Towards New Physics: Quantum Mechanics and "The Warping of Spacetime"

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Abstract

What appears to be the Higgs boson has been found, apparently completing the Standard Model. However there are still no signs of either Supersymmetry or extra dimensions. This paper proposes new physics possibly consistent with this. In a radically different approach, it builds fundamental particles as infinite superpositions, but all with non zero proper mass, by borrowing mass from a Higgs type scalar field. However energy is also borrowed from zero point vector fields. Just as the Standard Model divides the fundamental particles into two types…those with proper mass and those without and the Higgs mechanism providing the difference…infinite superpositions seem also to divide naturally into two sets: (a) those with only "infinitesimal" proper mass, or (b) those with significant proper mass (from micro electron volts upwards). In the infinitesimal set (a), photons, gluons and gravitons (to fit with cosmology and the expansion of the cosmos) appear to have $\approx 10^{-34}$ eV. proper mass, approximately the inverse of the causally connected horizon radius. These values are so close to zero the Standard Model symmetry breaking approach remains essentially valid. Particles travelling this close to the speed of light have virtually fixed helicity and the Higgs mechanism can still increase their proper masses from infinitesimal type (a) to significant or measureable type (b) values. Also the energy in the zero point fields (borrowed to build the fundamental particles) is limited, particularly at the extreme wavelengths of virtual gravitons interacting at near horizon radii. Any causally connected region grows with time after the big bang and the number of virtual gravitons with wavelengths similar to the size of the causally connected region increases approximately as the square of the causally connected mass. Space has to expand progressively with time after the big bang, increasing the zero point energy available to meet this increased requirement. For similar reasons the extra gravitons near mass concentrations change the metric in proportion to m/r (to the first order), as predicted by the Schwarzschild solution of Einstein's equations. The first and major part of this paper builds the fundamental particles from infinite virtual superpositions. It looks at the expanding Universe and connections with General Relativity only after attempting to show that infinite superpositions are virtually the same as the Standard Model, apart from infinitesimal (but important) differences.

1. Introduction

Since the weak and electromagnetic forces were unified in the 1960's physicists have wished to somehow unite all the fundamental forces. Initially it seemed that three of the forces (excluding gravity) could unite near the Planck scale. Higher energy experiments however ruled out the possibility of single energy unification. Supersymmetry was proposed as a possible solution addressing several problems, but also modifying high energy running constants of the three forces in such a manner they united near the Planck scale. None of the particles predicted has yet been seen. Supersymmetry is a stepping stone to String theory, which is seen by many as the most likely future path. Not all physicists are comfortable however with its non-testability and need for 10 or 11 dimensions. The enormous landscape of different universes or multiverse it proposes is also widely regarded as the solution to the minute amount of dark energy proposed to explain the current accelerating expansion of the universe. This suggests some important and relevant questions. For example:

1. Is it possible that the fundamental forces may connect in some different way?
2. Are the extra dimensions of Supersymmetry and String Theory really necessary?
3. Can the problems these theories were meant to solve be addressed differently?
4. Is "The Multiverse" the only explanation of accelerating cosmic expansion?
5. Alternatively "Is there a different approach to all this?"

This paper sets out to address these questions using only basic simple principles of quantum mechanics and relativity. It has no extra dimensions or supersymmetry. Apart from "infinitesimal" but still fundamental differences it appears compatible with the Standard Model. It requires the universe to expand after the big bang and possibly in an accelerating manner. It also changes the metric around mass concentrations in accordance with General Relativity; a possible stepping stone to linking it with quantum mechanics.

1.1 Summary

Papers modifying the Standard Model are too numerous to list, however we briefly touch on a small number of some early versions of these in section 1.1.2. The approach in this paper is very different from that in most of these earlier papers, with the main differences summarized below.

1.1.1 General Relativity as our starting point

General Relativity tells us that all forms of mass, energy and pressure are sources of the gravitational field. Thus to create gravitational fields all spin ½ leptons & quarks, spin 1 gluons, photons, $\mathrm{W}^{\pm}$ & Z^{0} particles etc. emit virtual gravitons. Virtual gravitons themselves however may behave differently as gravitational energy is not part of the Einstein tensor.

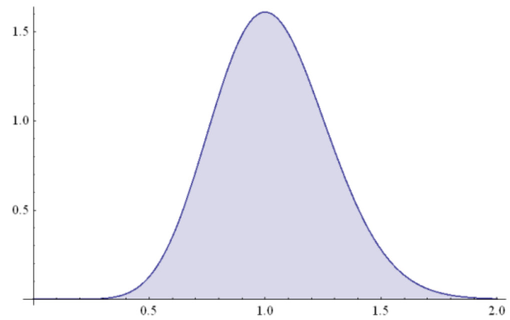
The starting point of this paper assumes there is a common thread uniting these fundamental particles making this possible. Equations are developed that unite the amplitudes of the colour and electromagnetic coupling constants with that of gravity. The precision required by quantum mechanics for half integral and integral angular momentum allows gravity to be included, despite the vast disparity in magnitude between gravity and the other two. This combination of colour, electromagnetic and gravitational amplitudes in the same equation is possible only because of a radically different approach taken in this paper: An approach using infinite superpositions of positive and negative integral $\hbar$ angular momentum virtual wavefunctions for spin ½, spin 1 and spin 2 particles. The final result is almost identical to the Standard Model, with infinitesimal but important differences.

The total angular momentum can be summed over all wavenumbers k; from $k=0$ to some cutoff value k_{cutoff}. We will assume (as with many unification theories) that the cutoff for these infinite superpositions is somewhere near Planck scale. Firstly imagine a universe where the gravitational constant $G \to 0$. As $G \to 0$, the Planck length $L_P \to 0$, the Planck energy $E_P \to \infty$ and $k_{cutoff} \to \infty$ also. If we sum the angular momentum of these infinite superpositions when $G \to 0$ (i.e. from $k=0$ to $k_{cutoff} \to \infty$) we get precisely half integral or integral $\hbar$ for the fundamental spin ½, spin 1 & spin 2 particles in appropriate m states. If we now put $G > 0$ the infinitesimal effect of including gravity can be balanced by an equal but opposite effect due to the non-infinite cutoff value in k. A near Planck scale superposition cutoff requires gravity to be included to get precisely half integral or integral $\hbar$. (Section 4.2)

These infinite superpositions have another very relevant property relating to the fact that all experiments indicate that fundamental particles such as electrons behave as point particles.

Each wavefunction with wavenumber k, which we label as ψ_k, has a maximum radial probability at $r \approx 1/k$ and they all look the same (Figure 1.1. 1.) Every wavefunction ψ_k of these infinite superpositions, interacts only with virtual photons (for example) of the same k; if superpositions representing say an electron are probed with such photons (that interact only with wavefunction ψ_k) the resolution possible is of the same order as the dimensions of ψ_k, both have $r \approx 1/k$. The higher the energy of the probing particle the smaller the ψ_k it interacts with, the resolution of an observing photon can never be fine enough to see any ψ_k dimensions. Even if this energy approaches the Planck value, with a matching ψ_k radius near the Planck length it is still not possible to resolve it. This behaviour is consistent with the quantum mechanical properties of point particles.

$\frac{4\pi}{k} R^* R \uparrow$



$kr \rightarrow$

Figure 1.1. 1 The radial probability of the dominant $n = 6$ for spin ½ wavefunction ψ_{6k}.

1.1.2 Primary interactions and Secondary interactions

Supposing that superpositions can in fact build the fundamental spin ½, spin 1, and spin 2 particles, then what builds the superpositions? Before answering that question, in this paper we divide the world of all interactions into two categories.

Secondary Interactions are those we are familiar with and are covered by the Standard Model, but with the addition of gravity, which is not included in the Standard Model. They take place between fundamental spin ½, spin 1 and spin 2 particles formed from infinite superpositions. They are the real world interactions on which all experiments are based.

Primary Interactions on the other hand are those that build infinite superpositions and are hidden to the real world of experiments.

The majority of this paper is about these primary interactions, and the superpositions they build representing the fundamental spin ½, spin 1 and spin 2 particles. Primary interactions are between spin zero particles borrowed from a Higgs type scalar field and the zero point vector fields. In the 1970's models were proposed with preons as common building blocks of leptons and quarks [1][2][3][4]. In contrast with the virtual particles of this paper some of these earlier models used real spin ½ building blocks. Real substructure has difficulties with large rest masses if compressed into the small volumes required to approach point particle behaviour. On the other hand with *virtual substructure borrowing energy from zero point fields the rest mass contribution at high* k *values can be cancelled* (section 3.2.1). As in earlier models this paper also calls the common building blocks preons, *but here the preons are both virtual and spin zero.* They also now build all spin ½ leptons and quarks, spin 1 gluons, photons, W & Z particles, plus spin 2 gravitons in contrast to only the leptons and quarks in the earlier models. (See Table 2.2. 1.)

As these preons have zero spin they possess no weak charge, primary interactions (section 2.2.1) can take place only with the zero point colour, electromagnetic and gravitational fields. The three primary coupling constants for each of these three zero point fields are different from (but related to) the secondary coupling constants. The behaviour of primary coupling is also entirely different from secondary coupling. Secondary coupling strengths vary (or run) with wavenumber k (the electromagnetic increasing with k and colour decreasing with k). In contrast, primary coupling strengths (or constants) do not run. In this paper virtual preons are continually born out of a scalar Higgs type field, existing only for time $\Delta t \approx \hbar / E$. At their birth, they interact while still bare with zero point vector fields at this instant of birth $t = 0$. The primary coupling constants consequently are fixed for all k: there is no time for charge canceling or reinforcing, which in secondary interactions forms around the bare charge progressively after its birth. The equations work only if this is true, and they also work only if the primary colour coupling constant is 1. This does not seem implausible as it simply means that primary colour coupling is certain (sections 2.2.2). The ratio between the primary and secondary colour coupling constants labelled χ_c is thus (*if primary colour coupling is 1*) the inverse of the secondary (or usual QCD $/\alpha_3^{-1}$) colour coupling constant *at the superposition cutoff near the Planck scale.* (Sections 3.3, 4.1.1 & 4.3)

To enable the primary coupling to colour, electromagnetic and gravitational zero point fields, preons need colour, electric charge and rest mass. Red green or blue coloured preons have positive electric charge; anticolour red, green or blue preons have negative electric charge. Their mass which is borrowed from some type of scalar Higg's field must always be non-zero, which is discussed further in section 1.1.3. As there are 8 gluon fields, superpositions are built with 8 virtual preons for each virtual wavefunction ψ_k. The nett sum of these 8 electric charges is $0, \pm 2, \pm 4, \pm 6$, and never $> \pm 6$. This leads to the usual $0, \pm 1/3, \pm 2/3, \pm 1$

electric charge seen in the real world. Various combinations of these 8 preons in appropriate superpositions can build leptons and quarks, colour changing and neutral gluons, neutral photons, neutral massive Z^0 photons and the charged massive $W^{\pm}$ photons. (Table 2.2. 1)

1.1.3 Photons with infinitesimal mass; "rest mass" versus "proper mass"

For many decades after the discovery of the neutrino in the 1930s it was thought to be massless, and to travel at velocity c. Towards the end of last century however evidence slowly accumulated that this may not in fact be quite true, and that the family of 3 neutrinos have rest masses in the sub electron volt ($\approx 10^{-2}eV.$) range. Due to this very low mass, and their normal emitted energies, they invariably travel at virtually the velocity of light c. Photons also have always been seen as massless traveling precisely at velocity c, except in the case of the massive $W^{\pm}$ & Z^0. Massless virtual photons have an infinite range, which has always been seen as an absolute requirement of the electromagnetic field. On the other hand, this paper requires some rest frame (even if this frame normally moves virtually at c) in which to build all the fundamental particles. So from here on we will frequently use the term *rest mass* for all fundamental particles. Section 6.3 suggests that the *infinitesimal rest mass set* have $\approx 10^{-34}eV$ energy with *a range of approximately the inverse of the causally connected horizon radius,* and velocities sufficiently close to that of light their helicity remains essentially fixed. This allows some form of Higgs mechanism to increase the mass of the infinitesimal set to the various values in the massive set. (Section 6.3.1).

The virtual wavefunction we use is $\psi_{nk} = C_{nk} r^3 \exp(-n^2k^2r^2/18)Y(\theta,\varphi)$ an $l=3$ wavefunction. This virtual $l=3$ property is normally hidden. In the same way as scattering experiments on spin 0 pions show spin 0 properties, and not the properties of the two canceling spin ½ component particles, this $l=3$ property of the virtual components of superpositions is not visible in the real world. Scattering experiments can exhibit only the spin properties of the resulting particle. The individual angular momentum vectors $|\mathbf{L}| = 2\sqrt{3}\hbar$ of the infinite superposition all sum to a resulting: $|\mathbf{L}_{Total}| = (\sqrt{3}/2)\hbar$, $\sqrt{2}\hbar$ or $\sqrt{6}\hbar$ for spin ½ , spin 1 or spin 2 respectively, in a similar way to two spin ½ particles forming spin 0 or spin 1 states.

The wavefunction $\psi_{nk} = C_{nk} r^3 \exp(-n^2k^2r^2/18)Y(\theta,\varphi)$ has Eigenvalues $\mathbf{P}_{nk}{}^2 = n^2\hbar^2k^2$ with $|\mathbf{P}_{nk}| = n\hbar k$, suggesting it borrows n parallel $|\hbar\mathbf{k}|$ quanta from zero point vector fields provided n is integral. We can see this by letting $k \to \infty$ allowing energy $E \to n\hbar\omega$ by absorbing n quanta $\hbar\omega$ from the zero point vector fields (section 2.3.2). As spin 3 needs at least 3 spin 1 particles to create it, the lowest integral number that n can be is 3. The virtual $l=3$ property can however be used to derive the magnetic moment of a charged spin ½, $m = \pm 1/2$ state as a function of n. Section 3.5 shows $g = 2$ Dirac electrons need an average (over integral n states) of $\bar{n} \approx 6.0135$. Three member superpositions $\psi_k = \sum c_{nk}\psi_{nk}$ with $n = 5, 6, \&7$ achieve this, creating Dirac spin ½ states. We also find that $n = 6$ is the dominant member and each superposition ψ_k needs at least 3 members to make all the equations consistent for Dirac particles. Secondary interactions at any wavenumber k can occur with ψ_k if integers n change by ± 1, thus changing the Eigenvalues $|\mathbf{P}| = n\hbar k$ by $\pm \hbar k$ where this can be only a temporary rearrangement of the triplets of values of n. This is true, whether the interaction is with leptons, quarks, photons, gluons, W & Z particles, or gravitons. (Section 3.3)

1.1.4 Superpositions require only squared vector potentials

The wavefunction $\psi_{nk} = C_{nk} r^3 \exp(-n^2k^2r^2/18) Y(\theta, \varphi)$ also requires a squared vector potential to create it: $Q^2A^2 = n^4\hbar^2k^4r^2/81$. There are no linear potential terms in contrast with secondary interactions. In our momentum operator for primary interactions we thus use $\hat{P}^2 = -\hbar^2\nabla^2 + Q^2A^2$, no linear potential terms are included and Q simply represents a collective symbol for all the effective charges concerned. As an example, the dominant $n = 6$ wavefunction of a spin ½ Dirac ψ_k requires a squared vector potential of $Q^2A^2 = n^4\hbar^2k^4r^2/81 = 16\hbar^2k^4r^2$ (section 2.3.1). Primary coupling between the 8 virtual preons and the colour, electromagnetic and gravitational zero point fields produces a vector potential squared value for all infinite superpositions which can be expressed as:

$$Q^2A^2 = \frac{\left[8 + 8\sqrt{\alpha_{EMP}} + im_0(1+\beta_k)\sqrt{G_p/(2s\hbar c)}\right]^2 (\hbar^2k^4r^2)}{3\pi(sN)(1+\varepsilon)} \left[\frac{(sN)(1+\varepsilon)dk}{k}\right]$$

(Where the length of the complex vector is squared here.) The significance of the cancelling top and bottom factors (sN) is explained in section 2.1.2. Also the cancelling $(1+\varepsilon)$ factors are due to gravity and explained in section 4.2. The primary colour coupling amplitude is 1 to each of the eight preons, and $\sqrt{\alpha_{EMP}}$ the primary electromagnetic coupling. This equation applies regardless of the individual preon colour or electric charge signs, whether positive or negative (section 2.2.3). The primary gravitational coupling is to the particle rest mass m_0, and its momentum $m_0\beta_k$. The primary gravitational constant is G_P divided by $\hbar c$ to put it in the same form as the other two coupling constants. The magnitude of the total angular momentum vector of the infinite superposition is $|\mathbf{L}_{Total}| = \sqrt{s(s+1)}$.) This Q^2A^2 without the gravity term generates superpositions with probability $(N \cdot s)dk/k$ where s is the superposition spin, $N = 1$ for massive superpositions and $N = 2$ for infinitesimal rest mass superpositions (see Table 4.3. 1). Section 4.2 includes gravity raising the superposition probability to $(1+\varepsilon)(N\cdot s)dk/k$ where the infinitesimal ε (not to be confused with infinitesimal rest mass) is $\varepsilon \approx m_0^2 G_P/(110s\hbar c) \approx 10^{-45}$ for electrons, and $\varepsilon \approx 10^{-34}$ for a Z^0 The ψ_k superpositions require at least three integral n members. The following three member superpositions seem to fit the Standard Model (see Table 4.3. 1)

Spin ½ massive $N=1$ fermion superpositions $\quad \psi_k = \sum_{n=5,6,7} c_{nk}\psi_{nk}$.

Spin 1 massive $N=1$ boson superpositions $\quad \psi_k = \sum_{n=4,5,6} c_{nk}\psi_{nk}$.

Spins 1 & 2 infinitesimal mass $N=2$ superpositions $\quad \psi_k = \sum_{n=3,4,5} c_{nk}\psi_{nk}$.

Below are infinite virtual superpositions $\psi_{\infty,s,m}$ for only *massive spins* 1 & 2. The symbol ∞ refers to the infinite sum, s the spin of the resulting real particle, m its angular momentum state, and ss a spherically symmetric state. Section 3.1.3 explains this format. Also square cutoffs in wavenumber k are used here for simplicity. Infinite exponential cutoffs in k are introduced in section 4.3. Infinitesimal rest mass superpositions are introduced in section 6.3.

$$\text{Massive } N=1 \text{ Spin } \tfrac{1}{2}, \psi_{\infty,1/2,m} = \sum_{n=5,6,7} c_n \int_0^{k(cutoff)} \left[\frac{1}{\gamma_{nk}}(\psi_{nk,ss}) + \beta_{nk}(\psi_{nk,4m})\right]\sqrt{\frac{1+\varepsilon}{2k}}dk$$
$$\text{Massive } N=1 \text{ Spin } 1, \psi_{\infty,1,m} = \sum_{n=4,5,6} c_n \int_0^{k(cutoff)} \left[\frac{1}{\gamma_{nk}}(\psi_{nk,ss}) + \beta_{nk}(\psi_{nk,2m})\right]\sqrt{\frac{(1+\varepsilon)}{k}}dk \qquad (1.1.\ 1))$$

In these infinite superpositions the probability that the wavefunction is spherically symmetric is $1/\gamma_{nk}{}^2 = 1-\beta_{nk}{}^2$ and the probability that it is an m state is $\beta^2{}_{nk}$ where β_{nk} is the magnitude of the velocity of the centre of momentum of the primary interactions that generate each ψ_{nk} . This is similar to the superposition of time and spatially polarized virtual photons in QED. For example spin ½ has probabilities of $1/\gamma_{nk}{}^2 = 1-\beta_{nk}{}^2$ spherically symmetric ψ_{nk} wavefunctions, and $\beta^2{}_{nk} \times (\psi_{nk}, m=\pm 2)$ wavefunctions. Each ψ_k is normalized to 1 but the infinite superpositions $\psi_{\infty,s,m}$ are not normalized, diverging logarithmically with k ; the same logarithmic divergence that applies to virtual photon emission. *Real wavefunctions have to be normalized to one as they refer to finding a real particle somewhere but this need not apply here.* Each member of these spin ½ superpositions has probability $dk(1+\varepsilon)/2k$, and if electrically charged emits virtual photons with probability $4\alpha/\pi$. Ignoring the factor of $(1+\varepsilon) \approx 1+\approx 10^{-45}$, the overall virtual scalar photon emission probability is the usual

$\left(2\alpha/\pi\right)dk/k$. (We discuss possible implications of the infinitessimal ε in section 6.4) We also find in section 3.1 that $m=+2$ virtual wavefunctions have $\beta^2{}_{nk}$ probability of leaving an $m=-2$ debt in the zero point fields. Integrating this over all k produces a total angular momentum for a spin ½ state of $(\hbar/2)(1-1/\gamma^2{}_{Cutoff})\left(1+\varepsilon\right)$, (section 3.2.2). When $1/k_{Cutoff}$ is near the Planck length $(1-1/\gamma^2{}_{Cutoff})=1/(1+\varepsilon)$.

A similar integration over all k for the rest energy of the infinite superposition also leads to $\pm m_0c^2(1-1/\gamma^2{}_{Cutoff})\left(1+\varepsilon\right)$, (section 3.2.1). The infinitesimal quantity ε vanishes in a zero gravity, zero Planck length universe where $k_{Cutoff}\ \&\ \gamma_{Cutoff}\to\infty$. In this paper each preon borrowed from a Higgs type scalar field has virtual rest mass. The superposition rest energy is obtained by summing squared momenta over all k. Each of these momenta is a sum of positive and negative quanta $\left|\hbar k\right|$.The equations are based on probabilities of these in a similar manner to those for angular momentum which is why the final rest mass energy equation above is of the same form (section 3.2). This implies that rest mass is both energy and mass borrowed from zero point vector, and Higgs type scalar fields. In section 5 this idea extended to the extreme wavelength virtual gravitons spanning the causally connected cosmos requires it to continually expand. It may also link quantum mechanics with General Relativity as the space at any radius *r* around a mass *m* has to expand proportionally to *m/r* in accordance with the Schwarzschild solution (section 5.2).

This paper is mainly about the primary interactions between spin zero preons and spin one quanta that build the fundamental particles. The Standard Model is about the secondary interactions between them. The weak force is only between spin ½ particles and thus a secondary interaction. It is not involved in primary interactions. Apart from various infinitesimal effects, such as infinitesimal photon rest masses, the properties of the fundamental particles covered in this paper seem to agree with their Standard Model counterparts. This paper relates the colour coupling constant of secondary (Standard Model) interactions at the near Planck length cutoff to the secondary (Standard Model) interaction electromagnetic coupling constant also at this cutoff. Section 4.3 suggests gravity cuts off all interactions exponentially somewhere between ≈ 2 and $2.1\times10^{18}GeV$. or $\approx 1/6$ of the Planck energy. The secondary electromagnetic coupling constant suggested at this cutoff is $\alpha^{-1}{}_{EMS}=\alpha^{-1}{}_{EM}\approx105.93\pm0.17$. This is in line with Standard Model predictions assuming three families of fermions and one Higgs field. (See Figure 4.1. 1 & Figure 4.1. 2). At this cutoff the Standard Model secondary colour coupling constant is $\alpha^{-1}{}_3\approx50.41\pm0.22$. This is also the primary to secondary coupling ratio χ.

Because it is entirely conjectural (and does not fit the Standard Model) we leave it to our conclusions to discuss a possible dark matter, massive $N=1$ spin 2 infinite superposition; it does not interact with spin 1 virtual photons or gluons, but with only virtual gravitons.

2 Building Infinite Virtual Superpositions

2.1 The possibility of Infinite Superpositions

2.1.1 Early ideas

After World War II there was still much confusion about QED. In 1947 at the Long Island Conference the results of the Lamb shift experiment were announced [5]. Some of the first early explanations that gave approximately correct answers used simple semi classical thinking to get a better understanding of what seemed to be going on. These early ideas helped to eventually lead to the QED of today, perhaps in a similar manner to the way Bohr's original simple semi classical explanation of quantized atomic energy levels played such a large part in the eventual development of full three dimensional wavefunction solutions of atoms, and quantum mechanics. We start this paper with an example of a semi classical Lamb shift explanation that seems to lead into the possibility of fundamental particles and infinite virtual superpositions being one and the same.

The density of transverse modes of waves at frequency ω is $\omega^2 d\omega/\pi^2 c^3$ and the zero point energy for each of these modes is $\hbar\omega/2$. The electrostatic and magnetic energy densities in electromagnetic waves are equal, thus for electromagnetic zero point fields:

$$\frac{\overline{\varepsilon_0 E^2}}{2}+\frac{\overline{\varepsilon_0 c^2 B^2}}{2}=\frac{\hbar\omega}{2}\left[\frac{\omega^2 d\omega}{\pi^2 c^3}\right] \quad \text{and} \quad \overline{\varepsilon_0 E^2}=\overline{\varepsilon_0 c^2 B^2}=\frac{\hbar\omega^4}{2\pi^2 c^3}\frac{d\omega}{\omega}.$$

For a fundamental charge e using $\alpha=e^2/4\pi\varepsilon_0\hbar c,$ and provided $\beta<<1,$ this gives an

$$\text{Average force squared of} \quad \overline{F^2}=\overline{e^2E^2}=\frac{2\alpha}{\pi}\frac{\hbar^2\omega^4}{c^2}\frac{d\omega}{\omega} \tag{2.1. 1}$$

Thinking semi classically, for an electron of rest mass m this can generate simple harmonic motion of amplitude r, where $F^2 = m^2\omega^4 r^2$ (if $\beta << 1$). Solving for r^2 (where r^2 is superimposed on the normal quantum mechanical electron orbit, $\lambda\!\!\!^{-}_c = \hbar / mc$ is the Compton wavelength, and $k = \omega / c$):

$$r^2 = \frac{\hbar^2}{m^2c^2}\frac{2\alpha}{\pi}\frac{d\omega}{\omega} \quad = \quad \left[\lambda\!\!\!^{-}_c{}^2\right].\left[\frac{2\alpha}{\pi}\frac{dk}{k}\right]$$

Integrating r^2:

$$r^2{}_{Total} = \lambda\!\!\!^{-}_c{}^2 \frac{2\alpha}{\pi}\int_{k\min}^{k\max}\frac{dk}{k} \quad = \quad \lambda\!\!\!^{-}_c{}^2 \frac{2\alpha}{\pi}\log(k_{\max} / k_{\min}) .$$

The minimum and maximum values for k are chosen to fit atomic orbits, and a root mean square value for r can be found. Combining this with the small probability that the electron will be found in the nucleus, this small root mean square deviation shifts the average potential by approximately the Lamb shift. This can also be thought of as simple harmonic motion of amplitude $\approx \lambda\!\!\!^{-}_c$, occurring with probability $(2\alpha / \pi)dk / k$. It can also be interpreted as the electron recoiling by $\approx \lambda\!\!\!^{-}_c$ (when $\beta << 1$) in random directions due to virtual photon emission with a probability of $(2\alpha / \pi)dk / k$.

2.1.2 Dividing probabilities into the product of two component parts

This probability $(2\alpha / \pi)dk / k$ can be thought of as the product of two terms A & B, where A includes the electromagnetic coupling constant α, B includes dk / k, and $AB = (2\alpha / \pi)dk / k$. This suggests that this same behaviour is possible if we have an appropriate superposition of virtual wavefunctions occurring with probability B, which emits virtual photons with probability A (by changing Eigenvalues $|\mathbf{p}_{nk}| = n\hbar k$ by $n = \pm 1$). For example, if a virtual superposition occurs with probability $B = (N \cdot s)dk / k$, and has a virtual photon emission probability for each member of these superpositions of $A = (N \cdot s)^{-1}(2\alpha / \pi)$, then the overall virtual photon emission probability remains as above at $AB = (2\alpha / \pi)dk / k$. This applies equally whether it is virtual gluon/photon/W&Z/graviton etc. emission. Provided A includes the appropriate coupling constant this same logic applies regardless of the type of boson emitted. As is usual to get integral or half integral total angular momentum $2s$ has to be integral and section 6.3 suggests the same applies for N. (This paragraph is simplified to illustrate the principle and will later be modified in section 3.3.)

In section 1.1.4 we said that these wavefunctions are built with squared vector potentials. If superpositions of them are to represent real particles they must be able to exist anywhere. This is possible only if they are generated by uniform fields. The only fields uniform in space-time are the zero point fields, and looking at the electromagnetic field first we can use section 2.1.1 above. Consider a vector $\mathbf{r}$ from some central origin O and a magnetic field vector $\mathbf{B}$ through origin O, then the vector potential at point $\mathbf{r}$ is $\mathbf{A} = (\mathbf{B} \times \mathbf{r})/2$ and the vector potential squared is $A^2 = (B^2 r^2 \sin^2\theta)/4$ where the angle between vectors $\mathbf{B}$ & $\mathbf{r}$ is θ.

$$\text{As } \sin^2\theta \text{ averages 2/3 over a sphere: } \overline{A^2} = B^2 r^2 / 6 \qquad (2.1.\ 2)$$

Here B^2 is the magnetic field squared at any point due to the cubic intensity of zero point EM also as in section 2.1.1. Putting Eq's. (2.1. 1) & (2.1. 2) together the vector potential squared is

$$\overline{e^2 A^2} = \frac{e^2 B^2 r^2}{6} = \frac{\alpha}{3\pi}\frac{\hbar^2 \omega^4 r^2}{c^4}\frac{d\omega}{\omega} = \frac{\alpha}{3\pi}\hbar^2 k^4 r^2 \frac{dk}{k} \qquad (2.1.\ 3)$$

As in section 2.1.2 we can divide this into two parts, noting the inclusion of spin s and integer N in the numerator and denominator:

$$\overline{e^2 A^2} = \left[\frac{\alpha}{3\pi sN}\hbar^2 k^4 r^2\right] . \left[\frac{sN \cdot dk}{k}\right] \qquad (2.1.\ 4)$$

But here a vector potential squared term $\left[\frac{\alpha}{3\pi sN}\hbar^2 k^4 r^2\right]$ *occurs with probability* $\left[\frac{sN \cdot dk}{k}\right]$. *Another way of looking at this is that a wavefunction* ψ_k *that is generated by a vector potential squared term* $\left[\frac{\alpha}{3\pi sN}\hbar^2 k^4 r^2\right]$ *can occur with* $\left[\frac{sN \cdot dk}{k}\right]$ *probability.*

This is similar reasoning to that used in the semi classical Lamb shift explanation of section 2.1.1. In the first bracketed term of Eq. (2.1. 4), α is the electromagnetic coupling constant, but the same logic applies for the eight gluon and gravitational zero point vector fields where we will sum appropriate amplitudes of these and square this total as our effective coupling constant in Eq. (2.1. 4). But first we need to look at groups of spin zero preons that could build our wavefunctions. What mixtures of colours and electrical charges end up with the appropriate final colour and electrical charge for each of the fundamental particles or at least the ones we know of?

2.2 Spin Zero Virtual Preons from a Higgs type Scalar Field

2.2.1 Groups of eight preons that form superpositions

In this paper preons have zero spin and can have no weak charge. The only fields they can interact with (via Primary Interactions that build superpositions as in section 1.1.2) are colour, electromagnetic and gravity. In the simplest world there would be just one type of preon that comes in three colours, always positively charged say, with their three anti colours all negatively charged. We will assume that this is true unless it does not work. Looking at Table 2.2. 1 we see that a minimum of 6 preons is required to get the correct charge ratios of 3:2:1 between electrons, and up and down quarks. To get vector potential squared values that make all our equations work however, we need to couple to all 8 gluon fields requiring a total of 8 preons. Table 2.2. 1 has all the basic properties required to build infinite superpositions for the fundamental particles. We need to remember when looking at this table that from section 1.1.2 the effective secondary charge is much less than the primary charge and we have no idea yet of just what effective value the primary preon electric charge is.

Particles only are addressed in the groups of preons in Table 2.2. 1. To get anti particles it would seem that we can just change the signs of each preon in the groups of 8, excepting those that are already their own antiparticle. The first point to notice however is that both the electron and the W^- are predominantly anti preons, yet they are both defined as particles. Have we got something wrong? When we look at relativistic rest masses in section 3.2.1 we get the usual plus and minus solutions and Feynman showed us how to interpret the negative solutions as antiparticles. If this also applies in anti preons then because they are zero spin, and the weak force discriminates between particles and antiparticles by their helicity, this discrimination can apply only in secondary interactions. The preon anti preon content of the groups in Table 2.2. 1 does not necessarily tell us whether they produce particles or antiparticles. We will discuss this further in section 3.2.1, also as of now there is still no good understanding of the predominance of matter over antimatter in our universe. In Table 2.2. 1 only one example of colour is given for quarks and gluons. Different colours can be obtained by simply changing appropriate preon colours. Various combinations of 8 preons in this table are borrowed from a scalar field for time $\Delta T \approx \hbar / \Delta E$, this process continually repeating in time. Conservation of charge normally allows only opposite sign pairs of electric charges to appear out of the vacuum. Let us imagine that these virtual preons are building an electron for example whose electric charge exists continually unless it meets a positron and is annihilated.

Fundamental Particles	Preon colour	Preon electric charge.	Group colour	Group electric charge.
Spin ½ Neutrino family. Spin 1 Photons, Z_0 & Neutral gluons. Spin 2 Gravitons.	Any colour + its Anticolour Red Antired Green Antigreen Blue Antiblue	1 -1 1 -1 1 -1 1 -1	Colourless	0
Spin ½ Electron family. Spin 1 W^-.	Any colour + its Anticolour Antired Antired Antigreen Antigreen Antiblue Antiblue	1 -1 -1 -1 -1 -1 -1 -1	Colourless	-6
Spin ½ Blue up quark family.	Red Antired Green Antigreen Green Blue Blue Red	1 -1 1 -1 1 1 1 1	Blue	+4
Spin ½ Red down quark family.	Green Antigreen Red Antired Green Antigreen Antiblue Antigreen	1 -1 1 -1 1 -1 -1 -1	Red	-2
Spin 1 Red to green Gluons.	Red Antigreen Red Antired Green Antigreen Blue Antiblue	1 -1 1 -1 1 -1 1 -1	Red plus antigreen	0

Table 2.2. 1 Groups of 8 virtual preons forming the fundamental particles.

This charged electron is thus due to a continuous appearance out of and back into the vacuum of virtual charged preons in a steady state process existing for the life of the superposition, and not conflicting with conservation of charge. If the electron itself does not conflict then neither do the borrowed preons that build it.

2.2.2 Primary coupling constants behave differently

Q.E.D. tells us that the bare (electric) charge of an electron for example increases logarithmically inversely with radius from its centre. Polarizations of the vacuum (of virtual charged pairs) progressively shield the bare charge from a radius of approximately one Compton radius $\lambdabar_c$ inwards towards the centre. When an electron (for example) is created in some interaction the full bare charge is exposed for an infinitesimal time. Instantaneously after its creation, shielding due to polarization of the vacuum builds progressively outward from the centre of its creation at the velocity of light. For radii $\geq \lambdabar_c$ we measure the usual fundamental charge e. There are similar but more complicated processes that occur to the colour charge. Camouflage is the dominant one where the colour charge grows with radius as the emitted gluons themselves have color charge. At the instant of their birth the preons are bare and at this time $t = 0$ say, all the zero point vector fields can act on these bare colour and electric charges as there is simply no time for shielding and other effects to build. The primary coupling constants that we use must consequently be the same for all values of k in complete contrast to those for secondary interactions. We don't know what this primary electromagnetic coupling constant is so we will just call it α_{EMP}. Also we will find that to get any sense out of our equations the primary colour coupling has to be approximately 1. A coupling of 1 is a natural number and simply reflects certainty of coupling. Provided the secondary colour coupling can be in line with the Standard Model and there does not seem to be any other good reason to pick a number less than 1, we will make the (apparently arbitrary) assumption that the bare primary colour coupling is exactly 1. (In section 4.1.1 we will find that this does seem to be consistent with the Standard Model.)

2.2.3 Primary interactions also behave differently

Let us define a frame in which the central origin of the wavefunctions ψ_k of our infinite superposition is at rest: The laboratory or rest frame we will refer to as the LF. The preons that build each ψ_k are born from a Higg's type scalar field with zero momentum in this frame. This has very relevant consequences as their *wavelength is infinite* in this rest frame at time $t = 0$, and after they build wavefunction ψ_k their wavelength is of the order $1/k$ for times $0 < t < \hbar / 2E$. This implies that there could possibly be significant differences in the way amplitudes are handled between primary and secondary interactions.

Let us consider secondary interactions first with an electron and positron for example located approximately distance r apart. For photon wavelengths $<< r$ both the electron and the positron each emit virtual photons with probabilities proportional to α, but for wavelengths $>> r$ their amplitudes cancel. Returning to primary interactions, zero momentum preons must always have an infinite wavelength which is greater than the wavelengths (or $1/k$ values) of the zero point quanta they interact with, for all $k \neq 0$. This implies that we cannot simply add or subtract amplitudes algebraically; the charges are always further apart than the wavelength of the interacting quanta (except in the special case where $k = 0$). In fact if algebraic addition of amplitudes did apply in primary interactions, infinite superpositions for colourless and electrically neutral neutrinos would be impossible. So how can infinitely far apart preons of differing charge generate wavefunctions of all dimensions down to Planck scale? This can happen only if the amplitudes of all 8 preons are somehow linked over infinite space, all at the same time $t = 0$ contributing to generating the wavefunction ψ_k. This non-local behaviour is not new. Recent experiments have confirmed that what Einstein struggled to come to terms with is in fact true; he called it “spooky action at a distance”. While these experiments are so far limited in the distance over which they demonstrate entanglement, there is now wide acceptance that it can reach across the Universe. In the same manner wavefunctions covering all space can instantly collapse. We want to suggest here that this same non-locality applies in primary interactions: our 8 virtual preons all unite instantaneously at time $t = 0$ across infinite space in generating each ψ_k. Also the vector potential squared equations that they generate must always be the same for all the preon combinations in Table 2.2. 1. This can happen only if the amplitudes of all 8 are added regardless of charge sign for primary interactions. This applies to both colour and electric charge.

The opposite is true for the secondary interactions. At time $t = 0$ all 8 preons instantaneously collapse into some sort of virtual composite particle that for times $0 < t < \hbar / 2E$ obeys wavefunction ψ_k. The dimensions of ψ_k are of the same order as the wavelength of the interacting quanta, and the usual algebraic total electric charge and nett colour charge must now apply as in the group charges in Table 2.2. 1. All of this may seem contrary to current thinking which has gradually been built up over several centuries of secondary interaction experiments; however it may not be so out of place when viewed in the context of the counter intuitive results of entanglement experiments. The key point to bear in mind is that the predictions of this paper must agree or at least be able to fit the Standard Model, or secondary interaction experiments, as we may never be able to look into virtual primary interactions, but only observe their effects.

Amplitudes to interact are complex numbers which we can draw as a vector. This applies to both colour and electric coupling, where these two vectors can be at the same complex angle or at different angles. The simplest case is if they are in line and we will assume this is true for both colour and electromagnetic primary interactions which are both spin 1. This seems to work and when we later include gravity, a spin 2 interaction, we find that the spin 2 vector only works if it is at right angles to the two in line spin 1 vectors. *Let us start in a zero gravity world* by simply adding the 8 preon colour vectors of amplitude 1 and the eight primary electromagnetic vectors of amplitude $\sqrt{\alpha_{EMP}}$ together, as all this only works if they are all in line.

The total colour plus electromagnetic primary amplitude is $8+8\sqrt{\alpha_{EMP}}$ (2.2. 1)

This equation is always true regardless of signs as in section 2.2.3

The colour plus electromagnetic primary coupling constant is $\left(8+8\sqrt{\alpha_{EMP}}\right)^2$ (2.2. 2)

Inserting this into Eq. (2.1. 4) we get

$$Q^2A^2 = \left[\frac{\left[8+8\sqrt{\alpha_{EMP}}\right]^2}{3\pi sN}\hbar^2k^4r^2\right]\cdot\left[\frac{sN\cdot dk}{k}\right] \tag{2.2. 3}$$

Again we interpret this just as we did in section 2.1.2 and Eq. (2.1. 4) as a vector potential squared term

$$Q^2A^2 = \frac{\left[8+8\sqrt{\alpha_{EMP}}\right]^2}{3\pi sN}\hbar^2k^4r^2 \text{ occurring with probability } = \frac{sN\cdot dk}{k} \tag{2.2. 4}$$

Where Q is a symbol representing all the 8 colour and 8 electric amplitudes combined, with s the spin and $N=1$ for massive and $N=2$ for infinitesimal mass infinite superpositions.

2.3 The Virtual Wavefunctions that form Infinite Superpositions

2.3.1 Infinite families of similar wavefunctions

Consider the family of wave functions where ignoring time:

$$\begin{aligned} \psi_{nk} &= U(nrk)Y(\theta\varphi) \\ U(nrk) &= C_{nk} r^{l} \exp(-n^2k^2r^2/18) \end{aligned} \tag{2.3. 1}$$

$U(nrk)$ is the radial part of ψ_{nk}, $Y(\theta\phi)$ the angular part, C_{nk} a normalizing constant, and we will find that l is the usual angular momentum quantum number. There is an infinite family of ψ_{nk}, one for each value k where $0 < k < \infty$ *in a zero gravity world.*

$$\text{Now put } R(nrk) = rU(nrk) = C_{nk} r^{l+1} \exp(-n^2k^2r^2/18) \tag{2.3. 2}$$

As we are dealing with zero spin preons we use Klein-Gordon equations [6]. The Klein-Gordon equation is based on the relativistic equation $\mathbf{p}^2 = E^2/c^2 - m_0{}^2c^2$ and in a squared vector potential the Time Independent Klein Gordon Equation is

$$\hat{P}^2\psi = -\hbar^2\nabla^2\psi + Q^2A^2\psi = \left[\frac{E^2}{c^2} - m_0{}^2c^2\right]\psi \tag{2.3. 3}$$

Using $\quad \dfrac{\nabla^2\psi}{\psi} = \dfrac{1}{R}\dfrac{\partial^2 R}{\partial r^2} - \dfrac{l(l+1)}{r^2} \quad$ we get the Time Independent

$$\text{Radial Klein Gordon Equation} \quad \frac{\hbar^2}{R}\frac{\partial^2 R}{\partial r^2} = \frac{l(l+1)\hbar^2}{r^2} + Q^2A^2 - \left[\frac{E^2}{c^2} - m_0c^2\right] \tag{2.3. 4}$$

For each ψ_{nk} the energy is E_{nk} a function of $n\,\&\,k$, and we will label the rest mass as m_{0snk} a function of spin s, $n\,\&\,k$, but also a function of the particle rest mass m_0 and this becomes

$$\frac{\hbar^2}{R}\frac{\partial^2 R}{\partial r^2} = \frac{l(l+1)\hbar^2}{r^2} + Q^2A^2 - \left[\frac{E_{nk}{}^2}{c^2} - m_{0snk}{}^2c^2\right] \tag{2.3. 5}$$

Differentiating $R(nrk) = rU(nrk) = C_{nk} r^{l+1} \exp(\frac{-n^2k^2r^2}{18})$ twice with respect to r, multiplying by $\hbar^2$ and dividing by R

$$\frac{\hbar^2}{R}\frac{\partial^2 R}{\partial r^2} = \frac{l(l+1)\hbar^2}{r^2} + \frac{n^4\hbar^2k^4r^2}{81} - \frac{(2l+3)n^2\hbar^2k^2}{9} \tag{2.3. 6}$$

Comparing Eq's. (2.3. 5) & (2.3. 6) we see that l is the usual angular momentum quantum number and the vector potential squared required to generate these wavefunctions is

$$Q^2A^2 = \frac{n^4\hbar^2k^4r^2}{81} = \left[\frac{n}{3}\right]^4 \hbar^2k^4r^2 \tag{2.3. 7}$$

The momentum squared is $\mathbf{p}_{nk}{}^2 = \frac{E_{nk}{}^2}{c^2} - m_{0snk}{}^2c^2 = \frac{(2l+3)n^2\hbar^2k^2}{9}$ (2.3. 8)

For $l = 3$ wavefunctions this becomes $\mathbf{p}_{nk}{}^2 = n^2\hbar^2k^2$ & $|\mathbf{p}_{nk}| = n\hbar k$ (2.3. 9)

2.3.2 Eigenvalues of these wavefunctions and parallel momentum vectors

From Eq.'s (2.3. 8) & (2.3. 9) as $k \to \infty$, the energy squared $E_{nk}{}^2 \to \mathbf{p}_{nk}{}^2c^2 = n^2\hbar^2\omega^2$ or

If $l = 3$ when $k \to \infty$ energy $E_{nk} \to n\hbar\omega$ (taking the positive case). (2.3. 10)

In virtual exchange processes $E_X{}^2 = \mathbf{p}_X{}^2c^2 + m_{0X}{}^2c^4$ (where subscript X refers to the virtual exchanged quantities) is not always true. Also there is no mass exchange here as the preons are born with a virtual rest mass at $t = 0$, followed instantaneously by this interaction. As $k \to \infty$ however, the virtual quanta exchanged start to behave as real; each quanta transferring energy $\hbar\omega$, and total energy $n\hbar\omega$, if n is integral. When $k << \infty$ however, the energy transferred $E_X{}^2 < \mathbf{p}_{nk}{}^2c^2$ and $< n^2\hbar^2\omega^2$, but as virtual quanta can transfer $|\mathbf{p}| = \hbar k$ and at the same time have $E_X < \hbar\omega$, this allows n to remain integral for all k. This can be thought of as n parallel momentum vector $|\mathbf{p}| = \hbar k$ quanta, transferring total momentum $|\mathbf{p}_{nk}| = n\hbar k$ and energy $E_X \le n\hbar\omega$ to wavefunction ψ_{nk}. This implies $\psi_{nk} = C_{nk}r^3\exp(-n^2k^2r^2/18)$ wavefunctions have (where $0 < k < \infty$ *until we introduce gravity* when $k_{Cutoff} < \infty$)

Eigenvalues $\mathbf{p}_{nk}{}^2 = n^2\hbar^2k^2$ if n is integral and $l = 3$. (2.3. 11)

Also there are no scalar potentials involved, only squared vector potentials, so this is a magnetic or vector type interaction. Particles in classical magnetic fields have a constant magnitude of linear momentum which is consistent with the squared momentum Eigenvalues of Eq. (2.3. 11).This also implies that each ψ_{nk} is formed from quanta of wave number k only and that secondary interactions with ψ_{nk} emit or absorb $|\hbar k|$ virtual quanta if n changes by ± 1. The wavefunction ψ_{nk} is virtual and in this sense both the energy E_{nk} and rest mass m_{0snk} in Eq. (2.3. 8) are also virtual quantities borrowed from some sort of scalar Higg's field. We use these virtual quantities to calculate the amplitude that ψ_{nk} is in an m state of angular momentum in section 3.1, and in section 3.2 to calculate total angular momentum and real rest mass. As in section 2.3.2 above, we can think of $|\mathbf{p}_{nk}| = n\hbar k$ as n parallel momentum vectors $|\mathbf{p}| = \hbar k$. As spin 3 (or $l = 3$) needs at least 3 spin 1 quanta to build it n must be at least 3. When $n = 3$ we can think of this as 3 of the 8 preons each absorbing quanta $|\hbar k|$ at time $t = 0$. We will find that a spin ½ state has a dominant $n = 6$ mode where 6 of the 8 preons each absorb quanta $|\hbar k|$. It needs at least two smaller side modes $n = 5$ & $n = 7$ with either 5 or 7 of the 8 preons each absorbing quanta $|\hbar k|$ respectively at $t = 0$ as in Figure 3.1. 4 where a positron superposition is illustrated.

From Eq. (2.3. 7) $Q^2A^2 = \left[\frac{n}{3}\right]^4 \hbar^2 k^4 r^2 \; = 16\hbar^2 k^4 r^2$ for this dominant $n = 6$ mode.

Thus from Eq. (2.2. 4) $Q^2A^2 = \dfrac{\left[8 + 8\sqrt{\alpha_{EMP}}\right]^2}{3\pi s}\hbar^2 k^4 r^2 \; = 16\hbar^2 k^4 r^2$ for an $n = 6$ mode.

Now s = ½ for spin ½ and $\dfrac{2\left[8 + 8\sqrt{\alpha_{EMP}}\right]^2}{3\pi} = 16$ if we have only an $n = 6$ mode.

Thus $8 + 8\sqrt{\alpha_{EMP}} = \sqrt{24\pi}$ and $\alpha_{EMP}^{-1} \approx 137.1$, but this is true for an $n = 6$ mode only, and we have a superposition where the amplitudes of the smaller side modes $n = 5$ & $n = 7$ determine the ratio between the primary to secondary (colour and electromagnetic) coupling amplitudes or the value of $\alpha_3^{-1} @ L_P$ (Section 3.3). The Q^2A^2 required to produce this superposition with amplitudes c_n is, using Eq. (2.3. 7)

$$Q^2A^2 = \sum_{n=5,6,7} c_n * c_n \frac{n^4\hbar^2 k^4 r^2}{81} \tag{2.3. 12}$$

Repeating the same procedure as above for these superpositions and using Eq. (2.3. 12) we find the strength of α_{EMP} required increases considerably (see sections 4.1.1 & 4.3). As the secondary electromagnetic coupling $\alpha_{EMS}{}^{-1} @ L_P$ must be constant for all spin ½ leptons and quarks the amplitudes of the smaller side modes $n=5$ & $n=7$ that determine this must also be constant for all the fermions, implying that Eq. (2.3. 12) must be the same for all fermions. The same arguments apply to the other groups of fundamental particles but we will return to this in sections 3.3 where we see that the same also applies with graviton emission.

3 Properties of Infinite Superpositions

3.1 What is the Amplitude that ψ_{nk} is in an *m* state?

3.1.1 Four vector transformations

The rules of quantum mechanics tell us that if we carry out any measurement on a real spherically symmetric $l=3$ wavefunction it will immediately fall into one of the seven possible states $l=3, m=0,\pm1,\pm2,\pm3$ [7]. But ψ_{nk} is a virtual $l=3$ wave function so we cannot measure its angular momentum. During its brief existence it must always remain in some virtual superposition of the above seven possible states and we can describe only the amplitudes of these. So is there any way to calculate these amplitudes as they must relate to the amplitudes of the angular momentum states of the spin 1 quanta it absorbs from the zero point vector fields? First consider the 4 vector wavefunction of a spin 1 particle and start with a time polarized state which has equal probability of polarization directions. It is thus spherically symmetric, which we will label as ss. Using 4 vector (*t, x, y, z*) notation:

In frame A, a time polarized or ss spin 1 state is (1,0,0,0).

Let frame B move along the z axis at velocity $\beta = v/c$ in the z direction.

In frame B the polarization state transforms to $(\gamma, 0, 0, \gamma\beta)$.

But this is γ^2 time polarized (ss states) minus $\gamma^2\beta^2 \times z$ polarized $(m=0$ states).

In frame B there are $\gamma^2 \times ss$ states $-\gamma^2\beta^2 \times m=0$ states.

Now $\gamma^2 - \gamma^2\beta^2 = \gamma^2(1-\beta^2) = 1$ is an invariant probability in all frames and in removing $\gamma^2\beta^2 \times (m=0)$ states from γ^2 ss states, the new ratio of spherical symmetry is $(\gamma^2 - \gamma^2\beta^2)/\gamma^2 = 1-\beta^2$. Thus a spherically symmetric state is transformed from probability

1 in frame A, to $1-\beta^2$ in frame B. Also removing $m=0$ states from spherically symmetric states leaves a surplus of $m=\pm1$ states, as spherically symmetric states are equal superpositions of $m=-1,\ m=0,\&\,m=+1$ states.

Thus in Frame B the probabilities are $(1-\beta^2)\times ss$ states $+\beta^2\times m=\pm1$ states. (3.1. 1)

We can describe this as a virtual superposition of $(\frac{1}{\gamma}\times ss,+\beta\times m=\pm1)$ states. (3.1. 2)

As $\beta^2\to1$ we have transverse polarized states, the same as real photons. Now transverse polarized spin 1 states can be either left $(m=-1)$, or right $(m=+1)$ circular polarization, or equal superpositions of $(1/\sqrt{2})L+(1/\sqrt{2})R$ as in $x\,\&\,y$ polarization. If we think of individual spin zero preons absorbing these spin 1 quanta at $t=0$ they must also have this same β^2 probability of transversely polarized spin 1 states. If they then merge into some composite $l=3$ particle (as in Figure 3.1. 4) for time $0<t<\hbar/2E$, the probability of it being in some particular state $(l=3,m=0)$, $(l=3,m=\pm1)$, $(l=3,m=\pm2)$ or $(l=3,m=\pm3)$, must be the same β^2. If we look at Eq.'s (1.1. 1)) we can see what is behind them. We have written the amplitudes in these three equations in terms of β_{nk} & γ_{nk} as this is the most convenient way to express them. Velocities however are not the quantum world language, momenta are. Velocity operators are momentum operators over relativistic masses. Our Eigenvalues are $\mathbf{p}_{nk}{}^2=n^2\hbar^2k^2$ for each $n\,\&\,k$, and this allows our velocity operators to give constant $\beta_{nk}{}^2$. Even though the mass in these operators is virtual, we can still use it to calculate $|\beta_{nk}|$. For each k and integral n there will be a constant $|\beta_{nk}|$ and $\gamma_{nk}=(1-\beta_{nk}{}^2)^{-1/2}$. As we will see β_{nk} can be thought of as the magnitude of the velocity of an imaginary centre of momentum frame in which these interactions take place. We will also draw our Feynman diagrams of these interactions in terms of $\beta_{nk}\,\&\,\gamma_{nk}$ for convenience, even though this is unconventional. To proceed from here however, we need to define two frames as follows:

1) The Laboratory Frame (LF) or Fixed Frame as in section 2.2.3

The infinite superposition has rest mass m_0 and zero nett momentum in this frame. Each ψ_{nk} is centered here with magnitude of momentum $|\mathbf{p}_{nk}|=n\hbar k$. Even though we have no idea of the direction of this momentum vector we will define it as the z direction. The eight preons are born in this frame with zero momentum and can thus be considered here as being at rest or with zero velocity and infinite wavelength at their birth. The Feynman diagram of the interaction in this frame that builds ψ_{nk} is illustrated in Figure 3.1. 3.

2) The Center of Momentum Frame (CMF)

This (imaginary) frame is the center of momentum of the interaction that builds ψ_{nk}. The CMF moves at velocity β_{nk} relative to the laboratory frame in the z direction or parallel to the unknown momentum vector direction $\mathbf{p}_{nk}$. In this CMF the momenta and velocities of the preons at birth and after the interaction are equal and opposite. This is illustrated in Figure 3.1. 2 again in terms of $m_0, \beta_{nk}, \& \gamma_{nk}$. In the LF the velocity of the preons at birth is zero, in the CMF this is $-\beta_{nk}$ and after the interaction $+\beta_{nk}$, where both $-\beta_{nk}$ and $+\beta_{nk}$ are in the unknown z direction. In the LF the particle velocity $\beta_{nk}(particle) = \beta_{nkp}$ is the simple relativistic addition of the two equal velocities β_{nk} as in Figure 3.1. 1.

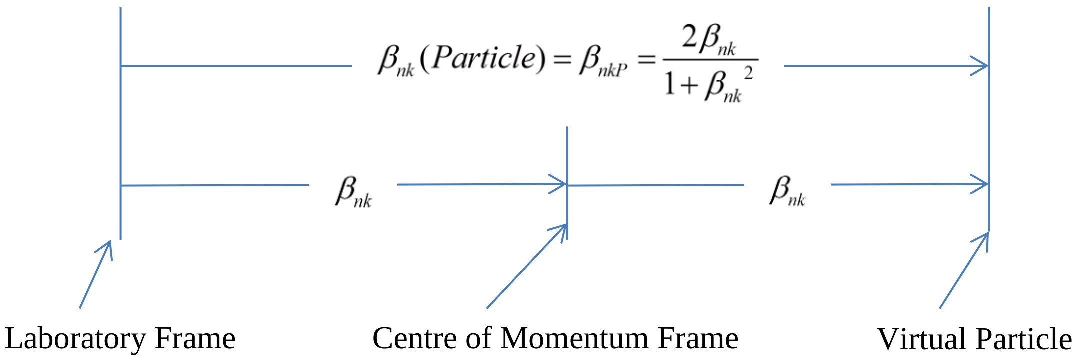


Figure 3.1. 1

3.1.2 Feynman diagrams of primary interactions

Let us start with

$$\beta_{nk}(Particle) = \beta_{nkP} = \frac{2\beta_{nk}}{1+\beta_{nk}^2} \text{ and } \gamma_{nkP} = (1-\beta_{nkp}^2)^{-1/2} = \gamma_{nk}^2(1+\beta_{nk}^2) \tag{3.1. 3}$$

If the particle rest mass is m_0 let each preon have a virtual rest mass $m_0/(8\gamma_{nk}\sqrt{2s})$.

The eight preons are effectively a virtual particle of rest mass $m_{0snk} = \dfrac{m_0}{\gamma_{nk}\sqrt{2s}}$ (3.1. 4)

The particle momentum in the LF is zero at birth. After the interaction using these equations

$$|\mathbf{p}_{nk}| = n\hbar k = m_{0snk}\beta_{nkP}\gamma_{nkP}c = \left[\frac{m_0}{\gamma_{nk}\sqrt{2s}}\right]\left[\frac{2\beta_{nk}}{1+\beta_{nk}^2}\right]\left[\gamma_{nk}^2(1+\beta_{nk}^2)\right]c$$

The particle momentum after the interaction in the LF $\left|\mathbf{p}_{nk}\right| = n\hbar k = \dfrac{2m_0\beta_{nk}\gamma_{nk}c}{\sqrt{2s}}$ (3.1. 5)

Using Eq. (3.1. 4), in the LF the particle energy at birth is

$$m_{0snk}c^2 = \frac{m_0c^2}{\gamma_{nk}\sqrt{2s}} \tag{3.1. 6}$$

In the LF the particle energy after the interaction is using Eq's. (3.1. 3)

$$m_{0snk}\gamma_{pnk}c^2 = \frac{m_0}{\gamma_{nk}\sqrt{2s}}\gamma_{nk}{}^2(1+\beta_{nk}{}^2)c^2 = \frac{m_0\gamma_{nk}}{\sqrt{2s}}(1+\beta_{nk}{}^2)c^2 \tag{3.1. 7}$$

In the CMF the momentum at birth is using Eq. (3.1. 4)

$$-m_{0snk}\gamma_{nk}\beta_{nk} = \frac{-m_0\beta_{nk}}{\sqrt{2s}} \tag{3.1. 8}$$

In the CMF the momentum after the interaction is equal but in the opposite direction

$$= \frac{+m_0\beta_{nk}}{\sqrt{2s}} \tag{3.1. 9}$$

In the CMF the energy at birth, and after the interaction is

$$m_{0snk}\gamma_{nk}c^2 = \frac{m_0c^2}{\sqrt{2s}} \tag{3.1. 10}$$

These values are all summarized in Figure 3.1. 2 and Figure 3.1. 3 but with $c = 1$.

Now from Eq. (3.1. 5) $\left|\mathbf{p}_{nk}\right| = n\hbar k = \dfrac{2m_0\beta_{nk}\gamma_{nk}c}{\sqrt{2s}}$ and $\beta_{nk}\gamma_{nk} = \dfrac{n\hbar k\sqrt{2s}}{2m_0c} = \dfrac{\lambda\!\!\!^{-}{}_c nk\sqrt{2s}}{2}$

(where $\lambda\!\!\!^{-}{}_c$ is the Compton wavelength) so let us now put:

$$K_{nk} = \beta_{nk}\gamma_{nk} = \frac{n\hbar k\sqrt{2s}}{2m_0c} = \frac{\lambda\!\!\!^{-}{}_c nk\sqrt{2s}}{2} \tag{3.1. 11}$$

In terms of K_{nk}: $\beta_{nk}{}^2 = \dfrac{K_{nk}{}^2}{1+K_{nk}{}^2}$ and $\gamma_{nk}{}^2 = 1+K_{nk}{}^2$ (3.1. 12)

Each infinite superposition has a fixed $\lambda\!\!\!^{-}{}_c$, also each wavefunction ψ_{nk} of this infinite superposition has fixed $n \,\&\, s$, thus $K_{nk} \propto k$. For example we can put

$$\frac{dK_{nk}}{K_{nk}} = \frac{dk}{k} \tag{3.1. 13}$$

These simple expressions and what follows are not possible if $m_{0snk} \neq m_0 / \gamma_{nk}\sqrt{2s}$, and *when we include gravity* we find $m_{0snk} = m_0 / (\gamma_{nk}\sqrt{2s})$ is essential (section 4.2).

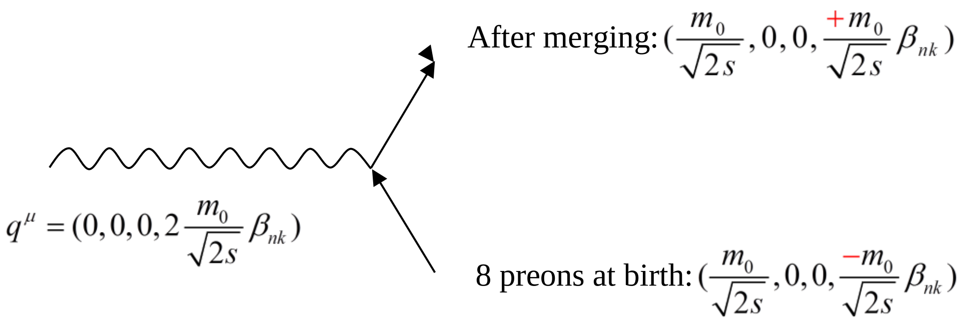


Figure 3.1. 2 Feynman diagram in an imaginary centre of momentum frame.

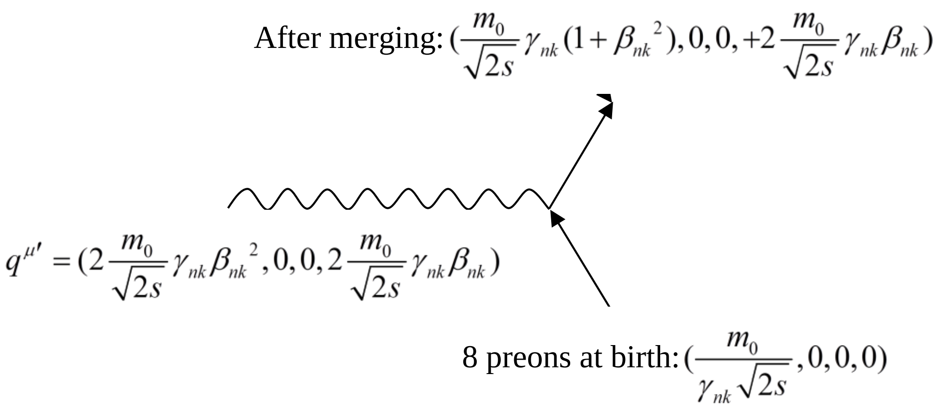


Figure 3.1. 3 Feynman diagram in the laboratory frame.

The interaction in the Feynman diagrams above is with spin 1 quanta. The Feynman transition amplitude of this interaction tells us that the polarization states of these exchanged quanta is determined by the sum of the components of the initial, plus final 4 momentum $(p_i + p_f)^{\mu}$. Ignoring all other common factors this tells us that the space polarized component is the sum of the momentum terms $(\mathbf{p}_i + \mathbf{p}_f)$ and the time polarized component is the sum of the energy terms $(p_i + p_f)^0$. We have defined our momentum as in an unknown z direction:

The ratio of z polarization to time polarization amplitudes is $\dfrac{(p_i+p_f)^z}{(p_i+p_f)^0}$ (3.1. 14)

In the CMF $(p_i+p_f)^z=0$, thus an interaction in the CMF exchanges only time polarized, or spherically symmetric $l=1$ states. In the LF the ratio of z (or $m=0$) polarization, to time polarization in the LF is $\beta_{nk}{}^2$,

$$\text{where} \quad \beta_{nk}=\frac{(p_i+p_f)^z}{(p_i+p_f)^0}=\frac{2m_0\gamma_{nk}\beta_{nk}}{2m_0\gamma_{nk}} \qquad (3.1.\ 15)$$

From section 3.1.1 these are probabilities of $\gamma_{nk}{}^2\ ss\ -\gamma_{nk}{}^2\beta_{nk}{}^2\times(m=0)$ states, or $(1-\beta_{nk}{}^2)\ ss\ +\beta_{nk}{}^2\times(l=1,m=\pm1)$ states.

In the LF this is a virtual superposition of $(\dfrac{1}{\gamma_{nk}}\times ss,+\beta_{nk}\times m=\pm1)$ states. (3.1. 16)

From section 3.1.1 as these quanta from the scalar and vector zero point fields build each ψ_{nk} this implies that:

In the LF ψ_{nk} has virtual superposition amplitudes $(\dfrac{1}{\gamma_{nk}}\times ss,+\beta_{nk}\times m$ states) (3.1. 17)

From section 3.1.1 appropriate $l=1,m=\pm1$ superpositions can build any $l=3,\ m$ state. Figure 3.1. 4 is an example of such a ψ_{nk} for $n=5,6,\&7\ (l=3,m=+2)$ states.

3.1.3 Different ways to express superpositions

We have expressed all superpositions here in terms of spherically symmetric and m states for convenience and simplicity. We could have expressed them in the form:

$$\frac{1}{\gamma_{nk}\sqrt{7}}\left[(m=-3),(m=-2),(m=-1),(m=0),(m=+1),m=+2),(m=+3)\right]+\beta_{nk}(m=+2)$$

This is equivalent to (ignoring complex number amplitude factors)

$$\psi_{nk}=\frac{1}{\gamma_{nk}}\times ss,+\beta_{nk}\times(m=+2) \text{ where we have put m}=+2 \text{ for example.}$$

Because all these wavefunctions are virtual they cannot be measured in the normal way that collapses them into any of these Eigenstates, it is more convenient to use the method adopted here which is similar to QED virtual photons superpositions.

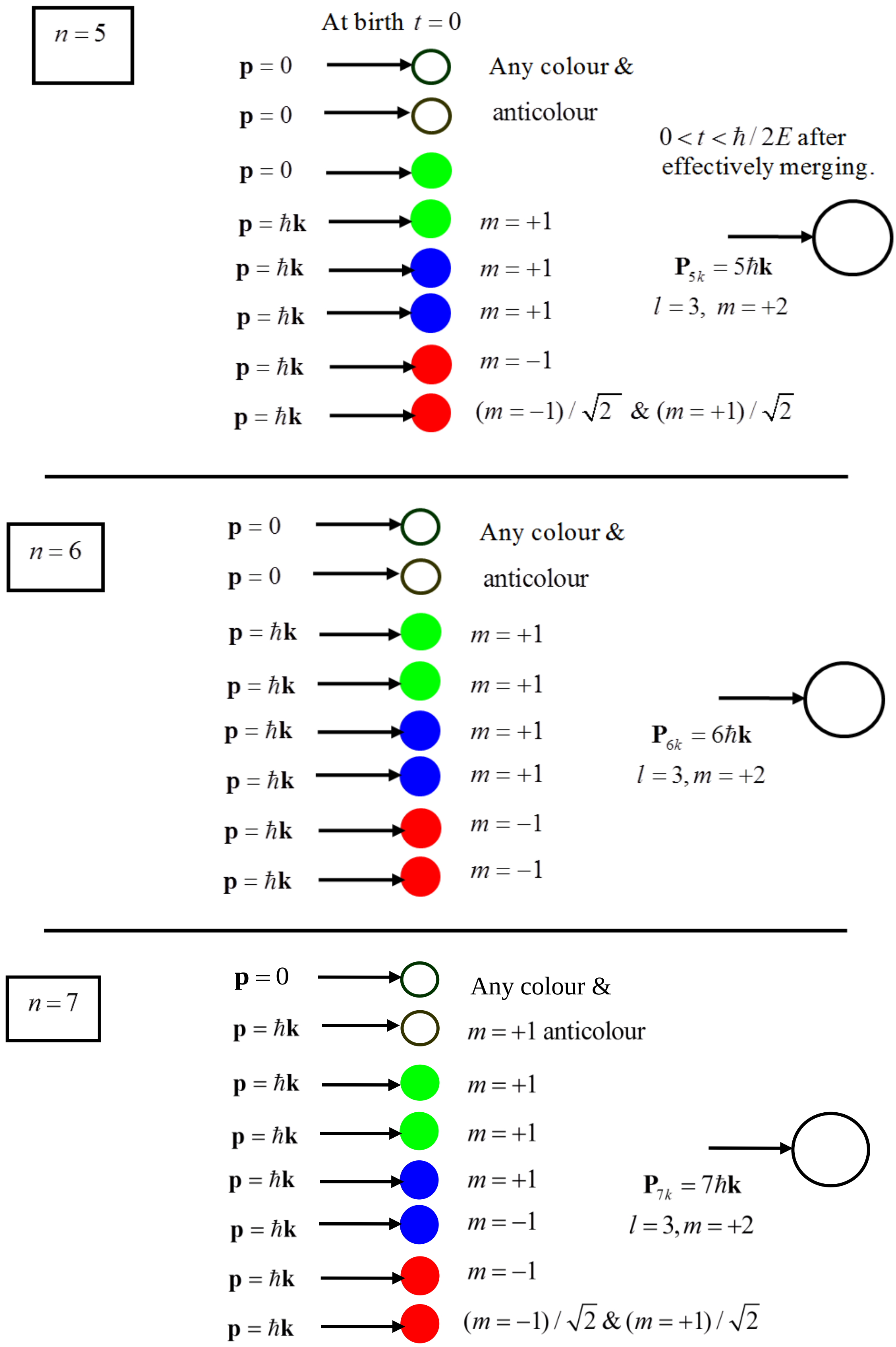


Figure 3.1. 4 Eight preons forming $m=+2$ states as part of a positron superposition.

There is no significance in which preons absorb quanta in the above.

3.2 Rest Mass and Total Angular Momentum of Infinite Superpositions

3.2.1 Real rest masses of massive infinite superpositions

We will consider the total rest mass first of the infinite assembly and consider one wavefunction ψ_{nk} at a time, temporarily assuming that the amplitude c_n of each ψ_{nk} has magnitude $|c_n| = 1$. Each time ψ_{nk} is born it borrows virtual rest masses and virtual energies from scalar and vector zero point fields. We are however going to focus solely on the Eigenvalues $\mathbf{p}_{nk}{}^2 = n^2\hbar^2k^2$, and treat only momentum as real. But do these momenta themselves also leave momentum debts in the vacuum?

From section 2.3.2 as $k \to \infty$, $E_{nk}{}^2 \to \mathbf{p}_{nk}{}^2c^2 = n^2\hbar^2\omega^2$ or $E_{nk} \to n\hbar\omega$ and n quanta of energy $\hbar\omega$ and momentum $|\hbar k|$ are absorbed. We know that in some unknown direction $\mathbf{p}_{nk} = n\hbar\mathbf{k}$, which implies these n absorbed quanta must leave a cancelling debt in the opposite direction of $\mathbf{p}_{nk}(debt) = -n\hbar\mathbf{k}$ in the vacuum. But this is true only as $k \to \infty$ & $\beta_{nk}{}^2 \to 1$ and the virtual quanta energy transferred $E_X \to \hbar\omega$. So what happens when $\beta_{nk}{}^2 << 1$? Our wavefunctions ψ_{nk} are generated from a vector potential squared term A^2 derived in section 2.1.2 which in turn came from the B^2 term of section 2.1.1. As discussed in section 2.3.2 the Eigenvalues $\mathbf{p}_{nk}{}^2 = n^2\hbar^2k^2$ confirm the constant momentum squared feature of magnetic interactions. Also in section 2.1.1 the scalar virtual photon emission probability is directly related to the force squared term $F^2 = \varepsilon^2E^2$. Magnetic coupling probabilities are related to a magnetic force squared term $F^2 = \beta^2\varepsilon^2B^2/c^2 = \beta^2\varepsilon^2E^2$, where from section 3.1.2 and Eq. (3.1. 14) the ratio of this scalar to magnetic coupling is $\beta_{nk}{}^2$. Thus when $k < \infty$ and the exchanged energy $E_X \neq \hbar\omega$, $\beta_{nk}{}^2n$ quanta $|\hbar k|$ are absorbed from the vacuum and:

$$\text{We can expect a momentum debt of } \mathbf{p}_{nk}(debt) = -\beta_{nk}{}^2 n\hbar\mathbf{k} \tag{3.2. 1}$$

We can now take the next step, as both the vectors $\mathbf{p}_{nk}$ and $\mathbf{p}_{nk}(debt)$ are parallel in the same unknown direction, to get a nett momentum:

$$\mathbf{p}_{nk}(nett) = \mathbf{p}_{nk} + \mathbf{p}_{nk}(debt) = (1-\beta_{nk}{}^2)n\hbar\mathbf{k} = \frac{n\hbar\mathbf{k}}{\gamma_{nk}{}^2} = \frac{\mathbf{p}_{nk}}{\gamma_{nk}{}^2} \text{ at wavenumber } k. \tag{3.2. 2}$$

Does this make sense using the relativistic energy expression $E_n{}^2 = \sum_{k=0}^{k=\infty}\mathbf{p}_{nk}(nett)^2c^2$ as we have simply ignored all other borrowed virtual energies and virtual rest masses?

We will initially look at only massive infinite superpositions where $N=1$ in Eq.(2.2. 4). Thus using probability $sN\cdot dk/k = s\cdot dk/k$, also Eq's.(3.1. 11), (3.1. 12),(3.1. 13),&(3.2. 2).

$$E_n{}^2 = c^2\int_{k=0}^{k=\infty}\mathbf{p}_{nk}(nett)^2\,\frac{s\cdot dk}{k} = c^2\int_0^\infty \frac{n^2\hbar^2k^2}{\gamma_{nk}{}^4}\frac{s\cdot dk}{k} = 4m_0{}^2c^4\int_0^\infty \frac{K_{nk}{}^2}{(1+K_{nk}{}^2)^2}\frac{dK_{nk}}{2K_{nk}}$$

$$E_n{}^2 = m_0{}^2c^4\left[\frac{-1}{1+K_{nk}{}^2}\right]_0^\infty = m_0{}^2c^4 \text{ or } E_n = \pm m_0c^2 \tag{3.2. 3}$$

This energy is due to summing momenta squared but it is real, with a real rest mass $\pm m_0$ for infinite superpositions of wavefunctions ψ_{nk}. These superpositions can form all the non infinitesimal rest mass fundamental particles. The equations do not work if the rest mass m_0 is zero. (We will look at infinitesimal rest masses in section 6.3.) The negative mass solutions in Eq. (3.2. 3) must be handled in the usual Feynman manner, and treated as antiparticles with positive energy going backwards in time. If they are spin ½ this also determines how they interact with the weak force.

3.2.2 Angular momentum of massive infinite superpositions

We will now use the same procedure for the total angular momentum of an infinite superposition with non infinitesimal rest mass where $N=1$ in Eq.(2.2. 4). Wavefunctions $\psi_{nk} = C_{nk}r^3\exp(-n^2k^2r^2/18)Y(\theta,\varphi)$ have angular momentum squared Eigenvalues $\mathbf{L}^2 = 12\hbar^2$ and m states of this have angular momentum Eigenvalues $\mathbf{L}_z = m\hbar$, thus we will treat both angular momentum and angular momentum debts as real just as we did for linear momentum. Even though m state wavefunctions are part of superpositions they still have probabilities just as the linear momenta squared above and it seemed to work. Using exactly the same arguments as in section 3.2.1 , if ψ_{nk} is in a state of angular momentum $\mathbf{L}_{zk} = m\hbar$, then it must leave an angular momentum debt in the vacuum of $\mathbf{L}_{zk}(debt) = -\beta_{nk}{}^2 m\hbar$ (or as in section) $\mathbf{L}_{zk}(nett) = \mathbf{L}_{zk} - \mathbf{L}_{zk}(debt)$.

$$\mathbf{L}_{zk}(nett) = (1-\beta_{nk}{}^2)m\hbar = (1-\beta_{nk}{}^2)\mathbf{L}_{zk} = \frac{\mathbf{L}_{zk}}{\gamma_{nk}{}^2} \quad (\text{if } \mathbf{L}_{zk} = m\hbar) \tag{3.2. 4}$$

But from Eq. (3.1. 16) the probability that $\mathbf{L}_{zk}$ is in an m state is also $\beta_{nk}{}^2$ so that

$$\text{Including } \beta_{nk}{}^2 \text{ probability } \mathbf{L}_{zk}(nett) = m\hbar\frac{\beta_{nk}{}^2}{\gamma_{nk}{}^2} \text{ at wavenumber } k. \tag{3.2. 5}$$

For an $N=1$ type infinite superposition $\mathbf{L}_z(Total) = \int\limits_{k=0}^{k=\infty} \mathbf{L}_{zk}(nett)\frac{s \cdot dk}{k}$. $= sm\hbar\int\limits_0^\infty \frac{\beta_{nk}^{\ 2}}{\gamma_{nk}^{\ 2}}\frac{dk}{2k}$

Using Eq's. (3.1. 11) to (3.1. 13) to $\mathbf{L}_z(Total) = sm\hbar\int\limits_0^\infty \frac{K_{nk}^{\ 2}}{(1+K_{nk}^{\ 2})^2}\frac{dK_{nk}}{K_{nk}} = \frac{sm\hbar}{2}\left[\frac{-1}{1+K_{nk}^{\ 2}}\right]_0^\infty$

$$\mathbf{L}_z(Total) = m'\hbar = \frac{sm\hbar}{2} \qquad \text{or} \qquad m' = \frac{s}{2}m \tag{3.2. 6}$$

Where m' *is the angular momentum state of the infinite superposition and* m *the state of* ψ_{nk} *.* Thus for spin ½ particles with $s=$ ½ in Eq.(3.2. 6) $m' = m/4$, but m' can be only ± ½, implying that the m state of ψ_{nk} that generates spin ½ must be $m = \pm 2$. Massive spin 1 particles have $s=1$ (& $N=1$) with $m' = m/2$. (There are no massive spin 2 particles in the Stansdard Model.) This is summarized in the following three member infinite superpositions.

$$(N=1)\ \text{Spin } ½ \qquad \psi_{\infty,1/2,\pm 1/2} = \sum_{n=5,6,7} c_n \int\limits_{k=0}^{k=\infty}\left[\frac{1}{\gamma_{nk}}(\psi_{nk,ss}) + \beta_{nk}(\psi_{nk,\pm 2})\right]\sqrt{\frac{1}{2k}}dk \tag{3.2. 7}$$

$$(N=1)\quad \text{Spin } 1 \qquad \psi_{\infty,1,m} = \sum_{n=4,5,6} c_n \int\limits_{k=0}^{k=\infty}\left[\frac{1}{\gamma_{nk}}(\psi_{nk,ss}) + \beta_{nk}(\psi_{nk,2m})\right]\sqrt{\frac{1}{k}}dk \tag{3.2. 8}$$

The spin vectors of each ψ_{nk} with $|\mathbf{L}| = 2\sqrt{3}\hbar$, and their spin vector debts in the zero point vector fields, have to be aligned such that the sum in each case is the correct value: $|\mathbf{L}| = \sqrt{3}\hbar/2$, $|\mathbf{L}| = \sqrt{2}\hbar$ or $|\mathbf{L}| = \sqrt{6}\hbar$ for spins ½ , 1 & 2 respectively. Gravity is also included in Eq. (1.1. 1)) in our summary, otherwise they are as above.

A spherically symmetric massive spin 1 state is a superposition of the three states $\frac{1}{\sqrt{3}}(m'=-1), \frac{1}{\sqrt{3}}(m'=0), \frac{1}{\sqrt{3}}(m'=+1)$, and using Eq. (3.2. 8) can be formed as follows

$$\text{Massive spin 1} \quad \left[\begin{array}{l} \frac{1}{\sqrt{3}}\psi_{\infty,1,m'=-1} = \frac{1}{\sqrt{3}}\sum\limits_{n=4,5,6} c_n \int\limits_{k=0}^{k=\infty}\left[\frac{1}{\gamma_{nk}}(\psi_{nk,ss}) + \beta_{nk}(\psi_{nk,m=-2})\right]\sqrt{\frac{1}{k}}dk \\ +\frac{1}{\sqrt{3}}\psi_{\infty,1,m'=0} = \frac{1}{\sqrt{3}}\sum\limits_{n=4,5,6} c_n \int\limits_{k=0}^{k=\infty}\left[\frac{1}{\gamma_{nk}}(\psi_{nk,ss}) + \beta_{nk}(\psi_{nk,m=0})\right]\sqrt{\frac{1}{k}}dk \\ +\frac{1}{\sqrt{3}}\psi_{\infty,1,m'=+1} = \frac{1}{\sqrt{3}}\sum\limits_{n=4,5,6} c_n \int\limits_{k=0}^{k=\infty}\left[\frac{1}{\gamma_{nk}}(\psi_{nk,ss}) + \beta_{nk}(\psi_{nk,m=+2})\right]\sqrt{\frac{1}{k}}dk \end{array}\right] \tag{3.2. 9}$$

3.2.3 Rest mass and angular momentum of complete infinite superpositions of ψ_k

In sections 3.2.1 & 3.2.2 to keep things simple we looked only at one wavefunction ψ_{nk} at a time. If we consider superpositions $\psi_k = \sum_n c_n{}^* c_n \psi_{nk}$, we simply need to replace $K_{nk}{}^2$ with $\langle K_{nk}\rangle^2$. Using Eq. (2.3. 9) we can say $|\mathbf{p}_{nk}| = n\hbar k$ and $\langle|\mathbf{p}_k|\rangle = \langle n\rangle \hbar k \;\; = \hbar k \sum_n c_n{}^* c_n \cdot n$, then using Eq. (3.1. 11)

$$\langle K_{nk}\rangle = \frac{\lambda\!\!\!^{-}_c k\sqrt{2s}}{2}\langle n\rangle \;\;\&\;\; \langle K_{nk}\rangle^2 = \frac{\lambda\!\!\!^{-}_c{}^2 k^2 s}{2}\langle n\rangle^2 \;\;\text{where}\;\; \langle n\rangle = \sum_n (c_n{}^* c_n) n \tag{3.2. 10}$$

Replacing $K_{nk}{}^2$ with $\langle K_{nk}\rangle^2 = \dfrac{\lambda\!\!\!^{-}_c{}^2 k^2 s}{2}\langle n\rangle^2$ in the key equations (3.2. 3) & (3.2. 6) we find there is no change to the final results.

The laws of quantum mechanics tell us that the total angular momentum has to be precisely either integral $\hbar$ or half integral $\hbar/2$. If we look at the above integrals used to derive the total angular momentum we can see that N must be 1, also s must be exactly ½ or 1 for spin ½ & spin 1 massive particles respectively, in Eq. (2.2. 4) our probability formula. Also these integrals are infinite sums of positive and negative integral $\hbar$ that are virtual and cannot be observed, and occur with probabilities between 0 & 1. If an infinite superposition for an electron is in a spin up state and flips to spin down in a magnetic field, a real $m = \pm 1$ photon is emitted carrying away the change in angular momentum. This is the only real effect observed from this infinity of $(l = 3, m = +2)$ virtual wavefunctions all flipping to $(l = 3, m = -2)$ states, plus an infinite flipping of the virtual zero point vector debts. Also Eq's. (3.2. 3) & (3.2. 6) are true *only if our high energy cutoff is at infinity*. We will look at the effect of including gravity on these equations in section 4.2 where we will find our high frequency cutoff is then near the Planck scale.

3.3 Ratios between Primary and Secondary Coupling

3.3.1 Initial simplifying assumptions

This section is based on a special case "thought experiment" to try and illustrate as simply as possible how superpositions interact with one another in the same way as virtual photons interact with electrons for example. It is also important to remember here that *because* *primary coupling constants are to bare charges (section 2.2.2), and thus fixed for all k, while secondary coupling constants run with k, that the coupling ratios can be defined only at the cutoff value of k applying to the bare charge (sections 4.1.1 & 4.3).* Initially it might seem there must be three ratios, χ_G for gravity, χ_C for colour and χ_{EM} for electromagnetism. However there is a very simple relationship between the colour and electromagnetic ratios and we will also find that $\chi_G = \chi_C$. We define these three ratios as follows:

$$\frac{\alpha_{Gravity(\text{Secondary})}}{\alpha_{Gravity(\text{Primary})}} = \frac{G_{\text{Sec}}}{G_{\text{Pri}}} = \frac{1}{\chi_G}, \quad \frac{\alpha_{Colour(\text{Secondary})}}{\alpha_{Colour(\text{Primary})}} = \frac{\alpha_{3S}}{\alpha_{3P}} = \frac{1}{\chi_C}, \quad \frac{\alpha_{EM(\text{Secondary})}}{\alpha_{EM(\text{Primary})}} = \frac{\alpha_{EMS}}{\alpha_{EMP}} = \frac{1}{\chi_{EM}}. \tag{3.3. 1}$$

From Table 2.2. 1 there are 6 fundamental primary charges for electrons and positrons. But electrons and positrons are defined as fundamental charges. In other words what we define as a fundamental electric charge is in reality 6 primary charges. Of course we never measure 6 as their effect is reduced by the ratio between primary and secondary coupling. Because electromagnetic and colour coupling are both via spin one bosons, their coupling ratios are essentially the same but related simply as $6^2 = 36:1$.

$$\left[\frac{\alpha_{EM(\text{Secondary})}}{\alpha_{EM(\text{Primary})}} = \frac{\alpha_{EMS}}{\alpha_{EMP}} = \frac{1}{\chi_{EM}}\right] = 36 \times \left[\frac{\alpha_{Colour(\text{Secondary})}}{\alpha_{Colour(\text{Primary})}} = \frac{\alpha_3}{\alpha_{3P}} = \frac{1}{\chi_C}\right] \tag{3.3. 2}$$

The secondary couplings α_{EMS} & α_{3S} are the bare charge values, and the gravitational coupling G_S is to Planck masses; all three are at the superposition cutoff near the Planck length. We will discuss in section 6.1.2 why the secondary gravitational coupling constant should not run with wavenumber *k* as the colour and electromagnetic do. Also we assumed in section 2.2.2 that $\alpha_{3P} = 1;$ thus from Eq. (3.3. 2)

$$\alpha_{3S}{}^{-1} = \chi_C \tag{3.3. 3}$$

In other words provided $\alpha_{3P} = 1,$ the ratio χ_C is also the inverse of the colour coupling constant α_3 at the superposition cutoff near the Planck length L_p (section 4.3).

From the above paragraphs to find the coupling ratios we need secondary interactions that are between bare charges. But this implies extremely close spacing where the effects of spins dominate. If the spacing is sufficiently large the effects of spin can be ignored but then we are not looking at bare charges. However we can ignore the effects of shielding due to virtual charged pairs by imagining as a simple thought experiment, an interaction between bare charges even at such large spacing. We can also simplify things further by considering only scalar or coulomb type interactions at this large spacing. We are also going to temporarily ignore Eq. (3.3. 2) and imagine that we have only one primary electric and or one colour charge. Consider two infinite superpositions and (*due to the above simplifying assumptions*) imagine them as spin zero charges. QED considers the interaction between them as a single covariant combination of two separate and opposite direction non-covariant interactions (a) plus (b) as in the Feynman diagram of Figure 3.3. 1 below. The Feynman transition amplitude is invariant in all frames [6]. So let us consider a special simple case in a CM frame where we have identical particles on a head on collision path with spatial momenta:

$$\mathbf{p}_a = -\mathbf{p}'_a = -\mathbf{p}_b = +\mathbf{p}'_b \tag{3.3. 4}$$

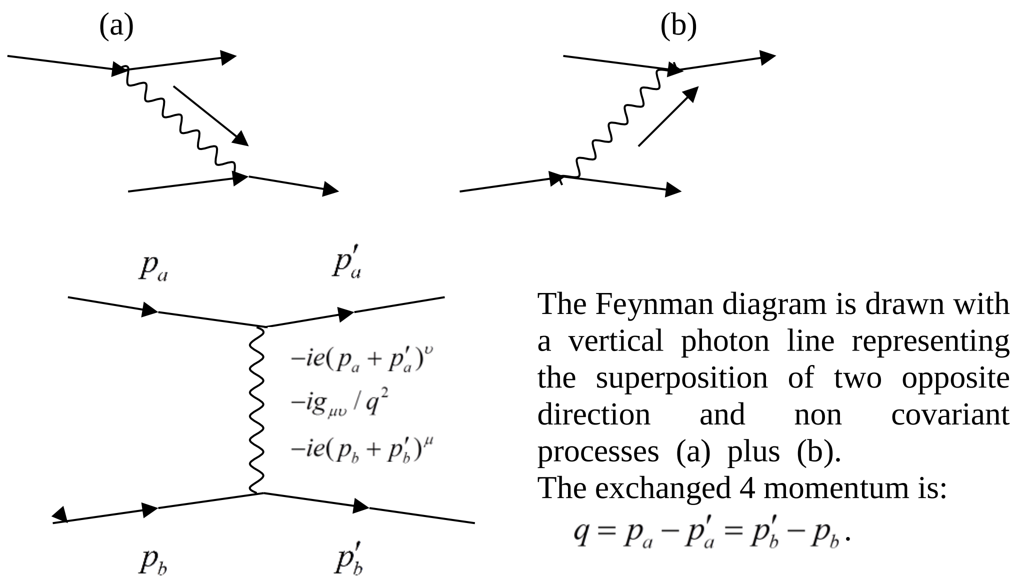


Figure 3.3. 1 Feynman diagram of virtual photon exchange between two spin zero particles of charge e.

From Eq. (3.3. 4) the initial and final spatial momenta are reversed with mirror images of each other at each vertex. Also in this simple special scalar case the transferred four momentum squared is simply the transferred three momentum squared.

$$q^2 = (p_a - p_a{}')^2 = (p_b - p_b{}')^2 = 4\mathbf{p}_a{}^2 = 4\mathbf{p}_b{}^2. \tag{3.3. 5}$$

If we look at Figure 3.3. 2 we see that at any fixed value of *k*, all modes ψ_{nk} in the groups of three overlapping superpositions for the various spins ½, 1 & 2 occupy very similar regions of space (provided they are all on similar centres.) The directions of their linear momenta are unknown but let us imagine some particular vector $\hbar\mathbf{k}$ that is parallel to the above vectors $\mathbf{p}_a = \mathbf{p}_b$. As we are considering only scalar interactions, all these modes must be spherically symmetric (as in section 3.2.2 for spins 1 & 2, and for spin ½ provided k or in turn β_{nk} is small enough the probability that it is not spherically symmetric can be extremely low) and at a fixed value of k they have momenta $\pm n\hbar\mathbf{k}$. Also as they overlap each other we can imagine units of $\pm\hbar\mathbf{k}$ quanta somehow transferring between these superpositions so that the values of n in each mode can change temporarily by ± 1 for times $\Delta T \approx \hbar / \Delta E$. The directions of these momentum transfers causing either repulsion or attraction depending on the charge signs of the superpositions at each vertex, whether the same or opposite.

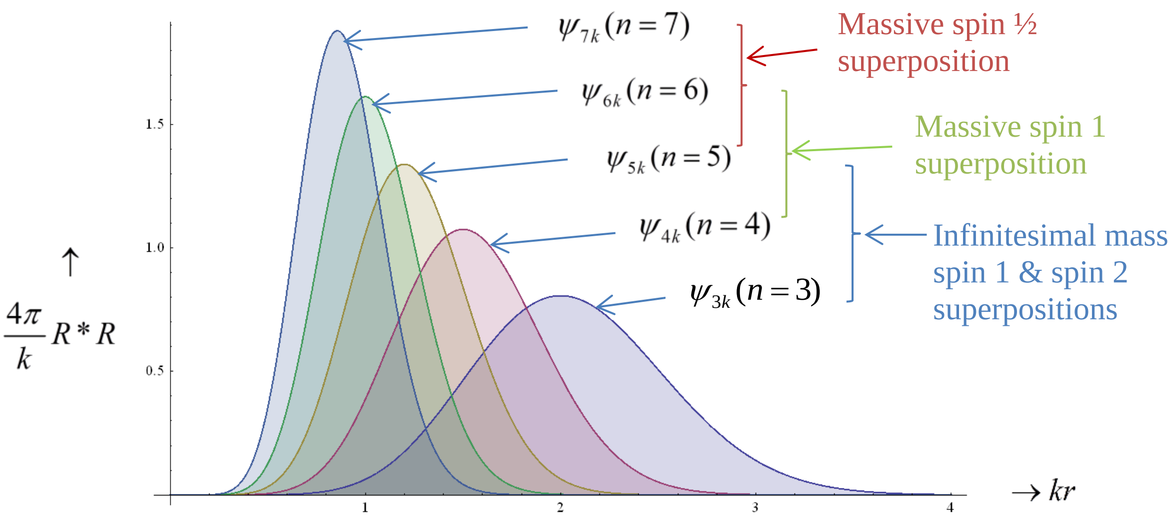


Figure 3.3. 2 All modes ψ_{nk} in the groups of three overlap at fixed wavenumber *k*.

3.3.2 Restrictions on possible Eigenvalue changes

Before we look at changing these Eigenvalues by $n = \pm 1$ we need to consider what restrictions there are on these changes.

From Eq. (2.3. 12) superposition ψ_k requires $Q^2A^2 = \sum_n c_n * c_n \frac{n^4\hbar^2k^4r^2}{81}$ and Eq.(2.2. 4) tells us the available $Q^2A^2 = \frac{\left[8+8\sqrt{\alpha_{EMP}}\right]^2}{3\pi sN}\hbar^2k^4r^2$ occurs with probability $= \frac{sN \cdot dk}{k}$.

For very brief periods the required value of Q^2A^2 can fluctuate, such as during these changes of momentum, but if its average value changes over the entire process then Eq. (2.2. 4) tells us that probability $sN \cdot dk / k$ changes also, and we have shown in section 3.2.1 that this is not allowed. For example in a spin ½ superposition $\psi_{5k}, \psi_{6k}, \psi_{7k}$, the average values of $|c_5|, |c_6|$ & $|c_7|$ must each remain constant. This can happen only if n remains within its pre-existing boundaries of $(5 \le n \le 7)$. For example if ψ_7 adds $+\hbar\mathbf{k}$, it can create ψ_8, but $|c_8|$ must average zero, which it can do only if it fluctuates either side of zero, and $|c_n|$ cannot be negative. Similarly $|c_4|$ must average zero, thus ψ_4 & ψ_8 are forbidden states. Keeping the average values of $|c_n|$ constant is also equivalent to a constant internal average particle energy (we have shown in section 3.2.1 that rest mass is a function of $\sum c_n * c_n \mathbf{p}_{nk}{}^2$. By changing these Eigenvalues by $n = \pm 1$ there are only four possibilities; ψ_6 & ψ_7 can both reduce by $-\hbar\mathbf{k}$ quanta, ψ_6 & ψ_5 can both increase by $+\hbar\mathbf{k}$ quanta. If ψ_6 becomes ψ_7, $|c_7|$ also increases and $|c_6|$ decreases, but then ψ_7 has to drop back becoming ψ_6, with $|c_7|$ decreasing back down and $|c_6|$ increasing back up in exact balance. If we view this as one overall process the average values of both $|c_6|$ and $|c_7|$ remain constant but fluctuate continuously. We can use exactly the same argument if ψ_5 increases which has to be followed by ψ_6 dropping, where if we view this as one process again, the average values of both $|c_5|$ and $|c_6|$ remain constant. This is similar to a particle not being able to absorb a photon in a covariant manner, it has to reemit within time $\Delta T \approx \hbar / E$. With spherical symmetry the momentum $\mathbf{p} = \pm n\hbar\mathbf{k}$. If we change n by ± 1 the sign of $\mathbf{p} = \pm n\hbar\mathbf{k}$ determines the direction of the momentum transfer $\Delta\mathbf{p}$. In the above if $\psi_{5k} \to \psi_{6k}$ then returns $\psi_{6k} \to \psi_{5k}$, and $\mathbf{p} = \pm n\hbar\mathbf{k}$ keeps the same sign during this process, there is no nett momentum transfer and there is a probability of this, but it is not the probability we need. However if this process is as in figure Figure 3.3. 3

$$\begin{array}{c} 6\hbar\mathbf{k} \to \\ \downarrow \\ 5\hbar\mathbf{k} \to \\ |a\rangle \end{array} \quad \text{produces } \Delta\mathbf{p} = -\hbar\mathbf{k}\text{, but there is another } \Delta\mathbf{p} = -\hbar\mathbf{k} \text{ if returning via} \quad \begin{array}{c} -5\hbar\mathbf{k} \leftarrow \\ \downarrow \\ -6\hbar\mathbf{k} \leftarrow \\ |b\rangle \end{array}$$

Figure 3.3. 3

To get a net momentum transfer the momenta have to be in opposite directions for each half of this process. (Conservation of momentum allows this only if there is an equal and opposite transfer of momentum at the other vertex of the interaction.) The problem with this is that a total transfer of $\Delta\mathbf{p} = -2\hbar\mathbf{k}$ implies superpositions ψ_k interact with virtual $2k$ photons. Section 3.5 shows that interactions only with virtual k photons give the correct Dirac spin ½ magnetic energy. However just as transversely polarized photons are equal left and right polarization superpositions $|L\rangle/\sqrt{2}+|R\rangle/\sqrt{2},$ we can perhaps regard the Figure 3.3. 3 process as a similar equal superposition $|a\rangle/\sqrt{2}+|b\rangle\sqrt{2}.$

The figure 3.3.3 process becomes the superposition $\dfrac{|a\rangle}{\sqrt{2}}+\dfrac{|b\rangle}{\sqrt{2}}=\dfrac{|-\hbar\mathbf{k}\rangle}{\sqrt{2}}+\dfrac{|-\hbar\mathbf{k}\rangle}{\sqrt{2}}$ (3.3. 6)

We have two equal 50% probabilities of states $|a\rangle\,\&\,|b\rangle$ producing the required total $\Delta\mathbf{p} = -\hbar\mathbf{k}$. Also as from the above paragraphs the average values of $|c_5|$ and $|c_6|$ remain constant:

The probability of transitions $\psi_5 \rightleftarrows \psi_6$ must be the same in either direction. (3.3. 7)

As spherically symmetric states have momentum $\mathbf{p} = \pm n\hbar\mathbf{k}$:

We can also think of $\mathbf{p} = \pm n\hbar\mathbf{k}$ as a superposition $\mathbf{p} = |+n\hbar\mathbf{k}\rangle/\sqrt{2}+|-n\hbar\mathbf{k}\rangle/\sqrt{2}.$ (3.3. 8)

3.3.3 Looking as just one vertex of the interaction first

In Table 4.3. 1 and section 6.3 we introduce infinitesimal rest mass photons and gluons as superpositions of $\psi_{3k},\psi_{4k},\psi_{5k}$ where $N=2$ in Eq. (2.2. 4). Consider just one vertex of an infinitesimal rest mass spin 1 photon superposition $\psi_{3k},\psi_{4k},\psi_{5k}$ interacting with a spin ½ superposition $\psi_{5k},\psi_{6k,}\psi_{7k}$ at the same k. Looking at one possibility first, ψ_{4k} & ψ_{5k} for spin 1 and ψ_{6k} & ψ_{7k} for spin ½, we can apply Figure 3.3. 3 to get a nett momentum transfer. For this combination of Eigenfunctions there are four possible ways of getting the momentum transfer as in Figure 3.3. 4. In each of these 4 cases the amplitude for this to happen includes the factors $c_4 \cdot c_5 \cdot c_6 \cdot c_7$. Let us temporarily imagine $|c_4 \cdot c_5 \cdot c_6 \cdot c_7.| = 1.$ Then $\mathbf{p} = +n\hbar\mathbf{k}$ as in $|a\rangle$ of Figure 3.3. 3 with an amplitude of $1/\sqrt{2}$ from Eq. (3.3. 8) transfers $\Delta\mathbf{p} = -\hbar\mathbf{k}$ also with an amplitude of $1/\sqrt{2}$, which is the required first half of our superposition Eq.(3.3. 6) $|a\rangle/\sqrt{2}+|b\rangle/\sqrt{2}.$ Similarly $\mathbf{p} = -n\hbar\mathbf{k}$ as in $|b\rangle$ of Figure 3.3. 3 gives the second half. It would thus seem that our amplitude is simply $c_5 \cdot c_6 \cdot c_6 \cdot c_7$. However from Eq. (3.3. 7) there is a 50% probability of the transitions $\psi_5 \rightleftarrows \psi_6$ in either direction, or an extra $1/\sqrt{2}$ amplitude factor for $\psi_5 \rightleftarrows \psi_6$ in either direction, similarly an extra $1/\sqrt{2}$ amplitude factor for $\psi_6 \rightleftarrows \psi_7$. These two extra $1/\sqrt{2}$ factors reduce the amplitude $c_4 \cdot c_5 \cdot c_6 \cdot c_7$ to $c_4 \cdot c_5 \cdot c_6 \cdot c_7 / (\sqrt{2} \times \sqrt{2}) = c_4 \cdot c_5 \cdot c_6 \cdot c_7 / 2$. Thus adding the four cases in Figure 3.3. 4 together and *treating all other factors as 1*:

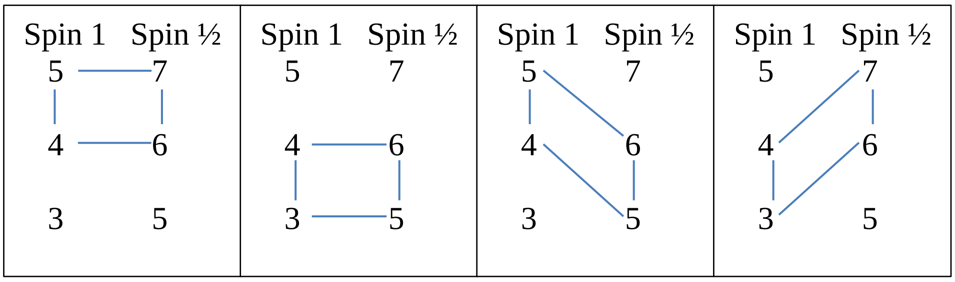
$$\text{Figure 3.3. 4 process amplitude factor is } 4 \times (c_4 \cdot c_5 \cdot c_6 \cdot c_7)/2 = 2c_4 \cdot c_5 \cdot c_6 \cdot c_7 \qquad (3.3.\ 9)$$

Spin 1 4 goes to 5 with $\mathbf{p} \leftarrow$ 5 returns to 4 with $\mathbf{p} \rightarrow$	Spin 1/2 7 goes to 6 with $\mathbf{p} \leftarrow$ 6 returns to 7 with $\mathbf{p} \rightarrow$	Spin 1 4 goes to 5 with $\mathbf{p} \leftarrow$ 5 returns to 4 with $\mathbf{p} \rightarrow$	Spin 1/2 6 goes to 7 with $\mathbf{p} \rightarrow$ 7 returns to 6 with $\mathbf{p} \leftarrow$
Spin 1 5 goes to 4 with $\mathbf{p} \rightarrow$ 4 returns to 5 with $\mathbf{p} \leftarrow$	Spin 1/2 7 goes to 6 with $\mathbf{p} \leftarrow$ 6 returns to 7 with $\mathbf{p} \rightarrow$	Spin 1 5 goes to 4 with $\mathbf{p} \rightarrow$ 4 returns to 5 with $\mathbf{p} \leftarrow$	Spin 1/2 6 goes to 7 with $\mathbf{p} \rightarrow$ 7 returns to 6 with $\mathbf{p} \leftarrow$

Figure 3.3. 4

The four possibilities in Figure 3.3. 4 are all between the same sets of Eigenfunctions ψ_{4k} & ψ_{5k} for spin 1, ψ_{6k} & ψ_{7k} for spin ½. But there are also four different sets of these between the groups of four Eigenfunctions A, B, C & D, as in Figure 3.3. 5; with their amplitudes from Eq. (3.3. 9) below each relevant box, which we also label as *A*, *B*, *C* & *D*. (Subscripts *a* refer to spin ½ and *b* to spin 1.)

A **B** **C** **D**



Amplitudes: $A = 2c_{4b}c_{5b}c_{6a}c_{7a}$, $B = 2c_{3b}c_{4b}c_{6a}c_{5a}$, $C = 2c_{4b}c_{5b}c_{6a}c_{5a}$, $D = 2c_{3b}c_{4b}c_{6a}c_{7a}$.

Figure 3.3. 5

3.3.4 Assumptions when looking at both vertexes of the interaction

Because we are looking at an interaction between identical spin ½ fermions each vertex has the same groups of Eigenfunctions A,B,C&D as in Figure 3.3. 5. From section 2.2.2 and Figure 3.1. 4 the three Eigenfunctions forming each of the interacting particles are born simultaneously. It is thus reasonable to assume that the amplitudes of each group of three Eigenfunctions have the same complex phase angle. The two fermions and one boson can be at different complex phase angles to each other but each one individually is a superposition of three Eigenfunctions at the same complex phase angle. Thus the four amplitudes *A,B,C &D* from Figure 3.3. 5 (*A,B,C &D* each comprising two fermion amplitudes and two boson amplitudes) must all have the same complex phase angle. Similarly the four amplitudes $A', B', C' \& D'$ of vertex 2 in Figure 3.3. 6 all have a common complex phase angle.

Eigenfunction Groups	A	B	C	D
Vertex 1	Amplitude A	Amplitude B	Amplitude C	Amplitude D
Vertex 2	Amplitude A'	Amplitude B'	Amplitude C'	Amplitude D'

Figure 3.3. 6

We are also going to assume that Eigenfunctions A of vertex 1 interact only with Eigenfunctions A of vertex 2 and Eigenfunctions B of vertex 1 interact only with Eigenfunctions B of vertex 2 etc. Eigenfunctions A of vertex 1 do not interact with Eigenfunctions B of vertex 2 etc. Thus *if all other amplitude factors are 1*:

$$\text{The total interaction amplitude } = AA' + BB' + CC' + DD' \qquad (3.3.\ 10)$$

Apart from a different complex phase angle this is equivalent to: ($A \& A', B \& B'$ etc. all differ by the same complex phase angle.)

$$\text{Total interaction amplitude} = A^2 + B^2 + C^2 + D^2 \qquad (3.3.\ 11)$$

$$\text{Interaction probability} = (A^2 + B^2 + C^2 + D^2)*(A^2 + B^2 + C^2 + D^2) \qquad (3.3.\ 12)$$

Using $(A^2 * A^2) = (A*A)(A*A)$ etc. this is equivalent to

$$\text{Interaction probability} = (A^*A + B^*B + C^*C + D^*D)^2 \tag{3.3. 13}$$

Using $P_{5a} = c_{5a}{}^*c_{5a}, P_{4b} = c_{4b}{}^*c_{4b}$ etc. & $A^*A = 4P_{4b}P_{5b}P_{6a}P_{7a}$ etc. this is equivalent to

$$16\left[P_{4b}P_{5b}P_{6a}P_{7a} + P_{3b}P_{4b}P_{6a}P_{5a} + P_{4b}P_{5b}P_{6a}P_{5a} + P_{3b}P_{4b}P_{6a}P_{7a}\right]^2$$

$$= 16\left[P_{4b}(P_{3b} + P_{5b})\right]^2 \left[P_{6a}(P_{5a} + P_{7a})\right]^2$$

Then using $c_{3b}{}^*c_{3b} + c_{4b}{}^*c_{4b} + c_{5b}{}^*c_{5b} = c_{5a}{}^*c_{5a} + c_{6a}{}^*c_{6a} + c_{6a}{}^*c_{6a} = 1.$

$$\text{Interaction Probability} = 2^4\left[c_{4b}{}^*c_{4b}(1 - c_{4b}{}^*c_{4b})\right]^2 \left[c_{6a}{}^*c_{6a}(1 - c_{6a}{}^*c_{6a})\right]^2 \tag{3.3. 14}$$

We have assumed to here that *all other amplitude factors are 1*. However at each vertex there are both fermion and boson superposition probabilities from Eq. (2.2. 4). Writing the superposition probability at each vertex $sN \cdot dk/k$ as $s_{1/2}N_1 dk/k,\ \ s_1 N_2 dk/k$ for clarity (where we already know that $s_{1/2} = 1/2$ & $N = 1$ for a massive spin ½ particle and $s_1 = 1$ & $N_2 = 2$ for an infinitesimal rest mass spin 1 boson) then including these factors (*if all other factors are one*) in Eq. (3.3. 14) our overall probability at wavenumber k is

$$\left[\frac{2s_{1/2}N_1 c_{6a}{}^*c_{6a}(1 - c_{6a}{}^*c_{6a})}{k}\right]^2 \left[\frac{2s_1 N_2 c_{4b}{}^*c_{4b}(1 - c_{4b}{}^*c_{4b})}{k}\right]^2$$

$$= \frac{\left[2s_{1/2}N_1 c_{6a}{}^*c_{6a}(1 - c_{6a}{}^*c_{6a})\right]^2 \left[2s_1 N_2 c_{4b}{}^*c_{4b}(1 - c_{4b}{}^*c_{4b})\right]^2}{(k)^4}.$$

The momentum per transfer is a total of $\pm\hbar\mathbf{k}$ and using Eq's. (3.3. 5), (3.3. 6) & Figure 3.3. 3 we have $(\pm\hbar\mathbf{k})^4 = q^4$ (then putting $\hbar = 1$) the interaction probability:

$$= \frac{\left[2s_{1/2}N_1 c_{6a}{}^*c_{6a}(1 - c_{6a}{}^*c_{6a})\right]^2 \left[2s_1 N_2 c_{4b}{}^*c_{4b}(1 - c_{4b}{}^*c_{4b})\right]^2}{q^4} \tag{3.3. 15}$$

This is the scalar interaction probability between two spin ½ fermions exchanging infinitesimal rest mass spin 1 bosons at very large spacings, where the fermions are effectively spin zero, imagining them as bare charges, *with all other factors being one*. Going through exactly the same procedure but similarly exchanging spin 2 infinitesimal rest mass scalar gravitons (with $N = 2 = N_2$ for clarity) the *gravitational interaction probability* between fermions becomes (using subscript *c* for spin 2) *if all other amplitude factors are 1*:

$$= \frac{\left[2s_{1/2}N_1 c_{6a} * c_{6a}(1 - c_{6a} * c_{6a})\right]^2 \left[2s_2 N_2 c_{4c} * c_{4c}(1 - c_{4c} * c_{4c})\right]^2}{q^4} \text{ for fermions.} \tag{3.3. 16}$$

And if for example two spin 1 photons exchange spin 2 gravitons (all infinitesimal rest mass with $N = 2 = N_2$) the *interaction probability* becomes *if all other amplitude factors are 1*:

$$= \frac{\left[2s_1 N_2 c_{4b} * c_{4b}(1 - c_{4b} * c_{4b})\right]^2 \left[2s_2 N_2 c_{4c} * c_{4c}(1 - c_{4c} * c_{4c})\right]^2}{q^4} \text{ for } N = 2 \text{ photons.} \tag{3.3. 17}$$

If now two massive photons (*which are a different superposition from* Figure 3.3. 2) exchange spin 2 gravitons the interaction probability becomes *if all other factors are one*

$$= \frac{\left[2s_1 N_1 c_{5b} * c_{5b}(1 - c_{5b} * c_{5b})\right]^2 \left[2s_2 N_2 c_{4c} * c_{4c}(1 - c_{4c} * c_{4c})\right]^2}{q^4} \text{ for } N = 1 \text{ photons.} \tag{3.3. 18}$$

We will discuss in section 5.1.1 why virtual gravitons cannot emit gravitons but also why real gravitons can. So if for example two spin 2 real gravitons exchange virtual gravitons (all infinitesimal rest mass with $N = 2 = N_2$) the interaction probability becomes *if all other amplitude factors are 1*:

$$= \frac{\left[2s_1 N_2 c_{4c} * c_{4c}(1 - c_{4c} * c_{4c})\right]^2 \left[2s_2 N_2 c_{4c} * c_{4c}(1 - c_{4c} * c_{4c})\right]^2}{q^4} \text{ for real gravitons.} \tag{3.3. 19}$$

General Relativity section 1.1.1 tells us the emission of gravitons is identical for both mass and energy. So keeping all other factors (such as mass) in Eq's. (3.3. 16), (3.3. 17) , (3.3. 18) & (3.3. 19) constant, the exchange probabilities must be the same in all four, implying:

$$2s_2N_2c_{4c}*c_{4c}(1-c_{4c}*c_{4c}) = 2s_1N_2c_{4b}*c_{4b}(1-c_{4b}*c_{4b}) = 2s_1N_1c_{5b}*c_{5b}(1-c_{5b}*c_{5b})$$
$$= 2s_{1/2}N_1c_{6a}*c_{6a}(1-c_{6a}*c_{6a}) \quad \text{or}$$

$$\underset{N=2\ \text{Spin } 2}{8c_{4b}*c_{4b}(1-c_{4b}*c_{4b})} = \underset{N=2\ \text{Spin } 1}{4c_{4b}*c_{4b}(1-c_{4b}*c_{4b})} = \underset{N=1\ \text{Spin } 1}{2c_{5b}*c_{5b}(1-c_{5b}*c_{5b})} = \underset{N=1\ \text{Spin } 1/2}{c_{6a}*c_{6a}(1-c_{6a}*c_{6a})}$$

(3.3. 20)

Assuming all other factors are 1, and remembering that we are simplifying by looking at spin ½ superpositions sufficiently far apart so we can treat them as approximately spherically symmetric or effectively spin zero even though they are supposed to be bare charges:
Under the same scalar exchange conditions QED tells us that with electrons for example:

The probability of scalar or coulomb exchange in Eq (3.3. 15). $= \frac{4\alpha^2}{q^4}.$ (3.3. 21)

Let us also temporarily ignore the fact that gluons have very limited range and imagine that our "thought experiment" also applies to colour charges exchanging gluons. Now treat *α as a general coupling constant regardless of type of charge or whether it is mass. Also temporarily ignore the fact that there can be up to six primary electric charges. Plus call the general coupling ratio in all three cases χ*. If the primary constant is one and putting $\alpha = \chi^{-1},\ 2s_{1/2} = 1,\ 2s_1 = 2,\ N_1 = 1\ \&\ N_2 = 2$ we can equate Eq's. (3.3. 15) & (3.3. 21):

$$\frac{[c_{6a}*c_{6a}(1-c_{6a}*c_{6a})]^2[4c_{4b}*c_{4b}(1-c_{4b}*c_{4b})]^2}{q^4} = \frac{4(\chi^{-1})^2}{q^4}$$
$$\text{or} \quad [c_{6a}*c_{6a}(1-c_{6a}*c_{6a})][c_{4b}*c_{4b}(1-c_{4b}*c_{4b})] = \chi^{-1}/2 \qquad (3.3.\ 22)$$

Combining Eq's. (3.3. 20) & (3.3. 22)

$$\underset{N=2\ \text{Spin } 2}{8c_{4c}*c_{4c}(1-c_{4c}*c_{4c})} = \underset{N=2\ \text{Spin } 1}{4c_{4b}*c_{4b}(1-c_{4b}*c_{4b})} = \underset{N=1\ \text{Spin } 1}{2c_{5b}*c_{5b}(1-c_{5b}*c_{5b})} = \underset{N=1\ \text{Spin } 1/2}{c_{6a}*c_{6a}(1-c_{6a}*c_{6a})}$$
$$= \sqrt{2/\chi}$$

(3.3. 23)

The coupling ratio is thus fundamentally the same for colour, electromagnetism and gravity. However reintroducing six primary electric charges Eq. (3.3. 2) and because of the way electric charge is defined

$$\frac{1}{\chi}=\left[\frac{1}{\chi_G}=\frac{G_{\text{Secondary}}}{G_{\text{Primary}}}\right]=\left[\frac{1}{\chi_C}=\frac{\alpha_{Colour(\text{Secondary})}}{\alpha_{Colour(\text{Primary})}}\right]=\frac{1}{36}\left[\frac{1}{\chi_{EM}}=\frac{\alpha_{EM(\text{Secondary})}}{\alpha_{EM(\text{Primary})}}\right] \tag{3.3. 24}$$

We also need to remember from Eq. (3.3. 3) that $\alpha_{3S}{}^{-1}=\chi_C$, or in other words provided $\alpha_{3P}=1,$ the ratio χ_C is also the inverse of the colour coupling constant at the superposition cutoff near the Planck length (section 4.3).

Equations (3.3. 22) & (3.3. 23) tell us that for any interactions between pairs of superpositions, the inverse coupling ratio always involves the product of the central superposition member probability, by one minus that probability, and the same product for the other superposition. In section 4.3 when we introduce gravity we will solve these ratios and show that despite all the simplifications the above equations seem to fit the Standard Model reasonably well provided there are only three families of fermions.

3.4 Electrostatic Energy between two Infinite Superpositions

3.4.1 Using a simple quantum mechanics pre QED era approach

In section 3.3 we have shown that fermion superpositions can exchange boson superpositions in the same way as electrons can exchange virtual photons for example. Providing the superposition amplitudes are appropriate, the coupling constants can be just as in QED, though we will look further at this in section 4.1.1. So it would initially seem that evaluating electrostatic energy between superpositions is unnecessary. However there are three key reasons for revisiting a quantum mechanics method used before the days of QED to find the scalar potentials between two charges (or infinite superpositions).

a) It facilitates a simplified solution to the magnetic energy between superpositions in section 3.5 where we modify relevant equations in a simple manner.

b) We also use some of the same equations when looking at why borrowing energy and mass from zero point fields requires the universe to expand after the Big Bang.

c) These same equations also help explain why General Relativity requires the metric to change near any mass concentration. (Actually for the same reason as (b).)

We assume spherically symmetric $l=3$ superpositions emit virtual scalar photons in this section and $l=3, m=\pm 2$ superpositions emit virtual $m=\pm 1$ photons in section 3.5. As section 3.3 has shown that we can achieve the same electromagnetic coupling constant α we can use the scalar photon emission probability $(2\alpha/\pi)(dk/k)$ covered in section 2.1.1. From section 3.3 we can also see that the effective average emission point has to be the center of superpositions. The probability of finding this interacting virtual photon (or spin 1 superposition) decays exponentially with distance travelled as the energy of the virtual photon is borrowed. The normalized wavefunction ψ for such a virtual scalar photon of wave number k emitted at $r = 0$ is:

$$\psi = \sqrt{\frac{2k}{4\pi}}\frac{e^{-kr}e^{+i\omega t}}{r} = \sqrt{\frac{2k}{4\pi}}\frac{e^{-kr}e^{+ikr}}{r}.$$

Wavefunction ψ is spherically symmetric as scalar photons are time polarized**.** Figure 3.4. 1 plots the radial probabilities of the exponential range of the virtual photon and the dominant $n = 6$ mode of its relating superposition ψ_k. The effective range of the interacting photon is of a similar order to the radial probability dimensions of ψ_{6k}.

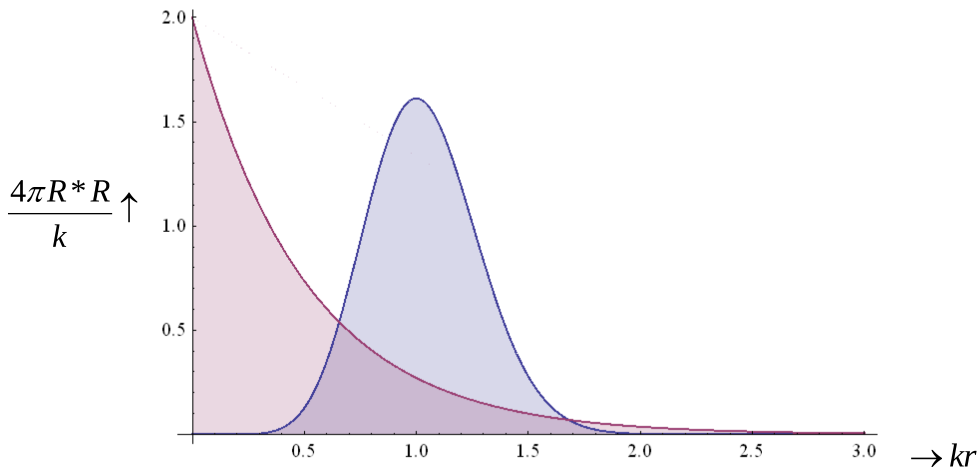


Figure 3.4. 1 Radial probability of ψ_{6k} and the exponential decay with radius of its interacting virtual boson $R^* R \propto 2ke^{-2kr}$. These curves are the same for all k, applying equally to virtual photons, gravitons and to large k value gluons etc.

For simplicity in what follows we locate two superpositions (which we refer to as sources) in cavities that are small in relation to the distance between them. The accuracy of our results depends on how far apart they are in relation to the cavity size. Consider two spherically symmetric sources distance $2C$ apart emitting virtual scalar photons as in Figure 3.4. 2 where point P is r_1 from source 1, & r_2 from source 2. Let ψ_1 be the amplitude from source 1, and ψ_2 be the amplitude from source 2.

Thus $\psi_1 = \sqrt{\frac{2k}{4\pi}} \frac{e^{-kr_1+ikr_1}}{r_1}$ & $\psi_2 = \sqrt{\frac{2k}{4\pi}} \frac{e^{-kr_2+ikr_2}}{r_2}$ (3.4. 1)

Consider $(\psi_1 + \psi_2)^*(\psi_1 + \psi_2) = \psi_1^*\psi_1 + \psi_1^*\psi_2 + \psi_2^*\psi_1 + \psi_2^*\psi_2$

Now $\psi_1^*\psi_1$ & $\psi_2^*\psi_2$ are just the normal probability densities around sources 1 & 2 as though they are infinitely far apart but the work done per pair of superpositions k on bringing 2 sources closer together is in the interaction term: $\psi_1^*\psi_2 + \psi_2^*\psi_1$.

$$\psi_1^*\psi_2 = \frac{2k}{4\pi r_1 r_2} e^{-kr_1} e^{-kr_2} e^{+ikr_1} e^{-ikr_1} = \frac{2k}{4\pi r_1 r_2} e^{-k(r_1+r_2)} e^{+ik(r_1-r_2)}$$

$$\psi_2^*\psi_1 = \frac{2k}{4\pi r_1 r_2} e^{-kr_2} e^{-kr_1} e^{+ikr_2} e^{-ikr_2} = \frac{2k}{4\pi r_1 r_2} e^{-k(r_1+r_2)} e^{-ik(r_1-r_2)}$$

$$\psi_1^*\psi_2 + \psi_2^*\psi_1 = \frac{2k}{4\pi r_1 r_2} e^{-k(r_1+r_2)} \left[e^{+ik(r_1-r_2)} + e^{-ik(r_1-r_2)} \right]$$

$$= \frac{4k}{4\pi r_1 r_2} e^{-k(r_1+r_2)} \cos k(r_1 - r_2)$$

Now put $(A = r_1 + r_2, B = r_1 - r_2)$ & $\psi_1^*\psi_2 + \psi_2^*\psi_1 = \frac{4k}{4\pi r_1 r_2} e^{-Ak} \cos kB$ (3.4. 2)

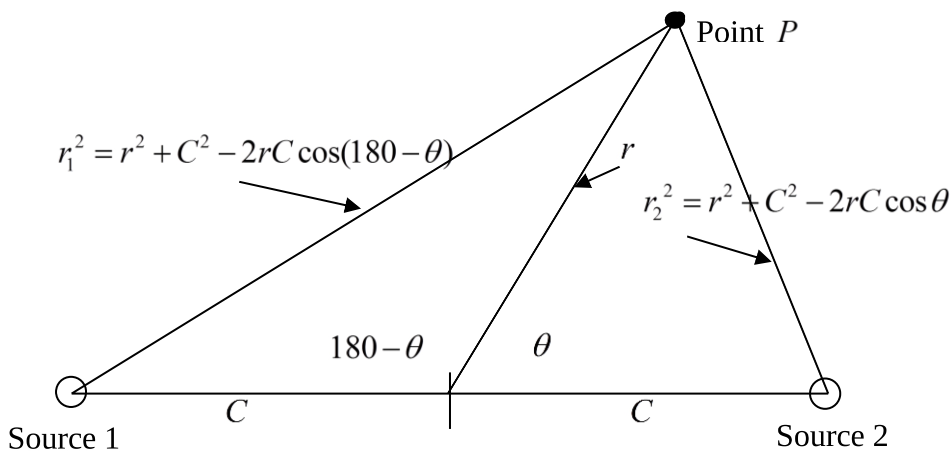


Figure 3.4. 2

Real work is done bringing superpositions together and we can treat these virtual photons as having real energy $\hbar\omega = \hbar kc$. (We will see in section 5.1.1 that virtual gravitons cannot have this energy.) Using virtual photon emission probability $(2\alpha/\pi)(dk/k)$ from section 2.1.1.

$$\text{Energy } \hbar\omega = \hbar kc \times \text{Probability } \frac{2\alpha}{\pi}\frac{dk}{k} = \frac{2\alpha\hbar c}{\pi} dk \tag{3.4. 3}$$

Our interaction energy $@\, k$ is thus $(\psi_1 {}^*\psi_2 + \psi_2 {}^*\psi_1 \;)\dfrac{2\alpha\hbar c}{\pi} dk$ which using Eq. (3.4. 2)

Interaction energy $@\, k$ is $\dfrac{2\alpha\hbar c}{\pi} dk \;\; \dfrac{4k}{4\pi r_1 r_2} e^{-Ak} \cos kB$. The total interaction energy density due to $\psi_1 {}^*\psi_2 + \psi_2 {}^*\psi_1$ for all k is

$$\frac{2\alpha\hbar c}{\pi}\frac{4}{4\pi r_1 r_2}\int_0^\infty k e^{-Ak}\cos(Bk)dk \tag{3.4. 4}$$

$$\int_0^\infty k e^{-Ak}\cos(Bk)dk = \frac{A^2 - B^2}{(A^2+B^2)^2} \tag{3.4. 5}$$

Where $\quad A^2 = (r_1 + r_2)^2 = r_1^2 + 2r_1 r_2 + r_2{}^2 \;\;\&\;\; B^2 = (r_1 - r_2)^2 = r_1^2 - 2r_1 r_2 + r_2{}^2$

$$\text{Thus} \quad A^2 = (r_1 + r_2)^2 = r_1{}^2 + 2r_1 r_2 + r_2{}^2 \;\;\& A^2 + B^2 = 2(r_1{}^2 + r_2{}^2) \tag{3.4. 6}$$

$$= 2(r^2 + C^2) \text{ as } \cos(180-\theta) = -\cos\theta$$

$$\text{and} \quad A^2 + B^2 = 4(r^2 + C^2) \tag{3.4. 7}$$

Putting Eq's. (3.4. 4), (3.4. 5), (3.4. 6) & (3.4. 7) together $\dfrac{A^2 - B^2}{(A^2+B^2)^2} = \dfrac{4r_1 r_2}{16(r^2 + C^2)^2}$

$$\int_0^\infty k e^{-Ak}\cos(Bk)dk = \frac{r_1 r_2}{4(r^2 + C^2)^2}$$

$$\frac{2\alpha\hbar c}{\pi}\frac{4}{4\pi r_1 r_2}\int_0^\infty k e^{-Ak}\cos(Bk)dk = \frac{2\alpha\hbar c}{\pi}\frac{4}{4\pi r_1 r_2}\;\frac{r_1 r_2}{4(r^2 + C^2)^2}$$

$$= \frac{2\alpha\hbar c}{\pi}\frac{1}{4\pi}\frac{1}{(r^2 + C^2)^2} \tag{3.4. 8}$$

This is the total interaction energy density of time polarized virtual photons at point P due to $\psi_1{}^*\psi_2+\psi_2{}^*\psi_1$ for all k and there are no directional vectors to take into account. We will use similar equations for the vector potential ($m=\pm 1$) photons for magnetic energies but will then need directional vectors. Equation (3.4. 8) is the energy due to the interaction of amplitudes at any radius r from the centre of the pair. It is independent of θ, and to get the total energy of interaction we multiply by $4\pi r^2 dr$ for any layer dr and integrate from $r=0\rightarrow\infty$.

The total interaction energy is $\quad \dfrac{2\alpha\hbar c}{\pi}\displaystyle\int_0^\infty\int_0^\infty(\psi_1{}^*\psi_2+\psi_2{}^*\psi_1)dk\ 4\pi r^2 dr$

Using Eq. (3.4. 8) $\quad = \dfrac{2\alpha\hbar c}{\pi}\dfrac{1}{4\pi}\displaystyle\int_0^\infty \frac{4\pi r^2 dr}{(r^2+C^2)^2}$

Thus $\quad \dfrac{2\alpha\hbar c}{\pi}\displaystyle\int_0^\infty\int_0^\infty(\psi_1{}^*\psi_2+\psi_2{}^*\psi_1)dkdv = \frac{2\alpha\hbar c}{\pi}\int_0^\infty \frac{r^2 dr}{(r^2+C^2)^2}$

$$\int_0^\infty \frac{r^2 dr}{(r^2+C^2)^2} = \frac{1}{2C}\frac{\pi}{2}$$

The interaction or potential energy is $\dfrac{\alpha\hbar c}{2C}=\dfrac{\alpha\hbar c}{R}$ (3.4. 9)

If $R=2C$ is the distance between the centres of our assemblies, this is the classical potential. The procedure used here with small changes, simplifies the derivation of the magnetic moment; we reuse some equations, but in a slightly modified form taking polarization vectors into account.

3.5 Magnetic Energy between two spin aligned Infinite Superpositions

In this section we are going to consider two infinite superpositions that form Dirac spin ½ states. We will look at the magnetic energy between them when they are both in a spin up state say along some z axis as in Figure 3.5. 1. We are not looking at the magnetic energy here when they are both coupled in a spin 0 or spin 1 state. That is, both Dirac spin ½ states have their $\sqrt{3}\hbar/2$ spin vectors randomly oriented around the z axis with $\hbar/2$ components aligned along this z axis. Also in this section we will be dealing with transversely polarized virtual photons and must take account of polarization vectors. In section 3.2.2 and Eq. (3.2. 7) spin ½ states are generated only from $l=3, m=2$ states and as transversely polarized photons are superpositions of $m=\pm1$ photons they can only be emitted from these $l=3, m=2$ states, the remaining states are spherically symmetric and cannot emit transversely polarized photons. We don't yet know the value of amplitudes $|c_n|$ so we will derive the magnetic energy in terms of these. We will then equate this energy to the Dirac values assuming a g value of 2 before QED corrections; this allows us to evaluate in section 4.4 the amplitudes $|c_n|$ in terms of the ratio χ_{EM} between primary and secondary electromagnetic coupling. We can then evaluate in section 4.1 the primary electromagnetic coupling constant α_{EMP} in terms of the ratio χ_{EM}. This section uses the same format as Chapter 18, "The Feynman Lectures on Physics" Volume 3, Quantum Mechanics [8].

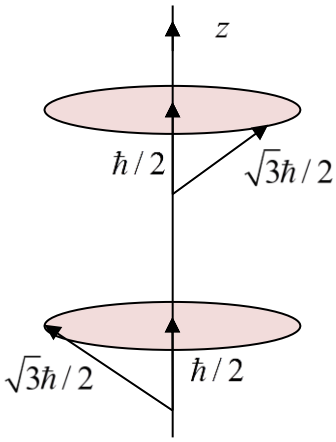


Figure 3.5. 1

An $l=3, m=2$ state can emit a right hand circularly (R.H.C.) polarized $(m=+1)$ photon in the $+z$ direction. Let the amplitude for this be temporarily $|R\rangle$.
An $l=3, m=-2$ state can emit a left hand circularly (L.H.C.) polarized $(m=-1)$ photon in the $+z$ direction. Let the amplitude for this also be temporarily $|L\rangle$.
First rotate the z axis about the y axis by angle θ (call this operation $S|R\rangle$) then use $\langle x'| = (1/\sqrt{2})\left[\langle R'| + \langle L'|\right]$ and multiply on the right by operation $S|R\rangle$.
The amplitude to emit a transversely polarized photon in the x' direction is thus

$$\langle x'|S|R\rangle = \frac{1}{\sqrt{2}}\left[\langle R'|S|R\rangle + \langle L'|S|R\rangle\right]$$

Where $\langle R'|S|R\rangle = \langle 3,+2'|S|3,+2\rangle = (1/4)\left[2+2\cos\theta - 4\sin^2\theta + 3\sin^2\theta\cos\theta\right]$ is the amplitude an $l=3, m=2$ state remains in an $l=3, m=2$ state after rotation by angle θ.

Also $\langle L'|S|R\rangle = -\langle 3,-2'|S|3,+2\rangle = (1/4)\left[2-2\cos\theta - 4\sin^2\theta - 3\sin^2\theta\cos\theta\right]$ is minus the amplitude that an $l=3, m=2$ state is in an $l=3, m=-2$ state after rotation by θ.

Putting this together $$\langle x'|S|R\rangle = \frac{1-2\sin^2\theta}{\sqrt{2}} = \frac{\cos 2\theta}{\sqrt{2}} \tag{3.5. 1}$$

An $l=3, m=2$ state can also emit an $(m=+1)$ photon in the $-z$ direction but it will now be left hand circularly polarized. Let this amplitude be temporarily: $|L\rangle$.
Similarly an $l=3, m=-2$ state can emit an $(m=-1)$ photon in the $-z$ direction which is right hand circularly polarized. Let this amplitude be temporarily: $|R\rangle$.

We can go through the same procedure as above to get $$\langle x'|S|L\rangle = \frac{\cos 2\theta}{\sqrt{2}} \tag{3.5. 2}$$

This amplitude Eq. (3.5. 2) is for a photon emitted in the opposite direction to amplitude Eq. (3.5. 1) but $\cos 2\theta = \cos 2(180+\theta)$ and we can simply add these two amplitudes. Let us assume however that an $l=3, m=2$ state has equal amplitudes to emit in the $+z$ & $-z$ directions of $|R\rangle/\sqrt{2}$ and $|L\rangle/\sqrt{2}$.

With these amplitudes; $$\frac{1}{\sqrt{2}}\left[\langle x'|S|R\rangle + \langle x'|S|L\rangle\right] = \frac{\cos 2\theta}{2} + \frac{\cos 2\theta}{2} = \cos 2\theta \tag{3.5. 3}$$

Eqation (3.5. 3) is the angular component of the amplitude for a transverse x' polarization in the new z' direction where $x \to x'$ & $z \to z' = \theta$. When $\theta = 0$ or 180 the on axis amplitude for transverse polarization is one as expected ignoring other factors. Using the same normalization factors (we check the validity of this in section 3.5.1 we can still use the amplitudes and phasing of our original time mode photons Eq's. (3.4. 1) but instead of including polarization vectors we will for simplicity just use the cosine of the angle $(\gamma-\delta)$ between them (as in Figure 3.5. 2) as a multiplying factor. Including the angular factor Eq. (3.5. 3) in our earlier scalar amplitudes Eq's. (3.4. 1) we have for our new wavefunctions:

$$\psi_1 = \cos 2\delta\sqrt{\frac{2k}{4\pi}}\frac{e^{-kr_1+ikr_1}}{r_1} \quad \& \quad \psi_2 = \cos 2\gamma\sqrt{\frac{2k}{4\pi}}\frac{e^{-kr_2+ikr_2}}{r_2} \tag{3.5. 4}$$

The transverse polarized photons from sources (1) & (2) have polarization vectors $|x_1\rangle$ and $|x_2\rangle$ at angle to each other $(\gamma-\delta)$, (Figure 3.5. 2) and the complex product becomes:

$$(\psi_1 + \psi_2)^*(\psi_1 + \psi_2) \quad = \psi_1{}^*\psi_1 + (\psi_1{}^*\psi_2 + \psi_2{}^*\psi_1)(\cos(\gamma - \delta) + \psi_2{}^*\psi_2$$

Where the interaction term is now: $(\psi_1{}^*\psi_2 + \psi_2{}^*\psi_1)\cos(\gamma - \delta)$ and as in the scalar case (section 3.4.1) but now using Eq's. (3.5. 4)

$$\psi_1{}^*\psi_2 \cos(\gamma - \delta) = \cos 2\delta \cos 2\gamma \frac{2k}{4\pi r_1 r_2} e^{-k(r_1+r_2)} e^{+ik(r_1-r_2)} \cos(\gamma - \delta)$$

$$\psi_2{}^*\psi_1 \cos(\gamma - \delta) = \cos 2\delta \cos 2\gamma \frac{2k}{4\pi r_1 r_2} e^{-k(r_1+r_2)} e^{-ik(r_1-r_2)} \cos(\gamma - \delta)$$

$$(\psi_1{}^*\psi_2 + \psi_2{}^*\psi_1)\cos(\gamma - \delta) = \cos 2\delta \cos 2\gamma \frac{4k}{4\pi r_1 r_2} e^{-Ak} \cos kB \cos(\gamma - \delta) \qquad (3.5.\ 5)$$

Where just the same as section 3.4.1, Eq. (3.4. 2) we have used: $A = r_1 + r_2 \,\&\, B = r_1 - r_2$.
In the laboratory frame ψ_{nk} has amplitude β_{nk} to be in an $m = +2$ state (section 3.1). For a superposition ψ_k we need the expectation value $\langle \beta_{nk} \rangle$.

Using Eq's. (3.1. 11) & (3.1. 12) $\beta_{nk} = \dfrac{K_{nk}}{\sqrt{1 + K_{nk}{}^2}}$ and $\langle \beta_{nk} \rangle = \left\langle \dfrac{K_{nk}}{\sqrt{1 + K_{nk}{}^2}} \right\rangle$.

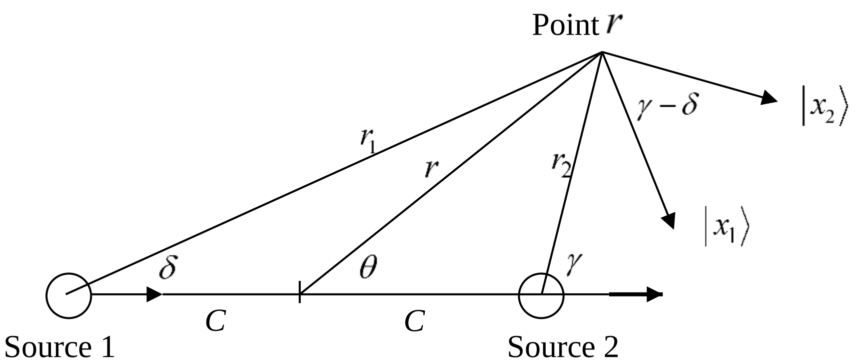


Figure 3.5. 2 Two sources $2C$ apart, both with $\beta_{nk}{}^2 \times (m = +2)$ states along the joining line, $\delta \,\&\, \gamma$ are the respective angles to P, $r_1 \,\&\, r_2$ are the respective distances to r.

To keep our integrals simple we will assume that $\langle \beta_{nk} \rangle <<<< 1$ or that our spacing is very large and that our interacting k values are also very small. (We can make a comparison with the Dirac values at any large spacing so our accuracy should not be affected.) Thus when $\left\langle \beta_{nk}{}^2 \right\rangle <<<< 1$ (and $\left\langle \gamma_{nk}{}^2 \right\rangle \approx 1$), we can approximate the above as: $\langle \beta_{nk} \rangle \approx \langle K_{nk} \rangle$ and from Eq. (3.2. 10) $\langle K_{nk} \rangle = \dfrac{\lambda\!\!\!^{-}_c k\sqrt{2s}}{2}\langle n \rangle \; = \dfrac{\lambda\!\!\!^{-}_c k}{2}\langle n \rangle$ as $s = ½$ for spin ½ and also $\langle n \rangle = \sum_{n=5,6,7} (c_n{}^* c_n) n$

.

The approximate expectation value for the superposition is thus

$$\langle \beta_{nk} \rangle \approx \frac{\lambda\!\!\!^{-}_c \langle n \rangle k}{2} \tag{3.5. 6}$$

We will use the same probability of virtual photon emission and energy $\hbar kc$ as in Eq.(3.4. 3) of the scalar case.

$$\frac{2\alpha\hbar c}{\pi} dk \tag{3.5. 7}$$

3.5.1 Checking our normalization factors

Let us pause and check the reasonableness of this and our normalization factors. From Eq's. (3.4. 1) for scalar photons $\psi^*\psi = \dfrac{2k}{4\pi}\dfrac{e^{-2kr}}{r^2}$ × emission probability $\dfrac{2\alpha}{\pi}\dfrac{dk}{k}$ gives a scalar ψ_k emission probability density $\psi^*\psi \dfrac{4\alpha}{\pi} = \dfrac{2k}{4\pi}\dfrac{e^{-2kr}}{r^2}\dfrac{2\alpha}{\pi}\dfrac{dk}{k}$. The probability density at k for the transversely polarized case, using Eq. (3.5. 4) then including Eq. (3.5. 7) and $\left\langle \beta_{nk}{}^2 \right\rangle$

$$\langle \beta_{nk} \rangle^2 \psi'^*\psi' \frac{2\alpha}{\pi}\frac{dk}{k} = \langle \beta_{nk} \rangle^2 \cos^2 2\delta \frac{2k}{4\pi}\frac{e^{-2kr}}{r^2}\frac{2\alpha}{\pi}\frac{dk}{k} \ .$$

If we now consider the on axis $\delta = 0$ case we see that the transverse polarized on axis emission probability density at k is:

$$\langle \beta_{nk} \rangle^2 \frac{2k}{4\pi}\frac{e^{-2kr}}{r^2}\frac{2\alpha}{\pi}\frac{dk}{k} = \langle \beta_{nk} \rangle^2 \;\; \psi^*\psi \frac{2\alpha}{\pi}\frac{dk}{k}$$

Just as in QED the factor $\langle \beta_{nk} \rangle^2$ is the factor we need for this on axis emission probability density ratio between transverse and scalar polarization. This justifies using the same normalization constant $\sqrt{2k/4\pi}$ for both the scalar and magnetic wavefunctions. We seem to be on the right track and putting Eq. (3.5. 6) and Eq. (3.5. 7) into Eq. (3.5. 5) we get the transverse interaction energy @ wavenumber k :

$$\langle \beta_{nk} \rangle^2 (\psi_1 {}^* \psi_2 + \psi_2 {}^* \psi_1) \cos(\gamma - \delta) \left[\frac{2\alpha\hbar c}{\pi} dk \right]$$

$$= \left[\frac{ƛ_c{}^2 \langle n \rangle^2 k^2}{4} \right] \cos 2\delta \cos 2\gamma \frac{4k}{4\pi r_1 r_2} e^{-Ak} \cos kB \cos(\gamma - \delta) \left[\frac{2\alpha\hbar c}{\pi} dk \right]$$

Rearranging this: $\langle \beta_{nk} \rangle^2 (\psi_1 {}^* \psi_2 + \psi_2 {}^* \psi_1) \cos(\gamma - \delta) \left[\frac{2\alpha\hbar c}{\pi} dk \right]$

$$= \frac{2\langle n \rangle^2 ƛ_c{}^2 \alpha\hbar c}{\pi} \frac{\cos 2\delta \cos 2\gamma \cos(\gamma - \delta)}{4\pi r_1 r_2} \left[k^3 e^{-Ak} \cos(kB) dk \right] \tag{3.5. 8}$$

As in the scalar case we integrate over k first but now with a k^3 term due to the inclusion of the $\langle \beta_{nk} \rangle^2$ factor which is approximately proportional to k^2 from Eq. (3.5. 6).

Using $A = r_1 + r_2$ & $B = r_1 - r_2$ and Eq's. (3.4. 6) & (3.4. 7)

$$\int_0^\infty \left[k^3 e^{-Ak} \cos(kB) dk \right] = \frac{3}{8} \left[\frac{2r_1^2 r_2^2 - (r^2 + C^2)^2}{(r^2 + C^2)^4} \right]$$

And thus: $$\int_0^\infty \langle \beta_{nk} \rangle^2 (\psi_1 {}^* \psi_2 + \psi_2 {}^* \psi_1) \cos(\gamma - \delta) \frac{2\alpha\hbar c}{\pi} dk$$

$$= \frac{2\langle n \rangle^2 ƛ_c{}^2 \alpha\hbar c}{\pi} \frac{\cos 2\delta \cos 2\gamma \cos(\gamma - \delta)}{4\pi r_1 r_2} \times \frac{3}{8} \left[\frac{2r_1^2 r_2^2 - (r^2 + C^2)^2}{(r^2 + C^2)^4} \right] \tag{3.5. 9}$$

Equation (3.5. 9) is the magnetic interaction energy density at point P for all wave numbers k. Figure 3.5. 2 is a plane of symmetry that can be rotated through angle 2π around the axis of symmetry (the joining line along the axis of the 2 spin aligned sources). To evaluate the total magnetic energy density over all space we just multiply by $4\pi r^2 \sin\theta d\theta dr$. We thus integrate Eq. (3.5. 9) × $4\pi r^2 \sin\theta d\theta dr$ =

$$\frac{3\langle n \rangle^2 ƛ_c{}^2 \alpha\hbar c}{4\pi} \int_0^\infty \int_0^{\pi/2} \frac{\cos 2\delta \cos 2\gamma \cos(\gamma - \delta)}{r_1 r_2} \left[\frac{2r_1^2 r_2^2 - (r^2 + C^2)^2}{(r^2 + C^2)^4} \right] r^2 \sin\theta d\theta dr \tag{3.5. 10}$$

Now $\int_0^\infty \int_0^{\frac{\pi}{2}} \frac{\cos 2\delta \cos 2\gamma \cos(\gamma - \delta)}{r_1 r_2} \left[\frac{2r_1^2 r_2^2 - (r^2 + C^2)^2}{(r^2 + C^2)^4} \right] r^2 \sin\theta d\theta dr$ can be reduced to the single integral: $\frac{1}{8C^3}\int_0^1 \sqrt{1-x^2}\left[\frac{(7-5x^2)}{x^3}\ln\frac{1+x}{1-x}-\frac{14}{x^2}+\frac{16}{3}\right]dx$ which can be also expressed as an infinite series in p (to not confuse with superposition value n):

$$\frac{1}{8C^3}\sum_{p=1}^{p=\infty}\left[\frac{14}{2p+3}-\frac{10}{2p+1}\right]\frac{(2p-1)!}{(p-1)!(p+1)!4^p}.\frac{\pi}{2} \quad =\frac{1}{8C^3}\frac{(160-51\pi)}{6}.\frac{\pi}{2}$$

$$\text{(Putting } R=2C\text{)} \quad =\frac{1}{R^3}\frac{(160-51\pi)}{6}.\frac{\pi}{2} \tag{3.5. 11}$$

$$\text{This infinite series is approximately } \approx -\frac{1}{R^3}\frac{\pi}{54(1.0045062....)} \tag{3.5. 12}$$

Putting Eq.(3.5. 12) into Eq. (3.5. 9) the total magnetic interaction energy over all frequencies and all space for 2 spin aligned infinite superpositions is:

$$U' \approx \frac{3\langle n\rangle^2 \lambda\!\!\!^{-}{}_c{}^2\alpha\hbar c}{4\pi}\left[-\frac{1}{R^3}\frac{\pi}{54(1.0045062....)}\right]$$

$$\text{We will call this } \quad U\left(\text{superpositions}\right)\approx -\left[\frac{\langle n\rangle^2 \lambda\!\!\!^{-}{}_c{}^2\alpha\hbar c}{72R^3(1.0045062....)}\right] \tag{3.5. 13}$$

We can equate this magnetic energy to the classical value assuming the Dirac value of $g=2$ for spin ½ (No QED corrections have been applied so it must be $g=2$). For the arrangement of spins as in Figure 3.5. 1 the Dirac magnetic energy between two spin ½ states is

$$U\left(\text{Dirac}\right)=-\left[\frac{2\mu^2}{4\pi\varepsilon_o c^2R^3}\right] \tag{3.5. 14}$$

Using the Dirac magnetic moment $\mu=\frac{e\hbar}{2m_0}=\frac{e\hbar c}{2m_0c}=\frac{ec\lambda\!\!\!^{-}{}_c}{2}$ the Dirac magnetic energy is

$$U(\text{Dirac}) \;=-\left[\frac{\lambda\!\!\!^{-}{}_c{}^2\alpha\hbar c}{2R^3}\right]$$

The approximation used in deriving Eq. (3.5. 6) $\gamma^2\beta^2\approx\beta^2$ for $\beta^2<<<1$ is true only when $R>>>\lambda\!\!\!^{-}{}_c$. This error in β^2 is of the order of $\lambda\!\!\!^{-}{}_c{}^2/R^2$ and rapidly tends to zero with R. There is no upper limit on the value of distance R we can choose. Thus comparing our estimate of the magnetic energy with Dirac's value when $R>>>\lambda\!\!\!^{-}{}_c$.

$$U(\text{Dirac}) = U(\text{Superpositions}) \text{ or } -\left[\frac{\lambda\!\!\!^{-}{}_c{}^2 \alpha\hbar c}{2R^3}\right] \approx -\left[\frac{\langle n\rangle^2 \lambda\!\!\!^{-}{}_c{}^2 \alpha\hbar c}{72R^3(1.0045062....)}\right] \tag{3.5. 15}$$

All terms cancel except $\langle n\rangle$ leaving: $\langle n\rangle^2 \approx 36(1.0045062.....)$

The expectation value $\langle n\rangle$ in our superposition is slightly more than $n=6$ our dominant mode. This is why we have used a three member superposition centred on this dominant $n=6$ mode. The two side modes $n=5$ & $n=7$ are smaller so that:

$$\langle n\rangle = \sum_{n=5,6,7.} (c_n c_n{}^*) n \approx \sqrt{36(1.0045062...)} \approx 6.01350345 \tag{3.5. 16}$$

This is for Dirac spin ½ particles. This mean value of n creates a $g=2$ fermion which *QED corrections* (*which are secondary interactions*) *increase slightly to the experimental value.* In section 4.1 we will solve the primary electromagnetic coupling constant in terms of the ratio χ_{EM} using Eq. (3.5. 16). It is important to remember that this derivation of the magnetic energy applies only to two infinite assemblies (or particles) localized in two cavities that are small in relation to their distance R apart. They must also be both on the z axis with their spins aligned (or anti aligned) along this same z axis as in Figure 3.5. 1 & Figure 3.5. 2. Also the agreement with Dirac and in what follows is possible *if superposition* ψ_k *interacts only with virtual photons of the same wavenumber* k.

4 High Energy Superposition Cutoffs

4.1 Primary Electromagnetic Coupling to Spin ½ Infinite Superpositions

Equation (3.5. 16) tells us what is required of spin ½ superpositions to correctly behave as Dirac fermions, allowing us to solve for $\alpha_{EMP}{}^{-1}$ as a function of the coupling ratio χ.

Starting with Eq. (3.5. 16) $\langle n\rangle = \sum_{n=5,6,7.} (c_n c_n{}^*) n \approx \sqrt{36(1.0045062...)} \approx 6.01350345$.

$$5c_5{}^*c_5 + 6c_6{}^*c_6 + 7c_7{}^*c_7 \approx 6.01350345723 \quad \text{but} \quad 6c_5{}^*c_5 + 6c_6{}^*c_6 + 6c_7{}^*c_7 = 6$$

$$\text{Thus } c_7{}^*c_7 - c_5{}^*c_5 \approx 0.01350340345273$$

As $c_7{}^*c_7 + c_5{}^*c_5 = 1 - c_6{}^*c_6$ we can solve for $c_7{}^*c_7$ & $c_5{}^*c_5$ in terms of $c_6{}^*c_6$.

$$c_7{}^*c_7 \approx 0.50675172 - \frac{c_6{}^*c_6}{2} \quad \& \quad c_5{}^*c_5 \approx 0.49324827 - \frac{c_6{}^*c_6}{2} \tag{4.1. 1}$$

From Eq. (2.3. 12) the Q^2A^2 required to produce this superposition with amplitudes c_n is

$$Q^2A^2 = \sum_{n=5,6,7} c_n * c_n \frac{n^4\hbar^2k^4r^2}{81} \text{ and using Eq.(4.1. 1)}$$

$$\sum_{n=5,6,7} c_n * c_n\ n^4 = 625c_5 * c_5 + 1296c_6 * c_6 + 2401c_7 * c_7 \approx 1524.991 - 217c_6 * c_6$$

Thus $Q^2A^2 = \sum_{n=5,6,7} c_n * c_n \frac{n^4\hbar^2k^4r^2}{81} \approx [18.82705 - 2.67901c_6 * c_6]\hbar^2k^4r^2$ is *the required vector potential squared to produce this spin ½ superposition*. From Eq. (2.2. 4) with $s =$ ½ & $N = 1$ for massive fermions $Q^2A^2 = \frac{2\left[8+8\sqrt{\alpha_{EMP}})\right]^2}{3\pi}\hbar^2k^4r^2$ *is the available* Q^2A^2.

Equating *required and available*: $2\left[8+8\sqrt{\alpha_{EMP}})\right]^2 \approx 3\pi[18.82705 - 2.67901c_6 * c_6]$

$$\left[1+\sqrt{\alpha_{EMP}})\right]^2 \approx [1.386256 - 0.197258c_6 * c_6]$$

$$\alpha_{EMP} \approx \left[\sqrt{1.386256 - 0.197258c_6 * c_6} - 1\right]^2 \qquad (4.1.\ 2)$$

From Eq's. (3.3. 23) & (3.3. 24) $c_6 * c_6(1 - c_6 * c_6) = \sqrt{2/\chi} = 6\sqrt{2/\chi_{EM}}$, we can now solve for α_{EMP} as a function of either χ_{EM} or χ. We then use Eq. (3.3. 24) again to get $\alpha_{EMS}{}^{-1} @ \approx L_P$.

Now both χ_{EM} and χ are fundamentally the same ratio differing only by 36:1, because electron superpositions have six primary charges whereas we define them as one fundamental charge (section 3.3.1) and quarks have only one colour charge (Table 2.2. 1). Because $\chi = \alpha_3{}^{-1}$ at the cutoff near L_P it is more convenient to work with.

From Eq. (3.3. 23) $c_6 * c_6 = \frac{1}{2} \pm \frac{1}{2}\sqrt{1 - 4\sqrt{\frac{2}{\chi}}}$ and there are two solutions for each χ; one with $c_6 * c_6$ dominant and two smaller $c_5 * c_5$ & $c_7 * c_7$ side modes, or the reverse with $c_6 * c_6$ the minor player and two larger $c_5 * c_5$ & $c_7 * c_7$ side modes. As the values for α_{EMP} with $c_6 * c_6$ dominant fit the Standard Model best, we include only these. Table 4.1. 1 shows these results for $\chi = \alpha_3{}^{-1}$ at possible cutoffs, but in the range $\chi = 50 \rightarrow 51$ only, as we will see in section 4.3 and Figure 4.1. 2 that the yellow highlighted $\chi \approx 50.41$ fits the Standard Model most closely.

Coupling Ratio χ	$c_6 * c_6$	$\alpha^{-1}{}_{EM\,Primary}$	$\alpha^{-1}{}_{EMSecondary}$ @ Cutoff
50.00	≈ 0.723607	≈ 75.4414	≈ 104.7798
50.20	≈ 0.724497	≈ 75.5447	≈ 105.3429
50.40	≈ 0.725378	≈ 75.6472	≈ 105.9060
50.41	≈ 0.725422	≈ 75.6523	≈ 105.9342
50.60	≈ 0.726250	≈ 75.7488	≈ 106.4692
50.80	≈ 0.727115	≈ 75.8497	≈ 107.0324
51.00	≈ 0.727970	≈ 75.9499	≈ 107.5956

Table 4.1. 1 Coupling ratio χ versus $\alpha^{-1}{}_{EMSecondary}$ highlighting $\chi = 50.41$.

4.1.1 Comparing this with the Standard Model

In the real world of secondary interactions the Standard Model splits the electromagnetic force into the two components $\alpha_1 \& \alpha_2$ for energies greater than the mass/energy of the Z_0 boson or $\approx 91.2\ GeV$. [9] Because primary interactions are with only spin zero preons, the coupling can also be electromagnetic only, as in the above equations.

The weak force split obeys $\alpha_{EM}{}^{-1} = \frac{5}{3}\alpha_1{}^{-1} + \alpha_2{}^{-1}$

Also $\alpha^{-1}{}_1 = \frac{3}{5}\alpha_{EM}{}^{-1}Cos^2\theta_W$ & $\alpha_2{}^{-1} = \alpha_{EM}{}^{-1}Sin^2\theta_W$ where θ_W is the Weinberg angle. (4.1. 3)

Assuming three families of fermions and one Higgs field the SM [10] predicts

$$\begin{aligned}
\alpha_1{}^{-1} &\approx 58.98 \pm 0.08 - \frac{4.1}{2\pi}\log_e \frac{Q}{91.2} \\
\alpha_2{}^{-1} &\approx 29.6 \pm 0.04 + \frac{3.16666}{2\pi}\log_e \frac{Q}{91.2} \\
\alpha_3{}^{-1} &\approx 8.47 \pm 0.22 + \frac{7}{2\pi}\log_e \frac{Q}{91.2}
\end{aligned} \tag{4.1. 4}$$

Combining Eq's. (4.1. 3) & (4.1. 4)

$$\alpha_{EM}{}^{-1} \approx 127.9 \pm 0.173 - \frac{3.66666}{2\pi}\log_e \frac{Q}{91.2} \tag{4.1. 5}$$

Figure 4.1. 1 plots these four inverse coupling constants. Figure 4.1. 2 plots the intersection of $\alpha^{-1}{}_{EMSecondary}$ predicted in Table 4.1. 1 and the Standard Model prediction for $\alpha^{-1}{}_{EM}$ in Eq. (4.1. 5). It also includes the high energy cutoff suggested by gravity Eq (4.3. 8). It would initially seem in Figure 4.1. 2 that there is an unusually large error band in the predicted

results. However $\Delta\alpha^{-1}{}_{EMSecondary} / \Delta\chi^{-1} \approx 2.8$ is approximately constant in this table and the error band in the Standard Model colour coupling α_3^{-1} of ± 0.22 in Eq's (4.1. 4) translates into the much larger error band for $\alpha^{-1}{}_{EMSecondary}$, of $\pm 0.22 \times 2.8 \approx \pm 0.62$ in Figure 4.1. 2. In section 4.3 we look at superposition cutoffs exponentially tailing off into the Planck region; but only after we have introduced the effect of gravity into our equations.

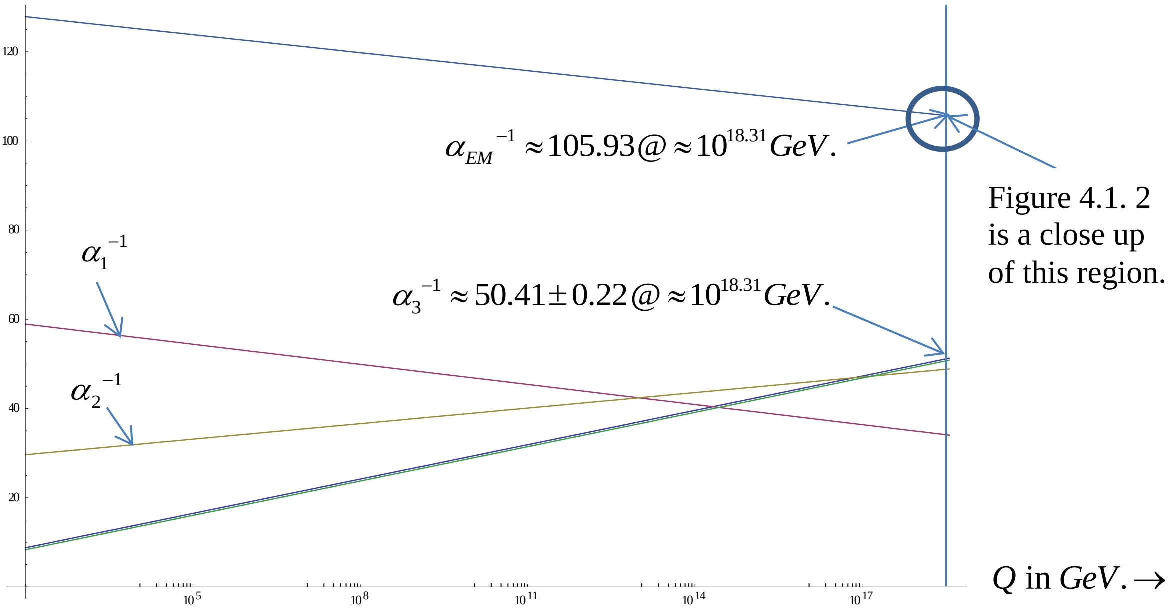


Figure 4.1. 1 Standard Model based on three families of fermions and one Higgs field.

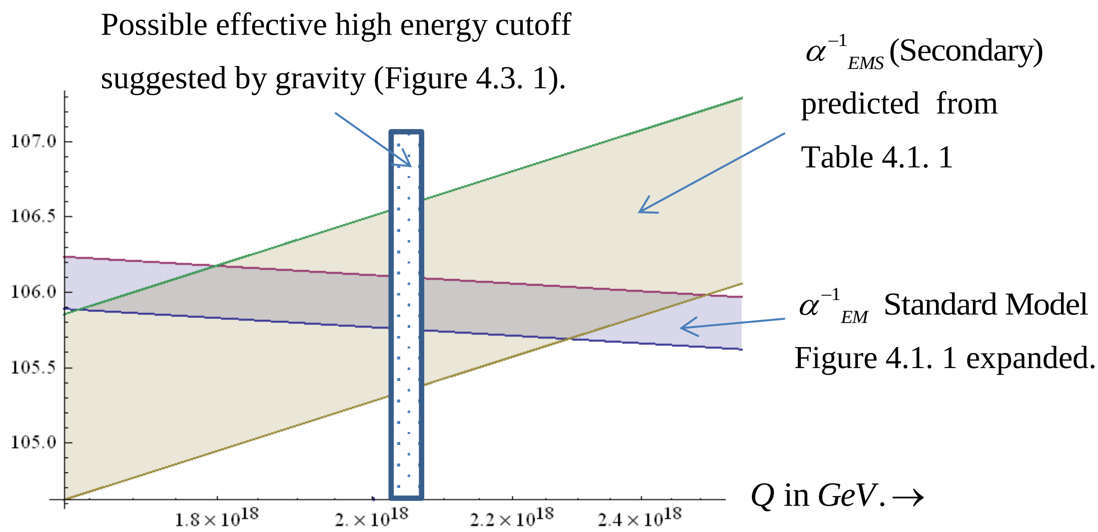


Figure 4.1. 2 Predicted range for $\alpha^{-1}{}_{EM}$ (Bare) $\approx 105.93 \pm 0.17$ in the intersecting region of the Standard Model and this paper.

4.2 Introducing Gravity into our Equations

4.2.1 Initially looking at square superposition cutoffs only

In section 3.2 it was easier to work with one wavefunction ψ_{nk} at a time, and we found later that a superposition of them did not affect our final results; we will do the same here. We also found in Eq's. (3.2. 3) & (3.2. 6) that the integrals for both angular momentum and rest masses are of identical form. They both ended up including the term

$\left[\frac{-1}{1+K_{nk}{}^2}\right]_0^\infty$ which if $K_{nk}cutoff < \infty$ becomes $\left[\frac{-1}{1+K_{nk}{}^2}\right]_0^{K_{nk}cutoff}$ which is equal to

$$1-\frac{1}{1+K_{nk}{}^2cutoff}=\frac{K_{nk}{}^2cutoff}{1+K_{nk}{}^2cutoff}=\frac{1}{1+1/K_{nk}{}^2(cutoff)}=\frac{1}{1+\varepsilon} \quad (4.2.\ 1)$$

where using Eq. (3.1. 11) the infinitesimal $\varepsilon=\frac{1}{K_{nk}{}^2cutoff}=\frac{2m_0{}^2c^2}{n^2\hbar^2(k_{cutoff})^2 s}$ (4.2. 2)

For integral or half integral $\hbar$ angular momentum absolute precision is required but Eq. (3.2. 6) now gives us $\mathbf{L}_z(Total)=\frac{sm\hbar}{2}\left[\frac{-1}{1+K_{nk}{}^2}\right]_0^{K_{nk}cutoff}=\frac{sm\hbar}{2}\frac{1}{1+\varepsilon}$. So can the effect of gravity increase our probabilities from $sN\cdot\frac{dk}{k}$ to $sN\cdot(1+\varepsilon)\frac{dk}{k}$? We need to also remember here that *these derivations are for massive infinite superpositions* where $N=1$ in Eq.(2.2. 4).

The first question we need to address is what is the effective preon mass to be used when coupling to gravity? In Eq. (3.1. 4) we said the preon rest mass is $m_0/(8\gamma_{nk}\sqrt{2s})$ for each of the 8 preons that build a spin ½ particle of rest mass m_0. Now gravity couples to the total mass including the kinetic energy. It also couples to the momentum in Einstein's energy-momentum tensor, but to keep things simple initially, we will temporarily ignore the momentum term. At the start of the interaction each preon mass is $m_0/(8\gamma_{nk}\sqrt{2s})$ and after the interaction (Figure 3.1. 3) it is $m_0\gamma_{nk}(1+\beta_{nk}{}^2)/(8\sqrt{2s})$. Let us think semi classically again and see where it leads us. We have been using magnitudes of velocities as they are the most convenient way to express our equations even if not the conventional language of quantum mechanics. The interaction with the zero point fields takes the momentum of each preon from zero to $2m_0\gamma_{nk}\beta_{nk}c/(8\sqrt{2s})$ (Figure 3.1. 3). While this happens as a quantum step change let us imagine it as a virtually infinite acceleration from zero velocity to $2\beta_{nk}/(1+\beta_{nk}{}^{2})$, which is the relativistic velocity addition (see Figure 3.1. 1) of 2 equal steps of β_{nk}. At the half way point after one step the velocity is β_{nk} (the velocity of the CMF, the preon mass has increased to $m_0/(8\sqrt{2s})$. We can imagine this as being like the central point of a quantum interaction.

We will conjecture this midway point preon mass $m_0/(8\sqrt{2s})$ is the mass value that gravity acts on and we will see that it is indeed the only value that makes all the equations fit. Also it does not make sense to choose either of the end point masses. We can also get reassurance from the properties of the Feynman transition amplitude which tells us in Eq. (3.1. 15)

$\dfrac{(p_i+p_f)^z}{(p_i+p_f)^0}=\dfrac{2m_0\gamma_{nk}\beta_{nk}}{2m_0\gamma_{nk}}=\beta_{nk}$ and the ratio of space to time polarization in the LF is $\beta_{nk}{}^{2}$.

This centre of momentum velocity tells us the key properties of the interaction. (Still ignoring coupling to this momentum term.) We will thus assume we have 8 preons in each ψ_{nk} of effective gravitational mass $m_0/(8\sqrt{2s})$ with effective total gravitational mass $m_0/\sqrt{2s}$. To put the gravitational constant in the same form as the other coupling constants we need to divide it by $\hbar c$. The gravitational coupling amplitude is thus $m_0\sqrt{G_P/(2s\hbar c)}$ to the gravitational zero point field, where G_P is the primary gravitational constant. Now this gravitational amplitude can be regarded as a complex vector just as colour and electromagnetism. We assumed for simplicity, as they are both spin 1 field particles, that colour and electromagnetism were parallel. Gravity is spin 2 so it can be expected that it may well be at a different complex angle to the other two. In fact if we put it at right angles to the other two the equations all seem to fit. We will thus conjecture using Eq.(3.3. 24) that (still ignoring gravitational coupling to the momentum component):

The gravitational coupling amplitude is $im_0\sqrt{G_P/(2s\hbar c)}=im_0\sqrt{\chi\cdot G_S/(2s\hbar c)}$ (4.2. 3)

From here on we are going to put the secondary gravitational coupling constant G_s to a bare mass equal to the measured gravitational constant G. We will elaborate on this further in section 6.1.2. So modifying Eq's. (2.2. 1) to (2.2. 3) by adding Eq. (4.2. 3)

$$Q^2A^2 = \left[\frac{\left| 8+8\sqrt{\alpha_{EMP}} + im_0\sqrt{\chi \cdot G/(2s\hbar c)} \right|^2}{3\pi sN} \hbar^2 k^4 r^2 \right] . \left[\frac{sN \cdot dk}{k} \right]$$

$$Q^2A^2 = \left[\frac{\left[8+8\sqrt{\alpha_{EMP}}) \right]^2}{3\pi sN} \hbar^2 k^4 r^2 \right] . \left[\frac{sN(1+\varepsilon')dk}{k} \right] \text{ where } \varepsilon' = \frac{m_0{}^2 \chi \cdot G}{2s\hbar c(8+8\sqrt{\alpha_{EMP}})^2}$$

Our previous wavefunctions ψ_k required $Q^2A^2 = \dfrac{\left[8+8\sqrt{\alpha_{EMP}}) \right]^2}{3\pi sN} \hbar^2 k^4 r^2$ from Eq.(2.2. 4).

Gravity has increased the probability of our previous wavefunctions ψ_k by $1+\varepsilon'$ as required to obtain precision in our integrals for $\hbar/2 \,\&\, \hbar$ if $K_{nk}cutoff < \infty$.

Using Eq.(4.2. 2) now put $\varepsilon' = \dfrac{m_0{}^2 \chi \cdot G}{2s\hbar c(8+8\sqrt{\alpha_{EMP}})^2} = \varepsilon = \dfrac{1}{K_{nk}{}^2 cutoff} = \dfrac{2m_0{}^2c^2}{sn^2\hbar^2(k_{cutoff})^2}$ (4.2. 4)

4.2.2 Including the momentum term from Einstein's Energy Momentum Tensor

If we now include the momentum term from Einstein's energy-momentum tensor the gravitational coupling amplitude Eq. (4.2. 3) becomes $im_0(1+\beta_{nk})\sqrt{\chi \cdot G/(2s\hbar c)}$, and ε' becomes $\varepsilon'' \to \varepsilon'(1+\beta_{nk})^2$ where β_{nk} is the velocity at the central point of this interaction which varies with *k*. We have to integrate a varying ε'' to find the averaged effect on the total angular momentum. Using the equations leading up to Eq. (3.2. 6) these integrals become:

$$\mathbf{L}_z(Total) = sm\hbar \int_0^{K_c} \frac{K_{nk}{}^2}{(1+K_{nk}{}^2)^2} \frac{dK_{nk}}{K_{nk}} [1+\varepsilon''] = sm\hbar \int_0^{K_c} \frac{K_{nk}{}^2}{(1+K_{nk}{}^2)^2} \frac{dK_{nk}}{K_{nk}} [1+\varepsilon'(1+\beta_{nk})^2]$$

From Eq. (3.1. 12) $\beta_{nk} = \dfrac{K_{nk}}{\sqrt{1+K_{nk}{}^2}}$ and with appropriate factors the averaged effect on ε'' is:

$$\varepsilon'' \to \varepsilon' \int_0^{K_c} \frac{2K_{nk}{}^2}{(1+K_{nk}{}^2)^2} \frac{dK_{nk}}{K_{nk}} [1+\frac{K_{nk}}{(1+K_{nk}{}^2)}]^2 \text{ or } \varepsilon'' \to \varepsilon'(1+\frac{1}{6}+\frac{\pi}{4}) \approx 1.952\varepsilon' \quad (4.2.\ 5)$$

Equation (4.2. 4) becomes $\varepsilon'' \approx \dfrac{1.952 m_0{}^2 \chi \cdot G}{2s\hbar c(8+8\sqrt{\alpha_{EMP}})^2} \approx \varepsilon = \dfrac{1}{K_{nk}{}^2 cutoff} = \dfrac{2m_0{}^2c^2}{sn^2\hbar^2(k_{cutoff})^2}$

or $\dfrac{1.952\chi \cdot G}{4\hbar c(8+8\sqrt{\alpha_{EMP}})^2} \approx \dfrac{c^2}{n^2\hbar^2(k_{cutoff})^2}$ and $\dfrac{1.952\chi}{256(1+\sqrt{\alpha_{EMP}})^2}\dfrac{G\hbar}{c^3} \approx \dfrac{1}{n^2(k_{cutoff})^2}$

But $L_P{}^2 = \dfrac{G\hbar}{c^3}$ and $\dfrac{1.952\chi \cdot L_P{}^2}{256(1+\sqrt{\alpha_{EMP}})^2} \approx \dfrac{1}{n^2(k_{cutoff})^2}$ (4.2. 6)

Now consider the radial probability of $\psi_{nk} = C_{nk} r^3 \exp(-n^2k^2r^2/18) Y(\theta,\varphi)$.

Ignoring factors C_{nk} & $Y(\theta,\varphi)$ and differentiating $P_r = r^8 \exp(-n^2k^2r^2/9)$ we get

$$\frac{dP_r}{dr} = \left[8 - \frac{2n^2k^2r^2}{9}\right].\left[r^7 \exp(-n^2k^2r^2/9)\right] = 0 \text{ when } r^2 = \frac{36}{n^2k^2} \text{ or } r_{peak} = \frac{6}{nk}$$

Thus for a superposition $\left\langle r_{peak} \right\rangle = \dfrac{6}{\left\langle n \right\rangle k}$ & & $\left\langle r_{peak} \right\rangle^2 = \dfrac{36}{\left\langle n \right\rangle^2 k^2}$ (4.2. 7)

Also for superpositions using Eq. (3.2. 10) , Eq. (4.2. 2**)** becomes:

$$\varepsilon = \frac{1}{\left\langle K_{nk} cutoff \right\rangle^2} = \frac{2m_0{}^2c^2}{s\left\langle n \right\rangle^2 \hbar^2 (k_{cutoff})^2} \tag{4.2. 8}$$

Putting equations (4.2. 6), (4.2. 7) & (4.2. 8) together

$$\frac{1.952\chi \cdot L_P{}^2}{256(1+\sqrt{\alpha_{EMP}})^2} \approx \frac{1}{\left\langle n \right\rangle^2 (k_{cutoff})^2} = \frac{\left\langle r_{peak@cutoff} \right\rangle^2}{36}. \tag{4.2. 9}$$

The expectation value of the maximum probability radius at this cutoff is independent of superposition value n. In section 4.1, from Eq's. (4.1. 2) we find $1+\sqrt{\alpha_{EMP}} \approx 1.115$ and thus:

$$\frac{\left\langle r_{peak@cutoff} \right\rangle}{L_P} \approx \frac{\sqrt{\chi}}{2.128} \tag{4.2. 10}$$

The peak radial probability at cutoff depends only on ratio χ for all fundamental particles represented as infinite superpositions. From Figure 4.1. 1 & Figure 4.1. 2 we can see that $\chi = \chi_C = \alpha_3{}^{-1} \approx 50.41$ at the cutoff and putting this into Eq. (4.2. 10) we get

$$r_{peak@cutoff} = 1/k_{cutoff} \approx 3.34 L_P. \tag{4.2. 11}$$

We have no idea what might happen inside the Planck region, so cutoffs outside the Planck length make intuitive sense. Figure 4.2. 1 illustrates the radial Gaussian probability distribution of such a square cutoff wavefunction just outside the Planck length. The Planck energy $E_P \approx 1.22 \times 10^{19} GeV$. and using Eq. (4.2. 11) assuming a square cutoff:

$$\text{Square cutoff energy is } \hbar k_{cutoff} c \approx 10^{18.56} GeV. \qquad (4.2.\ 12)$$

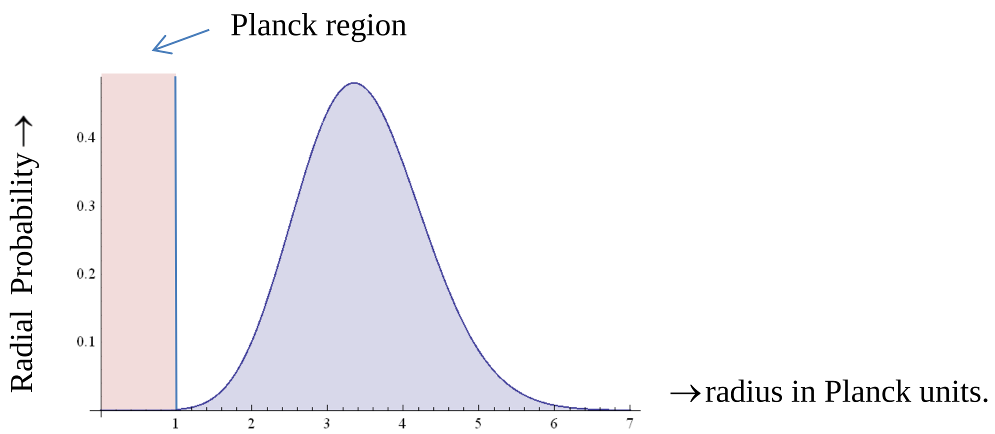


Figure 4.2. 1 Radial probability for a square cutoff wavefunction $\psi_k\ @ \approx 10^{18.56} GeV$.

4.3 Exponential Superposition Cutoffs

Because a square cutoff is close to the Planck length (Figure 4.2. 1) it is reasonable to assume that any cutoff should be as sharp as possible to minimize penetration of the Planck region. Squared exponentials cut off faster than linear. Our wavefunctions for superpositions are also squared exponentials, so we will try this path and also find that it seems to work.

Using $\int_0^{\infty} Exp\left(-\frac{\langle K_{nk(cutoff)}\rangle^2}{\langle K_{nk}\rangle^2}\right)\frac{2d\langle K_{nk}\rangle}{\langle K_{nk}\rangle^3} = \frac{1}{\langle K_{nk(cutoff)}\rangle^2}$ & $\langle K_{nk(cutoff)}\rangle >>>> 1$ we can show:

$$\int_0^{K_{nk}(cutoff)} \frac{2\langle K_{nk}\rangle}{(1+\langle K_{nk}\rangle^2)^2} d\langle K_{nk}\rangle \approx \int_0^{\infty}\left[1 - Exp - \frac{\langle K_{nk(cutoff)}\rangle^2}{\langle K_{nk}\rangle^2}\right]\frac{2\langle K_{nk}\rangle}{(1+\langle K_{nk}\rangle^2)^2} d\langle K_{nk}\rangle \qquad (4.3.\ 1)$$

Both give the result: $\approx \ 1 - \frac{1}{K^2_{nk(cutoff)}}$ but only if $K_{nk(cutoff)} \to \infty$. Also from Eq's (3.2. 10)

$$\frac{\left\langle K_{nk(cutoff)}\right\rangle}{\left\langle K_{nk}\right\rangle}=\frac{k_{cutoff}}{k} \quad \text{and thus} \quad 1-Exp\frac{-\left\langle K_{nk(cutoff)}\right\rangle^2}{\left\langle K_{nk}\right\rangle^2}=1-Exp\frac{-k_{cutoff}{}^2}{k^2} \tag{4.3. 2}$$

Where $\left\langle K_{nk(cutoff)}\right\rangle$ & k_{cutoff} are the same as for square cutoffs in Eq. (4.2. 8). Thus from section 4.2.1, an exponential cutoff of this form gives identical results to the original square cutoff for both angular momentum and rest masses. Using Eq's. (3.3. 15) and (3.3. 21) the scalar interaction probability between two spin ½ superpositions for example is:

$$\frac{\left[2s_{1/2}N_1c_{6a}*c_{6a}(1-c_{6a}*c_{6a})\right]^2\left[2s_1N_2c_{4b}*c_{4b}(1-c_{4b}*c_{4b})\right]^2}{q^4}=\frac{4\alpha^2}{q^4} \tag{4.3. 3}$$

If all superpostitions cutoff as in Eq. (4.3. 2) the relative probability of each of the components in all superpositions also cutoff at the same rate. The probabilities $C_{6a}*C_{6a}$ & $C_{4b}*C_{4b}$ in Eq. (4.3. 3) do also; thus reducing the overall interaction probability between superpositions by:

$$\begin{gathered}\text{Interaction probabilities between pairs of superpositions } \propto\left[1-Exp\frac{-k^2cutoff}{k^2}\right]^8 \\ \text{For example the coupling constant } \alpha \text{ cuts off as } \propto\left[1-Exp\frac{-k^2cutoff}{k^2}\right]^4\end{gathered} \tag{4.3. 4}$$

If we next consider the transition of an $N=2$ infinitesimal rest mass photon superposition into a massive $N=1$ photon superposition (as for example in the Higgs mechanism) each member of the groups of three transform into another member. We can use the same logic as in the first sentence of the above paragraph. For example an $N=2$ ψ_{4k} with probability $C_{4b}*C_{4b}$ can transform to an $N=1$ ψ_{5k} with probability $C_{5b}*C_{5b}$ (See Eq's. (3.3. 17) & (3.3. 18). But there are three such pairs so that (*if all other factors are one*) the overall transition probability between all three member superpositions cuts off proportionally as

$$\text{Transition probability cutoff } \propto\left[1-Exp\frac{-k^2cutoff}{k^2}\right]^6 \tag{4.3. 5}$$

Pair creation and the Higgs mechanism both cutoff as in Eq. (4.3. 5) and combine two powers of six. This suggests that vacuum polarization/ virtual photon emmission cuts off as

Alpha Higgs mechanism Pair creation Total

$$\left[1-Exp\frac{-k^2cutoff}{k^2}\right]^4\times\left[1-Exp\frac{-k^2cutoff}{k^2}\right]^6\times\left[1-Exp\frac{-k^2cutoff}{k^2}\right]^6\propto\left[1-Exp\frac{-k^2cutoff}{k^2}\right]^{16}$$

(4.3. 6)

Vacuum polarization is a logarithmic term in k proportional to $\int dk/k$ where k starts near the Compton wavelength. We can get an approximate idea of an effective exponential cutoff by equating

$$\int_{k(start)}^{k(effective)}\frac{dk}{k}=\log_e\frac{k(effective)}{k(start)}=\int_{k(start)}^{\infty}\frac{\left[1-Exp\frac{-k^2(square\ cutoff)}{k^2}\right]^{16}dk}{k} \qquad (4.3.\ 7)$$

Now Eq. (4.2. 12) showed that introducing gravity gave a square cutoff at $\approx 10^{18.56}GeV$ and inserting this into Eq. (4.3. 7) yields an

$$\text{Electromagnetic high energy cutoff } \hbar k(effective\ cutoff)c\approx 2.04\times 10^{18}GeV. \qquad (4.3.\ 8)$$

Similar arguments apply to quarks emitting gluons. Because gluons are also coloured they emit more gluons and so on, effectively spreading and increasing the colour charge. Each step of this process adds further powers of four as in Eq. (4.3. 4) in the cutoff region. But gluons also form quark antiquark pairs partially screening this extra colour charge. This involves powers of six as in Eq. (4.3. 5), thus quark antiquark screening cuts off at a power of six even faster than the colour spreading cutoff. Because of these opposing effects this is not a simple calculation. The cutoff for both electromagnetism and colour however must be very close to each other, so we will conjecture that they are both probably very close to Eq. (4.3. 8) $\approx 2.04\times 10^{18}GeV$. This approximate cutoff value is plotted in Figure 4.1. 2.

Figure 4.3. 1 plots various exponential cutoffs and would seem to indicate that the colour cutoff is highly likely to be in a similar region to the electromagnetic. The integrated area of the exponential superposition cutoff having greater than Planck Energy is approximately 0.023. Of course we have only conjectured a possible exponential cutoff here. However if Eq's. (4.3. 4), (4.3. 7) & (4.3. 8) are even approximately correct, the fit between the

gravitationally predicted cutoff, the Dirac equation requirements represented in Table 4.1. 1 and the Standard Model predictions of α_{EM}^{-1} seem to be within the experimental error bands.

If the cutoff energy is $\approx 2.04\times 10^{18}GeV.$ the bare charges using either Table 4.1. 1 predictions, or the Standard Model as in Eq's. (4.1. 4) & (4.1. 5), are approximately

$$\begin{aligned}\alpha_3^{-1} &= \chi \approx 50.41 \pm 0.22 @ \approx 10^{18.31} GeV.\\ \alpha_{EM} &\approx 105.93 \pm 0.17 \quad @ \approx 10^{18.31} GeV.\end{aligned} \tag{4.3. 9}$$

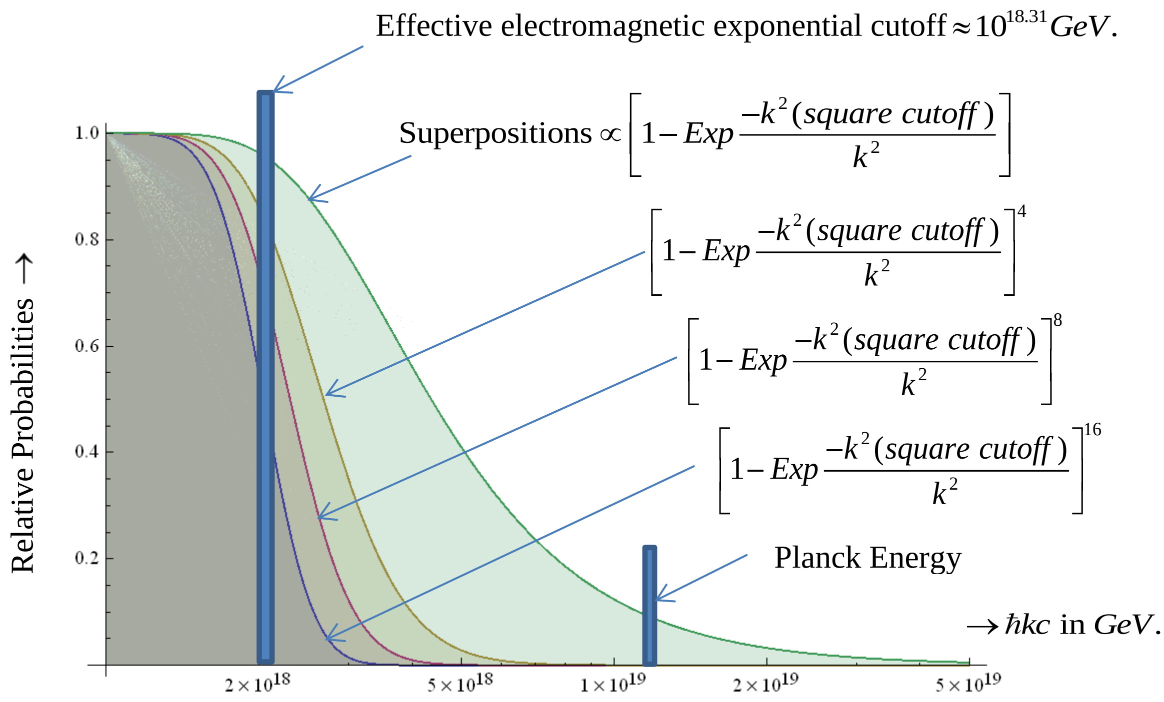


Figure 4.3. 1 Exponential cutoffs where $\hbar k(square\ cutoff)c \approx 10^{18.56} GeV.$ see Eq. (4.2. 12).

4.4 Solving for spin ½, spin 1 and spin 2 superpositions

Infinitesimal rest mass superpositions where $N = 2$ are covered further in section 6.3 but Eq. (4.3. 9) $\chi = \alpha_3^{-1} \approx 50.41$ & Eq. (3.3. 23) allow us to solve all $N = 1$ & $N = 2$ superpositions.

$$\begin{gathered} N=2 \ \ \text{Spin } 2 \qquad\qquad N=2 \ \ \text{Spin } 1 \qquad\qquad N=1 \ \ \text{Spin } 1 \qquad\qquad N=1 \ \ \text{Spin } 1/2 \\ 8c_{4c}{}^{*}c_{4c}(1-c_{4c}{}^{*}c_{4c}) = 4c_{4b}{}^{*}c_{4b}(1-c_{4b}{}^{*}c_{4b}) = 2c_{5b}{}^{*}c_{5b}(1-c_{5b}{}^{*}c_{5b}) = c_{6a}{}^{*}c_{6a}(1-c_{6a}{}^{*}c_{6a}) \\ = \sqrt{2/\chi} \approx \sqrt{2/50.41} \approx 0.1992 \end{gathered} \tag{4.4. 1}$$

Starting with spin ½ we can solve this to get $c_6 * c_6 \approx 0.7254$ as the dominant value. Putting $c_6 * c_6 \approx 0.7254$ into Eq. (4.1. 2) or alternatively use Table 4.1. 1

$$\alpha_{EMP} \approx \left[\sqrt{1.386256 - 0.197258 c_6 * c_6} - 1\right]^2 \approx 75.65^{-1} \tag{4.4. 2}$$

From Eq. (2.2. 4) the available $Q^2 A^2 = \dfrac{\left[8 + 8\sqrt{\alpha_{EMP}}\,)\right]^2}{3\pi sN} \hbar^2 k^4 r^2$ with probability $\dfrac{sN \cdot dk}{k}$ where we can ignore the infinitesimal factor of $(1+\varepsilon)$ due to gravity. And from Eq. (2.3. 12)

$$Q^2 A^2 = \sum_n c_n * c_n \frac{n^4 \hbar^2 k^4 r^2}{81} = \frac{\left[8 + 8\sqrt{\alpha_{EMP}}\,)\right]^2}{3\pi sN} \hbar^2 k^4 r^2$$

Inserting α_{EMP} from Eq. **(**4.4. 2) into this we find that: $\sum_n c_n * c_n \cdot n^4 \approx \dfrac{1367.58}{2sN}$

$$\begin{aligned} \sum_n c_n * c_n \cdot n^4 &\approx 1367.58 \text{ for spin 1/2} \quad \& \quad \approx 683.79 \text{ for spin 1 when } N = 1; \\ &\approx 341.9 \text{ for spin 1} \quad \& \quad \approx 170.95 \text{ for spin 2 when } N = 2. \end{aligned} \tag{4.4. 3}$$

The same primary electromagnetic coupling α_{EMP} builds all fundamental particles, allowing Eq. (4.4. 3) to be true. Using Eq's. (4.4. 1), (4.4. 3) & $\sum_n c_n * c_n = 1$ we can get Table 4.3. 1.

Mass Type	Spin	$c_3 * c_3$	$c_4 * c_4$	$c_5 * c_5$	$c_6 * c_6$	$c_7 * c_7$
Infinitesimal N=2	2	0.8173	0.0256	0.1571		
Infinitesimal N=2	1	0.4847	0.0526	0.4627		
Massive N=1	1		0.0134	0.8878	0.0988	
Massive N=1	½			0.1305	0.7254	0.1441

Table 4.3. 1 Approximate superposition probabilities.

Table 4.3. 1 is approximate only, as we have picked the midpoints of experimental results with error bands. In each case three member superpositions seem to fit the Standard Model best. There may possibly be significance in why massive $N = 1$ superpositions can have only a dominant central mode if they are to fit the Standard Model, whereas infinitesimal rest mass $N = 2$ superpositions can have only minor central mode solutions. We will be looking further at gravitons in section 5.

5 The Expanding Universe and General Relativity

5.1 Zero point energy densities are limited

To this point this paper has been about building all the fundamental particles from energy borrowed from zero point fields. But surely this borrowing from a limited energy source (particularly at very long wavelengths) must have implications such as on the maximum possible densities of these fundamental particles? In section 2.2.3 we discussed how the preons that build the fundamental particles are born from a Higg's type scalar field with zero momentum in the laboratory rest frame. In this frame they have an infinite wavelength and can thus be borrowed from anywhere in the universe. This would suggest that there should be little effect on localized densities, but possibly on overall average densities in any or all of these universes. So which fundamental particle is there likely to be most of? Working in Planck, or natural units with $G=1$, the gravitational coupling constant between Planck masses is one. As an example, say there are approximately $M \approx 10^{61}$ Planck masses within our causally connected region of the universe. They will have an average distance between them of approximately the radius R of this region. Thus there must be approximately $M^2 \approx 10^{122}$ virtual gravitons with wavelengths of the order of radius R within this same volume. (No other fundamental particle is likely to approach these values, for example the number of virtual photons *of this extreme wavelength* must be much smaller.) If this density of virtual gravitons needs to borrow more energy from the zero point fields than what is available at extreme wavelengths, does this somehow control the maximum possible density of any causally connected universe? There may be insufficient long wavelength virtual gravitons to allow gravity to apply. This would however be true only at vast cosmic scales such as the distance to the horizon. At smaller scales within these causally connected regions, where the wavelengths of the virtual gravitons involved are shorter, and the zero point energy densities (which vary as k^3 or $1/\lambda^3$) are much greater, gravity would apply in the usual manner.

5.1.1 Gravitational and Electromagnetic field energies may behave differently

In section 3.2.1 we showed that the momentum debt in the vacuum at wavenumber k is Eq. (3.2. 1) i.e. $\left|\mathbf{p}_{nk}(debt)\right| = -\beta_{nk}{}^2 n\hbar\mathbf{k},$ and the nett momentum at that k is Eq.(3.2. 2)

$$\mathbf{p}_{nk}(nett) = \mathbf{p}_{nk} + \mathbf{p}_{nk}(debt) = (1-\beta_{nk}{}^2)n\hbar\mathbf{k} = \frac{n\hbar\mathbf{k}}{\gamma_{nk}{}^2} = \frac{\mathbf{p}_{nk}}{\gamma_{nk}{}^2}.$$

We then summed the squares of these nett momenta to get the total rest mass/energy of all the fundamental particles. But this rest mass/energy must also have come from the zero point fields. We can perhaps thus think of the nett momentum in this equation (3.2. 2) times light velocity *c* as the total energy borrowed from the zero point fields at wavenumber *k.*

At wavenumber k the borrowed energy is $E_{nk}(borrowed) = \dfrac{n\hbar kc}{\gamma_{nk}{}^{2}} = \dfrac{nk}{\gamma_{nk}{}^{2}}.$ (5.1. 1)

(The final expression of this equation is in Planck units.) For a superposition ψ_k (*not a complete infinite superposition*) this becomes

For a superposition ψ_k $\langle E_k(borrowed)\rangle = \dfrac{\langle n\rangle k}{\langle \gamma_{nk}{}^{2}\rangle}$ in Planck units. (5.1. 2)

In sections 3.4 & 3.5 we evaluated the electrostatic and electromagnetic energies between charged infinite superpositions (or particles). Effectively we calculated the number of extra virtual photons in the electromagnetic field due to the interaction between them. We then multiplied by the energy of each virtual photon. We treated the energy of each wavenumber *k* virtual photon as a real $\hbar kc$ energy photon. This gave the correct interacting energies. In other words the work done in bringing two charges together is equal to the change in the interacting electromagnetic field energy. On the other hand in General Relativity there has always been uncertainty in just where the gravitational field energy is [11]. It is not included in Einstein's Energy- Momentum tensor and this puzzled the early pioneers. This raises the possibility that gravitational field energy is somehow different to the electromagnetic field energy.

In section 3.3 we discussed the exchange of virtual superpositions ψ_k as being equivalent to exchanging a virtual photon/boson of wavenumber *k.* Now there are two ways this can happen. Because it did not affect the results we got, we did not complicate things back then by discussing it. These two possibilities are:

(a) *A complete infinite superposition is exchanged* but the actual interaction at each vertex is with only its virtual superposition ψ_k member.

(b) *A single superposition ψ_k is exchanged* and not the full infinite superposition.

For electrostatic and magnetic field energies we treated wavenumber *k* virtual photons as having real energy $\hbar kc$, where $\hbar kc = m_0 c^2 / \sqrt{1-\beta^2}$. ($m_0 c^2$ is the infinitesimal rest energy of the virtual photon/infinite superposition and $\beta \to 1$ is its approximate light velocity). This implies that the electromagnetic field virtual photons have to be complete infinite superpositions as in type (a). On the other hand in what follows we find that in the case of virtual gravitons there is only sufficient zero point energy available at cosmic scale wavelengths to exchange a single member superposition ψ_k. This suggests exchanged virtual gravitons are type (b) with total borrowed energy @ *k* as in Eq's. (3.2. 2) & (5.1. 2), and only real gravitons would be complete spin 2 ($l = 2, m = \pm 2$) infinite superpositions with real energy $\hbar kc$. So if virtual gravitons are in fact type (b) what happens at cosmic scales? Firstly we need to find what $\langle \gamma_{nk}{}^2 \rangle$ & $\langle n \rangle$ are for the gravitons in Eq (5.1. 2). Modifying Eq's. (3.1. 11) & (3.1. 12) to Planck units:

$$\gamma_{nk}{}^2 = 1 + K_{nk}{}^2 \quad \& \quad K_{nk} = \frac{n\hbar k\sqrt{2s}}{2m_0 c} = \frac{nk\sqrt{2s}}{2m_0} \tag{5.1. 3}$$

We will find in section 5.2.3 that the Compton wavelength of gravitons is approximately $\approx 10^{122}$ Planck lengths at this point in Cosmic time and at cosmic scales the values of *k* we will be considering $K_{nk} >>> 1$. The only time it may not be true is at the very start of the big bang when the equations suggest the rest mass of the graviton approaches one Planck mass.

$$\text{Provided} \quad K_{nk} >>> 1, \qquad \langle \gamma_k{}^2 \rangle \approx \langle K_k{}^2 \rangle \approx \frac{\langle n^2 \rangle k^2 s_2}{2m_{G0}{}^2} \approx \frac{11.69k^2}{m_{G0}{}^2} \tag{5.1. 4}$$

Where we have put $s_2 = 2$ & $\langle n^2 \rangle = \sum_n c_n {}^* c_n \cdot n^2 \approx 11.69$ using Table 4.3. 1, m_{G0} is the rest mass of a graviton and we will use the subscript G for gravitons from here on. Also from Table 4.3. 1 for gravitons we obtain $\langle n \rangle = 3.34$ so that Eq. (5.1. 2) becomes

$$\langle E_k(borrowed) \rangle = \frac{\langle n \rangle k}{\langle \gamma_{nk}{}^2 \rangle} \approx \frac{3.34 k m_{G0}{}^2}{11.69k^2} \approx \frac{0.2856 m_{G0}{}^2}{k} \text{ per graviton at } k. \tag{5.1. 5}$$

5.1.2 Virtual graviton density at wavenumber k in a causally connected Universe

General Relativity predicts nonlinear fields near black holes, but in the low average densities of typical universes *we can assume approximate linearity*. We can also ignore momentum as the average local velocities are low (or $\beta << 1$). We should also be able to simply apply the equations in sections 3.4 & 3.5 to spin 2 virtual graviton emissions, as it seems they should apply equally to both spins 1 & 2. We will assume spherically symmetric $l = 3$ wavefunctions emit both spin 1 & 2 scalar virtual bosons, and $l = 3, m = \pm 2$ states emit both $m = \pm 1$, & $m = \pm 2$ bosons. Section 3.4 derived the electrostatic energy between infinite superpositions. In flat space we looked at the amplitude that each equivalent point charge emits a virtual photon, and then focused on the interaction terms between them. Now whether the virtual particle emitted is a complete infinite superposition as in (a) section 5.1.1, or just a single superposition member as in (b) and both with momentum $\hbar\mathbf{k}$ we will assume they both obey $\Delta T \approx \hbar / \Delta E \approx \hbar / \hbar kc,$ and both have the same exponentially decaying wavefunction with radius travelled. Thus we can use the same scalar wavefunctions Eq's. (3.4. 1) for virtual scalar gravitons as we did for virtual photons. For virtual photons in section 3.4.1 using $(\psi_1 + \psi_2)^*(\psi_1 + \psi_2) = \psi_1^*\psi_1 + \psi_1^*\psi_2 + \psi_2^*\psi_1 + \psi_2^*\psi_2$ we showed the interaction term

$$\psi_1^*\psi_2 + \psi_2^*\psi_1 = \frac{4k}{4\pi r_1 r_2} e^{-k(r_1+r_2)} \cos k(r_1 - r_2) \tag{5.1. 6}$$

This equation is strictly true only in flat space but it is still approximately true if the curvature is small or when $2m/r <<<< 1$*, which we will assume applies almost everywhere throughout the universe except in the infinitesimal fraction of space close to black holes*. In both sections 3.4 & 3.5, for simplicity and clarity, we delayed using coupling constants and emission probabilities in the wavefunctions until necessary. We will do the same here. In Eq. (5.1. 6) constant $r_1 + r_2$ describe ellipses, and constant $r_1 - r_2$ describe hyperbolae. Using elliptical coordinates in Eq. (5.1. 6) to integrate over all space with constant wavenumber k, and putting r as the distance between the two charges (or Planck masses):

$$\text{At any fixed wavenumber } k \quad \iiint \left(\psi_1^*\psi_2 + \psi_2^*\psi_1\right)\, dv = \frac{2e^{-kr}\sin(kr)}{kr} \tag{5.1. 7}$$

Now consider one Planck mass at any point P somewhere in the interior region of a typical universe, and let the average density be ρ_U (subscript U for homogeneous universe density)

Planck masses per unit volume. Consider spherical shells around point P of radius r and thickness dr with $dm = \rho dv = 4\pi r^2 dr \rho_U$. As in section 3.4.1 when we derived Eq. (3.4. 3) we used a scalar emission probability $(2\alpha / \pi)(dk / k)$ which here becomes $(2 / \pi)(dk / k)$ between Planck masses where $\alpha = 1$. We want to know how many virtual gravitons of wavenumber k will couple between this point Planck mass and the causally connected universe *where the radius of the horizon is* R_H. Using Eq. (5.1. 7) this becomes

$$\begin{aligned}
&(\frac{2}{\pi} \cdot \frac{dk}{k}) \int_{r=0}^{R_H} \frac{2e^{-kr} \sin(kr)}{kr} \cdot 4\pi r^2 dr \rho_U \\
&= \frac{16\rho_U dk}{k^2} \int_0^{R_H} re^{-kr} \sin(kr) dr \\
&= \frac{8\rho_U dk}{k^4} \left[1 - e^{-kR_H} \left\{ (1 + kR_H) \cos(kR_H) + kR_H \sin(kR_H) \right\} \right]
\end{aligned} \tag{5.1. 8}$$

This is the virtual graviton coupling between one Planck mass and all the others Planck masses inside the horizon at wavenumber k. In a homogeneous universe we can carry out this same integral at all points (at the same cosmic time t); all at the centre of a causally connected region of the same radius R_H. Thus to get the total virtual graviton density we just multiply Eq. (5.1. 8) by $\rho_U / 2$ (so as to not count all pairs of Planck masses twice).

$$\text{Graviton density at } k = \rho_{Gk} = \frac{4\rho_U{}^2 dk}{k^4} \left[1 - e^{-kR_H} \left\{ (1 + kR_H) \cos(kR_H) + kR_H \sin(kR_H) \right\} \right] \tag{5.1. 9}$$

5.1.3 Borrowed virtual graviton energy densities at wavenumber *k*

To get the virtual graviton energy density borrowed at wavenumber k we multiply Eq. (5.1. 5) by Eq. (5.1. 9) to get

$$\begin{aligned}
&\frac{0.2856 m_{G0}{}^2}{k} \times \frac{4\rho_U{}^2 dk}{k^4} \left[1 - e^{-kR_H} \left\{ (1 + kR_H) \cos(kR_H) + kR_H \sin(kR_H) \right\} \right] \\
&\approx \frac{1.143 m_{G0}{}^2 \rho_U{}^2 dk}{k^5} \left[1 - e^{-kR_H} \left\{ (1 + kR_H) \cos(kR_H) + kR_H \sin(kR_H) \right\} \right]
\end{aligned} \tag{5.1. 10}$$

There will be some minimum wavenumber k which *we will call* $k_{\min}$ where for all $k < k_{\min}$ there will be insufficient zero point energy density available. Also from here on we will define the variable Υ (upsilon) where

$$\Upsilon = k_{\min} R_{Horizon} = k_{\min} R_H \text{ (in radians).} \tag{5.1. 11}$$

The *graviton energy density* E_{Gk} borrowed from the zero point fields Eq. (5.1. 10) becomes

$$E_{Gk} \;@\; k_{\min} \approx \frac{1.143 m_{G0}{}^2 \rho_U{}^2 dk}{k^5}\left[1 - e^{-\Upsilon}\left\{(1+\Upsilon)\cos\Upsilon + \Upsilon\sin\Upsilon\right\}\right] \tag{5.1. 12}$$

Equation (5.2. 15) and Figure 5.2. 2 shows General Relativity restricts Υ to $\approx 0 < \Upsilon < 3.8$. Figure 5.1. 1 plots this range of the function $1 - e^{-\Upsilon}\left\{(1+\Upsilon)\cos\Upsilon + \Upsilon\sin\Upsilon\right\}$. If we consider that in the early stages of the expansion for example that $\Upsilon = k_m R_H \approx 2$ then $1 - e^{-\Upsilon}\left[(1+\Upsilon)\cos\Upsilon + \Upsilon\sin\Upsilon\right] \approx 0.923$ so that Eq. (5.1. 12) becomes

$$\textit{Graviton energy borrowed}: E_{Gk} @\, k_{\min} \approx \frac{1.055 m_{G0}{}^2 \rho_U{}^2 dk}{k^5} \quad (\text{when } k = k_{\min} \;\&\; \Upsilon = 2). \tag{5.1. 13}$$

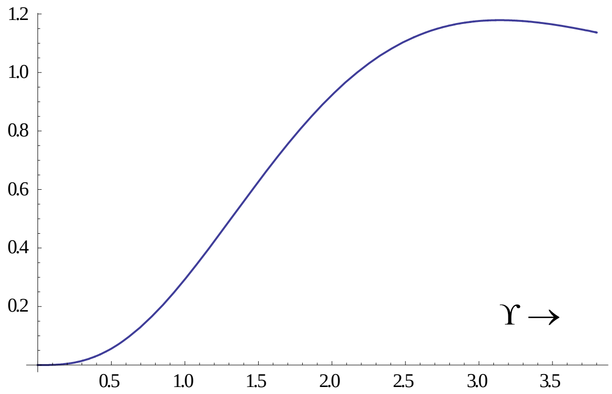


Figure 5.1. 1 plots $\left[1 - e^{-\Upsilon}\left\{(1+\Upsilon)\cos\Upsilon + \Upsilon\sin\Upsilon\right\}\right]$.

From section 2.1.1 the magnetic (vector) and electrostatic (scalar) zero point energy densities both $= k^3 dk / 4\pi^2$ in Planck units. Throughout this paper the primary interactions that build fundamental particles borrow energy from the 8 gluon and 1 photon zero point vector fields.

Ignoring the negligible contribution from gravity itself let us assume that due to these 9 vector fields.

The total zero point vector energy density available @ k is: $E_{ZPk} \approx 9 \times \frac{k^3 dk}{4\pi^2}$ (5.1. 14)

For all $k < k_{\min}$ the energy density borrowed (or required) Eq. (5.1. 12) is less than the energy density available in Eq. (5.1. 14) thus combining Eq's. (5.1. 11), (5.1. 13) & (5.1. 14)

$$\text{When } k = k_{\min},\quad E_{Gk}(borrowed) \approx E_{ZPk} \text{ thus } \frac{1.055 m_{G0}{}^2 \rho_U{}^2 dk}{k^5} \approx \frac{9k^3 dk}{4\pi^2}$$
$$\rightarrow m_{G0}{}^2 \rho_U{}^2 \approx 0.216 k^8 \;\rightarrow\; m_{G0}\rho_U \approx 0.465 k^4 \;\rightarrow\; m_{G0}\rho_U \approx \frac{0.465 \Upsilon^4}{R_H{}^4} \tag{5.1. 15}$$

$$\text{If in fact } \Upsilon = k_m R_H \approx 2 \text{ in the early era } m_{G0}\rho_U \approx \frac{0.465\Upsilon^4}{R_H{}^4} \approx \frac{7.44}{R_H{}^2 \times R_H{}^2} \tag{5.1. 16}$$

We have expressed it this way as we know that during the early expansion of the universe the density ρ_U was $\propto 1/R_H{}^2$ and thus the graviton rest mass m_{G0} was also $\propto 1/R_H{}^2$. We can also generalize this for any value of Υ by equating Eq's. (5.1. 12) & (5.1. 14) when $k = k_{\min}$

$$E_{Gk}(borrowed) \approx E_{ZPk} \quad \rightarrow \frac{1.143 m_{G0}{}^2 \rho_U{}^2 dk}{k^5}\left[1 - e^{-\Upsilon}\left\{(1+\Upsilon)\cos\Upsilon + \Upsilon\sin\Upsilon\right\}\right] \approx \frac{9k^3 dk}{4\pi^2}$$
$$\rightarrow m_{G0}{}^2 \rho_U{}^2 \approx \frac{0.1995 k^8}{1 - e^{-\Upsilon}\left\{(1+\Upsilon)\cos\Upsilon + \Upsilon\sin\Upsilon\right\}} \rightarrow m_{G0}\rho_U \approx \frac{0.447 k^4}{\sqrt{1 - e^{-\Upsilon}\left\{(1+\Upsilon)\cos\Upsilon + \Upsilon\sin\Upsilon\right\}}}$$
$$\rightarrow m_{G0}\rho_U \approx \frac{0.447\Upsilon^4}{R_H{}^4 \sqrt{1 - e^{-\Upsilon}\left\{(1+\Upsilon)\cos\Upsilon + \Upsilon\sin\Upsilon\right\}}} \tag{5.1. 17}$$

We will return to these equations after looking at how this relates to General Relativity.

5.2 Relating this to General Relativity

The first point to notice about Eq. (5.1. 16) is that provided $m_{G0} \propto 1/R_H{}^2$ the universe has to expand after the big bang to keep the average density $\rho_U \propto 1/R_H{}^2$. Also at any point in cosmic time there is always some value $k_{\min}$ where the borrowed energy density $E_{Gk} = E_{ZPk}$ the available zero point energy. We have assumed so far that the mass in the universe is like a perfect fluid and homogeneous, also that space is essentially flat on average. Thus all observers fixed relative to expanding cosmic coordinates must measure the same density of virtual gravitons $\rho_{Gk\min}$ at this minimum value $k_{\min}$, as in Eq. (5.1. 9). Using Eq. (5.1. 11)we can rewrite Eq. (5.1. 9) in terms of $k_{\min}$ & Υ

$$\text{Graviton density at } k_{\min} = \rho_{Gk\min} = \frac{4\rho_U{}^2 dk}{k_{\min}{}^4}\left[1 - e^{-\Upsilon}\left\{(1+\Upsilon)\cos\Upsilon + \Upsilon\sin\Upsilon\right\}\right] \qquad (5.2.\ 1)$$

This equation must be true for all such cosmic observers. We can also say that the total number of gravitons at $k_{\min}$ inside the horizon which we will label as $N_{Gk\min}$ is

$$N_{Gk\min} = \iiint \rho_{Gk\min} dv \text{ (over the volume inside the horizon)} \qquad (5.2.\ 2)$$

$N_{Gk\min}$ must be a fixed number at any cosmic time *t*. Let us now change the uniform fluid mass density ρ_U by putting a mass concentration $+m_1$ at some point. This increases the local mass density by $+\Delta\rho_U$. If at some other point there is a balancing drop in the local density $-\Delta\rho_U$ with an effectively negative mass concentration $-m_1$ at this other point, this keeps the overall total mass inside the horizon constant. Now the logic in the steps to Eq. (5.1. 16) is that any increase in the total borrowed graviton energy required, forces space to expand so that more zero point energy is available. Thus we should also expect space to expand locally around a mass concentration with an equal and opposite contraction around a negative mass (or local density reduction).

The total integrated volume $\iiint dv$ inside the horizon will not change because of these equal and opposite increases and decreases. Thus if the number of virtual gravitons at $k_{\min}$ i.e. $N_{Gk\min} = \iiint \rho_{Gk\min} dv$ (over the volume inside the horizon) is to remain constant

The local value of $\rho_{Gk\min}$ near any mass concentration must remain constant. (5.2. 3)

However when we are near a mass concentration we would expect it to locally increase this density $\rho_{Gk\min}$. But General Relativity tells us that near mass concentrations the metric changes, radial rulers shrink and local observers measure larger radial lengths. This expands volumes locally and should lower their measurement of the background $\rho_{Gk\min}$. Can these effects balance each other, so that the extra gravitons at $k_{\min}$ generated by a nearby mass concentration can bring this density back to the flat space or background $\rho_{Gk\min}$ of Eq.(5.2. 1)? (Changes in the metric will also change the local measurement of $k_{\min} \to k'_{\min} = k_{\min}(1-2m/r)^{-1/2}$ but this does not matter as from Eq's. (5.2. 2) & (5.2. 3) it is the density $\rho_{Gk\min}$ of the unchanged distant metric $k_{\min}$ that must remain constant.)

5.2.1 Approximations that are necessary with possibly important consequences

Firstly let us refer to Eq. (3.4. 2) and the steps to derive it; in particular

$$\psi_1 {}^*\psi_2 + \psi_2 {}^*\psi_1 = \frac{4k}{4\pi r_1 r_2} e^{-k(r_1+r_2)} \cos k(r_1 - r_2) \tag{5.2. 4}$$

and remember that *space has to be approximately flat* with errors $\propto 1-(1-2m/r)^{1/2} \approx m/r$. If we now focus on Figure 3.4. 2 then Eq. (5.2. 4) is the probability that (in this case) a virtual graviton is at the point *P if all other factors are one*. Let us now put a mass of m_1 Planck masses at the point *P*. Also assume that *point P is close to mass* m_1 at distance r_1 as in Figure 3.4. 2 and that *the vast majority of the rest of the mass inside the horizon* R_H *is at various radii* $r = r_2$ *where* $r_2 = r >>>> r_1$. *Only under these conditions can we approximate* Eq. (5.2. 4) as

$$\psi_1 {}^*\psi_2 + \psi_2 {}^*\psi_1 = \frac{4k}{4\pi r_1 r} e^{-kr} \cos k(-r) \tag{5.2. 5}$$

As we have assumed average particle velocities are low this is a scalar interaction (as in the electrostatic case) and there are no directional effects so we can consider simple spherical shells of thickness dr and radius *r* around point *P* which have mass $dm = \rho_U 4\pi r^2 dr$. At each radius *r* the coupling factor $(2\alpha/\pi)(dk/k)$ we used in deriving Eq. (3.4. 3) becomes (as the coupling constant $\alpha = 1$ between Planck masses)

$$\frac{2m_1}{\pi} dm \frac{dk}{k} = \frac{2m_1}{\pi} \frac{dk}{k} \rho_U 4\pi r^2 dr \tag{5.2. 6}$$

Including this factor Eq. (5.2. 5) becomes

$$(\frac{2m_1}{\pi}\frac{dk}{k}\rho_U 4\pi r^2 dr)(\psi_1 {}^*\psi_2 + \psi_2 {}^*\psi_1) \approx \frac{2m_1}{\pi}\frac{dk}{k}\rho_U 4\pi r^2 dr \frac{4k}{4\pi r_1 r} e^{-kr}\cos(-kr)$$
$$\approx \frac{2m_1}{r_1}\frac{dk}{\pi}\rho_U 4re^{-kr}\cos(-kr)dr \qquad (5.2.\ 7)$$

This is the virtual graviton density at P due to each spherical shell and the total graviton density is (putting $k = k_{\min}$, $\Upsilon = k_{\min} R_H$ from Eq. (5.1. 11) and integrating over radius r)

$$\frac{2m_1}{r_1}\frac{dk}{\pi}2\rho_U \int_0^{R_H} 2re^{-kr}\cos(-kr)dr \approx \frac{2m_1}{r_1}\frac{dk}{\pi}\frac{2\rho_U}{k_{\min}{}^2} e^{-\Upsilon}\left[(1+\Upsilon)\sin\Upsilon - \Upsilon\cos\Upsilon\right] \qquad (5.2.\ 8)$$

Thus the extra virtual graviton density $\Delta\rho_{Gk\min}$ at point P distance r_1 from mass m_1 is

$$\Delta\rho_{Gk\min} \approx \frac{2m_1}{r_1}\frac{dk}{\pi}\frac{2\rho_U}{k_{\min}{}^2} e^{-\Upsilon}\left[(1+\Upsilon)\sin\Upsilon - \Upsilon\cos\Upsilon\right] \qquad (5.2.\ 9)$$

Let us temporarily imagine we know nothing about General Relativity but assume from the logic behind Eq. (5.1. 16) that any mass requires a nearby volume of space to expand by $\Delta V / V$. We want the new background graviton density $\rho_{Gk_{\min}}{}'$ to remain unchanged and thus

$$\text{New } \rho_{Gk\min}{}' \approx \frac{\rho_{Gk_{\min}} + \Delta\rho_{Gk\min}}{1+\Delta V/V} \approx \text{ original } \rho_{Gk\min} \text{implying } \frac{\Delta\rho_{Gk\min}}{\rho_{Gk\min}} \approx \frac{\Delta V}{V} \qquad (5.2.\ 10)$$

Thus using Eq's. (5.2. 1) & (5.2. 9)

$$\frac{\Delta V}{V} \approx \frac{\Delta\rho_{Gk\min}}{\rho_{Gk\min}} \approx \frac{\frac{2m_1}{r_1}\frac{dk}{\pi}\frac{2\rho_U}{k_{\min}{}^2} e^{-\Upsilon}\left[(1+\Upsilon)\sin\Upsilon - \Upsilon\cos\Upsilon\right]}{\frac{4\rho_U{}^2 dk}{k_{\min}{}^4}\left[1 - e^{-\Upsilon}\left\{(1+\Upsilon)\cos\Upsilon + \Upsilon\sin\Upsilon\right\}\right]} \qquad (5.2.\ 11)$$

$$\frac{\Delta V}{V} \approx \left[\frac{m_1}{r_1}\right] \frac{k_{\min}{}^2 e^{-\Upsilon}\left[(1+\Upsilon)\sin\Upsilon - \Upsilon\cos\Upsilon\right]}{\rho_U \pi\left[1-e^{-\Upsilon}\left\{(1+\Upsilon)\cos\Upsilon + \Upsilon\sin\Upsilon\right\}\right]} \tag{5.2. 12}$$

But from Eq. (5.1. 11) $k_{\min}{}^2 = \Upsilon^2 / R_H{}^2$ and Eq. (5.2. 12) becomes

$$\frac{\Delta V}{V} \approx \left[\frac{m_1}{r_1}\right]\left[\frac{\Upsilon^2 e^{-\Upsilon}\left[(1+\Upsilon)\sin\Upsilon - \Upsilon\cos\Upsilon\right]}{\rho_U \pi R_H{}^2\left[1-e^{-\Upsilon}\left\{(1+\Upsilon)\cos\Upsilon + \Upsilon\sin\Upsilon\right\}\right]}\right] \tag{5.2. 13}$$

In cosmic coordinates, at any particular cosmic time, the red part of this equation is a fixed value at all points inside the horizon and the expansion of space around any mass is thus proportional to m_1 / r_1. Now the Schwarzschild solution to Einstein's equations tells us that

The radial metric around mass m_1 changes as $\frac{\Delta r_\infty}{\Delta r_{local}} = (1 - \frac{2m_1}{r_1})^{1/2} \approx 1 - \frac{m_1}{r_1}$ when $r_1 >>> m_1$

and the local change in volume $\Delta V / V \approx m_1 / r_1$. We have been approximating to the first order in m_1 / r_1 so to this first order we can say

$$\frac{\Delta V}{V} \approx \frac{m_1}{r_1} \approx \left[\frac{m_1}{r_1}\right]\left[\frac{\Upsilon^2 e^{-\Upsilon}\left[(1+\Upsilon)\sin\Upsilon - \Upsilon\cos\Upsilon\right]}{\rho_U \pi R_H{}^2\left[1-e^{-\Upsilon}\left\{(1+\Upsilon)\cos\Upsilon + \Upsilon\sin\Upsilon\right\}\right]}\right] \tag{5.2. 14}$$

General Relativity thus tells us that (provided we are working in Planck units) the red highlighted part is approximately one and so finally

$$\rho_U \approx \frac{1}{\pi R_H{}^2} \frac{\Upsilon^2 e^{-\Upsilon}\left[(1+\Upsilon)\sin\Upsilon - \Upsilon\cos\Upsilon\right]}{\left[1-e^{-\Upsilon}\left\{(1+\Upsilon)\cos\Upsilon + \Upsilon\sin\Upsilon\right\}\right]} \tag{5.2. 15}$$

5.2.2 The Expanding Universe

Equation (5.2. 15) is plotted in Figure 5.2. 2 showing the region consistent with General Relativity. The first point to notice is that if Υ remains constant the density of the universe decreases $\propto 1/R_H{}^2$ as expected in the initial stages at least. The second point is that the current approximate density for baryonic matter is $\approx 1\times10^{-124}$ and dark matter $\approx 4.5\times10^{-124}$

making a total of $\approx 5.5\times10^{-124}$ in Planck units, which is equivalent to $\rho_U R_H{}^2 \approx 0.4$ in Figure 5.2. 2 (using the current estimate for the horizon radius of $R_H \approx 2.66\times10^{61}$). Equation (5.1. 16) initially and Eq. (5.1. 17) later must also control the rate of expansion of the universe after the big bang. The rest mass of gravitons m_{G0} must be involved somehow in the possible values for Υ which in turn must be some sort of function of the horizon radius $\Upsilon(R_H)$ or alternatively of time $\Upsilon(t)$ after the big bang.

There are various possible scenarios of what might happen after the big bang. They must all however fit current observations by having Hubble constants $H(t)\approx 1/t$ at the present time and $H(t)\approx 2/3t$ in the early stages of the expansion. They must also fit the current red shift observations indicating accelerating expansion. Looking at Figure 5.2. 2 we can see that if $\Upsilon \approx 2$ initially, but then either increases or decreases constantly that the rate of expansion slowly becomes greater than $1/R_H{}^2$. (Because the shape of the curve is approximately sinusoidal with $\Upsilon \approx 2$ at its peak any departure from this value does not initially increase the $1/R_H{}^2$ rate.) We will consider just these two possibilities but in a very simplified manner. Assuming a starting point of $\Upsilon = 2 @ t = 0$, and a very simplified constant rate of change in Υ we can simply put $\Upsilon = 2 \pm t$ into Eq. (5.2. 15). From this we can derive scale factors as a function of time and also Hubble constants. These are plotted in Figure 5.2. 1 and they suggest that at the present time $\Upsilon \approx 0.9$ if it has decreased from its initial value of $\Upsilon = 2 @ t = 0$ or it is now $\Upsilon \approx 3$ if it has increased. At both these points the Hubble constant is currently $H(t)\approx 1/t$. The scale factors in both cases as a function of time look very similar to suggested current Dark energy solutions of the accelerating expansion of space. Also predicted densities are quite close to current baryonic plus dark matter values.

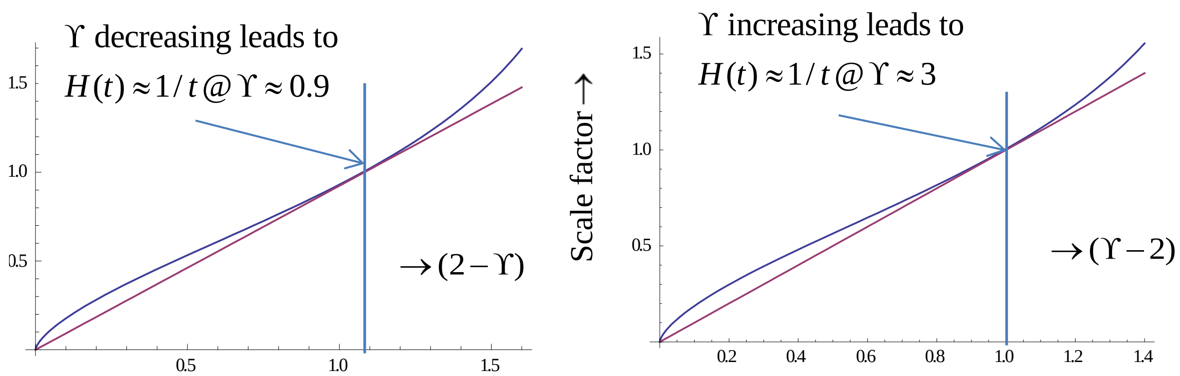


Figure 5.2. 1 showing scale factors and current Hubble constants for $\Upsilon = 2 \pm t$.

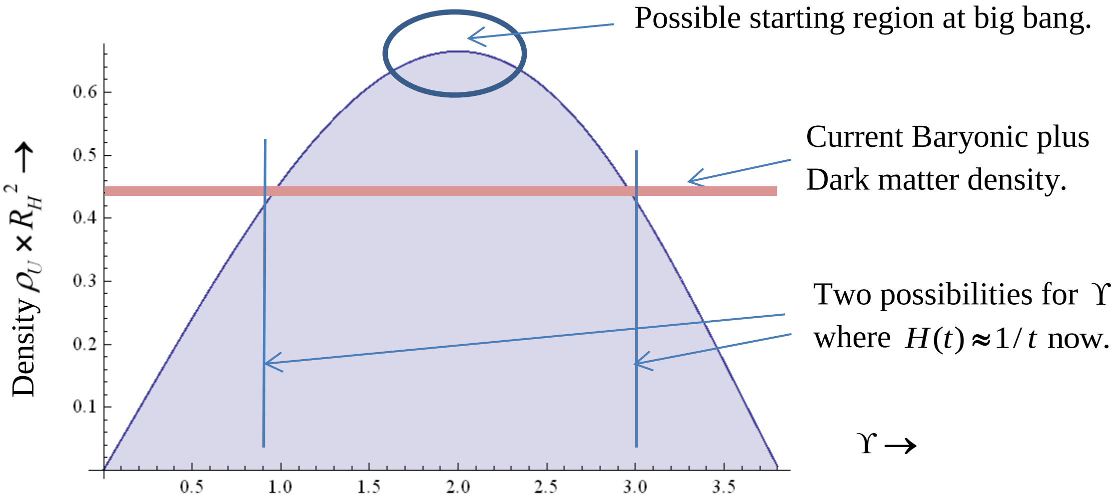


Figure 5.2. 2 plots Eq. (5.2. 15) for $0 < \Upsilon < 3.8$ showing the region consistent with General Relativity. It is almost sinusoidal peaking at $\rho_U \approx 0.664 / R_H{}^2$ when $\Upsilon \approx 2$.

While this may be oversimplified, it illustrates that it should be possible to find a suitable scenario that can match predicted red shifts based on Eq. (5.2. 15) with observational data. For example the starting point for Υ can vary either side of the top of the curve as in Figure 5.2. 2 delaying or advancing the transition point between acceleration and deceleration. Also the rate of change of Υ with time (which we assumed for a very simple first analysis to be constant) can also change.

5.2.3 Virtual graviton rest masses

Equation (5.1. 16) tells us when $\Upsilon \approx 2$ that $m_{G0}\rho_U \approx \dfrac{0.465\Upsilon^4}{R_H{}^4} \approx \dfrac{7.44}{R_H{}^2 \times R_H{}^2}$.

It might seem from this that in the first instants when $R_H \approx 1$ that both initial density and graviton rest mass are greater than Planck values. However both Eq's. (5.1. 16) & (5.2. 15) are unlikely to be true until the radius of the horizon $R_H >> 1$ because of the assumptions we have made. They could possibly be reasonably correct by the time that current cosmological models suggest quark-gluon plasma has formed. All we can surmise is that the rest mass of gravitons will possibly approach Planck values in the very early stages. This would limit their range but in the very early stages when R_H is also small this is probably irrelevant. What is more important is its current value.

Starting with Eq.(5.1. 17) $m_{G0}\rho_U \approx \dfrac{0.447\Upsilon^4}{{R_H}^4\sqrt{1-e^{-\Upsilon}\left\{(1+\Upsilon)\cos\Upsilon+\Upsilon\sin\Upsilon\right\}}}$ and putting in the density ρ_U derived from Eq. (5.2. 15) which in both the possibilities of Figure 5.2. 2 is $\rho_U \approx 0.4/{R_H}^2$ thus

$$m_{G0} \approx \frac{0.447\Upsilon^4}{{R_H}^4\sqrt{1-e^{-\Upsilon}\left\{(1+\Upsilon)\cos\Upsilon+\Upsilon\sin\Upsilon\right\}}}\frac{1}{\rho_U} \approx \frac{0.447\Upsilon^4}{{R_H}^4\sqrt{1-e^{-\Upsilon}\left\{(1+\Upsilon)\cos\Upsilon+\Upsilon\sin\Upsilon\right\}}}\frac{{R_H}^2}{0.4}$$

$$m_{G0} \approx \frac{1.12\Upsilon^4}{{R_H}^2\sqrt{1-e^{-\Upsilon}\left\{(1+\Upsilon)\cos\Upsilon+\Upsilon\sin\Upsilon\right\}}} \qquad (5.2.\ 16)$$

The two possible values for Υ from Figure 5.2. 1 are $\Upsilon \approx 0.9$ when Υ decreases and $\Upsilon \approx 3$ when it increases. With the current value of $R_H \approx 2.66\times10^{61}$ this becomes

Current virtual graviton rest masses (a) $m_{G0} \approx 10^{-123}$ or $\approx 10^{-95} ev$. (Υ decreasing)

(b) $m_{G0} \approx 1.3\times10^{-121}$ or $\approx 10^{-93} ev$. (Υ increasing) (5.2. 17)

Thus virtual gravitons appear to have Compton wavelengths of at least 10^{121} Planck lengths or about 10^{60} times the distance to the horizon. These values can only be very approximate as they depend on current dark matter plus baryonic matter densities, the radius of the horizon and the actual value of Υ. As we will see they are far smaller even than the infinitesimal rest masses of virtual photons and gluons.

5.2.4 Possible consequences from our approximations and assumptions

In section 5.2.1 the approximations we made meant that the agreement we got with General Relativity in Eq. (5.2. 14) is true only for points close to a mass concentration in relation to the distance to the horizon R_H. This implies that General Relativity or in turn the expansion of space around any mass concentration is locally true, and true to some substantial fraction of R_H, possibly some billions of light years, but not true at distances of the order of R_H. This agreement also is true only when we assumed that space is essentially flat on average. This does not seem unreasonable if in fact it is true that General Relativity does not reach to the horizon, as there would be no effective gravitational field acting at these vast distances trying to distort space on average. There will certainly be local distortions of the metric due to shorter range effects around naturally occurring mass concentrations.

Inflation was originally proposed to solve the flatness problem but it may in fact not be necessary if the above is true. Of course in the initial very early era, before the causally connected region was large enough to allow infinite superpositions, some form of inflation is not ruled out by the above and could well have occurred.

6 Further consequences of Infinite Superpositions

6.1 Gravitational Energy

In section 5.1.1 we discussed virtual gravitons being exchanged only as single member ψ_k superpositions, rather than the full infinite superpositions exchanges of virtual photons/gluons etc. The agreement we get for both General Relativity and the approximate density of the Universe would not be possible if full infinite superposition virtual gravitons are exchanged. There is just insufficient zero point energy available at the extreme wavelengths reaching towards the horizon. This has further consequences.

6.1.1 Gravitational energy must be due to the change in the metric

In section 5.2 and Eq. (5.2. 14) we showed that space has to expand in accord with the Schwarzschild change in the metric around a mass. To maintain constant light velocity in any free falling frame, time has to also change if space has changed. Thus the measurement of any mass and energy increases in a gravitational potential just as predicted by the Schwarzschild change in the metric. This is all so that the measured graviton density $\rho_{Gk\min}$ at $k_{\min}$ remains constant from Eq. (5.2. 1). Now the energy borrowed by single member superpositions is almost zero Eq. (5.1. 1) compared with full infinite superpositions and thus there is virtually zero gravitational energy in the field. The work done on pulling two masses apart appears to be due to the change in the metric only. It appears that it cannot equal the field energy as it does with full infinite superpositions exchange as with the electromagnetic field for example. There is insufficient zero point energy available at $k_{\min}$ to allow this. Perhaps this is why gravitational energy is not part of the Einstein tensor. This also affects the gravitational coupling constant. (These arguments do not apply to real gravitons which are full infinite superpositions.) In possible support of the thinking in this paragraph Roger Penrose (on page 467 of his "Road to Reality") believes there is not currently a complete understanding of gravitational energy [11].

6.1.2 The Gravitational coupling constant does not run with wavenumber k

If gravitons do not have field energy $\hbar kc$ and there is negligible gravitational field energy, then virtual gravitons also have negligible mass; so they interact only negligibly with other virtual gravitons. This implies that gravitons do not create more gravitons in the same way as for example the colour of gluons means that gluons create more gluons, resulting in the colour coupling constant increasing in strength with radius from a bare quark. This in turn implies that the gravitational coupling constant G is in fact a true constant and does not run with the wavenumber k in the same manner as the other coupling constants.

6.2 Low frequency Infinite Superposition cutoffs

In section 4.2 when we introduced gravity, for the lower limit in our integrals we assumed that $k=0$, and then in section 5 showed that there is a lower limit $k_{\min}>0$. We have been able to ignore this so far as its effect in the universe at present is negligible in comparison to the high frequency cutoff $k_{cutoff}<\infty$ which we showed gravity can address in section 4.2.

6.2.1 Quantifying the approximate effect of $k_{\min}>0$ on Infinite Superpositions

If we look again at section 4.2.1 we can repeat what we did there as follows.

Provided we can say that $K_{nkCutoff}\to\infty$ & $K_{nk\min}\to 0$

$$\left[\frac{-1}{1+K_{nk}{}^{2}}\right]_{K_{nk\min}}^{K_{nk}cutoff}=\frac{1}{1+K_{nk\min}{}^{2}}-\frac{1}{1+K_{nkCutoff}{}^{2}}\approx 1-\left[\frac{1}{K_{nkCutoff}{}^{2}}+K_{nk\min}{}^{2}\right]\approx\frac{1}{1+\varepsilon'} \tag{6.2. 1}$$

Our infinitesimal $\varepsilon\to\varepsilon'\approx\dfrac{1}{K_{nkCutoff}{}^{2}}+K_{nk\min}{}^{2}$ and from Eq. (3.1. 11) $K_{nk}{}^{2}=\dfrac{n^{2}s}{2}\lambda\!\!\!^{-}{}_{c}{}^{2}k^{2}$

For a spin ½ fermion for example $\left\langle\dfrac{n^{2}s}{2}\right\rangle\approx 9$ which we can ignore for a rough analyses.

Also $k_{cutoff}{}^{2}\approx 1/L_{P}{}^{2}$ and $k_{\min}{}^{2}\approx 1/R_{H}{}^{2}$ so that very approximately

$$\varepsilon'\approx\frac{1}{K_{nkCutoff}{}^{2}}+K_{nk\min}{}^{2}\approx\frac{L_{P}{}^{2}}{\lambda\!\!\!^{-}{}_{c}{}^{2}}+\frac{\lambda\!\!\!^{-}{}_{c}{}^{2}}{R_{H}{}^{2}}\approx\frac{(L_{P}R_{H})^{2}+(\lambda\!\!\!^{-}{}_{c}{}^{2})^{2}}{\lambda\!\!\!^{-}{}_{c}{}^{2}R_{H}{}^{2}}$$

The ratio of the extra contribution $\Delta\varepsilon$ to ε where $\varepsilon'=\varepsilon+\Delta\varepsilon$ is $\dfrac{\Delta\varepsilon}{\varepsilon}\approx\left[\dfrac{\lambda\!\!\!^{-}{}_{c}{}^{2}}{L_{P}R_{H}}\right]^{2}$

Working in Planck units $L_P R_H \approx 10^{61}$, but for electrons say $\lambdabar_c{}^2 \approx 6\times 10^{44}$, so the effect is of the order of $\Delta\varepsilon/\varepsilon \approx 10^{-32}$ which we have been ignoring. But we cannot ignore this in the case of infinitesimal rest mass photons for example if $K_{nk\min} >> 0$. For example if $\lambdabar_C \to R_H$ then $\Delta\varepsilon/\varepsilon \to R_H \approx 10^{61}$ in Planck units.

6.3 Infinitesimal rest masses

Looking again at angular momentum and rest masses in section 3.2 the key factor in our final integrals is in Eq. (6.2. 1). Using Eq. (3.1. 12) we can rewrite Eq. (6.2. 1) as

$$\left[\frac{-1}{1+K_{nk}{}^2}\right]_{K_{nk\min}}^{K_{nk}cutoff} = \frac{1}{\gamma_{nk\min}{}^2} - \frac{1}{\gamma_{nk\mathrm{Cutoff}}{}^2} \tag{6.3. 1}$$

With massive particles it is easy to show that the difference between $\gamma^2{}_{nk\min}$ & 1 is vanishingly small, i.e. $(\gamma^2{}_{nk\min}-1)\to 1/\infty$ and as in section 6.2.1 this first term is of much less significance than the $\gamma^2{}_{nk\mathrm{cutoff}}$ term. Let us now however give more meaning to the *N* of Eq's. (2.1. 4) & (2.2. 4) by defining it as follows using Eq. (3.1. 12)

$$N = 1 + \left\langle K_{nk\min}{}^2 \right\rangle = \left\langle \gamma_{nk\min}{}^2 \right\rangle \tag{6.3. 2}$$

In section 3.2 we derived angular momentum and rest masses for only massive or what we called $N=1$ particles. To get integral angular momentum we had to assume in deriving Eq. (3.2. 6) that the minimum value of K_{nk} or $K_{nk\min}=0$. For massive particles such as the fermions the error in this assumption (as in section 6.2.1) is $\approx 10^{-32}$ times smaller than ε, which for an electron is already $\varepsilon \approx 10^{-45}$ due to the high frequency cutoff $@\approx 10^{18.31} GeV$. (We allowed for this ε when we included gravity in section 4.2.) From section 6.2.1 above we approximated $K_{nk\min}$ as $\approx \lambdabar_c / R_H$. So we can express Eq. (6.3. 2) in terms of this approximation

$$N \approx 1 + \frac{\lambdabar_C{}^2}{R_H{}^2} \approx 1 \text{ as } \frac{\lambdabar_C{}^2}{R_H{}^2} \to 0 \tag{6.3. 3}$$

For example an electron $\dfrac{\lambdabar_C{}^2}{R_H{}^2} \approx 10^{-78}$

For all the known massive particles $N \to 1$. Even a neutrino mass as low as $10^{-2} eV$ has $N-1 \approx 10^{-63}$. *We will conjecture that for all massive particles* $N \to 1$, otherwise we cannot get integral angular momentum. If the mass is too small however we cannot get the correct angular momentum unless something else changes. Infinitesimal increases above 1 can be handled perhaps by a small change in the ninety degree angle (section 4.2) we found worked between the primary gravity vector and the inline colour plus electromagnetic pair of vectors. Even this does not however allow massive particles to be much less than micro electron volts. So if massive particles are a group with $N \approx 1$, then it would not seem unreasonable to imagine there could possibly be another group with $N = 1 + \langle K_{nk\min}{}^2 \rangle \approx 2$ implying that $\langle K_{nk\min}{}^2 \rangle \approx 1$. Repeating the derivation of Eq. (3.2. 6) but with $N = 1 + \langle K_{nk\min}{}^2 \rangle \to 2$ and for clarity and simplicity let $K_{nkcutoff} \to \infty$.

$$\mathbf{L}_z(Total) = s \cdot (N = 2) m\hbar \int_{K_{nk\min}}^{\infty} \frac{K_{nk}{}^2}{(1 + K_{nk}{}^2)^2} \frac{dK_{nk}}{K_{nk}} = sm\hbar \left[\frac{-1}{1 + K_{nk}{}^2} \right]_{K_{nk\min}}^{\infty}$$

$$\mathbf{L}_z(Total) = sm\hbar \left[\frac{1}{1 + K_{nk\min}{}^2} \right] = sm\hbar \left[\frac{1}{(N = 2)} \right] = \frac{sm\hbar}{2} \text{ as previously.} \tag{6.3. 4}$$

Provided we have doubled the probability of superpositions as in Eq. (2.1. 4) from $s \cdot (N = 1) dk / k$ to $s \cdot (N = 2) dk / k$, the final angular momentum results in Eq's. (3.2. 6) & (6.3. 4) are identical. The same is true for rest mass calculations. For complete infinite superpositions if $N = 2$ then the expectation value $\langle K_{nk\min}{}^2 \rangle = 1$. From Table 4.3. 1 an $N = 2$ infinitesimal rest mass spin 1 superposition has $\langle n^2 \rangle \approx 16.77$ and using Eq. (5.1. 11)

$$\langle K_{nk\min}{}^2 \rangle = \frac{\langle n^2 \rangle s}{2} ƛ_c{}^2 k_{\min}{}^2 \approx \frac{16.77}{2} ƛ_C{}^2 k_{\min}{}^2 \approx 8.4 \frac{\Upsilon^2}{R_H{}^2} ƛ_C{}^2 \approx 1$$

$$\text{or } ƛ_C \approx 0.34 \frac{R_H}{\Upsilon} \tag{6.3. 5}$$

$$ƛ_C \approx 0.38 R_H \text{ if } \Upsilon = 0.9 \text{ or } 0.11 R_H \text{ if } \Upsilon = 3$$

$$N = 2 \text{ infinitesimal rest masses are equivalent to } \approx 10^{-34} eV.$$

Whether we use $\Upsilon = 0.9 \text{ or } 3$ (from Figure 5.2. 1) the range of the electromagnetic force reaches only partway towards the horizon. *We can perhaps conjecture that all infinitesimal rest mass infinite superpositions have* $\langle K_{nk\min}{}^2 \rangle \approx 1$ *and in terms of* $N = 1 + \langle K_{nk\min}{}^2 \rangle = \langle \gamma_{nk\min}{}^2 \rangle$ *it may be more reasonable to think of them all having* $\langle \gamma_{nk\min}{}^2 \rangle = 2$. Real spin 2 gravitons would also be in this $N = 2$ group. The infinitesimal rest mass virtual gravitons of section 5.2.3 have $N = 2$ but do not need to have $\langle K_{nk\min}{}^2 \rangle \approx 1$ as the rest of this group do, this is because they are only single member ψ_k superpositions. Their Compton wavelengths can thus be thus much greater as we found in Eq. (5.2. 17).

6.3.1 Infinitesimal photon rest mass and symmetry breaking

The Standard Model assumes all particles are massless with the Higgs field breaking this symmetry. The logic behind this is based on weak interactions coupling to fermions of one helicity and not the other: Massless fermions travelling at light velocity cannot be caught and have fixed helicity. The Higgs field adds mass and the helicity is no longer fixed as particles with rest mass cannot travel at the speed of light. Photons with the rest masses of Eq. (6.3. 5) are so close to massless and (for all normal energies) travel so close to the velocity of light their helicity is virtually fixed, allowing this logic to remain essentially correct. The Higgs mechanism can still apply (via primary interactions) to such infinitesimal rest mass particles, giving them the rest mass of the massive bosons that we measure in secondary interactions.

A possible consequence of Eq. (6.3. 5) if correct in implying infinitesimal rest masses are always less than the inverse of the horizon radius, may apply in the early era of the big bang. When the horizon radius is smaller the behaviour of the weak force may have been different as infinitesimal rest masses would have been greater. Could this be a factor in the imbalance between particles and antiparticles? It might also mean that while we may approach the temperatures and energies of big bang eras in large accelerators we cannot change back to the rest masses of that time, if they are indeed controlled by the horizon radius R_H applying then.

6.4 Infinitesimal electric charge differences

There is also another infinitesimal but critical difference between this paper and the Standard Model. It has always been fundamental that the charge of protons and electrons are precisely equal and opposite to get a neutral universe. This paper suggests there is a minute difference.

The probability of virtual photon emission is proportional to the probability $\frac{sN\cdot dk(1+\varepsilon)}{k}$ of each superposition (section 4.2) and would only be equal for electrons and quarks if their rest masses are identical, as $\varepsilon=\left[\frac{7}{6}+\frac{\pi}{4}\right]\frac{{m_0}^2\chi_G\cdot G}{2s\hbar c(8+8\sqrt{\alpha_{EMP}})^2}=\frac{1}{\left\langle K_{nkcutoff}\right\rangle^2}$ and for an electron $\varepsilon\approx10^{-45}$. The bare masses of quarks average about ten times heavier than the electron so there is a difference of order $\Delta\varepsilon\approx10^{-43}$.

The number of protons in the Earth for example is of order $\approx10^{51}$, and if we assume there are equal numbers of electrons and protons there would be a nett effective excess of $\approx10^{8}$

positive fundamental charges. For a sphere the size of the Earth this amounts to an effective positive potential of $\approx 10^{-8}$ volts. In the case of a larger body such as the Sun this increases to an effective excess of $\approx 10^{13}$ positive fundamental charges and $\approx 10^{-5}$ volts positive potential. These are insignificant values. In the case of neutron stars, as their charge consists of mainly up and down quarks with reasonably similar bare rest masses, there may also be little difference in potential. The same is likely to apply in the case of massive black holes, but any difference there, must depend on the bare rest masses of the charges they are composed of. This effect may possibly show up only at the cosmic scale if at all, but it must always be many orders smaller than gravity if there are equal numbers of electrons and protons.

6.5 Early models of the Electron and Infinite Superpositions

Over a century ago there were various models of the electron. The Abraham-Lorenz was probably the most well-known [12][13]. All these models suffered from the problem that the electromagnetic mass in the field was 4/3 times the relativistic mass.

In 1906 Poincare showed that if the bursting forces due to charge were balanced by stresses (or forces) in the same rest frame as the particle, that these would cancel the extra 1/3 figure, thus restoring covariance [14]. In chapter 29 Volume II of his famous lectures on physics, Feynman, probably jokingly, suggested that if the electron is held together by strings that their resonances could explain the muon mass; he may have been right [15].

The equations for infinite superpositions in this paper apply equally to all massive particles. Also, as infinite superpositions are held together by interactions with zero point forces in the same rest frame, could these zero point interactions possibly be Feynman's strings? If they hold the virtual preons in orbit, they must also surely be able to balance any bursting forces due to electric charge. However this paper suffers from the same problem as the Standard Model: There is nothing in it suggesting the quantization of mass of massive particles; it suggests only the mass of infinitesimal rest mass particles.

7 Conclusions

- If fundamental particles are built from infinite superpositions then why do we never see any sign of this? It is important to remember here that all the members of infinite superpositions are virtual and only complete infinite superpositions behave as real particles. If infinite superpositions could be somehow decomposed into their virtual components this would destroy the resulting equivalent real particle. Could it be that particle conservation laws that control the behaviour of fundamental particles somehow prevent any sign of their virtual components? We have however suggested in this paper that virtual gravitons are such section 5.1.1 type (b) virtual components but real gravitons are not. Of course we cannot observe virtual gravitons but they make their presence felt through inertial or gravitational forces when they exchange momentum.

- The complete viability of this paper depends on primary interactions where spin zero preons can borrow mass from some possibly Higgs type scalar zero point field, and energy from colour and electromagnetic zero point vector fields. We have stressed that the behaviour of these primary interactions is very different to the secondary interactions that the Standard Model is all about. The usual Standard Model rules applying to borrowing mass and energy from scalar and vector zero point fields will almost certainly need modifications for primary interactions.
- This paper has attempted to show that fundamental particles built from infinite superpositions are (apart from infinitesimal differences) compatible with their Standard Model equivalents. The main difference being the minute rest masses of what the Standard Model calls massless particles. The Higgs mechanism originally proposed as the source of mass for the $W^{\pm} \& Z_0$ particles is now thought to be the source for all Standard Model particles that possess rest mass. If in fact photons do have a rest mass of $\approx 10^{-34} GeV$, we have assumed that the Higgs mechanism still applies, increasing their infinitesimal rest mass to the massive $W^{\pm} \& Z_0$. The Higg's field could in fact be the same scalar field that the preons of this paper borrow their rest mass from.
- We showed that spin ½ infinite superpositions have $g = 2$: they must obey Dirac's equation and so the QED corrections (which are secondary interactions) of the Standard Model must apply, giving the correct QED $g-2$ value.
- All long range forces including gravity do not reach past the horizon radius R_H.
- An entirely conjectural Dark Matter possibility could be massive $N = 1$ spin 2 electrically neutral and colourless, graviton type infinite superpositions as in Table 2.2. 1, which interact only with $N = 2$ infinitesimal mass virtual gravitons. Just as virtual and real $N = 2$ gravitons, it would be a $n = 3, 4, 5$ superposition, but with

exactly the same probabilities as an $N=2$ spin 1 infinitesimal mass virtual photon as in Table 4.3. 1. (Because $N \cdot s = [(N=1)\cdot(s=2)] = [(N=2)\cdot(s=1)] =$ the constant 2.)

- Another implication of this paper is that *secondary interaction virtual gravitons only interact between infinite superpositions.* Section 5 connected this with General Relativity by showing the metric changes to enable the available zero point energy to equal that required by these virtual gravitons. But the virtual graviton energy required is proportional to *only the mass/energy/momentum of infinite superpositions which the virtual gravitons are interacting with*. This can be virtual photons, virtual gluons etc. but only if they are in the form of infinite superpositions. This includes the electromagnetic and colour fields etc., as well as all the fundamental particles. The energy in the zero point fields is not in the form of infinite superpositions and will thus not curve spacetime. If the energy in the Higg's scalar field is not in the form of infinite superpositions it also cannot curve spacetime. *If this paper is correct only fields that can be represented as infinite superpositions can change the metric.*
- Finally *inertial mass is the same as gravitational mass*: Gravity is a fictitious force or acceleration, as Einstein in his "Equivalence Principle" always believed. He saw it as an *illusion due to the changing metric around mass concentrations,* but this paper suggests *"that changing metric"* is caused by Quantum Mechanics and also that General Relativity is thus a consequence of Quantum Mechanics.

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