The Analysis of McMonigal,Lewis and O'Byrne applied to the Natario Warp Drive Spacetime

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Abstract

Warp Drives are solutions of the Einstein field equations that allows superluminal travel within the framework of General Relativity. There are at the present moment two known solutions: The Alcubierre warp drive discovered in 1994 and the Natario warp drive discovered in 2001. Recently McMonigal,Lewis and OByrne presented an important analysis for the Alcubierre warp drive:A warp drive ship at superluminal speeds in interstellar space would trap in the warp bubble all the particles and radiations the ship encounters in its pathway and these trapped bodies would achieve immense energies due to the bubble superluminal speed.As far the ship goes by in interstellar space, more and more particles and radiations are trapped in the bubble generating in front of it a blanket with extremely large amounts of positive energy. The physical consequences of having this blanket of positive energies with enormous magnitudes in front of the negative energy of the warp bubble are still unknown. When the ship finishes the trip it stops suddenly releasing in a highly energetic burst all the trapped particles and radiations contained in the blanket damaging severely the destination point. In this work,we reproduce the same analysis for the Natario warp drive however using different mathematical arguments more accessible to beginners or intermediate students and we arrived exactly to the same conclusions. While in a long-term future some of the physical problems associated to the warp drive science(negative energy,Horizons) seems to have solutions(better shape functions for the negative energy problem and a theory that encompasses both General Relativity and non local quantum entanglements of Quantum Mechanics for the Horizons problem) and we discuss these solutions in our work,the analysis of McMonigal,Lewis and OByrne although entirely correct do not have a foreseeable solution and remains the most serious obstacle against the warp drive as a physical reality

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1 Introduction:

The Warp Drive as a solution of the Einstein field equations of General Relativity that allows superluminal travel appeared first in 1994 due to the work of Alcubierre.([1]) The warp drive as conceived by Alcubierre worked with an expansion of the spacetime behind an object and contraction of the spacetime in front.The departure point is being moved away from the object and the destination point is being moved closer to the object.The object do not moves at all[1].It remains at the rest inside the so called warp bubble but an external observer would see the object passing by him at superluminal speeds(pg 8 in [1])(pg 1 in [2]).

Later on in 2001 another warp drive appeared due to the work of Natario.([2]).This do not expands or contracts spacetime but deals with the spacetime as a "strain" tensor of Fluid Mechanics(pg 5 in [2]). Imagine the object being a fish inside an aquarium and the aquarium is floating in the surface of a river but carried out by the river stream.The warp bubble in this case is the aquarium whose walls do not expand or contract. An observer in the margin of the river would see the aquarium passing by him at a large speed but inside the aquarium the fish is at the rest with respect to his local neighborhoods.

Many other works appeared on the scientific literature about the warp drive outlining the problems of large negative energy requirements([12] , [13] , [14]) Doppler blueshifts ([2], [7], [10] , [11]) and Horizons(causally disconnected portions of spacetime)([2]).

While negative energies and Horizons are considered serious problems against the development of the warp drive as a real physical science, in this work we present two long-term possible future solutions one for the Horizons problem and another for the negative energy problem although both are outside the scope of our present knowledge in physics. Our solution for the negative energy(better shape functions) appears in the Appendix A and the solution for the Horizons(possible merge between General Relativity with the non-local quantum entanglements of Quantum Mechanics) appears in Section 2.

The main reason for this work is the recent analysis by McMonigal,Lewis and OByrne where a warp drive ship in real interstellar space would trap in the warp bubble all the particles and radiations in front of it and these particles or radiations would achieve high levels of energy due to the bubble superluminal speed.When the ship finishes the journey,if the ship stops instantaneously these highly energetic particles and radiations trapped are released and could affect the destination point.(pg 10 in [8]).McMonigal,Lewis and OByrne performed the calculations for the Alcubierre warp drive. In this work we reproduce their analysis for the Natario warp drive and we arrived exactly at the same conclusions however due to a different distribution of energy the Natario warp drive behaves slightly different when compared to its Alcubierre counterpart.

In order to make the McMonigal,Lewis and OByrne analysis more accessible to an audience of wide scope,we choose to adopt in this work a language more accessible to beginners or intermediate students with already some acquaintance in warp drive science and we present in Appendices the development of the geometry of the Natario warp drive,in order to do not make an unnecessary long or heavy main text. Those already familiarized with the warp drive but without knowledge of the Natario geometry will have no difficulties browsing the Appendices.

Natario in its warp drive uses the spherical coordinates rs and θ.In order to simplify our analysis we

[1]do not violates Relativity

consider motion in the $x - axis$ where $\theta = 0$ and $\cos(\theta) = 1$.(see pgs 4,5 and 6 in [2]).

We adopted as a frame of reference the Eulerian observer inside the Natario warp bubble(co-moving ship frame observer) and we consider in our work $c = G = 1$.

2 Horizons(Causally Disconnected portions of Spacetime) and Doppler Blueshifts in the Natario Warp Drive Spacetime

The warp drive spacetime according to Natario for the coordinates rs and θ is defined by the following equation:(see Appendix E for details)

$$ds^2 = [1 - (X^{rs})^2 - (X^\theta)^2]dt^2 + 2[X^{rs}drs + X^\theta rsd\theta]dt - drs^2 - rs^2d\theta^2 \quad (1)$$

The expressions for X^{rs} and X^θ are given by:(see pg 5 in [2],see also Appendix D for details)

$$X^{rs} = -2v_s n(rs)\cos\theta \quad (2)$$

$$X^{rs} = 2v_s n(rs)\cos\theta \quad (3)$$

$$X^\theta = v_s(2n(rs) + (rs)n'(rs))\sin\theta \quad (4)$$

$$X^\theta = -v_s(2n(rs) + (rs)n'(rs))\sin\theta \quad (5)$$

$n(rs)$ is the Natario shape function being $n(rs) = \frac{1}{2}$ for large rs(outside the warp bubble) and $n(rs) = 0$ for small rs(inside the warp bubble) while being $0 < n(rs) < \frac{1}{2}$ in the walls of the warp bubble also known as the Natario warped region(pg 5 in [2])

Considering a photon being sent by an Eulerian observer in the center of the Natario bubble to the front or the rear of the bubble according to pg 6 in [2] or pg 20 and 22 in [6] we consider the photon moving parallel to the $x - axis$ then $\theta = 0$ and $cos(\theta) = 1$.Our assumption is also valid for incoming photons.moving towards the bubble from outside the bubble in the front.

$$ds^2 = [1 - (X^{rs})^2]dt^2 + 2X^{rs}drsdt - drs^2 \quad (6)$$

For a photon we have a null-like spacetime interval $ds^2 = 0$.Then we have the following result:

$$ds^2 = 0 \dashrightarrow [1 - (X^{rs})^2]dt^2 + 2X^{rs}drsdt - drs^2 = 0 \quad (7)$$

From above we can see that we have two solutions:one for the photon sent towards the front of the ship($-$ sign) and another for the photon sent to the rear of the ship($+$ sign).(see Appendix F for details)

$$U_{front} = \frac{drs_{front}}{dt} = X^{rs} - 1 \quad (8)$$

$$U_{rear} = \frac{drs_{rear}}{dt} = X^{rs} + 1 \quad (9)$$

Also from the above expressions we can easily see that according to pg 6 in [2] or pg 20 and 22 in [6]

$$U_{generic} = \frac{drs}{dt} = X^{rs} \pm 1 \quad (10)$$

$$\parallel \frac{drs}{dt} - X^{rs} \parallel = 1 \quad (11)$$

$$X^{rs} = -2v_s n(rs)\cos\theta \tag{12}$$

$$X^{rs} = 2v_s n(rs)\cos\theta \tag{13}$$

But since we are considering the motion in the $x - axis$ only or parallel to the $x - axis$ where $\theta = 0$ or $cos(\theta) = 1$ then we are left with:

$$X^{rs} = -2v_s n(rs) \tag{14}$$

$$X^{rs} = 2v_s n(rs) \tag{15}$$

Examining the case of the photon sent to the front of the bubble from inside the bubble for the Natario vector $nX = vs(t)dx$ (pgs 4 , 5 and 6 in [2]):

$$U_{front} = \frac{drs_{front}}{dt} = X^{rs} - 1 \tag{16}$$

- photon sent to the front but still inside the bubble:$n(rs) = 0$ then $X^{rs} = 0$ and therefore $\frac{drs_{front}}{dt} = -1$
- photon sent to the front but already outside the bubble:$n(rs) = \frac{1}{2}$ then $X^{rs} = vs$ and therefore $\frac{drs_{front}}{dt} = vs - 1$

According with pg 6 in [2] or pg 20 and 22 in [6] if $X^{rs} = 0$ inside the bubble and $X^{rs} = vs$ outside the bubble being $0 < X^{rs} < vs$ in the Natario warped region and assuming a continuous growth of X^{rs} from 0 to vs then before X^{rs} reaches the value of vs in a certain given point $X^{rs} = 1$ and we have:

$$U_{front} = \frac{drs_{front}}{dt} = X^{rs} - 1 = 1 - 1 = 0 \tag{17}$$

The photon stops never reaching the end of the bubble which is causally disconnected from the Eulerian observer inside the center of the bubble:An Horizon is established.

In the Horizon, events inside the bubble cannot causally influence events on the outer side of the bubble because the photon stops in this point inside the Natario warped region(pg 6 in [2]).

The photon enters the Natario warped region and cross the portion of this region where $0 < X^{rs} < 1$ stopping effectively where $X^{rs} = 1$.

Of course we consider here only Classical General Relativity where light speed ($c = 1$ or $vs = 1$ or $X^{rs} = 1$) is the maximum speed allowed to send information.

We still do not have a Quantum Gravity theory that encompasses the non-Local entanglements of Quantum Mechanics with the spacetime geometry described by General Relativity.

This issue of the Horizon is still an open question in warp drive science and we will use here a pedagogical example to illustrate this:Imagine that we have two supersonic jet planes one in front of the another but with the radios turned off due malfunction and the only thing both planes have to communicate between each other are phonon[2] machines.Initially both planes are at subsonic speeds and a phonon sent by the rear plane can reach the plane in the front so both planes are "causally connected".They have synchronized clocks and in a given time both planes passes the speed of the sound and both enters in supersonic(Mach) speeds .When this happens a phonon from the rear plane can no longer reach the plane in the front because the phonon sent by the rear plane to reach the plane in the front is outrunned by the speed of the rear plane itself.Then from the point of view of the rear plane the front plane is "causally disconnected".But a phonon sent by the front plane will reach the plane in the rear.Anyone will figure out that this is a similar situation between our pairs of photons sent to the front or the rear part of the bubble.

But we also know that $n(rs) = 0$ inside the bubble and $0 < n(rs) < \frac{1}{2}$ in the Natario warped region while being $n(rs) = \frac{1}{2}$ outside the bubble.According with Appendix A,the derivatives of $n(rs)$ vanishes inside the bubble and outside the bubble but do not vanishes in the Natario warped region keeping there a non null negative energy density.Thene we have for the motion inside the Natario warped region in the $x - axis$ the following conditions:

$$0 < n(rs) < \frac{1}{2} \tag{18}$$

$$0 < X^{rs} < vs \tag{19}$$

When the photon enters the region where $0 < X^{rs} < vs$ it also enters the region where the derivatives of $n(rs)$ do not vanish.The negative energy density "deflect" the photon before reaching the Horizon.The photon is deflected before reaching $X^{rs} = 1$ because in front of the ship the space in the Natario warp drive is not empty.See also Appendix H.

The important point here is the Appendix A:the negative energy density in front of the ship is not null and the repulsive behavior of the negative energy deflect the photon before the point of the Horizon.

The case of incoming photons in front of the Bubble moving towards the Eulerian observer in the center of the bubble can be depicted by the equation given below

$$U_{incoming} = -X^{rs} - 1 \tag{20}$$

- incoming photon approaching the bubble but still outside the bubble:$n(rs) = \frac{1}{2}$ then $X^{rs} = vs$ and therefore $\frac{drs_{front}}{dt} = -vs - 1$
- incoming photon approaching the Eulerian observer but already inside the bubble:$n(rs) = 0$ then $X^{rs} = 0$ and therefore $\frac{drs_{front}}{dt} = -1$

Remember that we are analyzing the situation from the point of view of the Eulerian observer inside the bubble(ship frame observer) watching the approach of an incoming photon from outside the bubble then the incoming photon is being seen with a relative velocity $-vs - 1$ or $-1(1 + vs)$.The minus sign depicts motion in the opposite direction of the ship since the photon is approaching the bubble from the front.

[2]equivalent to the photon but for sound waves

From the equation of energy(pg 8 in [2]) we have the following energy for the photon outside the bubble $(X^{rs} = vs)$ [3,4] :

$$E = \frac{E_0}{(1 + n.X^{rs})} = \frac{E_0}{(1 + vs)} \tag{21}$$

$$E_0 = E(1 + n.X^{rs}) = E(1 + vs) = E_{outside-the-bubble} \tag{22}$$

In the above equations E is the energy of the photon seen by a distant observer(remote frame observer) and E_0 is the energy of the photon seen by the Eulerian observer inside the bubble(ship frame observer)

Hence the Eulerian observer measures a Doppler blueshift of $1 + vs$ which means to say that he(or she) measures an amount of energy extremely high and hazardous for the crew of the ship(pg 8 in [2],pg 9 and 11 in [7],pg 10 in [8].

At 200 times light speed a single photon is Doppler blueshifted to the energy of a solar photosphere (pg 11 in [7]) and we have plenty of COBE[5,6]photons in the empty space in front of the bubble

From the equation of energy inside the Natario warp bubble we have the following situation: $(X^{rs} = 0)$

$$E_0 = E(1 + n.X^{rs}) = E(1 + 0) = E = E_{inside-the-bubble} \tag{23}$$

So the photon decreases its energy by the amount of $1 + vs$ when reaching the region inside the bubble.This amount of energy is released and some authors says that this will raise the temperature of the bubble(see pgs 6 and 7 in [11]).

$$\Delta E = E_{released} = E_{outside-the-bubble} - E_{inside-the-bubble} \tag{24}$$

$$\Delta E = E_{released} = E(1 + vs) - E = E[(1 + vs) - 1) = E \times vs \tag{25}$$

However and again recalling Appendices A and H the negative energy in front of the ship in the Natario warped region deflects the photon never reaching the region inside the bubble

This energy released is also deflected and remains outside the bubble.[7].

- incoming photon approaching the bubble reaching the outermost layers of the Natario warped region:$0 < n(rs) < \frac{1}{2}$ then $X^{rs} = 2v_s n(rs)$ and therefore $\frac{drs_{front}}{dt} = -X^{rs} - 1 = -2v_s n(rs) - 1$

The Natario shape function is given by(pg 9 eq 38 in [5]):

$$n(rs) = [\frac{1}{2}][1 - f(rs)]^{WF} \tag{26}$$

[3]energy seen by an Eulerian observer in the center of the Natario warped region-ship frame observer
[4]since the photon is approaching the bubble from the front then $n = +1$
[5]Cosmic Background Radiation left from the Big Bang
[6]this is a very important point in the McMonigal Lewis and O'Byrne analysis
[7]But remains in front of it.This will be further discussed in the end of McMonigal,Lewis and O'Byrne analysis

According with pg 10 in [5] for a $WF = 200$[8] a bubble radius $R = 100meters$ and an Alcubierre parameter @ $= 20$ and making $rs = R$ the Natario shape function have a value of $n(rs) = 3,111 \times 10^{-61}$

Considering the speed of the bubble 200 times faster than light then $vs = 6 \times 10^{10}$ and we have the following result:

$$\frac{drs_{front}}{dt} = -X^{rs} - 1 = -2v_s n(rs) - 1 = -2 \times 6 \times 10^{10} \times 3,111 \times 10^{-61} - 1 \tag{27}$$

$$\frac{drs_{front}}{dt} = -2 \times 6 \times 3,111 \times 10^{-51} - 1 \cong -36 \times 10^{-51} - 1 \cong -1 \tag{28}$$

$$E_0 = E(1 + n.X^{rs}) = E(1 + 0) = E = E_{outermost_layers_of the_Natario_warped_region} \tag{29}$$

$$\Delta E = E_{released} = E_{outside-the-bubble} - E_{outermost-layers-of-the-Natario-warped-region} \tag{30}$$

$$\Delta E = E_{released} = E(1 + vs) - E = E[(1 + vs) - 1) = E \times vs \tag{31}$$

Note that the photon reaching the outermost layers of the Natario warped region behaves almost similar to its counterpart inside the bubble where the energy of the photon do not have the factor $1 + vs$.

Note also that this form of the Natario shape function not only lowers the negative energy density requirements to "low" and "affordable" levels (pg 13 in [5])but also forces the photon to release the Doppler blueshifted energy factor $1 + vs$ before reaching the regions inside the Bubble.

Also in this case and still according with Appendices A and H,the negative energy in front of the bubble deflects the photon and the released energy will never reach the regions inside the bubble and both photon and released energy remains outside the bubble.[9]

So in the Natario warp drive spacetime the crew of the ship are not being harmed by Doppler blueshifts.

Compare both Appendix A and Appendix C.[10]

The geodesic Lagrangian for the Natario warp drive is given by the following expression(pg 2 in [2])

$$L = \frac{1}{2}\left[-\dot{t}^2 + \sum_{i=1}^{3}\left(\dot{x}^i - X^i\dot{t}\right)^2\right] \tag{32}$$

$$L = \frac{1}{2}\left[-\dot{t}^2 + \left(\dot{rs} - X^{rs}\dot{t}\right)^2 + \left(rs\dot{\theta} - X^{\theta}\dot{t}\right)^2\right] \tag{33}$$

$$\frac{d}{d\tau}\left(\frac{\partial L}{\partial \dot{t}}\right) - \frac{\partial L}{\partial t} = 0 \Leftrightarrow \frac{d}{d\tau}\left(-\dot{t} - X^i\left(\dot{x}^i - X^i\dot{t}\right)\right) + \dot{t}\frac{\partial X^i}{\partial t}\left(\dot{x}^i - X^i\dot{t}\right) = 0; \tag{34}$$

$$\frac{d}{d\tau}\left(\frac{\partial L}{\partial \dot{x}^i}\right) - \frac{\partial L}{\partial x^i} = 0 \Leftrightarrow \frac{d}{d\tau}\left(\dot{x}^i - X^i\dot{t}\right) + \dot{t}\frac{\partial X^j}{\partial x^i}\left(\dot{x}^j - X^j\dot{t}\right) = 0. \tag{35}$$

[8] the warp factor WF is a dimensionless parameter

[9] This will be further discussed in the McMonigal,Lewis and O'Byrne analysis

[10] This affects some of the final results of the McMonigal,Lewis and O'Byrne analysis but not all ones

According with pg 2 in [2] it can be easily seen that any curve satisfying $\dot{t}=1, \dot{x}^i = X^i$ is a solution of these equations.See also eq $C.5$ pg 21 in [9] with $X^i = v_s f(rs)$ and $k_1 = 1$.Changing the metric signature from $(-,+,+,+)$ to $(+,-,-,-)$ we have :

$$L = \frac{1}{2}\left[\dot{t}^2 - \sum_{i=1}^{3}\left(\dot{x}^i - X^i \dot{t}\right)^2\right] \tag{36}$$

$$L = \frac{1}{2}\left[\dot{t}^2 - \left(\dot{rs} - X^{rs}\dot{t}\right)^2 - \left(rs\dot{\theta} - X^{\theta}\dot{t}\right)^2\right] \tag{37}$$

Inserting the above Lagrangian into the Euler-Lagrange equations we can see that the conditions $\dot{t}=1, \dot{x}^i = X^i$ remains valid.

The solutions of the geodesics for the Natario warp drive according to the condition $\dot{x}^i = X^i$ are given by:

$$\dot{rs} = \frac{drs}{dt} = X^{rs} \tag{38}$$

$$rs\dot{\theta} = rs\frac{d\theta}{dt} = X^{\theta} \tag{39}$$

$$X^{rs} = -2v_s n(rs)\cos\theta \tag{40}$$

$$X^{rs} = 2v_s n(rs)\cos\theta \tag{41}$$

$$X^{\theta} = v_s(2n(rs) + (rs)n'(rs))\sin\theta \tag{42}$$

$$X^{\theta} = -v_s(2n(rs) + (rs)n'(rs))\sin\theta \tag{43}$$

But since we are analyzing the case of a particle motion in the $xaxis$ only where $\theta = 0$ and $cos(\theta) = 1$ for the Natario vector $nX = vsdx$ we are left with the following equation:

$$X^{rs} = 2v_s n(rs) \tag{44}$$

According with pg 21 in [10] a geodesics depicts the shortest distance between two points.We must now examine what happens when a Natario warp drive encounters a particle at the rest in interstellar space[11] in front of it and runs towards such a particle to face a head-on collision at a superluminal speed v_s.

This is the main point of the analysis of McMonigal,Lewis and O'Byrne.(see end of pg 10 in [8])

[11] eg space dust or debris from supernova remnants etc

3 The Analysis of McMonigal,Lewis and O'Byrne Applied to the Natario Warp Drive Spacetime

We already stated the fact that we are considering the ship frame observer in our analysis:the Eulerian observer in the center of the bubble. Remember that inside the bubble $X^{rs} = 0$ because $n(rs) = 0$ and outside the bubble $X^{rs} = v_s$ because $n(rs) = \frac{1}{2}$(see fig 2 pg 8 in [2])

The Eulerian observer in the center of the bubble is at the rest inside the bubble while the bubble moves with a speed v_s with respect to a static observer outside the bubble watching the bubble passing by him or watching the bubble moving towards him for a head-on collision.

Then for our Eulerian observer at the rest inside the bubble it is the static external observer outside the bubble that would be seen coming towards the bubble with a relative speed v_s.This is the reason why $X^{rs} = 0$ inside the bubble and $X^{rs} = v_s$ outside the bubble.These measures are taken by the Eulerian observer.

Lets explain the situation better: Consider four particles A,B,C and D.

A is our Eulerian observer inside the bubble in the center and B is also inside the bubble but at a small distance from the center.B is also an Eulerian observer.Then for A $rs = 0$[12] and for B $rs > 0$ but $rs \cong 0$.

With respect to A,B is at the rest because both are co-moving Eulerian observers inside the bubble[13].

C and D are faraway observers at the rest outside the bubble however C and D sees A and B inside the bubble passing by them with a speed v_s(bubble speed). With respect to C,D is at the rest but C is directly in front of the bubble path and will suffer a head-on collision with the bubble that is approaching him with the speed v_s.D is at a safe distance in a point perpendicular to the bubble path.

A and B at the rest inside the bubble sees C and D outside the bubble coming towards them with a relative velocity v_s.

Then the following geodesics equation depicts the physical situation of C that is running to a head-on collision with the bubble,as seen by A and B.

- point C outside the bubble as seen by A and B:

$$X^{rs} = 2v_s n(rs) \dashrightarrow X^{rs} = v_s \dashrightarrow n(rs) = \frac{1}{2} \tag{45}$$

  The result above was expected from the previous statements.

- point C reaching the outermost layers of the bubble as seen by A and B:

[12] center of the bubble where the ship resides

[13] co-moving observers with respect to C and D

According with pg 10 in [5] for a $WF = 200$[14] a bubble radius $R = 100meters$ and an Alcubierre parameter @ $= 20$ and making $rs = R$ the Natario shape function have a value of $n(rs) = 3,111 \times 10^{-61}$

Considering the speed of the bubble 200 times faster than light then $vs = 6 \times 10^{10}$ and we have the following result:

$$X^{rs} = 2v_s n(rs) \dashrightarrow X^{rs} = 2 \times 6 \times 10^{10} \times 3,111 \times 10^{-61} \quad (46)$$

$$X^{rs} = 12 \times 3,111 \times 10^{-51} \dashrightarrow X^{rs} \cong 0 \dashrightarrow n(rs) = 3,111 \times 10^{-61} \quad (47)$$

According to the result above our Eulerian observer A sees the incoming particle C stopping when the particle reaches the outermost layers of the bubble.

This result is very important:the incoming particle C outside the bubble however approaching the bubble for a head-on collision as being seen by our Eulerian observer A with an incoming speed$X^{rs} = v_s$ is now seen with a speed $X^{rs} \cong 0$ when the particle C reaches the outermost layers of the bubble. This means to say that the incoming particle C is now stationary with respect to our Eulerian observer A and also stationary with respect to the Eulerian observer B.The relative velocity v_s almost disappeared.

The incoming particle C when reaching the outermost layers of the bubble behaves almost like a particle inside the bubble with $X^{rs} = 0$.

The observer D sees exactly the contrary:C was initially at the rest with respect to D while being seen by both A and B with a relative speed v_s.Now A and B sees C with almost a null relative speed.With respect to A and B C is almost not moving after all.But A and B inside the bubble are being seen by D outside the bubble with a bubble velocity v_s.This means to say that when a bubble approaches the particle C at the rest with respect to the observer D and the bubble comes with a bubble velocity v_s approaching to a head-on collision with the particle C,C gets itself trapped or caught in the Natario warped region and achieves the speed v_s as seen by the observer D.C achieves the same speed v_s of A and B.This is the reason why the relative speed of C with respect to A and B disappears.The Natario warped region acts like a "jet stream" and drags the particle C with it.

This is the main result of the McMonigal,Lewis and O'Byrne Analysis:particles initially at the rest are caught by the warp bubble as a "jet stream" and achieves the bubble velocity v_s.See pg 10 in [8].

Although we used the Natario warp drive and not the Alcubierre one as McMonigal,Lewis and O'Byrne did ,their analysis is entirely correct.

[14]the warp factor WF is a dimensionless parameter

From the Horizons section we can recall the following equations where we can see that a big amount of energy is released outside the bubble near to the outermost layers of the Natario warped region:

$$E_0 = E(1 + n.X^{rs}) = E(1 + vs) = E_{outside-the-bubble} \tag{48}$$

$$E_0 = E(1 + n.X^{rs}) = E(1 + 0) = E = E_{outermost-layers-of-the-Natario-warped-region} \tag{49}$$

$$\Delta E = E_{released} = E_{outside-the-bubble} - E_{outermost-layers-of-the-Natario-warped-region} \tag{50}$$

$$\Delta E = E_{released} = E(1 + vs) - E = E[(1 + vs) - 1) = E \times vs \tag{51}$$

Note that this energy released is directly proportional to vs.

Some authors says that this will raise the temperature of the bubble(see pgs 6 and 7 in [11]).In this case the amount of released energy remains outside the bubble never reaching the inside region.Again we recall the Appendices A and H

Remember that these equations were computed for the Eulerian observer A or B inside the bubble.

The point of view of photons coming to impact the bubble from the front as seen by C or D is very different:

Imagine that the observer C watching the bubble coming towards him however outside the bubble at the rest as seen by observer D send a photon in the direction of the bubble. Initially both observers C and D measures the photon with the energy E(see Appendix G for details).When the photon reaches the outermost layers of the bubble it inverts the direction of motion and now approaches the observer C with the energy $E(1 + vs)$.

This is a very important point in the McMonigal,Lewis and O'Byrne analysis:while in Alcubierre warp drive there are no negative energy in front to deflect photons and these Doppler blueshifted will harm the crew of a spaceship(pg 10 in [8])(see Appendix C),in the Natario case the photons do not reach the regions inside the bubble due to the presence of the negative energy in front and forms a highly energetic blanket in the outermost layers of the warp bubble.(see Appendices A and H).

This energetic blanket will add its influence to the rest particles caught by the motion of the warp bubble

If the bubble stops suddenly we have no difficulties to see that these particles and radiations will harm the destination point and this is the main result of the McMonigal,Lewis and OByrne work.

In an interstellar trip at 200 times light speed for a star at 20 light-years away some months are required(see abs of [5]).The ship cannot "drop out of warp" in the destination point at once otherwise the "planet" would certainly be affected.

A safe way to do the trip is the following:the ship starts the journey at subluminal speed,leaves the solar system and when already in interstellar space the ship "engages" the "warp drive" in order to achieve superluminal velocity.When approaching the destination point but still in interstellar space,the ship starts to decrease the value of vs but smoothly and slowly in order to dissipate this excess of energy and particles "dropping out of warp" in interstellar space but at a safe distance from the system being visited and terminates again its journey entering in the system with subluminal speeds.

Remember that both Alcubierre and Natario warp drives were constructed to work with a static speed vs.A dynamical speed being changed in time would affect the form of the spacetime metric for both models,affecting Christoffel symbois,Riemann,Ricci and Einstein tensors.

The Natario vector in this case would not be $nX = vdx$,it would be $nX = d(vsx) = vsdx + xdvs$

This is about to be done in a future work.

4 Conclusion:

In this work we demonstrated the analysis of McMonigal Lewis and OByrne applied to the Natario warp drive spacetime using a language that we believe is more accessible to beginners or intermediate students and we arrived exactly at the same conclusions.(see pg 10 in [8]).

The conclusion is the following:while for the warp drive science the problems of negative energies and Horizons seems to have solution in a long-term period (better shape functions and/or a theory that encompasses both General Relativity and the non-local quantum entanglements of Quantum Mechanics),the problem raised by McMonigal Lewis and OByrne remains without a foreseeable solution.

We mentioned a trip to a star at 20 light-years away with a speed 200 times faster than light:How many photons of COBE[15] radiation are scattered across each cubic centimeter of space waiting to be trapped by the blanket of photons in front of the outermost layers of the Natario warp bubble each one of these photons carrying the energy of a solar photosphere???.(see pg 11 in [7]).And how many cubic centimeters of space exists in 20 light-years of distance???? This blanket of photons and trapped particles in the outermost layers of the Natario warped region tend to grow and becomes more dense and more energetic as far the ship goes by.In the middle of the trip at 10 light-years of distance all the photons or particles trapped in the ship pathway would make a blanket of immense density and energy and we still do not know the physical consequences of these concentrations of high positive energies in front of the ship and how this would affect the behavior of the negative energy in the Natario warped region.

This problem as far as we know do not have a foreseeable solution right now but we are optimistic that in the future with a better theory of gravity (Quantum Gravity) the solution will arise.

[15]Cosmic Background Radiation left from the Big Bang

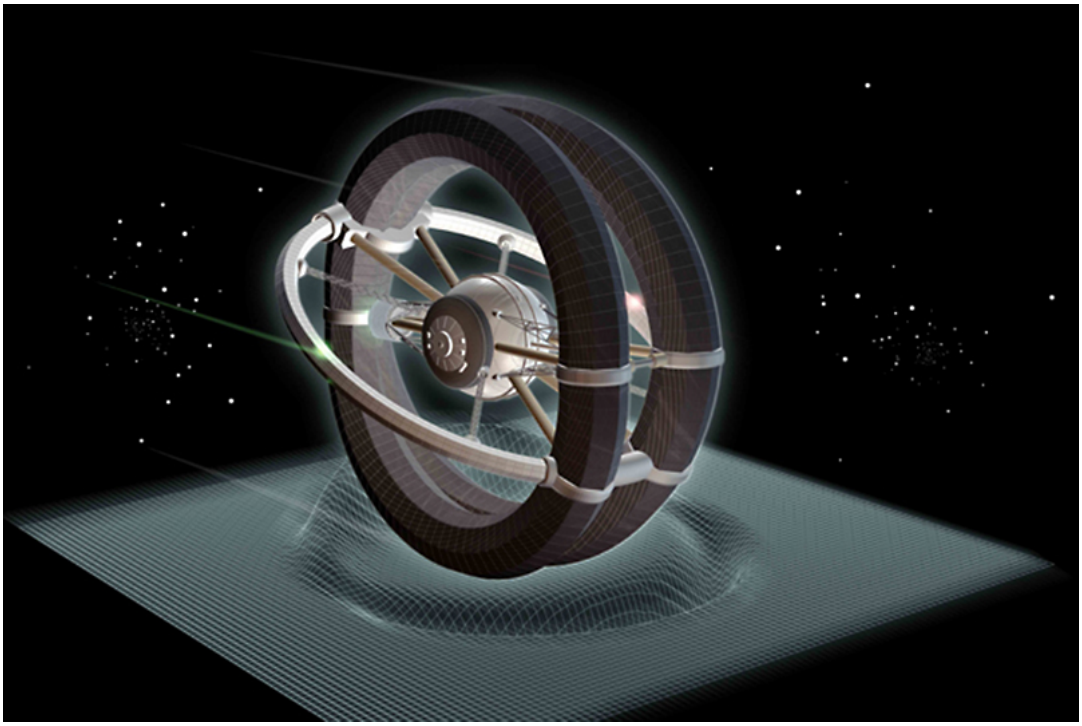

Figure 1: Artistic representation of the Natario Warp Drive .Note the Alcubierre Expansion of the Normal Volume Elements below.(Source:Internet)

5 Appendix A:Artistic Presentation of the Natario Warp Drive

Note that according to the geometry of the Natario warp drive the spacetime contraction in one direction(radial) is balanced by the spacetime expansion in the remaining direction(perpendicular).

Remember also that the expansion of the normal volume elements in the Natario warp drive is given by the following expressions(pg 5 in [2]). :

$$K_{rr} = \frac{\partial X^r}{\partial r} = -2v_s n'(r)\cos\theta \tag{52}$$

$$K_{\theta\theta} = \frac{1}{r}\frac{\partial X^\theta}{\partial\theta} + \frac{X^r}{r} = v_s n'(r)\cos\theta; \tag{53}$$

$$K_{\varphi\varphi} = \frac{1}{r\sin\theta}\frac{\partial X^\varphi}{\partial\varphi} + \frac{X^r}{r} + \frac{X^\theta\cot\theta}{r} = v_s n'(r)\cos\theta \tag{54}$$

$$\theta = K_{rr} + K_{\theta\theta} + K_{\varphi\varphi} = 0 \tag{55}$$

If we expand the radial direction the perpendicular direction contracts to keep the expansion of the normal volume elements equal to zero.

This figure is a pedagogical example of the graphical presentarion of the Natario warp drive.

The "bars" in the figure were included to illustrate how the expansion in one direction can be counterbalanced by the contraction in the other directions.These "bars" keeps the expansion of the normal volume elements in the Natario warp drive equal to zero.

Note also that the graphical presentation of the Alcubierre warp drive expansion of the normal volume elements according to fig 1 pg 10 in [1] is also included

Note also that the energy density in the Natario Warp Drive being given by the following expressions(pg 5 in [2]):

$$\rho = -\frac{1}{16\pi} K_{ij} K^{ij} = -\frac{v_s^2}{8\pi} \left[3(n'(r))^2 \cos^2\theta + \left(n'(r) + \frac{r}{2} n''(r) \right)^2 \sin^2\theta \right] . \tag{56}$$

$$\rho = -\frac{1}{16\pi} K_{ij} K^{ij} = -\frac{v_s^2}{8\pi} \left[3(\frac{dn(r)}{dr})^2 \cos^2\theta + \left(\frac{dn(r)}{dr} + \frac{r}{2} \frac{d^2 n(r)}{dr^2} \right)^2 \sin^2\theta \right] . \tag{57}$$

Is being distributed around all the space involving the ship(above the ship $\sin\theta = 1$ and $\cos\theta = 0$ while in front of the ship $\sin\theta = 0$ and $\cos\theta = 1$).The negative energy in front of the ship "deflect" photons so these will not reach the Horizon and the Natario warp drive will not suffer from Doppler blueshifts.The illustrated "bars" are the obstacles that deflects photons or incoming particles from outside the bubble never allowing these to reach the interior of the bubble

-)-Energy directly above the ship($y - axis$)

$$\rho = -\frac{1}{16\pi} K_{ij} K^{ij} = -\frac{v_s^2}{8\pi} \left[\left(\frac{dn(r)}{dr} + \frac{r}{2} \frac{d^2 n(r)}{dr^2} \right)^2 \sin^2\theta \right] . \tag{58}$$

-)-Energy directly in front of the ship($x - axis$)

$$\rho = -\frac{1}{16\pi} K_{ij} K^{ij} = -\frac{v_s^2}{8\pi} \left[3(\frac{dn(r)}{dr})^2 \cos^2\theta \right] . \tag{59}$$

Note that as fast as the ship goes by,then the negative energy density requirements grows proportionally to the square of the bubble speed vs.

For a bubble speed 200 times faster than light then ($vs = 6 \times 10^{10}$) and this affects the negative energy requirements by the factor vs^2 being ($vs^2 = 3,6 \times 10^{21}$) [16]

This raises the amount of negative energy to enormous levels making this a major concern when studying warp drive spacetimes.

The derivatives of the Natario shape function $n(rs)$ must be very low in order to obliterate the factor vs^2.

[16] the mass of the Earth according to Wikipedia is $M_\oplus = 5,9722 \times 10^{24}$ kilograms.In tons this reaches 10^{21} tons,exactly the factor vs^2 for 200 times light speed

The following Natario shape function(see pg 13 in [5]) can reduce these negative energy density requirements to "low" and "affordable" levels.

$$n(r) = [\frac{1}{2}][1 - f(r)]^{WF} \tag{60}$$

According with pg 10 in [5] for a $WF = 200$[17] a bubble radius $R = 100 meters$ and an Alcubierre parameter $@ = 20$ and making $rs = r$ the Natario shape function have a value of $n(rs) = 3,111 \times 10^{-61}$ and according with pg 12 in [5] the square of the first derivative of the Natario shape function have the value of $n'(r)^2 = 1,55 \times 10^{-114}$

This is more than enough to obliterate the factor 10^{21} arising from a bubble speed vs of 200 times light speed.

Note that in pg 10 in [13] Ford and Pfenning computes the total negative energy of a warp drive of being 10 times the total energy of the visible Universe but they also shows how to drop from this huge amount to a quarter of a solar mass by changing the form of the shape function

[17] the warp factor WF is a dimensionless parameter

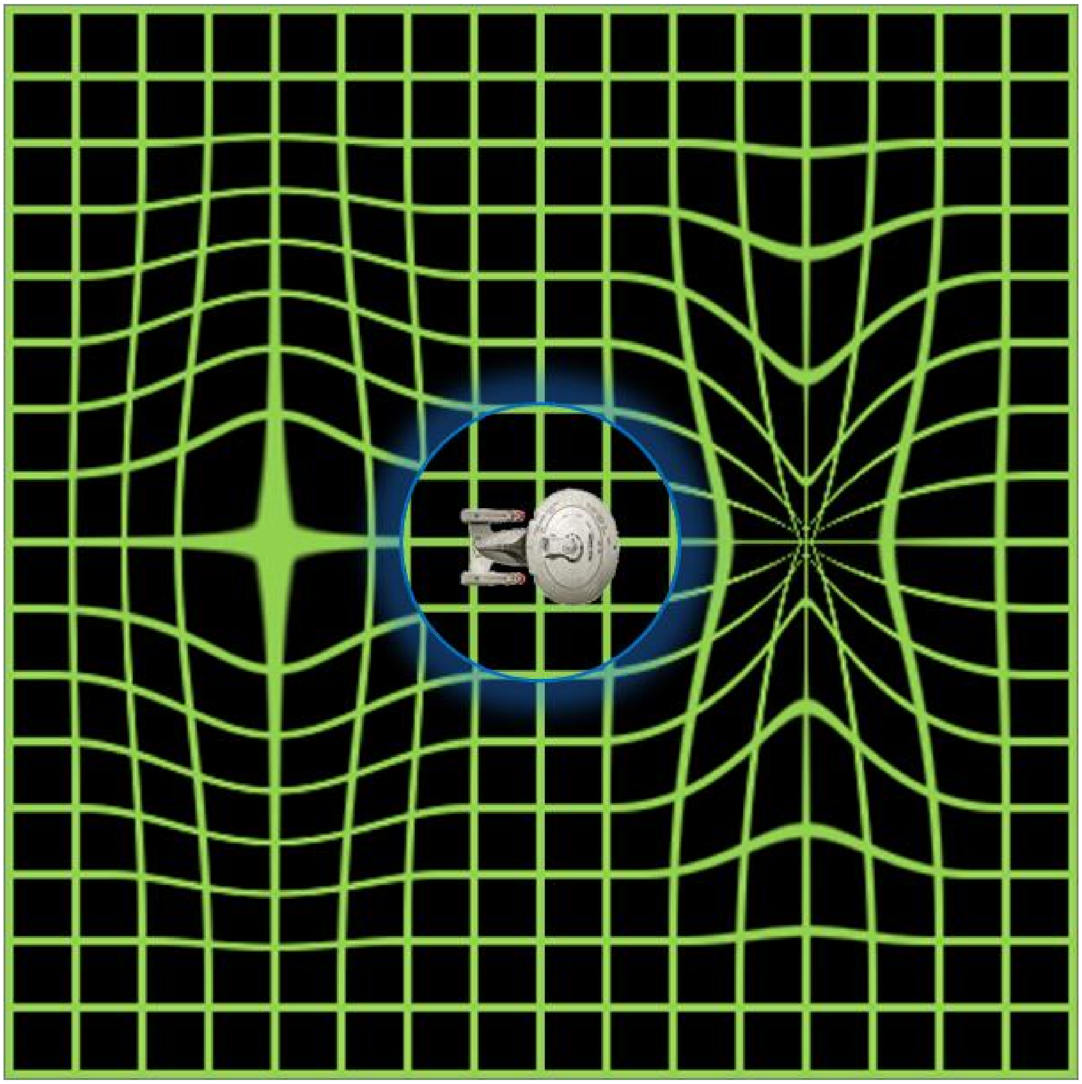

Figure 2: Artistic representation of the Natario Warp Bubble .(Source:Internet)

6 Appendix B:Artistic Presentation of the Natario Warp Bubble

According to the Natario definition for the warp drive using the following statement(pg 4 in [2]):

- 1)-Any Natario vector nX generates a warp drive spacetime if $nX = 0$ and $X = vs = 0$ for a small value of rs defined by Natario as the interior of the bubble and $nX = -vs(t)dx$ or $nX = vs(t)dx$ with $X = vs$ for a large value of rs defined by Natario as the exterior of the bubble with $vs(t)$ being the speed of the bubble(pg 5 in [2]).The blue region is the Natario warped region(bubble walls)

A given Natario vector nX generates a Natario warp drive Spacetime if and only if satisfies these conditions stated below:

- 1)-A Natario vector nX being $nX = 0$ for a small value of rs(interior of the bubble)
- 2)-A Natario vector $nX = -Xdx$ or $nX = Xdx$ for a large value of rs(exterior of the bubble)
- 3)-A shift vector X depicting the speed of the bubble being $X = 0$(interior of the bubble) while $X = vs$ seen by distant observers(exterior of the bubble).

The Natario vector nX is given by:

$$nX = -v_s(t)d\left[n(rs)rs^2\sin^2\theta d\varphi\right] \sim -2v_s n(rs)\cos\theta drs + v_s(2n(rs) + rsn'(rs))rs\sin\theta d\theta \tag{61}$$

This holds true if we set for the Natario vector nX a continuous Natario shape function being $n(rs) = \frac{1}{2}$ for large rs(outside the bubble) and $n(rs) = 0$ for small rs(inside the bubble) while being $0 < n(rs) < \frac{1}{2}$ in the walls of the bubble(pg 5 in [2])

The Natario vector $nX = -vs(t)dx = 0$ vanishes inside the bubble because inside the bubble there are no motion at all because $dx = 0$ or $n(rs) = 0$ or $X = 0$ while being $nX = -vs(t)dx \neq 0$ not vanishing outside the bubble because $n(rs)$ do not vanish.Then an external observer would see the bubble passing by him with a speed defined by the shift vector $X = -vs(t)$ or $X = vs(t)$.

The "spaceship" above lies in the interior of the bubble at the rest $X = vs = 0$ but the observer outside the bubble sees the "spaceship" passing by him with a speed $X = vs$.

See also pgs 14,15 and 16 in [16] and pgs 7,8 and 9 in [2] for more graphical presentations of the Natario warp bubble

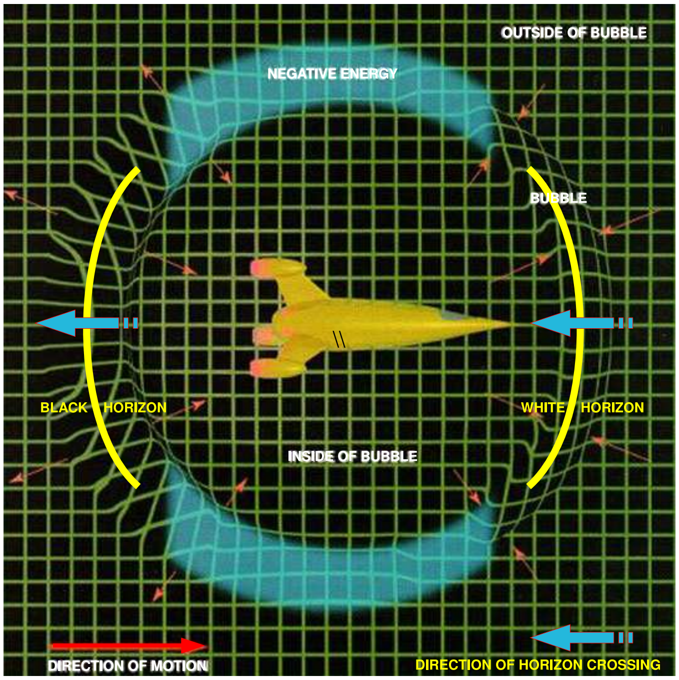

Figure 3: Artistic representation of the Energy Density distribution in the Alcubierre Warp Drive . (Source:fig 2 pg 4 in [11])

7 Appendix C:Artistic Presentation of the Energy Density distribution in the Alcubierre Warp Drive

Above is being presented the artistic graphical presentation of the energy density for the Alcubierre warp drive given by the following expressions(pg 4 in [2])(pg 8 in [1])(eq 140 pg 28 in [12])(eq 5.8 pg 75 in [14])(eq 8 pg 6 in [13]):

$$\rho = -\frac{1}{32\pi} v_s^2 \left[f'(r_s)\right]^2 [\frac{y^2+z^2}{r_s^2}] \tag{62}$$

$$\rho = -\frac{1}{32\pi} v_s^2 \left[\frac{df(rs)}{drs}\right]^2 [\frac{y^2+z^2}{r_s^2}] \tag{63}$$

Note that the negative energy density is located on a toroidal region above and below the ship and perpendicular to the direction of motion due to the term $y^2 + z^2 \neq 0$.The front of the ship is "empty" space where the contraction of spacetime in front occurs. There are no negative energy densities in front of the ship in the Alcubierre warp drive

In front of the ship in the $x - axis$ only we have $y^2 + z^2 = 0$ and then the Alcubierre energy density will have the following value:

$$\rho = -\frac{1}{32\pi} v_s^2 \left[\frac{df(rs)}{drs}\right]^2 [\frac{y^2+z^2}{r_s^2}] = 0 \tag{64}$$

Note that an observer stationary inside an Alcubierre warp drive that moves with a superluminal speed $vs > 1$ with respect to the rest of the Universe[18] will suffer from the Horizons and Doppler blueshift problems because there are no negative energies in front of the ship to "deflect" photons.This is the reason why the Hawking temperatures that compromises the physical stability of the Alcubierre warp drive(see pgs 6 and 7 in [11]) cannot be avoided .

Although Natario and Alcubierre warp drive spacetimes are members of the same family of the Einstein Field Equations of General Relativity,due to the different geometrical distribution of the negative energy densities ,one(Natario) will assume a different behavior when compared to the other(Alcubierre).

[18]Observer at rest inside the bubble $X = 0$ while outside the bubble $X = vs$

8 Appendix D:Differential Forms,Hodge Star and the Mathematical Demonstration of the Natario Vectors $nX = -vsdx$ and $nX = vsdx$ for a constant speed vs

This appendix is being written for novice or newcomer students on Warp Drive theory still not acquainted with the methods Natario used to arrive at the final expression of the Natario Vector nX

The Canonical Basis of the Hodge Star in spherical coordinates can be defined as follows(pg 4 in [2]):

$$e_r \equiv \frac{\partial}{\partial r} \sim dr \sim (rd\theta) \wedge (r\sin\theta d\varphi) \sim r^2 \sin\theta (d\theta \wedge d\varphi) \tag{65}$$

$$e_\theta \equiv \frac{1}{r}\frac{\partial}{\partial \theta} \sim rd\theta \sim (r\sin\theta d\varphi) \wedge dr \sim r\sin\theta (d\varphi \wedge dr) \tag{66}$$

$$e_\varphi \equiv \frac{1}{r\sin\theta}\frac{\partial}{\partial \varphi} \sim r\sin\theta d\varphi \sim dr \wedge (rd\theta) \sim r(dr \wedge d\theta) \tag{67}$$

From above we get the following results

$$dr \sim r^2 \sin\theta (d\theta \wedge d\varphi) \tag{68}$$

$$rd\theta \sim r\sin\theta (d\varphi \wedge dr) \tag{69}$$

$$r\sin\theta d\varphi \sim r(dr \wedge d\theta) \tag{70}$$

Note that this expression matches the common definition of the Hodge Star operator * applied to the spherical coordinates as given by(pg 8 in [4]):

$$*dr = r^2 \sin\theta (d\theta \wedge d\varphi) \tag{71}$$

$$*rd\theta = r\sin\theta (d\varphi \wedge dr) \tag{72}$$

$$*r\sin\theta d\varphi = r(dr \wedge d\theta) \tag{73}$$

Back again to the Natario equivalence between spherical and cartezian coordinates(pg 5 in [2]):

$$\frac{\partial}{\partial x} \sim dx = d(r\cos\theta) = \cos\theta dr - r\sin\theta d\theta \sim r^2 \sin\theta\cos\theta d\theta \wedge d\varphi + r\sin^2\theta dr \wedge d\varphi = d\left(\frac{1}{2}r^2\sin^2\theta d\varphi\right) \tag{74}$$

Look that

$$dx = d(r\cos\theta) = \cos\theta dr - r\sin\theta d\theta \tag{75}$$

Or

$$dx = d(r\cos\theta) = \cos\theta dr - \sin\theta rd\theta \tag{76}$$

Applying the Hodge Star operator * to the above expression:

$$*dx = *d(r\cos\theta) = \cos\theta(*dr) - \sin\theta(*rd\theta) \tag{77}$$

$$*dx = *d(r\cos\theta) = \cos\theta[r^2\sin\theta(d\theta\wedge d\varphi)] - \sin\theta[r\sin\theta(d\varphi\wedge dr)] \tag{78}$$

$$*dx = *d(r\cos\theta) = [r^2\sin\theta\cos\theta(d\theta\wedge d\varphi)] - [r\sin^2\theta(d\varphi\wedge dr)] \tag{79}$$

We know that the following expression holds true(see pg 9 in [3]):

$$d\varphi\wedge dr = -dr\wedge d\varphi \tag{80}$$

Then we have

$$*dx = *d(r\cos\theta) = [r^2\sin\theta\cos\theta(d\theta\wedge d\varphi)] + [r\sin^2\theta(dr\wedge d\varphi)] \tag{81}$$

And the above expression matches exactly the term obtained by Natario using the Hodge Star operator applied to the equivalence between cartezian and spherical coordinates(pg 5 in [2]).

Now examining the expression:

$$d\left(\frac{1}{2}r^2\sin^2\theta d\varphi\right) \tag{82}$$

We must also apply the Hodge Star operator to the expression above

And then we have:

$$*d\left(\frac{1}{2}r^2\sin^2\theta d\varphi\right) \tag{83}$$

$$*d\left(\frac{1}{2}r^2\sin^2\theta d\varphi\right) \sim \frac{1}{2}r^2 * d[(\sin^2\theta)d\varphi] + \frac{1}{2}\sin^2\theta * [d(r^2)d\varphi] + \frac{1}{2}r^2\sin^2\theta * d[(d\varphi)] \tag{84}$$

According to pg 10 in [3] the term $\frac{1}{2}r^2\sin^2\theta * d[(d\varphi)] = 0$

This leaves us with:

$$\frac{1}{2}r^2 * d[(\sin^2\theta)d\varphi] + \frac{1}{2}\sin^2\theta * [d(r^2)d\varphi] \sim \frac{1}{2}r^2 2\sin\theta\cos\theta(d\theta\wedge d\varphi) + \frac{1}{2}\sin^2\theta 2r(dr\wedge d\varphi) \tag{85}$$

Because and according to pg 10 in [3]:

$$d(\alpha+\beta) = d\alpha + d\beta \tag{86}$$

$$d(f\alpha) = df\wedge\alpha + f\wedge d\alpha \tag{87}$$

$$d(dx) = d(dy) = d(dz) = 0 \tag{88}$$

From above we can see for example that

$$*d[(\sin^2\theta)d\varphi] = d(\sin^2\theta)\wedge d\varphi + \sin^2\theta \wedge dd\varphi = 2sin\theta\cos\theta(d\theta\wedge d\varphi) \tag{89}$$

$$*[d(r^2)d\varphi] = 2rdr\wedge d\varphi + r^2\wedge dd\varphi = 2r(dr\wedge d\varphi) \tag{90}$$

And then we derived again the Natario result of pg 5 in [2]

$$r^2\sin\theta\cos\theta(d\theta\wedge d\varphi) + r\sin^2\theta(dr\wedge d\varphi) \tag{91}$$

Now we will examine the following expression equivalent to the one of Natario pg 5 in [2] except that we replaced $\frac{1}{2}$ by the function $f(r)$:

$$*d[f(r)r^2\sin^2\theta d\varphi] \tag{92}$$

From above we can obtain the next expressions

$$f(r)r^2 * d[(\sin^2\theta)d\varphi] + f(r)\sin^2\theta * [d(r^2)d\varphi] + r^2\sin^2\theta * d[f(r)d\varphi] \tag{93}$$

$$f(r)r^2 2sin\theta\cos\theta(d\theta\wedge d\varphi) + f(r)\sin^2\theta 2r(dr\wedge d\varphi) + r^2\sin^2\theta f'(r)(dr\wedge d\varphi) \tag{94}$$

$$2f(r)r^2 sin\theta\cos\theta(d\theta\wedge d\varphi) + 2f(r)r\sin^2\theta(dr\wedge d\varphi) + r^2\sin^2\theta f'(r)(dr\wedge d\varphi) \tag{95}$$

Comparing the above expressions with the Natario definitions of pg 4 in [2]):

$$e_r \equiv \frac{\partial}{\partial r} \sim dr \sim (rd\theta)\wedge(r\sin\theta d\varphi) \sim r^2\sin\theta(d\theta\wedge d\varphi) \tag{96}$$

$$e_\theta \equiv \frac{1}{r}\frac{\partial}{\partial\theta} \sim rd\theta \sim (r\sin\theta d\varphi)\wedge dr \sim r\sin\theta(d\varphi\wedge dr) \sim -r\sin\theta(dr\wedge d\varphi) \tag{97}$$

$$e_\varphi \equiv \frac{1}{r\sin\theta}\frac{\partial}{\partial\varphi} \sim r\sin\theta d\varphi \sim dr\wedge(rd\theta) \sim r(dr\wedge d\theta) \tag{98}$$

We can obtain the following result:

$$2f(r)\ cos\theta[r^2 sin\theta(d\theta\wedge d\varphi)] + 2f(r)\ sin\theta[r\sin\theta(dr\wedge d\varphi)] + f'(r)r\sin\theta[r\sin\theta(dr\wedge d\varphi)] \tag{99}$$

$$2f(r)\ cos\theta e_r - 2f(r)\ sin\theta e_\theta - rf'(r)\sin\theta e_\theta \tag{100}$$

$$*d[f(r)r^2\sin^2\theta d\varphi] = 2f(r)\ cos\theta e_r - [2f(r) + rf'(r)]\sin\theta e_\theta \tag{101}$$

Defining the Natario Vector as in pg 5 in [2] with the Hodge Star operator * explicitly written :

$$nX = vs(t) * d\left(f(r)r^2\sin^2\theta d\varphi\right) \tag{102}$$

$$nX = -vs(t) * d\left(f(r)r^2\sin^2\theta d\varphi\right) \tag{103}$$

We can get finally the latest expressions for the Natario Vector nX also shown in pg 5 in [2]

$$nX = 2vs(t)f(r)\ cos\theta e_r - vs(t)[2f(r) + rf'(r)]\sin\theta e_\theta \tag{104}$$

$$nX = -2vs(t)f(r)\ cos\theta e_r + vs(t)[2f(r) + rf'(r)]\sin\theta e_\theta \tag{105}$$

With our pedagogical approaches

$$nX = 2vs(t)f(r)\ cos\theta dr - vs(t)[2f(r) + rf'(r)]r\sin\theta d\theta \tag{106}$$

$$nX = -2vs(t)f(r)\ cos\theta dr + vs(t)[2f(r) + rf'(r)]r\sin\theta d\theta \tag{107}$$

9 Appendix E:Mathematical Demonstration of the Natario Warp Drive Equation for a constant speed vs

The warp drive spacetime according to Natario is defined by the following equation but we changed the metric signature from $(-,+,+,+)$ to $(+,-,-,-)$(pg 2 in [2])

$$ds^2 = dt^2 - \sum_{i=1}^{3}(dx^i - X^i dt)^2 \tag{108}$$

where X^i is the so-called shift vector.This shift vector is the responsible for the warp drive behavior defined as follows(pg 2 in [2]):

$$X^i = X, Y, Z \curvearrowright i = 1, 2, 3 \tag{109}$$

The warp drive spacetime is completely generated by the Natario vector nX(pg 2 in [2])

$$nX = X^i \frac{\partial}{\partial x^i} = X\frac{\partial}{\partial x} + Y\frac{\partial}{\partial y} + Z\frac{\partial}{\partial z}, \tag{110}$$

Defined using the canonical basis of the Hodge Star in spherical coordinates as follows(pg 4 in [2]):

$$e_r \equiv \frac{\partial}{\partial r} \sim dr \sim (rd\theta) \wedge (r\sin\theta d\varphi) \tag{111}$$

$$e_\theta \equiv \frac{1}{r}\frac{\partial}{\partial\theta} \sim rd\theta \sim (r\sin\theta d\varphi) \wedge dr \tag{112}$$

$$e_\varphi \equiv \frac{1}{r\sin\theta}\frac{\partial}{\partial\varphi} \sim r\sin\theta d\varphi \sim dr \wedge (rd\theta) \tag{113}$$

Redefining the Natario vector nX as being the rate-of-strain tensor of fluid mechanics as shown below(pg 5 in [2]):

$$nX = X^r \mathbf{e}_r + X^\theta \mathbf{e}_\theta + X^\varphi \mathbf{e}_\varphi \tag{114}$$

$$nX = X^r dr + X^\theta r d\theta + X^\varphi r\sin\theta d\varphi \tag{115}$$

$$ds^2 = dt^2 - \sum_{i=1}^{3}(dx^i - X^i dt)^2 \tag{116}$$

$$X^i = r, \theta, \varphi \curvearrowright i = 1, 2, 3 \tag{117}$$

We are interested only in the coordinates r and θ according to pg 5 in [2])

$$ds^2 = dt^2 - (dr - X^r dt)^2 - (rd\theta - X^\theta dt)^2 \tag{118}$$

$$(dr - X^r dt)^2 = dr^2 - 2X^r dr dt + (X^r)^2 dt^2 \tag{119}$$

$$(rd\theta - X^{\theta}dt)^2 = r^2 d\theta^2 - 2X^{\theta} r d\theta dt + (X^{\theta})^2 dt^2 \tag{120}$$

$$ds^2 = dt^2 - (X^r)^2 dt^2 - (X^{\theta})^2 dt^2 + 2X^r dr dt + 2X^{\theta} r d\theta dt - dr^2 - r^2 d\theta^2 \tag{121}$$

$$ds^2 = [1 - (X^r)^2 - (X^{\theta})^2]dt^2 + 2[X^r dr + X^{\theta} r d\theta]dt - dr^2 - r^2 d\theta^2 \tag{122}$$

making $r = rs$ we have the Natario warp drive equation:

$$ds^2 = [1 - (X^{rs})^2 - (X^{\theta})^2]dt^2 + 2[X^{rs} drs + X^{\theta} rs d\theta]dt - drs^2 - rs^2 d\theta^2 \tag{123}$$

According with the Natario definition for the warp drive using the following statement(pg 4 in [2]):any Natario vector nX generates a warp drive spacetime if $nX = 0$ and $X = vs = 0$ for a small value of rs defined by Natario as the interior of the warp bubble and $nX = -vs(t)dx$ or $nX = vs(t)dx$ with $X = vs$ for a large value of rs defined by Natario as the exterior of the warp bubble with $vs(t)$ being the speed of the warp bubble

The expressions for X^{rs} and X^{θ} are given by:(see pg 5 in [2])

$$nX \sim -2v_s n(rs)\cos\theta \mathbf{e}_{rs} + v_s(2n(rs) + (rs)n'(rs))\sin\theta \mathbf{e}_{\theta} \tag{124}$$

$$nX \sim 2v_s n(rs)\cos\theta \mathbf{e}_{rs} - v_s(2n(rs) + (rs)n'(rs))\sin\theta \mathbf{e}_{\theta} \tag{125}$$

$$nX \sim -2v_s n(rs)\cos\theta drs + v_s(2n(rs) + (rs)n'(rs))\sin\theta rs d\theta \tag{126}$$

$$nX \sim 2v_s n(rs)\cos\theta drs - v_s(2n(rs) + (rs)n'(rs))\sin\theta rs d\theta \tag{127}$$

But we already know that the Natario vector nX is defined by(pg 2 in [2]):

$$nX = X^{rs} drs + X^{\theta} rs d\theta \tag{128}$$

Hence we should expect for:

$$X^{rs} = -2v_s n(rs)\cos\theta \tag{129}$$

$$X^{rs} = 2v_s n(rs)\cos\theta \tag{130}$$

$$X^{\theta} = v_s(2n(rs) + (rs)n'(rs))\sin\theta \tag{131}$$

$$X^{\theta} = -v_s(2n(rs) + (rs)n'(rs))\sin\theta \tag{132}$$

10 Appendix F:Solution of the Quadratic Form $ds^2 = 0$

$$ds^2 = [1 - (X^{rs})^2]dt^2 + 2X^{rs}drsdt - drs^2 \tag{133}$$

$$ds^2 = 0 \dashrightarrow [1 - (X^{rs})^2]dt^2 + 2X^{rs}drsdt - drs^2 = 0 \tag{134}$$

$$[1 - (X^{rs})^2] + 2X^{rs}\frac{drs}{dt} - (\frac{drs}{dt})^2 = 0 \tag{135}$$

$$U = \frac{drs}{dt} \tag{136}$$

$$[1 - (X^{rs})^2] + 2X^{rs}U - U^2 = 0 \tag{137}$$

$$U^2 - 2X^{rs}U - [1 - (X^{rs})^2] = 0 \tag{138}$$

$$U = \frac{2X^{rs} \pm \sqrt{4(X^{rs})^2 + 4[1 - (X^{rs})^2]}}{2} \tag{139}$$

$$U = \frac{2X^{rs} \pm 2}{2} \tag{140}$$

$$U = \frac{drs}{dt} = X^{rs} \pm 1 \tag{141}$$

From above we can see that we have two solutions:one for the photon sent towards the front of the ship($-$ sign) and another for the photon sent to the rear of the ship($+$ sign).

$$U_{front} = \frac{drs_{front}}{dt} = X^{rs} - 1 \tag{142}$$

$$U_{rear} = \frac{drs_{rear}}{dt} = X^{rs} + 1 \tag{143}$$

11 Appendix G:The Doppler-Fizeau Equation and the observers C and D outside the Natario Warp Bubble

Using the Classical Doppler-Fizeau formula[19]

$$f = f_0 \frac{c + va}{c - vb} \tag{144}$$

The terms above are:

- 1)-f is the photon frequency seen by an observer
- 2)-f_0 is the original frequency of the emitted photon
- 3)-c is the light speed.in our case $c = 1$
- 4)-va is the speed of the light source approaching the observer.
- 5)-vb in the speed of the light source moving away from the observer.

When the observer C send a photon to the front of the bubble the photon is moving away from him.Since the photon is moving away from the observer C then $va = 0$.In our case $vb = 0$ because observer c as the source of the emitted photon is stationary.Hence we can rewrite the equation as:

$$f = f_0 \frac{1 + va}{1 - vb} = f_0 \frac{1 + 0}{1 - 0} = f_0 \tag{145}$$

However in the Natario case when the photon impacts the outermost layers of the bubble it is deflected by the negative energy in the front never reaching the innermost layers.The photon inverts the direction of motion[20] and starts to approach the observer C with the known Doppler factor $1 + vs$ because now the photon is being carried out by the outermost layers of the bubble acting as the source of emitted photons that moves towards the observer C with a speed vs,Then now $va = vs$ and $vb = 0$ and we have:

$$f = f_0 \frac{1 + va}{1 - vb} = f_0 \frac{1 + vs}{1 - 0} = f_0(1 + vs) \tag{146}$$

[19]warp drives do not obey Lorentz Transformation

[20]it is reflected by the negative energy as a mirror while the regions inside the bubble remains behind the mirror

12 Appendix H:The Repulsive Behavior of Negative Energy

Adapted from [15]:

"if we have a small object with equal inertial and passive gravitational masses falling in the gravitational field of an object with negative active gravitational mass (a small mass dropped above a negative-mass planet, say), then the acceleration of the small object is proportional to the negative active gravitational mass creating a negative gravitational field and the small object would actually accelerate away from the negative-mass object rather than towards it."

Instead of an object falling in the negative gravitational field of a planet think of an object in front of the outermost layers of the Natario warp bubble being deflected by the negative energy in the front.

13 Epilogue

- "The only way of discovering the limits of the possible is to venture a little way past them into the impossible."-Arthur C.Clarke[21]
- "The supreme task of the physicist is to arrive at those universal elementary laws from which the cosmos can be built up by pure deduction. There is no logical path to these laws; only intuition, resting on sympathetic understanding of experience, can reach them"-Albert Einstein[22][23]

14 Remarks

References 3,4,10 and 16 were taken from Internet although not from a regular available site like the one of almost all the other references(arXiv,HAL).

Reference 10 can be obtained from the web pages of Professor Eric Poisson at University of Guelph Ontario Canada as long as the site remains on-line. [24] [25] [26]

Reference 16 can be obtained from the web pages of Professor Jose Natario at Instituto Superior Tecnico Lisboa Portugal as long as the site remains on-line. [27] [28]

We can provide the Adobe PDF Acrobat Reader of these references 3,4,10 and 16 for those interested.

Reference 15 can be obtained from Wikipedia under the Creative Commons Attribution-ShareAlike License

[21]special thanks to Maria Matreno from Residencia de Estudantes Universitas Lisboa Portugal for providing the Second Law Of Arthur C.Clarke

[22]"Ideas And Opinions" Einstein compilation, ISBN $0-517-88440-2$, on page 226."Principles of Research" ([Ideas and Opinions],pp.224-227), described as "Address delivered in celebration of Max Planck's sixtieth birthday (1918) before the Physical Society in Berlin"

[23]appears also in the Eric Baird book Relativity in Curved Spacetime ISBN $978-0-9557068-0-6$

[24]http://www.physics.uoguelph.ca/poisson/research/

[25]http://www.physics.uoguelph.ca/poisson/research/notes.html

[26]http://www.physics.uoguelph.ca/poisson/research/agr.pdf

[27]http://www.math.ist.utl.pt/ jnatar/

[28]http://www.math.ist.utl.pt/ jnatar/seminario.pdf

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[15] *Negative energy-Wikipedia:The Free Enciclopedia.*

[16] *Warp Drive Seminar Lectures.Instituto Superior Tecnico,Lisboa Portugal,Natario J..*2003*.*

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