# Gravitational Separator of Isotopes 

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#### Abstract

In this work we show a gravitational separator of isotopes which can be much more effective than those used in the conventional processes of isotopes separation. It is based on intensification of the gravitational acceleration, and can generate accelerations tens of times more intense than those generated in the most powerful centrifuges used for Uranium enrichment.


Key words: Modified theories of gravity, Isotope separation and enrichment, Nonconventional mechanisms.
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## 1. Introduction

A conventional gas centrifuge is basically a cylinder that spins around its central axis with ultra-high angular speed while a gas is injected inside it. Under these conditions, the heavier molecules of the gas move towards the cylinder wall and the lighter ones remain close to the center. In addition, if one creates a thermal gradient in a perpendicular direction by keeping the top of the rotating column cool and the bottom hot, the resulting convection current carries the lighter molecules to the top while the heavier ones settle at the bottom, from which they can be continuously withdrawn.

An important use of gas centrifuges is for the separation of uranium- 235 from uranium-238. As a first step, the uranium metal is turned into a gas (uranium hexafluoride, $\mathrm{UF}_{6}$ ). Next, the $\mathrm{UF}_{6}$ is injected inside a gas centrifuge, which spins at about 100.000 rpm in order to produce a strong centrifugal force upon the $\mathrm{UF}_{6}$ molecules. Thus, the $\mathrm{UF}_{6}$ is separated by the difference in molecular weight between ${ }^{235} \mathrm{UF}_{6}$ and ${ }^{238} \mathrm{UF}_{6}$ [1]. The heavier molecules of the gas $\left({ }^{238} \mathrm{UF}_{6}\right)$ move towards the cylinder wall and the lighter ones $\left({ }^{235} \mathrm{UF}_{6}\right)$ remain close to the center. The convection current carries the lighter molecules $\left({ }^{235} \mathrm{UF}_{6}\right)$ to the top while the heavier ones $\left({ }^{238} \mathrm{UF}_{6}\right)$ settle at the bottom. However, the gas at the top is not composed totally by ${ }^{235} \mathrm{UF}_{6}$ it contains also ${ }^{238} \mathrm{UF}_{6}$, in such way that we can say that the gas at the top is only a gas rich in ${ }^{235} \mathrm{U}$. In practice, several of such centrifuges are connected in
series. A cascade of identical stages produces successively higher concentrations of ${ }^{235} \mathrm{U}$. This process is called Uranium enrichment.

Uranium occurs naturally as two isotopes: $99.3 \%$ is Uranium-238 and $0.7 \%$ is Uranium-235. Their atoms are identical except for the number of neutrons in the nucleus: Uranium- 238 has three more and this makes it less able to fission. Uranium enrichment is used to increase the percentage of the fissile U-235. Nuclear reactors typically require uranium fuel enriched to about $3 \%$ to $5 \%$ U-235. Nuclear bombs typically use 'Highly Enriched Uranium', enriched to $90 \% \mathrm{U}-235$ [2].

In order to extract the ${ }^{235} \mathrm{U}$ from the ${ }^{235} \mathrm{UF}_{6}$ it is necessary to add Calcium. The Calcium reacts with the gas producing a salt and pure ${ }^{235} \mathrm{U}$.

The conventional gas centrifuges used in the Uranium enrichment are very expensive, and they consume much energy during the process. Here, it is proposed a new type of separator of isotopes based on the intensification the gravitational acceleration * [3]. It can generate accelerations tens times more intense than those generated in the most powerful centrifuges used for Uranium enrichment.

[^0]From the quantization of gravity it follows that the gravitational mass $m_{g}$ and the inertial mass $m_{i}$ are correlated by means of the following factor [3]:

$$
\begin{equation*}
\chi=\frac{m_{g}}{m_{i 0}}=\left\{1-2\left[\sqrt{1+\left(\frac{\Delta p}{m_{i 0} c}\right)^{2}}-1\right]\right\} \tag{1}
\end{equation*}
$$

where $m_{i 0}$ is the rest inertial mass of the particle and $\Delta p$ is the variation in the particle's kinetic momentum; $c$ is the speed of light.

When $\Delta p$ is produced by the absorption of a photon with wavelength $\lambda$, it is expressed by $\Delta p=h / \lambda$. In this case, Eq. (1) becomes

$$
\begin{align*}
\frac{m_{g}}{m_{i 0}} & =\left\{1-2\left[\sqrt{1+\left(\frac{h / m_{i 0} c}{\lambda}\right)^{2}}-1\right]\right\} \\
& =\left\{1-2\left[\sqrt{1+\left(\frac{\lambda_{0}}{\lambda}\right)^{2}}-1\right]\right\} \tag{2}
\end{align*}
$$

where $\lambda_{0}=h / m_{i 0} c$ is the De Broglie wavelength for the particle with rest inertial mass $m_{i 0}$.

It has been shown that there is an additional effect - Gravitational Shielding effect - produced by a substance whose gravitational mass was reduced or made negative [4]. The effect extends beyond substance (gravitational shielding), up to a certain distance from it (along the central axis of gravitational shielding). This effect shows that in this region the gravity acceleration, $g_{1}$, is reduced at the same proportion, i.e., $g_{1}=\chi_{1} g \quad$ where $\chi_{1}=m_{g} / m_{i 0}$ and $g$ is the gravity acceleration before the gravitational shielding). Consequently, after a second gravitational shielding, the gravity will be given by $g_{2}=\chi_{2} g_{1}=\chi_{1} \chi_{2} g$, where $\chi_{2}$ is the value of the ratio $m_{g} / m_{i 0}$ for the second gravitational shielding. In a generalized way, we can write that after the $n$th gravitational shielding the gravity, $g_{n}$, will be given by

$$
\begin{equation*}
g_{n}=\chi_{1} \chi_{2} \chi_{3} \cdots \chi_{n} g \tag{3}
\end{equation*}
$$

This possibility shows that, by means of a battery of gravitational shieldings, we can make particles acquire enormous accelerations.

From Electrodynamics we know that when an electromagnetic wave with frequency $f$ and velocity $c$ incides on a material with relative permittivity $\varepsilon_{r}$, relative magnetic permeability $\mu_{r}$ and electrical conductivity $\sigma$, its velocity is reduced to $v=c / n_{r}$ where $n_{r}$ is the index of refraction of the material, given by [5]

$$
\begin{equation*}
n_{r}=\frac{c}{v}=\sqrt{\frac{\varepsilon_{r} \mu_{r}}{2}\left(\sqrt{1+(\sigma / \omega \varepsilon)^{2}}+1\right)} \tag{4}
\end{equation*}
$$

If $\sigma \gg \omega \varepsilon, \omega=2 \pi f$, Eq. (4) reduces to

$$
\begin{equation*}
n_{r}=\sqrt{\frac{\mu_{r} \sigma}{4 \pi \varepsilon_{0} f}} \tag{5}
\end{equation*}
$$

Thus, the wavelength of the incident radiation (See Fig. 1) becomes

$$
\begin{equation*}
\lambda_{\bmod }=\frac{v}{f}=\frac{c / f}{n_{r}}=\frac{\lambda}{n_{r}}=\sqrt{\frac{4 \pi}{\mu f \sigma}} \tag{6}
\end{equation*}
$$



Fig. 1 - Modified Electromagnetic Wave. The wavelength of the electromagnetic wave can be strongly reduced, but its frequency remains the same.

If a lamina with thickness equal to $\xi$ contains $n$ atoms $/ \mathrm{m}^{3}$, then the number of atoms per area unit is $n \xi$. Thus, if the electromagnetic radiation with frequency $f$ incides on an area $S$ of the lamina it reaches $n S \xi$ atoms. If it incides on the total area of the lamina, $S_{f}$, then the total number of atoms reached by the radiation is $N=n S_{f} \xi$. The number of atoms per unit of volume, $n$, is given by

$$
\begin{equation*}
n=\frac{N_{0} \rho}{A} \tag{7}
\end{equation*}
$$

where $N_{0}=6.02 \times 10^{26}$ atoms $/$ kmole is the Avogadro's number; $\rho$ is the matter density of the lamina (in $\mathrm{kg} / \mathrm{m}^{3}$ ) and $A$ is the molar mass(kg/kmole).

When an electromagnetic wave incides on the lamina, it strikes $N_{f}$ front atoms, where $N_{f} \cong\left(n S_{f}\right)_{m}, \phi_{m}$ is the "diameter" of the atom. Thus, the electromagnetic wave incides effectively on an area $S=N_{f} S_{m}$, where $S_{m}=\frac{1}{4} \pi \phi_{m}^{2}$ is the cross section area of one atom. After these collisions, it carries out $n_{\text {collisions }}$ with the other atoms (See Fig.2).


Fig. 2 - Collisions inside the lamina.

Thus, the total number of collisions in the volume $S \xi$ is

$$
\begin{align*}
N_{\text {collisions }} & N_{f}+n_{\text {collisions }} \overline{\bar{s}} \bar{n}_{l} S \phi_{m}+\left(n_{l} S \xi-n_{m} S \phi_{m}\right)= \\
& =n_{n t} S \xi \tag{8}
\end{align*}
$$

The power density, $D$, of the radiation on the lamina can be expressed by

$$
\begin{equation*}
D=\frac{P}{S}=\frac{P}{N_{f} S_{m}} \tag{9}
\end{equation*}
$$

We can express the total mean number of collisions in each atom, $n_{1}$, by means of the following equation

$$
\begin{equation*}
n_{1}=\frac{n_{\text {total photons }} N_{\text {collisions }}}{N} \tag{10}
\end{equation*}
$$

Since in each collision a momentum $h / \lambda$ is transferred to the atom, then the total momentum transferred to the lamina will be $\Delta p=\left(n_{1} N\right) h / \lambda$. Therefore, in accordance with Eq. (1), we can write that

$$
\begin{align*}
& \frac{m_{g(l)}}{m_{i 0(l)}}=\left\{1-2\left[\sqrt{1+\left[\left(n_{1} N\right) \frac{\lambda_{0}}{\lambda}\right]^{2}}-1\right]\right\}= \\
& =\left\{1-2\left[\sqrt{1+\left[n_{\text {total photons }} N_{\text {collisions }} \frac{\lambda_{0}}{\lambda}\right]^{2}}-1\right]\right\} \tag{11}
\end{align*}
$$

Since Eq. (8) gives $N_{\text {collisions }}=n_{l} S \xi$, we get

$$
\begin{equation*}
n_{\text {total photons }} N_{\text {collisions }}=\left(\frac{P}{h f^{2}}\right)\left(n_{l} S \xi\right) \tag{12}
\end{equation*}
$$

Substitution of Eq. (12) into Eq. (11) yields

$$
\begin{equation*}
\frac{m_{g(l)}}{m_{i 0(l)}}=\left\{1-2\left[\sqrt{1+\left[\left(\frac{P}{h f^{2}}\right)\left(n_{l} S \xi\right) \frac{\lambda_{0}}{\lambda}\right]^{2}}-1\right]\right\} \tag{13}
\end{equation*}
$$

Substitution of $P$ given by Eq. (9) into Eq. (13) gives
$\frac{m_{g(l)}}{m_{i O(l)}}=\left\{1-2\left[\sqrt{1+\left[\left(\frac{N_{f} S_{m} D}{f^{2}}\right)\left(\frac{n_{l} S \xi}{m_{i O(l)} c}\right) \frac{1}{\lambda}\right]^{2}}-1\right]\right\}$
Substitution of $N_{f} \cong\left(n_{l} S_{f}\right) \phi_{m} \quad$ and $\quad S=N_{f} S_{m}$ into Eq. (14) results

$$
\begin{equation*}
\frac{m_{g(l)}}{m_{i 0(l)}}=\left\{1-2\left[\sqrt{1+\left[\left(\frac{n_{l}^{3} S_{f}^{2} S_{m}^{2} \phi_{m}^{2} \xi D}{m_{i 0(l)} c f^{2}}\right) \frac{1}{\lambda}\right]^{2}}-1\right]\right\} \tag{15}
\end{equation*}
$$

where $m_{i 0(l)}=\rho_{(l)} V_{(l)}$.
Now, considering that the lamina is inside an ELF electromagnetic field with $E$ and $B$, then we can write that [6]

$$
\begin{equation*}
D=\frac{n_{r(l)} E^{2}}{2 \mu_{0} c} \tag{16}
\end{equation*}
$$

Substitution of Eq. (16) into Eq. (15) gives
$\frac{m_{g(l)}}{m_{i O(l)}}=\left\{1-2\left[\sqrt{1+\left[\left(\frac{n_{r(l)} n_{l}^{3} S_{f}^{2} S_{m}^{2} \phi_{m}^{2} \xi E^{2}}{2 \mu_{0} m_{i O(l)} c^{2} f^{2}}\right) \frac{1}{\lambda}\right]^{2}}-1\right]\right\}$
In the case in which the area $S_{f}$ is just the area of the cross-section of the lamina $\left(S_{\alpha}\right)$, we obtain from Eq. (17), considering that $m_{0(l)}=\rho_{l l} S_{\alpha} \xi$, the following expression
$\frac{m_{g(l)}}{m_{i O(l)}}=\left\{1-2\left[\sqrt{1+\left[\left(\frac{n_{r(l)} n_{l}^{3} S_{\alpha} S_{m}^{2} \phi_{m}^{2} E^{2}}{2 \mu_{0} \rho_{l( } c^{2} f^{2}}\right) \frac{1}{\lambda}\right]^{2}}-1\right]\right\}(18)$
According to Eq. (6) we have

$$
\begin{equation*}
\lambda=\lambda_{\bmod }=\frac{v}{f}=\frac{c}{n_{r(l)} f} \tag{19}
\end{equation*}
$$

Substitution of Eq. (19) into Eq. (18) gives

$$
\begin{equation*}
\frac{m_{g(l)}}{m_{i O(l)}}=\left\{1-2\left[\sqrt{1+\frac{n_{r(l)}^{4} n_{l}^{6} S_{\alpha}^{2} S_{m}^{4} \phi_{m}^{4} E^{4}}{4 \mu_{0}^{2} \rho_{(l)}^{2} c^{6} f^{2}}}-1\right]\right\} \tag{20}
\end{equation*}
$$

Note that $E=E_{m} \sin \omega t$. The average value for $E^{2}$ is equal to $1 / 2 E_{m}^{2}$ because $E$ varies sinusoidaly $\left(E_{m}\right.$ is the maximum value for $E)$. On the other hand, $E_{r m s}=E_{m} / \sqrt{2}$. Consequently we can replace $E^{4}$ for $E_{r m s}^{4}$, and the equation above can be rewritten as follows

$$
\begin{align*}
\chi & =\frac{m_{g(l)}}{m_{i 0(l)}}= \\
& =\left\{1-2\left[\sqrt{1+\frac{n_{r(l)}^{4} n_{l}^{6} S_{\alpha}^{2} S_{m}^{4} \phi_{m}^{4} E_{r m s}^{4}}{4 \mu_{0}^{2} \rho_{(l)}^{2}}-1}\right]\right\} \tag{21}
\end{align*}
$$

Now consider the system shown in Fig.3. It was originally designed to convert Gravitational Energy directly into Electrical Energy [7]. Here, it works as Separator of Isotopes. These systems are basically similar, except in the core. The core of the original
system has been replaced by the one shown in Fig. 3 and Fig. 4 (detailed).

Inside the Gravitational Separator of Isotopes there is a dielectric tube $\left(\varepsilon_{r} \cong 1\right)$ with the following characteristics: $\alpha=60 \mathrm{~mm}$, $S_{\alpha}=\pi \alpha^{2} / 4=2.83 \times 10^{-3} \mathrm{~m}^{2}$. Inside the tube there is an Aluminum sphere with 30 mm radius and mass $M_{g s}=0.30536 \mathrm{~kg}$. The tube is filled with air at ambient temperature and 1 atm . Thus, inside the tube, the air density is

$$
\begin{equation*}
\rho_{\text {air }}=1.2 \mathrm{~kg} . \mathrm{m}^{-3} \tag{22}
\end{equation*}
$$

The number of atoms of air (Nitrogen) per unit of volume, $n_{\text {air }}$, according to Eq.(7), is given by

$$
\begin{equation*}
n_{\text {air }}=\frac{N_{0} \rho_{\text {air }}}{A_{N}}=5.16 \times 10^{25} \mathrm{atoms} / \mathrm{m}^{3} \tag{23}
\end{equation*}
$$

The parallel metallic plates (p), shown in Fig. 3 are subjected to different drop voltages. The two sets of plates ( $D$ ), placed on the extremes of the tube, are subjected to $V_{(D) r m s}=16.22 \mathrm{~V}$ at $f=1 \mathrm{~Hz}$, while the central set of plates (A) is subjected to $V_{\text {(A)rms }}=191.98 \mathrm{~V}$ at $f=1 \mathrm{H}_{2}$. Since $d=98 \mathrm{~mm}$, then the intensity of the electric field, which passes through the 36 cylindrical air laminas (each one with 5 mm thickness) of the two sets $(D)$, is

$$
E_{(D) r m s}=V_{(D) r m s} / d=165.53 \mathrm{~V} / \mathrm{m}
$$

and the intensity of the electric field, which passes through the 7 cylindrical air laminas of the central set $(A)$, is given by

$$
E_{(A) r m s}=V_{(A) r m s} / d=1.959 \times 10^{3} \mathrm{~V} / \mathrm{m}
$$

Note that the metallic rings ( 5 mm thickness) are positioned in such way to block the electric field out of the cylindrical air laminas. The objective is to turn each one of these laminas into a Gravity Control Cells (GCC) [4]. Thus, the system shown in Fig. 3 has 3 sets of GCC. Two with 18 GCC each, and one with 7 GCC. The two sets with 18

GCC each are positioned at the extremes of the tube ( $D$ ). They work as gravitational decelerator while the other set with 7 GCC (A) works as a gravitational accelerator, intensifying the gravity acceleration produced by the mass $M_{g s}$ of the Aluminum sphere. According to Eq. (3), this gravity, after the $7^{\text {th }} \mathrm{GCC}$ becomes $g_{7}=\chi^{7} G M_{g s} / r_{0}^{2}$, where $\chi=m_{g(l)} / m_{i(l)}$ given by Eq. (21) and $r_{0}=35 \mathrm{~mm}$ is the distance between the center of the Aluminum sphere and the surface of the first GCC of the set (A).

The objective of the sets $(D)$, with 18 GCC each, is to reduce strongly the value of the external gravity along the axis of the tube. In this case, the value of the external gravity, $g_{\text {ext }}$, is reduced by the factor $\chi_{d}^{18} g_{\text {ext }}$, where $\chi_{d}=10^{-2}$. For example, if the base BS of the system is positioned on the Earth surface, then $g_{\text {ext }}=9.81 \mathrm{~m} / \mathrm{s}^{2}$ is reduced to $\chi_{d}^{18} g_{\text {ext }}$ and, after the set A, it is increased by $\chi^{7}$. Since the system is designed for $\chi=-308.5$ (See Eq. (26)), then the gravity acceleration on the sphere becomes $\chi^{7} \chi_{d}^{18} g_{\text {ext }}=2.6 \times 10^{-18} \mathrm{~m} / \mathrm{s}^{2}$. This value is much smaller than $g_{\text {sphere }}=G M_{g s} / r_{s}^{2}=2.26 \times 10^{-8} \mathrm{~m} / \mathrm{s}^{2}$.

Note that there is a uniform magnetic field, $B$, through the core of the Gravitational Separator of Isotopes (a cylindrical Dielectric Chamber with 60 mm external diameter; 50 mm internal diameter and 100 mm height).
The values of $\chi$ and $\chi_{d}$, according to Eq. (21) are given by

$$
\begin{align*}
\chi & =\left\{1-2\left[\sqrt{1+\frac{n_{r(l)}^{4} n_{l}^{6} S_{\alpha}^{2} S_{m}^{4} \phi_{m}^{4} E_{(A) r m s}^{4}}{4 \mu_{0}^{2} \rho_{(l)}^{2} c^{6} f^{2}}}-1\right]\right\}= \\
& =\left\{1-2\left[\sqrt{1+1.645 \times 10^{-9} E_{(A) r m s}^{4}}-1\right]\right\} \tag{24}
\end{align*}
$$

$$
\begin{align*}
& \chi_{d}=\left\{1-2\left[\sqrt{1+\frac{n_{r(l)}^{4} n_{l}^{6} S_{\alpha}^{2} S_{m}^{4} \phi_{m}^{4} E_{(D) r m s}^{4}}{4 \mu_{0}^{2} \rho_{(l)}^{2} c^{6} f^{2}}}-1\right]\right\}= \\
& =\left\{1-2\left[\sqrt{1+1.645 \times 10^{-9} E_{(D) r m s}^{4}-1}\right]\right\} \tag{25}
\end{align*}
$$

where $n_{r(\text { air })}=\sqrt{\varepsilon_{r} \mu_{r}} \cong 1$, since $(\sigma \ll \omega \varepsilon)^{\dagger}$; $n_{\text {air }}=5.16 \times 10^{25}$ atoms $/ \mathrm{m}^{3}, \phi_{m}=1.55 \times 10^{-10} \mathrm{~m}$, $S_{m}=\pi \phi_{m}^{2} / 4=1.88 \times 10^{-20} m^{2}$ and $f=1 \mathrm{~Hz}$. Since $E_{(A) r m s}=1.959 \times 10^{3} \mathrm{~V} / \operatorname{mand} E_{(D) r m s}=165.53 \mathrm{~V} / \mathrm{m}$ , we get

$$
\begin{equation*}
\chi=-308.5 \tag{26}
\end{equation*}
$$

and

$$
\begin{equation*}
\chi_{d} \cong 10^{-2} \tag{27}
\end{equation*}
$$

It is important to note that the set with 7 GCC (A) cannot be turned on before the magnetic field $B$ is on. Because the gravitational accelerations on the dielectric chamber and Al sphere will be enormous $\left(\chi^{7} G M_{g s} / r_{0}^{2} \cong 4.4 \times 10^{9} \mathrm{~m} / \mathrm{s}^{2}\right), \quad$ and will explode the device.

The isotopes inside the Dielectric Chamber are subjected to the gravity acceleration produced by the sphere, and increased by the 7 GCC in the region (A). Its value is

$$
\begin{equation*}
a_{i}=\chi^{7} g_{s}=\chi^{7} G \frac{M_{g s}}{r_{s}^{2}} \cong 6.0 \times 10^{9} \mathrm{~m} / \mathrm{s}^{2} \tag{28}
\end{equation*}
$$

Comparing this value with the produced in the most powerful centrifuges (at 100,000 rpm ), which is of the order of $10^{7} \mathrm{~m} / \mathrm{s}^{2}$, we conclude that the accelerations in the Gravitational Separator of Isotopes is about 600 times greater than the values of the centrifuges.

[^1]In the case of Uranium enrichment, the gas $\mathrm{UF}_{6}$ is injected inside this core where it is strongly accelerated. Thus, the $\mathrm{UF}_{6}$ is separated by the difference in molecular weight between ${ }^{235} \mathrm{UF}_{6}$ and ${ }^{238} \mathrm{UF}_{6}$ (See Fig.4). The heavier molecules of the gas $\left({ }^{238} \mathrm{UF}_{6}\right)$ move towards the cylinder bottom and the lighter ones ( ${ }^{235} \mathrm{UF}_{6}$ ) remain close to the center. The convection current, produced by a thermal gradient of about $300^{\circ} \mathrm{C}$ between the bottom and the top of the cylinder, carries the lighter molecules $\left.{ }^{235} \mathrm{UF}_{6}\right)$ to the top while the heavier ones $\left({ }^{238} \mathrm{UF}_{6}\right)$ settle at the bottom, from which they can be continuously withdrawn. The gas withdrawn at the top of the cylinder is a gas rich in ${ }^{235} \mathrm{U}$.

The gravitational forces due to the gravitational mass of the sphere $\left(M_{g s}\right)$ acting on electrons $\left(F_{e}\right)$, protons $\left(F_{p}\right)$ and neutrons $\left(F_{p}\right)$ of the dielectric of the Dielectric Chamber, are respectively expressed by the following relations

$$
\begin{align*}
& F_{e}=m_{g e} a_{e}=\chi_{B e} m_{e}\left(\chi^{\top} G \frac{M_{g s}}{r_{0}^{2}}\right)  \tag{29}\\
& F_{p}=m_{g p} a_{p}=\chi_{B p} m_{p}\left(\chi^{\top} G \frac{M_{g s}}{r_{0}^{2}}\right)  \tag{30}\\
& F_{n}=m_{g n} a_{n}=\chi_{B n} m_{n}\left(\chi^{\top} G \frac{M_{g s}}{r_{0}^{2}}\right) \tag{31}
\end{align*}
$$

In order to make null the resultant of these forces in the Dielectric Chamber (and also in the sphere) we must have $F_{e}=F_{p}+F_{n}$, i.e.,

$$
\begin{equation*}
m_{e} \chi_{\mathrm{Be}}=m_{p} \chi_{\mathrm{Bp}}+m_{n} \chi_{\mathrm{Bn}} \tag{32}
\end{equation*}
$$

In order to calculate the expressions of $\chi_{B e}, \chi_{B p}$ and $\chi_{B n}$ we start from Eq. (17), for the particular case of single electron in the region subjected to the magnetic field $B$. In this case, we must substitute $n_{r(l)}$ by $n_{r}=\left(\mu_{r} \sigma / 4 \pi \varepsilon_{0} f\right)^{\frac{1}{2}} ; n_{l}$ by $1 / V_{e}=1 / \frac{4}{3} \pi \pi_{e}^{3} \quad\left(r_{e}\right.$ is
the electrons radius), $S_{f}$ by $\left(S S A_{e}\right) \rho_{e} V_{e}$ ( $S S A_{e}$ is the specific surface area for electrons in this case: $\left.\operatorname{SSA}_{e}=\frac{1}{2} A_{e} / m_{e}=\frac{1}{2} A_{e} / \rho_{e} V_{e}=2 \pi r_{e}^{2} / \rho_{e} V_{e}\right)$, $S_{m}$ by $S_{e}=\pi r_{e}^{2}, \xi$ by $\phi_{m}=2 r_{e}$ and $m_{i 0(l)}$ by $m_{e}$. The result is

$$
\begin{equation*}
\chi_{\mathrm{Be}}=\left\{1-2\left[\sqrt{1+\frac{45.56 \pi^{2} r_{e}^{4} n_{r}^{2} E^{4}}{\mu_{0}^{2} m_{e}^{2} c^{4} f^{4} \lambda^{2}}}-1\right]\right\} \tag{33}
\end{equation*}
$$

Electrodynamics tells us that $E_{r m s}=v B_{r m s}=\left(c / n_{r}\right) B_{r m s}$, and Eq. (19) gives $\lambda=\lambda_{\text {mod }}=\left(4 \pi / \mu_{r} \sigma f\right)$. Substitution of these expressions into Eq. (33) yields

$$
\begin{equation*}
\chi_{\mathrm{Be}}=\left\{1-2\left[\sqrt{1+\frac{45.56 \pi^{2} r_{e}^{4} B_{r m s}^{4}}{\mu_{0}^{2} m_{e}^{2} c^{2} f^{2}}}-1\right]\right\} \tag{34}
\end{equation*}
$$

Similarly, in the case of proton and neutron we can write that

$$
\begin{align*}
& \chi_{\mathrm{Bp}}=\left\{1-2\left[\sqrt{1+\frac{45.56 \tau^{2} r_{p}^{4} B_{r m s}^{4}}{\mu_{0}^{2} m_{p}^{2} c^{2} f^{2}}}-1\right]\right\}  \tag{35}\\
& \chi_{\mathrm{Bn}}=\left\{1-2\left[\sqrt{1+\frac{45.56 \tau^{2} r_{n}^{4} B_{r m s}^{4}}{\mu_{0}^{2} m_{n}^{2} c^{2} f^{2}}}-1\right]\right\} \tag{36}
\end{align*}
$$

The radius of free electron is $r_{e}=6.87 \times 10^{-14} \mathrm{~m}$ (See Appendix A) and the radius of protons inside the atoms (nuclei) is $r_{p}=1.2 \times 10^{-15} \mathrm{~m}$, $r_{n} \cong r_{p}[\underline{9}, \underline{10}]$, then we obtain from Eqs. (34) (35) and (36) the following expressions:

$$
\begin{gather*}
\chi_{\mathrm{Be}}=\left\{1-2\left[\sqrt{1+8.49 \times 10^{4} \frac{B_{r m s}^{4}}{f^{2}}}-1\right]\right\}  \tag{37}\\
\chi_{\mathrm{Bn}} \cong \chi_{\mathrm{Bp}}=\left\{1-2\left[\sqrt{1+2.35 \times 10^{-9} \frac{B_{r m s}^{4}}{f^{2}}}-1\right]\right\} \tag{38}
\end{gather*}
$$

Then, from Eq. (32) it follows that

$$
m_{e} \chi_{\mathrm{Be}} \cong 2 m_{p} \chi_{\mathrm{Bp}}
$$

(39) In the case of $f=0.1 \mathrm{~Hz}$ the result is

Substitution of Eqs. (37) and (38) into Eq. (39) gives
$\frac{\left\{1-2\left[\sqrt{1+8.49 \times 10^{4} \frac{B_{r m s}^{4}}{f^{2}}}-1\right]\right\}}{\left\{1-2\left[\sqrt{1+2.35 \times 10^{-9} \frac{B_{r m s}^{4}}{f^{2}}}-1\right]\right\}}=3666.3$
For $f=0.1 \mathrm{~Hz}$, we get
$\left.\frac{\left\{1-2\left[\sqrt{1+8.49 \times 10^{6} B_{r m s}^{4}}-1\right]\right.}{\left\{1-2\left[\sqrt{1+2.35 \times 10^{-7} B_{r m s}^{4}}-1\right]\right.} \right\rvert\,=3666.3$
whence we obtain

$$
\begin{equation*}
B_{r m s}=0.793 \mathrm{~T} \tag{42}
\end{equation*}
$$

Consequently, Eq. (37) and (38) yields

$$
\begin{equation*}
\chi_{B e}=-3666.3 \tag{43}
\end{equation*}
$$

and

$$
\begin{equation*}
\chi_{\mathrm{Bn}} \cong \chi_{\mathrm{B} p} \cong 0.999 \tag{44}
\end{equation*}
$$

In order for the forces $F_{e}$ and $F_{p}$ have contrary direction (such as it occurs in the case, in which the nature of the electromotive force is electrical) we must have $\chi_{B e}<0$ and $\chi_{B n} \cong \chi_{B p}>0$ (See equations (29) (30) and (31)), i.e.,

$$
\begin{equation*}
\left\{1-2\left[\sqrt{1+8.49 \times 10^{4} \frac{B_{r m s}^{4}}{f^{2}}}-1\right]\right\}<0 \tag{45}
\end{equation*}
$$

and

$$
\begin{equation*}
\left\{1-2\left[\sqrt{1+235 \times 10^{9} \frac{B_{r m s}^{4}}{f^{2}}-1}\right]\right\}>0 \tag{46}
\end{equation*}
$$

This means that we must have

$$
\begin{equation*}
0.06 \sqrt{f}<B_{r m s}<151.86 \sqrt{f} \tag{47}
\end{equation*}
$$



Fig. 3 - Schematic Diagram of a Gravitational Separator of Isotopes (Based on a process of gravity control patented in July, 31 2008, PI0805046-5). In the case of Uranium enrichment, the gas $\mathrm{UF}_{6}$ is injected inside the core of the Gravitational Separator of Isotopes where it is strongly accelerated. Thus, the $\mathrm{UF}_{6}$ is separated by the difference in molecular weight between ${ }^{235} \mathrm{UF}_{6}$ and ${ }^{238} \mathrm{UF}_{6}$. The heavier molecules of the gas ( ${ }^{238} \mathrm{UF}_{6}$ ) move towards the cylinder bottom and the lighter ones $\left({ }^{235} \mathrm{UF}_{6}\right)$ remain close to the center. The convection current, produced by a thermal gradient of about $300^{\circ} \mathrm{C}$ between the bottom and the top of the cylinder, carries the lighter molecules $\left({ }^{235} \mathrm{UF}_{6}\right)$ to the top while the heavier ones $\left({ }^{238} \mathrm{UF}_{6}\right)$ settle at the bottom, from which they can be continuously withdrawn. The gas withdrawn at the top of the cylinder is a gas rich in ${ }^{235} \mathrm{U}$.


Fig. 4 - Details of the core of the Gravitational Separator of Isotopes. In the case of Uranium enrichment, the heavier molecules of the gas $\left({ }^{238} \mathrm{UF}_{6}\right)$ move towards the cylinder bottom and the lighter ones $\left({ }^{235} \mathrm{UF}_{6}\right)$ remain close to the center. The convection current, produced by a thermal gradient of about $300^{\circ} \mathrm{C}$ between the bottom and the top of the Dielectric Chamber, carries the lighter molecules $\left({ }^{235} \mathrm{UF}_{6}\right)$ to the top while the heavier ones $\left({ }^{238} \mathrm{UF}_{6}\right)$ settle at the bottom, from which they can be continuously withdrawn. The gas withdrawn at the top of the chamber is a gas rich in ${ }^{235} \mathrm{U}$.

## Appendix A: The "Geometrical Radii" of Electron and Proton

It is known that the frequency of oscillation of a simple spring oscillator is

$$
\begin{equation*}
f=\frac{1}{2 \pi} \sqrt{\frac{K}{m}} \tag{A1}
\end{equation*}
$$

where $m$ is the inertial mass attached to the spring and $K$ is the spring constant (in $\mathrm{N} \cdot \mathrm{m}^{-1}$ ). In this case, the restoring force exerted by the spring is linear and given by

$$
\begin{equation*}
F=-K x \tag{A2}
\end{equation*}
$$

where $x$ is the displacement from the equilibrium position.

Now, consider the gravitational force: For example, above the surface of the Earth, the force follows the familiar Newtonian function, i.e., $F=-G M_{g \oplus} m_{g} / r^{2}$, where $M_{g \oplus}$ is the mass of Earth, $m_{g}$ is the gravitational mass of a particle and $r$ is the distance between the centers. Below Earth's surface the force is linear and given by

$$
\begin{equation*}
F=-\frac{G M_{g \oplus} m_{g}}{R_{\oplus}^{3}} r \tag{A3}
\end{equation*}
$$

where $R_{\oplus}$ is the radius of Earth.
By comparing (A3) with (A2) we obtain

$$
\begin{equation*}
\frac{K}{m_{g}}=\frac{K}{\chi m}=\frac{G M_{g \oplus}}{R_{\oplus}^{3}}\left(\frac{r}{x}\right) \tag{A4}
\end{equation*}
$$

Making $x=r=R_{\oplus}$, and substituting (A4) into (A1) gives

$$
\begin{equation*}
f=\frac{1}{2 \pi} \sqrt{\frac{G M_{g \oplus \chi}}{R_{\oplus}^{3}}} \tag{A5}
\end{equation*}
$$

In the case of an electron and a positron, we substitute $M_{g \oplus}$ by $m_{g e}, \chi$ by $\chi_{e}$ and $R_{\oplus}$ by
$R_{e}$, where $R_{e}$ is the radius of electron (or positron). Thus, Eq. (A5) becomes

$$
\begin{equation*}
f=\frac{1}{2 \pi} \sqrt{\frac{G m_{g e} \chi_{e}}{R_{e}^{3}}} \tag{A6}
\end{equation*}
$$

The value of $\chi_{e}$ varies with the density of energy [3]. When the electron and the positron are distant from each other and the local density of energy is small, the value of $\chi_{e}$ becomes very close to 1 . However, when the electron and the positron are penetrating one another, the energy densities in each particle become very strong due to the proximity of their electrical charges $e$ and, consequently, the value of $\chi_{e}$ strongly increases. In order to calculate the value of $\chi_{e}$ under these conditions $\left(x=r=R_{e}\right)$, we start from the expression of correlation between electric charge $q$ and gravitational mass, obtained in a previous work [3]:

$$
\begin{equation*}
q=\sqrt{4 \pi \varepsilon_{0} G} m_{g(\text { imaginary })} i \tag{A7}
\end{equation*}
$$

where $m_{g(\text { (imaginary })}$ is the imaginary gravitational mass, and $i=\sqrt{-1}$.

In the case of electron, Eq. (A7) gives

$$
\begin{align*}
q_{e} & =\sqrt{4 \pi \varepsilon_{0} G} m_{\text {ge(imaginary })} i= \\
& =\sqrt{4 \pi \varepsilon_{0} G}\left(\chi_{e} m_{i e(\text { imaginary } i} i\right)= \\
& =\sqrt{4 \pi \varepsilon_{0} G}\left(-\chi_{e} \frac{2}{\sqrt{3}} m_{i 0 e(\text { real })} i^{2}\right)= \\
& =\sqrt{4 \pi \varepsilon_{0} G}\left(\frac{2}{\sqrt{3}} \chi_{e} m_{\text {ioe(real) }}\right)=-1.6 \times 10^{-19} \mathrm{C} \tag{A8}
\end{align*}
$$

where we obtain

$$
\begin{equation*}
\chi_{e}=-1.8 \times 10^{21} \tag{A9}
\end{equation*}
$$

This is therefore, the value of $\chi_{e}$ increased by the strong density of energy produced by the electrical charges $e$ of the two particles, under previously mentioned conditions.

Given that $m_{g e}=\chi_{e} m_{i 0 e}$, Eq. (A6) yields

$$
\begin{equation*}
f=\frac{1}{2 \pi} \sqrt{\frac{G \chi_{e}^{2} m_{i 0 e}}{R_{e}^{3}}} \tag{A10}
\end{equation*}
$$

From Quantum Mechanics, we know that

$$
h f=m_{i 0} c^{2}
$$

where $h$ is the Planck's constant. Thus, in the case of $m_{i 0}=m_{i 0 e}$ we get

$$
\begin{equation*}
f=\frac{m_{i 0 e} c^{2}}{h} \tag{A12}
\end{equation*}
$$

By comparing (A10) and (A12) we conclude that

$$
\begin{equation*}
\frac{m_{i 0 e} c^{2}}{h}=\frac{1}{2 \pi} \sqrt{\frac{G \chi_{e}^{2} m_{i 0 e}}{R_{e}^{3}}} \tag{A13}
\end{equation*}
$$

Isolating the radius $R_{e}$, we get:

$$
R_{e}=\left(\frac{G}{m_{i 0 e}}\right)^{\frac{1}{3}}\left(\frac{\chi_{e} h}{2 \pi c^{2}}\right)^{\frac{2}{3}}=6.87 \times 10^{-14} m \quad(\mathrm{Al} 4)
$$

Compare this value with the Compton sized electron, which predicts $R_{e}=3.86 \times 10^{-13} \mathrm{~m}$ and also with standardized result recently obtained of $R_{e}=4-7 \times 10^{-13} \mathrm{~m}[\underline{11]}$.

In the case of proton, we have

$$
\begin{align*}
q_{p} & =\sqrt{4 \pi \varepsilon_{0} G} m_{\text {gpp(imaginary }} i= \\
& =\sqrt{4 \pi \varepsilon_{0} G}\left(\chi_{p} m_{i 0 p(\text { imaginar } i} i\right)= \\
& =\sqrt{4 \pi \varepsilon_{0} G}\left(-\chi_{p} \frac{2}{\sqrt{3}} m_{i 0 p(\text { real })} i^{2}\right)= \\
& =\sqrt{4 \pi \varepsilon_{0} G}\left(\frac{2}{\sqrt{3}} \chi_{p} m_{i 0 p(\text { real })}\right)=-1.6 \times 10^{-19} \mathrm{C} \tag{A15}
\end{align*}
$$

$$
\begin{equation*}
\chi_{p}=-9.7 \times 10^{17} \tag{A16}
\end{equation*}
$$

Thus, the result is

$$
R_{p}=\left(\frac{G}{m_{i 0 p}}\right)^{\frac{1}{3}}\left(\frac{\chi_{p} h}{2 \pi c^{2}}\right)^{\frac{2}{3}}=3.72 \times 10^{-17} m \text { (A17) }
$$

Note that these radii, given by Equations (A14) and (A17), are the radii of free electrons and free protons (when the particle and antiparticle (in isolation) penetrate themselves mutually).

Inside the atoms (nuclei) the radius of protons is well-known. For example, protons, as the hydrogen nuclei, have a radius given by $R_{p} \cong 1.2 \times 10^{-15} \mathrm{~m}[\underline{9}, \underline{10}]$. The strong increase in respect to the value given by Eq. (A17) is due to the interaction with the electron of the atom.
where we obtain

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[^1]:    ${ }^{\dagger}$ The electrical conductivity of air, inside the dielectric tube, is equal to the electrical conductivity of Earth's atmosphere near the land, whose average value is $\sigma_{\text {air }} \cong 1 \times 10^{-14} \mathrm{~S} / \mathrm{m}$ [8].

