Derivation of the Pauli Exclusion Principle

Sylwester Kornowski

Abstract: In generally, the Pauli Exclusion Principle follows from the spectroscopy whereas its origin is not good understood. To understand fully this principle, most important is origin of quantization of the azimuthal quantum number i.e. the angular momentum quantum number. Here, on the base of the theory of ellipse and starting from very simple physical condition, I quantized the azimuthal quantum number. The presented model leads directly to the eigenvalue of the square of angular momentum and to the additional potential energy that appears in the equation for the modified wave function.

1. Introduction

The Pauli Exclusion Principle says that no two identical half-integer-spin fermions may occupy the same quantum state simultaneously. For example, no two electrons in an atom can have the same four quantum numbers. They are the principal quantum number n that denotes the number of the de Broglie-wave lengths λ in a quantum state, the azimuthal quantum number l (i.e. the angular momentum quantum number), the magnetic quantum number m and the spin s.

On the base of the spectrums of atoms, placed in magnetic field as well, follows that the quantum numbers take the values:

$n = 1, 2, 3, \ldots.$

$l = 0, 1, 2, \ldots. n - 1$

$m = -l, \ldots. +l$

$s = \pm 1/2.$

The three first quantum numbers n, l, and m are the integer numbers and define a state in which can be maximum two electrons with opposite spins.

The magnetic quantum number m determines the projection of the azimuthal quantum number l on the arbitrary chosen axis. This axis can overlap with a diameter of the circle $l = 0$.

To understand fully the Pauli Exclusion Principle we must answer following questions concerning the azimuthal quantum number l:

1.
What is physical meaning of this quantum number?
2.
Why the l numbers are the natural numbers only?
3.
Why the zero is the lower limit?
4.
Why the $n - 1$ is the upper limit?

To answer these questions we must apply the theory of ellipse, especially the formula for its circumference C and eccentricity e. When we use the complete elliptic integral of the second kind and the Carlson symmetric form [1], we obtain for circumference C of an ellipse following formula

$$C = 2\pi a[1 - (1/2)^2 e^2/1 - (1\cdot 3/(2\cdot 4))^2 e^4/3 - (1\cdot 3\cdot 5/(2\cdot 4\cdot 6))^2 e^6/5 - \ldots], \quad (1)$$

where a is the major radius and e is the eccentricity defined as follows

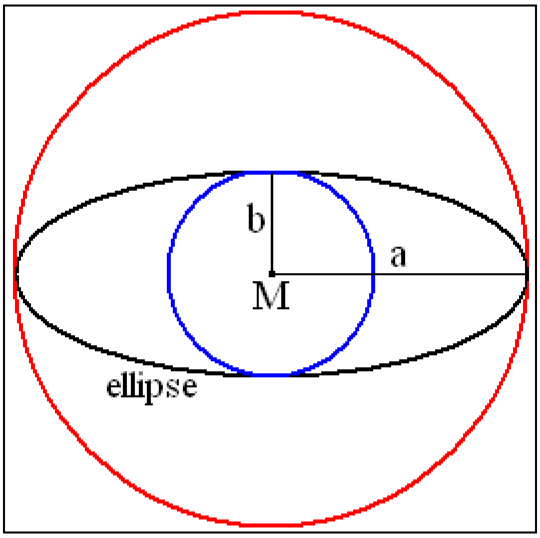
$$e = [\mathrm{sqrt}(a^2 - b^2)]/a, \quad (2)$$

where b is the minor radius.

2. Calculations

2.1 Angular momentum quantum number

In the figure, the circumference of the ellipse $C_{de\text{-}Broglie}$ is $C_{de\text{-}Broglie} = n\lambda = 2\pi n\lambda\!\!\!^-$, where the n is the principal quantum number whereas the $\lambda\!\!\!^-$ is the reduced de Broglie-wave length. Assume that there are allowed only ellipses that circumference is the arithmetic mean of the circumferences of two circles that radii are equal to the major and minor radii of the ellipse.



Similarly as for the circumference of the ellipse, the circumferences of the circles must be equal to a natural number multiplied by the de Broglie-wave length. This leads to following definitions

$$a = j\lambda\!\!\!^- \text{ and } b = k\lambda\!\!\!^-. \quad (3)$$

Notice that j = k = 0 has no sense.

Then, we can rewrite formula (2) as follows

$$e = [\mathrm{sqrt}(j^2 - k^2)]/j. \quad (4)$$

It is the natural assumption that the allowed circumferences of the ellipse should be the arithmetic mean of the sum of the circumferences of the two circles. It leads to following conclusion

$$(j + k)/2 = n. \quad (5)$$

Define some number l as follows

$$(j - k)/2 = l. \quad (6)$$

Formulae (5) and (6) lead to following relations

$$j = n + l, \quad (7)$$

$$k = n - l. \quad (8)$$

Since the j, k and n are the integers so the number l must be an integer as well.

On the base of formulae (7) and (8) we can rewrite formula (4) as follows

$$e = 2[\mathrm{sqrt}(nl)]/(n + l). \quad (9)$$

We can see that due to the square root, this formula has no real sense for $l < 0$. Since the l cannot be negative then from formulae (5) and (6) follows that $l < n$.

On the base of formulae (3) and (7), we can rewrite formula (1) as follows

$$C_K = 2\pi(n + l)\bar{\lambda}[1 - (1/2)^2e^2/1 - (1\cdot3/(2\cdot4))^2e^4/3 - (1\cdot3\cdot5/(2\cdot4\cdot6))^2e^6/5 - \ldots]. \quad (10)$$

Notice that for $n = l$ is e = 1 and then $C_{de\text{-}Broglie} > C_K$ i.e. l cannot be equal to n. For $l = 0$ is $C_{de\text{-}Broglie} = C_K$ and because l cannot be negative then the $l = 0$ is the lower limit for l.

Some recapitulation is as follows. We proved that the azimuthal quantum number l

1) is associated with transitions between the states j and k,
2) is the integer,
3) cannot be negative and the lower limit is zero,
4) the $n - 1$ is the upper limit.

Some abbreviation of it is as follows

$l = 0, 1, 2, \ldots n - 1$.

The Quantum Physics is timeless because a quantum particle disappears in one region of a field or spacetime and appears in another, and so on. There are no trajectories of individual quantum particles. Quantum Physics is about the statistical shapes and their allowed orientations. Such procedure simplifies considerably the Quantum Physics.

2.2 Eigenvalue of the square of angular momentum

An ellipse/electron-state we can resolve into two circles that radii are defined by the semi-axes of the ellipse. The two circles in a pair are entangled due to the exchanges of the binary systems of the closed strings (the entanglons [2]) the Einstein-spacetime components, from which are built up all the Principle-of-Equivalence particles, consist of [2]. Spin of the entanglons is 1 [2] and they are responsible for the infinitesimal transformations that lead to the commutators [3]. Calculate a change in the azimuthal quantum number l when the smaller circle or one of identical two circles emits one entanglon (since in this paper is $j \geq k$ so there is the transition $k \rightarrow k - 1$) whereas the second circle in the pair almost simultaneously absorbs the emitted entanglon (there is the transition $j \rightarrow j + 1$). Such transition causes that ratio of the major radius to the minor radius of the ellipse (or circle) increases. From formula (5) follows that such emission-absorption does not change the principal quantum number n whereas from formula (6) follows that there is following transition for the azimuthal quantum number l: $l \rightarrow l + 1$. The geometric mean is sqrt($l(l + 1)$) and this expression multiplied by $\hbar$ is the mean angular momentum L for the described transition. This leads to conclusion that eigenvalue of the square of angular momentum $\mathbf{L}^2$ is $l(l + 1)\hbar^2$.

The eigenvalue of the square of angular momentum leads to the additional potential energy E_A (it follows from the radial transitions i.e. from the changes in shape of the ellipses) equal to

$$E_A = L^2/(2mr^2) = l(l + 1)\hbar^2/(2mr^2). \quad (11)$$

The energy E_A appears in the equation for the modified wave function.

The theory of baryons [2] shows that inside the baryons are only the $l = 0$ states (i.e. there are only the circles) so the quantum mechanics describing baryons is much simpler than for atoms.

3. Summary

In generally, the Pauli Exclusion Principle follows from the spectroscopy whereas its origin is not good understood. To understand fully this principle, most important is origin of quantization of the azimuthal quantum number i.e. the angular momentum quantum number. Here, on the base of the theory of ellipse and starting from very simple physical condition, I quantized the azimuthal quantum number. The presented model leads directly to the eigenvalue of the square of angular momentum and to the additional potential energy that appears in the equation for the modified wave function.

The Quantum Physics is timeless because a quantum particle disappears in one region of a field or spacetime and appears in another, and so on. There are no trajectories of individual quantum particles. Quantum Physics is about the statistical shapes and their allowed orientations. Such procedure simplifies considerably the Quantum Physics.

References

[1] Carlson, B. C. (1995). "Computation of real or complex elliptic integrals". *Numerical Algorithms* **10**: 13-26. arXiv:math/9409227 .

[2] Sylwester Kornowski (3 December 2012). "The Everlasting Theory and Special Number Theory". http://www.rxiv.org/abs/1203.0021 [v2].

[3] Sylwester Kornowski (10 April 2013). "Quantum Physics in the Lacking Part of Ultimate Theory". http://www.rxiv.org/abs/1304.0043 .