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CHARMONIUM WITH AN EFFECTIVE MORSE MOLECULAR POTENTIAL

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The Morse molecular potential is used for the first time as an effective potential for the overall interaction in charmonium. This procedure allows the calculation of the rotational contributions of P states, the radii of some S states, and an absolute threshold for bound states. The calculation of the latter provides important information on the character of the recently found levels $\chi(3940)$, $\Psi(4040)$, $\chi(4050)$, $\chi(4140)$, $\Psi(4160)$, $Y(4260)$, $Y(4350)$, $\Psi(4415)$, $\chi(4430)$, and $\chi(4660)$.

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1. Introduction

The pioneering work of Eichten *et al.*[1], and many other subsequent works have shown that the static potential plays an important role in the description of heavy mesons. Despite the effort along the last 30 years, some important features of the static interaction are not yet fully understood. And that is why several approximate methods and effective potentials have been proposed for the description of the overall interaction of quarkonia. A partial list of the articles on these two subjects is found in reference [2].

The literature has established that for heavy mesons non-relativistic quantum mechanics with constituent masses for the quark and antiquark can be applied. In this work we use the Morse molecular potential for describing some low energy states of charmonium. This unique description allows to calculate the contribution of the rotational energy of P states, and the radii of some S states. It also permits the calculation of the absolute threshold for the bound states of charmonium that sheds light on the character of the recently found levels[3] $\chi(3940)$, $\Psi(4040)$, $\chi(4050)$, $\chi(4140)$, $\Psi(4160)$, $Y(4260)$, $Y(4350)$, $\Psi(4415)$, $\chi(4430)$, and $\chi(4660)$.

As it is well known, confinement is not well understood and there are models

that do not consider it inside hadrons. For example, the original MIT bag model treats confinement only at the wall by making the vector current null at it[4]. In the case of the chiral bag model[5], confinement is treated by means of the continuity of the axial vector current at the wall. We do not need to worry about confinement because we only deal with low energy levels.

All the experimental data used below for the energies of charmonium states were taken from the Particle Data Group[3]. All energy values below are in MeV unless noted otherwise. This work is based on the the recently published article by de Souza[6].

2. The Morse molecular potential

The Morse potential[7]

$$V(r) = D\left(e^{-2\alpha x} - 2e^{-\alpha x}\right) \tag{1}$$

is widely used in the description of diatomic molecules. In the above equation $-D$ is the minimum of the well, a is the distance where $V = -D$, and $x = (r-a)/a$. The leading term of the expansion for $x \ll 1$ is of the form $-C/r$ and, thus, it is a QCD-like term and its second term takes care of the well-known repulsion of the strong force for small distances[8].

For $|x| < 1$ this potential can be expanded about the minimum up to order 3 in x and produces the expression

$$V(x) = -D + \frac{1}{2}ka^2x^2 - \lambda ka^3x^3 \tag{2}$$

where $\lambda = \alpha/2a$ for the Morse potential.

The solution of the Schrödinger equation yields the expression[7,9]

$$\begin{aligned} E_{\upsilon L} = \hbar\omega\left(\upsilon + \frac{1}{2}\right) - A\left(\upsilon + \frac{1}{2}\right)^2 \\ + B_L L(L+1) - D_L L^2(L+1)^2 \\ - C_{\upsilon L}\left(\upsilon + \frac{1}{2}\right) L(L+1) + \ldots \end{aligned} \tag{3}$$

for the vibrational and rotational levels above the minimum of the potential, where $\upsilon, L = 0, 1, 2, 3, \ldots$. In this equation the first term describes harmonic vibrations, the second term takes into account the anharmonicity of the potential, the third term describes rotations with constant moment of inertia, the fourth term represents the centrifugal distortion and the fifth term represents the coupling between vibration and rotation. This expression can also be written as[9]

$$\begin{aligned} E_{\upsilon L} = \hbar\omega\left(\upsilon + \frac{1}{2}\right) - \frac{(\hbar\omega)^2}{4D}\left(\upsilon + \frac{1}{2}\right)^2 \\ + B_L L(L+1) - D_L L^2(L+1)^2 \\ - C_{\upsilon L}\left(\upsilon + \frac{1}{2}\right) L(L+1) + \ldots \end{aligned} \tag{4}$$

where m is the reduced mass of the constituent quark and antiquark, that is, $mc^2 = \frac{1}{2}M_c c^2$, where M_c is the mass of the c quark. The parameters ω and D are related to α, a and m as

$$\omega^2 = \frac{2\alpha^2 D}{ma^2} \tag{5}$$

The constant B_L is given by $B_L = \hbar^2/2ma^2$.

In the fitting below we disregarded the last term of Eqs. (3) and (4).

3. The fitting

The first two levels, $\eta_c(1S)$ and $J/\Psi(1S)$, are a hyperfine doublet because of the spin-spin interaction, but since our Hamiltonian does not depend on spin, we can use the S states $J/\Psi(1S)$ and $\Psi(2S) \equiv \Psi(3686)$ for the fitting. In the case of $\chi_c(1P)$ states, we should take out the spin-orbit interaction contribution which is given by

$$\Delta E_{SL} = \Delta\left[J(J+1) - L(L+1) - S(S+1)\right] \tag{6}$$

where $J = \left|\vec{J}\right| = \left|\vec{L} + \vec{S}\right|$ in which $S = 1$ for $\chi_c(P)$ states. Applying Eq. (6) to the states $\chi_{c0}(1P)$, $\chi_{c1}(1P)$ and $\chi_{c2}(1P)$ we obtain the average values $\Delta_{1P} = 27.7 \pm 13.6$, $E_{1P} = 3549.7 \pm 37.9$, where E_{1P} is the energy of the degenerate level $\chi_c(1P)$.

The state $\Psi(3770)$, which is assigned as 1^3D_1, is a well established state, experimentally found with an energy of 3778.1 ± 1.2. Table 1 presents a summary of the levels used in the fitting with the corresponding values of υ and L.

Table 1. The levels considered in the fitting of charmonium.

(υ, L)	Particle	Mass (MeV/c^2)
$(0,0)$	$J/\Psi(1S)$	3096.916 ± 0.011
$(0,1)$	$\chi_c(1P)$	3549.7 ± 37.9
$(1,0)$	$\Psi(3686)$	3686.09 ± 0.04
$(0,2)$	$\Psi(3770)$	3778.1 ± 1.2

4. Results and discussion

Fitting the levels of Table 1 to Eq.(3) we obtain the following values for its parameters: $\hbar\omega = 8062.0 \pm 0.1$; $A = 3736.4 \pm 0.1$; $B_L = 282.8 \pm 58.3$; $D_L = 28.3 \pm 5.1$. The constant D_L is a measure of the anharmonicity of the potential. In reference [6] it is reported that for bottomonium $D_L = 25.9 \pm 4.5$ MeV which is quite close to the above value, 25.5 ± 5.1 MeV. Therefore, the Morse effective potentials for both heavy mesons present the same anharmonicity.

From Eqs. (3) and (4) we obtain that $A = (\hbar\omega)^2/4D$, and thus $D = 4348.8 \pm 0.5$. This means that there is no charmonium bound state with energy above 4348 ± 0.5.

This is a very important result and shows that the recently found states $\Psi(4415)$, $\chi(4430)$, and $\chi(4660)$ are not bound states of charmonium. On the other hand this result shows that the recently found states $\Psi(4160)$, $\chi(4140)$, $\chi(4050)$, $\Psi(4040)$, $\chi(3940)$, $Y(4260)$, and $Y(4350)$ can be bound states of charmonium.

Using the values of D and $\hbar\omega$ above in Eq. (5) we obtain $a/\alpha = (13.15 \pm 0.05) \times 10^{-2}$ fm, and from the value of B_L we find that $a = (0.28 \pm 0.05)$ fm which is a very reasonable figure since the Compton wavelength of charmonium is about 0.69 fm if we use a constituent mass of 1.7 GeV/c^2. Of course, we obtain a very similar figure if we use the uncertainty principle. Using the above values of a and a/α we obtain $\alpha = 2.15 \pm 0.39$.

As we showed above, a molecular potential is harmonic about its minimum, and thus we can calculate the value for the constant $k = m\omega^2$ which can be written as $k = mc^2(\hbar\omega)^2/(\hbar c)^2$. Using the above values we obtain $k \approx 1.44 \times 10^3$ GeV/fm^2 $\approx 2.31 \times 10^{23}$ N/m which is a quite fair number. For a distance of 0.3 fm it produces a force $F \approx 10^8$ N. The following simple calculation shows that this is reasonable figure. If we calculate the average force necessary for producing a work of 4 GeV in 0.3 fm we obtain a force of about 10^6 N. As it was shown above the Morse potential,

Table 2. Radii of two states of charmonium calculated with the use of Eq. (7). The numbers in the particle names refer to the quantum number n of QCD models.

(υ, L)	Particle	Radius (fm)
$(0,0)$	$J/\Psi(1S)$, $\eta_c(1S)$	0.35 ± 0.06
$(1,0)$	$\Psi(3686)$, $\eta_c(2S)$	0.49 ± 0.09
$(2,0)$	$\Psi(3S)$	0.63 ± 0.11

when expanded about its minimum, yields Eq. (2). For such a potential Robinett[10] obtained the following equation for the average value of position for S states

$$\begin{aligned} \langle r \rangle_\upsilon &= a + \frac{3\alpha\hbar\omega}{2m\omega^2 a}\left(\upsilon + \frac{1}{2}\right) \\ &= a + \frac{3a\hbar\omega}{4\alpha D}\left(\upsilon + \frac{1}{2}\right) \end{aligned} \tag{7}$$

where we have taken into account that $\lambda = \alpha/2a$. We only calculate the radii of the states $J/\Psi(1S)$, $\eta_c(1S)$, $\Psi(3686)$, $\eta_c(2S)$, and $\Psi(3S)$ because the other upper states are far from equilibrium. Using the above values for the constants, we obtain the results shown in Table 2 for the radii of these states of charmonium.

5. Conclusion

The fitting of some energy levels of charmonium to the Morse potential makes possible the calculation of parameters of the effective molecular potentia, predic-

tion of the radii of two states and sheds some light on the character of the states $\chi(3940)$, $\Psi(4040)$, $\chi(4050)$, $\chi(4140)$, $\Psi(4160)$, $Y(4260)$, $Y(4350)$, $\Psi(4415)$, $\chi(4430)$, and $\chi(4660)$. Therefore, the above result add important information for the understanding of charmonium.

References

1. E. Eichten *et al.*, Phys. Rev. Lett. **34**, 369 (1975).
2. http://inspirehep.net/record/129333/references.
3. J. Beringer *et al.* (Particle Data Group), Phys.Rev. D **86**, 010001 (2012).
4. A. Chodos, R.L. Jaffe, K. Johnson, C.B. Thorn, and V.F. Weiskopf, Phys. Rev. D **9**, 3471 (1974).
5. A. Hosaka and H. Toki, Phys. Reports **277**(2-3), 65 (1996).
6. M. E. de Souza, Nucl. Phys. B Proc.Suppl.**00**, 1 (2012).
7. S. Flügge, *Practical Quantum Mechanics*, Vol. I , pp. 182-189, (Springer-Verlag, New York, 1974).
8. S. Aoki *et al.*(HAL QCD collaboration), Baryon-Baryon Interaction from Lattice QCD, 5th Int. Conf. on Quarks and Nuclear Physics (QNP09), Sept 2009, Beijing, Chinese Physics C, **34**, 1229 (2010).
9. MIT OpenCourseWare, Lecture # 3 Supplement, *Small-Molecule Spectroscopy and Dynamics*, Fall 2008.
10. R. W. Robinett, Am. J. Phys. **65**(3), 190 (1997).