Relations between Distorted and Original Angles in STR

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Using the Oblique-Length Contraction Factor, which is a generalization of Lorentz Contraction Factor, one shows several trigonometric relations between distorted and original angles of a moving object lengthsin the Special Theory of Relativity.

1 Introduction

The lengths at oblique angle to the motion are contracted with the Oblique-Length Contraction Factor $OC(v, \theta)$, defined as [3-4]:

$$OC(v, \theta) = \sqrt{C(v)^2 \cos^2\theta + \sin^2\theta} \qquad (1)$$

where $C(v)$ is just Lorentz Factor:

$$C(v) = \sqrt{1 - \frac{v^2}{c^2}} \in [0, 1]\, for\, v \in [0, c] \qquad (2)$$

Of course

$$0 \le OC(v, \theta) \le 1. \qquad (3)$$

The Oblique-Length Contraction Factor is a generalization of Lorentz Contractor $C(v)$, because: when $\theta = 0$, or the length is moving along the motion direction, then $OC(v, 0) = C(v)$. Similarly

$$OC(v, \pi) = OC(v, 2\pi) = C(v). \qquad (4)$$

Also, if $\theta = \pi/2$, or the length is perpendicular on the motion direction , then $OC(v, \pi/2) = 1$, i.e. no contraction occurs. Similarly $OC(v, \frac{3\pi}{2}) = 1$.

2 Tangential Relations between Distorted Acute Angles vs. Original Acute Angles of a Right Triangle

Let's consider a right triangle with one of its legs along the motion direction (Fig.1).

$$\tan\theta = \frac{\beta}{\gamma} \qquad (5)$$

$$\tan(180^o - \theta) = -\tan\theta = \frac{\beta}{\gamma} \qquad (6)$$

After contraction of the side AB (and consequently contraction of the oblique side BC) one gets (Fig.2):

$$\tan(180^o - \theta') = -\tan\theta' = -\frac{\beta'}{\gamma'} = -\frac{\beta}{\gamma C(v)} \qquad (7)$$

Then:

$$\frac{\tan(180^o - \theta')}{\tan(180^o - \theta)} = \frac{-\frac{\beta}{\gamma C(v)}}{-\frac{\beta}{\gamma}} = \frac{1}{C(v)} \qquad (8)$$

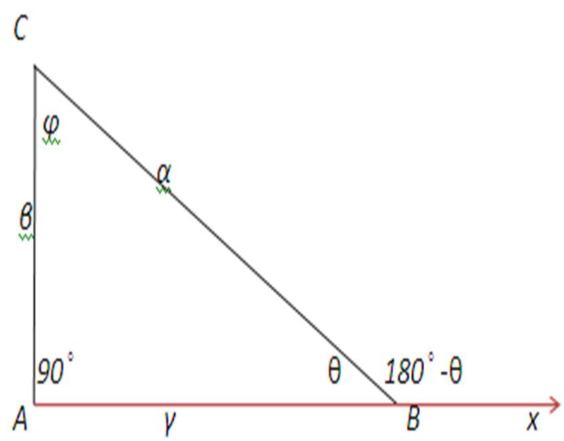

Fig. 1:

Therefore

$$\tan(\pi - \theta') = -\frac{\tan(\pi - \theta)}{C(v)} \qquad (9)$$

and consequently

$$\tan(\theta') = \frac{\tan(\theta)}{C(v)} \qquad (10)$$

or

$$\tan(B') = \frac{\tan(B)}{C(v)} \qquad (11)$$

which is the Angle Distortion Equation, where θ is the angle formed by a side travelling along the motion direction and another side which is oblique on the motion direction.

The angle θ is increased (i.e. $\theta' > \theta$).

$$\tan\varphi = \frac{\gamma}{\beta}\, and\, \tan\varphi' = \frac{\gamma'}{\beta'} = \frac{\gamma C(v)}{\beta} \qquad (12)$$

whence:

$$\frac{\tan\varphi'}{\tan\varphi} = \frac{\frac{\gamma C(v)}{\beta}}{\frac{\gamma}{\beta}} = C(v) \qquad (13)$$

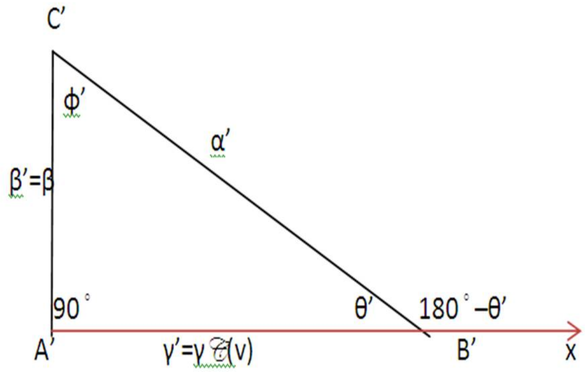

Fig. 2:

So we get the following Angle Distortion Equation:

$$\tan\varphi' = \tan\varphi \cdot C(v) \tag{14}$$

or

$$\tan C' = \tan C \cdot C(v) \tag{15}$$

where φ is the angle formed by one side which is perpendicular on the motion direction and the other one is oblique to the motion direction.

The angle φ is decreased (i.e. $\varphi' < \varphi$). If the traveling right triangle is oriented the opposite way (Fig.3)

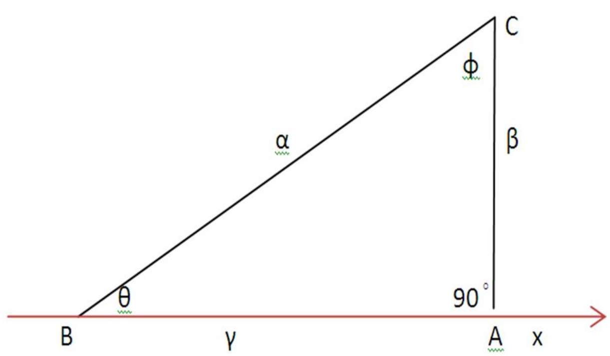

Fig. 3:

$$\tan\theta = \frac{\beta}{\gamma} \; and \; \tan\varphi = \frac{\gamma}{\beta} \tag{16}$$

Similarly, after contraction of side AB (and consequently contraction of the oblique side BC) one gets (Fig.4)

$$\tan\theta' = \frac{\beta'}{\gamma'} = \frac{\beta}{\gamma C(v)} \tag{17}$$

and

$$\tan\varphi' = \frac{\gamma'}{\beta'} = \frac{\gamma C(v)}{\beta} \tag{18}$$

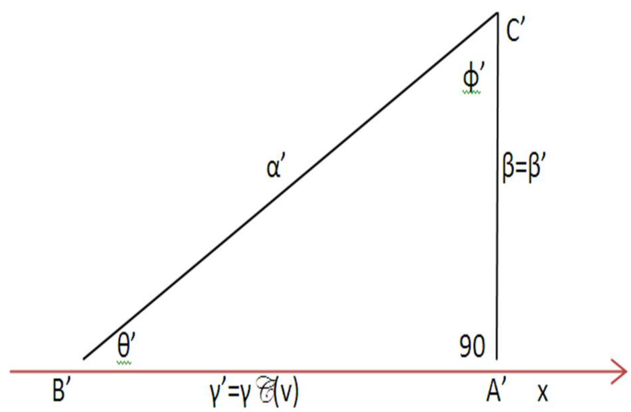

Fig. 4:

$$\frac{\tan\theta'}{\tan\theta} = \frac{\frac{\beta}{\gamma C(v)}}{\frac{\beta}{\gamma}} = \frac{1}{C(v)} \tag{19}$$

or

$$\tan\theta' = \frac{\tan\theta}{C(v)} \tag{20}$$

and similarly

$$\frac{\tan\varphi'}{\tan\varphi} = \frac{\frac{\gamma C(v)}{\beta}}{\frac{\gamma}{\beta}} = C(v) \tag{21}$$

or

$$\tan\varphi' = \tan\varphi \cdot C(v) \tag{22}$$

Therefore one got the same Angle Distortion Equations for a right triangle traveling with one of its legs along the motion direction.

3 Tangential Relations between Distorted Angles vs. Original Angles of A General Triangle

Let's suppose a general triangle ΔABC is travelling at speed v along the side BC as in Fig. 5. The height remains not contracted: $AM \equiv A'M'$.

We can split this figure into two traveling right sub-triangles as in Fig.6:

Similarly we can split this Fig.7 into two traveling right sub-triangles as in Fig.8: In the right triangles $\Delta A'M'B'$ and

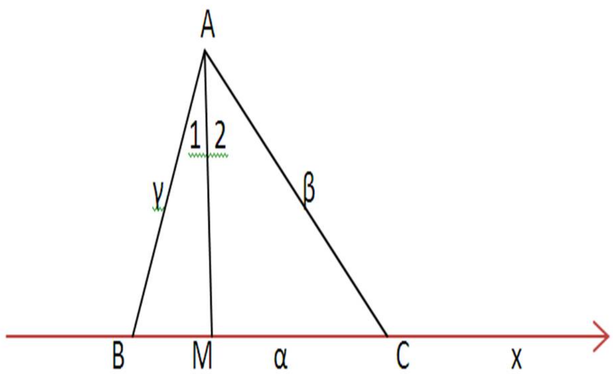

Fig. 5:

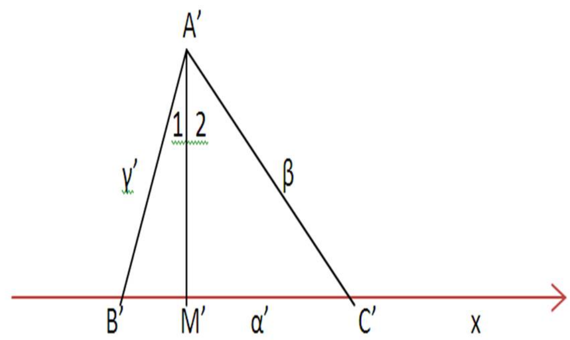

Fig. 7:

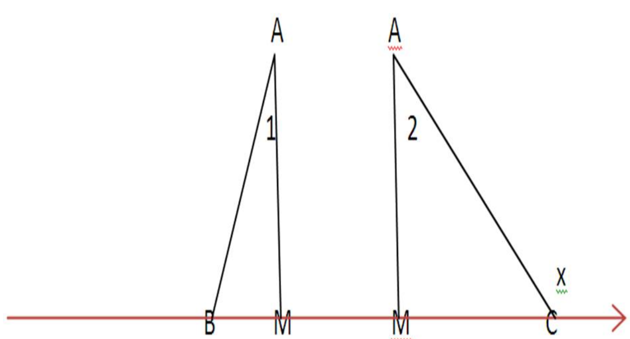

Fig. 6:

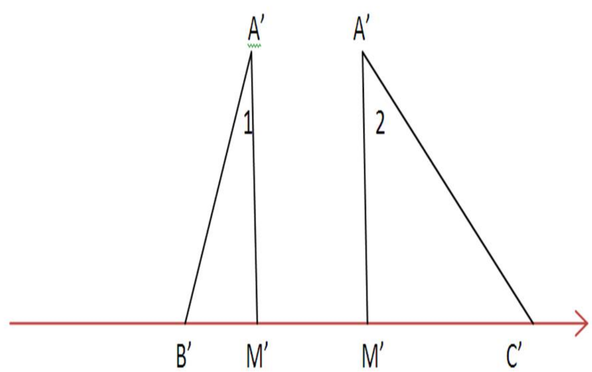

Fig. 8:

respectively $\Delta A'M'C'$ one has

$$\tan B' = \frac{\tan B}{C(v)} \; and \; \tan C' = \frac{\tan C}{C(v)} \tag{23}$$

Also

$$\tan A'_1 = \tan A_1 C(v) \; and \; \tan A'_2 = \tan A_2 C(v) \tag{24}$$

But

$$\tan A' = \tan(A'_1 + A'_2) = \frac{\tan A'_1 + \tan A'_2}{1 - \tan A'_1 \tan A'_2}$$

$$= \frac{\tan A_1 C(v) + \tan A_2 C(v)}{1 - \tan A_1 C(v) \tan A_2 C(v)}$$

$$= C(v) \cdot \frac{\tan A_1 + \tan A_2}{1 - \tan A_1 \tan A_2 C(v)^2}$$

$$= C(v) \cdot \frac{\frac{\tan A_1 + \tan A_2}{1 - \tan A_1 \tan A_2} \cdot (1 - \tan A_1 \tan A_2)}{1 - \tan A_1 \tan A_2 C(v)^2}$$

$$= C(v) \cdot \frac{\tan(A_1 + A_2)}{1} \cdot \frac{1 - \tan A_1 \tan A_2}{1 - \tan A_1 \tan A_2 C(v)^2}$$

$$= C(v) \cdot \tan(A) \cdot \frac{1 - \tan A_1 \tan A_2}{1 - \tan A_1 \tan A_2 C(v)^2} \tag{25}$$

We got

$$\tan A' = \tan(A) \cdot C(v) \cdot \frac{1 - \tan A_1 \tan A_2}{1 - \tan A_1 \tan A_2 C(v)^2} \tag{26}$$

4 Other Relations between the Distorted Angles and the Original Angles

1. Another relation uses the Law of Sine in the triangles ΔABC and respectively $\Delta A'B'C'$:

$$\frac{\alpha}{\sin A} = \frac{\beta}{\sin B} = \frac{\gamma}{\sin C} \tag{27}$$

$$\frac{\alpha'}{\sin A'} = \frac{\beta'}{\sin B'} = \frac{\gamma'}{\sin C'} \tag{28}$$

After substituting

$$\alpha' = \alpha C(v) \tag{29}$$

$$\beta' = \beta \mathscr{O}C(v, C) \tag{30}$$

$$\gamma' = \gamma \mathscr{O}C(v, B) \tag{31}$$

into the second relation one gets:

$$\frac{\alpha C(v)}{\sin A'} = \frac{\beta \mathscr{O}C(v, C)}{\sin B'} = \frac{\gamma \mathscr{O}C(v, B)}{\sin C'} \tag{32}$$

Then we divide term by term the previous equalities:

$$\frac{\frac{\alpha}{\sin A}}{\frac{\alpha C(v)}{\sin A'}} = \frac{\frac{\beta}{\sin B}}{\frac{\beta \mathscr{O}C(v,C)}{\sin B'}} = \frac{\frac{\gamma}{\sin C}}{\frac{\gamma \mathscr{O}C(v,B)}{\sin C'}} \tag{33}$$

whence one has:

$$\frac{\sin A'}{\sin A \cdot C(v)} = \frac{\sin B'}{\sin B \cdot \mathscr{O}C(v, C)} = \frac{\sin C'}{\sin C \cdot \mathscr{O}C(v, B)} \tag{34}$$

2. Another way:

$$A' = 180^o - (B' + C') \; and \; A = 180^o - (B + C) \tag{35}$$

$$\tan A' = \tan[180^o - (B' + C')] = -\tan(B' + C') = -\frac{\tan B' + \tan C'}{1 - \tan B' \cdot \tan C'} =$$

$$= -\frac{\frac{\tan B}{C(v)} + \frac{\tan C}{C(v)}}{1 - \tan B \cdot \tan C / C(v)^2}$$

$$= -\frac{1}{C(v)} \cdot \frac{\tan B + \tan C}{1 - \tan B \cdot \tan C / C(v)^2}$$

$$= -\frac{\tan(B + C)}{C(v)} \cdot \frac{1 - \tan B \tan C}{1 - \tan B \cdot \tan C / C(v)^2}$$

$$= -\frac{-\tan[180^o - (B + C)]}{C(v)} \cdot \frac{1 - \tan B \cdot \tan C}{1 - \tan B \cdot \tan C / C(v)^2} =$$

$$= \frac{\tan A}{C(v)} \cdot \frac{1 - \tan B \cdot \tan C}{1 - \tan B \cdot \tan C / C(v)^2}$$

We got

$$\tan A' = \frac{\tan A}{C(v)} \cdot \frac{1 - \tan B \cdot \tan C}{1 - \tan B \cdot \tan C / C(v)^2} = \tag{36}$$

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