The radii of baryons

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Abstract

Considering the model in which the effective interaction between any two quarks of a baryon can be approximately described by a simple harmonic potential, and making use of the expression of the energy obtained in Cartesian coordinates for the above mentioned model, we find a general expression for the radii of baryons. We then apply the expression to some baryons and find very consistent values for the radii of baryons and an experimental confirmation for the ground state of Ξ^-.

Keywords: Radii of baryons, baryon spectroscopy

1. Introduction

There is no theoretical work to date that has derived a formula for the radii of baryons. There are just a couple of works dealing with specific baryons such as the work of Sekihaka et al. [1] that deals with the $\Lambda(1405)$ resonance. The present work is an updated version of the pre-print [2]. It is based on reference [3] which calculated most energy levels of baryons by means of two simple formulas (one in Cartesian coordinates and another one in polar cylindrical coordinates) with which we can predict levels yet to be found. The formulas do not apply to levels resulting from hadronic molecules. One of the energy levels predicted was the 1st excited state of Λ_b (energy of 5.93 GeV) which has recently been found by CDF [4] with energy equal to (5919.5 ± 2.07) MeV.

2. Derivation of an equation for the radius of a baryon

We make use of Eq. (1) below for the energies of baryons [3]

$$E_{nml} = h\upsilon_1(n+1) + h\upsilon_2(m+1) + h\upsilon_3(l+1) \quad (1)$$

where $n, m, k = 0,1,2,3,4,...$ and $h\upsilon_1, h\upsilon_2, h\upsilon_3$ are the masses of quarks. On the other hand it is well known that the average potential energy of a harmonic oscillator is half of the total energy, that is,

$$\left\langle \frac{1}{2} k\xi^2 \right\rangle = \frac{h\nu}{2}\left(n + \frac{1}{2}\right) \tag{2}$$

where $n = 0,1,2,3,...$ but since the 3 quarks are in a plane and as there are two spatial degrees of freedom in the plane for each quark, we have

$$\begin{aligned} &E_{n,m,k} = h\upsilon_1(n+1) + h\upsilon_2(m+1) + h\upsilon_3(k+1) = \\ &2\times\left[\left\langle \frac{1}{2}k_1\eta_1^{\,2}\right\rangle + \left\langle \frac{1}{2}k_2\eta_2^{\,2}\right\rangle + \left\langle \frac{1}{2}k_3\eta_3^{\,2}\right\rangle\right] = \\ &k_1\left\langle \eta_1^{\,2}\right\rangle + k_2\left\langle \eta_2^{\,2}\right\rangle + k_3\left\langle \eta_3^{\,2}\right\rangle \end{aligned} \tag{3}$$

where 1,2,3 refer to the 3 quarks of the baryons and $\eta_i^{\,2} = \xi_{ij}^{\,2} + \xi_{iq}^{\,2}$ in which j and q are the two orthogonal directions. Eq. (1) above was obtained considering three independent oscillators. Thus we can make the association

$$h\upsilon_i(n+1) = k_i\left\langle \eta_i^{\,2}\right\rangle \tag{4}$$

And defining the radius of a baryon as

$$r_{\eta_1\eta_2\eta_3} = \frac{1}{3}\left(\sqrt{\left\langle \eta_1^{\,2}\right\rangle} + \sqrt{\left\langle \eta_2^{\,2}\right\rangle} + \sqrt{\left\langle \eta_3^{\,2}\right\rangle}\right) \tag{5}$$

we obtain

$$r_{nml} = \frac{1}{3}\left(\sqrt{\frac{h\upsilon_1(n+1)}{k_1}} + \sqrt{\frac{h\upsilon_2(m+1)}{k_2}} + \sqrt{\frac{h\upsilon_3(l+1)}{k_3}}\right) \tag{6}$$

The consistency of Eq. (6) is proven with the calculation of r_{nml} for the ground state of Ξ and its agreement with the experimental value.

For making easier the calculation of the radii of baryons we reproduce below the energy levels of baryons calculated by Eq. 1 [3]. On the tables below E_C is the calculated value by Eq. (1), E_M is the measured value and the error is given by $Error = 100\% \times \left|E_M - E_C\right| / E_C$.

Table 1. Energy levels of baryons N and Δ according to the formula $E_{nml} = 0.31(n+m+l+3)$.

$State(n,m,l)$	E_C (GeV)	E_M (GeV)	$Error(\%)$	$L_{2I,2J}$	$Parity$
$0,0,0$	0.93	0.938(N)	0.9	P_{11}	+
$n+m+l=1$	1.24	1.232(Δ)	0.6	P_{33}	+
$n+m+l=2$	1.55	1.44(N)	7.1	P_{11}	+
$n+m+l=2$	1.55	1.52(N)	1.9	D_{13}	−
$n+m+l=2$	1.55	1.535(N)	1.0	S_{11}	−
$n+m+l=2$	1.55	1.6(Δ)	3.1	P_{33}	+
$n+m+l=2$	1.55	1.62(Δ)	4.5	S_{31}	−
$n+m+l=2$	1.55	1.655(N)	6.7	S_{11}	−
$n+m+l=2$	1.55	1.675(N)	8.1	D_{15}	−
$n+m+l=2$	1.55	1.685(N)	8.7	F_{15}	+
$n+m+l=2$	1.55	1.70(N)	9.7	D_{13}	−
$n+m+l=2$	1.55	1.70(Δ)	9.7	D_{33}	−
$n+m+l=2$	1.55	1.72(N)	11.0	P_{13}	+
$n+m+l=3$	1.86	1.71(N)	8.1	P_{11}	+
$n+m+l=3$	1.86	1.90(N)	2.2	P_{13}	+
$n+m+l=3$	1.86	1.90(Δ)	2.2	S_{31}	−
$n+m+l=3$	1.86	1.905(Δ)	2.4	F_{35}	+
$n+m+l=3$	1.86	1.91(Δ)	2.7	P_{31}	+
$n+m+l=3$	1.86	1.92(Δ)	3.2	P_{33}	+
$n+m+l=3$	1.86	1.93(Δ)	3.8	D_{35}	−
$n+m+l=3$	1.86	1.94(Δ)	4.3	D_{33}	−
$n+m+l=3$	1.86	2.0(N)	7.5	F_{15}	+

Table 1. (Continued)

$n+m+l=4$	2.17	1.95(Δ)	10.1	F_{37}	+
$n+m+l=4$	2.17	1.99(N)	8.3	F_{17}	+
$n+m+l=4$	2.17	2.00(Δ)	7.8	F_{35}	+
$n+m+l=4$	2.17	2.08(N)	4.1	D_{13}	−
$n+m+l=4$	2.17	2.09(N)	3.7	S_{11}	−
$n+m+l=4$	2.17	2.10(N)	3.2	P_{11}	+
$n+m+l=4$	2.17	2.15(Δ)	0.9	S_{31}	−
$n+m+l=4$	2.17	2.19(N)	0.9	G_{17}	−
$n+m+l=4$	2.17	2.20(N)	1.4	D_{15}	−
$n+m+l=4$	2.17	2.20(Δ)	1.4	G_{37}	−
$n+m+l=4$	2.17	2.22(N)	2.3	H_{19}	+
$n+m+l=4$	2.17	2.225(N)	2.5	G_{19}	−
$n+m+l=4$	2.17	2.3(Δ)	6.0	H_{39}	+
$n+m+l=5$	2.48	2.35(Δ)	5.2	D_{35}	−
$n+m+l=5$	2.48	2.39(Δ)	3.6	F_{37}	+
$n+m+l=5$	2.48	2.40(Δ)	3.2	G_{39}	−
$n+m+l=5$	2.48	2.42(Δ)	2.4	$H_{3,11}$	+
$m+n+l=6$	2.79	2.60(N)	6.8	$I_{1,11}$	−
$m+n+l=6$	2.79	2.70(N)	3.2	$K_{1,13}$	+
$m+n+l=6$	2.79	2.75(Δ)	1.4	$I_{3,13}$	−
$m+n+l=7$	3.10	2.95(Δ)	4.8	$K_{3,15}$	+
$m+n+l=7$	3.10	3.10(N)	0	$L_{1,15}$	−
$m+n+l=8$	3.21	?	?	?	?

Table 2. Energy levels of Σ $(\Sigma^0, \Sigma^+, \Sigma^-)$ and Λ^0 according to the formula $E_{nml} = 0.31(n+m+2)+0.5(l+1)$.

$State(n,m,l)$	E_C (GeV)	E_M (GeV)	$Error(\%)$	$L_{2I,2J}$	$Parity$
$0,0,0$	1.12	$1.116(\Lambda)$	0.4	P_{01}	+
$0,0,0$	1.12	$1.189(\Sigma^{\pm})$	6.2	P_{11}	+
$0,0,0$	1.12	$1.193(\Sigma^0)$	6.5	P_{11}	+
$n+m=1, l=0$	1.43	$1.385(\Sigma)$	3.2	P_{13}	+
$n+m=1, l=0$	1.43	$1.405(\Lambda)$	1.7	S_{01}	−
$n+m=1, l=0$	1.43	$1.48(\Sigma)$	3.5	?	?
$0,0,1$	1.62	$1.52(\Lambda)$	6.2	D_{03}	−
$0,0,1$	1.62	$1.56(\Sigma)$	3.7	?	+
$0,0,1$	1.62	$1.58(\Sigma)$	2.5	D_{13}	−
$0,0,1$	1.62	$1.60(\Lambda)$	1.2	P_{01}	+
$0,0,1$	1.62	$1.62(\Sigma)$	0	S_{11}	−
$0,0,1$	1.62	$1.66(\Sigma)$	2.5	P_{11}	+
$0,0,1$	1.62	$1.67(\Lambda)$	3.1	S_{01}	−
$n+m=2, l=0$	1.74	$1.67(\Sigma)$	4.0	D_{13}	−
$n+m=2, l=0$	1.74	$1.69(\Lambda)$	2.9	D_{03}	−
$n+m=2, l=0$	1.74	$1.69(\Sigma)$	2.9	?	?
$n+m=2, l=0$	1.74	$1.75(\Sigma)$	0.6	S_{11}	−
$n+m=2, l=0$	1.74	$1.77(\Sigma)$	1.7	P_{11}	+
$n+m=2, l=0$	1.74	$1.775(\Sigma)$	2.0	D_{15}	−
$n+m=2, l=0$	1.74	$1.80(\Lambda)$	3.4	S_{01}	−
$n+m=2, l=0$	1.74	$1.81(\Lambda)$	4.0	P_{01}	+
$n+m=2, l=0$	1.74	$1.82(\Lambda)$	4.6	F_{05}	+
$n+m=2, l=0$	1.74	$1.83(\Lambda)$	5.2	D_{05}	−
$n+m=1, l=1$	1.93	$1.84(\Sigma)$	4.7	P_{13}	+
$n+m=1, l=1$	1.93	$1.88(\Sigma)$	2.6	P_{11}	+
$n+m=1, l=1$	1.93	$1.89(\Lambda)$	2.1	P_{03}	+
$n+m=1, l=1$	1.93	$1.915(\Sigma)$	0.8	F_{15}	+
$n+m=1, l=1$	1.93	$1.94(\Sigma)$	0.5	D_{13}	−

Table 2. (Continued)

$n+m=3, l=0$	2.05	2.00(Λ)	2.5	?	?
$n+m=3, l=0$	2.05	2.00(Σ)	2.5	S_{11}	−
$n+m=3, l=0$	2.05	2.02(Λ)	1.5	F_{07}	+
$n+m=3, l=0$	2.05	2.03(Σ)	1.0	F_{17}	+
$n+m=3, l=0$	2.05	2.07(Σ)	1.0	F_{15}	+
$n+m=3, l=0$	2.05	2.08(Σ)	1.5	P_{13}	+
$0, 0, 2$	2.12	2.10(Σ)	0.9	G_{17}	−
$0, 0, 2$	2.12	2.10(Λ)	0.9	G_{07}	−
$0, 0, 2$	2.12	2.11(Λ)	0.5	F_{05}	+
$n+m=2, l=1$	2.24	2.25(Σ)	0.5	?	?
$n+m=4, l=0$	2.36	2.325(Λ)	1.5	D_{03}	−
$n+m=4, l=0$	2.36	2.35(Λ)	0.4	H_{09}	+
$n+m=1, l=2$	2.43	2.455(Σ)	1.0	?	?
$n+m=3, l=1$	2.55	2.585(Λ)	1.4	?	?
$0, 0, 3$	2.62	2.62(Σ)	0	?	?
$n+m=5, l=0$	2.67	?	?	?	?
$n+m=2, l=2$	2.74	?	?	?	?
$n+m=4, l=1$	2.86	?	?	?	?
$n+m=1, l=3$	2.93	?	?	?	?
$n+m=6, l=0$	2.98	3.00(Σ)	0.7	?	?
$n+m=3, l=2$	3.05	?	?	?	?
$n=m=0, l=4$	3.12	?	?	?	?
$n+m=5, l=1$	3.17	3.17(Σ)	0	?	?
$n+m=2, l=3$	3.24	?	?	?	?
$n+m=2, l=3$	3.29	?	?	?	?

Table 3. Energy levels of Ξ (Ξ^0, Ξ^-) according to the formula $E_{nml} = 0.31(n+1) + 0.5(m+l+2)$.

$State(n,m,l)$	E_C (GeV)	E_M (GeV)	$Error(\%)$	$L_{2I,2J}$	$Parity$
$0,0,0$	1.31	$1.315\Xi^0$	0.5	P_{11}	+
$0,0,0$	1.31	$1.321\Xi^-$	0.8	P_{11}	+
$1,0,0$	1.62	1.53	5.6	P_{13}	+
$1,0,0$	1.62	1.62	0	?	?
$1,0,0$	1.62	1.69	4.3	?	?
$n=0, m+l=1$	1.81	1.82	0.6	D_{13}	−
$2,0,0$	1.93	1.95	1.0	?	?
$n=1, m+l=1$	2.12	2.03	4.2	?	?
$n=1, m+l=1$	2.12	2.12	0	?	?
$n=3, m=l=0$	2.24	2.25	0.5	?	?
$n=0, m+l=2$	2.31	2.37	2.6	?	?
$n=2, m+l=1$	2.43	?	?	?	?
$n=4, m=l=0$	2.55	2.5	2.0	?	?
$n=1, m+l=2$	2.62	?	?	?	?

Table 4. Energy levels of Ω^- according to the formula $E_{nml} = 0.5(n+m+l+3)$.

$State(n,m,l)$	E_C (GeV)	E_M (GeV)	$Error(\%)$
$0,0,0$	1.5	1.672	11.17
$n+m+l=1$	2.0	2.25	12.5
$n+m+l=2$	2.5	2.38	4.8
$n+m+l=2$	2.5	2.47	1.2
$n+m+l=3$	3.0	?	?

3. Calculation of the radii of baryons

The masses of quarks $h\upsilon_1, h\upsilon_2, h\upsilon_3$ were taken from Particle Data Group [5] as $m_u = m_d = 0.31\,\text{GeV}$, $m_s = 0.5\,\text{GeV}$, $m_c = 1.7\,\text{GeV}$, $m_b = 5\,\text{GeV}$, and $m_t = 174\,\text{GeV}$.

3.1. Baryons N, Δ^-, Δ^{++}

As shown above $E_{nml} = 0.31(n+m+l+3)$, and thus, as calculated above

$$r_{nml} = \frac{1}{3}\sqrt{\frac{h\upsilon}{k}}\left(\sqrt{n+1}+\sqrt{m+1}+\sqrt{l+1}\right) \tag{7}$$

Using the value $h\upsilon = 0.31\,\mathrm{GeV}$ and the experimental value $r_{000} = r_0 = \left(0.85 \pm 0.02\right)\mathrm{fm}$ [XXXX] for the proton radius, we obtain $k = \left(0.43 \pm 0.03\right)$ GeV/fm^2. Therefore, we have

$$r_{nml} = \frac{0.85 \pm 0.02}{3}\left(\sqrt{n+1}+\sqrt{m+1}+\sqrt{l+1}\right) \tag{8}$$

in fermis. Now we apply this above formula to all the other levels. Taking a look at Table 1 we observe that the excited states designated by (n,m,l) split into many levels. It is important to stress that with the above model there is not a way of determining to which split levels the radii correspond.

3.1.1. The level $n+m+l=1; E_{nml}=1.24$ GeV

As it is shown on Table 1 this is the first state of Δ and has the calculated energy 1.24 GeV and experimental energy 1.232 GeV. In this case for n,m,l there are the 3 possibilities $0,0,1; 0,1,0; 1,0,0$ and thus,

$$r_{001} = r_{010} = r_{100} = \left(0.85 \pm 0.02\right) \times \left(\frac{2+\sqrt{2}}{3}\right) = \left(0.97 \pm 0.02\right)\mathrm{fm}.$$

3.1.2. The level $n+m+l=2; E_{nml}=1.55$ GeV

As it is shown on Table 1 there are many states of N and Δ. For (n,m,l) there are the possibilities $(2,0,0),(0,2,0),(0,0,2)$ and $(1,1,0),(1,0,1),(0,1,1)$. The corresponding radii are

$$r_{200} = r_{020} = r_{002} = \left(0.85 \pm 0.02\right) \times \left(\frac{2+\sqrt{3}}{3}\right) = \left(1.06 \pm 0.02\right)\mathrm{fm} \text{ and}$$

$$r_{110} = r_{101} = r_{011} = \left(0.85 \pm 0.02\right) \times \left(\frac{2\sqrt{2}+1}{3}\right) = 1.09 \pm 0.02\,\mathrm{fm}.$$

3.1.3. The level $n+m+l=3; E_{nml}=1.86$ GeV

We see on Table 1 many states of N and Δ. For (n,m,l) there are the possibilities $(3,0,0),(0,3,0),(0,0,3)$; $(0,1,2),(0,2,1),(1,0,2),(1,2,0),(2,1,0),(2,0,1)$ and $(1,1,1)$. For the first set we represent the radius by r_{003}, and for the 2nd set by r_{012}. The corresponding radii are

$r_{003}=(0.85\pm0.02)\left(\frac{4}{3}\right)=(1.13\pm0.02)$ fm; $r_{012}=(0.85\pm0.02)\left(\frac{1+\sqrt{2}+\sqrt{3}}{3}\right)=(1.17\pm0.02)$ fm; $r_{111}=(0.85\pm0.02)\left(\frac{3\times\sqrt{2}}{3}\right)=(1.20\pm0.02)$ fm.

Doing the same for the other levels, and summarizing the results we obtain Table 5 below.

Table 5. Radii for levels of baryons N and Δ according to the formula $r_{nml}=\frac{0.85\pm0.02}{3}\left(\sqrt{n+1}+\sqrt{m+1}+\sqrt{l+1}\right)$.

$State(n,m,l)$	$E_C(GeV)$	r_{mnl} (fm)
$0,0,0$	$0.93(N)$	$r_{000}=0.85\pm0.02$
$n+m+l=1$	$1.24(\Delta)$	$r_{001}=0.97\pm0.02$
$n+m+l=2$	1.55	$r_{002}=1.06\pm0.03$
$n+m+l=2$	1.55	$r_{011}=1.09\pm0.03$
$n+m+l=3$	1.86	$r_{003}=1.13\pm0.03$
$n+m+l=3$	1.86	$r_{012}=1.17\pm0.03$
$n+m+l=3$	1.86	$r_{111}=1.20\pm0.03$
$n+m+l=4$	2.17	$r_{004}=1.20\pm0.03$
$n+m+l=4$	2.17	$r_{013}=1.25\pm0.03$
$n+m+l=4$	2.17	$r_{022}=1.26\pm0.03$
$n+m+l=4$	2.17	$r_{112}=1.29\pm0.03$
$n+m+l=5$	2.48	$r_{005}=1.26\pm0.03$
$n+m+l=5$	2.48	$r_{014}=1.32\pm0.03$
$n+m+l=5$	2.48	$r_{023}=1.34\pm0.03$
$n+m+l=5$	2.48	$r_{113}=1.37\pm0.03$
$n+m+l=5$	2.48	$r_{122}=1.38\pm0.03$

Table 5. (Continued)

$m+n+l=6$	2.79	$r_{006}=1.32\pm0.03$
$m+n+l=6$	2.79	$r_{015}=1.38\pm0.03$
$m+n+l=6$	2.79	$r_{024}=1.41\pm0.03$
$m+n+l=6$	2.79	$r_{033}=1.42\pm0.03$
$m+n+l=6$	2.79	$r_{114}=1.44\pm0.03$
$m+n+l=6$	2.79	$r_{123}=1.46\pm0.03$
$m+n+l=7$	3.10	$r_{007}=1.37\pm0.03$
$m+n+l=7$	3.10	$r_{016}=1.43\pm0.03$
$m+n+l=7$	3.10	$r_{025}=1.47\pm0.04$
$m+n+l=7$	3.10	$r_{034}=1.48\pm0.04$
$m+n+l=7$	3.10	$r_{115}=1.50\pm0.04$
$m+n+l=7$	3.10	$r_{124}=1.52\pm0.04$
$m+n+l=7$	3.10	$r_{133}=1.53\pm0.04$
$m+n+l=7$	3.10	$r_{223}=1.55\pm0.04$

3.2. Baryons Σ and Λ

As shown above $E_{nml}=0.31(n+m+2)+0.50(l+1)$, and thus, according to Eq. (6) we have

$$r_{nml}=\frac{1}{3}\left(\sqrt{\frac{h\upsilon_1}{k_1}}\left(\sqrt{n+1}+\sqrt{m+1}\right)+\sqrt{\frac{h\upsilon_3}{k_3}}\sqrt{l+1}\right) \quad (9)$$

Using the experimental value $r_{000}=\left(0.79\pm0.02\right)$fm for the ground state of Σ^- and the above value of $k_1=\left(0.43\pm0.03\right)$GeV/fm^2, we obtain from Eq. (9) $k_3=\left(1.11\pm0.03\right)$ GeV/fm^2. Therefore, the equation for the radii of these baryons is given by

$$r_{nml}=\frac{1}{3}\left[\left(0.85\pm0.02\right)\left(\sqrt{n+1}+\sqrt{m+1}\right)+\left(0.67\pm0.02\right)\sqrt{l+1}\right] \quad (10)$$

3.2.1. The level $n+m=1, l=0; E_{nml}=1.43$ GeV

The possibilities for (n,m,l) are $(0,1,0),(1,0,0)$ and thus, Eq. (10) yields

$$r_{010} = r_{100} = r_{nml} = \frac{1}{3}\left[(0.85 \pm 0.02)\left(1+\sqrt{2}\right) + (0.67 \pm 0.02) \times 1\right] = (0.91 \pm 0.02)\ \text{fm.}$$

3.2.2. The level $(0,0,1); E_{nml} = 1.62$ GeV

We have for (n,m,l) the set $(0,0,1)$, and hence, the radius

$$r_{001} = \frac{1}{3}\left[(0.85 \pm 0.02)(1+1) + (0.67 \pm 0.02)\sqrt{2}\right] = (0.88 \pm 0.02)\ \text{fm.}$$

Calculating the other levels in the same way and ordering the set *(n,m,l)* from smaller to larger, and arranging the results in a table, we obtain Table 6 below.

Table 6. Radii for levels of baryons Σ and Λ according to the formula $r_{nml} = \frac{1}{3}\left[(0.85 \pm 0.02)\left(\sqrt{n+1} + \sqrt{m+1}\right) + (0.67 \pm 0.02)\sqrt{l+1}\right]$.

$State(n,m,l)$	$E_C\,(GeV)$	r_{mnl} (fm)
$0,0,0$	1.12	$r_{000} = 0.79 \pm 0.02$
$n+m=1, l=0$	1.43	$r_{010} = 0.91 \pm 0.02$
$0,0,1$	1.62	$r_{001} = 0.88 \pm 0.02$
$n+m=2, l=0$	1.74	$r_{020} = 1.00 \pm 0.02$
$n+m=2, l=0$	1.74	$r_{110} = 1.02 \pm 0.02$
$n+m=1, l=1$	1.93	$r_{011} = 1.00 \pm 0.02$
$n+m=3, l=0$	2.05	$r_{030} = 1.07 \pm 0.03$
$n+m=3, l=0$	2.05	$r_{120} = 1.11 \pm 0.02$
$0,0,2$	2.12	$r_{002} = 0.95 \pm 0.02$
$n+m=2, l=1$	2.24	$r_{021} = 1.09 \pm 0.03$
$n+m=2, l=1$	2.24	$r_{111} = 1.12 \pm 0.03$
$n+m=4, l=0$	2.36	$r_{040} = 1.14 \pm 0.03$
$n+m=4, l=0$	2.36	$r_{130} = 1.19 \pm 0.03$
$n+m=4, l=0$	2.36	$r_{220} = 1.20 \pm 0.03$
$n+m=1, l=2$	2.43	$r_{012} = 1.07 \pm 0.03$

Table 6. (Continued)

$m+n=3, l=1$	2.55	$r_{031}=1.16\pm0.03$
$m+n=3, l=1$	2.55	$r_{121}=1.21\pm0.03$
$0,0,3$	2.62	$r_{003}=1.01\pm0.02$
$m+n=6, l=0$	2.98	$r_{060}=1.26\pm0.03$
$m+n=6, l=0$	2.98	$r_{150}=1.32\pm0.03$
$m+n=6, l=0$	2.98	$r_{240}=1.35\pm0.03$
$m+n=6, l=0$	2.98	$r_{330}=1.36\pm0.03$
$m+n=5, l=1$	3.17	$r_{051}=1.29\pm0.03$
$m+n=5, l=1$	3.17	$r_{141}=1.35\pm0.03$
$m+n=5, l=1$	3.17	$r_{231}=1.37\pm0.03$

3.3. Baryons Ξ

Using Eq. (6) and what was calculated above , we have that the radii are described by

$$r_{nml}=\frac{1}{3}\left[(0.85\pm0.02)\sqrt{n+1}+(0.67\pm0.02)\left(\sqrt{m+1}+\sqrt{l+1}\right)\right] \quad (11)$$

3.3.1. The ground state level $(0,0,0)$; $E_{nml}=1.31$ GeV

From Eq. (11) we obtain $r_{000}=(0.73\pm0.02)\,\text{fm}$ which is very close to the experimental value of $(0.74\pm0.02)\,\text{fm}$ [6]. This result confirms the validity of the general formula

$$r_{nml}=\frac{1}{3}\left(\sqrt{\frac{h\upsilon_1(n+1)}{k_1}}+\sqrt{\frac{h\upsilon_2(m+1)}{k_2}}+\sqrt{\frac{h\upsilon_3(l+1)}{k_3}}\right).$$

Using Eq. (11) we obtain that the other levels of Ξ have the radii summarized on Table 7 below.

Table 7. Radii for levels of baryons Ξ according to the formula $r_{nml} = \frac{1}{3}\left[(0.85 \pm 0.02)\sqrt{n+1} + (0.67 \pm 0.02)\left(\sqrt{m+1} + \sqrt{l+1}\right)\right]$.

$State(n,m,l)$	$E_C(GeV)$	r_{mnl} (fm)
$0,0,0$	1.31	$r_{000} = 0.73 \pm 0.02$
$1,0,0$	1.62	$r_{100} = 0.85 \pm 0.02$
$n=0, m+l=1$	1.81	$r_{001} = 0.82 \pm 0.02$
$2,0,0$	1.93	$r_{200} = 0.94 \pm 0.02$
$n=1, m+l=1$	2.12	$r_{101} = 0.94 \pm 0.02$
$3,0,0$	2.24	$r_{300} = 1.01 \pm 0.02$
$n=0, m+l=2$	2.31	$r_{002} = 0.89 \pm 0.02$
$n=0, m+l=2$	2.31	$r_{011} = 0.92 \pm 0.02$
$4,0,0$	2.55	$r_{400} = 1.08 \pm 0.02$

3.4. Baryons Ω

From what was calculated above, and with the use of Eq. (6), we obtain that the radii are described by

$$r_{nml} = \frac{1}{3}\left[(0.67 \pm 0.02)\left(\sqrt{n+1} + \sqrt{m+1} + \sqrt{l+1}\right)\right]. \tag{12}$$

we can, thus, predict that the ground state has a radius of about $r_{000} = (0.67 \pm 0.02)$ fm. The calculation of the other levels produce Table 8 below.

Table 8. Predicted radii for levels of baryons Ω according to the formula $r_{nml} = \frac{1}{3}\left[(0.67 \pm 0.02)\left(\sqrt{n+1} + \sqrt{m+1} + \sqrt{l+1}\right)\right]$.

$State(n,m,l)$	$E_C(GeV)$	r_{mnl} (fm)
$0,0,0$	1.50	$r_{000} = 0.67 \pm 0.02$
$n+m+l=1$	2.00	$r_{001} = 0.76 \pm 0.02$
$n+m+l=2$	2.00	$r_{002} = 0.84 \pm 0.02$
$n+m+l=2$	2.00	$r_{011} = 0.81 \pm 0.02$

4. Conclusion

Considering that the effective interaction between any two quarks of a baryon can be approximately described by a simple harmonic oscillator, we derive a general formula that describes the energy levels of baryons and using it we can obtain a general formula for the description of the radii of baryons. The calculation is very consistent and agrees very well with the experimental value for the ground state of Ξ^-. Since the formula for the radius was deduced from the expression for the energy in Cartesian coordinates there is not a way at the moment of identifying the radii in terms of J and L.

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