

Anti-matter and black holes have a space-like spacetime geometry

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Abstract: If matter has a spherical or elliptical time-like spacetime, through geometrical symmetry it makes sense that anti-matter, the converse of matter, has a hyperbolic space-like spacetime. The author theorizes that such a space-like spacetime is the spacetime for anti-matter and for the natural phenomena referred to as *black holes*. Also such a spacetime shortens spacetime distances and could eventually lead to feasible space travel.

To support the theory here is presented an extension to the Schwarzschild derivation for the Einstein vacuum field equations corresponding to the gravitational field surrounding a pseudo-spherical point mass. The same restrictions employed by Schwarzschild are used here but an interesting property of the spacetime is elicited in that it is revealed to be gravitationally repulsive to matter as opposed to gravitationally attractive. The pseudo-sphere point mass solution complements the spherical Schwarzschild solution and provides an elegant symmetry to the original Schwarzschild solution. Also *via Occam's razor*, this theory could potentially reduce the notion of *dark* or *exotic* matter to familiar anti-matter.

Keywords: Schwarzschild radius, general relativity, Einstein vacuum field equations, pseudo-sphere, anti-matter, black holes, feasible space travel

1.) Introduction

Mankind has been seeking the answers to how flight and space travel could be possible since his beginnings. It does not make sense that we live in a Universe so vast yet we are unable to explore it. In this paper, a simple notion is put forward and the physics and mathematics are revealed to back up this notion: that anti-matter has a space-like spacetime and shortens spacetime distances. Anti-matter is space-like because it is created when matter transitions from a region of time-like spacetime to space-like spacetime. According to general relativity, space-like spacetime shortens spacetime

distances. So if this theory holds true, then it could lead to feasible space travel, once it is figured out how to generate and isolate enough anti-matter to power a craft.

In 1916, K. Schwarzschild introduced the first full solution to Einstein's vacuum field equations [1]. Since that time, there have been numerous other solutions such as the Kerr solution [6], the Anti-de-Sitter solution, the work in cosmology by Lemaître, Friedman, Robertson and Walker, Gödel [4] and numerous others. In fact since any geometry can lead to a vacuum field solution, there exist an infinite number of such solutions. But since the Schwarzschild solution represents something observed in nature (the planets in the solar system orbiting the sun) it remains fundamental.

Something else that is observed in nature and that is fundamental is the tractrix (or "*hound curve*") which was first introduced by Claude Perrault in 1670 and further studied by Leibniz, Newton and others. By revolving the tractrix around an axis, one obtains the pseudo-sphere which is a surface of constant negative curvature that can be used as a geometry for hyperbolic space-like spacetime. Since the spherical Schwarzschild solution is so fundamental to our understanding of gravity and general relativity, the symmetrical pseudo-spherical solution could also play a fundamental role.

More recently, Dr. Massimo Villata has theorized using CPT transformations on the equations for general relativity that anti-matter is in fact repulsive to matter but attractive to itself [8]. This paper extends that notion by revealing that in order to be repulsive to matter, anti-matter would need to have a pseudo-spherical geometry because the weak-field approximation changes in sign and a pseudo-spherical geometry corresponds to this change in sign. In the case that the weak-field approximation changes in sign from negative to positive which it has to do if anti-matter is gravitationally repulsive to matter, a spherical geometry would not lead to a vacuum solution of the Einstein field equations.

2.) Assumptions

The same assumptions used in the Schwarzschild derivation are used here:

- 1.) The space-time surrounding the point mass is static.
- 2.) The space-time surrounding the point mass is isotropic.
- 3.) At a sufficient distance away from the point mass, the space-time becomes flat Minkowski space-time.
- 4.) The metric at the surface must match the metric at the interior (boundary condition.)

3.) Derivation

The geodesic equation typically used for the Schwarzschild derivation is:

$$ds^2 = -e^{N(r)}c^2dt^2 + e^{P(r)}dr^2 + r^2d\theta^2 + r^2\sin^2(\theta)d\phi^2, \quad (1)$$

where the constants $N(r)$ and $P(r)$ are determined using Einstein's vacuum field equations. Note that in the canonical Schwarzschild derivation for a spherical space-time metric with signature $[-1, +1, +1, +1]$, this geodesic yields the familiar equation:

$$ds^2 = -\left(1 - \frac{2GM}{rc^2}\right)c^2dt^2 + \frac{dr^2}{1 - (2GM/rc^2)} + r^2d\theta^2 + r^2\sin^2(\theta)d\phi^2, \quad (2)$$

The geodesic equation corresponding to a pseudo-sphere used in the derivation for this paper is similar to (1) and appears as:

$$ds^2 = -e^{N(r)}c^2dt^2 + e^{P(r)}dr^2 + r^2\operatorname{sech}^2(\phi)d\theta^2 + r^2\tanh^2(\phi)d\phi^2, \quad (3)$$

The only difference between this geodesic and (1) are the third and fourth terms.

The diagonal metric $g_{a,b}$ used for the derivation in this paper has signature $[-1, +1, +1, +1]$ and appears in tensor form as:

$$g_{a,b} = \begin{pmatrix} -e^{N(r)} & 0 & 0 & 0 \\ 0 & e^{P(r)} & 0 & 0 \\ 0 & 0 & r^2 \operatorname{sech}^2(\phi) & 0 \\ 0 & 0 & 0 & r^2 \tanh^2(\phi) \end{pmatrix}, \quad (4)$$

Using Maple 14 to solve for the Einstein tensor and simplifying, we find that the only Einstein tensor non-zero components are:

$$G_{11} = \frac{e^{N(r)}(-P'(r)r + e^{P(r)} + 1)}{r^2 e^{P(r)}} = 0, \quad (5)$$

$$G_{22} = -\frac{(N'(r)r + e^{P(r)} + 1)}{r^2} = 0, \quad (6)$$

$$G_{33} = -\frac{1}{4} \frac{r(2(N'(r)) - 2(P'(r)) + r(N'(r))^2 - r(N'(r))(P'(r)) + 2r(N''(r)))}{e^{P(r)}} = 0, \quad (7)$$

$$G_{44} = G_{33} \sinh^2(\theta) = 0, \quad (8)$$

Solving for $N(r)$ and $P(r)$ yields:

$$P(r) = -N(r), \quad (9)$$

$$e^{P(r)} = \frac{Cr}{1 - Cr}, \quad (10)$$

where C is some as yet undetermined constant.

If one compares equation (10) with the equation that is derived from the spherical solution, the only difference is a sign change for Cr in the denominator from positive to negative but this is an important difference because it necessitates a sign change for the weak field approximation implying that the pseudo-spherical point mass is gravitationally repulsive to matter as opposed to attractive. Using the weak field approximation with the

corresponding sign change to determine the constant C , the constant is equivalent to $c^2 / 2GM$.

So we have our full solution for the pseudo-spherical geodesic equation:

$$ds^2 = \left(1 - \frac{2GM}{rc^2}\right) c^2 dt^2 - \frac{dr^2}{1 - (2GM/rc^2)} + r^2 \operatorname{sech}^2(\phi) d\theta^2 + r^2 \tanh^2(\phi) d\phi^2, \quad (11)$$

Comparing equation (11) with equation (2), the first term and the second term of both equations are equivalent except for a sign change from equation (2) to equation (11) for both of the terms. The author wishes to further emphasize that such a spacetime is space-like as opposed to time-like and theorizes that this is the spacetime for anti-matter particles and atoms such as the anti-hydrogen atom as well as for *black holes*. Just as planets and stars are regions of aggregated matter, black holes are regions in spacetime composed of aggregated anti-matter.

4.) References

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5.) Appendix

Simple Maple 14 script follows to reinforce and demonstrate pseudo-spherical derivation. All Einstein tensor values being zero means that we have a valid vacuum solution to the Einstein field equations. Note that a spherical solution of the Einstein field equations does not lead to a vacuum solution for a point mass that is gravitationally repulsive. Here all constants (G , c , and M and the multiplier 2 in $2GM/c^2$) are set to one for simplicity. These values could take on any positive constant values and the result would be the same with all Einstein tensor values coming back as zeroes (which is the requirement for a vacuum field solution):

```
> restart;
> with(tensor);
> coords :=[t,r,theta,phi];
> g := array(symmetric, sparse, 1..4, 1..4);
> g[1, 1] := 1-1/r; g[2, 2] := -1/g[1, 1]; g[3, 3] := r^2*sech(phi)^2; g[4, 4] :=
r^2*tanh(phi)^2;
> metric := create([-1,-1], eval(g))
```

$$\text{table} \left(\text{compts} = \begin{pmatrix} 1 - \frac{1}{r} & 0 & 0 & 0 \\ 0 & -\frac{1}{1 - \frac{1}{r}} & 0 & 0 \\ 0 & 0 & r^2 \operatorname{sech}(\phi)^2 & 0 \\ 0 & 0 & 0 & r^2 \tanh(\phi)^2 \end{pmatrix}, \text{index_char} = [-1, -1] \right)$$

```
> tensorsGR(coords, metric, contra_metric, det_met, C1, C2, Rm, Rc, R, G, C) :
> displayGR(Einstein, G)
```

The Einstein Tensor

non-zero components :

None

character : [-1, -1]