

## Correlation of Neutrosophic Data

I.M. Hanafy, A.A.Salama and K. Mahfouz

*Egypt, Port Said University, Faculty of Sciences, Department of Mathematics and Computer Sciences*

**ABSTRACT:** Neutrosophy has been introduced by Smarandache [3, 4] as a new branch of philosophy. Salama et al. [5] in 2012 introduced and studied the operations on neutrosophic sets. The purpose of this paper is to introduce a new type of data called the neutrosophic data. After given the fundamental definitions of neutrosophic set operations due to Salama [5], we obtain several properties, and discussed the relationship between neutrosophic sets and others. Finally, we discuss and derived a fomula for correlation coefficient, defined on the domain of neutrosophic sets.

**Keywords:** Neutrosophic Data; Correlation Coefficient.

### I. INTRODUCTION

The fuzzy set was introduced by Zadeh [9] in 1965, where each element had a degree of membership. The intuitionistic fuzzy set (Ifs for short) on a universe X was introduced by K. Atanassov [1] in 1983 as a generalizaion of fuzzy set, where besides the degree of membership and the degree of non- membership of each element.

After the introduction of the neutrosophic set concept [3,4]. In recent years neutrosophic algebraic structures have been investigated. Neutrosophy has laid the foundation for a whole family of new mathematical theories generalizing both their classical and fuzzy counterparts, such as a neutrosophic set theory. It is very common in statistical analysis of data to find the correlation between variables or attributes, the correlation coefficient defined on ordinary crisp sets, fuzzy sets [2] and intuitionistic fuzzy sets [6,7,8] respectively. In this paper we discuss and derived a fomula for correlation coefficient, defined on the domain of neutrosophic sets.

### II. TERMINOLOGIES

We recollect some relevant basic preliminaries, and in particular, the work of Smarandache in [3, 4], and Atanassov in [1]. Smarandache introduced the neutrosophic components T, I, F which represent the membership, indeterminacy, and non-membership values respectively, where  $\left]0^-, 1^+\right]$  is non-standard unit interval.

#### 2.1 Definition. [3]

Let T, I, F be real standard or non\_standeard subsets of  $\left]0^-, 1^+\right]$ , with

$$\text{Sup}_T = t_{\text{sup}}, \text{inf}_T = t_{\text{inf}}$$

$$\text{Sup}_I = i_{\text{sup}}, \text{inf}_I = i_{\text{inf}}$$

$$\text{Sup}_F = f_{\text{sup}}, \text{inf}_F = f_{\text{inf}}$$

$$n\text{-sup} = t_{\text{sup}} + i_{\text{sup}} + f_{\text{sup}}$$

$$n\text{-inf} = t_{\text{inf}} + i_{\text{inf}} + f_{\text{inf}},$$

T, I, F are called neutrosophic components

### III. On NEUTROSOPHIC SETS

Salama et al [5] considered some possible definitions for basic concepts of the neutrosophic set and its operations by the following :

#### 3.1 Definition

Let X be a non-empty fixed set. A neutrosophic set ( NS for short) A is an object having the form

$$A = \left\{ \left\langle x, \mu_A(x), \sigma_A(x), \gamma_A(x) \right\rangle : x \in X \right\}. \text{ Where } \mu_A(x), \sigma_A(x) \text{ and } \gamma_A(x) \text{ which represent the degree of}$$

member ship function (namely  $\mu_A(x)$ ), the degree of indeterminacy (namely  $\sigma_A(x)$ ), and the degree of non-member ship (namely  $\gamma_A(x)$ ) respectively of each element  $x \in X$  to the set A .

#### 3.1 Example

Every intuitionistic fuzzy set  $A$  a non empty set is obviously a neutrosophic set having the form  $A = \{ \langle x, \mu_A(x), \sigma_A(x), 1 - \mu_A(x) \rangle : x \in X \}$

**3.1 Remark**

A neutrosophic set  $A = \{ \langle x, \mu_A(x), \sigma_A(x), \gamma_A(x) \rangle : x \in X \}$  can be defined an ordered triple  $\langle \mu_A, \sigma_A, \gamma_A \rangle$  in  $]^{-}0, 1^{+}[^3$  on  $X$ .

**3.2 Remark**

For the sake of simplicity, we shall use the symbol  $A = \langle x, \mu_A, \sigma_A, \gamma_A \rangle$  for the  $NS$

**3.2 Example**

Every Ifs  $A$  an non-empty set  $X$  is obviously on  $NS$  having the

$$\text{form } A = \{ \langle x, \mu_A(x), 1 - (\mu_A(x) + \gamma_A(x)), \gamma_A(x) \rangle : x \in X \}$$

Since our main purpose is to construct the tools for developing neutrosophic set and neutrosophic data, we must introduce the two neutrosophic sets  $0_N$  and  $1_N$  in  $X$

as follows:

$0_N$  may be defined as:

$$(0_1) \ 0_N = \{ \langle x, 0, 0, 1 \rangle : x \in X \}$$

$$(0_2) \ 0_N = \{ \langle x, 0, 1, 1 \rangle : x \in X \}$$

$$(0_3) \ 0_N = \{ \langle x, 0, 1, 0 \rangle : x \in X \}$$

$$(0_4) \ 0_N = \{ \langle x, 0, 0, 0 \rangle : x \in X \}$$

$1_N$  may be defined as:

$$(1_1) \ 1_N = \{ \langle x, 1, 0, 0 \rangle : x \in X \}$$

$$(1_2) \ 1_N = \{ \langle x, 1, 0, 1 \rangle : x \in X \}$$

$$(1_3) \ 1_N = \{ \langle x, 1, 1, 0 \rangle : x \in X \}$$

$$(1_4) \ 1_N = \{ \langle x, 1, 1, 1 \rangle : x \in X \}$$

**3.2 Definition**

Let  $A = \langle x, \mu_A(x), \gamma_A(x), \sigma_A(x) \rangle$  a  $NS$  on  $X$ , then the complement of the set  $A$  ( $C(A)$ , for short) may be defined as three kinds of complements

$$(C_1) \ C(A) = \{ \langle x, 1 - \mu_A(x), 1 - \sigma_A(x), 1 - \nu_A(x) \rangle : x \in X \},$$

$$(C_2) \ C(A) = \{ \langle x, \nu_A(x), \sigma_A(x), \mu_A(x) \rangle : x \in X \}$$

$$(C_3) \ C(A) = \{ \langle x, \nu_A(x), 1 - \sigma_A(x), \mu_A(x) \rangle : x \in X \}$$

One can define several relations and operations between two neutrosophic sets follows:

**3.3 Definition**

Let  $X$  be a non-empty set, and two neutrosophics  $A$  and  $B$  in the form  $A = \langle x, \mu_A(x), \sigma_A(x), \gamma_A(x) \rangle$ ,

$B = \langle x, \mu_B(x), \sigma_B(x), \gamma_B(x) \rangle$ , then we may consider two possible definitions for subsets ( $A \subseteq B$ )

( $A \subseteq B$ ) may be defined as

$$(1) \ A \subseteq B \Leftrightarrow \mu_A(x) \leq \mu_B(x), \gamma_A(x) \geq \gamma_B(x)$$

$$\text{and } \sigma_A(x) \leq \sigma_B(x) \ \forall x \in X$$

$$(2) \ A \subseteq B \Leftrightarrow \mu_A(x) \leq \mu_B(x), \gamma_A(x) \geq \gamma_B(x) \text{ and}$$

$$\sigma_A(x) \geq \sigma_B(x)$$

**3.1 Proposition**

For any neutrosophic set  $A$  the following are holds

$$(1) \ 0_N \subseteq A, \ 0_N \subseteq 0_N$$

(2)  $A \subseteq 1_N, 1_N \subseteq 1_N$

**3.4 Definition**

Let  $X$  be a non-empty set, and  $A = \langle x, \mu_A(x), \gamma_A(x), \sigma_A(x) \rangle, B = \langle x, \mu_B(x), \sigma_B(x), \gamma_B(x) \rangle$  are NSS . Then

(1)  $A \cap B$  may be defined as:

(I<sub>1</sub>)  $A \cap B = \langle x, \mu_A(x) \wedge \mu_B(x), \sigma_A(x) \wedge \sigma_B(x), \gamma_A(x) \wedge \gamma_B(x) \rangle$

(I<sub>2</sub>)  $A \cap B = \langle x, \mu_A(x) \wedge \mu_B(x), \sigma_A(x) \wedge \sigma_B(x), \gamma_A(x) \vee \gamma_B(x) \rangle$

(I<sub>3</sub>)  $A \cap B = \langle x, \mu_A(x) \wedge \mu_B(x), \sigma_A(x) \vee \sigma_B(x), \gamma_A(x) \vee \gamma_B(x) \rangle$

(2)  $A \cup B$  may be defined as:

(U<sub>1</sub>)  $A \cup B = \langle x, \mu_A(x) \vee \mu_B(x), \sigma_A(x) \vee \sigma_B(x), \gamma_A(x) \wedge \gamma_B(x) \rangle$

(U<sub>2</sub>)  $A \cup B = \langle x, \mu_A(x) \vee \mu_B(x), \sigma_A(x) \wedge \sigma_B(x), \gamma_A(x) \wedge \gamma_B(x) \rangle$

(3)  $[A] = \langle x, \mu_A(x), \sigma_A(x), 1 - \mu_A(x) \rangle$

(4)  $\langle \rangle A = \langle x, 1 - \gamma_A(x), \sigma_A(x), \gamma_A(x) \rangle$

We can easily generalize the operations of intersection and union in Definition 3.4 to arbitrary family of neutrosophic sets as follows:

**3.5 Definition**

Let  $\{A_j : j \in J\}$  be a arbitrary family of neutrosophic sets in  $X$  , then

(1)  $\cap A_j$  may be defined as:

(i)  $\cap A_j = \langle x, \wedge_{j \in J} \mu_{A_j}(x), \wedge_{j \in J} \sigma_{A_j}(x), \vee_{j \in J} \gamma_{A_j}(x) \rangle$

(ii)  $\cap A_j = \langle x, \wedge_{j \in J} \mu_{A_j}(x), \vee_{j \in J} \sigma_{A_j}(x), \vee_{j \in J} \gamma_{A_j}(x) \rangle$

(2)  $\cup A_j$  may be defined as:

(i)  $\cup A_j = \langle x, \vee_{j \in J} \mu(x), \wedge_{j \in J} \sigma(x), \wedge_{j \in J} \gamma(x) \rangle$

(ii)  $\cup A_j = \langle x, \vee_{j \in J} \mu(x), \vee_{j \in J} \sigma(x), \wedge_{j \in J} \gamma(x) \rangle$

**3.6 Definition**

Let  $A$  and  $B$  are two neutrosophic sets then

$A|B$  may be defined as

$A|B = \langle x, \mu_A \wedge \mu_B, \sigma_A(x) \sigma_B(x), \gamma_A \vee \mu_B(x) \rangle$

**3.2 Proposition**

For all  $A, B$  two neutrosophic sets then the following are true

(1)  $A \subseteq B \Rightarrow C(B) \subseteq C(A), C(C(A)) = A$

(2)  $C(1_X) = O_X, C(O_X) = 1_X$

(3)  $C(A \cap B) = C(A) \cup C(B)$

(4)  $C(A \cup B) = C(A) \cap C(B)$

**3.1 Corollary**

Let  $A, B, C$  be are neutrosophic in  $X$  . Then

i)  $A \subseteq B$  and  $C \subseteq D \Rightarrow A \cup C \subseteq B \cup D$  and  $A \cap C \subseteq B \cap D$

ii)  $A \subseteq B$  and  $A \subseteq C \Rightarrow A \subseteq B \cap C$

iii)  $A \subseteq C$  and  $B \subseteq C \Rightarrow A \cup B \subseteq C$

iv)  $A \subseteq B$  and  $B \subseteq C \Rightarrow A \subseteq C$

**Proof.** It is clear from the definition.

#### IV. Correlation of two neutrosophic sets.

If we have a random from a crisp set with corresponding of triple membership grads of two neutrosophic sets we have interest in very likely we will compare the grads of membership functions of neutrosophic sets to see if there is any linear relationship between the two neutrosophic sets, we need a formula for the sample correlation coefficient of two neutrosophic sets to show the relationship between them.

##### 4.1 Definition

For A and B are two neutrosophic sets in a finite space  $X = \{x_1, x_2, \dots, x_n\}$ , we define the correlation of neutrosophic sets A and B as follows:

$$CN(A, B) = \sum_{i=1}^n [(\mu_A(x_i)\mu_B(x_i) + \sigma_A(x_i)\sigma_B(x_i) + \nu_A(x_i)\nu_B(x_i))]$$

and the correlation coefficient of A and B given by

$$R(A, B) = \frac{CN(A, B)}{(T(A).T(B))^{\frac{1}{2}}}$$

Where

$$T(A) = \sum_{i=1}^n (\mu^2_A(x_i) + \sigma^2_A(x_i) + \nu^2_A(x_i))$$

$$T(B) = \sum_{i=1}^n (\mu^2_B(x_i) + \sigma^2_B(x_i) + \nu^2_B(x_i))$$

The following Proposition is immediate from the definition

##### 4.1 Proposition

For all A, B are two neutrosophic sets in a finite space X we have

$$i) CN(A, B) = CN(B, A), R(A, B) = R(B, A)$$

$$ii) \text{If } A = B, \text{ then } R(A, B) = 1$$

The following Theorem generalizes both Theorem 1.2. [2] and Proposition 2.3. [5].

##### 4.1 Theorem

For neutrosophic sets A and B in X, we have

$$R(A, B) = \frac{CN(A, B)}{(T(A).T(B))^{\frac{1}{2}}} \in [0, 1^+]$$

**Proof.**

Since  $CN(A, B) \geq 0$ , we need only to show that  $CN(A, B) \leq (CN(A, A))^{\frac{1}{2}}(CN(B, B))^{\frac{1}{2}}$  If we denoted by the usual dot product in  $\mathfrak{R}^2$ , then

$$\begin{aligned} CN(A, B) &= \sum_{i=1}^n [(\mu_A(x_i)\mu_B(x_i) + \sigma_A(x_i)\sigma_B(x_i) + \nu_A(x_i)\nu_B(x_i))] \leq \\ &\sum_{i=1}^n \left( (\mu^2_A(x_i) + \sigma^2_A(x_i) + \nu^2_A(x_i))^{\frac{1}{2}} \left( (\mu^2_B(x_i) + \sigma^2_B(x_i) + \nu^2_B(x_i))^{\frac{1}{2}} \right) \right) \leq \\ &\left( \sum_{i=1}^n (\mu^2_A(x_i) + \sigma^2_A(x_i) + \nu^2_A(x_i)) \right)^{\frac{1}{2}} \cdot \left( \sum_{i=1}^n (\mu^2_B(x_i) + \sigma^2_B(x_i) + \nu^2_B(x_i)) \right)^{\frac{1}{2}} = \\ &((C(A, A))^{\frac{1}{2}}(C(B, B))^{\frac{1}{2}} \end{aligned}$$

where the first inequality comes from Schwarz' s inequality for  $\mathfrak{R}^2$  and the second for  $L^2(X)$ . This completes the proof.

##### 4.1 Remark

From the following counterexample, we can easily check that

$$R(A, B) = 1, \text{ but } A \neq B.$$

##### 4.1 Example

Let  $X = \{x_1, x_2\}$  and the neutrosophic sets  $A, B$  given by  $\mu_A(x_i) = \nu_A(x_i) = \sigma_A(x_i) = \frac{1}{2}$  and  $\mu_B(x_i) = \nu_B(x_i) = \sigma_B(x_i) = \frac{1}{4}, i = 1, 2,$

$$R(A, B) = 1 \text{ but } A \neq B.$$

#### 4.2 Example

In this example we will estimate the correlation coefficient between the neutrosophic data .

Let  $A, B$  are two neutrosophic sets in a finite space  $X = \{a, b\}$

defined by  $A = \{\langle a, (0.3, 0.2, 0.5) \rangle, \langle b, (0.5, 0.4, 0.2) \rangle\}, B = \{\langle a, (0.2, 0.4, 0.6) \rangle, \langle b, (0.4, 0.3, 0.6) \rangle\}$

Then we have  $CN(A, B) = 0.88, T(A) = 0.83, T(B) = 1,$  then  $R(A, B) = 0.896.$

This value gives us the information that the neutrosophic sets A and B are positively and closely related with Strength 0.896.

#### References

- [1] K.T. Atanassov (1999), intuitionistic Fuzzy Sets, Physica-Verlag, Heidelberg, New York
- [2] Ding. A. Chiang and Nancy.P. Lin (1999), Correlation of fuzzy sets, Fuzzy Sets and Systems 221-226.
- [3] F.Smarandach, Nutrosophy (2002), A New Branch of Philosophy, in Multiple valued logic, An International Journal, Vol.8, No.3, 297-384.
- [4] F.Smarandach(2003), Neutrosophic set a generalization of the intuitionistic fuzzy sets, Proceedings of the third conference of the European Society for fuzzy logic and Technolgye, EUSFLAT, Septamper Zittau Geamany; Univ. of Applied Sciences at Zittau Goerlit 2, 141-146.
- [5] A.A. Salama and S.A. AL- Blowli(2012); NEUTROSOPHIC SET AND NEUTROSOPHIC TOPOLOGICAL SPACES, *IOSR Journal of Mathematics (IOSR-JM) ISSN: 2278-5728. Volume 3, Issue 4 (Sep-Oct. 2012), 31-35*
- [6] C.Yu (1993), Correlation of fuzzy numbers, Fuzzy Sets and Systems 55303-307.
- [7] Dug Hun Hong and Seak Yoon Hwang(1995), Correlation of intuitionistic fuzzy sets in Probability Spaces, Fuzzy Sets an Systems 7577-81.
- [8] Wen-Liany Hung and Jong Wuu(2002), Correlation of intuitionistic fuzzy sets by Centroid method, Information Sciences 144 219-226.
- [9] L.A. Zadeh,(1965), Fuzzy Sets, Information and Control, 8, pp.338-353.