A non-vanishing cosmological constant is geometrodynamically prohibited

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If one solves the electromagnetic wave equation in the Lorenz gauge for a charged particle moving with a uniform velocity, and then inserts this solution into the gauge condition, mathematically one gets the result that the current density must be the charge density times the velocity. Thus the field equations of electromagnetism through the gauge condition imply a correct velocity requirement for the current density four-vector, much like the field equations of General Relativity imply through the Bianchi Identities a correct acceleration requirement.

Likewise we can construct a velocity requirement for the stress-energy tensor from the gauge condition of the Linear Field Equations of gravitation. However, the correct velocity relationship will be implied only if the cosmological constant vanishes. Thus the cosmological constant must vanish.

I. Introduction

The possibility of a cosmological constant was introduced by Einstein in 1917 in an attempt to cause the Field Equations to support a stable universe [1]. Ironically, shortly thereafter, observational data [2,3] indicated that the Universe was expanding rather than being maintained in a stable state.

Indeed, even if the Universe was in equilibrium, introduction of a cosmological constant would be somewhat questionable in that while it could produce equilibrium, the equilibrium would be unstable.

The currently accepted accelerated expansion of the Universe [4-12] cannot be explained by the standard Einstein Field Equations involving normal matter, and has led to renewed interest in adding a cosmological constant into the Field Equations. A non-zero cosmological constant is now considered to be a leading contender [4,13,14] for the purported "dark energy".

Theoretical justification for modification of the Field Equations by addition of a cosmological constant has been based on the fact that because the metric has a vanishing covariant derivative the addition of a cosmological constant term to the Einstein Tensor would not tamper with the covariant derivative of the stress-energy tensor being zero, a property necessary for the Field Equations to imply that matter acted upon by a gravitational field accelerates in a way consistent with the acceleration expected from the action of gravity.

However, while correctly implied acceleration is a necessary condition for a cosmological constant to be acceptable, it is not a sufficient condition. We will show that field equations can

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not only imply accelerations of sources, but can indeed also imply velocities of sources. For example, we will show that Maxwell's Equations require that the current density must be the charge density multiplied by the velocity.   We can also show that the Linear Field Approximation equations for General Relativity require the correct velocity relationships between the components of the stress-energy tensor, but only if the value of the cosmological constant is zero. Thus, contrary to previous belief, the equations of General Relativity cannot really allow a cosmological constant.

II. Example From Electromagnetism

i. Case of a Single Particle

In the Lorenz Gauge, the equations of electromagnetism are

$$\partial_\alpha \partial^\alpha A^\mu = \frac{4\pi}{c} J^\mu \tag{1a}$$

$$\partial_\mu A^\mu = 0 \tag{1b}$$

We of course "know" that $J^i = J^0 \frac{dx^i}{dt}$ for a point charge, but here we will not assume it but rather just treat J^i as the quantity appearing in Equation 1a, and then prove that Equations 1 *imply* J^i must be of the form $J^0 \frac{dx^i}{dt}$.

For a point charge moving with a uniform velocity, the solution to the Lienard-Wiechert formula takes on the surprisingly simple form [15]:

$$A^0(x) = \gamma \frac{Q}{|x'-x|}, \ A^i(x) = \gamma \frac{R}{|x'-x|} \tag{2}$$

Where $\gamma = \frac{1}{\sqrt{1-v^2/c^2}}$, $Q \equiv \int \rho(x') d^3x'$, and $R \equiv \int J^i(x') d^3x'$

Inserting Equations 2 into Equation 1b, we get

$$\frac{\partial}{\partial t}\left(\frac{\gamma Q}{|x'-x|}\right) + \frac{\partial}{\partial x^i}\left(\frac{\gamma R}{|x'-x|}\right) = 0 \tag{3}$$

implying

$$\left(\frac{\partial}{\partial x^{i}{}'}\frac{\gamma Q}{|x'-x|}\right)\frac{dx^{i}{}'}{dt}+\frac{\partial}{\partial x^{i}}\left(\frac{\gamma R}{|x'-x|}\right)=0 \tag{4}$$

Since

$$\frac{\partial}{\partial x'}\left(\frac{1}{|x'-x|}\right)=\frac{-\partial}{\partial x}\left(\frac{1}{|x'-x|}\right) \tag{5}$$

Equation 4 can be written

$$\left(\frac{-\partial}{\partial x^{i}}\frac{\gamma Q}{|x'-x|}\right)\frac{dx^{i}{}'}{dt}+\frac{\partial}{\partial x^{i}}\left(\frac{\gamma R}{|x'-x|}\right)=0 \tag{6}$$

From Equation 6, we see that it *must* be the case that $Q\frac{dx^{i}{}'}{dt}$ is equal to R which of course implies that $J^{i}=J^{0}\frac{dx^{i}{}'}{dt}$. The field equations imply that J^{i} moves with the physically required velocity. Had the field equations implied otherwise, they could not be acceptable to describe what physically is believed about the J^{μ} four-vector.

The result that the standard equations of electromagnetism imply a correct velocity relationship between J^{0} and J^{i} might seem trivial, but in reality it is highly non-trivial. We can see this by considering a concrete example. If the equations had been, for example, $\partial_{\alpha}\partial^{\alpha}A^{\mu}=\frac{4\pi}{c}J^{\mu}$ with $\partial_{\mu}A^{\mu}=A^{\mu}J_{\mu}$, rather than $\partial_{\alpha}\partial^{\alpha}A^{\mu}=\frac{4\pi}{c}J^{\mu}$ with $\partial_{\mu}A^{\mu}=0$, it would not be possible for J^{i} to be equal to $J^{0}\frac{dx^{i}{}'}{dt}$.

ii. Case of a Continuum of Rigidly Attached Particles

Let us derive the result for a general distribution of charge, not just a point charge, moving with a unique velocity—i.e. all of the charges moving with the same velocity, as if rigidly attached to each other. For the cases of two such charges, a treatment like that above shows in a straightforward way that $(Q_1+Q_2)\frac{dx^{i}{}'}{dt}=(R_1+R_2)$. For a continuum of charges moving with the same velocity as if rigidly attached to each other, more specifically, a system where

$A^{\mu}(x)=\int\frac{\gamma J^{\mu}(x')}{|x'-x|}d^3x'$ and where the charges move with the same velocities, the gauge condition implies

$$\left(\int\rho(x')d^3x'\right)\frac{dx^{i}{}'}{dt}=\left(\int J^{i}(x')d^3x'\right)\tag{7}$$

iii. Generalization

Our result that

1) a charge distribution is moving rigidly with all the pieces moving at the same velocity

and

2) $A^{\mu}(x)=\int\frac{\gamma J^{\mu}(x')}{|x'-x|}d^3x'$

and

3) $\partial_{\mu}A^{\mu}=0$

is required by mathematical identity to conform to

$$J^{0}\frac{dx^{i}{}'}{dt}=J^{i}.$$

can trivially be generalized to, for example, indicate that:

1) If a matter distribution is moving rigidly with all the pieces moving at the same velocity
2) and the quantities $M^{\mu\nu}$ and $L^{\mu\nu}$ exist such that $L^{\mu\nu}(x)=\int\frac{\gamma M^{\mu\nu}(x')}{|x'-x|}d^3x'$

   and
3) $\partial_{\mu}L^{\mu\nu}(x)=0$

Then

$$M^{0\nu}\left(\frac{dx^{i}{}'}{dt}\right)=M^{i\nu}\tag{8}$$

III. Example From Gravitation

For the Linear Field Equations of gravitation in the gauge analogous to the Lorenz Gauge of electromagnetism, the equations are:

$$\partial_\alpha \partial^\alpha \omega^{\mu\nu} = \kappa T^{\mu\nu} \tag{9a}$$

$$\partial_\mu \omega^{\mu\nu} = 0 \tag{9b}$$

where $g^{\mu\nu} = \eta^{\mu\nu} + h^{\mu\nu}$, and $\omega^{\mu\nu} = h^{\mu\nu} - \frac{1}{2}\eta^{\mu\nu} h$.

Applying Equation 8, where the $M^{\mu\nu}$ is $T^{\mu\nu}$, and the and $L^{\mu\nu}$ is $\omega^{\mu\nu}$, we get that

$$T^{0\nu}\left(\frac{dx^{i\,'}}{dt}\right) = T^{i\nu}.$$

Thus just as we demonstrated that the field equations of electromagnetism imply the correct velocity relationships between the components of the current-density for a point charge with uniform velocity, the linear field equations of gravitation imply the correct velocity relationships between the components of the stress-energy tensor for a point mass with uniform velocity. Had they done otherwise they could not be acceptable to describe what physically is believed about the stress-energy tensor. So we see that just as the Einstein equations for gravitation require (via the Bianchi Identities) that a particle in a gravitational field moves with the appropriate acceleration, the equations for electromagnetism and gravitation also turn out to require that a particle move with the appropriate velocity.

IV. A Modification of the Linear Field Equations

Before considering the behavior of the Linear Field Equations with a cosmological constant it will be pedagogically useful to examine the behavior of the Linear Field Equations with a more simple modification. Let us add a term $\Lambda\omega^{\mu\nu}$ to the left-hand side of the linearized equations of gravitation.[1]

Working in the equivalent of the Lorenz gauge[2] we have the equations:

$$\partial_\alpha \partial^\alpha \omega^{\mu\nu} + \Lambda\omega^{\mu\nu} = \kappa T^{\mu\nu} \tag{10a}$$

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[1] This system however of course does not represent the linear field equations with a cosmological constant--the cosmological constant term is $\Lambda g^{\mu\nu}$, not $\Lambda\omega^{\mu\nu}$. The quantity $\omega^{\mu\nu}$ is most certainly not $g^{\mu\nu}$. Our use of an equation with the additional $\Lambda\omega^{\mu\nu}$ term is done only to make the treatment of the Cosmological Constant situation which we will engage in later in the paper easier to analyze.

[2] The linear field equation with the added $\Lambda\omega^{\mu\nu}$ term are not gauge invariant, and thus it is not immediately obvious that splitting of that equation into (10a) and (10b) is really justified. However, such a splitting will turn out to yield a solution, and clearly such a solution will satisfy our modified linear field equation-- and thus justification is eventually demonstrated.

$$\partial_{\mu}\omega^{\mu\nu} = 0 \tag{10b}$$

Although Equation 10a is a Yukawa Equation, for our purposes we do not want to analyze it via that route. It will be useful to us to use a method of successive approximations.

Let us move the $\Lambda\omega^{\mu\nu}$ term to the right-hand side, and treat the situation as one where the $\Lambda\omega^{\mu\nu}$ term acts as if it was a source along with the $T^{\mu\nu}$ source.

$$\partial_{\alpha}\partial^{\alpha}\omega^{\mu\nu} = \kappa T^{\mu\nu} - \Lambda\omega^{\mu\nu} \tag{11a}$$

$$\partial_{\mu}\omega^{\mu\nu} = 0 \tag{11b}$$

Consider a situation where $T^{\mu\nu}$ is sufficiently small that $\omega^{\mu\nu}$ is small enough that a second-order approximation is sufficiently accurate. (The method though could be extended in a straightforward way to work for all orders of approximation.)

The first order value of $\omega^{\mu\nu}$, which we will call $\omega^{\mu\nu}_{(1)}$ comes from the action of the $T^{\mu\nu}$.

$$\partial_{\alpha}\partial^{\alpha}\omega^{\mu\nu}_{(1)} = \kappa T^{\mu\nu} \tag{12}$$

The solution for the case of mass distribution moving with uniform velocity is

$$\omega^{\mu\nu}_{(1)} = \int \frac{\gamma\kappa T^{\mu\nu}(x')}{|x - x'|} d^3x' \tag{13}$$

We will assume that $T^{\mu\nu}$ has the correct velocity relationships — i.e. that $T^{0\nu}\dfrac{dx^{i}{}'}{dt} = T^{i\nu}$ — and see if that leads to a contradiction. Therefore, proceeding under that assumption, Equation 13 indicates that the $\omega^{\mu\nu}_{(1)}$ field moves rigidly through space with the same velocity as the $T^{\mu\nu}$ field.

Next we calculate the second order approximation $\omega^{\mu\nu}_{(2)}$ from Equation 11a, where we use $\omega^{\mu\nu}_{(1)}$ as the approximate value of $\omega^{\mu\nu}$ in Equation 11a

$$\partial_{\alpha}\partial^{\alpha}\omega^{\mu\nu}_{(2)} = \kappa T^{\mu\nu} - \Lambda\omega^{\mu\nu}_{(1)} \tag{14}$$

Equation 14 indicates that mathematically the $\Lambda\omega^{\mu\nu}_{(1)}$ field acts like a supplemental $T^{\mu\nu}$ field. The solution to Equation 14 is thus

$$\omega^{\mu\nu}_{(2)} = \int \frac{\gamma\kappa T^{\mu\nu}(x') - \gamma\Lambda\omega^{\mu\nu}_{(1)}(x')}{|x - x'|} d^3x' \tag{15}$$

And as noted in our analysis of Equation 13, $\omega^{\mu\nu}_{(1)}$ moves rigidly attached to the $T^{\mu\nu}$ field with the same velocity as the $T^{\mu\nu}$ field. Therefore Equation 14 is a special case of the Equation 8 situation, where $\kappa T^{\mu\nu}(x') - \Lambda\omega^{\mu\nu}_{(1)}(x')$ is the $M^{\mu\nu}(x')$ and $\omega^{\mu\nu}_{(2)}(x)$ is the $L^{\mu\nu}(x)$. Thus, applying Equation 8, we get

$$\left(\kappa T^{0\nu} - \Lambda\omega^{0\nu}_{(1)}\right)\frac{dx^{i}{}'}{dt} = \left(\kappa T^{i\nu} - \Lambda\omega^{i\nu}_{(1)}\right) \tag{16}$$

Substituting Equation 13 into Equation 16, we get

$$\left(\kappa T^{0\nu}(x) - \Lambda\int \frac{\gamma\kappa T^{0\nu}(x')}{|x - x'|} d^3x'\right)\frac{dx^{i}{}'}{dt} = \left(\kappa T^{i\nu}(x) - \Lambda\int \frac{\gamma\kappa T^{i\nu}(x')}{|x - x'|} d^3x'\right) \tag{17}$$

Equation 11 is clearly not inconsistent with $T^{0\nu}\dfrac{dx^{i}{}'}{dt} = T^{i\nu}$ and thus a putative linearized system of gravitation given by Equations 11 is not ruled out by our geometrodynamic velocity requirement.

However, Equations 11 are neither tensor equations, nor do they satisfy the Bianchi Identities. The equations of General Relativity with a cosmological constant are tensor equations and do satisfy the Bianchi Identities, so we now apply our methodology to them.

V. Gravitation with a Cosmological Constant

The linearized equations in the equivalent of the Lorenz gauge for General Relativity with a cosmological constant are:[3]

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[3] Again we face a situation where a modified linear field equation is not actually gauge invariant, and thus the splitting is not immediately obviously justified. Our reasoning here is similar to our reasoning in the footnote in the previous section,--we will be able to show that field equations with a cosmological constant do have solutions in the gauge choice we are making, and thus those solutions are solutions to the general equations. These solutions will be solutions with unphysical velocity behavior, thus leading us to conclude that a non-vanishing cosmological constant cannot exist.

$$\partial_\alpha \partial^\alpha \omega^{\mu\nu} + \Lambda\left(\eta^{\mu\nu} + h^{\mu\nu}\right) = \kappa T^{\mu\nu} \tag{18a}$$

$$\partial_\mu \omega^{\mu\nu} = 0 \tag{18b}$$

We re-write them as

$$\partial_\alpha \partial^\alpha \omega^{\mu\nu} = \kappa T^{\mu\nu} - \Lambda\left(\eta^{\mu\nu} + \omega^{\mu\nu} - \tfrac{1}{2}\omega\eta^{\mu\nu}\right) \tag{19a}$$

$$\partial_\mu \omega^{\mu\nu} = 0 \tag{19b}$$

We must digress to address something that could become a serious point of confusion. When the weak-field approximation for General Relativity is expressed in the form of Equation 19a and Equation 19b, there actually is no de Sitter effect. Consider the (0,0) Einstein Field Equation (with cosmological constant) written in the form $R^{00} = 8\pi\kappa\left(T^{00} - \tfrac{1}{2}g^{00}T - \Lambda g^{00}\right)$. The left hand side, if the fields are weak enough for linearity to be a reasonable approximation, can be thought of as being composed of two pieces. One piece is essentially $\nabla^2 g_{00}$, and produces the sort of Newtonian gravitation that causes planets to orbit the Sun and objects to accelerate downwards on Earth. The other piece contains second derivatives with respect to time of g_{11}, g_{22} and g_{33}. This piece causes expansion of the Universe effects. If there is a cosmological constant, the second piece causes the de Sitter effects. So we have $T^{00} - \tfrac{1}{2}g^{00}T$ as well as a supplementary Λg_{00} term driving two effects—the Newtonian gravity effect and the expansion of the Universe effect. However when we made the gauge transformation to get Equation 19a, the expansion of the Universe effect became no longer present. In the Equation 19b gauge, the $\nabla^2 g_{00}$ term becomes a four-dimensional wave equation (i.e. the three dimensional ∇^2 becomes the four-dimensional $\nabla^2 - \dfrac{d^2}{dt^2}$ *and the expansion of the Universe term disappears*. Thus, in this gauge, the equations do not contain any de Sitter effect, and an assumption of such an effect would be due to an erroneous prejudice not consistent with the actual operative equations.

Noting the similarity between Equation 19a and Equation 11a – the only difference is that the $\Lambda\omega^{\mu\nu}$ on the right-hand side of Equation 11a is replaced by $\Lambda\left(\eta^{\mu\nu} + \omega^{\mu\nu} - \tfrac{1}{2}\omega\eta^{\mu\nu}\right)$. We therefore start to proceed analogously.

The first order value of $\omega^{\mu\nu}$, which we will call $\omega_{(1)}^{\mu\nu}$, comes from the action of the $T^{\mu\nu}$.

$$\partial_\alpha \partial^\alpha \omega_{(1)}^{\mu\nu} = \kappa T^{\mu\nu} \tag{20}$$

The solution for the case of mass distribution moving with uniform velocity is

$$\omega_{(1)}^{\mu\nu} = \int \frac{\gamma\kappa T^{\mu\nu}(x')}{|x - x'|} d^3x' \tag{21}$$

Like before will assume that $T^{\mu\nu}$ has the correct velocity relationships — i.e. that $T^{0\nu}\dfrac{dx^{i}{}'}{dt} = T^{i\nu}$ — and see if that leads to a contradiction. We see that Equation 21 implies, as Equation 13 had implied in the previous section, that the $\omega_{(1)}^{\mu\nu}$ field moves rigidly through space with the same velocity as the $T^{\mu\nu}$ field.

Next we calculate the second order value of $\omega^{\mu\nu}$.

$$\partial_\alpha \partial^\alpha \left(\omega_{(2)}^{\mu\nu}\right) = \kappa T^{\mu\nu} - \Lambda\left(\eta^{\mu\nu} + \omega_{(1)}^{\mu\nu} - \tfrac{1}{2}\omega_{(1)}\eta^{\mu\nu}\right) \tag{22}$$

Equation 22 indicates that mathematically the $\Lambda\left(\eta^{\mu\nu} + \omega_{(1)}^{\mu\nu} - \tfrac{1}{2}\omega_{(1)}\eta^{\mu\nu}\right)$ field acts like a supplemental $T^{\mu\nu}$ field. The solution to Equation 22 is thus, analogous to Equation 15 in the previous section,

$$\omega_{(2)}^{\mu\nu} = \int \frac{\gamma\kappa T^{\mu\nu}(x') - \gamma\Lambda\left(\eta^{\mu\nu}(x') + \omega_{(1)}^{\mu\nu}(x') - \tfrac{1}{2}\omega_{(1)}(x')\eta^{\mu\nu}\right)}{|x - x'|} d^3x' \tag{23}$$

In order for the $T^{\mu\nu}$ to have the proper velocity relationships we must be able to apply Equation 8 to Equation 23, where $M^{\mu\nu}$ would be $\kappa T^{\mu\nu}(x') - \Lambda\left(\eta^{\mu\nu}(x') + \omega_{(1)}^{\mu\nu}(x') - \tfrac{1}{2}\omega_{(1)}(x')\eta^{\mu\nu}\right)$. To do so we need to be assured that the $\kappa T^{\mu\nu}(x') - \Lambda\left(\eta^{\mu\nu}(x') + \omega_{(1)}^{\mu\nu}(x') - \tfrac{1}{2}\omega_{(1)}(x')\eta^{\mu\nu}\right)$ field moves through space rigidly with a single velocity. Let's proceed by assuming that the $T^{\mu\nu}$ field moves rigidly with a single velocity and that it has the proper velocity relationships between its components (which will lead us to later conclude that in reality it actually *cannot* have the proper velocity relationships). From Equation 21 this assumption about $T^{\mu\nu}$ causes $\omega_{(1)}^{\mu\nu}$ to also move rigidly with the velocity of the $T^{\mu\nu}$ field. By virtue of $\omega_{(1)}^{\mu\nu}$ moving rigidly with the velocity of the $T^{\mu\nu}$ field, so will $\omega_{(1)}$, and by virtue of $\omega_{(1)}$ moving that way, so will $\omega_{(1)}\ \eta^{\mu\nu}$. But what about the $\eta^{\mu\nu}$ term?

Is the field generated by the $\eta^{\mu\nu}$ term, a field of the form $\int \frac{\gamma\eta^{\mu\nu}(x')}{|x-x'|} d^3x'$ a moving field produced by a moving $\eta^{\mu\nu}$ that moves rigidly with the $T^{\mu\nu}$ field? Is it instead a constant background field, in which case it can be ignored when we apply Equation 8 to Equation 23?

Since the $\eta^{\mu\nu}$ field is a spatially uniform field, whether it moves or not is physically moot. So the answer to whether we should consider it to be moving is both “yes” and “no”.

One can argue from symmetry considerations that the field generated by $\eta^{\mu\nu}$ field would need to be itself spatially uniform since there is no preferred direction in the distribution of the $\eta^{\mu\nu}$ field, and that the spatial uniformity of the generated field makes the answer to the question of whether this generated field moves both “yes” and “no”, just as was the situation regarding whether the $\eta^{\mu\nu}$ field moves. However one can instead argue that the generated field is not uniform, as is very often implicitly done. Consider a point mass at the origin of a coordinate system in a linear field equation scenario with a cosmological constant. The situation is often treated as if there was an effective mass equal to the sum of the point mass’ mass plus the volume integral of $\Lambda\eta^{\mu\nu}$. The strength of the gravitational field would actually increase with distance as compared to a situation with no cosmological constant. In the limit of the point mass’ mass going to zero we actually have a very disturbing asymmetry in the induced gravitational field, an asymmetry occurring for no physical reason—the origin of the coordinate system, though arbitrary, becomes a special position. Indeed whether light travelling from one point to another point is red-shifted or blue-shifted would depend on which point we arbitrarily assign to be the origin of the coordinate system—a physically intolerable state of affairs. Likewise, we can expect that in a zero mass case with a cosmological constant the system should behave like a deSitter Universe, a behavior disturbingly non-consistent with the case of a finite mass of unlimitedly decreasing magnitude.

It is important to note that these ambiguities are not problems with our analysis, but rather are problems inherent with having a non-vanishing cosmological constant. Even without the result we will formally reach later in this paper, the red-shift/blue-shift ambiguity is enough to make the possibility of a non-vanishing cosmological constant dubious.

If we choose the “ x ” in $\int \frac{\gamma\eta^{\mu\nu}(x')}{|x-x'|} d^3x'$ to be the origin then in that special case we can unambiguously drop the $\eta^{\mu\nu}$ term in the gauge condition calculation, because $\partial_\mu \int \frac{\gamma\eta^{\mu\nu}(x')}{|x-x'|} d^3x'$ will contain only vanishing terms—i.e. $\frac{\partial}{\partial t}\int \frac{\gamma\eta^{0\nu}(x')}{|x-x'|} d^3x'$ will be zero because $\eta^{0\nu}$ is constant in time, and $\frac{\partial}{\partial x^i}\int \frac{\gamma\eta^{i\nu}(x')}{|x-x'|} d^3x'$ will vanish because $\int \frac{\gamma\eta^{\mu\nu}(x')}{|x-x'|} d^3x'$, the metric generated by the cosmological constant goes as r^2 about the origin. So we can set up a special case where we can unambiguously drop the $\eta^{\mu\nu}$ term from Equation 23. *However, these unpleasant considerations actually are of no importance, being that it turns out that* *regardless of whether we include the* $\eta^{\mu\nu}$ *term in Equation 23 or do not include it, it turns out that we would get the same final conclusion that the cosmological constant must vanish.* We will proceed dropping the $\eta^{\mu\nu}$ term.

Applying Equation 8 to Equation 23, we get

$$\left[\kappa T^{0\nu} + \Lambda\left(\omega_{(1)}^{0\nu}(x') - \tfrac{1}{2}\omega_{(1)}(x')\eta^{0\nu}\right)\right]\frac{dx^{i}{}'}{dt}$$

$$= \left[\kappa T^{i\nu} + \Lambda\left(\omega_{(1)}^{i\nu}(x') - \tfrac{1}{2}\omega_{(1)}(x')\eta^{i\nu}\right)\right]. \tag{24}$$

Substituting Equation 21 into Equation 24 we get

$$\left[\kappa T^{0\nu} + \Lambda\left(\int \frac{\gamma\kappa T^{0\nu}(x')}{|x-x'|}d^3x' - \tfrac{1}{2}\int \frac{\gamma\kappa T(x')}{|x-x'|}d^3x'\eta^{0\nu}\right)\right]\frac{dx^{i}{}'}{dt}$$

$$= \left[\kappa T^{i\nu} + \Lambda\left(\int \frac{\gamma\kappa T^{i\nu}(x')}{|x-x'|}d^3x' - \tfrac{1}{2}\int \frac{\gamma\kappa T(x')}{|x-x'|}d^3x'\eta^{i\nu}\right)\right] \tag{25}$$

where $T \equiv \eta_{\mu\nu}T^{\mu\nu}$.

To see if this is consistent with the correct velocity relationship of $T^{0\nu}\left(\frac{dx^{i}{}'}{dt}\right) = T^{i\nu}$ we substitute $T^{0\nu}\frac{dx^{i}}{dt} = T^{i\nu}$ into Equation 25 and, cancelling terms, we get

$$\kappa\Lambda\left(-\tfrac{1}{2}\int \frac{\gamma T(x')}{|x-x'|}d^3x'\eta^{0\nu}\frac{dx^{i}{}'}{dt}\right) = \kappa\Lambda\left(-\tfrac{1}{2}\int \frac{\gamma T(x')}{|x-x'|}d^3x'\eta^{i\nu}\right). \tag{26}$$

Let us consider what happens in Equation 26 when ν is 0. Noting that $\eta^{00} = 1$ and $\eta^{i0} = 0$, we get

$$\kappa\Lambda\left(-\tfrac{1}{2}\int \frac{\gamma T(x')}{|x-x'|}d^3x'\frac{dx^{i}{}'}{dt}\right) = 0 \tag{27}$$

This cannot be true for a non-vanishing $\frac{dx^{i}{}'}{dt}$ unless Λ vanishes! Thus we have shown that in order for the field equations with a cosmological constant to have the gauge condition consistent with $T^{i\nu} = T^{0\nu}\frac{dx^{i}{}'}{dt}$ the value of the cosmological constant must be zero.

VI. Conclusion

As is well-known, adding a cosmological constant to the Einstein Equations term does not produce a conflict with the Bianchi Identities. Compliance with the Bianchi Identities ensures that a cosmological constant term will not imply unphysical acceleration characteristics of a particle in a gravitational field. However, this is not a sufficient condition to allow such a term in the Einstein Equations. We have shown the addition of a cosmological constant term implies unphysical velocity characteristics.

Appendix

We have derived the vanishing of the cosmological constant by assuming that the stress-energy tensor must have the accepted velocity relationships. One might wonder if maybe a cosmological constant does exist, and that the real velocity relationships for the stress-energy might be slightly different from assumed. We can show that is moot.

If we define $\kappa\overline{T}^{\mu\nu}$ as $\kappa T^{\mu\nu} - \Lambda\left(\eta^{\mu\nu} + h^{\mu\nu}\right)$, then Equation 18a will become $\partial_{\alpha}\partial^{\alpha}\omega^{\mu\nu} = \kappa\overline{T}^{\mu\nu}$

We see that the quantity $\overline{T}^{\mu\nu}$ will obey the relationship $\overline{T}^{i\nu} = \overline{T}^{0\nu}\frac{dx^{i}{}'}{dt}$, will obey the Bianchi Identities, and will generate a gravitational field without cosmological constant term effects. So even if the equations have a cosmological constant, the geometrodynamical *physical* behavior will be as if no cosmological constant term were present, with the new quantity $\overline{T}^{\mu\nu}$ behaving just like a stress-energy tensor is assumed to behave both in terms of velocity and acceleration, and in terms of field generation. So we are led back to General Relativity without a cosmological constant.

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