internal and external "synchronization" procedures, respectively. $\overline{AB} = \overline{BC} =$ 1800 km; $v = 0.6c$.

exactly the same of a photon emitted by an observer in S, so that the synchronization of clocks in S is reliable, even if it is made from the observers in S'. Thus, within the weak formulation of the postulate of the constancy of the two-way speed of light, we define simultaneity from the comparison of the time readings of synchronized clocks. According to this definition, the answer is "yes."

In any case, whatever definition of "simultaneity" is adopted, physical reality is not changed. What is modified is merely the way to *describe* it. In fact, the observers in S' can describe any physical phenomenon using their synchronized and/or their Lorentzian clocks. Physical phenomena are independent on the way one chooses to set his own clocks and are precisely the same regardless of the description adopted. The picture 1a) of a conflict of philosophies corresponds to a big extent to implicitly consider that physical reality is indeed modified when we have just changed its description (*cf.* as well section 5). The picture 1b) of a full compatibility corresponds to noting that we can even adopt two definitions of "simultaneity," as long as we distinguish between *simultaneity* (or synchronized simultaneity), from *Einstein simultaneity*, as given, respectively, by the comparison of the time readings of synchronized and Lorentzian clocks.

Finally, it is consensual that a hypothetical signal with infinite speed would solve the problem of assigning physical meaning to any synchronization scheme once for all, as no element of convention would ever be needed in this case, in any frame. It is interesting to note that if such signal would be emitted from clock A at $t = 0$, then it would set all other clocks in figure 2 (B, C, D and E) exactly in the same way as the synchronized clocks. This is not a definitive argument, as infinite speed signals may be non-existing in nature. Nevertheless, the basic notion of synchronization is not affected by this observation and it does not prevent the use of infinite speed signals in a *Gedankenexperiment* like this one.

4. The IST transformation

The IST transformation (Inertial [15, 31, 32]–Synchronized [2, 4, 23, 29, 33]–Tangherlini [34]) has been first proposed by Tangherlini [34] and used by various other authors [10,35,36]. It emerges naturally when the external synchronization procedure delineated in the previous section is used. Its derivation is quite straightforward and has been outlined in [2]. In the usual configuration where the axis of the rest frame S and a moving frame $S'$are aligned, the origin of S' moves along the x-axis of S with speed v in the positive direction, and the reference event is the overlapping of the origins of both frames at time zero, the IST transformation is given by [2]

$$\begin{aligned} x' &= \gamma(x - vt) \\ t' &= \frac{t}{\gamma} \ , \end{aligned} \tag{1}$$

with

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \ . \tag{2}$$

The IST transformation is not symmetrical, as the inverse transformation, expressing x' and t' as functions of x and t, is given by,

$$\begin{aligned} x &= \frac{1}{\gamma}(x' + \gamma^2 v t') \\ t &= \gamma t' \ . \end{aligned} \tag{3}$$

Notice that the position of the origin of S, $x = 0$, is given in S' by $x' = -\gamma^2 v t'$. This means that S' sees S passing with speed $v' = -\gamma^2 v$, and not just $-v$ as one could think at first sight. One factor γ accounts for the fact that rulers are shorter in S', while the second γ factor comes from the fact that clocks run slower there [5]. Note as well that we can have $|v'| > c$, which may look surprising. However, c is a limit speed *in the rest system* [1], *i.e.*, no object can travel at a higher speed *in S*. The corresponding limit speeds in S' are obtained in section 6 [*cf.* equations (11) and (12)]. They are different for objects moving in the positive and negative x-directions, and one of them is larger than c. Besides, the notion of "Einstein speed" is introduced in the same section, where it is further shown that its value in S' is limited to c, in accordance with the standard formulation of special relativity.

When the speeds involved are low, γ is very close to one. In this case, the transformation of coordinates between the rest and the moving frame reduce to

$$\begin{aligned} x' &\simeq x - vt \\ t' &\simeq t \end{aligned} \tag{4}$$

Galileo's transformation, as it should be.

If a second reference frame, S'', is moving with speed w in the rest system, then it is not difficult to show that the quantities in S' and S'' relate through

$$x' = \frac{\gamma_v}{\gamma_w}\left[x'' - {\gamma_w}^2(v-w)t''\right]$$
$$t' = \frac{\gamma_w}{\gamma_v}t'' \; , \tag{5}$$

where γ_v and γ_w are the γ factors associated with speeds v and w, respectively. Of course that the inverse relations are simply obtained by interchanging the roles of w and v and the $'$ quantities with $''$ ones. Moreover, when $w = v$ the identity transformation is obtained.

Finally, the Lorentz transformation is readily obtained from the IST transformation (1) by introducing the offset factor mentioned in section 3,

$$t'_L = t' - \frac{v}{c^2}x' \tag{6}$$

and substituting t' in (1) [2, 4]. Additional remarks about the Lorentz transformation are made in section 7.

5. Clock rhythms and time readings

One simple – and yet critical – issue that has to be clarified before engaging any discussion on the interpretation of special relativity or of special relativity results is to state the difference between clock *rhythms* (or tick rates) and clock time readings (or time coordinates). Contrary to time readings, clock rhythms do *not* depend on any particular form of "synchronization." Surprisingly, the failure to make this basic distinction is at the origin of several misunderstandings surrounding the theory, including, *e.g.*, the discussion of the twin paradox, to be adressed in a subsequent publication.

Let us consider the time-evolution of the situation depicted in figure 2 and check how does it look like at $t = 10$ ms. The result is shown in figure 3. As it can be verified, *all moving clocks* D and E, both synchronized and Lorentzian, *tick at the same rate*: for each of them 8 milliseconds have passed from figure 2 to figure 3 (for instance, for the Lorentzian clock D 3.5-(-4.5)=8 ms have passed, the same as for the synchronized clock D). Thus, the clock rhythms are independent of the adopted "synchronization." In turn, 10 ms have passed for each of the clocks at rest A, B and C. All moving clocks, both synchronized and Lorentzian, tick slower, in this case by a factor of $10/8 = 1.25$, than the clocks at rest. Reversely, all clocks at rest tick faster than the moving clocks. Time dilation is related to the clock rhythms and, as expected, does not depend on the initial adjustment or "synchronization" that is made to the moving clocks.

The principle of relativity and, in particular, "relativity of time dilation," can also be addressed with the help of figures 2 and 3. If the observers in S' decide not to look outside and to use only their Lorentzian clocks, then, according to the principle of relativity, their description of the phenomenon has to be the same *as if* they were

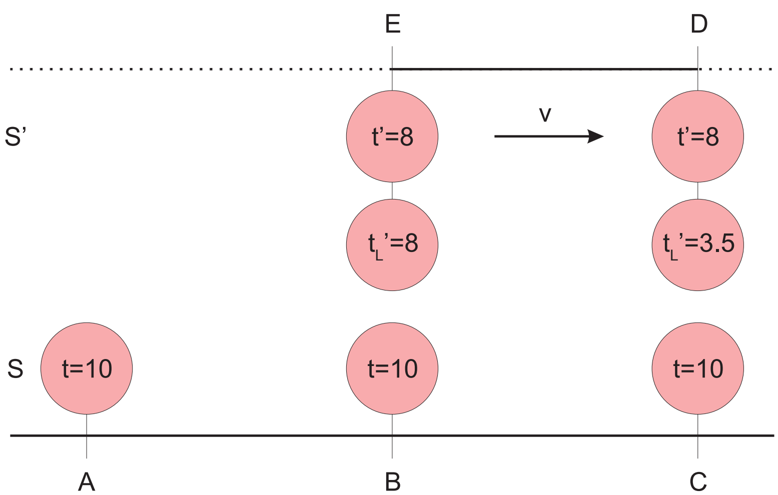

Figure 3. Evolution of the situation represented in figure 2, with Lorentzian (t'_L) and synchronized (t') clocks in an inertial frame S'. $\overline{AB} = \overline{BC} = 1800$ km; $v = 0.6c$.

at rest. This is just standard special relativity, but still let us verify explicitly how it works in this example. An observer with the Lorentzian clock D says he has seen a clock B from S just in front of him when this clock B was reading $t = 0$ while his clock D was marking $t'_L = -4.5$ ms (figure 2). Later on, an observer co-punctual with clock E sees the same clock B showing $t = 10$ ms, while his own clock E exhibits $t'_L = 8$ ms (figure 3). The observers from S' could then (erroneously) conclude that while for them, $8 - (-4.5) = 12.5$ ms have passed, in the "moving" frame S only $10 - 0 = 10$ ms have passed. Therefore, they could consider themselves "at rest" and the "moving" clocks from S would appear to run slower. In this case, by the same factor as before, $12.5/10 = 1.25$, as it had to be. As long as one does not look outside, there is a symmetry in the *description* of physical phenomena. However, this "relative time dilation" has nothing to do with the clocks rhythms and it is merely a result of comparing *time coordinates* using Lorentzian clocks. In fact, if the observers in S' perform the same comparison of time readings using their synchronized clocks instead of the Lorentzian ones, they arrive at the opposite (and correct) conclusion, namely, that their clocks are running slower.

The example above illustrates quite clearly the apparent "conflict of philosophies" and the claim that there is only one theory. Similarly to the discussion of simultaneity in section 3, when we set apart the symbols t' and t'_L, related by (6), and denote them by different words, distinguishing "time" (or "synchronized time") from "Lorentzian time," it becomes evident there is no formal incompatibility between the assertions of both "philosophies" and that the main problem lays on the language (in this case the use of the word "time" to denote two different concepts, in section 3 the use of the word "simultaneity" to denote two different concepts).

One important remark is the following. When observers in two inertial frames cross, if it is not possible to look outside and to perform some kind of external synchronization or, equivalently, to measure the one-way speed of light, then we *do not know* in which frame the clocks are actually running slower [*cf.* as well equation (26) in section 8]. This is an alternative way of pointing out the indeterminacy of special relativity thoroughly debated in [4]. We do know, however, that the description made with the Lorentzian clocks between one particular inertial frame and any other is the same as between the rest system and any inertial frame. This is the dangerous beauty of the study of relative motion.

6. Velocity addition and Einstein speed

The velocity addition formula can be obtained from the IST transformation without difficulty. As before, let S denote the rest system and S' an inertial frame moving with speed v. If an object is moving at speed w in S, then its speed in the inertial frame, w'_v, is simply

$$x' = w'_v t' \ . \tag{7}$$

Using the synchronized transformation (1) to substitute x' and t',

$$\gamma_v(x - vt) = w'_v \frac{t}{\gamma_v} \ , \tag{8}$$

where γ_v is the γ factor associated with speed v, given by (2). Rearranging the different terms, $x = \left(v + \frac{w'_v}{\gamma_v{}^2}\right) t$, and since, by definition, $x = wt$, one gets $w = v + \frac{w'_v}{\gamma_v{}^2}$, which can be written in the form

$$w'_v = \gamma_v{}^2(w - v) = \frac{w - v}{1 - \frac{v^2}{c^2}} \ . \tag{9}$$

This is the final form of the velocity addition expression. Keep in mind that the speed v''_w of an object moving with speed v, in a frame S'' moving with speed w, is different from w'_v (where v and w are the speeds in S). In fact,

$$v''_w = \gamma_w{}^2(v - w) = \frac{v - w}{1 - \frac{w^2}{c^2}} = -\left(\frac{\gamma_w}{\gamma_v}\right)^2 w'_v \ . \tag{10}$$

It may look surprising at first sight, but it has been seen already that the rest system is seen from the moving frame S' passing with speed $-\gamma^2 v$ and not merely $-v$.

It is now possible to calculate the one-way speed of light in any inertial frame. If a photon is emitted in the positive direction of the x-axis, the one-way speed of this photon in S' is given by (9) with $w = c$,

$$c_v{}^+ = \gamma_v{}^2(c - v) \ . \tag{11}$$

If the photon is emitted in the negative direction of the x-axis, $w = -c$ its speed in S' is given by

$$c_v{}^- = \gamma_v{}^2 (c + v) \ . \tag{12}$$

Thus, in the moving frame the one-way speed of light is not the same in different directions, although the two-way speed of light remains equal to c. Moreover, it is clear that the one-way speed of light is isotropic only in the frame corresponding to $v = 0$. In addition, when the speeds involved are low, $\gamma_v \simeq 1$ and, as expected

$$\begin{aligned} c_v{}^+ &\simeq c - v \\ c_v{}^- &\simeq c + v \ . \end{aligned} \tag{13}$$

As it is well known, this classical limit is *not* obtained from the velocity addition formula associated with the Lorentz transformation [*cf.* equation (17)].

In the same way as it is vital to distinguish clock rhythms from clock time readings, it is essential to distinguish *speed* (or "synchronized speed") from *Einstein speed* [2,4,5]. Speed is defined as in the previous equations and can be calculated from $w'_v = \Delta x'/\Delta t'$. In turn, the Einstein speed is defined from

$$w'_E = \frac{\Delta x'}{\Delta t'_L} \ , \tag{14}$$

i.e., its value is calculated with the difference of the *time readings* of Lorentizan clocks. Substituting relation (6) into (7), one obtains, $x' = w'_v t' = w'_v \left(t'_L + \frac{v}{c^2} x' \right)$, from where, $x' \left(1 - \frac{v w'_v}{c^2} \right) = w'_v t'_L$ and $x' = \frac{w'_v}{1 - \frac{v w'_v}{c^2}} t'_L$. In this way, the Einstein velocity w'_E, measured in a frame moving with speed v (in S) of an object which has speed w, is

$$w'_E = \frac{w'_v}{1 - \frac{v w'_v}{c^2}} \ . \tag{15}$$

This last expression can be rewritten replacing w'_v using (9),

$$w'_E = \frac{w - v}{1 - \frac{v w}{c^2}} \ . \tag{16}$$

The Einstein speed of light, c'_E, exhibits a very interesting property. As a matter of fact, since the speed of light in the rest system is always c, c'_E is obtained directly from (16) with $w = c$:

$$c'_E = \frac{c - v}{1 - \frac{v}{c}} = c \ . \tag{17}$$

Therefore, *the one-way Einstein speed of light is always c in any moving inertial frame, independently of the speed of the moving frame.* The cycle is thus closed and, reversely, we can confirm that if we "synchronize" moving clocks with a value c for the one-way "speed" of light, then we are using "Einstein speeds" and will get "Lorentzian clocks." An illustration with the explicit calculation of both the (synchronized) speed of light and the Einstein speed of light in precisely the same conditions was given in [2].

The procedure of "synchronization" using the average two-way value of the speed of light as its one-way speed is similar to the "formula 1 car synchronization of clocks" described in [4], where the average speed of a F_1 car along a racing track is used to "synchronize" clocks along the track. It is odd that although this procedure is easily recognized to be false in the case of the F_1 car, assuming that we are interested in a true synchronization and not simply in one synchronization procedure to be used for operational purposes, somehow it is accepted with very little criticism in the case of the speed of light.

7. A word on the Lorentz transformation

As the Lorentz transformation can be deduced from the IST transformation just with the change of coordinates (6) [2], by the very construction of the theory we can ensure that *all* the results described by the Lorentz transformation, for instance the muon decay, can also be described by the IST transformation. From the mathematical point of view, this statement corresponds to saying that physical laws cannot depend on the system of coordinates chosen. In even simpler terms, it is once more the observation that physical laws do not depend on the way the observers set their clocks.

The Lorentz transformation is symmetrical and lacks any explicit reference to the "external" rest system. In particular, the transformation of Lorentzian coordinates between two inertial frames, S' and S'', when written with Einstein speeds, *takes the same form* as between the rest system and a moving inertial frame,

$$\begin{aligned} x' &= \gamma_E(x'' + v_E t''_L) \\ t'_L &= \gamma_E\left(t''_L + \frac{v_E}{c^2}x''\right) \ , \end{aligned} \tag{18}$$

with γ_E given by $\gamma_E = 1/\sqrt{1-\frac{v_E{}^2}{c^2}}$ and v_E is the *relative* Einstein speed between both moving frames given by equation (16). The velocity addition expression (16) remains valid when w and v are themselves Einstein speeds in relation to any other inertial frame, playing in practice the role of the rest system. The explicit deduction of expressions (18) from the offset factors (6) is rather easy [23], albeit long and uninteresting.

These remarks show that for operational purposes all moving inertial frames are somewhat "equivalent" to the rest system. Physics can then be described only according to the *perception* of relative motion, without any reference to absolute motion. In this sense, the rest system is superfluous, as it is the knowledge of the value of the one-way speed of light [4]. But the group property of the Lorentz transformation masks the underlying assumptions and the very starting points. It evades the indeterminacy of special relativity, but does not solve it [4]. It is worth noting that a similar conclusion has been drawn by Zbigniew Oziewicz, who demonstrated, with a completely different approach, that the Lorentz transformation has implicit a reference to an "external" preferred reference system [37].

If the rest system is experimentally inaccessible, then we cannot remove the indeterminacy in the theory and may have no other option than to proceed through internal schemes and based only on the perception of relative motion. But the wider perspective gained from the weak formulation of both postulates proposed here reveals that the "equivalence" among inertial frames associated with the Lorentz transformation is purely formal. It emerges as a consequence of using Lorentzian clocks and Einstein speeds. What is more, it does not change the meaning of the symbols in the equations nor of the quantities defined.

8. General expression for the rhythms of two clocks in relative motion

Developing the ideas and definitions presented in the previous section, we can now derive the relation between the rhythms of clocks in relative motion. As a corollary, we will obtain in a more formal way one of the main results already discussed, namely, that the indeterminacy of special relativity [4] implies that just with Lorentzian clocks it is not possible to know in which of two inertial frames clocks are actually running slower (see figure 3 and respective discussion).

Consider two inertial frames, S' and S'', moving in the x direction respectively with speeds v and w in the rest system, S. Clock 1 is at the origin of S' and clock 2 is at the origin of S''. The speed of clock 2 in S' is given by (9), the Einstein speed of clock 2 in S' is given by (16). Let us further define the *proper time*, τ, in the usual way, as the time elapsed for one particular observer. Since the proper time is measured by a single clock, it is indeed associated with the clock *rhythm* and does *not* depend on the initial adjustment of distant clocks. The proper time of a clock in S'' relates to the time elapsed in the rest system S by the time equation in (1), which can be written in the form

$$d\tau'' = \frac{dt}{\gamma_w} \; , \tag{19}$$

where dt are the differential times marked by the different clocks in the rest system S that are co-punctual with clock 2 at each instant. The relation of the rhythm of a clock 1 in S' with the rhythms of clocks in S is given by a similar expression, namely

$$d\tau' = \frac{dt}{\gamma_v} \; . \tag{20}$$

Note that as the one-way speed of light in S is known, clocks in S are synchronized without ambiguity, without any element of convention. Therefore, dt can be eliminated and the relation of the proper times of clocks in S'' and in S' is

$$d\tau'' = d\tau' \frac{\gamma_v}{\gamma_w} \; . \tag{21}$$

Equation (21) establishes the relation of clock *rhythms* in two inertial frames. However, one final step is still missing. From (20) and (1) it directly follows

$$d\tau' = dt'. \tag{22}$$

Here, the equality is completely general and is valid whether or not $dx' = 0$, as x' does not appear in the second equation (1). In turn, as a consequence of (6), in general

$$d\tau' = dt' \neq dt'_L \; , \tag{23}$$

although, if $dx' = 0$, then $d\tau' = dt' = dt'_L$. Similarly, we can write $d\tau'' = dt'' \neq dt''_L$, except when $dx'' = 0$. Substituting (22) in (21), we get

$$d\tau'' = dt' \frac{\gamma_v}{\gamma_w} \ . \tag{24}$$

The inverse relation is simply $d\tau' = dt''(\gamma_w/\gamma_v)$. Take note that expression (24) can be deduced immediately from the second equation (5). In fact, the identification $d\tau'' = dt''$ issues straightaway from (5), as the proper time τ'' can be calculated imposing the condition $\Delta x'' = 0$ in the coordinate transformation and, contrary to the second equation (18), x'' does not appear in the second equation (5).

The findings expressed by (23) can yet be obtained, may be more intuitively, from inspection of the configurations depicted in figures 2 and 3. Suppose an observer at the origin of S'', moving with speed $w > v$, is co-punctual with clocks A and E at $t = 0$ (figure 2). In the situation depicted in figure 3 he is then located somewhere between clocks E and D. In order to compare with the proper time in S'', the differential time intervals dt' and dt'_L in S' in equation (23) refer to the time readings of clocks in S' co-punctual with clock 2 from S'' in successive instants, whereas $d\tau'$ is the differential proper time of any observer in S' (*e.g.*, clock 1 at E). Thus, dt' is the differential (synchronized) time in S', and dt'_L is the differential *Lorentzian* time in S'. The conclusion (23) is then a direct outcome of the very construction of the theory, all the remarks in section 5 on the independence of clock rhythms from "synchronization," and the direct analysis of the figures in the situation described.

As it is well-known from standard special relativity – and it is extremely easy to deduce from the Lorentz transformation (18) imposing $dx'' = 0$ – the proper time of clock 2 relates with the differential Lorentzian times through the Einstein speed,

$$d\tau'' = \frac{dt'_L}{\gamma_E} \ . \tag{25}$$

Contrary to (21), this expression corresponds to comparisons of *time readings* of Lorentzian clocks in S' and does *not* correspond to a relation with the clock rhythms in S'. Furthermore, the reverse equation gives account of the symmetric description of time dilation when Lorentzian clocks are used (*cf.* section 5), $d\tau' = dt''_L/\gamma_E$.

By noting that $dx' = w'_E dt'_L$ and $dx' = w'_v dt'$, and using (22), equation (25) can finally be rewritten as

$$d\tau'' = d\tau' \frac{w'_v}{w'_E} \frac{1}{\gamma_E} \ . \tag{26}$$

This is the result we were searching for, confirming that Lorentzian clocks and Einstein speeds are not enough to determine in which frame clocks are running faster, since the relation between rhythms additionally involves the speed w'_v.

The indeterminacy of special relativity [4] has thus been expressed in an alternative way. If the rest system is inaccessible, then we *do not know* the value of the one-way speed of light in one inertial frame, nor can we know the value of w'_v in (26). As an outcome, *we cannot know in which of two intertial frames clocks are ticking slower.*

This is the answer to "The Question" raised by Dingle [38], presented and discussed in [39, 40].

9. Slow transport "synchronization"

Let us finish this paper with the analysis of the "slow transport" method of clock "synchronization." As already pointed out in section 3, this scheme gives the same outcome as Einstein's procedure, in accordance with our weak formulation of the principle of relativity [4]. Indeed, as slow transport is an "internal" procedure, it must be equivalent to any methodology were we operationally proceed in an inertial frame as if the one-way speed of light were c. The result was demonstrated by several authors [10, 26–30], although not all of them have seen its real implication and its connexion with the principle of relativity. It can be obtained in a straightforward way from the present formalism.

We present here the derivation made by Gustavo Homem [29]. As in previous examples, consider two inertial frames, S' and S'', moving with speeds v and w (in S), respectively. We will try to "synchronize" two distant clocks in S', separated by a distance $\Delta x'$, with the help of the moving clock at the origin of S''. The duration of the trip in S' is simply $\Delta t' = \Delta x'/w'_v$. Therefore, using (24) and (9), we have,

$$\begin{aligned} \Delta t' - \Delta\tau'' &= \Delta t' \left(1 - \frac{\gamma_v}{\gamma_w}\right) \\ &= \frac{\Delta x'}{w'_v} \left(1 - \frac{\gamma_v}{\gamma_w}\right) \\ &= \Delta x' \frac{1}{\gamma_v^2 (w - v)} \left(1 - \frac{\gamma_v}{\gamma_w}\right) \\ &= \Delta x' \frac{(1 - v^2/c^2) - \sqrt{1 - v^2/c^2}\sqrt{1 - w^2/c^2}}{w - v} \ . \end{aligned} \tag{27}$$

We are interested in the slow transport limit, *i.e.*, $w \to v$, which can be easily calculated from l'Hôpital's rule,

$$\begin{aligned} \lim_{w\to v} (\Delta t' - \Delta\tau'') &= \lim_{w\to v} \Delta x' \frac{(1 - v^2/c^2) - \sqrt{1 - v^2/c^2}\sqrt{1 - w^2/c^2}}{w - v} \\ &= \lim_{w\to v} \Delta x' \frac{\sqrt{1 - v^2/c^2}}{\sqrt{1 - w^2/c^2}} \frac{w}{c^2} \\ &= \frac{v}{c^2} \Delta x' \ . \end{aligned} \tag{28}$$

Hence, the proper time of the slowly travelling clock at the origin of S'', after covering the distance $\Delta x'$ (in S'), has advanced by

$$\Delta\tau'' = \Delta t' - \frac{v}{c^2} \Delta x' \ . \tag{29}$$

This is precisely the offset factor (6), which proofs that if distant clocks in S' are set with the slow transport method, then they mark Lorentzian times. In other words, a clock that moves very slowly along an inertial frame gets delayed and loses the synchronization from that frame.

The current demonstration demystifies the role of slow transport as a possible alternative and supposedly independent synchronization scheme. Specifically, any experiment designed to measure the one-way speed of light based or involving slow-transport, such as the one proposed in [41], will be measuring the one-way Einstein speed of light (17), which is c.

10. Conclusions

In this article a general formulation of special relativity is proposed, where the postulates are formulated in a weaker form than in the traditional presentation, while keeping fully compatible with all experimental evidence. The starting point is the assumption that there exists a reference frame where the one-way speed of light in vacuum is isotropic and equal to c, denoted by "rest system." No claims are required regarding the possible uniqueness and/or experimental detectability of this frame. The theory is subsequently built from the postulates of the constancy of the two-way speed of light in inertial frames and the principle of relativity.

Synchronization of distant clocks is a central issue in special relativity. Despite its importance, we note that physics does not depend on the way one decides to set his own clocks. Therefore, one observer must be able to use, at the same time, clocks adjusted in different ways, and study physics, in a consistent way, with all of them. This is easily done, as long as he knows how to relate the time readings of clocks adjusted in one particular way with the time readings of clocks adjusted in another way. From the mathematical point of view, each particular initial adjustment of the clocks, often and somewhat misleadingly designated as "synchronization," is associated with one particular choice of system of space-time coordinates to describe physical reality. Consequently, the space-time coordinates associated different clock settings are trivially related by a transformation of coordinates.

Several examples are analyzed following the idea of using simultaneously multiple clock settings, in particular by using "synchronized clocks" and "Lorentzian clocks." Synchronized clocks are set with an "external" reference to the rest system. They are associated with the IST transformation. Lorentzian clocks are set "internally," without any reference to an external system, by proceeding in the inertial frames as if the one-way speed of light were c. They are associated with the Lorentz transformation.

Our approach, based on the IST transformation, reveals that the traditional development of special relativity, grounded on the Lorentz transformation, corresponds to a minimal view of the theory, to an operational procedure to study relative motion without any reference to the rest system. In fact, the IST and the Lorentz transformations only differ from a coordinate transformation and the latter is easily deduced from the former. Therefore, by the very construction of the formalism, the present view immediately encompasses all the results obtained within the standard view, with a clear and direct interpretation.

A critical issue in special relativity is the confusion between clock *time readings* and clock *rhythms*. Clock rhythms do not depend on the initial adjustment of distant clocks, while clock time readings do. Thus, synchronized and Lorentzian clocks display different time readings, but have the same rhythm. Indeed, the time readings of synchronized and Lorentzian clocks in the same position of one inertial frame just differ from a *constant offset* factor, proportional to the distance to the origin of the frame. Similarly, the phenomenon of time dilation is related with clock rhythms and, as such, is independent of the way the clocks are set. However, the *description* of time dilation does depend on the time readings of the various clocks involved and, hence, of the "synchronization" adopted. It is shown that the description of time dilation made with Lorentzian clocks is symmetrical among two inertial observers in relative motion, as it is well-known from the standard interpretation of special relativity. Nevertheless, this reciprocal relation does not relate the clock rhythms. It is further shown that when observers in two inertial frames cross, just with Lorentzian clocks it is impossible to know in each of the inertial frames the clocks are running slower.

The indeterminacy of special relativity [4] may forbid the identification of the rest system and prevent the practical utilization of synchronized clocks. However, such indeterminacy should be taken with humbleness and with the recognition that we simply do not know the value of the one-way speed of light in an inertial frame. This is not a big problem. For operational reasons we can use Einstein speeds and Lorentzian times and do Physics.

The lack of knowledge of the value of the one-way speed of light does not change the *physical meaning* of the different quantities, nor "promotes" any of them to another status. In particular, one should neither confuse synchronized time with Lorentzian time, nor speed with Einstein speed, where the latter is the speed calculated from the comparison of the time readings of Lorentzian clocks. That being so, from the physical point of view one cannot accept an arbitrary stipulation for the value of the one-way speed of light in an inertial frame, but instead acknowledge the tautological assertion "the one-way Einstein speed of light in vacuum is c in any inertial frame," which is an immediate consequence from its very definition. Moreover, nothing is changed and the theory does not collapse whether or not it is eventually possible to identify the rest system, for instance, with a measurement of the one-way speed of light. Recent suggestions to achieve this measurement have been put forward by Consoli and co-workers [42–44].

The slow transport method of clock "synchronization" was analyzed and discussed. This is an "internal" method, in which observers from one inertial frame do not make use of any external reference. According to the weak formulation of the principle o relativity, this procedure must then give the same result as Einstein's "synchronization" and the time readings of clocks adjusted in this way must be Lorentzian times. It is proved this is indeed the case, using the present formalism. As an outcome, no experiment making use of slow transport can ever be used to measure the one-way speed of light, as it is certain it will determine the Einstein one-way speed of light which, by virtue of its own definition, it is known to be c.

The present formulation shows that old conflicts, such as the ones opposing the Lorentz-Poincaré and Einstein's ideas, reside more on the language adopted than on the calculated quantities. The broader view of special relativity herein developed reveals that these seemingly irreconcilable ideas refer to different aspects of one and the same general theory. Despite raising hot debates, the apparent conflicts are clearly solved by making the distinction between clock rhythms and clock time readings and by accepting there are quantities we may be unable to know. The implications of the current approach in the interpretation of the twin paradox will be discussed in an ensuing paper.

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