Twisted Radio Waves and Twisted Thermodynamics

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Abstract. We point out that the assumption that more than two spatially orthogonal wave modes (the actual two polarization modes) can leave the antenna and propagate in the free space violates the second law of thermodynamics and is thus incorrect.

Recently, papers about a new, interesting idea and its experimental demonstration have appeared in the literature and the media, for example [1-4]: that it is possible to generate radio waves with more spatially orthogonal modes than the usual two polarization modes. Such radio waves would have different angular momentums, in a similar way as the orthogonal (l) wave modes of electrons belonging to the same main quantum number (n) exist at the same frequency. Communication utilizing such modes would expand the available frequency band by a factor given by the number N of such spatially orthogonal modes. Experimental demonstration for N=2 has been carried out and published.

However, the main question is; can modes indeed exist in free space? The situation of N=2 is obvious. It represents the two polarization modes of an electromagnetic wave. If the polarization is circular, which means the waves in the two polarization modes are phase shifted by 90^o, indeed, there is a non-zero angular momentum of the wave.

However, the assumption that there can be more than the two polarization modes is counter-intuitive. In the atom, the reason for the existence of waves with different angular momentum at the same energy originates from the potential and the resulting localized nature of the wave. For a charge revolving in a Coulomb potential field, there are an infinite number of different classical physical paths with the same energy. Bohr-Sommerfeld quantization will select a finite number of allowed states within quantum theory. However, no such state component exist for free electron waves. In the light of this intuitive argument, the existence of such spatially orthogonally modes of electromagnetic waves is fine for photons propagating in under spatially confined conditions (boundary conditions of optical constant correspond to potential change) such as wave guides or optical fibers [5], however it is difficult to imagine them in them free space.

Thus the ultimate question for potential applications in wireless communication [1] is: can such waves with $N>2$ exist and be radiated by an antenna in the free space? The existing experimental demonstrations hold for $N=2$. Note, a published experiment about polarization multiplexing, which is a standard multiplexing operation into two polarization channels is irrelevant here because it does not allow compressing more information into the channels, similarly to regular multiplexing in a single channel.

Instead of analyzing the theoretical treatments for potential errors, we choose another way: the Second Law of Thermodynamics, in other words, the impossibility of constructing a perpetual motion machine (of the second kind).

According to Planck, on a black body (maximal emissivity) each of the spatial orthogonal modes will radiate with $I(f)$ spectral intensity:

$$I(f)=\frac{4\pi hf^{3}}{c^{2}}\frac{1}{e^{hf/kT}-1} \tag{1}$$

This means that a unit surface area of the black body, in an infinitesimally small frequency band Δf around frequency f will emit an amount of power

$$P(f,\Delta f)=I(f)\Delta f=\frac{4\pi hf^{3}}{c^{2}}\frac{\Delta f}{e^{hf/kT}-1} \tag{2}$$

in each of the spatially orthogonal modes.

The total radiated power over a unit area is:

$$P(f,\Delta f)=NI(f)\Delta f=N\frac{4\pi hf^{3}}{c^{2}}\frac{\Delta f}{e^{hf/kT}-1} \tag{3}$$

where N is the number of spatially orthogonal modes. In Planck's work and the related radiation measurements $N=2$ because the two spatially orthogonal modes are the two polarization modes. So, for $N=2$, we obtain the standard Planck formula:

$$P(f,\Delta f)=\frac{8\pi hf^{3}}{c^{2}}\frac{\Delta f}{e^{hf/kT}-1} \tag{4}$$

Inspired by Nyquist's treatment of Johnson noise, we can set up the following thought experiment: a large box is located in a thermal reservoir. Inside the box, the walls, for the sake of simplicity, are ideally black. There is a thermally isolated resistor and a twisted-wave antenna with N spatially orthogonal modes within the box. The resistor is connected to the electrodes of the antenna. We start from thermal equilibrium, uniform temperature within the box including the walls, the inherent thermal radiation, the antenna, the resistor and its thermal isolation.

If N=2, the classical Nyquist/Planck case exists: the energy supplied by the resistor and radiated by the antenna gets absorbed in the walls. However, the antenna will absorb the same average amount of energy from the radiation field (originating from the walls) and transforms this energy in into voltage, so the resistor will dissipate the same amount of energy. The reason for this is the principle of detailed balance, which guarantees that each of the two polarization modes of the antenna keeps a separate dynamical equilibrium with the corresponding polarization mode of the black-body radiation of the walls. Thus the Second Law of Thermodynamics is not violated: the temperature stays homogeneous in the system.

If $N > 2$, the antenna will radiate more energy than it can absorb because the antenna radiation would have N-2 extra orthogonal modes that do not match the radiation originating from the walls. Thus the resistor will cool down; which means, temperature inhomogenity will be induced in the system: a direct violation of the Second Law.

There are only two ways to avoid violation of the Second Law with this setup:

1) We suppose that the black-body radiation of the walls also has at least as many (N>2) spatially orthogonal modes as the antenna. This assumption would preserve the Second Law but it would directly violate the theory and experiments on the intensity of black body radiation. Note, these experiments are part of the foundation of quantum physics and one of the most thoroughly checked experimental effects in the history of physics. For example, N=20 would result in a ten-fold speed of radiation cooling of bodies and would impact not only everyday experience and thermal engineering but also cosmology. Thus we must discard this assumption.

2) The only remaining way to save the Second Law of Thermodynamics is to suppose that N cannot be greater than 2.

In conclusion, we showed that the assumption that more than two spatially orthogonal wave modes (the actual two polarization modes) can leave the antenna and propagate in free space violates the second law of thermodynamics thus it cannot be correct. If some experiments seemingly support an assumption that violates a law of physics, then either the experiment and/or its interpretation is incorrect or that particular law of physics is invalid for this situation. However, we see no reason to doubt the validity of the Second Law of Thermodynamics. The existence of more than two spatially orthogonal polarization modes in free space would allow the construction a perpetual motion machine.

References

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