

# Variable Mass Cosmology Simulating Cosmic Acceleration

Alan M. Kadin

Princeton Junction, NJ 08550 USA

Email [amkadin@alumni.princeton.edu](mailto:amkadin@alumni.princeton.edu)

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## **Abstract:**

The cosmic Big Bang was long believed to be associated with a decelerating expansion of the universe, until recent observations of distant supernovae indicated accelerating expansion that requires dark energy, which is still not fully understood. It is proposed here that cosmic acceleration may be an illusion caused by gradually increasing rest mass, which produces an additional contribution to the apparent cosmological red shift. A novel simple model is presented that illustrates this effect without dark energy, whereby there is a smooth transition from an early universe of massless particles to the present universe of massive particles, stars, and galaxies, mediated by the decreasing gravitational potential of the expanding universe. Implications of this model for the cosmic microwave background and the early universe are discussed.

## I. Introduction

Modern observational cosmology is based primarily on measurements of red shifts of distant astronomical objects due to Hubble's law expansion [1]. The expansion of the universe can be characterized in terms of a scale factor  $R(t)$ , which is typically expressed as a Taylor series expansion around the present time,  $t_0$ :

$$R(t) = R(t_0)[1 + H_0(t-t_0) - \frac{1}{2} q_0 H_0^2(t-t_0)^2 \dots] \quad (1)$$

Here  $H_0$  is the Hubble constant indicating the present expansion rate, and  $q_0$  is the current acceleration parameter, written for historical reasons as a deceleration parameter. But  $R(t)$  is not measured directly; rather the spectral red shifts are measured in terms of a parameter  $z$ , and  $H_0$  and  $q_0$  inferred:

$$z+1 \equiv \lambda_1/\lambda_0 = R_0/R_1 \quad (2)$$

where  $\lambda_0$  is the emission wavelength of the distant object at present time  $t_0$ ,  $\lambda_1$  is the observed wavelength emitted at much earlier time  $t_1$ ,  $R_0 = R(t_0)$ , and  $R_1 = R(t_1)$ . The red shift parameter can then be written as

$$z = H_0(t_0-t_1) + (1+q_0/2) H_0^2(t_0-t_1)^2 \dots \quad (3)$$

Recent precision measurements of red shifts based on the "standard candle" of distant Type 1a supernovae have shown that this expansion is not slowing as was earlier expected, but rather is accelerating [2,3]. Present estimates yield  $1/H_0 = 14$  billion years and  $q_0 = -0.6$  [3]. This acceleration requires the presence of some sort of dark energy or equivalent cosmological constant, producing a weak anti-gravity effect that can account for this cosmic acceleration. The measurements giving rise to this acceleration have been carefully confirmed, and indeed have been recognized by the 2011 Nobel Prize in Physics [4]. Still, some researchers continue to question the need for Dark Energy to explain these results [5,6].

The present paper suggests that a simple modification of gravity can lead to a non-accelerating expansion that exhibits a red shift that simulates cosmic acceleration. The use of red shift observations to measure  $R(t)$  implicitly assumes that the emission wavelength of the distant object at time  $t_1$ , in its own reference frame, was indeed the same  $\lambda_0$  as if this object were emitting at the present time near the observer. If, on the contrary, the standard candle has an emission wavelength  $\lambda_0(t)$  which changes on the cosmic timescale, then a correction factor must be applied to Eq. (2):

$$z+1 \equiv \lambda_1/\lambda_0(t_0) = [\lambda_1/\lambda_0(t_1)] [\lambda_0(t_1)/\lambda_0(t_0)] = (R_0/R_1) [\lambda_0(t_1)/\lambda_0(t_0)] \quad (4)$$

So the key question is why the emission wavelength of the standard candle might change, and whether such a correction factor could be sufficient to change the corrected cosmic expansion from acceleration to deceleration.

It is suggested here that if the rest masses of all elementary particles were to increase gradually on the cosmic timescale according to a universal mass scale  $M(t)$ , this could cause the characteristic emission wavelength of objects such as supernovae to shift. Such a variation in mass scale is *not* part of the standard Big Bang cosmology or general relativity. But if all rest energies  $mc^2$  scale by the same function, and we further assume that all other energies scale in the same way, including those of emitted photons, then one obtains  $\lambda_0(t_1)/\lambda_0(t_0) = M_0/M_1$ , and Eq. (4) would become:

$$z+1 = (R_0M_0)/(R_1M_1) \quad (5)$$

The mass variation  $M(t)$  would also be expected to modify the absolute luminosity of supernovae and other standard candles, so that the apparent distance calibration of distant supernovae would also need to be modified. The inferred values of  $H_0$  and  $q_0$  in Eq. (3) would then need to be corrected for these effects.

There is as yet no evidence for such a variation in rest mass, but it is reasonable to suggest that this might reflect the gravitational interaction of the expanding universe. In

order to account for this, we need a set of coupled equations that follow the variations of both  $R(t)$  and  $M(t)$  as the universe expands. In the section below, we propose a simplified model that provides one illustration of these coupled equations.

It is important to understand that  $M(t)$  does *not* represent the total mass of a finite universe, since that would suggest a varying total energy in violation of energy conservation. Instead,  $M(t)c^2$  represents a rest energy, with variation that is compensated by changes in total kinetic energy of the expanding universe.

In special relativity, all forms of internal energy, both kinetic and potential, contribute to the rest mass of a body. But in general relativity, it is assumed that the gravitational potential locally affects space-time, but does not directly alter the rest mass. There have been several modified theories of gravity that incorporate variable gravity, either through a varying gravitational constant or through a rest mass that depends on the gravitational field [7-10]. One of the simplest such theories was recently presented by Ben-Amots [7,8], whereby the rest energy of a given particle is the sum of the unmodified rest energy and the gravitational potential energy, as shown in Eq. (6) for a small unperturbed test mass  $m_0$  in the presence of a large mass  $M$ . Here the inertial mass  $m_i$  is on the left, and the gravitational mass  $m_g$  on the right.

$$m_i c^2 = m_0 c^2 - m_g M G / r \quad (6)$$

From the principle of equivalence, one has that  $m_i = m_g = m$ , and this is solved to yield

$$m = m_0 / (1 + M G / r c^2) \quad (7)$$

For the symmetric case where  $m = M$ , one obtains a quadratic formula which can be written in the form

$$m^2 / m_1 + m - m_0 = 0 \quad (8)$$

where  $m_1 = rc^2/G$  is dimensionally a characteristic mass of the gravitational field.

$$m = (-m_1/2) + [(m_1/2)^2 + m_0m_1]^{1/2} \quad (9)$$

This looks odd, but is quite well behaved. For small  $r$ ,  $m \rightarrow (m_0m_1)^{1/2} \propto r^{1/2}$ , while for large  $r$  (i.e., large  $m_1$ ),  $m \rightarrow m_0$ . This prevents the total energy from going negative, while reducing to the standard theory for weak gravitational energies.

This gives rise to some surprising results [7]. Since rest masses go to zero asymptotically as densities get large and distances approach zero, this avoids singularities and event horizons, and eliminates black holes from the theory. There are still gravitationally condensed bodies, but they are not black and can radiate energy. Although this theory is unconventional, it is apparently in agreement with standard general relativity to lowest order, and consistent with experimental tests to date.

It was indicated in the treatment of Ben-Amots [7] that these variable mass effects are significant only near a gravitationally condensed body, on the scale that would otherwise be identified as near an event horizon, i.e., distances  $r \sim GM/c^2$ , and cosmological issues were not addressed. However, the scale of the universe has long been known to be  $R \sim GM/c^2$ , where  $R$  is the radius of the observable universe and  $M$  is its mass [10], so that similar variable mass effects should be relevant here as well. This cosmological variable mass would change slowly, on the timescale of billions of years, although it would be expected to change more quickly during the early era of the Big Bang.

This cosmological mass factor  $M(t)$  would be expected to be fairly uniform at any given time, with spatial variation only near gravitationally condensed bodies. It would also be expected to apply to all masses down to microscopic levels. So this would also provide a characteristic energy scale for phenomena such as stellar emission spectra and luminosities.

Such a cosmological mass dependence would not only alter inferred values of  $H_0$  and  $q_0$ , but would also affect other cosmological phenomena such as the cosmic microwave background. These other implications will be discussed after the model below is presented.

## II. Model for Big Bang Expansion

Consider first a homogeneous universe with a uniform mass density  $\rho$ . The gravitational potential energy for a test mass  $\delta m$  due to spherical shells at a distance  $r$  is

$$U_g = -\delta m \int 4\pi r^2 G \rho / r dr \propto \int r dr \propto r_{\max}^2 \quad (10)$$

Note that the potential is dominated by the most distant shells included in the integral, and in fact would diverge strongly for an infinite universe. For a dynamically expanding universe, the field contribution from the more distant shells would come from the past, and the Big Bang itself provides a natural cutoff. So a fully self-consistent calculation (with a properly retarded potential) would be complicated. For simplicity consider instead the distance  $R(t)$  that represents the size scale of the universe since the Big Bang, and take the total gravitational potential energy to be

$$U_g = -M^2 G / 2R, \quad (11)$$

where  $M(t) = \rho(t) (4\pi/3) R^3$  is the effective mass of the universe up to this cutoff. The factor of 2 prevents double-counting.

Consider now the effect of the gravitational potential on the rest mass. Adapting Eq. (6), we have

$$Mc^2 = M^*c^2 + U_g = M^*c^2 - M^2 G / 2R, \quad (12)$$

where  $M^*$  is the unmodified mass of the universe. We can express this equation in terms of dimensionless units  $m = M/M^*$  and  $r = R/R^*$ , where  $R^* = M^*G/2c^2$  is the characteristic length scale.

$$m = 1 - m^2/r. \quad (13)$$

This is a simple quadratic equation with solution

$$m = (r/2)[(1+4/r)^{1/2} - 1] \quad (14)$$

which goes from  $\sqrt{r}$  for small  $r$  asymptotically to 1 for large  $r$  – see Fig. 1. The crossover is  $r \sim 1$ .

In order to use this as the basis for a dynamical expansion of the universe, we take the usual energy-momentum relation from special relativity:

$$E^2 = (Mc^2)^2 + (pc)^2 \quad (15)$$

Note that this includes no dark energy or other exotic anti-gravity effect. Also, it does not include curved-space effects present in general relativity; however, the best current evidence suggests that the universe is practically flat [3], so that this may be sufficient. Let us consider the case with  $E = M^*c^2$  as the constant total energy of the expanding universe. Then Eq. (15) can be written in terms of velocity  $v = pc/E$ , in reduced units, as

$$m^2 + v^2 = 1 \quad (16)$$

where  $m$  is in units of  $M^*$  and  $v$  in units of  $c$ . By construction, the total energy is enhanced by the usual relativistic factor of  $\gamma = (1-v^2/c^2)^{-1/2}$  above the rest energy.

This then yields a critical expansion from  $M = 0$  and  $v = c$  for small  $R$  to  $M = M^*$  for  $R \gg R^*$ , as the expansion slows to zero velocity – see Fig. 1. (Choosing a smaller value of  $E$  would yield a universe with a maximum scale  $R_{\max}$  that would then start to collapse.)

One can reconstruct the time dependence by using  $v = dr/dt$ , where  $t$  is the dimensionless time in units of  $\tau^* = R^*/c = M^*G/2c^3$ . If one numerically integrates the  $v(r)$  relation in

Fig. 1, one can obtain  $t(r)$ . This enables one to plot  $r(t)$ ,  $m(t)$ , and  $v(t)$  – see Fig. 2. Clearly, this corresponds to decelerating expansion.

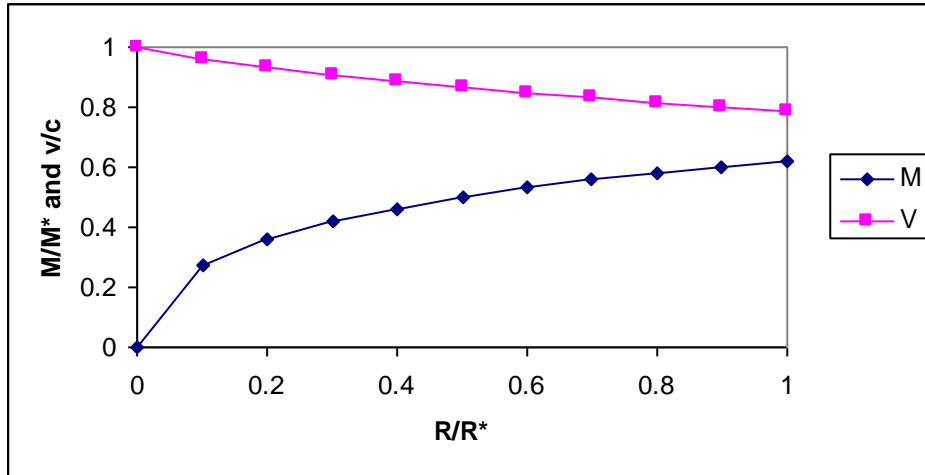


Fig. 1. Mass and velocity as function of expansion scale in simple Big Bang Model with variable mass.

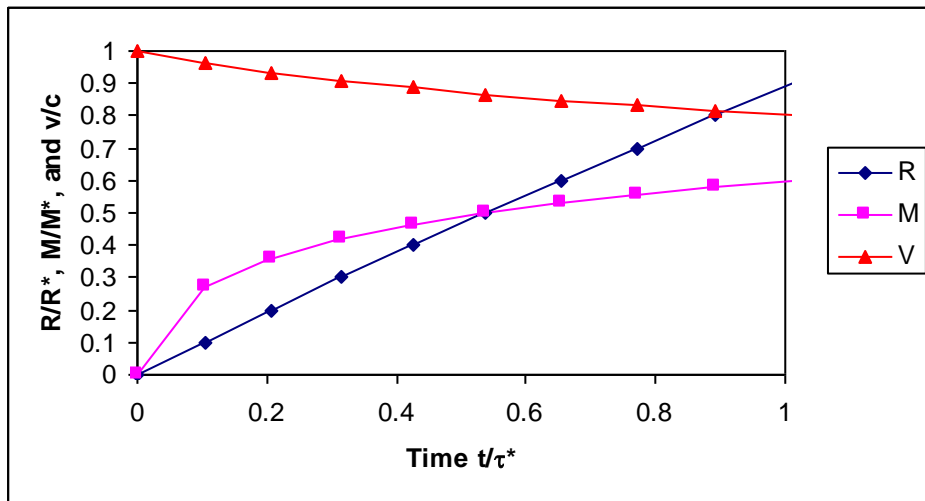


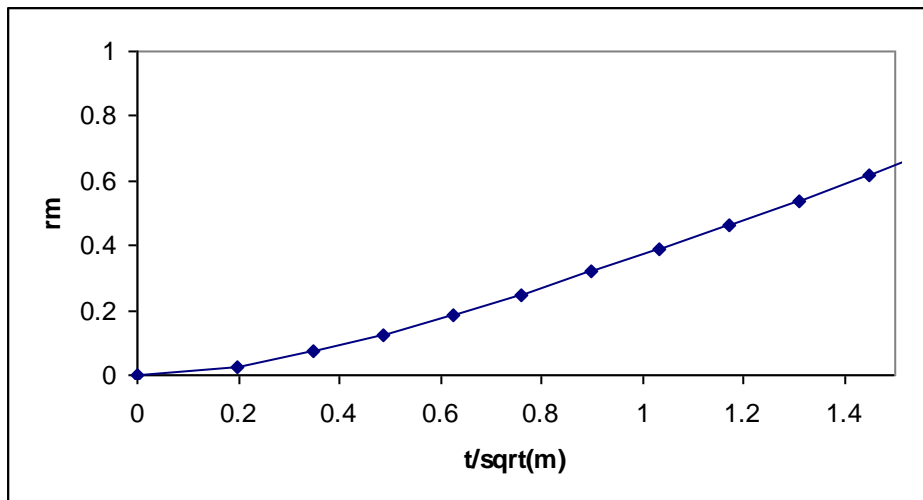
Fig. 2.  $R$ ,  $M$ , and  $v$  vs. time  $t$  in simple Big Bang model with variable mass.

However, as mentioned above, the astronomical observations do not directly measure the expansion. Instead, they measure the spectral shift of objects that are believed to have a standard absolute spectrum and luminosity. On the contrary, if we assume that the variation in  $M(t)$  establishes the energy scale for all astronomical objects, then one would



expect that the emitted spectrum (i.e. the energy scale of the emitted photons) would also scale as  $M(t)$ , and therefore the received spectrum would scale as the product  $MR$ , instead of just  $R(t)$ . This would lead to overestimating the expansion rate. Similarly, the apparent distance  $d_L$  is actually based on measurement of the received luminosity  $\ell = L/4\pi d_L^2$ , assuming a constant value of the absolute luminosity  $L_0$ , so that  $d_L \propto 1/\sqrt{\ell}$ . But if on the contrary  $L(t)$  scales with  $M(t)$ , then  $d_L$  should scale as  $d/\sqrt{M}$ , where  $d$  is the true distance of the object.

In order to simulate the inferred red shift measurements in a way that incorporates  $M(t)$ , Fig. 3 shows a (normalized) plot of  $R/M$  vs.  $t/\sqrt{M}$  for the data of Fig. 2 above. This should be compared with the  $R$  vs.  $t$  plot of Fig. 2. Note that this now shows an accelerating expansion (with upward curvature), similar to what the astronomical observations indicate. But this is an illusion; there is no dark energy or cosmological constant in this model, and the universe is decelerating rather than accelerating. While this model may be simplistic, it illustrates the point that astronomical evidence for cosmic acceleration, although based on accurate measurements, may be deceptive.



*Fig. 3. Normalized plot of  $R/M$  vs.  $t/\sqrt{M}$  from simple Big Bang model with variable mass, simulating observations of cosmic acceleration in distant supernovae.*

### III. Discussion

The present cosmological model obtains a mass parameter that varies in time due to gravitational interaction with distant matter near the edge of the observable universe. This is reminiscent of Mach's principle, in which inertia is defined relative to the total mass in the universe. But unlike theories based on Mach's principle such as Brans and Dicke [10], where the rest energy and gravitational potential energy are constrained to remain comparable as the universe expands, in the present model the rest energy increases as the universe expands, while the potential energy decreases.

Returning to the analysis above, in order to correct the red shift data, one needs to separate the RM product into  $R(t)$  and  $M(t)$ , which in turn requires information from other observations to identify the present time  $t_0$ . For  $t \gg 1$ , the mass variation saturates and the correction factor  $M_0/M_1$  goes to unity. For very small  $t$  ( $\ll 0.1$ ) the red shift would be dominated by changes in  $M$ , while for larger  $t$  it would be dominated by expansion, with small corrections due to mass changes. But in general, the correction will lead to a reduction in the Hubble constant  $H_0$ , and an increase of  $q_0$  from an apparent negative value toward a positive value. This, in turn, would require a recalibration of various parameters such as the critical density ( $\rho_c \propto H_0^2$ ), and the present size and mass of the observable universe  $R_0$  and  $M_0$ . For example, if one selects the present time  $t_0 = 0.5$  (corresponding to  $r_0 = 0.5$  and  $m_0 = 0.5$ ), one would expect the corrected  $H_0$  to be reduced about 30%, and  $\rho_c$  reduced by about 50%.

There are other astronomical observations that also have been interpreted in terms of dark energy, which should be reexamined in the light of the proposed model. The strongest such evidence is the cosmic microwave background [3,11] (CMB), corresponding to black body radiation at a temperature of 2.7 K. This is about a factor of 1100 lower than the expected temperature of  $\sim 3000$  K at which it was believed to be emitted, corresponding to a red shift  $z \sim 1100$ . In contrast, assume that the large red shift is actually measuring the product  $MR$ , and that the present time corresponds to  $r_0 = 0.5$  and  $m_0 = 0.5$ . Then the red shift associated with the expansion would be reduced to about 130, with the remaining shift factor  $\sim 8$  corresponding to a reduced temperature ( $\sim 360$

K) and proportionally longer wavelength at the time of emission. This would be a major change from the conventional CMB theory, and it is not clear that all observations would be compatible with such a picture.

Another consideration related to CMB is that the observed homogeneity and lack of anisotropy of the CMB radiation indicates that the universe is (and was) essentially flat (without gravitational curvature), suggesting that the average mass density  $\rho_0$  should be equal to the critical density  $\rho_c$ . A similar conclusion has been derived from the widely accepted inflationary cosmology model [12]. However, the estimated mass density from galactic dynamics, even including “dark matter”, is about a factor of 4 below  $\rho_c$  as determined from measurements of the Hubble constant  $H_0$  [2,3]. The conventional theory of dark energy asserts that the dark energy can supply the missing density to achieve criticality. Can one account for this apparent discrepancy without invoking dark energy?

It was pointed out above that a corrected value of  $H_0$  in the present picture would be somewhat reduced, and so would  $\rho_c = 3H_0^2/8\pi G$ . Furthermore, one might argue that the proper density to include in this picture (with variable mass) should be the density associated with the constant total energy  $M^*c^2$ , including the kinetic enhancement of the rest energy. For the above example with  $t_0 \sim 0.5$ , this would approximately double  $\rho_0$  and cut  $\rho_c$  in half, thus fully accounting for the apparent factor of 4 discrepancy. This is preliminary and somewhat speculative, but it suggests that perhaps there may be no need for dark energy in the theory of CMB either. (This would not appear to address the origin of the dark matter, however.)

This analysis can be more quantitative, using conventional literature estimates of measured quantities  $H_0 = 2.3 \times 10^{-18} \text{ s}^{-1}$  [equivalent to (14 billion years)<sup>-1</sup>] and  $\rho_c = 1 \times 10^{-26} \text{ kg/m}^3$  [3]. First, correct these as suggested above to  $H_0 = 1.8 \times 10^{-18} \text{ s}^{-1}$  and  $\rho_c = 0.5 \times 10^{-26} \text{ kg/m}^3$ . Now combine the following two relations for  $M^*$  and  $R^*$ :

$$M^* = 2c^2R^*/G \quad (17)$$

and

$$M^* = \rho_c (4\pi R^3/3) \approx \rho_c (\pi R^{*3}/6) \quad (18)$$

(taking  $R \approx 0.5R^*/2$ ), to obtain

$$\begin{aligned} R^* &\approx 2c/(G\rho_c)^{1/2} = 10^{27} \text{ m} = 100 \text{ billion light years} \\ \tau^* &= R^*/c = 100 \text{ billion years} \\ M^* &= 3 \times 10^{54} \text{ kg} \end{aligned} \quad (19)$$

So this simplified model suggests that the current universe corresponds to  $R \approx 50$  billion light years and  $t \approx 50$  billion years since the Big Bang. These values are somewhat larger than those generally accepted, but still quite reasonable.

It is worth noting that the proposed cosmological model proposes an early universe composed of fully relativistic, virtually massless particles. While the conventional picture of the early universe exhibits extremely high temperatures at which exotic massive particles would be present, this new picture would be even more dominated by exotic particles and fields. In this context, this new picture might provide a more natural environment for the generation of cosmic inflation as well as other novel regimes of high-energy physics. Even beyond early cosmology, this concept of variable mass from the gravitational potential may be more compatible with fundamental principles of quantum field theory, where the total relativistic energy corresponds to a (non-negative) oscillation frequency of the quantum wave. Together with the avoidance of singularities (black holes and event horizons), this might provide a more consistent foundation for developing a quantum theory of gravity,

#### **IV. Conclusion**

In summary, a novel theory of gravitation with a time-varying rest mass (but no dark energy) has been applied to cosmological expansion, using a simplified model based on the varying gravitational potential of the universe. The results suggest that the observed cosmological red shift may also include a significant component associated with changes in mass, and that the inferred cosmic acceleration seen for distant supernovae may be illusory. Further analysis also suggests that the condition of critical density, needed for a

flat universe without curvature, may be achieved without the need for dark energy. It remains to be seen whether this theory and simplified cosmological model are compatible with other observed phenomena in cosmology and astrophysics. But if this approach is valid, it offers important new insights into the foundations of gravitation and cosmology.

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