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Spin Must Be Added to the Moment of Poynting Vector

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Abstract: Calculating of absorption of a circularly polarized light beam and an analysis of the celebrated Beth's optics experiment show that the standard electrodynamics needs a concept of classical spin for completing. The spin is presented.

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There is a *casus* in the standard electrodynamics. Two examples are presented here.

1. Angular momentum of a light beam. Problem of optical spanners

According to the standard electrodynamics, a circularly polarized light beam of power P without an azimuthal phase structure carries angular momentum flux, i.e. torque,

$$\tau = \mathsf{P}/\omega\,. \qquad (1)$$

But, calculate the torque acting on the dielectric absorbing the beam, according to the standard formula

$$\tau = \int [\mathbf{r}\times(\mathbf{P}\cdot\nabla)\mathbf{E} + \mathbf{r}\times(\mathbf{j}\times\mathbf{B}) + \mathbf{P}\times\mathbf{E}]dV$$

[see, for example, [1] eqns. (5.1) and (7.18)]. Here $\mathbf{P} = (\varepsilon - 1)\mathbf{E}$ is the polarization, $\mathbf{j} = \partial_t\mathbf{P}$ is the displacement current, $\mathbf{r}\times(\mathbf{P}\cdot\nabla)\mathbf{E} + \mathbf{r}\times(\mathbf{j}\times\mathbf{B})$ is the moment of the total Lorentz force per unit volume, and $\mathbf{P}\times\mathbf{E}$ is the torque on electric dipoles per unit volume [2]. The point is the accurate calculation gives [3]

$$\tau = 2\mathsf{P}/\omega\,. \qquad (2)$$

In my opinion, this result confirms that angular momentum of a circularly polarized beam consists of two terms:

$$J^{\lambda\mu} = \int (2x^{[\lambda}T^{\mu]\nu} + \Upsilon^{\lambda\mu\nu})dV_\nu\,. \qquad (3)$$

$T^{\mu\nu}$ is the Maxwell energy-momentum tensor, and $\Upsilon^{\lambda\mu\nu}$ is an unacknowledged spin tensor [4,5]. The first term in (3) is the common one,

$$\mathbf{J} = \int \mathbf{r}\times(\mathbf{E}\times\mathbf{B})dV\,, \qquad (4)$$

but, I think, it represents an orbital angular momentum only, not spin. The second term in (3) represents spin of the beam. The sense of the spin tensor $\Upsilon^{\lambda\mu\nu}$ is as follows. The component Υ^{ij0} is a volume density of spin. This means that $dS^{ij} = \Upsilon^{ij0}dV$ is the spin of electromagnetic field inside the spatial element dV. The component Υ^{ijk} is a flux density of spin flowing in the direction of the x^k axis. For example, $dS_z/dt = dS^{xy}/dt = d\tau^{xy} = \Upsilon^{xyz}da_z$ is the *z*-component of spin flux passing through the surface element da_z per unit time, i.e. the torque acting on the element. The explicit expression for the spin tensor is

$$\Upsilon^{\lambda\mu\nu} = A^{[\lambda}\partial^{|\nu|}A^{\mu]} + \Pi^{[\lambda}\partial^{|\nu|}\Pi^{\mu]}, \qquad (5)$$

where A^{λ} and Π^{λ} are magnetic and electric vector potentials which satisfy $2\partial_{[\mu}A_{\nu]} = F_{\mu\nu}$, $2\partial_{[\mu}\Pi_{\nu]} = -e_{\mu\nu\alpha\beta}F^{\alpha\beta}$, where $F^{\alpha\beta} = -F^{\beta\alpha}$, $F_{\mu\nu} = F^{\alpha\beta}g_{\mu\alpha}g_{\nu\beta}$ is the field strength tensor of a free electromagnetic field. A relation between Π and F can be readily obtained in the vector form as follows.

If $\mathrm{div}\mathbf{E} = 0$, then $\mathbf{E} = \mathrm{curl}\,\Pi$. And if $\partial\mathbf{E}/\partial t = \mathrm{curl}\mathbf{H}$, then $\partial\Pi/\partial t = \mathbf{H}$. This reasoning is analogous to the common: if $\mathrm{div}\mathbf{B} = 0$, then $\mathbf{B} = \mathrm{curl}\,\mathbf{A}$. And if $\partial\mathbf{B}/\partial t = -\mathrm{curl}\mathbf{E}$, then $\partial\mathbf{A}/\partial t = -\mathbf{E}$.

It must be noted that Loudon [1] using a quantum generalization of a theory obtained the conventional result (1), but his calculation is difficult to verify. Simpson et al. [6] seemed to confirm the statement (1) experimentally, but their work was not quantitative one. In their case, 98% of the LG-mode light beam passed through their particle, and nobody knew what percent of this light was converted into HG modes inside the particle. But a recent work [7] confirms rather the formula (2). In this work a linearly polarized $\mathrm{LG}^{l=2}_{p=0}$ beam of λ = 1064 nm and power P = 20 mW rotates a trapped particle with the rotational rate 2.4 Hz, and, when circularly polarized, the beam rotates the particle with 2.9 Hz. This increase in the angular velocity, $\Delta\Omega$ = $2\pi\,0.5$/sec, causes the corresponding increase in the drag torque acting on the rotating particle (formula (3) from [7]): $\Delta\tau = 12\pi\eta a^3\Delta\Omega = 1.2\cdot10^{-19}$ J (here $\eta = 10^{-3}$ kg/m sec is the viscosity and $a = 10^{-6}$ m is the particle parameter). At the same time, the increase in the drag torque is provided with change in the circular polarization σ of the beam passing through the particle. This change is determined by signals of detectors. The output polarization is 0.9982 – 0.0012 = 0.997 (see Figure 2(b) from [7]). I.e. $\Delta\sigma$ = 0.003. These results yield $\Delta\sigma\mathsf{P}/\omega = 0.32\cdot10^{-19}$ J (here P = 20 mW and $\omega = 2\pi c/\lambda = 1.9\cdot10^{-15}$ /sec). So, we have, according to [7], $\Delta\tau = 3.7\Delta\sigma\mathsf{P}/\omega$ instead of $\Delta\tau = 2\Delta\sigma\mathsf{P}/\omega$, according to eqn. (2), and instead of $\Delta\tau = \Delta\sigma\mathsf{P}/\omega$, according to eqn. (1). This sizeable polarization contribution to the total torque confirms our statement (2).

2. Celebrated Beth's optics experiment is a puzzle in the frame of the standard electrodynamics

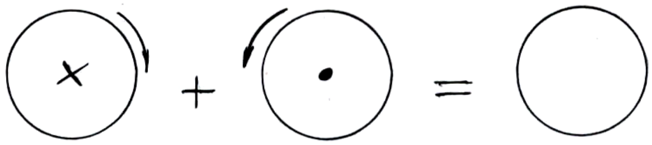
In the Beth's experiment [2] a half-wave plate is suspended with its plane horizontal from a quartz fiber, and a circularly polarized beam travels upwards passing through the plate from below upwards. Because the plate reverses the handedness of the circular polarization of the beam, according to eqn. (1), the torque acting on the plate must be $\tau = 2\mathsf{P}/\omega$. However, and this is the main point, in order to redouble the torque, the beam is reflected and passes through the plate a second time on the way back. For this, about 4 millimeters above the plate a fixed quartz quarter-wave plate is mounted. The *top* side of the upper plate was coated by evaporation with a reflecting layer of aluminum. So, the torque exerting on the half-wave plate is

$$\tau = 4\mathsf{P}/\omega. \qquad (6)$$

Meanwhile it is evident that the reflected beam cancels the energy flow in the Beth's apparatus. I.e. the Poynting vector is zero in the experiment, $\mathbf{E}\times\mathbf{B} = 0$. Add together the incident, $\mathbf{E}_1, \mathbf{B}_1$, and the reflected, $\mathbf{E}_2, \mathbf{B}_2$ beams, $\mathbf{E}_1 = e^{iz-it}[\mathbf{x} + i\mathbf{y} + \mathbf{z}(i\partial_x - \partial_y)]u$, $\mathbf{E}_2 = e^{-iz-it}[\mathbf{x} - i\mathbf{y} + \mathbf{z}(-i\partial_x - \partial_y)]u$, $\mathbf{B} = -i\mathbf{E}$, and get the total:

$$\mathbf{E}_{tot} = 2[(\mathbf{x}\cos z - \mathbf{y}\sin z) - \mathbf{z}(\sin z\partial_x + \cos z\partial_y)]u\cos t\,,$$
$$\mathbf{B}_{tot} = -2[(\mathbf{x}\cos z - \mathbf{y}\sin z) - \mathbf{z}(\sin z\partial_x + \cos z\partial_y)]u\sin t\,.$$

The total fields $\mathbf{E}_{tot}$, $\mathbf{B}_{tot}$ are parallel to each other everywhere. So, the Poynting vector is zero. Thus, according to equation (4), no angular momentum is contained in the double beam. So, no torque must act on the Beth plate according to the standard electrodynamics. It is important that not only the Poynting vector, i.e. $T^{0j} = T^{j0}$ components of the Maxwell tensor $T^{\lambda\mu}$, is zero, but time averaging of the flux densities T^{xz} and T^{yz} are zero as well. Thus, the only way to explain the Beth's result is to use the spin tensor (5). The tensor leads to the torque (6) [5,8]. We show that the Beth's double beam contains spin flux and energy without energy flux and spin. This is natural because two beams of the same handedness, which propagate in the opposite direction, are added together.



In this picture we add together two rifle bullets or two photons moving in opposite directions and spinning in opposite directions. As a result we have neither mass flux nor spin (the spin flux is invisible).

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At the same time, materials of this paper were rejected more than 350 times by scientific journals. For example (I show an approximate number of the rejections in parentheses): JETP Lett. (8), JETP (13), TMP (10), UFN (9), RPJ (75), AJP (16), APP (5), EJP (4), EPL (5), IJTP (2), JOSAA (2), JOSAB (3), MPEJ (3), PRA (5), PRD (4), PRE (2), PRL (2), FP (6), PLA (9), OC (5), JPA (4), JPB (1), JMP (7), JOPA (4), JMO (2), CJP (1), OL (4), NJP (5), arXiv (75).

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