On the origin of macroquantization in astrophysics and celestial motion

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ABSTRACT. Despite the use of Bohr radius formula to predict celestial quantization has led to numerous verified observations, the cosmological origin of this macroquantization remains an open question. In this article various plausible approaches are discussed. Further observation to verify or refute this proposition is recommended, in particular for exoplanets.

RÉSUMÉ: En dépit de l'utilisation de la formule de rayon de Bohr de prévoir la quantification céleste a mené aux nombreuses observations vérifiées, l'origine cosmologique de ce macroquantization est une question en suspens. En cet article de diverses approches plausibles sont discutées. Promouvez l'observation pour vérifier ou réfuter cette proposition est recommandée, en particulier pour des exoplanets.

1 Introduction

It is known that the use of Bohr radius formula [1] to predict celestial quantization has led to numerous verified observations [2][3]. This approach was based on Bohr-Sommerfeld quantization rules [4][5]. Some implications of this quantum-like approach include exoplanets prediction, which has become a rapidly developing subject in recent years [6][7]. While this kind of approach is not widely accepted yet, this could be related to a recent suggestion to reconsider Sommerfeld's conjectures in Quantum Mechanics [8].

While this notion of macroquantization seems making sense at least in the formation era of such celestial objects, i.e. "*all structures in the Universe, from superclusters to planets, had a quantum mechanical origin in its earliest moments*" [9], a question arises as to how to describe the physical origin of wave mechanics of such large-scale structures [5].

A plausible definition of the problem of quantization has been given by Grigorescu [10]: "select an infinite, discrete number of quantum possible real motions, from the continuous manifold of all mechanically possible motions." While this quantization method has been generally acceptable to describe physical objects at molecular scale, there is not much agreement why shall we also invoke the same notion to describe macrophenomena, such as celestial orbits. Nonetheless, there are plenty efforts in the literature in attempt to predict planetary orbits in terms of wave mechanics, including a generalisation of Keplerian classical orbits [11].

In this article we discuss some plausible approaches available in the literature to describe such macroquantization in astrophysics, in particular to predict celestial motion:

a. Bohr-Sommerfeld's conjecture;
b. Macroquantum condensate, superfluid vortices;
c. Cosmic turbulence and logarithmic-type interaction;
d. Topological geometrodynamics (TGD) approach.

While these arguments could be expected to make the notion of macroquantization a bit reasonable, it is beyond the scope of this article to conclude which of the above arguments is the most consistent with the observed data. There is perhaps some linkage between all of these plausible arguments. It is therefore recommended to conduct further research to measure the reliability of these arguments, which seems to be worthwhile in our attempt to construct more precise cosmological theories.

2 Bohr-Sommerfeld's quantization rules

In an attempt to describe atomic orbits of electron, Bohr proposed a conjecture of quantization of orbits using analogy with planetary motion. From this viewpoint, the notion of macroquantization could be considered as returning Bohr's argument back to the celestial orbits. In the meantime it is not so obvious from literature why Bohr himself was so convinced with this idea of planetary quantization [12], despite such a conviction could be brought back to Titius-Bode law, which suggests that celestial orbits can be described using simple series. In fact, Titius-Bode were also not the first one who proposed this kind of simple series [13], Gregory-Bonnet started it in 1702.

In order to obtain planetary orbit prediction from this hypothesis we could begin with the Bohr-Sommerfeld's conjecture of quantization of angular momentum. As we know, for the wavefunction to be well defined and unique, the momenta must satisfy Bohr-Sommerfeld's quantization condition [14]:

$$\oint_{\Gamma} p.dx = 2\pi.n\hbar \tag{1}$$

for any closed classical orbit Γ. For the free particle of unit mass on the unit sphere the left-hand side is

$$\int_0^T v^2.d\tau = \omega^2.T = 2\pi.\omega \tag{2}$$

where $T=2\pi/\omega$ is the period of the orbit. Hence the quantization rule amounts to quantization of the rotation frequency (the angular momentum): $\omega = n\hbar$. Then we can write the force balance relation of Newton's equation of motion:

$$GMm / r^2 = mv^2 / r \tag{3}$$

Using Bohr-Sommerfeld's hypothesis of quantization of angular momentum (2), a new constant g was introduced:

$$mvr = ng / 2\pi \tag{4}$$

Just like in the elementary Bohr theory (before Schrödinger), this pair of equations yields a known simple solution for the orbit radius for any quantum number of the form:

$$r = n^2.g^2 / (4\pi^2.GM.m^2) \tag{5}$$

or

$$r = n^2.GM / v_o^2 \tag{6}$$

where r, n, G, M, v_o represents orbit radii (semimajor axes), quantum number (n=1,2,3,…), Newton gravitation constant, and mass of the nucleus of orbit, and specific velocity, respectively. In this equation (6), we denote

$$v_o = (2\pi / g).GMm \quad (7)$$

The value of m is an adjustable parameter (similar to g).

Nottale [1] extends further this Bohr-Sommerfeld quantization conjecture to a gravitational-Schrödinger equation by arguing that the equation of motion for celestial bodies could be expressed in terms of a scale-relativistic Euler-Newton equation. For a Kepler potential and in the *time independent* case, this equation reads (in Ref [1c] p. 380):

$$2D^2 \Delta\Psi + (E / m + GM / r).\Psi = 0 \quad (8)$$

Solving this equation, he obtained that planetary orbits are quantized according to the law:

$$a_n = GMn^2 / v_o^{\,2} \quad (9)$$

where a_n,G,M,n,v_o each represents orbit radius for given n, Newton gravitation constant, mass of the Sun, quantum number, and specific velocity (v_o=144 km/sec for Solar system and also exoplanet systems), respectively. These equations (8)-(9) form the basis of Nottale's Scale Relativity prediction of planetary orbits [1]; and equation (9) corresponds exactly with equation (6) because both were derived using the same Bohr-Sommerfeld's quantization conjecture. Another known type of observed quantization in astronomy is Tifft's 72 km/sec quantization [13].

3 Macroquantum condensate, superfluid vortices

Provided the above Bohr-Sommerfeld description of macroquantization corresponds to the facts, then we could ask further what kind of physical object could cause such orbital quantization. Thereafter we could come to the macroquantum condensate argument. In this regard, astrophysical objects could be seen as results of vacuum condensation [15][16]. For instance Ilyanok & Timoshenko [17] took a further step by hypothesizing that the universe resembles a large Bose Einstein condensate, so that the distribution of all celestial bodies must also be quantized. This conjecture may originate from the fact that according to BCS theory, superconductivity can exhibit macroquantum phenomena [18]. There is also a known suggestion that the vacua consist of hypercrystalline: *classical spacetime coordinate and fields are parameters of coherent states* [19].

It is perhaps interesting to remark here that Ilyanok & Timoshenko do not invoke argument of *non-differentiability* of spacetime, as Nottale did [1]. In a macroquantum condensate context, this approach appears reasonable because Bose-Einstein condensate with Hausdorff dimension D_H~2 could exhibit fractality [20], implying that non-differentiability of spacetime conjecture is not required. The same fractality property has been observed in various phenomena in astrophysics [21], which in turn may also correspond to an explanation of the origin of multifractal spectrum as described by Gorski [22]. In this regard, Antoniadis *et al.* have discussed CMBR temperature (2.73^o K) from the viewpoint of conformal invariance [23], which argument then could be related to Winterberg's hypothesis of superfluid Planckian phonon-roton aether [24].

Based on previous known analogy and recent research suggesting that there is neat linkage between gravitation and condensed matter physics [25][26], we could also hypothesize that planetary quantization is related to quantized vortex. In principle, this hypothesis starts with observation that in quantum fluid systems like superfluidity, it is known that such vortexes are subject to quantization condition of integer multiples of 2π, or $\oint v_s.dl = 2\pi.n\hbar / m_4$. Furthermore, such quantized vortexes are distributed in equal distance, which phenomenon is known as vorticity [4]. In large superfluid system, usually we use Landau two-fluid model, with normal and superfluid component. The normal fluid component always possesses some non-vanishing amount of viscosity and mutual friction. Similar approach with this proposed model has been considered in the context of neutron stars [27], and this quantized vortex model could also be related to Wolter's vortex [28].

4 Cosmic turbulence and logarithmic type interaction

Another plausible approach to explain the origin of quantization in astronomy is using turbulence framework. Turbulence is observed in various astrophysical phenomena [21], and it is known that such turbulence could exhibit a kind of self-organization, including quantization.

Despite such known relations, explanation of how turbulence could exhibit orbital quantization is not yet clear. If *and only if* we can describe such a flow using Navier-Stokes equation [29], then we can use R.M. Kiehn's suggestion that there is exact mapping from Schrödinger equation to Navier-Stokes equation, using the notion of quantum vorticity [30]. But for fluid which cannot be described using Navier-Stokes equation, such exact mapping would not be applicable anymore. In fact, according to Kiehn the Kolmogorov theory of turbulence is based on assumption that the turbulent state consists of “vortices” of all “scales” with random intensities, but it is not based on Navier-Stokes equation explicitly, in fact “*the creation of the turbulent state must involve discontinuous solutions of Navier-Stokes equations*.” [31] However, there is article suggesting that under certain conditions, solutions of 3D Navier-Stokes equation could exhibit characteristic known as Kolmogorov length [32]. In this kind of hydrodynamics approach, macroquantization could be obtained from solution of diffusion equation [33].

In order to make this reasoning of turbulence in astrophysics more consistent with the known analogy between superfluidity and cosmology phenomena [26], we could also consider turbulence effect in quantum liquid. Therefore it seems reasonable to consider *superfluid turbulence* hypothesis, as proposed for instance by Kaivarainen [34]. There are also known relations such as discrete scale invariant turbulence [35], superstatistics for turbulence [36], and conformal turbulence. Furthermore, such a turbulence hypothesis could lead to logarithmic interaction similar to Kolmogorov-type interaction across all scales [28].

Another way to put such statistical considerations into quantum mechanical framework is perhaps using Boltzmann kinetic gas approach. It is known that quantum mechanics era began during Halle conference in 1891, when Boltzmann made a remark: “*I see no reason why energy shouldn’t also be regarded as divided atomically*.” Due to this reason Planck subsequently called the quantity $2\pi h$ after Boltzmann – ‘Boltzmann constant.’ Using the same logic, Mishinov *et al*. [37] have derived Newton equation from TDGL:

$$m^* d_t V_p(t) = e^* . E - m^* V_p(t) / \tau_p \qquad (10)$$

This TDGL (time-dependent Ginzburg-Landau) equation is an adequate tool to represent the *low-frequency* fluctuations near T_c, and it can be considered as more universal than GPE (Gross-Pitaevskii equation).

5 TGD viewpoint on the origin of macroquantization in astrophysics and celestial motion

Topological geometrodynamics (TGD) viewpoint on this macro-quantization subject [38] was based on recognition that this effect could be considered as simple substitution of Planck constant:

$$\hbar \to \hbar_{gr} = GMm / v_0 \qquad (11)$$

provided we assert that $\hbar = c = 1$. The motivation is the earlier proposal inspired by TGD [39] that the Planck constant is dynamical and quantized. As before v_o=144.7±0.7 km/sec, giving $v_0 / c = 4.82x10^{-4} km/\sec$. This value is rather near to the peak orbital velocity of stars in galactic halos. As a sidenote, this is not the only plausible approach to make extension from geometrodynamics to Planck scale, and *vice versa* [41].

A distinction of TGD viewpoint [42] from Nottale's fractal hydrodynamics approach is that many-sheeted spacetime suggests that astrophysical systems are not only quantum systems at larger space-time sheets but correspond to a gigantic value of gravitational Planck constant. The Bohr's rules for the visible matter reflect the quantum dynamics of the dark matter at larger space-time sheets. Furthermore, TGD predicts the value of the parameter v_o appearing in equation (9) and explains its harmonic and subharmonics. There is also a plausible linkage between hydrodynamics approach and Kähler structure to describe the Schrödinger equation [43].

5.1. Consistency with TGD model of galactic dark matter

The first step is to see whether the TGD based model for dark matter is consistent with the gravitational Schrödinger equation. The following argument was based on Bohr quantization rules [41].

a. The gravitational potential energy V(r) for a mass distribution M(r)=xTr (T denotes string tension) is given by:

$$V(r) = Gm\int_r^{Ro} M(r).dr / r^2 = GmxT \log(r / R_o) \tag{12}$$

Here R_o corresponds to a large radius so that the potential is negative, as it should in the region where binding energy is negative.

b. The Newton equation for circular orbit:

$$mv^2 / r = GmxT / r \tag{13}$$

which gives

$$v = xGT \tag{14}$$

c. Bohr quantization condition for angular momentum by equation (11) reads as

$$mvr = n\hbar_{gr} \tag{15}$$

and gives:

$$r_n = n\hbar_{gr} / (mv) = n.r_1 \tag{16}$$

$$r_1 = GM / (vv_o) \tag{17}$$

where v is rather near to v_o.

d. Bound state energies are given by

$$E_n = mv^2 / 2 - xT \log(r_1 / R_o) + xT \log(n) \tag{18}$$

The energies depend only weakly on the radius of the orbit.

e. The centrifugal potential $l(l+1)/r^2$ in the Schrödinger equation is negligible as compared to the potential term at large distances so that one expects that degeneracies of orbits with small values of *l* do not depend on the radius.

5.2. TGD based model of planetary system

The magnetic flux quanta (shells and flux tubes) are the carriers of the quantum coherent dark matter and behave effectively like quantum rigid bodies. This leads to a simple model for the generation of planetary system via a breaking of rotational symmetry. For inner planets this process leads from spherical shells with a full rotational symmetry to flux tubes with reduced rotational symmetry inside with planet are eventually formed. Earth and outer planets were formed by a splitting of a flattened flux tube in the common orbital plane to 5 flux tubes corresponding to Earth and outer planets except Pluto, which indeed has orbital parameters differing dramatically from those of other planets. The replacement of v_o by its subharmonic $v_o/5$ for these Jovian planets corresponds topologically to the splitting of a magnetic flux tube to five separate tubes.

Flux tubes and spherical cells containing quantum dark matter are predicted to be still there. The amazing finding is that the quantum time scales associated with Bohr orbits seem to correspond to important *biological* time scales. For instance, the time scale

$$T = \hbar_{gr} / E \tag{19}$$

associated with n=1 orbit is precisely 24 hours. This apparently supports the prediction of TGD based theory of living matter in with quantum coherent dark matter plays a fundamental role [40].

The inclinations of planetary orbits could be a test problem for the hypothesis outlined above. The prediction is not merely statistical like the predictions given by Nottale and others [1d][1e]. The minimal value of inclination for a given principal quantum number n follows from semiclassical view about angular momentum quantization for maximal value of z-component of angular momentum m=j=n [38]:

$$\cos(\phi) = n / \sqrt{n(n+1)} \tag{20}$$

where ϕ is the angle between angular momentum and quantization axis and thus also between orbital plane and (x,y)-plane. This angle defines the tilt angle between the orbital plane and (x,y)-plane. For n=3,4,5 (Mercury, Earth, Venus) this equation gives $\phi = 30.0^o, 26.6^o, 24.0^o$ respectively. Only the relative tilt angle can be compared with the experimental data. Taking Earth's orbital plane as reference will give 'inclination' angle, i.e. 6 degrees for Mercury, and 2.6 degrees for Venus. The observed values are 7.0 and 3.4 degrees, respectively, which are in good agreement with prediction.

Bohr-Sommerfeld rules allow also estimating eccentricities and the prediction is [38]:

$$e^2 = 2.(\sqrt{1 - m^2 / n^2}) / (1 + \sqrt{1 - m^2 / n^2}) \tag{21}$$

The eccentricities are predicted to be very large for m<n unless n is very large and the only possible interpretation is that planets correspond in the lowest order approximation to m=n and e=0 whereas comets with large eccentricities could correspond to m<n orbits. In particular, for m<n comets in Oort Clouds

(n<700) the prediction is e>0.32. This could be a good test problem for further astronomical observation.

6 Concluding remarks

In this article, some plausible approaches to describe the origin of macroquantization in astrophysics and also celestial motion are discussed. While all of these arguments are interesting, it seems that further research is required to verify which arguments are the most plausible, corresponding to the observed astrophysics data.

After all, the present article is not intended to rule out the existing methods in the literature to predict quantization of celestial motion, but instead to argue that perhaps this macroquantization effect in various astronomy phenomena requires a new kind of theory to describe its origin.

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