A PROPERTY OF THE CIRCUMSCRIBED OCTOGON

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Abstract

In this article we'll obtain through the duality method a property in relation to the contact cords of the opposite sides of a circumscribable octagon.

In an inscribed hexagon the following theorem proved by Blaise Pascal in 1640 is true.

Theorem 1 (Blaise Pascal)

The opposite sides of a hexagon inscribed in a circle intersect in collinear points.

To prove the Pascal theorem one may use [1].

In [2] there is a discussion that the Pascal's theorem will be also true if two or more pairs of vertexes of the hexagon coincide. In this case, for example the side AB for $B \to A$ must be substituted with the tangent in A. For example we suppose that two pairs of vertexes coincide. The hexagon $AA'BCC'D$ for A'→ A, C'→ C becomes the inscribed quadrilateral $ABCD$. This quadrilateral viewed as a degenerated hexagon of sides $AB, BC, CC' \to$ the tangent in $C, C'D \to CD, D'A \to DA, AA' \to$ the tangent in A and the Pascal theorem leads to:

Theorem 2

In an inscribed quadrilateral the opposite sides and the tangents in the opposite vertexes intersect in four collinear points.

Remark 1

In figure 1 is presented the corresponding configuration of theorem 2.

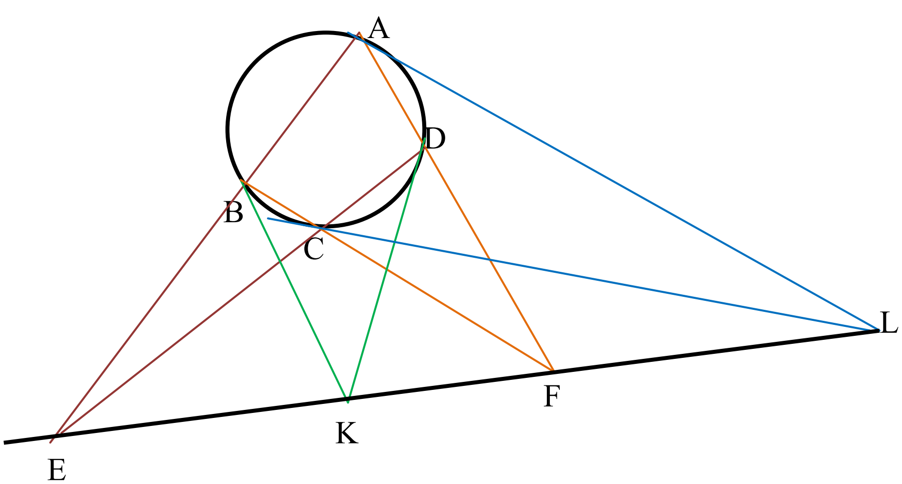

Fig. 1

For the tangents constructed in B and D the property is also true if we consider the $ABCD$ as a degenerated hexagon $ABB'CDD'A$.

Theorem 3

In an inscribed octagon the four cords determined by the contact points with the circle of the opposite sides are concurrent.

Proof

We'll transform through reciprocal polar the configuration from figure 1. To point E will correspond, through this transformation the line determined by the tangent points with the circle of the tangents constructed from E (its polar). To point K corresponds the side BD.

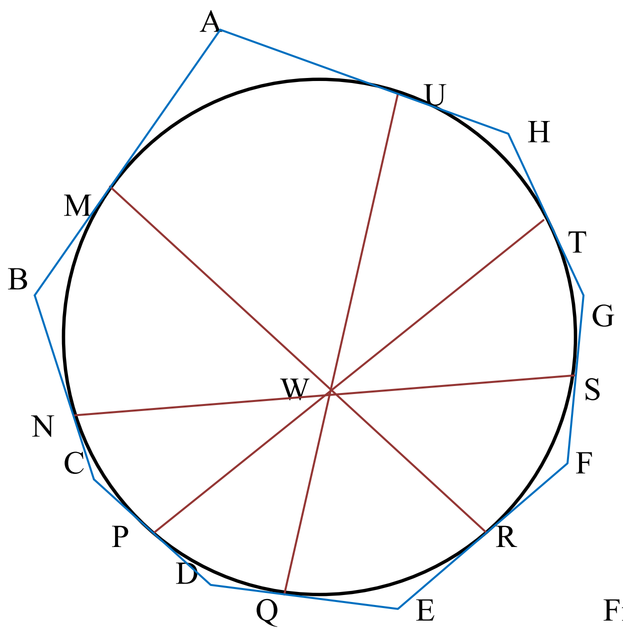

Fig.2

To point F corresponds the line determined by the contact points of the tangents constructed from F to the circle. To point L corresponds its polar AC. To point A corresponds, by duality, the tangent AL, also to points B, C, D correspond the tangents BK, CL, DK. These four tangents together with the tangents constructed from E and F (also four) will contain the sides of an octagon circumscribed to the given circle.

In this octagon (AC) and (BD) will connect the contact points of two pairs of opposite sides with the circle; the other two lines determined by the contact points of the opposite sides of the octagon with the circle will be the polar of the points E and F. Because the polar transformation through reciprocal polar leads to the fact that to collinear points correspond concurrent lines; the points' polar E, K, F, L are concurrent; these lines are the cords to which the theorem refers to.

Remark 2

In figure 2 we represented an octagon circumscribed $ABCDEFGH$. As it can be seen the cords MR, NS, PT, QU are concurrent in the point W.

References

[1] Roger A Johnson – *Advanced Euclidean Geometry*, Dover Publications, Inc. Mineola, New-York, 2007

[2] N. Mihăileanu – *Lecţii complementare de geometrie*, Editura Didactică şi Pedagogică, Bucureşti, 1976

[3] Florentin Smarandache – *Multispace & Multistructure, Neutrosophic Trandisciplinarity (100 Collected Papers of Sciences)*, Vol IV, 800 p., North-European Scientific Publishers, Hanko, Finland, 2010