Stochastic field theory and Terascale physics

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Both theory and experiment strongly suggest that new phenomena await discovery above the energy range of the standard model for particle physics (SM). In this brief study we argue that a correct description of physics in the Terascale sector needs to account for the inherent randomness induced by short-distance fluctuations. The alleged existence of "un-particles" beyond SM is motivated by a dynamic setting that is far-of-equilibrium and able to sustain a rich spectrum of complex phenomena.

Towards a paradigm shift in contemporary field theory

Quantum Field Theory (QFT) is a framework whose methods and ideas have found successful applications in many domains, from particle physics and condensed matter to cosmology, statistical physics and critical phenomena [1, 2]. As a fundamental synthesis of quantum mechanics and special relativity, QFT forms the foundation for SM, a body of knowledge that describes the behavior of all known particles and their interactions except gravity. Feynman diagrams are well-established tools for computing transition amplitudes in QFT [1, 2]. As particle physics enters the era of high-energy experiments at the Large Hadron Collider (LHC) and International Linear Collider (ILC), one is compelled to ask the following question: How reliable is the apparatus of perturbation theory in the Terascale sector of field theory? To answer this question, it is important to properly define the domain of validity for the path integral (PI) formalism of QFT and the technique of Feynman diagrams. In particular,

a) The superposition principle implied by PI does not hold for strongly coupled nonlinear dynamical systems. Strictly speaking, amplitudes computed using PI formalism are fully reliable only for models consisting exclusively of abelian fields (quantum electrodynamics) or weakly coupled non-abelian fields (UV limit of Yang-Mills theories) [1, 2].

b) PI assumes unitary evolution as embodied in the Schrödinger or Heisenberg pictures. More general, PI is rooted on equilibrium dynamics and echoes the principles of Boltzmann-Gibbs statistical physics. Effective field theory (EFT) is based on the explicit hypothesis that microscopic fields (quantum corrections contributed by super-heavy excitations) can be coarse-grained and absorbed into a re-definition of the coupling coefficients defining the Lagrangian [3]. This conjecture assumes that microscopic fields are stable and can be effectively shielded from interfering with macroscopic fields. However, overlap continues to exist in the so-called crossover region where critical fluctuations cannot be fully suppressed [4].

c) Quantum processes maintain coherence. This ansatz fails in the presence of fast fluctuations that rapidly decohere the system and drive the transition from “quantum” to the “classical” behavior [5].

d) Evolution is assumed to be regular and described by everywhere differentiable functions. According to [6], Hamiltonian systems are carriers of chaos. The phase space of an arbitrary Hamiltonian system contains regions where motion occurs with a mixing of trajectories. In this instance, the hypothesis of regular evolution and “smooth” trajectories breaks down.

e) Compliance with special relativity demands that particle processes are strictly local. But the "locality" ansatz is bound to fail near second order phase transitions following the manifest loss of scale associated with critical phenomena. Critical behavior involves cooperative phenomena that evolve on vastly different length scales *while still remaining compliant with relativity*. In this instance, the concept of "locality" cannot be separated from the concept of observation scale: self-similarity enables one to map a non-local process into a local one by an appropriate scale transformation.

It is our view that all these arguments call for a paradigm shift in how field theory is approached beyond SM. A natural question is then: What is the best way to initiate this change of perspective? Owing it to the significant progress in this field, we believe that a promising avenue is the *complex dynamics of nonlinear systems*. Pattern formation and self-organized criticality are typical examples of phenomena that display complex behavior [7, 8]. Recent years have taught us that complex phenomena seem to show "universality" across vastly different energy regions. Collective behavior is prone to develop in *nonlinear* systems that are *open* to environmental or internal fluctuations. Since QFT is essentially based on nonlinear gauge models and its ultra-short distance regime describes phenomena that unfold under large perturbations in momentum, it is reasonable to assume that complexity will play a key role in explaining upcoming experiments at LHC, ILC and next generation accelerators [8-10]. By the same token, analytic tools offered by stochastic dynamics and non-equilibrium statistical physics will most likely be of great utility to this undertaking [25].

Recently, the possibility of a scale-invariant hidden sector of particle physics extending beyond SM has attracted a lot of attention [11-15]. A strange consequence of this hypothesis is the emergence of a continuous spectrum of massless fields having non-integral scaling dimensions called “un-particles”. Drawing from arguments pertaining to the behavior of Renormalization Group in the presence of random fluctuations [16-18], we suggest herein that the would-be “un-particles” arise due to a dynamic setting that is manifestly stochastic and out-of-equilibrium. We also argue that this picture enables a natural explanation for breaking of space-time symmetries in weak interactions. The violation of space-time symmetries has recently been identified as a promising candidate signal for physics beyond SM [19].

<u>Terascale physics as source of “un-particles”</u>

Following [11, 12], we begin with the hypothesis that there is a hidden sector lying beyond SM whose existence is likely to be uncovered at LHC, ILC or future accelerators. To streamline the derivation, we use the EFT prescription [3] and model this sector using a single light field operator $O(\mu)$ in interaction with a single heavy state that emerges in the deep UV region ($\Lambda \gg \Lambda_{SM}$). Here, $\Lambda_{SM} = O(G_F^{-1/2}) \approx 300\, GeV$ stands for the uppermost bound of SM corresponding to the weak interaction scale. The EFT is then defined by the Lagrangian

$$L_{EFT}(\mu) = c(\mu, \Lambda) O(\mu) = \frac{c_0(\mu)}{\Lambda^{d_O - 4}} O(\mu) \tag{1}$$

Here, μ is the sliding scale and d_0 the mass dimension of operator $O(\mu)$

$$[O(\mu)] = \mu^{d_O} \tag{2}$$

Lagrangian (1) contains only the light field operator and the effect of the heavy field is encoded in the coupling constant $c(\mu,\Lambda)$. Our aim is to study the behavior of the theory near its infrared fixed point $\mu_{IR} \approx \Lambda_{SM}$.

According to [16] the light field operator acts as a random entity in momentum space. Without any loss of generality, let us define the coarse-grained operator

$$O_R(\mu) = \frac{1}{K}\int O(\eta)W(\mu-\eta)d\eta \qquad (3)$$

in which μ stands for the sliding scale and K is a normalization constant. The kernel function $W[\mu]$ is linearly related to the coarse-grained probability density of locating a specific value in momentum space $p[O(\mu),c_0(\mu_0)]$. It can be shown that the asymptotic form of the coarse-grained probability density near the IR point is given by [16]

$$\lim_{\mu\to\Lambda_{SM}} p[O(\mu),c_0(\mu_0)] \approx \mu^{-\delta}F[\frac{O(\mu_0)}{\mu_0^{d_O}}(\frac{\mu}{\mu_0})^{-\delta};c_0^*] \qquad (4)$$

where

$$\delta = d_O + \frac{1}{2}\gamma(c_0^*) \qquad (5)$$

Here, the theory is assumed to be massless for simplicity, c_0^* is a fixed point of $c_0(\mu)$, μ_0 denotes an arbitrary reference scale and $\gamma(...)$ represents the so-called anomalous dimension. This universal result indicates that the large scale asymptotic form of the coarse-grained probability density represents a non-trivial power of the sliding scale times a certain dimensionless function $F[...]$. Replacing in (3) yields

$$\lim_{\mu\to\Lambda_{SM}} O_R(\mu) \propto \int O(\eta)(\mu-\eta)^{-\delta}F[\frac{O(\mu_0)}{\mu_0^{d_O}}(\frac{\mu-\eta}{\mu_0})^{-\delta}]d\eta \qquad (6)$$

Since there is no restriction regarding the choice of μ_0, it is convenient to assume

$$|\mu - \eta| \square \; \mu_0 \tag{7}$$

On account of (7), a reasonable approximation of (6) can be presented as

$$\lim_{\mu \to \Lambda_{SM}} O_R(\mu) \propto \int (\mu - \eta)^{-\delta} \frac{\partial \Omega_1}{\partial \eta} d\eta + \int (\mu - \eta)^{1-\delta} \frac{\partial \Omega_2}{\partial \eta} d\eta \tag{8}$$

where

$$O(\eta) \square \frac{\partial \Omega_1(\eta)}{\partial \eta}$$

$$O(\eta) \frac{\partial F(\eta)}{\partial \eta}\bigg|_{\mu = \eta} \square \frac{\partial \Omega_2(\eta)}{\partial \eta} \tag{9}$$

Finally, using the expression of differential operator from fractional calculus, we arrive at

$$\boxed{\lim_{\mu \to \Lambda_{SM}} O_R(\mu) \propto D_C^{\delta} \Omega_1(\mu) + D_C^{\delta - 1} \Omega_2(\mu)} \tag{10}$$

where the Caputo derivative of order α is defined by [20]

$$D_C^{\alpha} f(x) \square \frac{1}{\Gamma(1-\alpha)} \int (x - \tau)^{-\alpha} \left(\frac{df(\tau)}{d\tau}\right) d\tau \tag{11}$$

This result shows that, near the weak interaction scale Λ_{SM}, conventional differential operators need to be replaced by fractional operators. Our conclusion agrees with [16], where it is argued that Renormalization Group in the presence of random fluctuations and interactions describes fractional Brownian motion and complex behavior. We also direct the reader to [9], in which a similar motivation is articulated in greater detail.

There are two important consequences that can be drawn from our model:

1) fractional operators lead to the emergence of non-integer numbers of particles and antiparticles. These fields were dubbed "complexons" in [9] and, at variance with the approach taken in [11, 12], they are directly related to stochastic dynamics

driven by Terascale fluctuations. It is instructive to note that "complexons" bear resemblance to the so-called "un-matter" particles discussed in [21-23].

2) fractional operators defined on space-time (rather than momentum space) have a built-in asymmetry to the inversion of coordinates. This fact enables a natural explanation for breaking of parity and CP symmetries in weak interactions [24].

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