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Fractional Modified Special Relativity

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Abstract

Fractional calculus represents a natural tool for describing relativistic phenomena in pseudo-Euclidean space-time. In this study, fractional modified special relativity is presented. We obtain fractional generalized relation for the time dilation.

Keywords: Special relativity; Time dilation; Fractional special relativity; Fractional calculus

1- Introduction

The theory of special relativity was first published as a paper in 1905[1]. Recently, some modifications on this theory have been proposed. For example, in ref. [3] some aspects of nonlocal special relativity have been studied. Dissipative Lagrangian for a relativistic particle also has been discussed in ref. [4].

In more recent years attention has been attracted to the application of Fractional Calculus (FC) to special (and general) relativity [9-13]. Fractional calculus can be used to model the nonlocal dissipative phenomena in space (-time) [7, 8]. In particular FC represents a natural tool for describing relativistic phenomena in pseudo-Euclidean space-time [9, 11].

The aim of this short paper is to introduce fractional modified special relativity as a new nonlocal dissipative model for the special theory of relativity. In the following, fractional calculus [5, 6] is briefly reviewed in Sec. 2. Fractional generalized relation for the time dilation is presented in Sec. 3. Finally, in Sec. 4, we will present some conclusions and discussions.

2-Fractional calculus: Riemann-Liouvill fractional integral

Fractional calculus is the calculus of derivatives and integrals with arbitrary order, which unify and generalize the notions of integer order differentiation and n-fold integration. In contrast with the definition of the ordinary derivative and integration, the definition of the fractional ones is not unique. In fact, there exists several definitions including, Grünwald - Letnikov, Riemann - Liouville, Weyl, Riesz and Caputo for fractional derivative and integral. For the purpose of this study we use the left-sided Riemann-Liouville fractional integral operator. The left-sided Riemann -Liouvill fractional integral of order α of a function $f(t)$ is defined by [5, 6]

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$$ {}_{0}^{RL}I_{t}^{\alpha} f(t) = \frac{1}{\Gamma(\alpha)} \int_{0}^{t} (t-\tau)^{\alpha-1} f(\tau) d\tau, \quad t > 0, \alpha \in \mathbb{R}^{+} \tag{1} $$

Where, $\mathbb{R}^{+}$ is the set of positive real numbers and $\Gamma(\alpha)$ [*]denotes the gamma function.

3-Time dilation for fractional special relativity

We start with the following expression for the fractional generalized position

$$ x(t) = {}_{0}^{RL}I_{t}^{\alpha} v_{\alpha}, \quad 0 < \alpha \le 1 \tag{2} $$

By use of the above equation for the fractional generalized position we can easily derive dilated time formula according to the famous light clock experiment [2] as below

$$ t = \frac{\sqrt{({}_{0}^{RL}I_{t}^{\alpha} v_{\alpha})^{2} + (ct_{0})^{2}}}{c} \tag{3} $$

Where c is the speed of light in vacuum, t_0 is proper time. By use of Eq. (1), we can obtain the following relationship between t_0 (proper time) and t (dilated time):

$$ t^{2} - \frac{\beta_{\alpha}^{2} t^{2\alpha}}{\alpha^{2} \Gamma^{2}(\alpha)} = t_{0}^{2} \tag{4} $$

We can write Eq. (4) as

$$ t^{2} - \frac{\beta_{\alpha}^{2} t^{2\alpha}}{\Gamma^{2}(\alpha+1)} = t_{0}^{2} \tag{5} $$

where $\beta_{\alpha} = \frac{v_{\alpha}}{c}$. As $\alpha \to 1$, Eq. (4) gives

$$ t^{2} - \beta^{2} t^{2} = t_{0}^{2} \tag{6} $$

where $\beta = \frac{v}{c}$.

This leads to well-known time dilation formula of ordinary special relativity

$$ t = \frac{t_{0}}{\sqrt{1-\beta^{2}}}. \tag{7} $$

[*] Definition of Gamma function is given by $\Gamma(\alpha) = \int_{0}^{\infty} e^{-t} t^{\alpha-1} dt, \alpha \in \mathbb{R}^{+}$ .For this function we have $\Gamma(\alpha+1) = \alpha\Gamma(\alpha)$.

If we plot t (dilated time) with respect to t_0 for the special case of $v = 0.99c$ we will have

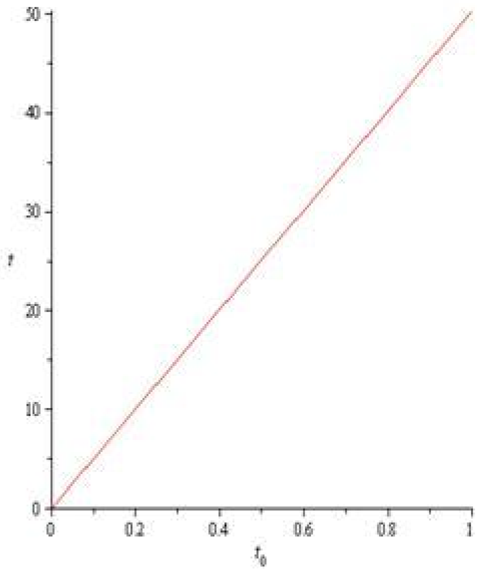

Fig. 1. t with respect to t_0 for the special case of $v = 0.99c, \alpha = 1$

With the aid of expression (5) we can find the t (dilated time) versus t_0 (proper time) for the arbitrary case of α. In particular, for the special case of $\alpha = \frac{1}{2}$ we can draw t versus t_0

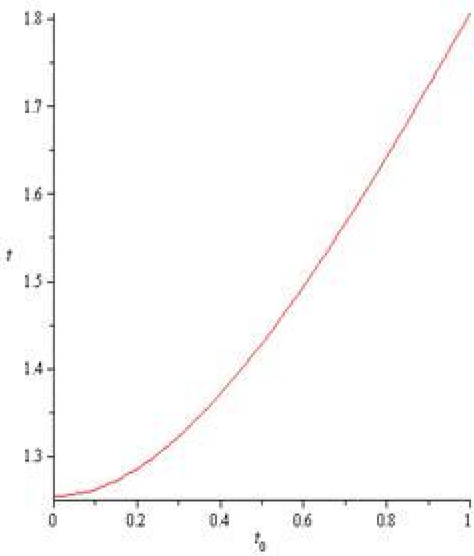

Fig. 2. t with respect to t_0 for the special case of $v = 0.99c, \alpha = \frac{1}{2}$

As shown in above figure, there is a major difference between Fig.1 and Fig.2 near the origin of coordinate system. We can interpret this anomalous behaviour as an immediate consequence of applications of fractional calculus in our model.

4- Conclusion and discussion

Fractional calculus can be used to model the nonlocal dissipative phenomena in space (-time). In particular FC represents an effective tool for describing relativistic phenomena in pseudo-Euclidean space-time. With this motivation we obtain fractional generalized relation for the well-known time dilation formula, to show the possibilities for future studies on fractional modified special theory of relativity. As we shown in Fig. (2), there is a memory like behavior when we plot t with respect to t_0 for the arbitrary case of α ($0 < \alpha < 1$) near the origin of coordinate system. We hope to study some other aspects of fractional modified special relativity in future. We also hope that it can give some new insights about some promising topic for future research such as dissipative relativistic fluid dynamics [14].

Rrefrences

[1] A. Einstein, Zur Elektrodynamik bewegeter Körper, *Annalen der Physik* **17**(1905) 891.
[2] R. Resnick, Introduction to special relativity (*J. Wiley and Sons*, 1968).
[3] B. Mashhoon, *Ann. Phys.* **17** (2008) 11705.
[4] G. Gonzalez, *Int. J. Theor. Phys.* **46** (2007) 417-423.
[5] I. Podlubny, Fractional differential equations (*Academic Press*, 1999).
[6] R. Herrmann, Fractional calculus (*World Scientific Press*, 2011).
[7] L. Vázquez, NATO Science Series, *Kluwer Academic Publishers* 2004.
[8] V. E. Tarasov , *Ann. Phys.* **323** (2008) 2756-2778 .
[9] E. Goldfain, *Comm. Non. Sci. Num. Siml.* **13** (2008) 666-676.
[10] E. Goldfain, *Chaos Solit. Fract.* **22** (2004) 513-520.
[11] V. E. Tarasov, *Int. J. Theor. Phys.* **49** (2010) 293-303.
[12] R. A. EL-Nabulsi, *Chaos Solit. Fract.***42** (2009) 2924-2933.
[13] R. A. EL-Nabulsi, *Chaos Solit. Fract.* **42** (2009) 377-384.
[14] Y. Minami, T. Kunihiro*, Prog.Theor.Phys.***122** (2010) 881-910.