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Conjecture on Time Dilation, Gravity and Inertia

Abstract

This conjecture postulates the existence of a field of particles in space that I'll call the ***temporal-inertial (TI) field***, but will refer to simply as space or the field. This field provides a frame of reference for motion such that the motion of an object (sub-atomic particle, atom, clock, etc.) with respect to that field causes two effects. The velocity of an object with respect to the field causes time dilation for that object and the acceleration of an object with respect to the field causes the familiar inertial reaction force. It is argued that the successful functioning of the Global Positioning System requires that the Earth not move with respect to this TI field. This assertion demands that the TI field move with Earth and that the field must therefore be subject to gravity and orbit the Sun just as the Earth and planets. The response of the TI field to gravity requires particles of the field in the vicinity of Earth to accelerate toward the center of the Earth. As a result, gravitational time dilation and gravitational redshift in a reference frame within the field are caused by the velocity of the particles of this field relative to the reference frame, not directly by gravity itself. Lastly, it is argued that the gravitational force is mediated indirectly by the acceleration of the TI field relative to matter particles in the field. In response to the acceleration of the TI field by gravity, the field asserts a force on matter particles in the direction of the acceleration of the field relative to the matter particles, just as for the inertial force. Accordingly, matter particles are not directly subject to the gravitational force.

Time Dilation

Time dilation is defined as the decrease in the rate of flow of time in a frame moving with respect to an outside observer. Time dilation in the frame moving relative to an outside observer is given by Cutner [1]:

$$t_2 / t_1 = 1 / (1 - v_2^2 / c^2)^{1/2} \qquad (1)$$

where

t_2 / t_1 is the ratio of period t_2 measured by the moving clock with respect to the period t_1 measured by the clock of the outside observer.

v_2 is the velocity of the moving clock relative to that of the outside observer.

Let me restrict the validity of Eq. (1) by requiring the clock of the outside observer, measuring the value of t_1, to be stationary relative to space, or more specifically in this conjecture, the TI field.

A Thought Experiment

Imagine two spaceships, A and B, located in space far away from any gravitational masses. The two ships contain identical, accurate clocks. Ship B moves at speed, v_2, away from ship A ***which is stationary with respect to the temporal-inertial field***. Onboard ship B, clock B runs more slowly than its counterpart on ship A in accordance with Eq. (1). Ship B cruises along for awhile and its clock continues to run more slowly than clock A. Ship B then begins to slow down. As it slows down, its clock speeds up until, when ship B's velocity with respect to ship A is zero, clock B ticks at the same rate as clock A. This observation is valid and is in accord with Eq. (1).

Now, reverse the process and send ship B back toward ship A. As ship B's velocity increases, its clock again slows down. Approaching ship A, ship B decelerates and as it does, the rate of its clock again speeds up until as ship B comes to a stop next to ship A its clock is again running at the same rate as clock A. Measurements confirm that clock B, having lost considerable time with respect to clock A in ship A, is now ticking at the same rate as clock A.

At no time during the test did clock A influence the tick rate of clock B. Had clock A not been a part of the test, the variation in clock B's rate would have changed as described. Clock A provided only a means of comparison and measurement of the time 'lost' by clock B. Accordingly, the variation of clock B's rate was determined solely by its motion with respect to the TI field itself. Clock B's tick rate slows as its velocity increases relative to the TI field. We can conclude from this thought experiment and Eq. (1) that clock B's tick rate would be at its maximum when the clock is stationary relative to the TI field.

Motion Through Space is the Root Cause of Time Dilation

It's a logical conclusion from the thought experiment that any motion with respect to the TI field causes time dilation. ***Accordingly, motion with respect to the temporal-inertial field, not just relative to another frame of reference, must be included in evaluating the time dilation of a moving object***.

To support the assertion of the highlighted sentence above, continue our thought experiment, but ignore the caveat that ship A is stationary with respect to space. Let ship A move with velocity v_1 with respect to the TI field. We may let space itself be our frame of reference, because this is, after all, a thought experiment. Let spaceship B move at velocity v_2 with respect to space. For simplicity, assume that v_2 is in the same direction as v_1 and is greater than v_1. Both clocks will now experience time dilation in accordance with Eqs. (2) and (3).

$$t_1 / t_0 = 1 / (1 - v_1^2 / c^2)^{1/2} \tag{2}$$

$$t_2 / t_0 = 1 / (1 - v_2^2 / c^2)^{1/2} \tag{3}$$

where

v_1 is the velocity of clock A with respect to the TI field.

v_2 is the velocity of clock B with respect to the TI field.

t_1 / t_0 is the ratio of period t_1 measured by clock A with respect to the period t_0 that would be measured by an identical clock that is stationary relative to the TI field.

t_2 / t_0 is the ratio of period t_2 measured by clock B with respect to the period t_0 measured by the stationary clock

The ratio of the periods measured by the two clocks is obtained by dividing Eq. (3) by Eq. (2):

$$t_2 / t_1 = (1 - v_1^2 / c^2)^{1/2} / (1 - v_2^2 / c^2)^{1/2} \tag{4}$$

Compare the result of Eq. (4) with that of Eq. (1) repeated below:

$$t_2 / t_1 = 1 / (1 - v_2^2 / c^2)^{1/2}$$

These two expressions equate only if v_1 in Eq. (4) is zero, that is, only if the clock measuring t_1 is stationary relative to the TI field.

Let me summarize the meaning of the foregoing arguments:

1. The TI field is the absolute reference frame for motion of particles or objects in the field.
2. Time dilation of an object moving in space is a function of its velocity relative to the TI field. The faster a clock moves relative to theTI field, the greater is its period and the slower its clock ticks.

3. Comparison of the time dilation between two clocks moving in space must be based on each clock's velocity relative to the TI field as expressed in Eq. (4), not on the difference of their velocities relative to each other.

The contention that any calculation of time dilation of a moving object must include its velocity with respect to space is supported.

A Second Thought Experiment With a Third Clock

Let's now add a third spaceship and third clock to examine another effect of time dilation. This effect is the increase in tick rate as a clock moves away from a reference clock. This effect is in opposition to the conventional notion that the tick rate of a clock that moves away from a reference clock decreases. The apparent anomaly is caused, in this example, by the fact that the clock that moves away from the reference clock decreases its velocity relative to the TI field while the reference clock retains its velocity relative to the TI field.

Call this third spaceship ship C and let this ship and its clock be the reference by which the motion of ship B and its clock are reckoned. Let this third ship move alongside ship B at the same velocity. Clocks B and C aboard ships B and C tick more slowly than clock A that is stationary relative to space as in our first experiment. Now, as before, let ship B slow down relative to ship A. As ship B slows down, the tick rate of clock B increases so that when ship B's velocity relative to ship A has decreased to zero, clock B and clock A tick at the same rate. Consider the situation from the perspective of ship C, our reference ship. As ship B slows down, observers aboard ship C see ship B accelerating in the direction of ship A. Accordingly, observers on ship C would see the tick rate of clock B slowing down when in actual fact the tick rate of clock B is increasing. How can this be true? This behavior occurs because a clock's tick rate increases as its velocity relative to space decreases. In this experiment, clock B slows down and is moving slower, relative to space, than clock C.

Rewrite Eq (4) as Eq (5) where t_2 and t_3 represent the periods of clocks B and C, respectively, and v_2 and v_3 represent the velocities of ships B and C, respectively.

$$t_3 / t_2 = (1 - v_2^2 / c^2)^{1/2} / (1 - v_3^2 / c^2)^{1/2} \qquad (5)$$

As ship B slows down, its velocity v_2 relative to space is less than v_3 of ship C. (Ship C continues to move as ship B slows down.) The value of t_3 / t_2 is thus greater than 1, and the period of clock C is greater than that of clock B. Clock B is ticking faster than clock C. This result is the inverse of the conventional notion that a clock moving away from an outside observer ticks more slowly. In this example, clock B, accelerating away from clock C, ticks faster and thus has a shorter period than clock C.

In this example, both clocks are moving relative to the TI field. The velocity of one clock relative to the TI field then decreases and its tick rate increases. From the perspective of the first clock that retains its velocity relative to the TI field, the second clock appears to move away. The conventional notion is that the tick rate of the second clock should decrease. In fact, because the second clock's velocity relative to the TI field decreases, its tick rate increases.

Equation (5) (identical to Eq (4) except for the subscripts) replaces Eq (1) to express the time dilation between two clocks in which the velocity of each clock relative to the TI field must be considered, rather than the difference in their velocities relative to each other.

How can we determine an object's velocity with respect to space? Is it even possible? Let's see how this question is handled in the Global Positioning System.

GPS, The Real Experiment

The Global Positioning System (GPS) provides an unequivocal test of relativity. Two phenomena described in the theories of relativity affect the clock rates of GPS satellites: gravitational time dilation and time dilation caused by motion through space. We'll discuss gravitational time dilation briefly in the section on that subject. Neglecting gravitational time dilation for the moment, the Global Positioning System compensates for the time dilation of each GPS satellite's clock based on the difference between the satellite clock's orbital velocity and the velocity of the ground-based reference clock. The validity of this calculation requires that the velocity of the ground-based clock relative to the TI field to be zero. The exception to this statement is discussed in the section on gravitational time dilation. Considering Earth's many motions including its orbit about the Sun, about the galactic center and its motion with the galaxy relative to the Cosmic Microwave Background (CMB), requires that the TI field to be moving with Earth. Conversely, Earth is moving with the TI field with no velocity difference between the two.

Consider what would happen if this were not true. If Earth moved with respect to space, the velocity with respect to space of an orbiting GPS satellite would be greater when a component of the satellite's velocity is in the same direction as the Earth's velocity and less when a component of the satellite's velocity is opposite the direction of the Earth's velocity. Then the rate of its clock due to time dilation would vary continuously during the orbit. Such behavior is not observed. (A sinusoidal variation of timing is observed because of the slight eccentricity of the orbit of a given GPS satellite, but that's a different effect [2]). Given that the Global Positioning System works flawlessly and that our interpretation of the thought experiment is correct, only one conclusion can be drawn: Earth does not move with respect to space. This concordant motion of the Earth and the TI field implies strongly that the TI field is subject to gravity.

A cascade of implications results from space being subject to gravity. The TI field will move with the galaxy, not just the Milky Way, but each galaxy individually. Space will move with the Sun in its orbit around the galaxy and space will orbit the Sun just as the planets, its velocity profile in keeping with each planet as expressed by Eq. (6).

$$v_{space} = (GM / R)^{1/2} \tag{6}$$

where

v_{space} = the orbital velocity about the Sun of the TI field of space

G = the gravitational constant

R = distance from the Sun

In addition, if the Earth is stationary with respect to space in its orbit around the Sun, and space is subject to the gravitational force, it follows that in the vicinity of Earth space will fall into the Earth radially from all directions. The acceleration profile (acceleration vs distance from the Earth) will be the same for the TI field as that of a particle in free fall in the gravitational field. At the surface of the Earth, the velocity of this infall will reach the escape velocity at the surface of Earth. The escape velocity is expressed in the next section. This inflow will continue toward the center of the planet. Particles of the TI field will not flow through the Earth to emerge on the opposite side of the planet and continue on back into space. The rationale for this assertion is discussed in the section on 'The Inertial Force as Gravitation' below.

Gravitational Time Dilation

The question to be answered in this section is this: How can the phenomenon of time dilation occur in two separate and apparently quite different circumstances? The two different circumstances are 1) the velocity of an object through space and 2) the action of a gravitational field on that object.

My own sense is that a phenomenon that occurs in two apparently different circumstances must have a common cause.

Schutz [3], p.286 expresses the dilation of time in a gravitational field:

$$t_1 / t_0 = 1 / (1 - 2GM / Rc^2)^{1/2} \tag{7}$$

where

t_1 / t_0 represents the time dilation caused by the gravitational field at a distance R from a center of a mass, M.

Expressed another way, t_1 is the period of a clock measured at a distance R from the gravitational mass M while t_0 is the period of an identical clock measured at a distance where there is no gravitational influence from the mass.

Now, I'll compare the expression of Eq. (7) with one derived based on the infall velocity of particles of the TI field in response to gravity. The escape velocity of a particle from a distance R from the center of a spherical body such as a planet or star with a mass M can be determined by equating the kinetic energy of the particle with its gravitational potential [4], p282:

$$1/2\ v_{escape}^2 = GM / R \tag{8}$$

The escape velocity is then

$$v_{escape} = (2GM / R)^{1/2} \tag{9}$$

Remembering that in Eq. (9) R is the distance from the center of the gravitational body that is greater than the radius of the body itself.

Substituting the value of v in Eq. (9) into Eq. (1) gives:

$$t_1 / t_0 = 1 / (1 - 2GM / Rc^2)^{1/2} \tag{10}$$

where again

> t_1 / t_0 represents the time dilation caused by the gravitational field at a distance R from the center of the mass M.

Equation (10) is identical to Eq. (7). Does the appearance of the value of the escape velocity in the expression for gravitational time dilation suggest another mechanism at work? The conjecture asserts that it does and it is this: gravitation does not cause time dilation directly, but does so only through its acceleration of the TI field. As mentioned in the previous section on the Global Positioning System, space is accelerated by the force of gravity and flows radially into the Earth from all directions and reaches the escape velocity from the Earth at the Earth's surface. The total time dilation of an orbiting GPS clock results from its own motion through space and its presence in a gravitational field. Does it not make sense that the effects have a common cause: the velocity of the object with respect to space? In orbit, clearly, the clock moves, in the gravitational field, space moves past the clock as space falls toward the center of the Earth.

The measurement of gravitational time dilation is in accord with Eqs. (7) and (10). It follows that the infall of space must account for the measurement of gravitational time dilation.

The total velocity of the GPS clock with respect to space is the vector sum of its orbital velocity and the velocity of space inflowing toward the Earth at the GPS clock altitude. If the satellite is in a circular orbit the two velocities are orthogonal and the velocities can be added as in Eq. (11) as the squares of their velocities. Accordingly, the sum of their squares equals the square of their vector sum (Pythagorean Theorem).

$$t_1 / t_0 = 1 / (1 - (v_{escape}^2 + v_{orbit}^2) / c^2)^{1/2} \tag{11}$$

This is not the case for a satellite in an eccentric orbit. The vector sum then depends on the angle between the orbital velocity at a given point in the orbit and the radial velocity of space at that orbital point.

Measurement of the time dilation of a clock in a satellite in a highly eccentric orbit would provide a falsifiable test of the conjecture that space is accelerated toward the Earth and is responsible for gravitational time dilation. If the conjecture is correct, adding algebraically the time dilation of the orbital velocity and that of the gravitational time dilation will be in error. If not, the conjecture is in error.

Gravitational Redshift

The right side of Eq. (10) also expresses the ***gravitational redshift*** (shown as wave length instead of frequency) of light emitted by a source located a distance R from the mass M [3], p. 288.

$$\lambda_{obs} / \lambda_{emit} = 1 / (1 - 2GM / Rc^2)^{1/2} \qquad (12)$$

where

λ_{obs} = the wavelength of light seen by a distant observer

λ_{emit} = the wavelength of light emitted at the source located a distance R from the gravitational body

The valuations of Eqs. (10) and (12) expressing gravitational time dilation and gravitational redshift, respectively, are identical. The expressions for time dilation and redshift are often used interchangeably.

A Falling Clock Loses No Time

An example consistent with relativity and the conjecture that space falls in to the Earth is that of an object falling radially toward the Earth in the absence of air resistance. A clock dropped straight toward the center of a gravitational mass (say the Earth) will accelerate with space itself and consequently will not change velocity with respect to space. Accordingly there will be no change in time dilation during its descent. The clock's velocity as it falls will be its initial velocity plus its acceleration, g, times the time of the fall.

$$v_{clock} = v_{0clock} + gt$$

where

v_{0clock} = velocity of the clock when it's released (nominally, $v_{0clock} = 0$)

The velocity of the infall of the TI field of space will be

$$v_{space} = v_{0space} + gt$$

where

> v_{0space} = velocity of the TI field of space at the point where the clock is released

The velocity of the clock with respect to the TI field of space as it falls remains constant and is

$$v_{clock} - v_{space} = v_{0clock} + gt - (v_{0space} + gt) = v_{0clock} - v_{0space} \quad (13)$$

Thus, during its fall the clock will retain the same period it had when it started its descent. The ratio of this period to that of a distant clock is expressed by Eq. (14), a reformulation of Eqs. (9) and (10).

$$t_1 / t_0 = 1 / (1 - v_{escape}^2 / Rc^2)^{1/2} \quad (14)$$

where

> $v_{escape} =$ escape velocity from Earth at a radius R from the Earth. This value is the radial velocity of the TI field of space at R as it falls into the Earth.

The clock's period will change to the value of its rest position when it stops at the surface of the gravitational mass. (It will tick more slowly than when it started its descent.) This behavior is in accord with Schutz's [4], p115 analysis and conclusion that "there is no redshift in a freely falling frame." The conjecture of space infall toward the Earth is consistent with this example.

The Cause of Gravitational Time Dilation and Redshift

The falling clock does not experience a change in time dilation, so time dilation is not caused by graviton flux nor by acceleration. The fact that neither graviton flux nor acceleration causes time dilation supports the conclusion that gravity does not directly cause time dilation. This, in turn, requires an alternative mechanism associating time dilation with gravity. My assertion is that this mechanism is the acceleration of the TI field by gravity. The reasoning behind this assertion is described in the section on *Gravitational Time Dilation* above.

Similarly, it is concluded that gravity does not directly cause redshift.

Tests of the Conjecture on Time Dilation

1. A falsifiable test of the conjecture that the infall of the TI field is the proximate cause of gravitational time dilation is to measure the time dilation of a clock in a highly eccentric orbit. If the contributions to time dilation of orbital velocity and gravity are additive algebraically, the conjecture fails. If the contributions must be added vectorially to yield the correct result, the conjecture is affirmed.

2. Place two clocks in solar orbit, one matching the Earth's orbit and one matching Mars' orbit. There will be no time dilation between the two as they are each moving

in the TI field that orbits the Sun just as the named planet. There will be no gravitational time dilation from the Sun's gravitational field because, again, there is no relative motion between either clock and its position in the local TI field. The two clocks will tick at the same rate. There will be two other effects to consider, the blueshift of radio waves from the outer orbit to the inner orbit and the Doppler shift caused by the clock's velocities relative to one another.

The Inertial Force as Gravity

We have argued that the TI field is subject to gravity and is accelerated toward a gravitational body by the same amount as a matter particle in free fall in the gravitational field of that body. Accordingly the TI field is subject to the same acceleration profile as matter particles in the gravitational field of the Sun. Newton's Second Law of Gravitation, Eq 15, expresses this relationship for a matter particle in the gravitational field of a massive body and, hence, also for a particle of the TI field.

$$a = GM/R^2 \tag{15}$$

where:

G is the gravitational constant

a is the acceleration of space at a distance of R from a mass, M.

Inertial reaction forces are repulsive and proportional to the relative acceleration of a matter particle, or object comprising matter particles, and the TI field. This is expressed by the familiar formula:

$$F = Ma \tag{16}$$

When an object comprising matter particles is accelerated relative to the TI field the reaction force is called inertia. The reactive force of the TI field on accelerated matter does not confer mass on matter, but acts only as a force resisting such acceleration. The gravitational acceleration of space with respect to a matter particle or an object composed of matter particles applies a force to that matter particle or object. This force is the familiar gravitational force applied indirectly through the intermediary of the acceleration of space.

To reiterate, matter particles experience a reactive force proportional to their acceleration with respect to the TI field. When the TI field is accelerated by gravity, a force is conveyed to matter particles in the field. ***Thus the gravitational force on a matter particle is conveyed indirectly through the acceleration of the TI field of space. Matter particles and the objects composed of matter particles are not directly subject to the gravitational force.*** In this new paradigm, the Principle of Equivalence between gravity and inertia is served by the mediation of a single mechanism, the relative acceleration of matter and the TI field, to accommodate both gravitational and inertial forces.

In the section, GPS, The Real Experiment', we described the infall of the particles of the TI field toward the center of the Earth, but asserted that these particles would not pass through the Earth to continue on into space. If, as we contend, the acceleration of the particles of the TI field conveys the gravitational force to matter particles, then that activity literally holds the planet and, indeed, every gravitational body, together. If the particles of the TI field passed through the Earth and continued out the other side, there would be no net gravitational force conveyed to matter particles.

Conclusions

1. The TI field provides an absolute reference frame for motion.
2. Time dilation of a moving frame is a function of the velocity of that frame relative to the TI field.
3. Time dilation of a moving particle or object composed of particles is intrinsic, absolute and not dependent on the observation of an outside observer.
4. The ratio of time dilation experienced by two moving clocks is a function of each clock's velocity relative to the TI field, and is not a function of their velocity relative to each other. This ratio is expressed by Eq. (4).
5. Proper operation of the Global Positioning System (GPS) indicates that the Earth does not move with respect to the TI field.
6. Accepting the premise of Item 5 and the fact that the Earth does move (about the Sun, the galaxy, etc.) implies that the TI field moves exactly in the same fashion as the Earth.
7. Item 6 implies that the acceleration of the TI field is in accord with Newton's Second Law of Motion in a gravitational field as expressed in Eq (15): $a = GM/R^2$.
8. When a matter particle or an object composed of matter particles is accelerated by an external force, its motion is resisted by its acceleration relative to the TI field. This reactive force of space is the familiar inertial force.
9. The gravitational acceleration of the TI field relative to a matter particle or an object composed of matter particles applies a force to that matter particle or object. This force is the familiar gravitational force applied indirectly through the intermediary of the acceleration of the TI field of space.
10. The reactive force of the TI field on accelerated matter does not confer mass on matter, but acts only as a force resisting such acceleration.
11. The TI field is accelerated by gravity directly toward the center of each planet (or massive body) just as a test particle would be and reaches the escape velocity of such a particle at a given radius from the planet (or massive body) as expressed by Eq. (9).

12. Gravitational time dilation is not caused directly by gravity, but by the velocity of the TI field at the point in space where the time measurement takes place. At the surface of the Earth, this velocity is the escape velocity. Gravitational time dilation is expressed by Eq. (10).
13. As in Item 12 above, gravitational redshift is not caused directly by gravity, but by the velocity of the TI field at the location of the emitting object. At the surface of the Earth, this velocity is the escape velocity. At a distance, R, from the Earth the redshift is expressed by Eq. (11).
14. Time dilation is zero in a reference frame that is stationary relative to the TI field.
15. The corollary to Item 14 is that the tick rate of a clock is maximum when the clock is stationary relative to the TI field.
16. In the GPS, the effects of time dilation caused by the orbital speed of a satellite clock and the time dilation caused by gravity should be added vectorially, not algebraically. The algebraic addition introduces a timing error for the GPS clock of a satellite in an eccentric orbit, but does not for a GPS clock in a circular orbit.
17. It follows from Items 8 and 9 that gravitational and inertial forces are not only equivalent, but are identical by virtue of their mediation by the acceleration of the TI field relative to matter.

Summary

We have seen that motion of an object with respect to the TI field causes time dilation and that when comparing the time dilation of one object with respect to another the motion of each object with respect to the TI field must be considered. If only their velocity relative to each other is considered in measuring the time dilation between the two, the calculation will be in error unless one of the objects is stationary with respect to the TI field.

The Global Positioning System must compensate for time dilation of its satellite clocks that is caused by their orbital motion and by gravity. These calculations do not account for the Earth's motion through space and yet the GPS works flawlessly. One of the fundamental conclusions of this conjecture is that the TI field moves in orbit around the Sun just as the Earth. In addition, the TI field orbits the Milky Way Galaxy just as the Sun and moves with the galaxy as well. Accordingly, there is no relative motion between the Earth and the TI field except for the infall velocity of the TI field that accounts for gravitational time dilation.

To account for its orbital motion about the Sun, the TI field must be subject to the force of gravity. Space thus will orbit the Sun in the same profile (orbital velocity vs distance from the Sun) as the planets. In addition, space will be accelerated by gravity toward the center of Earth and reach the escape velocity at the surface of Earth. Earth should not be a special case, so this behavior should prevail at each planet and, indeed, for each gravitating body in the solar system. I speculate that this circulatory model for the solar system transitions to an infall model in the 'vicinity' of the Sun.

The infall of the TI field toward the Earth's center imposes a velocity component on an orbiting satellite that contributes to the time dilation experienced by its clock. The contribution to the time dilation of this component is the same magnitude as that which would be contributed by gravity. Time dilation is not caused by graviton flux nor by acceleration. The conclusion of this conjecture is that gravitational time dilation is not caused directly by gravity, but indirectly by gravity's acceleration of space toward the Earth (or any massive body).

The components of orbital velocity and the TI field infall velocity must be added vectorially not algebraically to calculate the combined time dilation. If added algebraically there will be no error introduced for a perfectly circular orbit, but there will be an error introduced for an elliptical orbit. The greater the eccentricity of the orbit, the greater the error.

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