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Correlation Coefficients of Neutrosophic Sets by Centroid Method

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Abstract In this paper, we propose another method to calculate the correlation coefficient of neutrosophic sets. The value which obtained from this method tells us the strength of relationship between the neutrosophic sets and the whether the neutrosophic sets are positively or negatively related. Finally we give some proposition and examples.

Keywords Correlation Coefficient, Fuzzy Set, Neutrosophic Sets, Intuitionistic Fuzzy Sets, Centroid Method

1. Introduction

In 2012 Hanafy and Salama[8, 9, 10] introduced and studied some operations on neutrosophic sets and investigated the correlation of neutrosophic data[8]. Correlation plays an important role in statistics and engineering sciences, by correlation analysis, the joint relationship of two variables can be examined with the aid of a measure of interdependency of the two variables. The correlation coefficient is one of the most frequently used tools in statistics. Several authors have discussed and investigated the concept of correlation in fuzzy set[4, 14]. For example, Murthy and Pal[12] studied the correlation between two fuzzy membership functions, Chiang and Lin[4] studied the correlation and partial correlation of fuzzy sets, Chaudhuri and Bhattacharya[3] investigated the correlation between two fuzzy sets on the same universal support. Yu[14] defined the correlation of fuzzy numbers A, B in the collection F([a,b]) of all fuzzy numbers whose supports are included in a closed interval. After two decades Turksen[13] proposed the concept of interval-valued fuzzy set. But for some applications it is not enough to satisfy to consider only the membership-function supported by the evident but also have to consider the non-membership function against by the evident. Atanassov[1, 2] introduced another type of fuzzy sets that is called intuitionistic fuzzy set (IFS) which is more practical in real life situations. Intuitionistic fuzzy sets handle incomplete information i.e., the grade of membership function and non-membership function but not the indeterminate information and inconsistent information which exists obviously in belief system. Several authors have discussed the concept of correlation in intuitionistic fuzzy set. Smarandache and Salama[10, 11] introduced another concept of imprecise data called neutrosophic sets. neutrosophic set is a part of neutrosophy which studied the origin, nature and scope of neutralities, as well as their interactions with ideational spectra. The neutrosophic set generalized the concept of classical fuzzy set[16, 17, 18], interval-valued fuzzy set, intuitionistic fuzzy set[1, 2], etc. In this paper we discuss a concept of correlation for data represented as neutrosophic sets adopting the concept from statistics. We calculate it by showing both positive and negative relationship of the sets, and showing that it is important to take into account all three terms describing neutrosophic sets.

2. Terminologies

2.1. Correlation Coefficient of Fuzzy Sets

In 1965[17], Zadeh first introduce the concept of fuzzy sets as follow: Let X be a fixed set. A fuzzy set A of X is an object having the form $A=\{(x,\mu_A(x)),x\in X\}$ where the function $\mu_A : X\to[0,1]$ define the degree of membership of the element $x\in X$ to the set A, which is a subset of X.

Suppose there is a fuzzy set $A\subseteq F$, where F is a fuzzy space. The fuzzy sets A defined on a crisp set X with a membership function μ_A, the probability of fuzzy set has been defined by Zadeh[17, 18].

$$P(A)=\int_X \mu_A(x)dP=E(\mu_A), \tag{1}$$

where P is the probability measure over X, and $E(\mu_A)$ is the mean of the membership function of a fuzzy event A.

When we have a random sample $(x_1,x_2,\ldots,x_n)$ from a crisp set X, with the membership function μ_A of some

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specific fuzzy set A, then the sample mean and sample variance of the membership function of A, defined on X, can be written as,

$$\bar{\mu}_A = \frac{\sum_{i=1}^{n} \mu_A(x_i)}{n}, \qquad S_A^2 = \frac{\sum_{i=1}^{n} \left(\mu_A(x_i) - \bar{\mu}_A\right)^2}{n-1}, \quad (2)$$

where $\bar{\mu}_A$ and S_A^2 are the average and the degree of variations of membership function of fuzzy set A, then the correlation coefficient, $r_{A,B}$, between the fuzzy sets A and B which defined by Chiang and Lin [4]:

$$r_{A,B} = \frac{\sum_{i=1}^{n} \left(\mu_A(x_i) - \bar{\mu}_A\right)\left(\mu_B(x_i) - \bar{\mu}_B\right) \Big/ n-1}{S_A S_B} \quad (3)$$

2.2. Correlation Coefficient of Intuitionistic Fuzzy Sets

Atanassov[1, 2] introduced another type of fuzzy sets that is called intuitionistic fuzzy set (IFS) which is more practical in real life situations as follow: Let X be a fixed set. An intuitionistic fuzzy set A of X is an object having the form $A = \{(x, \mu_A(x), \nu_A(x)), x \in X\}$ where the function: $\mu_A : X \to [0,1]$ and $\nu_A : X \to [0,1]$ define respectively the degree of membership and degree of non-membership of the element $x \in X$ to the set A, which is a subset of X and for every $x \in X$, $0 \le \mu_A(x) + \nu_A(x) \le 1$.

Yu [14] defined the correlation of A and B in the collection $F([a,b])$ of all fuzzy numbers whose supports are included in a closed interval$[a,b]$ as follows:

$$C_Y(A,B) = \frac{1}{b-a} \int_a^b \mu_A(x)\mu_B(x) + \nu_A(x)\nu_B(x)dx, \quad (4)$$

where $\mu_A(x) + \nu_A(x) = 1$ and the correlation coefficient of fuzzy numbers A,B was defined by

$$\rho_Y = \frac{C_Y(A,B)}{\sqrt{C_Y(A,A) \times C_Y(B,B)}}. \quad (5)$$

In 1991, Gerstenkorn and Manko[16] defined the correlation of intuitionistic fuzzy sets A and B in a finite set $X = \{x_1, x_2, \ldots, x_n\}$ as follows:

$$C_{GM}(A,B) = \sum_{i=1}^{n} (\mu_A(x_i)\mu_B(x_i) + \nu_A(x_i)\nu_B(x_i)) \quad (6)$$

and the correlation coefficient of fuzzy numbers A,B was given by:

$$\rho_{GM} = \frac{C_{GM}(A,B)}{\sqrt{T(A) \cdot T(B)}} \quad (7)$$

where

$$T(A) = \sum_{i=1}^{n} (\mu^2{}_A(x_i) + \nu^2{}_A(x_i)). \quad (8)$$

In 1995, Hong and Hwang[5] defined the correlation of intuitionistic fuzzy sets A and B in a probability space (X,B,P) as follows:

$$C_{HH}(A,B) = \int_X (\mu_A\mu_B + \nu_A\nu_B)dP \quad (9)$$

and the correlation coefficient of intuitionistic fuzzy numbers A,B was given by

$$\rho_{HH} = \frac{C_{HH}(A,B)}{\sqrt{C_{HH}(A,A) \cdot C_{HH}(B,B)}}. \quad (10)$$

Hung and Wu[5,6,7] introduce the concept of positively and negatively correlated and used the concept of centroid to define the correlation coefficient of intuitionistic fuzzy sets which lies in the interval[-1,1], and the correlation coefficient of intuitionistic fuzzy sets A and B was given by:

$$\rho_{HW} = \frac{C_{HW}(A,B)}{\sqrt{C_{HW}(A,A) \cdot C_{HW}(B,B)}}, \quad (11)$$

where

$$C_{HW} = m(\mu_A)m(\mu_B) + m(\nu_A)m(\nu_B) \quad (12)$$

$$m(\mu_A) = \frac{\int x\mu_A(x)dx}{\int \mu_A(x)dx} \quad m(\nu_A) = \frac{\int x\nu_A(x)dx}{\int \nu_A(x)dx}$$

$$m(\mu_B) = \frac{\int x\mu_B(x)dx}{\int \mu_B(x)dx} \quad m(\nu_B) = \frac{\int x\nu_B(x)dx}{\int \nu_B(x)dx}$$

In 2012 Salama et al[10] introduced and studied some operations on neutrosophic sets

Definition.2.1. [10]

Let X be a non-empty fixed set. A neutrosophic set (*GNS* for short) A is an object having the form $A = \{\langle x, \mu_A(x), \sigma_A(x), \gamma_A(x)\rangle : x \in X\}$

Where $\mu_A(x), \sigma_A(x)$ and $\gamma_A(x)$ which represent the degree of member ship function (namely $\mu_A(x)$), the degree of indeterminacy (namely $\sigma_A(x)$), and the degree of non-member ship (namely $\gamma_A(x)$) respectively of each element $x \in X$ to the set A .

The correlation coefficient between neutrosophic sets ranges in[-1, 1], which can correlate neutrosophic concepts. Thus, we propose the following correlation measure.

Definition. 2.2. [8, 10]

Let X be a fixed set. A neutrosophic set A of X is an object having the form $A = \{(x, \mu_A(x), \nu_A(x), \gamma_A(x)), x \in X\}$ where the function: μ_A, ν_A and γ_A are real standard or non-standard subsets of] $^-0,1^+$ [define respectively the degree of membership, degree of non-membership and degree of indeterminacy of the element $x \in X$ to the set A, which is a subset of X and for every $x \in X$, $^-0 \le \mu_A(x) + \nu_A(x) + \gamma_A(x) \le ^+3$.

Definition. 2.3. [8].

For A and B are two neutrosophic sets in a finite space $X=\{x_1,x_2,...,x_n\}$, we define the correlation of neutrosophic sets A and B as follows:

$$CN(A,B)=\sum_{i=1}^{n}\left[\left(\mu_A(x_i).\mu_B(x_i)+\sigma_A(x_i).\sigma_B(x_i)+\nu_A(x_i).\nu_B(x_i)\right)\right]$$

and the correlation coefficient of A and B given by

$$R(A,B)=\frac{CN(A,B)}{(T(A).T(B))^{\frac{1}{2}}}$$

Where

$$T(A)=\sum_{i=1}^{n}\left(\mu^2{}_A(x_i)+\sigma^2{}_A(x_i)+\nu^2{}_A(x_i)\right)$$

$$T(B)=\sum_{i=1}^{n}\left(\mu^2{}_B(x_i)+\sigma^2{}_B(x_i)+\nu^2{}_B(x_i)\right)$$

3. Correlation Coefficient of Neutrosophic Sets

Definition 3.1.

Let A, B are two neutrosophic sets, we define

$$C(A,B)=m(\mu_A)m(\mu_B)+m(\nu_A)m(\nu_B)+m(\gamma_A)m(\gamma_B) \tag{13}$$

and we call it correlation formula of A and B. furthermore, we call

$$\rho(A,B)=\frac{C(A,B)}{M(A)\cdot M(B)}, \tag{14}$$

The correlation coefficient of A and B, where

$$m(\mu_A)=\begin{cases}\dfrac{\int_X x\mu_A(x)dx}{\int_X \mu_A(x)dx}, & X \text{ is Continous}\\[2ex] \dfrac{\sum_{i=1}^{n} x_i\mu_A(x_i)}{\sum_{i=1}^{n}\mu_A(x_i)}, & X \text{ is Finite}\end{cases},$$

$$m(\mu_B)=\begin{cases}\dfrac{\int_X x\mu_B(x)dx}{\int_X \mu_B(x)dx}, & X \text{ is Continous}\\[2ex] \dfrac{\sum_{i=1}^{n} x_i\mu_B(x_i)}{\sum_{i=1}^{n}\mu_B(x_i)}, & X \text{ is Finite}\end{cases}$$

$$m(\nu_A)=\begin{cases}\dfrac{\int_X x\nu_A(x)dx}{\int_X \nu_A(x)dx}, & X \text{ is Continous}\\[2ex] \dfrac{\sum_{i=1}^{n} x_i\nu_A(x_i)}{\sum_{i=1}^{n}\nu_A(x_i)}, & X \text{ is Finite}\end{cases},$$

$$m(\nu_B)=\begin{cases}\dfrac{\int_X x\nu_B(x)dx}{\int_X \nu_B(x)dx}, & X \text{ is Continous}\\[2ex] \dfrac{\sum_{i=1}^{n} x_i\nu_B(x_i)}{\sum_{i=1}^{n}\nu_B(x_i)}, & X \text{ is Finite}\end{cases}$$

$$m(\gamma_A)=\begin{cases}\dfrac{\int_X x\gamma_A(x)dx}{\int_X \nu_A(x)dx}, & X \text{ is Continous}\\[2ex] \dfrac{\sum_{i=1}^{n} x_i\gamma_A(x_i)}{\sum_{i=1}^{n}\gamma_A(x_i)}, & X \text{ is Finite}\end{cases},$$

$$m(\gamma_B)=\begin{cases}\dfrac{\int_X x\gamma_B(x)dx}{\int_X \nu_A(x)dx}, & X \text{ is Continous}\\[2ex] \dfrac{\sum_{i=1}^{n} x_i\gamma_B(x_i)}{\sum_{i=1}^{n}\gamma_B(x_i)}, & X \text{ is Finite}\end{cases}$$

are the centroid of $\mu_A, \nu_A, \gamma_A, \mu_B, \nu_B$ and γ_B, respectively and

$$M(A)=\sqrt{m^2(\mu_A)+m^2(\nu_A)+m^2(\gamma_A)}$$

$$M(B)=\sqrt{m^2(\mu_B)+m^2(\nu_B)+m^2(\gamma_B)}$$

Proposition 3.1.

For all A and B are neutrosophic sets, we have

1). $C(A,B)=C(B,A)$.

2). $\rho(A,B)=\rho(B,A)$.

3). If A=B, then $\rho(A,B)=1$.

Theorem 3.1.

For all A and B are neutrosophic sets, we have $|\rho(A,B)|\le 1$.

Proof

By the Cauchy-Schwarz inquality, Therefore, we have $|\rho(A,B)| \le 1$.

Corollary 3.1.

Let A and B are neutrosophic sets satisfied with $\mu_A = c\mu_B$, $\nu_A = c\nu_B$ and $\gamma_A = c\gamma_B$ for an arbitrary real number $c>0$, then $\rho(A,B)=1$.

4. Comparative Examples

We give some examples to compare with several correlation of neutrosophic sets.

Example 4.1.

For a finite universal set $X=\{x_1,x_2,x_3\}$, if two neutrosophic sets are written, repectively.

A={(1,0,0.4),(0.8,0,0.6),(0.7,0.1,0.6)},

B={(0.5,0.3,0.7),(0.6,0.2,0.7),(0.8,0.1,0.6)},

Therefore, we have:

$$\rho\ (A,B)=0.89871.\ R(A,B)=0.95.$$

It shows that neutrosophic sets A and B have a good positively correlated.

Example 4.2

For a continuous universal set X =[1,2], if two neutrosophic sets are written, respectively.

$$A=\{(x,\mu_A(x),\nu_A(x),\gamma_A(x))|x\in[1,2]\},$$

$$B=\{(x,\mu_B(x),\nu_B(x),\gamma_A(x))|x\in[1,2]\},$$

where

$$\mu_A(x)=0.5(x-1)\quad,\qquad 1\le x\le 2,$$

$$\mu_B=0.3(x-1)\quad,\qquad 1\le x\le 2$$

$$\nu_A(x)=1.9-0.9x\quad,\qquad 1\le x\le 2,$$

$$\nu_B=1.4-0.4x\quad,\qquad 1\le x\le 2$$

$$\gamma_A(x)=(5-x)/6\quad,\qquad 1\le x\le 2,$$

$$\gamma_B(x)=0.5x-0.3\quad,\qquad 1\le x\le 2$$

Thus, we have $\rho(A,B)=0.99938$

It shows that neutrosophic sets A and B have a good positively correlated.

5. Conclusions

Our main goal of this work is propose a method to calculate the correlation coefficient of neutrosophic sets by means of "centroid" which lies in [-1,1], give us information for the degree of the relationship between neutrosophic sets and the fact that these two sets are positively or negatively related.

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