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A SHORT DISCUSSION OF RELATIVISTIC GEOMETRY

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ABSTRACT

The relativists have not understood the geometry of Einstein's gravitational field. They have failed to realise that the geometrical structure of spacetime manifests in the geometrical relations between the components of the metric tensor. Consequently, they have foisted upon spacetime quantities and geometrical relations which do not belong to it, producing thereby, grotesque objects, not due to Nature, but instead, to faulty thinking. The correct geometry and its consequences are described herein.

Key words: Relativistic geometry, Einstein's gravitational field

1. INTRODUCTION

The specific characteristics of the geometry of Einstein's gravitational field must be determined by the geometrical structure of the associated spacetime manifold; in other words, by the geometrical relations between the components of the metric tensor. These relations are definite and inviolable. The relativists have not understood this and have therefore failed to solve the problem of Einstein's gravitational field.

The alleged solutions obtained by the relativists are all invalid, owing to their transgression of the inviolable geometry.

2. THE GEOMETRY OF THE GRAVITATIONAL FIELD

Consider the standard Minkowski metric,

$$ds^2 = dt^2 - dr^2 - r^2(d\theta^2 + \sin^2\theta d\varphi^2)\,, \tag{1}$$

$$0 \leqslant r < \infty\,.$$

The spatial component of (1) describes a sphere of radius r centred at $r = 0$. Compare it with the generalised metric,

$$ds^2 = A(r)dt^2 - B(r)dr^2 - C(r)(d\theta^2 + \sin^2\theta d\varphi^2)\,, \tag{2}$$

$$A, B, C, > 0 \;\forall\; r > r_0\,,$$

where r_0 is an entirely arbitrary lower bound on the real variable r. On (2) I identify the radius of curvature R_c, the proper radius R_p, the real-valued r-parameter, the surface area A_s of the associated sphere, and the volume V of the said sphere, thus

$$R_c = \sqrt{C(r)}\,,$$

$$R_p = \int_{r_0}^{r} \sqrt{B(r)}\; dr\,,$$

$$\text{the real-valued } r - \text{parameter is just the variable } r\,, \tag{3}$$

$$A_s = C(r) \int_0^{\pi} \sin\theta\, d\theta \int_0^{2\pi} d\varphi\,,$$

$$V = \int_{r_0}^{r} C(r)\sqrt{B(r)}\, dr \int_0^{\pi} \sin\theta\, d\theta \int_0^{2\pi} d\varphi\,.$$

I remark that I could already generalise equation (2) further, so that r_0 can be approached from above and below, but I will not include that complication at this point. Now I also remark that the geometrical relations between the components of the metric tensor of (1) are precisely the same as those between the components of the metric tensor of (2). This is entirely a matter of geometry.

Comparing (1) to (2), in the terms of relations (3), it is easily seen that for (1),

$$R_c = r\,,$$

$$R_p = \int_0^{r} dr = r\,,$$

$$A_s = r^2 \int_0^{\pi} \sin\theta\, d\theta \int_0^{2\pi} d\varphi = 4\pi r^2 = 4\pi R_c^2 = 4\pi R_p^2\,,$$

$$V = \int_0^{r} r^2\, dr \int_0^{\pi} \sin\theta\, d\theta \int_0^{2\pi} d\varphi = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi R_c^3 = \frac{4}{3}\pi R_p^3\,,$$

so $R_c \equiv R_p \equiv r$, owing to the pseudo-Euclidean nature of (1) and the associated lower bound on r at $r_0 = 0$.

Next consider a transformation of (1), which I write as,

$$r = (r^{*3} + a^3)^{\frac{1}{3}}\,, \tag{4}$$

and following the incorrect practice of the relativists, I immediately drop the *, so that (1) becomes,

$$ds^2 = dt^2 - \frac{r^4}{(r^3 + a^3)^{\frac{4}{3}}} dr^2 - (r^3 + a^3)^{\frac{2}{3}} (d\theta^2 + \sin^2\theta d\varphi^2)\,. \tag{5}$$

But, owing to (4),

$$-a \leqslant r^* < \infty\,,$$

i. e. on (5),

$$-a \leqslant r < \infty\,.$$

The relativists think that r in (5) is still a radius as defined on (1), and that $r = 0$ (i.e. $r^* = 0$) is an "origin" on (5). This is not correct. The r (correctly r^*) in (5) is no longer a radius, but is instead a real-valued parameter for the true radius on (5). Indeed,

$$R_c = (r^3 + a^3)^{\frac{1}{3}}\,,$$

$$R_p = \int_{-a}^{r} \frac{r^2}{(r^3 + a^3)^{\frac{2}{3}}}\, dr = (r^3 + a^3)^{\frac{1}{3}} \equiv R_c\,,$$

$$A_s = (r^3 + a^3)^{\frac{2}{3}} \int\limits_0^{\pi} \sin\theta \; d\theta \int\limits_0^{2\pi} d\varphi$$

$$= 4\pi (r^3 + a^3)^{\frac{2}{3}} = 4\pi R_c^2 = 4\pi R_p^2 \,,$$

$$V = \int\limits_{-a}^{r} r^2 \; dr \int\limits_0^{\pi} \sin\theta \; d\theta \int\limits_0^{2\pi} d\varphi = \frac{4}{3}\pi (r^3 + a^3)$$

$$= \frac{4}{3}\pi R_c^3 = \frac{4}{3}\pi R_p^3 \,.$$

Once again, $R_p \equiv R_c$ owing to the pseudo-Euclidean nature of (5). Note however that $R_p \equiv R_c \neq r$. The variable r in (5) is not a radial coordinate on (5), contrary to relativist claims. It is nothing more than a parameter for the determination of the true radial quantities R_c and R_p according to the geometrical relations between the components of the metric tensor, given in definitions (3).

In the case of (2), the relativists transgress the rules of mathematics in precisely the same fashion as I have illustrated in relation to (5). Here now is what they do.

Let

$$r^* = \sqrt{C(r)} \,. \tag{6}$$

They then transform (2) and immediately drop the * to get,

$$ds^2 = M(r)dt^2 - N(r)dt^2 - r^2(d\theta^2 + \sin^2\theta d\varphi^2) \,.$$

They then solve this in the usual way to obtain,

$$ds^2 = \left(1 - \frac{2m}{r}\right) dt^2 - \left(1 - \frac{2m}{r}\right)^{-1} dr^2 - r^2(d\theta^2 + \sin^2\theta d\varphi^2) \,, \tag{7}$$

which they incorrectly call the "Schwarzschild" solution. In truth, this is *not* Schwarzschild's solution at all. Schwarzschild's actual solution, which can be easily confirmed by reading his original paper [1], is,

$$ds^2 = \left(1 - \frac{\alpha}{R}\right) dt^2 - \left(1 - \frac{\alpha}{R}\right)^{-1} dR^2 - R^2(d\theta^2 + \sin^2\theta d\varphi^2) \,,$$

$$R = \left(r^3 + \alpha^3\right)^{\frac{1}{3}} \,, \quad \alpha = 2m, \quad 0 < r < \infty \,.$$

Schwarzschild's original paper testifies to the little known fact that all the claims attributed to him by the relativists, are completely false.

Equation (7) is actually due to J. Droste [2], who emphatically maintained that on (7), $2m < r < \infty$. It was also obtained by H. Weyl [3], who too emphatically maintained the same domain of definition on r as did Droste. Equation (7), obtained in the way described above, defined, without proof, on $0 < r < \infty$, was due to D. Hilbert. Unlike Hilbert, the relativists maintain, without proof, (i. e. by mere invalid assumption) that there are two domains for r,

$$0 < r < 2m, \quad \text{and} \quad 2m < r < \infty \,.$$

The interval $0 < r < 2m$ gives rise to the nonsensical Kruskal-Szekeres extension, which incorrectly treats of r in (7) as a radius in the gravitational field, and is therefore claptrap.

The allegations of Hilbert and the relativists are all demonstrably false. Their claims are derived from mere invalid assumption, not mathematical rigour. Any attempt to dismiss the issues as only of historical relevance is also inadmissible, because a history of errors is still erroneous. Furthermore, the fact that Schwarzschild worked with Einstein's penultimate version of the theory, requiring him to meet the condition $\det ||g|| = -1$, is quite irrelevant.

According to (6),

$$r_0^* = \sqrt{C(r_0)}\,,$$

the value of which must be determined by a boundary condition. One cannot just assume on (7) that r (in place of r^*) denotes a radius in the gravitational field, and one cannot just assume some lower bound on r (in place of r^*), as Hilbert did, and as the relativists have done ever since. That transgresses the rules of mathematics.

Now in the general solution for the gravitational field of the point-mass [4, 5], $R_p \neq R_c$, and r is merely a real-valued parameter for the determination of R_p and R_c, thus

$$ds^2 = \left(1 - \frac{\alpha}{\sqrt{C(r)}}\right) dt^2 - \left(1 - \frac{\alpha}{\sqrt{C(r)}}\right)^{-1} [d\sqrt{C(r)}]^2 - C(r)(d\theta^2 + \sin^2\theta d\varphi^2)\,, \tag{8a}$$

$$= \left(1 - \frac{\alpha}{\sqrt{C(r)}}\right) dt^2 - \left(1 - \frac{\alpha}{\sqrt{C(r)}}\right)^{-1} \frac{[C'(r)]^2}{4C(r)} dr^2 - C(r)(d\theta^2 + \sin^2\theta d\varphi^2)\,, \tag{8b}$$

$$R_c = \sqrt{C(r)}\,,$$

$$R_p = \int\limits_{r_0}^{r} \sqrt{\frac{\sqrt{C(r)}}{\sqrt{C(r)} - \alpha}}\; \frac{C'(r)}{2\sqrt{C(r)}}\; dr$$

$$= \int\limits_{\sqrt{C(r_0)}}^{\sqrt{C(r)}} \sqrt{\frac{\sqrt{C(r)}}{\sqrt{C(r)} - \alpha}}\; d\sqrt{C(r)}\,,$$

$$A_s = C(r) \int\limits_0^{\pi} \sin\theta\; d\theta \int\limits_0^{2\pi} d\varphi = 4\pi C(r)\,,$$

$$V = \int\limits_{\sqrt{C(r_0)}}^{\sqrt{C(r)}} \sqrt{\frac{\sqrt{C(r)}}{\sqrt{C(r)} - \alpha}} C(r)\; d\sqrt{C(r)} \int\limits_0^{\pi} \sin\theta\; d\theta \int\limits_0^{2\pi} d\varphi\,,$$

where I have shown elsewhere [4, 5] that,

$$\sqrt{C(r_0)} = \alpha = 2m \;\forall\; r_0\,,$$

i. e.

$$\sqrt{C(r_0)} \equiv \alpha = 2m\,.$$

Note that the value of r_0 is entirely arbitrary. It is the admissible form of $C(r)$ which is important, and which must be determined by the conditions of the gravitational field. Clearly, $R_p \neq R_c$, owing to the non-Euclidean nature of equations (8).

I shall now generalise (1) so that the origin is located at any arbitrary r_0, in which case the radius no longer takes the same value as the coordinate r, thus [6],

$$ds^2 = dt^2 - (d|r - r_0|)^2 - |r - r_0|^2(d\theta^2 + \sin^2\theta d\varphi^2) \tag{9a}$$

$$= dt^2 - \frac{(r - r_0)^2}{|r - r_0|^2} dr^2 - |r - r_0|^2(d\theta^2 + \sin^2\theta d\varphi^2) \tag{9b}$$

$$= dt^2 - dr^2 - |r - r_0|^2(d\theta^2 + \sin^2\theta d\varphi^2)\,, \tag{9c}$$

$$R_c = |r - r_0|\,,$$

$$R_p = \int\limits_0^{|r-r_0|} d|r - r_0| = \int\limits_{r_0}^{r} \frac{(r - r_0)}{|r - r_0|}\, dr = |r - r_0| \equiv R_c\,,$$

$$A_s = |r - r_0|^2 \int\limits_0^{\pi} \sin\theta d\theta \int\limits_0^{2\pi} d\varphi = 4\pi |r - r_0|^2 = 4\pi R_p^2$$

$$= 4\pi R_c^2\,,$$

$$V = \int\limits_0^{|r-r_0|} |r - r_0|^2\, d|r - r_0| \int\limits_0^{\pi} \sin\theta d\theta \int\limits_0^{2\pi} d\varphi$$

$$= \int\limits_{r_0}^{r} |r - r_0|^2 \frac{(r - r_0)}{|r - r_0|}\, dr \int\limits_0^{\pi} \sin\theta d\theta \int\limits_0^{2\pi} d\varphi$$

$$= \frac{4}{3}\pi |r - r_0|^3 = \frac{4}{3}\pi R_p^3 = \frac{4}{3}\pi R_c^3\,.$$

These equations clearly render Euclidean forms, owing to the pseudo-Euclidean nature of equations (9). Note that $R_p \equiv R_c$ but $R_c \neq r$, and more, that $R_p(r_0) = R_c(r_0) = 0$, irrespective of the value of r_0. The parametric origin for equations (9) is at the arbitrary r_0, and r_0 can be approached from above or below. Furthermore, $r = 0$ is not an origin unless $r_0 = 0$, in which case equations (9) reduce to equation (1). There is nothing special about $r = 0$ that makes it always the origin. This amplifies the fact that only the *distance* between two points is important, not the particular value of a variable coordinate. Equation (1) is merely a special case of equations (9). The radius of the sphere associated with equations (9) must be determined by the geometrical relations (3), which are common to all forms (2).

It is a rather trivial matter now to generalise (2), and therefore (8) [6]. One need only replace r there with $D = |r - r_0|$, and so the domain of the r-parameter becomes $\{r \mid r \in \Re, r \neq r_0\}$. Then r_0 can be approached from above and below, giving rise to a general mapping of a Euclidean distance in parameter space into a non-Euclidean distance in the gravitational field.

In the case of the metric for the gravitational field for the simple point-mass, equations (8), the fact that $R_c(r_0) = \sqrt{C(r_0)} \equiv \alpha = 2m$ when $R_p(r_0) = 0$, i. e. $R_p(r_0) \equiv 0$, is an inescapable consequence of Einstein's geometry. There is nothing more point-like in the gravitational field. The usual conception of a point in Minkowski space, manifest as $R_p(r_0) \equiv R_c(r_0) \equiv 0$, does not exist in Einstein's gravitational field. Notwithstanding, point-masses and point-charges are fictitious and so point-mass and point-charge solutions are all nonsense [7]. The correction of the geometrical error of the relativists leads directly to the following results in a very simple manner.

(a) The Hilbert solution and its charged and rotating extensions are all invalid [4, 5].

(b) Schwarzschild's true solution is a correct *particular* solution for the simple point-mass [4].

(c) The Droste/Weyl solution is a correct *particular* solution for the simple point-mass [4].

(d) Kepler's laws are modified by General Relativity [8].

(e) Black holes have no theoretical basis whatsoever [4, 5].

(f) The Kruskal-Szekeres extension is humbug [4, 5, 6].

(g) All solutions to Einstein's field equations purporting an expanding Universe are incorrect, hence the Friedmann solution, the Lemaître-Robertson solution, the Robertson-Walker solution, etc., are nothing more than mathematical gibberish - meaningless concoctions of mathematical symbols [9].

(h) Einstein's so-called "cylindrical universe" actually consists of a single world-line [9].

(i) de Sitter's so-called "spherical universe" actually consists of a static point [9].

(j) The conventional interpretation of the Hubble relation and the CMB are not consistent with General Relativity [9].

(k) The Big Bang hypothesis has no basis in theory whatsoever [9].

(l) Einstein's cosmological constant is precisely zero [9].

(m) Cosmologically, Einstein's field equations admit only of the flat, infinite, static, empty spacetime of Special Relativity, which, being devoid of matter, cannot describe the Universe other than locally [9].

(n) The general relativistic prediction of the deflection of light at the limb of the Sun is 1.65 arcseconds [10].

(o) Neither Special Relativity nor General Relativity can form the basis for a cosmology, being only theories of *local* phenomena. In other words, they admit of no "large-scale" spacetime [10].

The explicit form of the function $C(r)$ is given in my cited papers.

I finally remark that Einstein's pseudotensor is certainly incorrect, but that is another story.

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