First Draft

Dynamic Instability of the Standard Model and the Fine-Tuning Problem

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Abstract

The Standard Model for particle physics (SM) is a nonlinear field theory in which both Yang-Mills and Higgs bosons are self-interacting objects. Their classical or quantum evolution is inevitably sensitive to the transition from order to chaos. With few noteworthy exceptions, the mainstream of theoretical particle physics has ignored the dynamical contribution of chaos in Quantum Field Theory. Here we point out that quantum corrections to the classical interaction of the Higgs with gauge bosons may lower the threshold for the onset of chaos and destabilize the vacuum in the *low* or *intermediate* TeV scale. The inability of the vacuum to survive in this energy region provides a straightforward solution for the fine-tuning problem. It also implies that perturbative estimates on vacuum stability well above the LHC scale are likely to be invalid.

Key words: Quantum Field Theory, Standard Model, vacuum stability, fine-tuning problem, gauge fields, Higgs boson, order-to-chaos transition.

1. Introduction and motivation

By construction, the SM represents a *nonlinear field theory* in which Yang-Mills (YM) and Higgs bosons are self-interacting. Nonlinear dynamics of such objects is present at both classical and quantum levels. The chaotic attributes of YM fields have been known and studied since the beginning of the eighties. Chaos was first analyzed in the classical limit of the YM theory [1-3] and it was shown to exist in both the continuum and lattice formulations of the theory. Particularly, for homogeneous gauge field configurations [4], it was found that spontaneous symmetry breaking triggers the *transition to chaos* (TC) with the rise of the energy density [5,6], whereas the dynamics of YM fields in the absence of spontaneous symmetry breaking remains chaotic at any density of energy [7]. The emergence of chaos in classical dynamics of the *SU(2) x U(1)* theory was numerically explored in [8]. Follow-up research was focused on understanding the TC in the semi-classical regime of quantum mechanics (QM) [9, 10] as well as in quantum field theory (QFT) [11, 12]. The investigation of chaos in classical gauge theory has later targeted on a number of specific problems. One of them was confirming the effect induced by the Higgs scalar on the chaotic dynamics of classical YM theory. It was shown that the Higgs scalar regularizes the dynamics of gauge fields at low energy densities [13]. It was also discovered that quantum fluctuations of gauge fields leading to symmetry breaking via the Coleman–Weinberg mechanism [14] tend to stabilize chaotic dynamics of spatially homogeneous systems of YM and Higgs fields at low energy densities [15]. The connection between the chaotic dynamics of a classical field theory and the instability of the one-loop effective action of the associated QFT was analyzed in [16].

Surprisingly, except for isolated studies like the ones previously cited, mainstream theoretical models in particle physics have largely ignored the implications of chaos in QFT. The goal of our report is to contribute to a reversal of this trend. We emphasize here that one-loop corrections to the classical interaction of the Higgs with W, Z bosons or photons are likely to lower the threshold for the TC and destabilize the vacuum in the low or intermediate TeV scale. *By default, a rapidly decaying vacuum in this energy region offers straightforward explanations for the long-standing problems of fine-tuning and ultraviolet stability of the SM.*

Our report is organized as follows: section two reviews the Higgs potential and the fine-tuning problem of the SM. Estimates on vacuum stability based on extrapolation of the SM near the Planck scale are briefly addressed in section three. Section four highlights details on the TC in the di-boson and di-photon Higgs channels. Implications for the fine-tuning problem and for the ultraviolet stability of the vacuum are elaborated upon in the last section.

This work represents a continuation of several studies initiated by the author in [17-20]. It is preliminary in nature and calls out for further clarifications and revisions. Concurrent efforts may refute, amend or consolidate our findings.

2. Stability of the Higgs potential and the fine-tuning problem

Electroweak (EW) symmetry in the SM is broken by a scalar field having the following doublet structure [20]:

$$\Phi = \begin{pmatrix} G^{+} \\ \frac{1}{\sqrt{2}}[(H+\mathrm{v})+iG^{0}] \end{pmatrix} \tag{2.1}$$

Here, G^{+} and G^{0} represent the charged and neutral Goldstone bosons arisen from spontaneous symmetry breaking, H is the SM Higgs boson, $\mathrm{v} \approx O(M_{EW}) = 246\, GeV$ is the Higgs vacuum expectation value (vev) and M_{EW} stands for the EW scale. Symmetry breaking is caused by the Higgs potential, whose form satisfies the requirements of renormalizability and gauge-invariance

$$V = \mu^{2}\Phi^{+}\Phi + \lambda(\Phi^{+}\Phi)^{2} \tag{2.2}$$

with $\lambda \approx O(1)$ and $\mu^{2} \approx O(M_{EW}^{2})$. A vanishing quartic coupling ($\lambda = 0$) represents the critical value that separates the ordinary EW phase from an unphysical phase where the Higgs field assumes unbounded values. Likewise, the coefficient μ^{2} plays the role of an order parameter whose sign describes the transition between a symmetric phase and a broken phase. Minimizing the Higgs potential yields a vev given by

$$v^2 = -(\mu^2/\lambda) \tag{2.3}$$

where the physical mass of the Higgs is

$$M_H^2 = -2\lambda v^2 = 2\mu^2 \tag{2.4}$$

The renormalized mass squared of the Higgs scalar contains two contributions

$$\mu^2 = \mu_0^2 + \Delta\mu^2 \tag{2.5}$$

in which μ_0^2 represents the ultraviolet (bare) value. This mass parameter picks up quantum corrections $\Delta\mu^2$ that depend quadratically on the ultraviolet cutoff Λ of the theory. Consider for example the contribution of radiative corrections to μ^2 from top quarks. The complete one-loop calculation of this contribution reads

$$\Delta\mu^2 = \frac{N_c\lambda_t^2}{16\pi^2}[-2\Lambda^2 + 6M_t^2\ln(\frac{\Lambda}{M_t}) + ...] \tag{2.5}$$

in which λ_t and M_t are the Yukawa coupling and mass of the top quark. If the bare Higgs mass is set near the cutoff $\mu_0^2 = O(\Lambda^2)$, then $\Delta\mu^2 \approx -10^{35}$ GeV. This large correction must precisely cancel against μ_0^2 to protect the EW scale. This is the root cause of the *fine-tuning problem*, which boils down to the implausible requirement that μ_0^2 and $\Delta\mu^2$ should offset each other to about 31 decimal places.

Closely related to the fine-tuning problem is the question of whether the SM remains valid all the way up to the Planck scale (M_{Pl}). This question is non-trivial because it depends on how the Higgs quartic coupling λ behaves at high energy scales. Competing trends are at work here, namely [21, 22]:

1) Radiative corrections from top quarks *drop* λ at higher scales, while those from the self-interacting Higgs *grow* λ at higher scales.
2) If λ is too large at the EW scale, the Higgs loops dominate and λ diverges at some intermediate scale called the *Landau pole*. However, if λ is too small at the EW scale, the top loops dominate, λ runs negative at some intermediate scale which, in turn, makes the potential unbounded from below and destabilizes the vacuum.

3. Radiative corrections and vacuum stability

Within the SM, the value of the physical Higgs mass $M_H \approx 125\ GeV$ hinted by recent LHC data falls at the border of vacuum stability which, in turn, implies a vanishing quartic coupling near the M_{Pl} . A recent study [23] has undertaken a complete perturbative analysis on the vacuum stability, including the two-loop threshold correction to λ at the EW scale due to QCD and top quark couplings. It led to a couple of outcomes:

1) Vacuum instability develops around an RG scale of $\mu_{cr} = 10^{11}$ GeV.

2) Both parameters of the Higgs potential (2.2) assume near quasi-critical values about μ_{cr}

$$\mu^2 << M_{Pl}, \quad \lambda(\mu_{cr}) \approx 0 \tag{3.1}$$

It was concluded that 2) hints at the possibility that the SM behaves as a statistical system approaching criticality near μ_{cr} [23].

The next section attempts to refute this conclusion. We find that critical behavior and the approach to chaos in the SM are bound to occur at a scale *significantly lower* than μ_{cr}.

4. Transition to chaos in Higgs interactions (*to follow*)

4.1) Ref. [8]:

$$\boxed{\bar{\varepsilon} = (\frac{4m_W^2}{\sqrt{2}G_F})\varepsilon} \tag{4.1}$$

4.2) Ref. [16]:

$$\boxed{\varepsilon_c \approx \frac{11}{108}\lambda \mathrm{v}^4 - \frac{4}{81\pi^2}(\lambda^2 + \frac{3e^4}{10})\mathrm{v}^4} \tag{4.2}$$

4.3) Ref. [13-15]

$$\boxed{\varepsilon_c^0 = \frac{3\mu^4}{64\pi^2}\exp[2(\alpha_{\mathrm{w}} - \frac{\lambda}{g^4}\beta_{\mathrm{w}})](1 + \frac{1}{2\cos^4\theta_{\mathrm{w}}})[1 - 7\exp(-48\Lambda_{\mathrm{w}}\beta_{\mathrm{w}})]} \tag{4.3}$$

Relation	Context	Critical energy density (GeV^4)	Critical energy(GeV)
(4.1)	classical YM-Higgs	1.529×10^8	$O(10^2)$
(4.2)	quantum M-Higgs	4.658×10^7	$O(10)$
(4.3)	quantum YM-Higgs	4.581×10^8	$O(10^2)$

4. Conclusions and open issues (*to follow*)

References (*to follow*)