Mass of Higgs Boson, Branching Ratios and Holographic Principle

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Abstract: Within the Everlasting Theory I described mass spectrum of the sham Higgs boson 125.00 GeV. Due to the quadrupole symmetry characteristic for the weak interactions and due to the interactions of the Higgs-boson pairs with the dominant gluon balls 3.30 GeV, there appear two masses 126.65 ± 0.31 GeV and 123.35 ± 0.31 GeV. Due to the confinement characteristic for the weak interactions, there arise the pairs of Higgs bosons. In their decays appear groups of photons composed of two photon pairs, i.e. composed of four photons, or quadrupoles of leptons. The decays of the Higgs boson pairs into 4 photons lead to the mean central mass of Higgs boson equal to 126.65 GeV whereas the decays into the quadrupoles of leptons lead to the mean central mass 123.35 GeV or 125.00 GeV. The reformulated Theory of Branching Ratios leads to conclusion that the relative signal strength of the decays into two photons to the decays into two Z bosons should be in approximation 1.87 times higher than predicted within the Standard Model. Since there is the pairing of the Higgs bosons then for the decays into two photons, the relative signal strength in relation to the Standard Model is 1.732 whereas for ZZ channel is 0.926. The upper and lower limits for branching ratios follow from the holographic principle, entanglement and range of the gravitational interactions and from the atom-like structure as well.

1. Introduction and mass spectrum

The internal structure of the core of baryons and the confinement (the confinement follows from the weak interactions) of the Einstein spacetime components produced by electromagnetic energy, lead to the mass of the sham Higgs boson m_o = 125.00 GeV [1] (see Chapter "Electroweak Interactions, Non-Abelian Gauge Theories and Origin of $E = mc^2$"). I will call the sham Higgs boson 125 GeV, the Higgs boson. On the other hand, from the formulae (214)-(216) derived within the reformulated QCD [1] (see Chapter "Reformulated Quantum Chromodynamics") follows that there is an increase in number of gluon balls (the amplification) carrying mass equal to 3.304 GeV. Just such gluon balls produce identical gluon balls. Such gluon ball I will refer to as the amplifier. The unified formula of the formulae (214)-(216) is as follows

$$M[\mathrm{GeV}] = (C/E_N[\mathrm{GeV}] + D)^{10}, \quad (1)$$

C = 0.52294,

D = 0.96868,

E_N is the energy of collision per nucleon,

M is the mass of produced gluon ball.

Due to the internal structure of the cores of baryons (there is torus and ball in its centre composed of the Einstein spacetime components) there is valid the quadrupole symmetry for the weak interactions [1] (see Chapter "Neutrino Speed"). This means that we should observe the decays into quadrupoles of leptons or into groups composed of two photon pairs but such decays should be characteristic for Higgs-boson pairs, not for single Higgs bosons. Since there dominate the amplifiers 3.304 GeV, so such mass can be absorbed by one of the two components and then the mean central mass of the components in a Higgs-boson pair is 126.65 GeV. But some amplifier can force emission of amplifier by one of the two components of a Higgs-boson pair. Then the mean central mass is 123.35 GeV. It looks as a Gluon Amplification by Stimulated Emission (the GASE). There is a broadening of mass of the amplifier 3.304 GeV which follows from the decays inside the baryons [1] (see the description below the formulae (214)-(216)). The upper limit for the broadened mass we obtain multiplying the mass 3.304 GeV by $2^{1/4} = 1.1892$ so we obtain 3.929 GeV. This means that we can write the mass of the amplifier as follows: 3.304 ± 0.313 [GeV]. The two modified mean masses of the Higgs bosons should be as follows

$M_1 = 126.654 \pm 0.313$ GeV and $M_2 = 123.350 \pm 0.313$ GeV.

It is consistent with experimental data [2].

We can see that the real mass of the Higgs boson is 125.00 GeV because the observed mass distance 3.304 GeV is due to the interactions of the Higgs-boson pairs with the amplifiers.

Analyse thoroughly the equation (1). For $E_N \rightarrow \infty$ the mass of gluon ball M is 727.44 MeV. It is the mass of the core of baryons [1]. Since mass of gluon ball M decreases when energy of collision per nucleon E_N increases (the energy E_N I will refer to as the gluon propagator) then we can assume that at low energy of gluon propagator, there arise the virtual gluon-ball pairs carrying the mass ±727.44 MeV = 0 as well. On the other hand, the equation (1) has physical sense also for negative energy of gluon propagator. This follows from the fact that negative energy of gluon propagator or negative mass of gluon ball means the creation of an energy/mass depression in the strong field. We can see that the mass of gluon ball(s) is equal to zero for negative energy of virtual gluon propagator equal to –539.9 MeV, i.e. for the rest mass of four confined virtual neutral pions $E_N = - 4m_{pion(o)}$. Due to the quadrupole symmetry for the weak interactions [1] that follows from the internal structure of the core of baryons, there can appear objects containing four confined neutral pions. At low energy there appear mostly the virtual pairs carrying energy ±539.9 MeV = 0. The negative propagator carrying energy –539.9 MeV creates the virtual gluon-ball pairs carrying mass ±727.44 MeV = 0. We can see that due to the equation (1), which follows from the internal structure of the core of baryons [1], there is broken symmetry between the behaviour of the virtual negative and positive propagators. The negative propagator (energy = –539.9 MeV) leads to resultant mass of virtual gluon pair (±727.44 MeV) equal to zero whereas the positive (energy = +539.9 MeV) to mass of single gluon-ball equal to 745 GeV. Such gluon ball cannot be created at low energy.

The above analysis is consistent with results presented here [3]. The conclusion is that the infrared data can be associated with a massive propagator up to momenta in approximation 500 MeV. There appears a constant gluon mass of 723(11) MeV. We obtain such result when we exclude the zero-momentum gluon propagator from the analysis. We can see that the formula (1) has no physical sense for zero-momentum gluon propagator as well.

At low energy, the all gluon balls have the same mass Y = 424.1 MeV [1]. They all are the black holes in respect of the weak interactions [1]. Due to the internal structure of the gluon balls (the hedgehog-like structure – see the next Paragraph), the gluon-balls are the scalars. In the nature the pairing is widespread. This follows from the fact that binary systems have additional angular momentum and in interactions this angular momentum can change if it is necessary. The massive gluon balls appear as the gluon-ball pairs i.e. probability of creation of Higgs-boson pairs or gluon-ball pairs is much higher than of single object. The gluon balls can transform into loops and then into tori [1]. The mean distance between the carriers of gluons in a gluon ball is a little shorter than in the Einstein spacetime [1]. This means that absorption of a gluon ball by some other gluon ball leads to decay into photons whereas an emission causes that the confinement is still valid so the gluon balls decay into leptons. Due to the quadrupole symmetry for the weak interactions [1], in the decays of the pairs of Higgs bosons there appear four photons as the two pairs of photons ($\gamma\gamma$) or two quadrupoles of leptons (llll). From the formula (1) follows that there is higher number density of the gluon balls carrying energy 3.30 GeV (the G_o). This causes that the decays of the Higgs boson pairs into 4 photons $2(\gamma\gamma)$ lead to the mean central mass 126.65 GeV whereas the decays into the final states 2(llll) (the two pairs of four leptons) lead to the mean central mass 123.35 GeV or 125.00 GeV. Some summary for these two selected types of decays of the Higgs-boson-pair (the HH) is as follows.

$HH + G_o \rightarrow 2(\gamma\gamma)$: mean mass of H is 126.65 ± 0.31 GeV,

$HH - G_o \rightarrow$ 2(llll): mean mass of H is 123.35 ± 0.31 GeV,

$HH \pm G_o(= 0) \rightarrow$ 2(llll): mean mass of H is 125.00 GeV.

In the last case there is at first the amplifier-antiamplifier annihilation.

We can see that the observed differences in masses of the Higgs boson follow from its interactions. The real mass of the Higgs boson is 125.00 GeV. We can compare these theoretical results with experimental results observed with the ATLAS Detector at the LHC [2], [4] and the CMS Experiment at the LHC [5].

Due to the confinement [1], the mass of gluon balls can accumulate. We can see that the sum of the masses of the characteristic mass of gluon propagator 539.9 MeV and the constant gluon mass 727.44 MeV, is in approximation 1267 MeV. It is the mass of the charm quark. When in the equation (1) we use the mass of the charm quark 1.267 GeV as the mass M of gluon ball then we obtain for the gluon propagator energy in approximation E_N = 9.460 GeV. This energy is the mass of the boson Y(1S, 9460 MeV). There, as well, appear masses of other known particles [1]. This follows from the atom-like structure of baryons.

The core of the baryons consists of the torus and ball in its centre [1]. The torus is the strong/electric charge. For distances shorter than about 3 fm the torus behaves as the strong charge whereas for distances longer than about 3 fm the torus behaves as the electric charge [1].

2. The reformulated Theory of Branching Ratios (the RTBR) for Higgs bosons and pairs of gluon balls and holographic principle

The branching ratios B_k depend on number k of succeeding decays in a decay channel. Define the branching ratios as follows

$$B_k = B_{\gamma\gamma}(2^k - 1), \qquad (2)$$

where k = 1, 2, 3, 4, 5, 6, 7 and 8.

The value of $B_{\gamma\gamma} = B_1$ we can calculate from assumption that total branching ratio for all decay channels, which is the sum of the all B_k, is equal to 1. On the other hand, the smallest branching ratio is for the decays of the Higgs boson m_o = 125 GeV into two photons and is in approximation $B_{\gamma\gamma} \approx 0.002$. For $B_{\gamma\gamma} = 0.001992$, the total branching ratio is $B_t = \Sigma B_k = 1$. This means that there are 8 different decay channels and energies of initial gluon balls.

Define mass H_n of the components of the initial gluon-ball pairs as follows

$$H_n = 2 \cdot 2^{n/4} m_o/2, \qquad (3)$$

where n = 0, 1, 2, 4, 8, 16, 32 and 64 (they are the numbers in the Titius-Bode law [1]).

The branching ratios for the two selected decay channels are as follows: $B_{\gamma\gamma} = B(H_o \rightarrow \gamma\gamma) \approx$ 0.0020 and $B_3 = B_{ZW} = B(H_2 \rightarrow a(ZZ) + b(W^+W^-)) \approx 0.014$. The relative value is $B_{\gamma\gamma}/B_{ZW} \approx 0.0020/0.014 \approx 0.143$. In the Standard Model the $B_{ZZ,SM}$ is close to the B_{ZW} in the reformulated Theory of Branching Ratios but the assumption that these two branching ratios have the same value is incorrect. It is the reason that experimental results for the relative signal strength of the decays into two photons to the decays into two Z bosons are inconsistent with the value obtained within the Standard Model. The Standard Model leads to following value $\sigma_{SM} = B_{\gamma\gamma,SM}/B_{ZZ,SM} = 0.0023/0.016 \approx 0.144$ [6]. In reality, the B_{ZW} consists of two parts and abundance of the ZZ decay channel is a = m(Z)/(m(W) + m(Z)) = 0.5315. This leads to conclusion that the real branching ratio for the ZZ decay channel is $B_{ZZ} = aB_{ZW} = 0.007411$. We can see that the relative signal strength should be $\sigma_{RTBR} = B_{\gamma\gamma}/B_{ZZ} = 0.001992/0.007411 \approx 0.269$. This value is $\sigma_{RTBR}/\sigma_{SM} = 1.87$ times higher than the value predicted within the Standard Model. Since there is the pairing of the Higgs bosons then for the decays into two photons, the relative signal strength in relation to the Standard Model is $2B_{\gamma\gamma}/B_{\gamma\gamma,SM} = 2 \cdot 0.001992/0.0023 = 1.732$ whereas for ZZ channel is $2B_{ZZ}/B_{ZZ,SM} = 2 \cdot 0.007411/0.016 = 0.926$. We can compare these theoretical results with experimental data obtained in the CMS and ATLAS experiments [2], [4] and [5].

The threshold mass $H_{64}H_{64} = 1.64 \cdot 10^4$ TeV that appears in the reformulated Theory of Branching Ratios, must have a physical meaning. The groups of Higgs bosons arise due to the entanglement [1]. Calculate the lower limit for time T that is needed to transfer energy due to the entanglement from beyond the sphere that radius is equal to the range of the gravitational and electromagnetic interactions $R = 2 \cdot 10^{36}$ m [1] to the centre of the sphere. For the entanglement are responsible the binary systems of closed strings the Einstein-spacetime components consist of and their speed is $v = 0.727 \cdot 10^{68}$ m/s [1]

$$T = R/v = 2.75 \cdot 10^{-32} \text{ s.} \tag{4}$$

This time leads to following upper limit for the mass M_t of a binary system of gluon balls

$$M_t = 2\pi\hbar/(Tc^2) = 1.5 \cdot 10^5 \text{ TeV.} \tag{5}$$

This mass is greater than the mass $H_{64}H_{64}$. The next mass is $H_{128}H_{128} = 1.1 \cdot 10^9$ TeV and is above the calculated upper limit for the mass M_t. We proved that there indeed can be maximum 8 different branching ratios. Knowing the number of different branching ratios, we can from formula (2) calculate the $B_{\gamma\gamma}$ i.e. this quantity is not a parameter in the reformulated Theory of Branching Ratios.

Table 1 *Branching Ratios for gluon balls*

Branching ratios B_k for decay channels	**First decay**	**Mass H_n**
$B_{\gamma\gamma} = 0.001992 \approx 0.002$	$H_oH_o \rightarrow 2(\gamma\gamma) = 2 \cdot 1H_o$	$H_o = 2m_\gamma = 2 \cdot 62.5$ GeV
$B_2 \approx 0.006$	$H_1H_1 \rightarrow 2(llll)$	$H_1 = 2 \cdot 74.3$ GeV
$B_3 \approx B_{ZW} \approx 0.014$*	$H_2H_2 \rightarrow a2(ZZ) + b2(W^+W^-)$*	$H_2 = 2 \cdot 88.4$ GeV
$B_4 \approx 0.030$	$H_4H_4 \rightarrow 2 \cdot 2^1H_o$	$H_4 = 2 \cdot 125$ GeV
$B_5 \approx 0.062$	$H_8H_8 \rightarrow 2 \cdot 2^2H_o$	$H_8 = 2 \cdot 250$ GeV
$B_6 \approx 0.125$	$H_{16}H_{16} \rightarrow 2 \cdot 2^4H_o$	$H_{16} = 2 \cdot 1$ TeV
$B_7 \approx 0.253$	$H_{32}H_{32} \rightarrow 2 \cdot 2^8H_o$	$H_{32} = 2 \cdot 16$ TeV
$B_8 \approx 0.508$	$H_{64}H_{64} \rightarrow 2 \cdot 2^{16}H_o$	$H_{64} = 2 \cdot 4096$ TeV
***B_{ZZ} = 0.007411** $\Sigma B_k = 1.000$	*a = 0.5315 and b = 1 – a 1/a = 1.88	
Due to the pairing of the Higgs bosons, the relative signal strength is: **γγ**: $2B_{\gamma\gamma}/B_{\gamma\gamma,SM} = 2 \cdot 0.001992/0.0023 =$ **1.732** **ZZ**: $2B_{ZZ}/B_{ZZ,SM} = 2 \cdot 0.007411/0.016 =$ **0.926**		

The above description is the foundations of the holographic principle. There are the boundaries for the electromagnetic and gravitational interactions of particles and the Universe. Due to the entanglement, the particles and the matter inside the Universe can interact with the spacetime outside the boundary. When nucleons collide then their relativistic mass [1] transforms into the confined mass of the gluon balls. But when some part of confined mass deconfines then there must appear massless energy equal to the deconfined mass. Such massless energy is transferred from the boundary and from beyond this boundary. When a particle decays then the surplus energy is transferred on the boundary and farther away. We can say that information of the processes in centre of the sphere/boundary is encoded on the boundary and beyond it. We can see that, for example, to explain the hadronization we need the holographic principle and that the mainstream holographic principle must be reformulated but the main ideas are correct. The main ideas are presented here [7], [8], [9]. Similar holographic principle concerns the point mass Y in centre of baryons that is the black hole in respect of the weak interactions (its mass is 424.1249 MeV) [1]. The surface of the point mass is the boundary. Since the radius of such point mass is $0.871 \cdot 10^{-17}$ m and the characteristic speed is the speed of light c then the upper limit for energy arising in the centre of the point mass cannot be greater than in approximation 142 GeV. This means that in the collisions of baryons (there is valid the pairing so the maximum total mass can be 284 GeV) there, on the surface/boundary of the point mass and outside it can be created the electron-positron pairs, muon-antimuon pairs, the ZZ and W^+W^- pairs and the Higgs-boson pairs as well. The matter arises on the boundary and outside the point mass Y because the mass of the black hole in respect of the weak interactions cannot change. The masses of the black holes in respect of the gravitational, weak and strong interactions are quantized so they emit the surplus energy [1].

The theoretical results concerning the RTBR are collected in Table 1.

The formula (3) must follow from some phenomena associated with the atom-like structure of baryons [1]. The gluon balls consist of the Einstein-spacetime components. Due to the exchanged Einstein-spacetime components between the points of the torus inside the core of baryons [1], there arises the hedgehog-like confinement in the point mass Y. I will refer to such confinement the Y-confinement. Due to the polarization of the confined components in the spikes/spokes, the hedgehog-like confinement looks as divergent magnetic lines but the lines of forces (the spokes) have length equal to the radius of the Y. We cannot say that the Y is a magnetic monopole because the total spin of the Y is zero, there is not a magnetic charge and range of interactions is small. The gluon balls are the scalars so there is valid the theory of scalar field. The mass of gluon balls, when the components are confined in the same way as in the point mass Y in the centre of baryons, is directly proportional to their radius and the factor of proportionality is the same as for the Y (the factor is f = 48.689 GeV/femtometre). They are the black holes in respect of the weak interactions [1]. The mass density of the Einstein spacetime is x = 40,363 times higher than the mass Y so there is the second factor F = x f = 1,965.2 TeV/femtometre. I will refer to such confinement the E-confinement. It is the hedgehog-like confinement as well. The spins of the components, i.e. the carriers of the gluons, can rotate and it is the massless rotational energy of the proto-gluon-balls that transforms into the mass H_n defined by formula (3). The whole or a part of the confined mass, i.e. the spokes, can transform into the Wilson loops/vortices [10]. Then, the action of the Wilson loop depends on its circumference [10]. Due to the laws of conservation of spin and charge, the spacetime inside the torus of the core of baryons cannot be confined. Inside the torus appear the smallest Wilson loops so they deconfine from the confined field. Since the smallest loops (their radius is constant and is equal to 0.465 fm [1]; they are the loops responsible for the strong interactions at low energies) deconfine from the gluon balls then volume of the gluon balls and their mass decreases. The Einstein-spacetime components in the smallest Wilson loops are not confined, they are entangled. The whole or a part of confined mass of a spoke transforms into the entangled mass of the smallest Wilson loop/vortex. To do it, the total massless rotational energy E of the gluons the loop/vortex consists of must be equal to the mass of the loop/vortex mc^2 [1]. Such massless energy decreases the local pressure inside the loops/vortices [1] so there are the inflows of the additional Einstein-spacetime components into the loops/vortices. The loops/vortices acquire the massless rotational energy due to the holographic principle. On base of the atom-like structure of baryons [1] we can calculate the mass 125 GeV of the Higgs boson [1] but also following masses characteristic for the reformulated Theory of Branching Ratios. It is easy to calculate that the space between the Schwarzschild surface for the strong interactions [1] (the radius $r_1 = 1.395$ fm) and the range of strong interactions [1] (the radius $r_2 = 2.9214$ fm; it is the circumference of the smallest Wilson loop), that is confined similarly as the mass Y (its mass m = 0.4241249 GeV; its radius $r_3 = 0.008710945$ fm), has mass $H = m(r_2 – r_1)/r_3 = f(r_2 – r_1) = 74.32$ GeV and it is exactly the half of the mass of H_1 calculated from formula (3). The quadrupole symmetry characteristic for the weak interactions, leads to the mass of the pair H_1H_1. The E-confinement (we must use the factor F) between the d = 0 state [1] (the radius is about 0.7 fm) and the d = 4 state [1] (the radius is about 2.7 fm) has energy about F(2.705–0.6974) ≈ 4,000 TeV and it is in approximation the mass equal to the half of the maximum mass of the gluon ball H_{64} that appears in the reformulated Theory of Branching Ratios. The quadrupole symmetry leads to the upper limit for energy that appears in the RTBR. We can see that the spokes cannot be longer than about 2 fm. This means that applying the second method we proved that there are the 8 different branching ratios only. We can see that the H_nH_n pairs arise due to the interactions of four baryons.

3. Summary

Within the Everlasting Theory I described mass spectrum of the sham Higgs boson 125.00 GeV. Due to the quadrupole symmetry characteristic for the weak interactions and due to the interactions of the Higgs-boson pairs with the dominant gluon balls carrying mass 3.30 GeV, there appear two different masses $M_1 = 126.65 \pm 0.31$ GeV and $M_2 = 123.35 \pm 0.31$ GeV. The decays of the Higgs boson pairs into 4 photons lead to the mean central mass 126.65 GeV whereas the decays into quadrupoles of leptons lead to the mean central mass 123.35 GeV or 125.00 GeV.

At the low energy of interacting nucleons there appears constant gluon energy equal to 727.44 MeV and asymmetry in interactions of negative and positive virtual gluon propagators. Due to confinement of gluon balls and gluon propagators there can appear mass of the charm quark. Due to the atom-like structure of baryons there can appear the other masses of quarks as well [1].

The Theory of Branching Ratios leads to conclusion that the relative signal strength of the decays into two photons to the decays into two Z bosons should be in approximation 0.269 i.e. about 1.87 times higher than predicted within the Standard Model $B_{\gamma\gamma,SM}/B_{ZZ,SM} = 0.0023/0.016 = 0.144$. This follows from the fact that in reality there are the two parts in the decay channel k = 3 so there is less the decays into two Z bosons. There is 1/a = 1.88 times less such decays. We can see that the factor $1/a = 1.88 \neq 1$, follows from the wrong initial conditions in the Standard Model for the ZZ decay channel, not for the γγ decay channel. But due to the neglected pairing and quadrupole symmetry in the Standard Model, characteristic for the Higgs bosons as well, the experimental branching ratio for the ZZ decay channel is close to the theoretical result obtained within the Standard Model. We can see that these two wrong assumptions almost cancel each other.

The theoretical result for the branching ratio for the decay of the Higgs boson into two photons is $B_{\gamma\gamma} = 0.001992$ whereas into two Z bosons is $B_{ZZ} = 0.007411$. Since there is the pairing of the Higgs bosons then for the decays into two photons, the relative signal strength in relation to the Standard Model is 1.732 whereas for ZZ channel is 0.926.

The upper and lower limits for branching ratios follow from the holographic principle and from the atom-like structure of baryons as well.

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