Dynamic Spherical Casimir Effect as a Contributing Mechanism for Pair Production

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Abstract

A simple calculation is made where the zero-point energy is obtained for a spherical Casimir cavity the size of the classical electron radius. The result is found to be roughly equivalent to the rest mass-energy of an electron-positron pair. A discussion is provided from this that suggests a possible contributing mechanism for pair production. It is suggested how the virtual spherical cavity could come about in the presence of a background E&M field and that such cavities could be viewed as a dynamic virtual potential energy field.

Calculation

The following calculation may prove insightful upon examination. Suppose we take a quantum spherical conducting cavity chosen to have the radius of a classical electron. The intension is to apply this to electron-positron vacuum polarization. Therefore, we assume for the moment a background electromagnetic field (described in more detail below) exist that sets-up electron-positron fluctuations acting like a spherical cavity with a conducting surface. We can imagine a virtual spherical electron-positron dipole type geometry with alternating charging on its surface that simulates such a cavity produced by background radiation. The figure below provides a view of the polarized spinning electron-positron action that one might envision. Note that pair-production has not yet occurred.

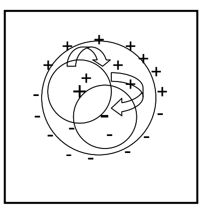

Figure Virtual dynamic spherical dipole vacuum cavity

The change in energy due to such a virtual dynamic cavity can then be summed in terms of Q.E.D. vacuum energy

$$\Delta E(t) \ = E_0 - E_{Cavity}(t) \tag{1}$$

Here $E_{Cavity}(t)$ is the sum over all discrete allowed modes due to the dynamic cavity, while E_o is the vacuum energy in the same space without external vacuum disturbance creating the cavity. The zero-point energy due to a spherical cavity can be evaluated. We use the classical electron radius for the evaluation as discussed. (Although electron radius is controversial, where high energy physics reports smaller and smaller point like observations, the classical electron radius is interesting for this evaluation). The arguments for such a virtual radius may be rational, but here we take it to be a modeling assumption for a feasible dimension. Then

$$\Delta r_{e-p} = k\frac{e^2}{mc^2} \tag{2}$$

where $k=1/4\pi\varepsilon_0$, ε_0 is the permittivity of free space, *e* and *m* are the electric charge and the mass of the electron, and *c* is the speed of light. The zero-point energy due to a dynamic spherical cavity is taken as [1-5]

$$\Delta E_{Casmir\,sphere}(t) = 0.09235\,\frac{\eta c}{2a} \tag{3}$$

where the time varying radius, *a*, is taken as constant in size. Engineering the equations, by inserting Eq. 2 provides

$$\Delta E_{Casmir\,sphere} = \frac{0.09235}{4\alpha}(2mc^2) \approx \pi\,(2mc^2) \tag{4}$$

where α is the fine structure constant. The result is then approximated by the mass-energy equivalence of an electron-positron pair to within a geometric factor of roughly π. The classical radius shows interesting results as a theoretical engineering estimate. Some discussion is appropriate.

Discussion

Casimir's original result later prompted his suggestion [6] that the original attractive force he found between parallel conducting plates could play a significant role in electron stability. Later it was noted in a calculation by Boyer [2], and verified by others [3-5] that the Casimir energy for the sphere was positive. Therefore, stability would not occur with a positive force. However, these authors did not consider this Casimir effect as a potential positive pressure for pair production. The instability of a virtual pair may be due to such a positive force and could be a mechanism in vacuum excitation. Although the scenario sited here may be specific, it is perhaps a reasonable model. By conservation of energy, the energy to create such a sphere could be given back in the excitation process partly due to this Casimir energy. Therefore, a possible "Casmir pair production energy" may exist.

Consider for a moment if we equate the energy difference in Eq. 1, as an overall quantum beat for a virtual positron-electron wave packet to obtain some insight

$$E_0 - E_{Cavity}(t) = \eta\Delta\omega_{beat} \tag{5}$$

From the equivalency in Eq. 4, we can write

$$\Delta\omega_{beat} \approx \pi\frac{2mc^2}{\eta} = \frac{1}{\Delta t_{Uncertainty}} \tag{6}$$

This as described above to within the π factor, equates to the minimum frequency for electron-positron pair production. The frequency provides an expected energy.

The work to produce the cavity of course comes from a background electromagnetic field. Such energy is then stored and may be viewed as potential energy until converted back to kinetic energy of pair production or removed by the background radiation. The result here shows some greater uncertainty in the energy than expected since Eq. 4 approximation is larger than the energy needed for pair production.

The fact that it can be viewed as a kind of potential energy vacuum field, suggest some ability to engineer the vacuum.

Background Field

The virtual electron-positron cavity may come about in a number of ways. It could be engineered with superimposed background dynamic fields with the correct polarization. Alternately a combined dynamic and a static E&M field might be used with the static field creating an initial electron-positron virtual separation and dynamic fields to induce spin. We might postulate that this may be consistent with what actually occurs during initial vacuum fluctuations in the presence of background radiation just prior to pair-production. In all cases the basic concept remains the same, such a Casimir geometric cavity could be a possible contributing factor in pair production due to a positive separation pressure.

Conclusions

From an engineering perspective a scenario has been suggested where an electron-positron virtual Casimir cavity may aid in pair production process in the presence of background radiation. From this perspective, it can be viewed as a virtual potential energy field suggesting some ability to further explore engineering vacuum pair production.

Reference:

1. M. E. Bowers and C. R. Hagen, Casimir Energy of a Spherical Shell, Arxiv 1998
2. T. H. Boyer, Phys. Rev. 174, 1764 (1968)
3. B. Davies, J. Math. Phys. 13, 1324 (1972)
4. R. Balian and B. Duplantier, Ann. Phys. (N.Y.) 112, 165 (1978)
5. K. A. Milton, L. L. DeRaad, Jr., and J. Schwinger, Ann. Phys. (N.Y.) 115, 388 (1978)
6. H. G. B. Casimir, Physics 19, 846 (1956)

Appendix – Proof of the Dynamic Casmir Effect

For the reader's interest, a proof of Dynamic Casmir Energy is provided here. The proof (Ref. 1) simply demonstrates that the zero point energy can be time dependent as

$$E(t) \cong \sum_k \frac{h\omega_k(t)}{2}$$

To show that the ground state energy could have this time dependence, consider the Hamiltonion having a time dependent frequency modulated harmonic oscillator

$$H_{k\lambda}(t) \cong \frac{1}{2}\sum_K \left(\frac{P_{k\lambda}^2}{m} + m\omega_k^2(t) Q_{k\lambda}^2 \right)$$

Let the polarization vary localized to within

$$\Delta Q = d$$

then from the uncertainty principal

$$\Delta P \ \Delta Q \approx \frac{h}{2\pi}$$

the energy of the oscillator is then

$$E(t,d) = \frac{1}{2}\sum_k \frac{h^2}{md^2 2\pi^2} + \frac{m\omega_k^2(t) d^2}{4}$$

Then the ground state energy can be estimated as the minimum value of E(d,t) as d varies. Thus if

$$E'(d,t) = 0$$

so that

$$\frac{m\omega_k^2(t) d}{2} - \frac{h^2}{d^3 m 2\pi^2} = 0$$

solving for d reads

$$d = \sqrt{\frac{h}{\pi m \omega(t)}}$$

inserting this value into E(d,t) reads

$$E(t) = \sum_k \frac{h\omega_k(t)}{2}$$

as required.

The results can alternately be obtained using the creation and annihilation operator with the requirement that

$$\langle h\omega\, a^+ a \rangle \;\geq\; 0$$

where

$$a(t)^+ = \frac{1}{\sqrt{2h\omega(t)}}\left(\omega(t)\boldsymbol{Q} - i\boldsymbol{P}\right)$$

and

$$a(t) = \frac{1}{\sqrt{2h\omega(t)}}\left(\omega(t)\boldsymbol{Q} + i\boldsymbol{P}\right)$$

we find that

$$\langle h\omega\;\, a(t)^+ a(t) \rangle \;\;=\;\; H - \frac{h\omega(t)}{2} \geq 0$$

or

$$H \geq \frac{h\omega(t)}{2}$$

as required

References

1. A. Feinberg, Proof of the Dynamic Casimir Effect, private communication to A. Widom, Northeastern University, Nov. 1987.