On Superluminal particles and the Extended Relativity theories

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Abstract

Superluminal particles are studied within the framework of the Extended Relativity theory in Clifford spaces (C-spaces). In the simplest scenario, it is found that it is the contribution of the Clifford scalar component π of the poly-vector-valued momentum which is responsible for the superluminal behavior in ordinary spacetime due to the fact that the *effective* mass $\mathcal{M} = \sqrt{M^2 - \pi^2}$ is imaginary (tachyonic). However, from the point of view of C-space, there is *no* superluminal (tachyonic) behavior because the true physical mass still obeys $M^2 > 0$. Therefore, there are *no* violations of the Clifford-extended Lorentz invariance and the extended Relativity principle in C-spaces. Furthermore, to lowest order, there is *no* contribution of terms involving powers of the Planck mass ($1/m_P^2$) indicating that quantum gravitational effects do *not* play a role at this order. A Born's Reciprocal Relativity theory in Phase Spaces leads to modified dispersion relations involving *both* coordinates and momenta, and whose *truncations* furnish Lorentz-violating dispersion relations which appear in Finsler Geometry, rainbow-metrics models and Double (deformed) Special Relativity. These models also admit superluminal particles. A numerical analysis based on the recent OPERA experimental findings on alleged superluminal muon neutrinos is made. For the average muon neutrino energy of 17 Gev, we find a value for $\pi = 119.7$ Mev that, coincidentally, is close to the mass of the muon $m_\mu = 105.7$ Mev.

1 The Extended Relativity in Clifford Spaces

In the past years an Extended Relativity Theory in C-spaces (Clifford spaces) and Clifford-Phase spaces were developed [1], [2]. The poly-vector valued coordinates $x^\mu, x^{\mu_1\mu_2}, x^{\mu_1\mu_2\mu_3}, ...$ are now linked to the basis vectors generators γ^μ, bi-vectors generators $\gamma_\mu \wedge \gamma_\nu$, tri-vectors generators $\gamma_{\mu_1} \wedge \gamma_{\mu_2} \wedge \gamma_{\mu_3}$, ... of the Clifford algebra, including the Clifford algebra unit element (associated to a scalar coordinate). These poly-vector valued coordinates can be interpreted as the quenched-degrees of freedom of an ensemble of p-loops associated with the dynamics of closed p-branes, for $p = 0, 1, 2, ..., D-1$, embedded in a target D-dimensional spacetime background [3].

The C-space poly-vector-valued momentum is defined as $\mathbf{P} = d\mathbf{X}/d\Sigma$ where $\mathbf{X}$ is the Clifford-valued coordinate corresponding to the $Cl(1,3)$ algebra in $D = 4$

$$\mathbf{X} \;=\; \sigma\,\mathbf{1} + x^\mu\,\gamma_\mu + x^{\mu\nu}\,\gamma_\mu\wedge\gamma_\nu + x^{\mu\nu\rho}\,\gamma_\mu\wedge\gamma_\mu\wedge\gamma_\rho + x^{\mu\nu\rho\tau}\,\gamma_\mu\wedge\gamma_\mu\wedge\gamma_\rho\wedge\gamma_\tau \quad (1.1)$$

σ is the Clifford scalar component of the poly-vector-valued coordinate and $d\Sigma$ is the infinitesimal C-space proper "time" interval which is *invariant* under $Cl(1,3)$ transformations which are the Clifford-algebra extensions of the $SO(1,3)$ Lorentz transformations [1]. One should emphasize that $d\Sigma$, which is given by the square root of the quadratic interval in C-space, is *not* equal to the proper time Lorentz-invariant interval ds in ordinary spacetime $(ds)^2 = g_{\mu\nu}dx^\mu dx^\nu$.

We begin by writing the C-space poly-vector-valued momentum $\mathbf{P} = d\mathbf{X}/d\Sigma$ in the form described in [5]

$$\mathbf{P} \;=\; \pi\;\mathbf{1} \;+\; p^\mu\;\gamma_\mu \;+\; p^{\mu\nu}\;\gamma_\mu\wedge\gamma_\nu \;+\; \pi^\mu\;\gamma_5\;\gamma_\mu \;+\; p^{0123}\;\gamma_5. \qquad (1.2)$$

where $(\gamma_5)^2 = -\mathbf{1},\ \{\gamma^\mu, \gamma^5\} = 0$. The C-space invariant norm-squared of a momentum poly-vector is defined by the scalar part of the Clifford geometric product of $<\ \mathbf{P}^\sim\ \mathbf{P}\ >$ where $\mathbf{P}^\sim$ is the reversal-conjugate of $\mathbf{P}$ obtained by reversing the order of the gamma factors in the decomposition of the poly-vector $\mathbf{P}$ [6]. The norm-squared is

$$||\ \mathbf{P}\ ||^2 \;=\; \pi^2 \;+\; p_\mu\;p^\mu \;+\; \frac{1}{2}\;p_{\mu\nu}\;p^{\mu\nu} \;+\; \frac{1}{3!}\;p_{\mu\nu\rho}\;p^{\mu\nu\rho} \;+\frac{1}{4!}\;p_{\mu\nu\rho\tau}\;p^{\mu\nu\rho\tau} \;=$$

$$\pi^2 \;+\; p_\mu\;p^\mu \;+\; \frac{1}{2}\;p^{\mu\nu}\;p_{\mu\nu} \;+\; \pi_\mu\;\pi^\mu \;+\; (p_{0123})(p^{0123}). \qquad (1.3)$$

it is necessary to introduce suitable powers of the Planck mass (that is set to unity) in order to match the units in the terms of eqs-(1, 2). The spin bi-vector $S^{\mu\nu}$ can be represented by the momentum bi-vector $p^{\mu\nu}$ (up to a power of $m^2_{Planck} = 1$) as explained in detail by [4]. A natural coupling of the classical spin (spin bi-vector $S^{\mu\nu}$) to the linear motion of the particle providing a new derivation of the Papapetrou equations can be found in [4] .

To simplify the calculations one may study a poly-particle in $D = 2$ spacetime dimensions, associated to a linear one-dimensional motion along the x^1 axis, so that

$$||\ \mathbf{P}\ ||^2 \;=\; \pi^2 \;+\; p_\mu\;p^\mu \;+\; \frac{1}{2m_P^2}\;p^{\mu\nu}\;p_{\mu\nu} \;=\; M^2 \;\Rightarrow$$

$$\pi^2 \; + \; (E^2 - p_1^2) \; + \; \frac{1}{2m_P^2} \; g_{00} \; g_{11} \; p^{01} \; p^{01} \; = $$

$$\pi^2 \; + \; (E^2 - p_1^2) \; - \; \xi^2 \; (E^2 - p_1^2)^2 \; = \; M^2 \tag{1.4}$$

after one sets on dimensional grounds

$$\frac{1}{2m_P^2} \; (p^{01})^2 \; = \; \frac{\lambda^2}{m_P^2} \; \mathbf{p}^4 \; \equiv \; \xi^2 \; \mathbf{p}^4 \; = \; \xi^2 \; (E^2 - p_1^2)^2; \quad \frac{\lambda^2}{m_P^2} = \xi^2 \tag{1.5}$$

The ratio $\frac{|\vec{p}|}{E}$, in natural units of $\hbar = c = 1$, is a measure of subluminal/superluminal velocity behavior. In this section we will follow the wave picture of particle propagation and focus on the group velocities, instead. Hence, after setting $E = \omega$ and $p_1 = k$ eq-(1.4) becomes

$$\pi^2 \; + \; \omega^2 \; - \; k^2 \; - \; \xi^2(\omega^4 - 2\omega^2 k^2 + k^4) \; = \; M^2 \; \Rightarrow$$

$$\omega^2 \; = \; \frac{1 \; + \; 2\xi^2 k^2 \; \pm \; \sqrt{1 - 4\xi^2(M^2 - \pi^2)}}{2\xi^2} \tag{1.6}$$

The ambiguity in the $\pm$ sign under the square root is *resolved* by noticing that by taking the *minus* sign it yields the ordinary Minkowski space dispersion relation $\omega^2 - k^2 = (M^2 - \pi^2) = \mathcal{M}^2$, in the limit $\xi \to 0$; namely this limit is attained by setting the bivector momentum components of the poly-particle to zero.

In order to obtain the group velocity $d\omega/dk$, instead of taking the square root of the expression in eq-(1.6), it is more convenient to differentiate eq-(1.4) directly

$$\pi \; d\pi \; + \; (\omega \; d\omega - k \; dk) \; [\; 1 \; - \; 2\xi^2 \; (\omega^2 - k^2) \;] \; = \; 0 \; \Rightarrow$$

$$\frac{\omega}{k} \; \frac{d\omega}{dk} \; = \; 1 \; - \; \frac{(\pi/k) \; (d\pi/dk)}{1 - 2\xi^2(\omega^2 - k^2)} \tag{1.7}$$

After some straightforward algebra one can recast eq-(1.7) in the equivalent form

$$\frac{\omega}{k} \; \frac{d\omega}{dk} \; = \; 1 \; + \; \frac{(\mathcal{M}/k) \; (d\mathcal{M}/dk)}{\sqrt{1 - 4\xi^2 \mathcal{M}^2}}; \quad \mathcal{M}^2 \equiv M^2 - \pi^2. \tag{1.8}$$

As discussed in [1], [5], one can have tachyonic (superluminal) behavior in ordinary spacetime while having *non*-tachyonic behavior in C-space. Hence from the C-space point of view there is *no* violation of the Clifford-extended Lorentz symmetry. This can easily be seen by noticing that despite setting the tachyonic condition $\omega^2 - k^2 < 0$ in eq-(1.4), one may still maintain $M^2 > 0$ in the right hand side of eq-(1.4) due to the *crucial* presence of the Clifford scalar component π^2 of the poly-momentum in eq-(1.4). If this is the case, after reinserting the proper units, one can write the product of the phase an group velocities as

$$\frac{\omega}{k}\,\frac{d\omega}{dk} \;=\; c^2\,\left(1\;-\;\frac{(\pi/k)\;(d\pi/dk)}{1-2\xi^2(\omega^2-k^2)}\right)\;=\;c^2\,\left(1\;-\;\frac{(\pi/k)\;(d\pi/dk)}{1+2\xi^2(k^2-\omega^2)}\right)\;\equiv\;c^2_{eff}(\omega,k) \tag{1.9}$$

where the $effective$ speed of light c_{eff} is now energy-momentum dependent (fact that may be related to the emitted photons in gamma ray bursts). There are two interesting cases to consider in eq-(1.9)

(i) when $(\pi/k)(d\pi/dk) < 0$ and $k^2-\omega^2 > 0$, one infers that $c^2_{eff}(\omega,k) > c^2$. (ii) when $1-2\xi^2(\omega^2-k^2) < 0$ and $(\pi/k)(d\pi/dk) > 0$, one could have also $c^2_{eff}(\omega,k) > c^2$. This second interesting case would require trans-Planckian masses $(\omega^2-k^2) > m_P^2$ (quantum gravity regime), and/or very high values for the numerical parameter λ appearing in the definition of $\xi^2 = \lambda^2/2m_P^2$.

To derive the expression for the derivative $d\pi/dk$ requires a knowledge of the actual poly-particle dynamics in C-space. We shall focus on two possibilities. In the first case, to simplify matters, we shall take $d\pi/dk = (d\pi/d\Sigma)/(dk/d\Sigma) = 0$, implying that the Clifford scalar component of the poly-momentum π does not depend on the C-space Σ proper "time". In this case one has, as to be expected, a *superluminal* group velocity, greater than *unity* (in $c=1$ units) because $k/\omega > 1$ as a result of the condition $k^2-\omega^2 > 0$.

Therefore, when $d\pi/dk = 0$, the expression for the group velocity as a function of k is given by

$$\frac{d\omega}{dk}\;=\;\frac{k}{\omega}\;=\;\xi\;k\;\left(\frac{1+2\xi^2k^2\;\pm\;\sqrt{1-4\xi^2(M^2-\pi^2)}}{2}\right)^{-1/2} \tag{1.10}$$

In order to write the group velocity as a function of ω (Energy), one may expand eq-(1.6) to lowest order in ξ in a Taylor series leading to

$$\omega^2\;=\;k^2+(M^2-\pi^2)\;+\;8\xi^2(M^2-\pi^2)^4\;+\ldots\ldots \tag{1.11}$$

when $\omega^2-k^2 < 0$, one learns from eq-(1.4) that $M^2-\pi^2 = \mathcal{M}^2 < 0$, so the $effective$ mass $\mathcal{M}$ is tachyonic (imaginary), but *not* M. Inserting the expression (1.11) into (1.10) yields

$$v_{group}\;=\;\frac{k}{\omega}\;\sim\;\frac{\sqrt{\omega^2\;-\;\mathcal{M}^2\;-8\xi^2\;\mathcal{M}^4}}{\omega}\;=$$

$$1\;-\frac{1}{2}\,\frac{\mathcal{M}^2}{\omega^2}\;-\;4\xi^2\,\frac{\mathcal{M}^4}{\omega^2}\;-\;\ldots\ldots\ldots. \tag{1.12}$$

to sum up, because the effective mass is tachyonic $\mathcal{M}^2 < 0 \Rightarrow -\mathcal{M}^2 > 0$, and $4\xi^2\mathcal{M}^4/\omega^2$ is a very small number, the $v_{group} > 1$ (in $c=1$ units) is superluminal. By equating $\omega = E$ in (1.12), in the case that $d\pi/dk = 0$, one has finally

$$v_{group} \sim 1 - \frac{1}{2} \frac{\mathcal{M}^2}{E^2} - 4\xi^2 \frac{\mathcal{M}^4}{E^2} - .... \quad with \ \mathcal{M}^2 < 0 \qquad (1.13)$$

if one neglects the $4\xi^2 \mathcal{M}^4/E^2$ terms in (1.13), one may obtain a numerical value for the quantity $\pi^2 - M^2 = -\mathcal{M}^2 > 0$

$$\frac{v_{group} - c}{c} = -\frac{1}{2} \frac{\mathcal{M}^2}{E^2} = \frac{1}{2} \frac{\pi^2 - M^2}{E^2} = 2.48 \times 10^{-5}, \ when \ E = 17 \ Gev \qquad (1.14)$$

directly from the muon neutrinos OPERA experiment [7] after inserting the average muon neutrino energy of $< E > \ = \ 17$ GeV into eq-(1.14). One then arrives at

$$\sqrt{\pi^2 - M^2} = 0.1197 \ GeV = 119.7 \ Mev \qquad (1.15)$$

if $M = m_{\nu_\mu}$ is identified with the very small and real-valued muon-neutrino ν_μ mass, one has then that the value of the Clifford scalar component π of the poly-momentum is $\pi \sim 119.7$ MeV. It is an interesting *coincidence* that this value for $\pi = 119.7$ Mev is close to the mass of the muon $m_\mu = 105.7$ Mev.

Since the condition $d\pi/dk = 0$ is too restrictive, in order to attain compatible results with other MINOS, SN 1987A, Fermilab, T2K, ... neutrino experiments, one should have in general that $d\pi/dk \neq 0$ so the group velocity becomes now a more complicated expression (in $c = 1$ units) of the form

$$\frac{d\omega}{dk} = \left(1 - \frac{(\pi/k) \ (d\pi/dk)}{1 + 2\xi^2(k^2 - \omega^2)} \right) \frac{k}{\omega} \sim [\ 1 - (\pi/k) \ (d\pi/dk) \ ] \ (\frac{k}{\omega}) \qquad (1.16)$$

where we have neglected the ξ^2 terms. If $(\pi/k)(d\pi/dk) < 0$, the right hand side of (1.16) is always greater than unity. By recasting (1.16) in terms of the frequency (energy) one gets to *lowest* order

$$\frac{d\omega}{dk} \sim \frac{1}{1 + (\pi/\omega)(d\pi/d\omega)} \frac{\sqrt{\omega^2 - \mathcal{M}^2}}{\omega} > 1; \quad \mathcal{M}^2 < 0 \qquad (1.17)$$

due to $k/\omega \sim (\sqrt{\omega^2 - \mathcal{M}^2}/\omega) > 1$ and $(\pi/\omega)(d\pi/d\omega) < 0$. If negative values of $(d\omega/dk)$ are excluded by assuming the particle moves along the positive x^1 direction, one must have that $1 + (\pi/\omega)(d\pi/d\omega) \geq 0$. In the limiting case $(\pi/\omega)(d\pi/d\omega) = -1 \Rightarrow (d\omega/dk) = \infty$. Infinite superluminal velocity solutions to the Dirac-Barut-Hestenes fermionic wave equation have been studied by [11].

Concluding, by defining the function

$$f(\omega) \equiv \frac{1}{1 + (\pi/\omega)(d\pi/d\omega)} > 1, \quad \omega = E \qquad (1.18a)$$

one arrives at the functional form of the group velocity in terms of the frequency ω (Energy) and the Clifford scalar component of the poly-momentum

$\pi = \pi(\omega) = \pi(E)$, given by

$$v_{group}(E) \sim f(E) \left(1 - \frac{1}{2} \frac{\mathcal{M}^2(E)}{E^2} \right) > 1; \quad \mathcal{M}^2(E) = M^2 - \pi^2(E) < 0 \tag{1.18b}$$

Different neutrino experiments correspond to different physical conditions. For example, neutrinos moving through inter stellar space (SN 1987A) will experience different dynamics and have a different functional form for $\pi(\omega), f(\omega), \mathcal{M}(\omega)$ than the neutrinos moving through the earth's crust (OPERA). Hence one expects to have different values for their group velocities; i.e different functional dependence on their frequencies (energies). Because the energies of the different neutrino experiments *varies* considerably, one could adopt a more ambitious view and explore the possibility in finding a set functions $\pi(\omega), f(\omega), \mathcal{M}(\omega)$ that fit *all* of the neutrino experiments irrespective of their physical conditions.

For the time being, we have to wait for further neutrino experiments to make sure that the alleged superluminal behavior of muon neutrinos is not due to a mundane or unforeseen technical issue, or to other physical reasons that perhaps were not taken into account by the experimenters like frame-dragging due to the earth's rotation, gravitational-redshift, motion of the GPS satellite and other effects [20]; eccentricity effects due to the elliptical trajectory of the GPS [22], [23], [24]; time dilation or contraction in the earth's gravitational field [16]; quadrupole corrections due to the oblateness of the earth (not a perfect sphere), speed of light measurements in non-inertial frames and the correct synchronization of clocks in the two laboratories [21], [24] etc....

To our knowledge, the possibility that neutrinos might be tachyons was first proposed long ago by [8], and later on by [9], among others. Superluminal behavior is also a feature of Topological Geometro-Dynamics [10]. The limitations of the special and general relativities and their isotopic generalizations has been raised by [12]. Background-dependent Lorentz Violations in String Theory laeding to superluminality can be found in [15], [14] and references therein. For mass-dependent Lorentz violation and neutrino velocity see [18]; the apparent Lorentz violation with superluminal Majorana neutrinos due to imaginary mass terms was advanced by [19]; superluminality due to local effects caused by a scalar field sourced by the earth can be found [13], and many other plethora of proposals have been put forward by others.

To conclude this section, it is the contribution of the Clifford scalar component π of the poly-momentum which is responsible for the superluminal behavior $\mathcal{M}^2 < 0$ in ordinary spacetime due to the fact that the $effective$ mass $\mathcal{M}$ is imaginary. From the point of view of C-space, there is no superluminal (tachyonic) behavior because $M^2 > 0$. Therefore, there is *no* violation of the Clifford-extended Lorentz invariance and the Relativity principle in C-space. Furthermore, to lowest order, there is *no* contribution of terms involving the Planck mass $(1/m_P^2)$ indicating that quantum gravitational effects do *not* play a role at this order.

2 Born's Reciprocal Relativity in Phase Spaces and Finsler Geometry

Born's reciprocal ("dual") relativity [25] was proposed long ago based on the idea that coordinates and momenta should be unified on the same footing, and consequently, if there is a limiting speed (temporal derivative of the position coordinates) in Nature there should be a maximal force as well, since force is the temporal derivative of the momentum. A *curved* velocity space case scenario has been analyzed by Brandt [33] within the context of the Finsler geometry of the $8D$ tangent bundle of spacetime where there is a limiting value to the proper acceleration and such that generalized $8D$ gravitational equations reduce to ordinary Einstein-Riemannian gravitational equations in the $infinite$ acceleration limit. A maximal acceleration principle was also suggested long ago by [26]. A pedagogical monograph on Finsler geometry can be found in [31] where, in particular, Clifford/spinor structures were defined with respect to nonlinear connections associated with certain nonholonomic modifications of Riemann–Cartan gravity.

Born's reciprocal "duality" principle is nothing but a manifestation of the large/small tension duality principle reminiscent of the T-duality symmetry in string theory; i.e. namely, a small/large radius duality, a winding modes/ Kaluza-Klein modes duality symmetry in string compactifications and the ultraviolet/infrared entanglement in noncommutative field theories. The generalized velocity and acceleration boosts (rotations) transformations of the $8D$ Phase space, where $X^i, T, E, P^i; i = 1,2,3$ are *all* boosted (rotated) into each-other, were given by [27], based on the group $U(1,3)$ and which is the Born version of the Lorentz group $SO(1,3)$. The $U(1,3) = SU(1,3) \otimes U(1)$ group transformations leave invariant the symplectic 2-form $\Omega = - dt \wedge dp^0 + \delta_{ij} dx^i \wedge dp^j;\ i,j = 1,2,3$ and also the following Born-Green line interval in the $8D$ phase-space (in natural units $\hbar = c = 1$)

$$(d\Upsilon)^2 = (dt)^2 - (dx)^2 - (dy)^2 - (dz)^2 + \frac{1}{b^2} \left((dE)^2 - (dp_x)^2 - (dp_y)^2 - (dp_z)^2 \right) \tag{2.1}$$

the rotations, velocity and force (acceleration) boosts leaving invariant the symplectic 2-form and the line interval in the $8D$ phase-space are rather elaborate, see [27] for details. These transformations can be simplified drastically when the velocity and force (acceleration) boosts are both parallel to the x-direction and leave the transverse directions y, z, p_y, p_z intact. There is now a subgroup $U(1,1) = SU(1,1) \otimes U(1) \subset U(1,3)$ which leaves invariant the following line interval

$$(d\chi)^2 = (dT)^2 - (dX)^2 + \frac{(dE)^2 - (dP)^2}{b^2} =$$

$$(d\tau)^2 \left(1 + \frac{(dE/d\tau)^2 - (dP/d\tau)^2}{b^2} \right) = (d\tau)^2 \left(1 - \frac{F^2}{F^2_{max}} \right); \; F_{max} = b \tag{2.2}$$

where one has factored out the proper time infinitesimal $(d\tau)^2 = dT^2 - dX^2$ in (2.2). The proper force interval $(dE/d\tau)^2 - (dP/d\tau)^2 = -F^2 < 0$ is "spacelike" when the proper velocity interval $(dT/d\tau)^2 - (dX/d\tau)^2 > 0$ is timelike. The analog of the Lorentz relativistic factor in eq-(2.2) involves the ratios of two proper *forces*.

If (in natural units $\hbar = c = 1$) one sets the maximal proper-force to be given by $b \equiv m_P A_{max}$, where $m_P = (1/L_P)$ is the Planck mass and $A_{max} = (1/L_p)$, then $b = (1/L_P)^2$ may also be interpreted as the maximal string tension. The units of b would be of $(m_P)^2$. In the most general case there are four scales of time, energy, momentum and length that can be constructed from the three constants $b, c, \hbar$ as follows [29]

$$\lambda_t = \sqrt{\frac{\hbar}{bc}}; \;\; \lambda_l = \sqrt{\frac{\hbar\, c}{b}}; \;\; \lambda_p = \sqrt{\frac{\hbar\, b}{c}}; \;\; \lambda_e = \sqrt{\hbar\, b\, c} \tag{2.3}$$

The gravitational constant can be written as $G = \alpha_G\, c^4/b$ where α_G is a dimensionless parameter to be determined experimentally. If $\alpha_G = 1$, then the four scales (2.3) coincide with the *Planck* time, length, momentum and energy, respectively. An interesting numerical relation involving the Planck scale and Hubble radius is $\mathcal{F}_{\max} = m_P\, \frac{c^2}{L_P} \sim M_{Universe}\, \frac{c^2}{R_H}$, hence in [2], we suggested that a certain large (Hubble) /small (Planck) scale *duality* was operating in this Born's reciprocal relativity theory reminiscent of the T-duality in string theory compactifications.

We provided in [28] *six* specific results resulting from Born's reciprocal Relativity and which are *not* present in Special Relativity. These are : momentum-dependent time delay in the emission and detection of photons; energy-dependent notion of locality; superluminal behavior; relative rotation of photon trajectories due to the aberration of light; invariance of areas-cells in phase-space and modified dispersion relations. In particular, given the null condition in a $flat$ phase-space

$$(d\chi)^2 = (dT)^2 - (dX)^2 + \frac{(dE)^2 - (dP)^2}{b^2} = 0 \tag{2.4}$$

dividing by $(dT)^2$, yields

$$1 - (\frac{dX}{dT})^2 + (\frac{1}{b})^2 (\frac{dE}{dT})^2 - (\frac{1}{b})^2 (\frac{dP}{dT})^2 = 0 \Rightarrow$$

$$1 - (v)^2 + (\frac{1}{b})^2 (f_o)^2 - (\frac{1}{b})^2 (f_1)^2 = 0 \Rightarrow v = \pm \sqrt{1 + (\frac{1}{b})^2 (f_0)^2 - (\frac{1}{b})^2 (f_1)^2} \tag{2.5}$$

where $v = \frac{dX}{dT}$ is the coordinate velocity; the analog of power and force are respectively $f_o = \frac{dE}{dT} \neq \frac{dE}{d\tau} = F_0$; $f_1 = \frac{dP}{dT} \neq \frac{dP}{d\tau} = F_1$. Reinserting the speed of light c (that was set to unity) one arrives at

$$v \ = \ \pm \ c \sqrt{ 1 \ + \ (\frac{1}{bc})^2 \ (f_0)^2 \ - \ (\frac{1}{b})^2 \ (f_1)^2 } \ = \ \pm \ c \sqrt{ 1 \ + \ (\frac{c}{b})^2 \ (\frac{d\mathbf{M}}{dT})^2 } \tag{2.6}$$

where the infinitesimal mass-displacement is defined as

$$c^2 \ (d\mathbf{M})^2 \ = \ (\frac{1}{c^2}) \ (dE)^2 \ - \ (dP)^2 \tag{2.7}$$

Taking the positive sign under the square root, when $(\frac{c}{b})^2 \ (\frac{d\mathbf{M}}{dT})^2 < 0$, one arrives at the interesting conclusion that at the *null* hypersurface in a *flat* $phase - space$ one can have points such that $v < c$. However, if $(\frac{c}{b})^2 \ (\frac{d\mathbf{M}}{dT})^2 > 0$ one can have *superluminal* $v > c$ behavior in this case, despite having a null hypersurface in a *flat* phase-space. When $(\frac{c}{b})^2 \ (\frac{d\mathbf{M}}{dT})^2 = 0$, one recovers $v = c$ as it occurs in Special Relativity.

Estimates of $(d\mathbf{M}/dT) = (d\mathbf{M}/dE)/(dT/dE) = F(E)$ can be found in the superluminal muon neutrino case by equating

$$\frac{v - c}{c} \ = \ \sqrt{ 1 \ + \ (\frac{c}{b})^2 \ (\frac{d\mathbf{M}}{dT})^2 } \ - 1 \ = \\ \sqrt{ 1 \ + \ (\frac{c}{b})^2 \ [\ F(E = 17Gev) \ ]^2 } \ - 1 \ \sim \ 2.48 \times 10^{-5} \tag{2.8}$$

to the OPERA experiment finding. In $\hbar = c = 1$ units , a Taylor expansion of eq-(2.8) yields

$$\frac{1}{2} \ [\ \frac{1}{b} \ F(E = 17Gev) \ ]^2 \ \sim \ 2.48 \times 10^{-5} \ \Rightarrow \ F(E = 17Gev) \ \sim \ 7.04 \times 10^{-3} \ m_P^2 \tag{2.9}$$

when the maximal proper force b is given by m^2_{Planck}. The value in eq-(2.9) for $F(E = 17Gev)$ is relatively high in this case. Because the gravitational constant was written as $G = \alpha_G \ c^4/b$, where α_G is a dimensionless parameter to be determined experimentally, lower values for $F(E = 17Gev)$ can be obtained by lowering the value of the maximal proper force $b < m^2_{Planck}$, meaning then that $\alpha_G \neq 1$, and such that now the four scales in eq-(2.3) do *not* longer necessarily coincide with the *Planck* time, length, momentum and energy, respectively. This could be a plausible scenario.

A particular modified dispersion relation can be obtained from a *truncation* of a more general $U(1,3)$-invariant dispersion relation in the $8D$ phase space [27], [28] involving both coordinates and momenta. The truncation given by

$$g_{\mu\nu}(E, p^i) \, p^\mu \, p^\nu \ = \ (\ \eta_{\mu\nu} + h_{\mu\nu}(E, p^i) \ ) \, p^\mu \, p^\nu \ = \ E^2 - p_i p^i + h_{\mu\nu}(E, p^i) \, p^\mu \, p^\nu \ = \ M^2 \tag{2.10}$$

breaks the $U(1,3)$ symmetry. If the perturbation of the flat phase space metric $h_{\mu\nu}(E, p^i)$ happens to depend on the energy and on the magnitude-squared of the momentum variables, it admits a Taylor series expansion of the form

$$h_{\mu\nu}(E, p_i p^i) \;=\; \sum_n \; h^{(n)}_{\mu\nu}(E) \; \frac{(p_j p^j)^n}{m_P^{2n}} \tag{2.11}$$

and it furnishes the following modified dispersion relations, after setting $|\vec{p}|^2 = p_i p^i$,

$$E^2 \;-\; |\vec{p}|^2 \;+\; \left(\sum_n \; h^{(n)}_{\mu\nu}(E) \; \frac{|\vec{p}|^{2n}}{m_P^{2n}} \; p^\mu \; p^\nu \right) \;=\; M^2 \tag{2.12}$$

In the special case that the only nonzero components $h^{(n)}_{\mu\nu}(E)$ are given by

$$h^{(n)}(E) \;=\; h^{(n)}_{00}(E) \;=\; -\; h^{(n)}_{11}(E) \;=\; -\; h^{(n)}_{22}(E) \;=\; -\; h^{(n)}_{33}(E) \tag{2.13}$$

one gets

$$(\; E^2 \;-\; |\vec{p}|^2 \;) \; \left(1 \;+\; \sum_n \; h^{(n)}(E) \; \frac{|\vec{p}|^{2n}}{m_P^{2n}} \right) \;=\; M^2 \tag{2.14}$$

In the other very special case that the only nonzero components are given by

$$h^{(n)}_{00}(E) \;=\; 0, \quad h^{(n)}_{11}(E) \;=\; h^{(n)}_{22}(E) \;=\; h^{(n)}_{33}(E) \;=\; -\; f^{(n)}(E) \tag{2.15}$$

one arrives at the modified dispersion relations

$$E^2 \;-\; |\vec{p}|^2 \;-\; \left(\sum_n \; f^{(n)}(E) \; \frac{|\vec{p}|^{2n+2}}{m_P^{2n}} \right) \;=\; M^2 \tag{2.16}$$

like those appearing in Finsler Geometry [17], [30], rainbow metrics [34] and Double (deformed) Special Relativity [35], [36].

The leading order behavior in eq-(2.16) is

$$E^2 \;-\; |\vec{p}|^2 \;-\; f^{(0)}(E) \; |\vec{p}|^2 \;=\; E^2 \;-\; |\vec{p}|^2 (\; 1 \;+\; f^{(0)}(E) \;) \sim \; M^2 \tag{2.17}$$

which is similar to the results in [17] when $f^{(0)}(E) < 0$. The ratio

$$v \;=\; \frac{|\vec{p}|}{E} \;\sim\; \sqrt{\frac{E^2 - M^2}{E^2}} \; \frac{1}{\sqrt{1 + f^{(0)}(E)}} \tag{2.18}$$

is a measure of subluminal/superluminal behavior. To attain superluminality (in $c = 1$ units) one must have that $f^{(0)}(E) < 0$ and such that $\sqrt{\frac{E^2 - M^2}{E^2}} > \sqrt{1 - |f^{(0)}(E)|}$ yielding, after expanding the square roots in a Taylor series,

$$v(E) \;\sim\; 1 \;-\frac{1}{2} \; \frac{M^2}{E^2} \;+\; \frac{1}{2} \; |f^{(0)}(E)| \;-\; \frac{1}{4} \; \frac{M^2}{E^2} \; |f^{(0)}(E)| \tag{2.19}$$

assuming the last term in the right hand side is negligible compared to the first three terms one has for the OPERA experiment, at an average energy of 17 Gev,

$$v \ - \ 1 \ \sim \ \frac{1}{2} \ (\ |f^{(0)}(17Gev)| \ - \ \frac{M^2}{(17Gev)^2} \ ) \ \sim \ 2.48 \times 10^{-5} \qquad (2.20)$$

giving an estimate of the very small value of $|f^{(0)}(17Gev)|$ of the order of 4.96×10^{-5} obtained by neglecting the mass term in (2.20). If we had used a different dispersion relation (2.14) we would have obtained different results for the value of $h^{(0)}(E = 17$ Gev). In the general case one must solve the generalized gravitational equations in the 8-dim phase space (cotangent bundle of spacetime) using the techniques of Lagrange-Finsler and Hamilton-Cartan spaces [32], [31]. This is a very difficult task. The solutions will determine the phase space metric $g_{AB}(x^\mu, p_\mu)$, $A, B = 1, 2, 3....., 8$ whose momentum components are the ones which appear in the *truncation* leading to the modified dispersion relations (2.10), and which in turn, furnish the functional behavior of the velocities in terms of the energies.

The advantage of using a non-truncated expression for the phase-space interval and which determines the functional form of the superluminal velocity in eq-(2.6), is that it is *uniquely* defined for a $flat$ phase-space metric. For this reason we could focus on eq-(2.6) and the numerical analysis which follows in eqs-(2.8, 2.9) to try to fit the experimental data on all neutrino experiments. The Clifford-space approach leading to eqs-(1.18a, 1.18b) is also equally valid to fit the superluminal neutrino data.

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