POLYVECTOR SUPER-POINCARE ALGEBRAS, M, F THEORY ALGEBRAS AND GENERALIZED SUPERSYMMETRY IN CLIFFORD-SPACES

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Abstract

Starting with a review of the Extended Relativity Theory in Clifford-Spaces, and the physical motivation behind this novel theory, we provide the generalization of the nonrelativistic Supersymmetric point-particle action in Clifford-space backgrounds. The relativistic Supersymmetric Clifford particle action is constructed that is invariant under generalized supersymmetric transformations of the Clifford-space background's polyvector-valued coordinates. To finalize, the Polyvector Super-Poincare and M, F theory super-algebras, in $D = 11, 12$ dimensions, respectively, are discussed followed by our final analysis of the novel Clifford-Superspace realizations of generalized Supersymmetries in Clifford spaces.

1 INTRODUCTION

In recent years we have argued [1] that the underlying fundamental physical principle behind string theory, not unlike the principle of equivalence and general covariance in Einstein's general relativity, might well be related to the existence of an invariant minimal length scale (Planck scale) attainable in nature. A scale relativistic theory involving spacetime *resolutions* was developed long ago by Nottale [25] where the Planck scale was postulated as the minimum observer independent invariant resolution in Nature. Since "points" cannot be observed physically with an ultimate resolution, they are fuzzy and smeared out into fuzzy balls of Planck radius of arbitrary dimension. For this reason one must construct a theory that includes all dimensions (and signatures) on the equal footing. Because the notion of dimension is a topological invariant, and the concept of a fixed dimension is lost due to the fuzzy nature of points, dimensions are resolution-dependent, one must also include a theory with *all* topologies as well. It is our belief that this may lead to the proper formulation of string and M theory.

In [1] we applied this Extended Scale Relativity principle to the quantum mechanics of p-branes which led to the construction of C-space (a dimension *category*) where all p-branes were taken to be on the same footing; i.e. transformations in C-space reshuffled a string history for a five-brane history, a membrane history for a string history, for example. It turned out that Clifford algebras contained the appropriate algebro-geometric features to implement this principle of polydimensional transformations .

Clifford algebras have been a very useful tool for a description of geometry and physics [23]. For a detailed discussion on the algebraic unification avenue of all forces in Nature based on Clifford, Exceptional and Division algebras see [11,27,29,31]. A Clifford group unification program was outlined in [5,27]. In [3,4] it was proposed that every physical quantity is in fact a *polyvector*, that is, a Clifford number or a Clifford aggregate. Many important aspects of Clifford algebra are described in [19]. Using these methods the bosonic p-brane propagator, in the quenched mini superspace approximation, was constructed in [20]; the logarithmic corrections to the black hole entropy based on the geometry of Clifford space (in short C-space) were obtained in [22]; the action for a higher derivative gravity with *torsion* was obtained directly from the geometry of C-spaces [21] and how the Conformal agebra of spacetime emerges also from the Clifford algebra was described in [24]; the resolution of the ordering ambiguities of QFT in curved spaces was resolved by [3].

In this new physical theory the arena for physics is no longer the ordinary spacetime, but a more general manifold of Clifford algebra valued objects, polyvectors. Such a manifold has been called a pan-dimensional continuum [4] or C-space [1]. The latter describes on a unified basis the objects of various dimensionality:

not only points, but also closed lines, surfaces, volumes,.., called 0-loops (points), 1-loops (closed strings) 2-loops (closed membranes), 3-loops, etc.. It is a sort of a *dimension* category, where the role of functorial maps is played by C-space transformations which reshuffles a p-brane history for a p'-brane history or a mixture of all of them, for example.

The above geometric objects may be considered as to corresponding to the well-known physical objects, namely closed p-branes. Technically those transformations in C-space that reshuffle objects of different dimensions are generalizations of the ordinary Lorentz transformations to C-space. In that sense, the C-space is roughly speaking a sort of generalized Penrose-Twistor space from which the ordinary spacetime is a *derived* concept. In [1] we derived the minimal length uncertainty relations as well as the full blown uncertainty relations due to the contributions of *all* branes of *every* dimensionality, ranging from $p = 0$ all the way to $p = \infty$. Most recently, an extended Relativity theory in Born-Clifford Phase spaces was constructed involving both an UV (ultraviolet) and IR (infrared) cutoff [5]. The Noncommutative Yang's spacetime algebra [28], where coordinates and momenta do not commute, was extended to the full C-space [8] and allowed the construction of generalized Noncommutative branes in Clifford-space backgrounds based on a novel Moyal-Yang star products deformation quantization of Nambu-Poisson brackets involving the UV-IR cutoffs. Noncommutative Riemann-Finsler geometries using Clifford algebras were studied by [26]. For further details of the Extended Relativity Theory in Clifford spaces we refer to the review [2] and [5] .

In the past years there has been a revival in the study of Clifford algebras within the context of the M, F theory superalgebras in $D = 11, 12$ dimensions, respectively [7,9]. Important recent applications of multivectors (polyvectors) in Physics that we shall be discussing is the work on Polyvector Super-Poincare Algebras and its relation to the M, F-theory superalgebras [9] . Formulations of conformal Higher Spin theories[12] based on twistor-particle dynamics in *tensorial* spaces [6], initiated by Fronsdal, have captured a lot of interest recently. Fronsdal conjectured that four-dim conformal higher spin field theory can be realized as an ordinary field theory on a ten-dim tensorial manifold parametrized by the coordinates $x^{\alpha\beta} = \frac{1}{2}x^\mu \gamma_\mu^{\alpha\beta} + \frac{1}{4}y^{\mu\nu}\gamma_{[\mu\nu]}^{\alpha\beta}$, where x^μ are associated with the four coordinates of conventional $4D$ spacetime and $y^{\mu\nu} = -y^{\nu\mu}$ describe six spinning degrees of freedom. An infinite tower of fields of increasing spin is obtained rather than an infinite tower of massive states as in the conventional Kaluza-Klein mechanism. In $D = 3, 4, 6, 10$ dimensions the conformal higher spin fields constitute the quantum spectrum of a twistor-like particle propagating in tensorial spaces of corresponding dimensions [6] . One can notice that a string propagating in the latter dimensions, has for transverse degrees of freedom $D - 2 = 1, 2, 4, 8$ which precisely match the degrees of freedom of the real, complex, quaternion and octonion normed-division algebras . The role of enlarged superspace coordinates in the context of super p-branes, Born-Infeld and M-theory has recently been investigated by [10] . Clifford Spaces are more *fundamental* than these tensorial spaces (have a richer structure) because they require polyvector coordinates (antisymmetric tensors) of variable rank (greater than two) until saturating the values of the spacetime dimensions.

There are fundamental differences among this present work with other approaches to understand the Grassmanian calculus and Supersymmetry within the realm of the Clifford Geometric Calculus [3,17] developed by Hestenes [33] . It will become clear why this work is very different than other previous approaches. Clifford-Superspace is very different than ordinary Clifford-space and generalized supersymmetries in the former are very different than polyrotations in the latter . It is well known that the particle content of supersymmetric theories fall under irreducible representations of Clifford algebras. The N extended supersymmetry algebra in $4D$ Minkowski spacetime is based mainly on the anticommutators $\{Q^i_\alpha, Q^j_\beta\} = 2\delta^{ij}(\gamma^\mu \mathcal{C})_{\alpha\beta}P_\mu$, for $i, j = 1, 2, 3, ...N$; and $\mathcal{C}$ is the charge conjugation matrix. In the rest frame for massive particles $m \neq 0$, the anticommutator takes the form of an algebra of $2n$ fermionic creation and annihilation operators isomorphic to the Clifford algebra $Cl(4n)$. Its unique *irreducible* representation is 2^{2n} dimensional and contains both boson and fermions as required by supersymmetry [15,30]. In the massless case, there is no rest frame and there are only 2^n states that are classified according to helicity, rather than spin [15, 30].

The Extended Relativity theory in Clifford-spaces (C-spaces) is a natural extension of the ordinary Relativity theory . A natural generalization of the notion of a space-time interval in Minkwoski space to C-space is :

$$dX^2 = d\sigma^2 + dx_\mu dx^\mu + dx_{\mu\nu}dx^{\mu\nu} + ... \tag{1.1}$$

The Clifford valued poly-vector:

$$X = X^M E_M = \sigma \ \mathbf{1} + x^\mu \gamma_\mu + x^{\mu\nu} \gamma_\mu \wedge \gamma_\nu + ... x^{\mu_1 \mu_2 .... \mu_D} \gamma_{\mu_1} \wedge \gamma_{\mu_2} .... \wedge \gamma_{\mu_D}. \tag{1.2a}$$

denotes the position of a polyparticle in a manifold, called Clifford space or C-space. The series of terms in (1.2a) terminates at a *finite* value depending on the dimension D. A Clifford algebra $Cl(r,q)$ with $r+q=D$ has 2^D basis elements. For simplicity, the gammas γ^μ correspond to a Clifford algebra associated with a flat spacetime :

$$\frac{1}{2}\{\gamma^\mu, \gamma^\nu\} = \eta^{\mu\nu} \mathbf{1}. \tag{1.2b}$$

but in general one could extend this formulation to curved spacetimes with metric $g^{\mu\nu}$. The multi-graded basis elements E_M of the Clifford-valued poly-vectors are

$$E_M \ \equiv \ \mathbf{1}, \quad \gamma^\mu, \quad \gamma^{\mu_1} \wedge \gamma^{\mu_2}, \quad \gamma^{\mu_1} \wedge \gamma^{\mu_2} \wedge \gamma^{\mu_3}, \quad \gamma^{\mu_1} \wedge \gamma^{\mu_2} \wedge \gamma^{\mu_3} \wedge ..... \wedge \gamma^{\mu_D}. \tag{1.2c}$$

It is convenient to order the collective M indices as $\mu_1 < \mu_2 < \mu_3 < ...... < \mu_D$.

The connection to strings and p-branes can be seen as follows. In the case of a closed string (a 1-loop) embedded in a target flat spacetime background of D-dimensions, one represents the projections of the closed string (1-loop) onto the embedding spacetime coordinate-planes by the variables $x_{\mu\nu}$. These variables represent the respective *areas* enclosed by the projections of the closed string (1-loop) onto the corresponding embedding spacetime planes. Similary, one can embed a closed membrane (a 2-loop) onto a D-dim flat spacetime, where the projections given by the antisymmetric variables $x_{\mu\nu\rho}$ represent the corresponding *volumes* enclosed by the projections of the 2-loop along the hyperplanes of the flat target spacetimr background.

This procedure can be carried to all closed p-branes (p-loops) where the values of p are $p = 0, 1, 2, 3, ...D-2$. The $p=0$ value represents the center of mass and the coordinates $x^{\mu\nu}, x^{\mu\nu\rho}....$ have been *coined* in the string-brane literature [32] as the *holographic* areas, volumes, ...projections of the nested family of p-loops (closed p-branes) onto the embedding spacetime coordinate planes/hyperplanes.

The classification of Clifford algebras $Cl(r,q)$ in $D=r+q$ dimensions (modulo 8) for different values of the spacetime signature r,q is discussed, for example, in the book of Porteous [19]. All Clifford algebras can be understood in terms of $CL(8)$ and the $CL(k)$ for k less than 8 due to the modulo 8 Periodicity theorem $CL(n) = CL(8) \times Cl(n-8)$ where $Cl(r,q)$ is a matrix algebra for even $n = r+q$ or the sum of two matrix algebras for odd $n = r+q$. Depending on the signature, the matrix algebras may be real, complex, or quaternionic. For furher details we refer to [19] .

If we take the differential dX and compute the scalar product among two polyvectors $< dX^T dX >_{scalar}$ we obtain the C-space extension of the particles proper time in Minkwoski space. The symbol X^T denotes the *reversion* operation and involves reversing the order of all the basis γ^μ elements in the expansion of X . The C-space proper time associated with a polyparticle motion is then :

$$< dX^T dX >_{scalar} = d\Sigma^2 = (d\sigma)^2 + \Lambda^{2D-2} dx_\mu dx^\mu + \Lambda^{2D-4} dx_{\mu\nu} dx^{\mu\nu} + .. \tag{1.3}$$

Here we have explicitly introduced the Planck scale Λ since a length parameter is needed in order to tie objects of different dimensionality together: 0-loops, 1-loops,..., p-loops. Einstein introduced the speed of light as a universal absolute invariant in order to "unite" space with time (to match units) in the Minkwoski space interval:

$$ds^2 = c^2 dt^2 - dx_i dx^i. \tag{1.4}$$

A similar unification is needed here to "unite" objects of different dimensions, such as x^μ, $x^{\mu\nu}$, etc... The Planck scale then emerges as another universal invariant in constructing an extended scale relativity theory in C-spaces [1].

To continue along the same path, we consider the analog of Lorentz transformations in C-spaces which transform a poly-vector X into another poly-vector X' given by $X' = RXR^{-1}$ with

$$R = e^{\omega^A E_A} = exp \ [(\omega \mathbf{1} + \omega^\mu \gamma_\mu + \omega^{\mu_1 \mu_2} \gamma_{\mu_1} \wedge \gamma_{\mu_2} .....)]. \tag{1.5}$$

and

$$R^{-1} = e^{-\omega^A E_A} = exp\ [-(\omega \mathbf{1} + \omega^\nu \gamma_\nu + \omega^{\nu_1 \nu_2} \gamma_{\nu_1} \wedge \gamma_{\nu_2} .....)]. \tag{1.6}$$

where the ω parameters also belong to a Clifford-valued quantity

$$\omega; \omega^\mu; \omega^{\mu\nu}; .... \tag{1.7}$$

they are the C-space version of the Lorentz rotations/boosts parameters.

Since a Clifford algebra admits a matrix representation, one can write the norm of a poly-vectors in terms of the trace operation as: $||X||^2 = Trace\ X^2$ Hence under C-space Lorentz transformation the norms of poly-vectors behave like follows:

$$Trace\ {X'}^2 = Trace\ [R X^2 R^{-1}] = Trace\ [R R^{-1} X^2] = Trace\ X^2. \tag{1.8}$$

These norms are invariant under C-space Lorentz transformations due to the cyclic property of the trace operation and $RR^{-1} = 1$. Another way of rewriting the inner product of polyvectors is by means of the reversal operation that reverses the order of the Clifford basis generators : $(\gamma^\mu \wedge \gamma^\nu)^T = \gamma^\nu \wedge \gamma^\mu$, etc... Hence the inner product can be rewritten as the scalar part of the geometric product $< X^T X >_s$. The analog of an orthogonal matrix in Clifford spaces is $R^T = R^{-1}$ such that

$$< X'^T X' >_s = < (R^{-1})^T X^T R^T R X R^{-1} >_s = < R X^T X R^{-1} >_s = < X^T X >_s = invariant. \tag{1.9a}$$

This condition $R^T = R^{-1}$, of course, will *restrict* (constrain) the type of terms allowed inside the exponential defining the rotor R in eq-(3-5) because the *reversal* of a p-vector obeys

$$(\gamma_{\mu_1} \wedge \gamma_{\mu_2} ..... \wedge \gamma_{\mu_p})^T = \gamma_{\mu_p} \wedge \gamma_{\mu_{p-1}} ..... \wedge \gamma_{\mu_2} \wedge \gamma_{\mu_1} = (-1)^{p(p-1)/2} \gamma_{\mu_1} \wedge \gamma_{\mu_2} ..... \wedge \gamma_{\mu_p} \tag{1.9b}$$

Hence only those terms that *change* sign (under the reversal operation) are permitted in the exponential defining $R = exp[\omega^A E_A]$.

Another possibility is to *complexify* the C-space polyvector valued coordinates $= Z = Z^A E_A = X^A E_A + i Y^A E_A$ (which is *not* the same as *complexifying* the Clifford algebra) as well as the boost/rotation parameters ω^A in order to allow the unitarity condition $U^\dagger = U^{-1}$ to hold . The generalized Clifford unitary transformations $Z' = U Z U^\dagger = U Z U^{-1}$ associated with the complexified polyvector $Z = Z^A E_A$ must be such so the interval

$$< \ dZ^\dagger\ dZ\ \ >_s = \ d\bar{\sigma} d\sigma + d\bar{z}^\mu dz_\mu + d\bar{z}^{\mu\nu} dz_{\mu\nu} + d\bar{z}^{\mu\nu\rho} dz_{\mu\nu\rho} + ... \tag{1.9c}$$

remains invariant under these unitary transformations above (upon setting the Planck scale $\Lambda = 1$).

The unitary condition $U^\dagger = U^{-1}$, under the *combined* reversal and complex-conjugate operation, will constrain the form of the complexified boosts/rotation parameters ω^A appearing in : $U = exp[\ \omega^A E_A\ ]$. The parameters ω^A must be either purely real, or purely imaginary, depending if the reversal ${E_A}^T = \pm E_A$, to ensure that an overall *change* of sign occurs in the terms $\omega^A E_A$ inside the exponential defining U so that $U^\dagger = U^{-1}$ actually holds, and the norm $< Z^\dagger Z >_s$ remains invariant under the analog of unitary transformations in *complexified* C-spaces. These techniques are not very different from Penrose Twistor spaces. As far as we know a Clifford-Twistor space construction of C-spaces has not been performed so far.

Another alternative is to define the unitary transformations by $U = exp\ (\Omega^{AB}[E_A, E_B])$ where the commutator $[E_A, E_B] = F^C_{AB} E_C$ is the C-space analog of the $i[\gamma_\mu, \gamma_\nu]$ commutator which is the generator of the Lorentz algebra, and the parameters Ω^{AB} are the C-space analogs of the rotation/boots parameters. The diverse parameters Ω^{AB} are purely real or purely imaginary depending whether the reversal $[E_A, E_B]^T = \pm[E_A, E_B]$ to ensure that $U^\dagger = U^{-1}$ such that the scalar part $< Z^\dagger Z >_s$ remains invariant under the transformations $Z' = U Z U^{-1}$. This last alternative seems to be more physical because a polyrotation should map the E_A direction into the E_B direction in C-spaces, hence the meaning of the generator $[E_A, E_B]$ which is the extension of the $i[\gamma_\mu, \gamma_\nu]$ Lorentz generator. We refer to the review [16] for further details about the Extended Relativity Theory in Clifford spaces. In particular, why Relativity in *curved* Clifford-spaces is equivalent to a higher derivative gravity with torsion associated with the underlying spacetime [21[.

The purpose of this work is to explore the features of Clifford-Superspaces. In section **2** we provide the generalization of the nonrelativistic supersymmetric point-particle action in Clifford space backgrounds. In section **3** the Relativistic Supersymmetric Clifford Particle action is given. We must remark that the results of sections **2, 3** are *new* and to our knowledge have not appeared before in the literature. Finally, in section **4** , Polyvector Super-Poincare and M, F theory superalgebras in $D = 11, 12$ dimensions are discussed followed by our analysis and construction of the *novel* Clifford-Superspace realizations of generalized Supersymmetries in Clifford spaces . We show in the final Appendix that the generalized superalgebra in Clifford spaces does close for a particular example in $D = 4$ Minkowski space and which we foresee as being valid to other dimensions and signatures.

2. The Nonrelativistic Supersymmetric Clifford Particle

The ordinary *nonrelativistic* supersymmetric point-particle action (not to be confused with the relativistic superparticle) is obtained after introducing a bosonic time coordinate t and a fermionic Grassmanian time coordinate τ such $\tau^2 = 0$. We shall follow very closely the discussion of the book [15] . The nonrelativistic superparticle is described by the coordinates

$$Z^m(t,\tau) = x^m + \theta^m(t)\ \tau = x^m - \tau\theta^m(t). \tag{2.1}$$

where $m = 1, 2, 3, ...d$ and θ^m are the Grassmanian coordinates : $(\theta_1)^2 = (\theta_2)^2 = ... = (\theta_d)^2 = 0$.

The supersymmetry generator is :

$$Q = i\tau\frac{\partial}{\partial t} - \frac{\partial}{\partial \tau}. \tag{2.2}$$

and the algebra is

$$\{Q,\ Q\} = 2Q^2 = -2H = -2i\frac{\partial}{\partial t}. \quad [Q, H] = [H, H] = 0. \tag{2.3}$$

The supersymmetric point-particle of mass M is

$$S = \frac{1}{2}\ M\ \int dt\ \int d\tau\ DZ^m\ D\ (DZ_m). \tag{2.4}$$

where D is the supertranslation generator :

$$D = i\tau\frac{\partial}{\partial t} + \frac{\partial}{\partial \tau}. \quad \{D,\ Q\} = 0. \tag{2.5}$$

The action becomes after using the rules of Grassmanian integration

$$\int d\tau\ f(\tau) = \frac{\partial f(\tau)}{\partial \tau}. \tag{2.6}$$

$$S = \frac{1}{2}\ M\ \int dt\ \int d\tau\ (-\theta^m + i\frac{dx^m}{dt})\ (i\frac{dx_m}{dt} + i\frac{d\theta_m}{dt}) =$$
$$-\frac{1}{2}M\ \int dt\ [\ (\frac{dx^m}{dt})(\frac{dx_m}{dt}) + i\theta^m\frac{d\theta_m}{dt}\ ]. \tag{2.7}$$

The *active* supersymmetry transformations laws, by definition, affect only the bosonic (fermionic) x^m, θ^m coordinates and leave the action invariant up to *total derivatives*. In the absence of boundaries the latter don't contribute. Hence, the transformation rules are

$$\delta Z^m = \delta x^m - \tau\delta\theta^m =\ \epsilon\ Q\ Z^m =$$

$$\epsilon(i\tau\frac{\partial}{\partial t}-\frac{\partial}{\partial\tau})\ (x^m(t)-\tau\theta^m(t))=i\epsilon\tau\frac{dx^m}{dt}+\epsilon\theta^m(t)=-i\tau\epsilon\frac{dx^m}{dt}+\epsilon\theta^m(t) \quad (2.8)$$

where one is taking into account the Grassmanian nature of the fermionic time $\tau^2=0$; the ϵ parameter is also Grassmanian $\epsilon\tau=-\tau\epsilon$ like the odd fermionic coordinates $(\theta_1)^2=(\theta_2)^2=.....(\theta_m)^2=0$. From eq-(2.8) one can read-off the supersymmetry transformations of the coordinates x^m,θ^m by matching the two expressions that do (and don't) depend on τ respectively :

$$\delta Z^m=\delta x^m\ -\tau\ \delta\theta^m=(\epsilon\theta^m)\ -\tau\ (i\epsilon\frac{dx^m}{dt})\ \Rightarrow$$

$$\delta x^m=\epsilon\ \theta^m.\quad \delta\theta^m=i\epsilon\ \frac{dx^m}{dt}. \quad (2.9)$$

where ϵ is a Grassmanian parameter.

The invariance of the action S under the symmetry transformation laws (2.9), up to a *total derivative* , can be verified by simple inspection after using the property $\delta(d/dt)=(d/dt)\delta$ and integrating by parts :

$$\int dt\ \delta(i\theta^m\frac{d\theta_m}{dt})=\int dt\ [\ i(\delta\theta^m)\frac{d\theta_m}{dt}+i\theta^m\delta(\frac{d\theta_m}{dt})\ ]=-\int dt\ [\ \epsilon(\frac{dx^m}{dt})(\frac{d\theta_m}{dt})+\theta^m\epsilon\frac{d}{dt}(\frac{dx^m}{dt})\ ]=$$

$$-\int dt\ [\ \epsilon(\frac{dx^m}{dt})(\frac{d\theta_m}{dt})-\epsilon\theta^m\frac{d}{dt}(\frac{dx^m}{dt})\ ]=\int dt\ [\ -2\epsilon(\frac{dx^m}{dt})(\frac{d\theta_m}{dt})+\frac{d}{dt}(\epsilon\theta_m\frac{dx^m}{dt})\ ]. \quad (2.10a)$$

due to the Grassmanian property $\theta^m\epsilon=-\epsilon\theta^m$. The variation of the bosonic terms are

$$\int dt\ \delta\ (\frac{dx^m}{dt}\frac{dx_m}{dt})=\int dt\ 2\frac{dx^m}{dt}\frac{d}{dt}(\delta x_m)=\int dt\ 2\epsilon(\frac{dx^m}{dt})(\frac{d\theta_m}{dt}). \quad (2.10b)$$

Thus the variation of the action is a total derivative

$$\delta S=-\frac{M}{2}\int dt\ \frac{d}{dt}(\epsilon\theta_m\frac{dx^m}{dt}). \quad (2.10c)$$

and if there are no boundaries the action is invariant under the global (rigid) supersymmetry transformations $\delta S=0$.

The *nonrelativistic* supersymmetric point particle action in ordinary spaces can be generalized to Clifford spaces in a straightforward way once a length scale is introduced . The extended Relativity in Clifford-spaces are endowed with two fundamental constants, the speed of light and the Planck scale. Quantization in C-spaces yields a minimal Planck length, Planck area, Planck volume, etc.... [1] This theory can also be extended to Born-Clifford Phase spaces where an additional infrared scale is introduced [5] . For this reason it is important to discuss the dimensions of the bosonic and fermionic variables. The dimensions of θ^m and ϵ are $\frac{1}{2}$ the dimensions of x^m and the dimension of τ is $\frac{1}{2}$ the dimension of t.

The C-space extension of the *nonrelativistic* supersymmetric point particle requires to introduce the Grassmanian-valued antisymmetric tensor partners coordinates of X^M :

$$\sigma\leftrightarrow\theta;\quad x^m\leftrightarrow\theta^m;\quad x^{m_1m_2}\leftrightarrow\theta^{m_1m_2};\quad x^{m_1m_2m_3}\leftrightarrow\theta^{m_1m_2m_3},...... \quad (2.11)$$

The supersymmetry transformations for the *nonrelativistic* Clifford particle are :

$$\delta\sigma=\epsilon\ \theta.\quad \delta\theta=i\epsilon\ \frac{d\sigma}{dt}. \quad (2.12a)$$

$$\delta x^m=\epsilon\ \theta^m.\quad \delta\theta^m=i\epsilon\ \frac{dx^m}{dt}. \quad (2.12b)$$

$$\delta x^{m_1m_2}=\epsilon\ \theta^{m_1m_2}\ (L_P)^{1/2}.\quad \delta\theta^{m_1m_2}=i\epsilon\ \frac{dx^{m_1m_2}}{dt}\ (L_P)^{-1/2}. \quad (2.12c)$$

$$\delta x^{m_1m_2m_3} = \epsilon\ \theta^{m_1m_2m_3}\ L_P. \quad \delta\theta^{m_1m_2m_3} = i\epsilon\ \frac{dx^{m_1m_2m_3}}{dt}\ (L_P)^{-1}. \tag{2.12d}$$

etc..... As usual, the powers of $(L_P)^{1/2}$ are required in (2.12) in order to match units. The Clifford-valued *nonrelativistic* super-coordinates (in units $L_p = 1$) are :

$$Z = Z^M E_M = (\ \sigma(t) - \tau\ \theta(t)\ )\mathbf{1}\ +\ (\ x^m(t) - \tau\ \theta^m(t)\ )\ \gamma_m\ +\ (\ x^{m_1m_2}(t) - \tau\ \theta^{m_1m_2}(t)\ )\ \gamma_{m_1}\wedge\gamma_{m_2} + \ ....$$

$$+\ (\ x^{m_1m_2....m_d}(t) - \tau\ \theta^{m_1m_2....m_d}(t)\ )\ \gamma_{m_1}\wedge\gamma_{m_2}\wedge.....\wedge\gamma_{m_d}. \tag{2.13}$$

One does not include the time component of the gamma matrices γ^0 in (2.13). The summation is only restricted to the *spatial* (and scalar) components of the gammas.

Concluding, after setting the value $L_P = 1$, the C-space extension of the *nonrelativistic* supersymmetric point particle action that is invariant under the transformation laws (2.12), up to a sum of total derivatives, is given by :

$$S = -\frac{\mathcal{M}}{2}\int dt\ [\ (\frac{d\sigma}{dt})^2 + i\theta\frac{d\theta}{dt} + \frac{dx^m}{dt}\frac{dx_m}{dt} + i\theta^m\frac{d\theta_m}{dt} + \frac{dx^{m_1m_2}}{dt}\frac{dx_{m_1m_2}}{dt} + i\theta^{m_1m_2}\frac{d\theta_{m_1m_2}}{dt}\ +\ ....]. \tag{2.14}$$

3. The Relativistic Supersymmetric Clifford Particle

There are two fundamental differences between the relativistic superparticle and the nonrelativistic supersymmetric point particle actions. One being the explicit relativistic Lorentz invariance of the action and the other is that one has supersymmetry in the target spacetime background, rather than supersymmetry on the world-line (spinning particle) like in the previous section. Likewise, spinning strings and spinning membranes have supersymmetry on the worldsheet (world volume) [17,18] , while superstrings and superbranes have supersymmetry in the target spacetime background.

The Lorentz and reparametrization invariant action for the superparticle is [16]

$$S = \frac{1}{2}\int\ ds\ e^{-1}(\frac{dx^\mu}{ds} - i\bar\theta^\alpha\gamma^\mu_{\alpha\beta}\frac{d\theta^\beta}{ds})^2. \tag{3.1}$$

where θ is a spacetime spinorial coordinate, e is an auxiliary field (the einbein) and the supermomentum is now defined by

$$\Pi^\mu = \frac{dx^\mu}{ds} - i\bar\theta^\alpha\gamma^\mu_{\alpha\beta}\frac{d\theta^\beta}{ds}. \tag{3.2}$$

The equations of motion are

$$\Pi_\mu\Pi^\mu = 0. \quad \frac{d\Pi^\mu}{ds} = 0. \quad (\gamma^\mu\Pi_\mu)\frac{d\theta}{ds} = 0. \quad (\gamma^\mu\Pi_\mu)^2 = -\Pi^\mu\Pi_\mu = 0. \tag{3.3}$$

The supersymmetry transformations that leave invariant the action are

$$\delta\theta^\alpha = \epsilon^\alpha. \quad \delta\bar\theta^\alpha = \bar\epsilon^\alpha. \quad \delta x^\mu = i\bar\epsilon^\alpha\gamma^\mu_{\alpha\beta}\theta^\beta. \quad \delta e = 0. \tag{3.4}$$

where ϵ is a *constant* spinorial parameter. The commutator $[\delta_1, \delta_2]x^\mu$ yields a complex translation in general $i\bar\epsilon_1\gamma^\mu\epsilon_2 - 1 \leftrightarrow 2$. Notice that shifting the real coordinates x^μ by a *constant* complex (real) vector $i\bar\epsilon_1\gamma^\mu\theta$ does not alter the dx^μ/ds terms and the real action remains invariant under translations. The Clifford polyvector basis elements $\Gamma^M_{\alpha\beta}$ are comprised of

$$\mathbf{1}_{\alpha\beta};\quad \gamma^\mu_{\alpha\beta};\quad \Gamma^{\mu\nu}_{\alpha\beta} = (\gamma^\mu\wedge\gamma^\nu)_{\alpha\beta}.\quad \Gamma^{\mu\nu\rho}_{\alpha\beta} = (\gamma^\mu\wedge\gamma^\nu\wedge\gamma^\rho)_{\alpha\beta},\ etc..... \tag{3.5}$$

In the most general case when the coordinates X^M of C-space are complex, the putative C-space extension of the superparticle action is

$$S = \frac{1}{2} \int d\Sigma \ E^{-1} \ \Pi^M (\Pi_M)^\dagger = \frac{1}{2} \int d\Sigma \ E^{-1} \ (\frac{dX^M}{d\Sigma} - i\bar\theta^\alpha \Gamma^M_{\alpha\beta} \frac{d\theta^\beta}{d\Sigma}) \ (\frac{dX_M}{d\Sigma} - i\bar\theta^\alpha \Gamma_{M\alpha\beta} \frac{d\theta^\beta}{d\Sigma})^\dagger \tag{3.6}$$

E is the analog of the einbein e field necessary to implement reparametrization invariance along the C-space wordline trajectory. Another reason one has to write the real action in terms of the product $\Pi^M(\Pi_M)^\dagger$ rather than $\Pi^M \Pi_M$ is because the terms $i\bar\theta^\alpha \Gamma^M_{\alpha\beta} \frac{d\theta^\beta}{d\Sigma}$ are not necessarily self-adjoint (Hermitian) for all values of the Clifford polyvector basis Γ^M elements. The γ matrices can be chosen to be Hermitian and anti-Hermitian depending on the spacetime signature. Choosing the signature $(+,-,-,-)$ in $4D$ allows a matrix representation such that $(\gamma^0)^\dagger = \gamma^0$ and $(\gamma^i)^\dagger = -\gamma^i$. The operator $(id/d\Sigma)$ is self-adjoint because $(d/d\Sigma)^\dagger = -(d/d\Sigma)$.

The global supersymmetry transformations that leave the action (3.6) invariant are :

$$\delta\theta^\alpha = \epsilon^\alpha. \quad \delta\bar\theta^\alpha = \bar\epsilon^\alpha. \quad \delta X^M = i\bar\epsilon^\alpha \Gamma^M_{\alpha\beta} \theta^\beta. \tag{3.7}$$

Complex X^M coordinates were instrumental in section **1** when we displayed the generalized Lorentz transformations in C-spaces. For real coordinates one could have used another action of the type :

$$S = \frac{1}{2} \int d\Sigma \ E^{-1} (\frac{dX^M}{d\Sigma} - i\bar\theta^\alpha \Gamma^M_{\alpha\beta} \frac{d\theta^\beta}{d\Sigma})^2 \ + Hermitian \ conjugate. \tag{3.8}$$

As usual, it is also required to introduce suitable powers of L_P to match units. For simplicity we set $L_P = 1$ but we must always keep in mind that physics and relativity in C-spaces requires always the introduction of an invariant minimal length scale (Planck) to be able to combine objects of different dimensions [1]

The Clifford-valued supermomentum is now defined by

$$\Pi^M = \frac{dX^M}{d\Sigma} - i\bar\theta^\alpha \Gamma^M_{\alpha\beta} \frac{d\theta^\beta}{d\Sigma}. \tag{3.9}$$

and the C-space infinitesimal proper time parameter $d\Sigma$ (in units $L_p = 1$) is defined by

$$d\Sigma^2 = dX^M dX_M = (d\sigma)^2 + (dx_\mu dx^\mu) + (dx_{\mu_1\mu_2} dx^{\mu_1\mu_2}) + (dx_{\mu_1\mu_2\mu_3} dx^{\mu_1\mu_2\mu_3}) + ... + (dx_{\mu_1\mu_2......\mu_d} dx^{\mu_1\mu_2....\mu_d}). \tag{3.10}$$

However there are several problems with the naive action (3.6) . Firstly one can see that the number of bosonic degrees of freedom does *not* match the number of fermionic degrees of freedom. The number of bosonic C-space coordinates is $2^D > D$, whereas the number of components of a spinor θ in D even dimensions are $2^{D/2} < 2^D$.

In the ordinary superparticle action the number of ($on-shell$) bosonic degrees of freedom matches the number of fermionic ones after implementing the Siegel's fermionic kappa symmetry related to the nonlinear constraints of the phase space variables associated with the action. For example, in $D = 10$, the Majorana-Weyl conditions and the Siegel' kappa symmetry reduce the number of fermionic degrees of freedom from 32 (complex components) to 8 real components which is precisely the number of transverse real bosonic degrees of freedom ($10 - 2 = 8$) of a massless superparticle in $D = 10$. Despite that the covariant quantization of the superparticle is notoriously difficult due to the presence of nonlinear constraints one can still count on-shell degrees of freedom. In the case of superstrings and superbranes the situation is even more restricted and one finds that an equality of bosonic and fermionic degrees of freedom (on-shell) severely constrains the values of the target spacetime dimensions D as well as the number of worldsheet and woldvolume dimensions, respectively, for a given spacetime signature (S, T) and worldvolume signature (s, t) [15] .

Since the standard superparticle action (3.1) has an *equal* number of on-shell bosonic and fermionic degrees of freedom, it is clear that the C-space naive action (2.22) is $flawed$ because one is *enlarging* the number of bosonic coordinates x^μ by introducing the additional polyvector valued bosonic coordinates. Therefore, there is a clear *mismatch* in the number of bosonic and fermionic degrees of freedom (on-shell and off shell) in the naive C-space action (3.6).

Secondly, before constructing any actions and counting degrees of freedom, one has to properly define what are the Clifford polyvector extensions of the super Poincare algebras (super Lie algebras) in diverse dimensions and different signatures. For example, one has to verify that the graded Jacobi identities are satisfied (nontrivial task). Thirdly, one must find what the enlarged polyvector superspaces look like, which are very different than the tensorial superspaces described in [10] . We will discuss these issues in the next section.

There are two solutions to these problems. One solution is to introduce the anticommuting multi-spinor valued coordinates $\Theta^{\alpha_1\alpha_2...\alpha_k}$ as the Grassmanian partners of the polyvectors X^M; including the Grassmanian scalar partner Θ to the scalar component σ of the Clifford polyvector. This is the proper way to define a Clifford-Superspace.

Another solution that is related to Polyvector Super Poincare algebras [9] is furnished by recurring to N extended supersymmetries ! Simply introduce N anticommuting spinor coordinates $\theta^{A\alpha}$, $A = 1, 2, 3, ...N$. The index α denotes a spacetime spinor in D dimensions. A Dirac spinor in D dimensions has $2^{[D/2]}$ complex components where $[D/2]$ denotes the integer part.

Therefore, the C-space extension of the superparticle action is :

$$S = \frac{1}{2}\int d\Sigma\ E^{-1}\ \Pi^M(\Pi_M)^\dagger = \frac{1}{2}\int d\Sigma\ E^{-1}\ (\frac{dX^M}{d\Sigma} - i\bar\theta^{A\alpha}\Gamma^M_{\alpha\beta}\frac{d\theta^{A\beta}}{d\Sigma})\ (\frac{dX_M}{d\Sigma} - i\bar\theta^{A\alpha}\Gamma_{M\alpha\beta}\frac{d\theta^{A\beta}}{d\Sigma})^\dagger \quad (3.11a)$$

where $A = 1, 2, 3, ...N$ and the coordinates X^M are complex-valued for the reasons explained earlier in section **1** . For real coordinates (involving restricted polyrotations discussed in section **1**) we have

$$S = \frac{1}{2}\int d\Sigma\ E^{-1}\ (\frac{dX^M}{d\Sigma} - i\bar\theta^{A\alpha}\Gamma^M_{\alpha\beta}\frac{d\theta^{A\beta}}{d\Sigma})^2\ + Hermitian\ conjugate. \quad (3.11b)$$

It is well known that spinors are the elements of the left/right ideals of a Clifford algebra. In $D = 3+1$ for example one has $4D$ Majorana real spinors (versus complex Dirac spinors) and a Majorana representation for all of the *gamma* 4×4 matrices with *real* entries. This allows us to envision the left/right ideals elements of the Clifford algebra as columns/rows of a 4×4 real matrix. In particular, a Clifford polyvector admits an expansion in the standard Γ_M basis (comprised of antisymmetrized products of the *gammas* and the unit element) and in the spinorial basis $\xi_{A\alpha}$ as follows:

$$\Phi = \Phi^M\Gamma_M = \theta^{A\alpha}\xi_{A\alpha}. \quad A = 1, 2, 3, 4. \quad (3.12)$$

For futher details see [3]. Hence, a real polyvector Φ can be rewritten as a direct sum of *four* $4D$ Majorana spinors giving a total of $4 \times 4 = 16 = 2^4$ real degrees of freedom. When one has real X^M bosonic polyvector coordinates $\sigma, x^\mu, x^{\mu_1\mu_2}, ..$ they span a $2^4 = 16$ real-dimensional C-space and there is a match among bosonic polyvector and fermionic degrees of freedom (off-shell) $X^M \leftrightarrow \theta^{A\alpha}$. When the coordinates X^M are *complex*-valued then one must use four complex Dirac spinors $\theta^{A\alpha}$ instead of four Majorana (real) spinors to properly match degrees of freedom (off-shell) .

The extended-supersymmetry transformation rules in $D = 4$ that leave the action (3.11) invariant are

$$\delta\theta^{A\alpha} = \epsilon^{A\alpha}. \quad \delta\bar\theta^{A\alpha} = \bar\epsilon^{A\alpha}. \quad \delta X^M = i\bar\epsilon^{A\alpha}\Gamma^M_{\alpha\beta}\theta^{A\beta}. \quad A = 1, 2, 3, 4. \quad \delta E = 0. \quad (3.13)$$

where the $\epsilon^{A\alpha}$ are now *four* infinitesimal anticommuting constant spinorial parameters of the same type as the spinor coordinates $\theta^{A\alpha}$. The $\epsilon^{A\alpha}$ parameters are constant (they don't depend on the proper time). When one promotes the global (rigid) supersymmetry to a local one (supergravity) the $\epsilon^{A\alpha}$ parameters are no longer constant. Supergravity in Clifford spaces is a more complicated matter. Gravity in curved Clifford spaces was studied by us [21] where it was shown that it leads to a higher derivative gravity with torsion.

Notice that there is one similarity with the steps taken in section **2** . We introduced the Grassmanian-partners $\theta^m, \theta^{m_1m_2}, \theta^{m_1m_2m_3}...$ of the spatial $x^m, x^{m_1m_2}, x^{m_1m_2m_3}, ...$ coordinates of the ordinary C-space with the *fundamental* difference that these *thetas* behaved like *anticommuting* vectors and antisymmetric tensors, from the spacetime perspective, instead of truly spacetime *spinors* like the $\theta^{A\alpha}$ coordinates in this section. The same occurs in the RNS string that has world sheet supersymmetry with anticommuting spacetime vectors Ψ^μ (but a multiplet of spinors from the worldsheet point of view) versus the GS superstring with target spacetime supersymmetry realized in terms of spinorial spacetime coordinates.

4. Polyvector-valued Super Poincare Algebras and Clifford-space Supersymmetry

Polyvector Super-Poincare algebras as extensions of ordinary Super-Poincare algebras have been studied by [9] . The former Lie superalgebras (involving commutators and anti-commutators) should not be confused with the Z_2-graded extensions of ordinary Lie algebras, in particular with Z_2-graded extensions of Clifford algebras [13] involving only commutators. The Polyvector Super Poincare algebras have the form of $\mathbf{g} = \mathbf{g}_0 + \mathbf{g}_1$, where the even sector is $\mathbf{g}_0 = so(V) + W_0$ and the odd sector $\mathbf{g}_1 = W_1$ consists of a spinorial representation of $so(V) = so(p,q)$; i.e. W_1 is an $so(p,q)$-spinorial-module where V is a vector space of signature p, q.

The algebra of generalized translations $W = W_0 + W_1$ is the maximal solvable ideal of $\mathbf{g}$. W_0 is generated by W_1 : $[W_1, W_1] \subseteq W_0$ and $[W_0, W_1] = 0$; $[W_0, W_0] = 0$. For example, in the ordinary Super-Poincare algebra, the translations are generated by the supersymmetry generators : $\{Q, \bar{Q}\} \sim P$ and $[Q, P] = [P, P] = 0$. Choosing W_1 to be a spinorial $so(V)$-module consisting of a sum of spinors and semispinors (chiral spinors) the authors [9] proved that W_0 consists of *polyvectors*. They provided the classification of all polyvector Lie superalgebras, for all dimensions and signatures, after analysing all the $so(V)$-invariant polyvector-valued bilinear forms that can be defined on the spinor modules. N-extended polyvector super Poincare algebras were also classified in [9].

The anti-commutator [9] is :

$$\{S_\alpha, S_\beta\} = \sum_k (\mathcal{C}\Gamma^{\mu_1\mu_2....\mu_k})_{\alpha\beta} \; W^{(k)}_{0\;\mu_2\mu_2....\mu_k} \tag{4.1}$$

where α, β denote spinor indices and the summation over k must obey certain crucial restrictions to match degrees of freedom with the terms in the l.h.s. The matrix $\mathcal{C}$ is the charge conjugation matrix. Depending on the given spacetime and its signature there are at most two charge conjugation matrices $\mathcal{C}_S, \mathcal{C}_A$ given by the product of all symmetric and all antisymmetric gamma matrices, respectively. In special spacetime signatures they collapse into a single matrix [7,9] . These charge conjugation matrix $\mathcal{C}$ are essential in order to satisfy the nontrivial graded super Jacobi identities.

For example, the M-theory superlagebra in $D = 11$ is :

$$\{Q_\alpha, Q_\beta\} = (\mathcal{A}\Gamma^\mu)_{\alpha\beta} P_\mu + (\mathcal{A}\Gamma^{\mu_1\mu_2})_{\alpha\beta} Z_{\mu_1\mu_2} + (\mathcal{A}\Gamma^{\mu_1\mu_2....\mu_5})_{\alpha\beta} Z_{\mu_1\mu_2....\mu_5}. \tag{4.2}$$

P_μ is the usual momentum operator; the antisymmetric tensorial central charges $Z_{\mu_1\mu_2}, \; Z_{\mu_1\mu_2....\mu_5}$ are of ranks $2, 5$ respectively. The matrix $\mathcal{A}$ plays the role of the timelike γ^0 matrix in Minkowskian spacetimes and is used to introduced barred-spinors. In spacetimes of signature (s, t) $\mathcal{A}$ is given by the products of all the timelike gammas, up to an overall sign [7,9] . Notice that the summation over the k indices in the r.h.s is very restricted since the $k = 1, 2, 5$ sectors of the r.h.s yield in $D = 11$ a total number of $11 + 55 + 462 = 528$ components which precisely match the number of independent components of a 32×32 symmetric real matrix in the l.h.s given by $(32 \times 33)/2 = 528$.

The 12-dim Euclidean generalized supersymmetric F algebra was

$$\{Q_\alpha, Q_\beta\} = (\mathcal{C}\Gamma^\mu)_{\alpha\beta} P_\mu + (\mathcal{C}\Gamma^{\mu_1\mu_2})_{\alpha\beta} Z_{\mu_1\mu_2} + (\mathcal{C}\Gamma^{\mu_1\mu_2....\mu_5})_{\alpha\beta} Z_{\mu_1\mu_2....\mu_5}. \tag{4.3}$$

together with its complex conjugation [7] . Other Hermitian versus holomorphic complex and quaternionic generalized supertranslations (" supersymmetries") of M-theory were classified by [7]

Therefore, by studying the Polyvector Super Poincare algebras, the M and F theory superalgebras (4.1, 4.2, 4.3) one concludes that these cannot be incorporated into Clifford-superspaces because one cannot have a *restricted* summation in the k rank of the terms appearing in the $\{Q_\alpha, Q_\beta\}$ (anti)commutators, like in eqs-(4.1, 4.2, 4.3). Unless one adds further spinorial degrees of freedom, like we did in the Clifford superparticle case by recurring to N extended supersymmetries, one will not be able to match the number of degrees of freedom in a satisfactory manner.

N extended Polyvector Super Lie Algebras which were also studied by [9] . This means that the odd sector W_1 consists of N copies of the irreducible spinor module S. There are cases where there are two inequivalent copies (complex even dimensional, or real with spatial signatures $s = 0, 4$) involving N_+ chiral generators and N_- anti-chiral ones. For further details we refer to [9] . Hence, by introducing a judicious number of extra spinorial degrees of freedom in superspace, depending on the dimensions and spacetime signatures, one can acommodate for the larger number of polyvector coordinates associated with C-spaces.

In section **1** we have defined the extended Lorentz generators J_{MN} in C-space, associated with the C-space Lorentz transformations $X' = RXR^{-1}$, as

$$R = e^{\omega^{MN} J_{MN}} = e^{\omega^{MN} \ [\Gamma_M, \Gamma_N]}. \quad J_{MN} = [\Gamma_M, \Gamma_N] = F_{MN}^C \Gamma_C. \tag{4.4a}$$

For example, in $D = 4$, one has the Jacobi indentities

$$\{ \ [\mathcal{M}_{\mu_1\mu_2}, Q_\alpha], Q_\beta \ \} \ + \{ \ [\mathcal{M}_{\mu_1\mu_2}, Q_\beta], Q_\alpha \ \} \ = [\ \mathcal{M}_{\mu_1\mu_2}, \ \{Q_\alpha, Q_\beta \ \} \ ]. \tag{4.4b}$$

and

$$\{ \ [\mathcal{M}_{\mu_1\mu_2\mu_3\mu_4}, Q_\alpha], Q_\beta \ \} \ + \{ \ [\mathcal{M}_{\mu_1\mu_2\mu_3\mu_4}, Q_\beta], Q_\alpha \ \} \ = [\ \mathcal{M}_{\mu_1\mu_2\mu_3\mu_2}, \ \{Q_\alpha, Q_\beta \ \} \ ]. \tag{4.4c}$$

with

$$[M_{\mu_1\mu_2}, \ P_{\rho_1\rho_2}] = -\eta_{\mu_1\rho_1} P_{\mu_2\rho_2} + \eta_{\mu_2\rho_1} P_{\mu_1\rho_2} \pm .... \quad [\mathcal{M}_{\mu_1\mu_2}, Q_\alpha] = -\frac{1}{2}(\gamma_{\mu_1\mu_2})_\alpha^{\ \delta} Q_\delta. \tag{4.4d}$$

where

$$\{Q_\alpha, Q_\beta \ \} = \frac{1}{2} C\gamma^\mu P_\mu + \frac{1}{2} C\gamma^{\mu\nu} P_{\mu\nu} \tag{4.4e}$$

the spinorial charges Q_α behave under poly-rotations as follows

$$[\mathcal{M}_{\mu_1\mu_2\mu_3\mu_4}, Q_\alpha] = -\frac{1}{2}(\gamma_{\mu_1\mu_2\mu_3\mu_4})_\alpha^{\ \delta} \ Q_\delta. \tag{4.4f}$$

and

$$[\mathcal{M}_{\mu_1\mu_2\mu_3\mu_4}, \ P_{\nu_1\nu_2}] = \ \eta_{\mu_1\mu_2\nu_1\nu_2} P_{\mu_3\mu_4} + \eta_{\mu_3\mu_4\nu_1\nu_2} P_{\mu_1\mu_2} \pm .... \quad [\mathcal{M}_{\mu_1\mu_2\mu_3\mu_4}, \ P_{\nu_1}] = 0. \tag{4.4g}$$

In the Appendix we will prove that this algebra closes and satisfies the Jacobi identities.

G_{MN} is the *flat* C-space generalized metric $\eta_{\mu_1\nu_1\mu_2\nu_2.....\mu_n\nu_n}$ given by the determinant of the $N \times N$ matrix $\mathbf{\Upsilon}_{mn}$ whose entries are $\eta_{\mu_m\nu_n}$. For instance :

$$\eta_{\mu_1\nu_1\mu_2\nu_2.....\mu_n\nu_n} = det \ \mathbf{\Upsilon}_{mn} = \frac{1}{N!} \epsilon^{i_1 i_2 ....i_n} \epsilon^{j_1 j_2 ....j_n} \eta_{\mu_{i_1}\nu_{j_1}} \eta_{\mu_{i_2}\nu_{j_2}} ....\eta_{\mu_{i_n}\nu_{j_n}}. \tag{4.5}$$

so that

$$\eta_{\mu_1\nu_1\mu_2\nu_2} = \eta_{\mu_1\nu_1}\eta_{\mu_2\nu_2} - \eta_{\mu_1\nu_2}\eta_{\mu_2\nu_1} \quad etc.... \tag{4.6}$$

Similar results apply to the definition of $\eta_{i_1 j_1 ....i_n j_n}$.

The first *question* is whether or not the alleged (anti) commutators displayed in eqs-(4.4) truly constitute a Super-Algebra which obeys the graded super Jacobi identities (nontrivial matter) in C-space due to the nontrivial algebraic relations obtained from the (geometric) product of two polyvector basis elements $\Gamma^M \Gamma^N$ that involves a sum of terms with polyvectors of mixed grade :

$$< \Gamma^M \Gamma^N >_{m+n} \quad < \Gamma^M \Gamma^N >_{m+n-2} \quad < \Gamma^M \Gamma^N >_{m+n-4} \quad .... \quad < \Gamma^M \Gamma^N >_{|m-n|} . \tag{4.7}$$

Using the standard notation

$$\gamma^{\nu_1\nu_2........\nu_p} \equiv \gamma^{\mu_1} \wedge \gamma^{\mu_2} \wedge .... \ \wedge \gamma^{\mu_p}. \tag{4.8}$$

where the anti-symmetrization of indices is performed with unit weight, one has for example :

$$\gamma^\mu \ \gamma^\nu = \frac{1}{2}\{\gamma^\mu, \gamma^\nu\} \ + \frac{1}{2}[\gamma^\mu, \gamma^\nu] = \eta^{\mu\nu}\mathbf{1} \ + \frac{1}{2}\gamma^{\mu\nu}. \tag{4.9a}$$

$$\gamma^{\mu_1\mu_2....\mu_p}\ \gamma^{\mu_{p+1}} = \gamma^{\mu_1\mu_2.....\mu_p\mu_{p+1}} + p\ \gamma^{[\mu_1\mu_2.....\mu_{p-1}}\eta^{\mu_p]\mu_{p+1}}. \tag{4.9b}$$

$$\gamma^{\mu}\gamma^{\nu_1\nu_2........\nu_p} - (-1)^p\gamma^{\nu_1\nu_2........\nu_p}\gamma^{\mu} = 2p\ \eta^{\mu[\nu_1}\gamma^{\nu_2\nu_3.....\nu_p]}. \tag{4.9c}$$

For these reasons it is highly nontrivial to verify the graded super Jacobi identities of the alleged generalized superalgebra (4.4). If, and only if, one truly has a generalized superalgebra as such in eqs-(4.4), the next step will be to find realizations of such superalgebra in Clifford-Superspace, a highly nontrivial extension of ordinary superspace.

Let us review very briefly the properties of ordinary superspace [30] where ordinary supersymmetries are realized. The standard $N = 1$ supersymmetry algebra in $3 + 1$ written in four-component Majorana spinor notation is

$$\{Q_\alpha, \bar{Q}_\beta\} = -2i\gamma^\mu_{\alpha\beta}\frac{\partial}{\partial x^\mu}. \quad [Q_\alpha, P_\mu] = [\bar{Q}_\alpha, P_\mu] = 0. \quad [Q_\alpha, \mathcal{M}^{\mu\nu}] = \frac{1}{2}(\gamma^{\mu\nu})^\beta_\alpha Q_\beta. \tag{4.10}$$

where $\mathcal{M}^{\mu\nu}$ is the Lorentz and $P_\mu = -i\partial_\mu$ is the translation generator. Even if one were to define the putative generalized supersymmetry operator $\mathcal{Q}_{A\alpha}$ in Clifford-superspace as

$$\mathcal{Q}_{A\alpha} = \frac{\partial}{\partial\theta^{A\alpha}} - i\gamma^{\mu_1\mu_2...\mu_k}_{\alpha\beta}\ \bar{\theta}^{A\beta}\frac{\partial}{\partial x^{\mu_1\mu_2...\mu_k}}. \quad A = 1, 2, 3, ...N. \tag{4.11}$$

which is the naive natural extension of the ordinary N extended supersymmetry charge generator :

$$Q_{A\alpha} = \frac{\partial}{\partial\theta^{A\alpha}} - i\gamma^\mu_{\alpha\beta}\ \bar{\theta}^{A\beta}\frac{\partial}{\partial x^\mu}. \quad A = 1, 2, 3, ...N. \tag{4.12}$$

it won't work unless one verifies that the Clifford-Superspace Supersymmetry generator $\mathcal{Q}_{A\alpha}$ forms part of a generalized supersymmetry algebra which obeys the graded super-Jacobi identities and satisfies the generalized superalgebra (anti) commutation relations in Clifford-superspaces (associated with the Clifford algebras in different spacetime dimensions and signatures). This is a highly nontrivial matter to begin with, let alone in trying to construct irreducible representations of the generalized superalgebras, field theory realizations; generalized superstring /superbrane actions in C-spaces; to pursue a quantization program, etc.....

As a reminder [30] we recall that a $4D$ Majorana spinor Ψ_M can be written in a Weyl basis in terms of two Weyl spinors $\chi_\alpha, \bar{\chi}^{\dot{\alpha}}$ where $\alpha = 1, 2$ and $\dot{\alpha} = 1, 2$. Spinor indices are raised and lower by the $\epsilon_{\alpha\beta}$, ... antisymmetric 2×2 matrices. This decomposition of a $4D$ Majorana spinor into two-component Weyl spinors and the 4×4 γ matrices in terms of blocks consisting of σ^μ Pauli 2×2 matrices (where σ^0 is the unit matrix, up to a sign) is very convenient. The chiral (antichiral) covariant differential operators in $N = 1$ superspace are

$$D_\alpha = \frac{\partial}{\partial\theta^\alpha} + i\sigma^\mu_{\alpha\dot{\alpha}}\ \bar{\theta}^{\dot{\alpha}}\frac{\partial}{\partial x^\mu}. \quad \bar{D}_{\dot{\alpha}} = -\frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}} - i\theta^\alpha\sigma^\mu_{\alpha\dot{\alpha}}\ \frac{\partial}{\partial x^\mu}. \tag{4.13}$$

and the chiral (antichiral) supersymmetry generators are

$$Q_\alpha = \frac{\partial}{\partial\theta^\alpha} - i\sigma^\mu_{\alpha\dot{\alpha}}\ \bar{\theta}^{\dot{\alpha}}\frac{\partial}{\partial x^\mu}. \quad \bar{Q}_{\dot{\alpha}} = -\frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}} + i\theta^\alpha\sigma^\mu_{\alpha\dot{\alpha}}\ \frac{\partial}{\partial x^\mu}. \tag{4.14}$$

The Q's and the D's anticommute among themselves and the only nonzero anticommutator among the D and the $\bar{D}$ is :

$$\{D_\alpha, \bar{D}_{\dot{\alpha}}\} = -i\sigma^\mu_{\alpha\dot{\alpha}}\ \frac{\partial}{\partial x^\mu}. \tag{4.15}$$

Superfields form linear representations of supersymetry algebras. In general these representations are highly reducible. The extra components can be eliminated by imposing covariant constraints like $\bar{D}_{\dot{\alpha}}\Phi = 0$, $D_\alpha\Phi = 0$ leading to chiral (anti-chiral) superfields, respectively. For example the chiral superfield in $D = 2$ can be expanded into components

$$\Phi(x^\mu;\theta^1,\theta^2) = \phi(x^\mu) + \bar\theta\ \Psi(x^\mu) + F(x^\mu)\ \bar\theta\theta. \tag{4.16}$$

since the expansion in powers of θ's terminates due to the Grassmanian nature $\theta\theta = 0$ where θ is a $2D$ Majorana spinor with real Grassmanian-valued (anticommuting) entries θ^1, θ^2 . After some straightforward algebra and using the Grassmanian integration rules in chiral superspace , the action becomes :

$$S = -\frac{i}{4\pi}\int [d^2x]\ [d^2\theta]\ \bar D_\alpha \Phi(x^\mu,\theta^1,\theta^2)\ D^\alpha \Phi(x^\mu,\theta^1,\theta^2) =$$

$$-\frac{1}{2\pi}\int [d^2x]\ [\ (\partial_\mu\phi)\ (\partial^\mu\phi) - i\bar\Psi\gamma^\mu\partial_\mu\Psi - FF)\ ]. \tag{4.17}$$

The *global* supersymmetry transformations laws for the chiral scalar multiplet in $D = 2$ that leave invariant the action, up to total derivatives, are

$$\delta\phi = \bar\epsilon\Psi. \quad \delta\Psi = -i(\gamma^\mu\partial_\mu\phi)\epsilon + F\epsilon. \quad \delta F = -i\bar\epsilon\gamma^\mu\partial_\mu\Psi. \tag{4.18}$$

If the alleged generalized superalgebra in C-spaces (4.4) does not obey the graded super Jacobi identities then one must search for other possibilities. If the superalgebra (4.4) is indeed adequate then we must verify that the supercharges $\mathcal{Q}_{A\alpha}$ truly admit the realizations postulated in (4.11). In this case one would have for generalized superfields the following

$$\mathbf{\Phi}(\sigma, x^\mu, x^{\mu_1\mu_2}, ..., x^{\mu_1\mu_2...\mu_d}; \theta^{A\alpha}, \bar\theta^{A\alpha}). \quad A = 1,2,3, ...N. \tag{4.19}$$

and follow similar steps as one does in ordinary superspace to construct covariant derivatives and invariant actions, up to total derivatives, as outline above.

If the supercharges $\mathcal{Q}_{A\alpha}$ are not satisfactory, the other proposal will be to introduce the anticommuting multi-spinor valued coordinates $\Theta^{\alpha_1\alpha_2...\alpha_k}$ as the Grassmanian partners of the polyvectors X^M in C-spaces. This may turn out to be the proper way to define a Clifford-Superspace realization of the generalized Superalgebras in Clifford-spaces. A lot remains to be done. The Polyvector Super-Poincare algebras [9] and the M, F theory superalgebras [7] are encouraging findings that should propel us to search for the proper generalized superalgebras in Clifford-spaces and to unravel the physical principle behind M, F theories [1] . The findings of sections **2, 3** were satisfactory. This is a positive sign that we are in the right track. A generalized Supersymmetry based on p-form coordinates in ordinary spacetimes (not in Clifford spaces) was suggested a while ago by [34].

APPENDIX : CLOSURE OF THE CLIFFORD SPACE SUPERSYMMETRY

The classification of the family of symmetric matrices $(C\gamma^{\mu_1\mu_2\cdots\mu_n})_{\alpha\beta}$ is what restricts the type of terms that appear in the $\{Q_\alpha, Q_\beta\}$ anticommutator and depends on the number of space time dimensions D, the signatures (s, t) and the rank n. A table of the allowed values of D, s, t, n can be found in [34] . In particular, when $D = 4 = 3 + 1$, the $\{Q_\alpha, Q_\beta\ \}$ is a symmetric matrix in α, β with 10 independent components and which matches the degrees of freedom in $P^\mu, P^{\mu\nu}$ given by $4 + 6 = 10$. Let us study the closure of

$$\{\ [\mathcal{M}_{\mu_1\mu_2\mu_3\mu_4}, Q_\alpha], Q_\beta\ \}\ + \{\ [\mathcal{M}_{\mu_1\mu_2\mu_3\mu_4}, Q_\beta], Q_\alpha\ \}\ = [\ \mathcal{M}_{\mu_1\mu_2\mu_3\mu_2},\ \{Q_\alpha, Q_\beta\ \}\ ]. \tag{A-1}$$

where

$$\{Q_\alpha, Q_\beta\ \} = \frac{1}{2}C\gamma^\mu P_\mu + \frac{1}{2}C\gamma^{\mu\nu}P_{\mu\nu} \tag{A-2}$$

In $D = 4$, with signatures $-,+,+,+$ one can find a charge conjugation matrix C and its transpose C^T obeying the properties

$$(C\gamma^\mu)^T = (C\gamma^\mu). \quad (C\gamma^{\mu\nu})^T = (C\gamma^{\mu\nu}) \tag{A-3}$$

$$C^T = -C, \quad C\gamma_\mu C^{-1} = -\gamma_\mu^T. \quad C^\dagger C = CC^\dagger = 1, \quad C^{-1}\gamma_{\mu\nu}C = -\gamma_{\mu\nu}^T. \qquad (A-4)$$

It is convenient to use a Majorana representation where the charge conjugation matrix is given by $C = \gamma_0$ and $\gamma_5^T = -\gamma_5$ is a hermitian matrix that has zero entries along the diagonal and $-i\sigma_1, i\sigma_1$ off the diagonal.

We must verify that (A-1) is obeyed. This requires that the spinorial charges Q_α behave under poly-rotations as follows

$$[\mathcal{M}_{\mu_1\mu_2\mu_3\mu_4}, Q_\alpha] = -\frac{1}{2}(\gamma_{\mu_1\mu_2\mu_3\mu_4})_\alpha^{\ \delta}\ Q_\delta. \qquad (A-5)$$

and

$$[\mathcal{M}_{\mu_1\mu_2\mu_3\mu_4},\ P_{\nu_1\nu_2}] = \ \eta_{\mu_1\mu_2\nu_1\nu_2}P_{\mu_3\mu_4} + \eta_{\mu_3\mu_4\nu_1\nu_2}P_{\mu_1\mu_2} \pm .... \qquad (A-6)$$

the $\pm$ signs in the r.h.s of (A-6) depend on the permutation of indices w.r.t to the initial combination $\mu_1\mu_2\mu_3\mu_4, \nu_1\nu_2$. There are 6 terms in (A-6). The l.h.s of (A-1) is

$$-\frac{1}{4}\gamma_5(C\gamma^\mu P_\mu + C\gamma^{\mu\nu}P_{\mu\nu})\ -\frac{1}{4}\ [\ \gamma_5(C\gamma^\mu P_\mu + C\gamma^{\mu\nu}P_{\mu\nu})\ ]^T =$$

$$-\frac{1}{4}\gamma_5(C\gamma^\mu P_\mu + C\gamma^{\mu\nu}P_{\mu\nu})\ -\frac{1}{4}\ [(C\gamma^\mu)^T\gamma_5^T P_\mu + (C\gamma^{\mu\nu})^T\gamma_5^T P_{\mu\nu}] =$$

$$-\frac{1}{4}\gamma_5(C\gamma^\mu P_\mu + C\gamma^{\mu\nu}P_{\mu\nu})\ +\frac{1}{4}\ (C\gamma^\mu P_\mu + C\gamma^{\mu\nu}P_{\mu\nu})\gamma_5. \qquad (A-7)$$

where we have used the conditions (A-3) and $\gamma_5^T = -\gamma_5$.

Multiplying (A-7) from the left by C^{-1} and using $C^{-1}\gamma_5 C = -\gamma_5$ yields

$$\frac{1}{4}(\gamma_5\gamma^\mu + \gamma^\mu\gamma_5)P_\mu\ +\frac{1}{4}\ (\gamma_5\gamma^{\mu\nu} + \gamma^{\mu\nu}\gamma_5)P_{\mu\nu} =$$

$$\frac{1}{4}(\gamma_5\gamma^{\mu\nu} + \gamma^{\mu\nu}\gamma_5)P_{\mu\nu} = \frac{1}{2}\gamma_5\gamma^{\mu\nu}P_{\mu\nu} = \frac{1}{2}\gamma_{[\mu_1}\gamma_{\mu_2}\gamma_{\mu_3}\gamma_{\mu_4]}\gamma^{\nu_1\nu_2}P_{\nu_1\nu_2} =$$

$$\frac{1}{2}[\ \gamma_{[\mu_1\mu_2]}\eta^{\nu_1\nu_2}_{\mu_3\mu_4} + ...\ ]P_{\nu_1\nu_2} =$$

$$\frac{1}{2}[\ \gamma_{[\mu_1\mu_2]}P_{\mu_3\mu_4} + \gamma_{[\mu_3\mu_4]}P_{\mu_1\mu_2} \pm ....\ ]. \qquad (A-8)$$

one may notice that due to the condition $\{\gamma_5, \gamma_\mu\} = 0$ there are no P_μ terms in (A-8). The r.h.s of (A-1) is

$$\frac{1}{2}[\ \mathcal{M}_{\mu_1\mu_2\mu_3\mu_4},\ (C\gamma^{\nu_1}P_{\nu_1}) + (C\gamma^{\nu_1\nu_2})P_{\nu_1\nu_2}] =$$

$$\frac{1}{2}(C\gamma^{\nu_1\nu_2})\ [\ \eta_{\mu_1\mu_2\nu_1\nu_2}P_{\mu_3\mu_4} + \eta_{\mu_3\mu_4\nu_1\nu_2}P_{\mu_1\mu_2} + ...\ ]. \qquad (A-9)$$

where

$$[\ \mathcal{M}_{\mu_1\mu_2\mu_3\mu_4}, (C\gamma^{\nu_1}P_{\nu_1})] = C\gamma^{\nu_1}[\ \mathcal{M}_{\mu_1\mu_2\mu_3\mu_4}, P_{\nu_1}] = 0. \qquad (A-10)$$

Multiplying (A-9) on the left by C^{-1} yields

$$\frac{1}{2}\gamma^{\nu_1\nu_2}\ [\ \eta_{\mu_1\mu_2\nu_1\nu_2}P_{\mu_3\mu_4} + \eta_{\mu_3\mu_4\nu_1\nu_2}P_{\mu_1\mu_2} + ...\ ] = \frac{1}{2}\ [\ \gamma_{[\mu_1\mu_2]}P_{\mu_3\mu_4} + \gamma_{[\mu_3\mu_4]}P_{\mu_1\mu_2} \pm ......\ ]. \quad (A-11)$$

We have seen that a left-multiplication of the r.h.s and l.h.s of (A-1) by C^{-1}, leads to the equality of (A-8) with (A-11), which implies that (A-1) is indeed satisfied .

The Jacobi identity

$$\{\ [\mathcal{M}_{\mu_1\mu_2}, Q_\alpha], Q_\beta\ \}\ +\{\ [\mathcal{M}_{\mu_1\mu_2}, Q_\beta], Q_\alpha\ \}\ = [\ \mathcal{M}_{\mu_1\mu_2},\ \{Q_\alpha, Q_\beta\ \}\ ]. \qquad (A-12)$$

when

$$[M_{\mu_1\mu_2},\ P_{\rho_1\rho_2}] = -\eta_{\mu_1\rho_1}P_{\mu_2\rho_2} \pm ... ; [\mathcal{M}_{\mu_1\mu_2}, Q_\alpha] = -\frac{1}{2}(\gamma_{\mu_1\mu_2})_\alpha{}^\delta Q_\delta. \quad \{Q_\alpha, Q_\beta\ \} = \frac{1}{2}C\gamma^\nu P_\nu + \frac{1}{2}C\gamma^{\nu_1\nu_2}P_{\nu_1\nu_2} \quad (A-13)$$

involves terms containing P_μ and $P_{\mu\nu}$. We know that the Jacobi identity is satisfied for the P_μ terms since this is what the ordinary supersymmetry algebra entails.

The $P_{\mu\nu}$ terms involve the commutator

$$-[\gamma_{\mu_1\mu_2},\ \gamma_{\nu_1\nu_2}]P^{\nu_1\nu_2} = (\eta_{\mu_1\nu_1}\gamma_{\mu_2\nu_2} \pm ....\ )P^{\nu_1\nu_2}. \quad (A-14)$$

Each one of the four terms in (A-14) , for example, like the term $\eta_{\mu_1\nu_1}\gamma_{\mu_2\nu_2}P^{\nu_1\nu_2}$ can be rewritten as :

$$\eta_{\mu_1\nu_1}\gamma_{\mu_2\nu_2}P^{\nu_1\nu_2} = \eta_{\mu_1\nu_1}\ \gamma^{\rho_1\rho_2}\eta_{\rho_1\rho_2\mu_2\nu_2}\ \eta^{\nu_1\nu_2\mu_2\rho_2}P_{\mu_2\rho_2} = -\eta_{\mu_1\nu_1}\delta_{\rho_1}^{\ \nu_1}\gamma^{\rho_1\rho_2}P_{\mu_2\rho_2} = -\eta_{\mu_1\rho_1}\gamma^{\rho_1\rho_2}P_{\mu_2\rho_2}. \quad (A-15)$$

and similarly one can rewrite the other three terms of (A-14), so that the Jacobi idenity (A-12) is satisfied due to the equality in (A-15)

$$\gamma^{\rho_1\rho_2}[M_{\mu_1\mu_2},\ P_{\rho_1\rho_2}] = \gamma^{\rho_1\rho_2}(-\eta_{\mu_1\rho_1}P_{\mu_2\rho_2} \pm ...) = -[\gamma_{\mu_1\mu_2},\ \gamma_{\nu_1\nu_2}]P^{\nu_1\nu_2} = P^{\nu_1\nu_2}(\eta_{\mu_1\nu_1}\gamma_{\mu_2\nu_2}\ \pm ...). \quad (A-16)$$

i.e, the equality among the terms of (A-16) can be seen effectively as exchanging $\gamma \leftrightarrow P$ and $(\nu_1, \nu_2) \leftrightarrow (\rho_1, \rho_2)$.

One must have as well :

$$[Q_\alpha, P_\mu] = [Q_\alpha, P_{\mu\nu}] = 0. \quad [P_\mu, P_\nu] = [P_{\mu_1\mu_2}, P_{\nu_1\nu_2}] = 0.... \quad (A-17)$$

This example in $D = 4$ should be valid in other dimensions and signatures provided we have the appropriate list of symmetric $(C\gamma^{\mu_1\mu_2\cdots\mu_n})_{\alpha\beta}$ matrices.

One has the remaining commutators :

$$[M_{\mu_1\mu_2}, M_{\nu_1\nu_2}] = -\eta_{\mu_1\nu_1}M_{\mu_2\nu_2} + \eta_{\mu_2\nu_1}M_{\mu_1\nu_2} \pm ... \quad (A-18)$$

$$[M_{\mu_1\mu_2\mu_3\mu_4}, M_{\nu_1\nu_2\nu_3\nu_4}] = \eta_{\mu_1\mu_2\nu_1\nu_2}M_{\mu_3\mu_4\nu_3\nu_4} \pm ...... \quad (A-19)$$

$$[M_{\mu_1\mu_2}, M_{\nu_1\nu_2\nu_3\nu_4}] = -\eta_{\mu_1\nu_1}M_{\mu_2\nu_2\nu_3\nu_4} \pm ......\ + \eta_{\mu_1\mu_2\nu_1\nu_2}M_{\nu_3\nu_4} \pm ..... \quad (A-20)$$

One must verify the Jacobi identities involving triplets comprised of the $M_{\mu_1\mu_2}, M_{\nu_1\nu_2\nu_3\nu_4}$ generators. The trivial commutator

$$[M_{\mu_1\mu_2}, M_{\nu_1\nu_2\nu_3\nu_4}] = 0. \quad (A-21)$$

obeys automatically the Jacobi identities. The nontrivial commutator (A-20) also obeys the Jacobi identities after laborious but straightforward algebra.

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