Heisenberg's Uncertainty : an Ill-Defined Notion ?

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Dedicated to Marie-Louise Nykamp

Abstract

The often cited book [11] of Asher Peres presents Quantum Mechanics without the use of the Heisenberg Uncertainty Principle, a principle which it calls an "ill-defined notion". There is, however, no argument in this regard in the mentioned book, or comment related to the fact that its use in the realms of quanta is not necessary, let alone, unavoidable. A possible comment in this respect is presented here. And it is related to certain simple, purely logical facts in axiomatic theories, facts which are disregarded when using "physical intuition" and "physically meaningful" axioms or principles in the development of mathematical models of Physics, [16-18].

> "... creativity often consists of finding hidden assumptions. And removing those assumptions can open up a new set of possibilities ..."
>
> Henry R Sturman

"History is written with the feet ..."

Chinese Ex-Chairman Mao,
of the Long March fame ...

Science is not done scientifically, since it is mostly done by non-scientists ...

Anonymous

Physics is too important to be left to physicists ...

Anonymous

Is the claim about the validity of the so called "physical intuition" but a present day version of medieval claims about the sacro-sant validity of theological revelations ?

Anonymous

1. Heisenberg's Uncertainty Principle, the Axioms of Quantum Mechanics, and "Physical Intuition"

It may come as a major surprise to see an often cited book like [11] of the recently passed away well known quantum physicist Asher Peres, a book which presents Quantum Mechanics without any use of the Heisenberg Uncertainty Principle. On top of that, the book does not make any comment why the use of that principle is not necessary in building up Quantum Mechanics, and as the book shows it, why it is in fact avoidable. The only related comment is on the back cover of the book where that principle is lumped together with other "ill-defined notions" ...

On the other hand, a recent excellent popularizing book, [2], makes an exceptional use of the Heisenberg Uncertainty Principle in explaining a large variety of rather counter-intuitive quantum phenomena, some of them truly subtle and fundamental.

One possible answer to the perplexity of the above two sharply different, if not in fact, conflicting developments is attempted here, and it is but an instance of a larger phenomenon recently approached in [16-18].

That phenomenon is about the insufficiently careful use of what goes in Physics by the often celebrated name of "physical intuition", an intuition which - especially lately - keeps pushing for a reformulation of Quantum Mechanics based on axioms which have a clear physical meaning.
Needless to say, there is nothing wrong with such a tendency. And no doubt, one can - and should - consider it perfectly legitimate in Physics. After all, in each discipline of science, the respective "intuitions" can - and do - often bring with them major breakthroughs ...

And then, what may be going on ?

Well, as analyzed in [16-18], "physical intuition", or for that matter, any other specialized scientific intuition, while it may - and in fact, should - be freely used, there is however, a special situation when its use is related to the *axiomatization* process. Details in this regard are presented briefly in section 2 below, while further arguments can be found in [16-18].

Suffice it here to make the following brief comment :

What happens in the mentioned book of Peres regarding the Heisenberg Uncertainty Principle is simply the fact that an alternative axiomatic build up of Quantum Mechanics is presented, one in which this principle is not used.

The fact, however, that Quantum Mechanics, as many other theories, can have more than one single way to be built up from axioms is a

rather elementary phenomenon in Mathematical Logic, the discipline of science to which the theory of axiomatic theories happens to belong. Therefore, it should not come as a surprise that one or another of the basic principles of Quantum Mechanics may be avoided when building up that theory of Physics.
And whenever such a principle turns out to be avoidable in the axiomatic build up of Quantum Mechanics - or for that matter, of any other scientific theory - that does not necessarily reflect negatively on the importance of the respective principle within the specific scientific theory, once that theory has been built up.

In Mathematics itself, there are various theories, each of which can be built up from more than one system of axioms. This fact, nevertheless, surprises mathematicians, and it is certainly not considered as impacting negatively in any way on one or another of the axioms which happens not to be used in one or another of the axiom systems.

However, here the discussion being about Physics, we shall present next in section 2 a physical example of an axiomatic system where similar phenomena seem to occur, see further details in [16-18].

2. An Axiomatic Story with Special Relativity ...

Regarding the axioms of Special Relativity, the following situation - long known, yet not widely familiar among physicists - has occurred, [1,5-10,16-18]. The usual two axioms, [4], which have a clear physical meaning are :

- Galilean Relativity : the Laws of Physics are the same in all inertial reference frames.

- Constancy of the Speed of Light : in all inertial reference frames the speed of light has the same value c.

However, as shown in [10,6,16-18], in order to obtain the Lorentz Transformations of inertial reference frames, which as is known, contain the essence of Special Relativity, one only needs the following weaker axioms :

- the homogeneity and isotropy of space,
- the homogeneity of time,
- the Axiom of Reciprocity which means that, given two inertial reference frames S and S', and a speed $v \in \mathbb{R}$, the laws of Physics are the same whether S' moves related to S with speed v, or with speed $-v$,
- the upper limit of all physical speeds, which already results from the above three axioms for all inertial reference frames, [9,5], is the speed c of light in void.

Here we note that the homogeneity and isotropy of space and the homogeneity of time are assumed as well when one axiomatizes Special Relativity with mentioned two usual axioms.

It follows that if we denote by $\mathcal{A}$ the above four axioms, while by $\mathcal{B}$ we denote the mentioned two usual axioms of Special Relativity, plus the two above axioms about space and time, then the corresponding theory, see Appendix

$$(2.1) \qquad \mathcal{T}_{\mathcal{R}}(\mathcal{A})$$

already contains the Lorentz Transformations of inertial reference frames.

Thus the following questions arise :

- are the axioms in $\mathcal{B}$ independent ?

and in case they are not, as suggested by [1,5-10,16-18], then :

- what is the point in stating the two usual axioms of Special Relativity in such a redundant manner, just in order to have an obvious physical meaning ?

Here one can, of course, note that the axioms $\mathcal{A}$ themselves have already an obvious physical meaning. Not to mention that both sets of axioms have the same number, namely, four, of axioms, thus none of them is shorter than the other one.

3. Back to Heisenberg's Uncertainty Principle

The fact in section 2 above that the axioms $\mathcal{A}$ already all in themselves give the Lorentz Transformations of inertial reference frames, thus one does not have to use for that purpose any of the two usual axioms of Special Relativity has never been considered to imply anything negative about these two usual axioms.

A similar situation, therefore, may be seen as happening with the Heisenberg Uncertainty Principle.

Appendix

The aim of the axiomatic method, as already envisaged and attained by Euclid in his Geometry more than two millennia ago, is to organize a set $\mathcal{T}$ of *theorems* which are supposed to formulate a given theory, and do it in the following way. Certain theorems, constituting a set $\mathcal{A}$, preferably a small subset of $\mathcal{T}$, are considered to be true, while the rest of the theorems in $\mathcal{T}$ are supposed to be obtainable as purely logical consequences of the theorems in $\mathcal{A}$. And whenever such a program can be achieved, then the theorems in $\mathcal{A}$ are called the *axioms* of the theory given by the theorems $\mathcal{T}$.

Clearly, for a given set $\mathcal{T}$ of theorems there may in general exist more than one set of axioms $\mathcal{A}$.

However, equally clearly, one simply *cannot* impose on the set $\mathcal{A}$ of axioms absolutely arbitrary requirements, no matter how much one's intuition would impel one to do so. Indeed, the two requirements any set of axioms $\mathcal{A}$ can be asked to satisfy are :

- $\mathcal{A}$ has to be a subset of $\mathcal{T}$, that is, $\mathcal{A} \subseteq \mathcal{T}$
- $\mathcal{T}$ has to be a logical consequence of $\mathcal{A}$

And what is crucial to understand is that any other conditions required on a set of axioms $\mathcal{A}$ may make it impossible for such a set to exist, or alternatively, may lead to another set of theorems $\mathcal{T}$. Details in this regard are presented now.

Let us recall briefly and in its essence the way axiomatic systems are conceived in Mathematics. One starts with a setup of a *formal deductive system*. Namely, let A be an *alphabet* which can be given by any nonvoid finite or infinite set. Then a procedure is given according to which one constructs - by using the symbols in A - a set $\mathcal{F}$ of *well formed formulas*, or in short wff-s. Next, one chooses a set $\mathcal{R}$ of *logical deduction rules* which operate as follows

(A.1) $\quad \mathcal{F} \supseteq P \stackrel{\mathcal{R}}{\longmapsto} Q \subseteq \mathcal{F}$

that is, from any set P of wff-s which are the *premises*, it leads to a corresponding set Q of wff-s which are the *consequences*.

And now come the *axioms* which can be any subset $\mathcal{A} \subseteq \mathcal{F}$ of wff-s.

Once the above is established, the respective *axiomatic theory* follows easily as being the *smallest* subset $\mathcal{T} \subseteq \mathcal{F}$ with the properties

(A.2) $\quad \mathcal{A} \subseteq \mathcal{T}$

(A.3) $\quad \mathcal{T} \supseteq P \stackrel{\mathcal{R}}{\longmapsto} Q \subseteq \mathcal{T}$

in which case the wff-s in $\mathcal{T}$ are called the *theorems* of the axiomatic system $\mathcal{A}$.

Of course, one should not forget that the set $\mathcal{T}$ of theorems depends essentially not only on the axioms in $\mathcal{A}$, but also on the logical deduction rules $\mathcal{R}$. Consequently, it is appropriate to write

(A.4) $\quad \mathcal{T}_{\mathcal{R}}(\mathcal{A})$

for the set $\mathcal{T}$ of theorems.

Here are some of the relevant questions which can arise regarding such axiomatic systems :

- are the axioms in $\mathcal{A}$ *independent* ?
- are the axioms in $\mathcal{A}$ *consistent* ?
- are the axioms in $\mathcal{A}$ *complete* ?

Independence means that for no axiom $P \in \mathcal{A}$, do we have $\mathcal{T}_{\mathcal{R}}(\mathcal{A}) = \mathcal{T}_{\mathcal{R}}(\mathcal{B})$, where $\mathcal{B} = \mathcal{A} \setminus \{P\}$. In other words, the axioms in $\mathcal{A}$ are minimal in order to obtain the theorems in $\mathcal{T}_{\mathcal{R}}(\mathcal{A})$. This condition can be formulated equivalently, but more simply and sharply, by saying that for no axiom $P \in \mathcal{A}$, do we have $P \in \mathcal{T}_{\mathcal{R}}(\mathcal{B})$, where $\mathcal{B} = \mathcal{A} \setminus \{P\}$.

As for consistency, it means that there is no $P \in \mathcal{T}_{\mathcal{R}}(\mathcal{A})$, such that for its negation *non* P, we have *non* $P \in \mathcal{T}_{\mathcal{R}}(\mathcal{A})$.

Completeness, in one possible formulation, means that, given any additional axiom $P \in \mathcal{F} \setminus \mathcal{A}$ which is independent from $\mathcal{A}$, the axiom system $\mathcal{B} = \mathcal{A} \cup \{P\}$ is inconsistent.

In the case of axiomatic systems for various theories of Physics, one is interested in the independence and consistency of the respective axioms. Independence means that the set of axioms is *minimal*, thus no axiom is a consequence of the other ones, therefore, no axiom can be eliminated without losing certain theorems. Consistency means that one cannot obtain contradictory consequences of the axioms.

The completeness of an axiomatic system for a given theory of Physics apparently has not yet been considered in the literature.

Back now to physically meaningful axioms

Let us see now in some more detail what can happen in case we start requiring certain conditions on the axioms of an axiomatic system. Namely, let be given a set $\mathcal{A} \subseteq \mathcal{F}$ of axioms. Then, as in (A.4), we have the corresponding theorems $\mathcal{T}_{\mathcal{R}}(\mathcal{A})$ of the axiomatic system $\mathcal{A}$.

Now, assume that we are not happy with the axioms in $\mathcal{A}$, and therefore, we want to replace them with another set $\mathcal{B} \subseteq \mathcal{F}$ of axioms.

Of course, in case such a replacement is not supposed to lead to another theory, then we must have

(A.5) $\quad \mathcal{T}_{\mathcal{R}}(\mathcal{B}) = \mathcal{T}_{\mathcal{R}}(\mathcal{A})$

Let us now look more closely to what can happen in such a process of replacement of axioms.

The fact that we are not happy with the initial axioms in $\mathcal{A}$ means that we want to choose the axioms not from the whole set $\mathcal{F}$ of wff-s, but only from a special subset $\mathcal{F}_{Phys} \subset \mathcal{F}$, say for example, the subset $\mathcal{F}_{Phys}$ of so called "physically meaningful" wff-s in $\mathcal{F}$.
In other words, in addition to condition (A.5), and in fact, prior to it, we also require the condition that

(A.6) $\quad \mathcal{B} \subseteq \mathcal{F}_{Phys}$

And now, it is obvious that, in general, conditions (A.5) and (A.6) may happen to be incompatible.

And in such a case, the natural way to proceed is to *weaken* the requirement (A.5), by replacing it with

(A.7) $\quad \mathcal{T}_{\mathcal{R}}(\mathcal{B}) \supseteq \mathcal{T}_{\mathcal{R}}(\mathcal{A})$

in case we do not wish to lose anything from the theory $\mathcal{T}_{\mathcal{R}}(\mathcal{A})$, when we replace the axioms $\mathcal{A}$ with the axioms $\mathcal{B}$.

In other words, instead of (A.5) and (A.6) which are incompatible, we have now (A.6) and (A.7). This means that the new set $\mathcal{B}$ of axioms certainly recovers all the theorems $\mathcal{T}_{\mathcal{R}}(\mathcal{A})$ in the axiomatic system $\mathcal{A}$ which was replaced. However, we risk to have *additional* theorems in $\mathcal{T}_{\mathcal{R}}(\mathcal{B})$, which were not among the theorems in $\mathcal{T}_{\mathcal{R}}(\mathcal{A})$. Thus in (A.7), we may in fact have the situation

$$\text{(A.8)} \qquad \mathcal{T}_{\mathcal{R}}(\mathcal{A}) \subsetneqq \mathcal{T}_{\mathcal{R}}(\mathcal{B})$$

due to the restriction (A.6).

Consequently, no matter how much justified by “physical intuition” may be a “physically meaningful” set of axioms, one has to be aware of the above.

In other words, “physical intuition” and being ”physically meaningful” need *not* automatically be in tune with the pure mathematical logic of axiomatic systems.

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