Weyl's Geometry solves the Riddle of Dark Energy

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Abstract

We rigorously prove why the proper use of Weyl's Geometry within the context of Friedman-Lemaitre-Robertson-Walker cosmological models can account for *both* the *origins* and the *value* of the observed vacuum energy density (dark energy). The source of dark energy is just the dilaton-like Jordan-Brans-Dicke scalar field that is required to implement Weyl invariance of the most *simple* of all possible actions. A *nonvanishing* value of the vacuum energy density of the order of $10^{-123} M^4_{Planck}$ is derived in agreement with the experimental observations. The full theory involving the dynamics of Weyl's gauge field A_μ is very rich and may explain the anomalous Pioneer acceleration and the temporal variations (over cosmological scales) of the fundamental constants resulting from the expansion of the Universe. This is consistent with Dirac's old idea of the plausible variation of the physical constants but with the advantage that it is *not* necessary to invoke extra dimensions.

The problem of dark energy is one of the most challenging problems facing us today, see [1], [3] for a review. In this letter we will show how Weyl's geometry (and its scaling symmetry) is instrumental to solve this problem. Weyl's geometry main feature is that the norm of vectors under parallel infinitesimal displacement going from x^μ to $x^\mu + dx^\mu$ change as follows $\delta ||V|| \sim ||V|| A_\mu dx^\mu$ where A_μ is the Weyl gauge field of scale calibrations that behaves as a connection under Weyl transformations :

$$A'_\mu = A_\mu - \partial_\mu \Omega(x). \quad g_{\mu\nu} \rightarrow e^{2\Omega} g_{\mu\nu}. \tag{1}$$

involving the Weyl scaling parameter $\Omega(x^\mu)$.

The Weyl covariant derivative operator is $D_\mu = \nabla_\mu + A_\mu$; where the derivative operator $\nabla_\mu = \partial_\mu + \Gamma_\mu$ involves a connection Γ_μ which is comprised of the ordinary Christoffel symbols plus extra A_μ terms in order for the metric to obey the condition $D_\mu(g_{\nu\rho}) = 0$.

The Weyl covariant derivative acting on a scalar ϕ of Weyl weight $\omega(\phi) = -1$ is defined by

$$D_\mu \phi = \partial_\mu \phi + \omega(\phi) A_\mu \phi = \partial_\mu \phi - A_\mu \phi. \tag{2}$$

The Weyl scalar curvature in D dimensions and signature $(+,-,-,-....)$ is

$$\mathcal{R}_{Weyl} = \mathcal{R}_{Riemann} - (D-1)(D-2)A_\mu A^\mu + 2(D-1)\nabla_\mu A^\mu. \tag{3}$$

For a signature of $(-,+,+,+,....)$ there is a *sign* change in the second and third terms due to a sign change of $\mathcal{R}_{Riemann}$.

The Jordan-Brans-Dicke action involving the scalar ϕ and $\mathcal{R}_{Weyl}$ is

$$S = -\int d^4x \sqrt{|g|} \; [\; \phi^2 \; \mathcal{R}_{Weyl} \;]. \tag{4}$$

Under Weyl scalings,

$$\mathcal{R}_{Weyl} \;\rightarrow\; e^{-2\Omega} \; \mathcal{R}_{Weyl}; \quad \phi^2 \;\rightarrow\; e^{-2\Omega} \; \phi^2. \tag{5}$$

to compensate for the Weyl scaling (in $4D$) of the measure $\sqrt{|g|} \rightarrow e^{4\Omega} \sqrt{|g|}$ in order to render the action (4) Weyl invariant.

When the Weyl integrability condition is imposed $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu = 0 \;\Rightarrow\; A_\mu = \partial_\mu \Omega$, the Weyl gauge field A_μ does not have dynamical degrees of freedom; it is pure gauge and barring global topological obstructions, one can choose the gauge in eq-(4)

$$A_\mu = 0; \quad \phi_0^2 = \frac{1}{16\pi G_N} = constant. \tag{6}$$

such that the action (4) reduces to the standard Einstein-Hilbert action of Riemannian geometry

$$S = -\frac{1}{16\pi G_N} \int d^4x \sqrt{|g|} \; [\mathcal{R}_{Riemann}(g)]. \tag{7}$$

The Weyl integrability condition $F_{\mu\nu} = 0$ means physically that if we parallel transport a vector under a closed loop, as we come back to the starting point, the *norm* of the vector has not changed; i.e, the rate at which a clock ticks does not change after being transported along a closed loop back to the initial point; and if we transport a clock from A to B along different paths, the clocks will tick at the same rate upon arrival at the same point B. This will ensure, for example, that the observed spectral lines of identical atoms will not change when the atoms arrive at the laboratory after taking different paths (histories) from their coincident starting point. If $F_{\mu\nu} \neq 0$ Weyl geometry may be responsible for the alleged variations of the physical constants in recent Cosmological observations. A study of the Pioneer anomaly based on Weyl geometry was made by [8]. The literature is quite extensive on this topic.

Our starting action is

$$S = S_{Weyl}(g_{\mu\nu}, A_\mu) \; + S(\phi). \tag{8}$$

with

$$S_{Weyl}(g_{\mu\nu}, A_\mu) \;=\; -\int d^4x \sqrt{|g|} \;\; \phi^2 \;\; [\; \mathcal{R}_{Weyl}(g_{\mu\nu}, A_\mu) \;]. \tag{9}$$

where we define $\phi^2 = (1/16\pi G)$. The Newtonian coupling G is spacetime dependent in general and has a Weyl weight equal to 2. The term $S(\phi)$ involving the Jordan-Brans-Dicke scalar ϕ is

$$S_\phi = \int d^4x \sqrt{|g|} \; [\; \frac{1}{2} g^{\mu\nu} \; (D_\mu\phi)(D_\nu\phi) \; - \; V(\phi) \;]. \tag{10}$$

where $D_\mu\phi = \partial_\mu\phi - A_\mu\phi$. The FRW metric is

$$ds^2 = dt^2 - a^2(t) \; (\frac{dr^2}{1 - k(r/R_0)^2} + r^2(d\Omega)^2). \tag{11a}$$

where $k = 0$ for a 3-dim spatially flat region; $k = \pm 1$ for regions of positive and negative constant spatial curvature, respectively. The de Sitter metric belongs to a special class of FRW metrics and it admits different forms depending on the coordinates chosen. The Friedman-Einstein-Weyl equations in the gauge $A_\mu = (0,0,0,0)$ (in units of $c = 1$)

$$G_{\mu\nu} \;=\; 8\pi G \; T_{\mu\nu}; \quad \phi^2 = \frac{1}{16\pi \; G}. \quad T_{\mu\nu} \;=\; - \frac{2}{\sqrt{|g|}} \; \frac{\delta S_{matter}}{\delta g^{\mu\nu}}. \tag{11b}$$

read

$$3(\frac{(da/dt)}{a})^2 \; + \; (\frac{3k}{a^2 R_0^2}) \;=\; 8\pi G(t)\rho. \tag{12}$$

and

$$-2 \; (\frac{(d^2a/dt^2)}{a}) - (\frac{(da/dt)}{a})^2 \; - \; (\frac{k}{a^2 R_0^2}) = \; 8\pi G(t) \; p. \tag{13a}$$

From eqs-(12-13a) one can infer the important relation :

$$- \; (\frac{(d^2a/dt^2)}{a}) \;=\; \frac{4\pi G(t)}{3} \; (\rho + 3p). \tag{13b}$$

Eqs-(12-13) are the ones one must use instead of the erroneous equations posed by [8] in the partial gauge $A_t = H(t), \; A_i = 0, \; i = 1,2,3$:

$$(\frac{(da/dt)}{a})^2 \;=\; H^2(t) \;=\; - \; (\frac{k}{a^2 R_0^2}) \; - 3(\; A_t(x) \; A^t(x) \; - \; \frac{1}{\sqrt{|g|}}\partial_t(\sqrt{|g|}A^t) \;) \; + \frac{8\pi G(t)}{3}\rho. \tag{14a}$$

and

$$- \; (\frac{(d^2a/dt^2)}{a}) \;=\; - \; (\; H^2(t) + \frac{dH}{dt} \;) \;=\; \frac{4\pi G(t)}{3} \; (\rho + 3p). \tag{14b}$$

The density and pressure terms should be given in terms of Weyl covariant derivatives of the scalar ϕ and the potential density $V(\phi)$. The scalar ϕ must be chosen to depend solely on time , $\phi(t)$, because this is the relevant case suitable for the FRW cosmologies due to the fact that the geometry is spatially homogeneous and isotropic . The gauge choice condition imposed by [8] : $A_t = H(t); \; A_i = 0, \; i = 1,2,3$ is compatible with the spatial isotropy and homogeneity of the FRW models. However, despite that a non-zero value A_t was chosen by [8] there is a *residual* symmetry that is still available to gauge A_t to zero. As mentioned earlier, Weyl's integrability condition $F_{\mu\nu} = 0$ physically means that A_μ is pure gauge, a total derivative, whence it does not have true dynamical degrees of freedom and *all* of its components can be gauged to zero $A_\mu = (0, 0, 0, 0)$.

However, if one partially fixes the gauge $A_t = H(t); A_i = 0$ like it was done in [8], one arrives at a *caveat* that was overlooked by [8] . One would arrive at a deep *contradiction* and *inconsistency* between the left hand side (l.h.s) and the right hand side (r.h.s) of the Friedman-Einstein-Weyl equations (for example in eq-(14b)) in the partially fixed gauge $A_t = H(t)$ because the l.h.s does *not* transform homogeneously under Weyl scalings, whereas the r.h.s does; if the quantities ρ and p were to transform properly under Weyl scalings, homogeneously, this behaviour would be *incompatible* with the transformation properties of the $A_t = H(t)$ terms appearing in the l.h.s of eqs-(14b).

In order to *reconcile* this *incompatibility* between the *inhomogeneous* transformation properties of the l.h.s of eq-(14b) with the homogeneous transformation properties of the r.h.s of (14b), *one must fix* the gauge $A_\mu = 0$ fully in the Einstein-Friedman-Weyl equations as shown in eqs-(12-13). The latter equations *are* the physically relevant and *not* eqs-(14). One may be inclined to say : if one is going to fix the gauge $A_\mu = 0$ anyway, then *what* is the role of Weyl's geometry and symmetry in all of this ? We will show below why despite fixing the gauge $A_\mu = 0$ one cannot forget the *constraint* which arises from the variations of the action w.r.t the Weyl's field A_μ ! This constraint holds the key to see why the density and pressure associated with the scalar ϕ obey the sought after relation $\rho(\phi) = -p(\phi)$ (which is the hallmark of dark energy) as we intend to prove next.

The Jordan-Brans-Dicke scalar ϕ must obey the generalized Klein-Gordon equations of motion

$$(\ D_\mu D^\mu \ + \ 2\mathcal{R}_{Weyl} \ ) \ \phi \ + \ (\frac{dV}{d\phi}) = 0 \tag{15}$$

combined with the crucial *constraint* equation obtained from the variation of the action w.r.t to the A^μ field :

$$\frac{\delta S}{\delta A^\mu} \ = \ 6 \ (A_\mu \phi^2 \ + \ \partial_\mu(\phi^2)) \ + \ \frac{1}{2} \ (A_\mu \phi^2 \ - \ \partial_\mu(\phi)^2 \ ) \ = \ 0. \tag{16}$$

The last constraint equation in the gauge $A_\mu = 0$, forces $\partial_\mu \phi = 0 \ \Rightarrow \phi = \phi_o = constant$. Consequently $G \sim \phi^{-2}$ is also constrained to a constant G_N and one may set $16\pi \ G_N \ \phi_o^2 = 1$, where G_N is the observed Newtonian constant today.

Furthermore, in the gauge $A_\mu = 0$, due to the constraint eq-(16), one can infer that $D_\mu \phi = 0, \ \ \Rightarrow D^\mu D_\mu \phi = 0$ because $D_t \ \phi(t) = \partial_t \ \phi - A_t \ \phi = \partial_t \ \phi = 0$, and $D_i \phi(t) = -A_i \phi(t) = 0$. These results will be used in the generalized Klein-Gordon equation.

Therefore, the stress energy tensor $T^\mu_\mu = diag \ (\rho, -p, -p, -p)$ corresponding to the constant scalar field configuration $\phi(t) = \phi_o$, in the $A_\mu = 0$ gauge, becomes :

$$\rho_\phi = \frac{1}{2}(\partial_t \ \phi - A_t \ \phi)^2 + V(\phi) \ = \ V(\phi); \quad p_\phi = \frac{1}{2}(\partial_t \ \phi - A_t \ \phi)^2 - V(\phi) \ = \ -V(\phi). \tag{17}$$

$$\rho + \ 3p = 2\ (\partial_t\ \phi - A_t\ \phi)^2\ -\ 2V(\phi)\ = -2V(\phi). \tag{18}$$

This completes the proof why the above ρ and p terms, in the gauge $A_\mu = 0$, become $\rho(\phi) = V(\phi) = -p(\phi)$ such that $\rho + 3p = -2V(\phi)$ (that will be used in the Einstein-Friedman-Weyl equations (13b)). This is the *key* reason why Weyl's geometry and symmetry is essential to explain the origins of a $non-vanishing$ vacuum energy (dark energy). The latter relation $\rho(\phi) = V(\phi) = -p(\phi)$ is the *key* to derive the vacuum energy density in terms of $V(\phi = \phi_o)$!., because such relation resembles the dark energy relation $p_{DE} = -\rho_{DE}$. Had one not had the constraint condition $D_t\ \phi(t) = (\partial_t - A_t)\phi = \partial_t\ \phi = 0$, and $D_i\phi(t) = -A_i\phi(t) = 0$, in the gauge $A_\mu = 0$, enforcing $\phi = \phi_o$, one would not have been able to deduce the crucial condition $\rho(\phi = \phi_o)\ =\ -p(\phi = \phi_o)\ =\ V(\phi = \phi_o)$ that will furnish the observed vacuum energy density today !

We will find now solutions of the Einstein-Friedman-Weyl equations in the gauge $A_\mu = (0,0,0,0)$ after having explained why A_μ can (and must) be gauged to zero. The most relevant case corresponding to de Sitter space :

$$a(t) = e^{H_o t};\ \ A_\mu = (0,0,0,0);\ \ k = 0;\ \mathcal{R}_{Weyl} = \mathcal{R}_{Riemann} = -12\ H_0^2;\ \ . \tag{19}$$

where we will show that the potential is

$$V(\phi) = 12 H_0^2 \phi^2\ + V_o. \tag{20}$$

one learns in this case that $V(\phi = \phi_o) \neq 0$ since this non-vanishing value is precisely the one that shall furnish the observed vacuum energy density today !!! (as we will see below) . We shall begin by solving the Einstein-Friedman-Weyl equations eq-(12-13) in the gauge $A_\mu = (0,0,0,0)$ for a spatially flat universe $k = 0$ and $a(t) = e^{H_0 t}$, corresponding to de Sitter metric :

$$ds^2 = dt^2\ -\ e^{2H_o t}\ \ (dr^2 + r^2 (d\Omega)^2). \tag{21}$$

the Riemannian scalar curvature when $k = 0$ is

$$\mathcal{R}_{Riemann} = -\ 6\ [\ (\frac{(d^2a/dt^2)}{a}) + (\frac{(da/dt)}{a})^2\ ]\ = -12\ H_0^2 \tag{22}$$

(the negative sign is due to the chosen signature $+,-,-,-$).

To scalar Weyl curvature $\mathcal{R}_{Weyl}$ in the gauge $A_\mu = (0,0,0,0)$ is the same as the Riemannian one $\mathcal{R}_{Weyl} = \mathcal{R}_{Riemann} = -12\ H_0^2$. Inserting the condition $D_\mu\phi = D_t\phi(t) = (\partial_t\phi - A_t\phi) = \partial_t\ \phi = 0$, in the gauge $A_\mu = 0$, the generalized Klein-Gordon equation (3.20) will be satisfied if, and only if, the potential density $V(\phi)$ is chosen to satisfy

$$(\ 12\ H_0^2)\ \phi\ =\ \frac{1}{2}\ (\frac{dV}{d\phi})\ \Rightarrow\ V(\phi)\ =\ 12\ H_0^2\ \phi^2\ +\ V_o \tag{23}$$

One must firstly differentiate w.r.t the scalar ϕ , and only afterwards, one may set $\phi = \phi_o$. $V(\phi)$ has a Weyl weight equal to -4 under Weyl scalings in order to ensure that the full action is Weyl invariant. H_0^2 and ϕ_o^2 have both a Weyl weight of -2, despite being constants, because as one performs a Weyl scaling of these quantities (a change of a scales) they will acquire then a spacetime dependence. H_0^2 is a masslike parameter, one may interpret H_0^2 (up to numerical factors) as the "mass" squared of the Jordan-Brans-Dicke scalar. We will see soon why the integration constant V_o plays the role of the "cosmological constant".

An important remark is in order. Even if we included other forms of matter in the Einstein-Fredmann-Weyl equations, in the very large t regime, their contributions will be *washed* away due to their scaling behaviour. We know that ordinary matter ($p = 0$); dark matter ($p_{DM} = w\rho_{DM}$ with $-1 < w < 0$) and radiation terms ($p_{rad} = \frac{1}{3}\rho_{rad}$) are all *washed* away due to their scaling behaviour :

$$\rho_{matter} \sim R(t)^{-3}. \quad \rho_{radiation} \sim R(t)^{-4}. \quad \rho_{DM} \sim R(t)^{-3(1+w)}. \tag{24}$$

where $R(t) = a(t)R_0$. The dark energy density remains *constant* with scale since $w = -1$ and the scaling exponent is zero, $\rho_{DE} \sim R^0 = costant$. For this reason it is the only contributing factor at very large times.

Now we are ready to show that eqs-(12-13) are indeed satisfied when $a(t) = e^{H_0 t}$; $k = 0$; $A_\mu = 0$; $\phi = \phi_o \neq 0$. Eq-(13b), due to the conditions $\rho + 3p = -2V(\phi)$ and $\phi(t) = \phi_o$ (resulting from the constraint eq-(16) in the $A_\mu = 0$ gauge) gives :

$$-\left(\frac{(d^2a/dt^2)}{a}\right) = -H_0^2 = \frac{4\pi G_N}{3}(\rho + 3p) =$$

$$-\left(\frac{8\pi\ G_N\ V(\phi = \phi_o)}{3}\right) = -\left(\frac{8\pi\ G_N\ 12\ H_0^2\ \phi_o^2}{3}\right) - \frac{8\pi G_N\ V_o}{3}. \tag{25}$$

Eq-(12) (with $k = 0$) is just the same as eq-(13b) but with an overall *change* of sign because $\rho(\phi = \phi_o) = V(\phi = \phi_o)$. Using the definition $16\pi\ G_N\ \phi_o^2 = 1$ in (25) one gets

$$-H_0^2 = -\left(\frac{8\pi\ G_N\ 12\ H_0^2\ \phi_o^2}{3}\right) - \frac{8\pi\ G_N\ V_o}{3} = -2\ H_0^2 - \frac{8\pi\ G_N\ V_o}{3} \Rightarrow$$

$$-\frac{8\pi\ G_N\ V_o}{3} = H_0^2 \Rightarrow -8\pi\ G_N\ V_o = 3\ H_0^2 \tag{26}$$

Therefore, we may identify the term $-V_o$ with the vacuum energy density so the quantity $3H_0^2 = -8\pi\ G_N\ V_o = \Lambda$ is nothing but the cosmological constant. It is *not surprising* at all to obtain $\Lambda = 3\ H_0^2$ in de Sitter space ! . One knew it long ago. What is most *relevant* about eq-(26) is that the observed vacuum energy density is minus the *constant* of integration V_o corresponding to the potential density $V(\phi) = 12H^2\phi^2 + V_o$!. Hence one has from the last term of eq-(26) :

$$-V_o = \rho_{vacuum} = \frac{3H_0^2}{8\pi\ G_N}. \tag{27}$$

and finally, when we set $H_0^2 = (1/R_0^2) = (1/R_{Hubble}^2)$ and $G_N = L_{Planck}^2$ in the last term of eq-(26), as announced, the vacuum density ρ_{vacuum} observed today is *precisely* given by :

$$-V_o = \rho_{vacuum} = \frac{3H_0^2}{8\pi\ G_N} = \frac{3}{8\pi}\ (L_{Planck})^{-2}\ (R_{Hubble})^{-2} =$$

$$\frac{3}{8\pi}\ (\frac{1}{L_{Planck}})^4\ (\frac{L_{Planck}}{R_{Hubble}})^2\ \sim 10^{-123}\ (M_{Planck})^4. \qquad (28)$$

This completes our third derivation of the vacuum energy density given by the formula (26-28). The first derivation was attained in [4], while the second derivation was attained in section [5] .

Concluding this analysis of the Einstein-Friedman-Weyl eqs-(12-13) : By invoking the principle of Weyl scaling symmetry in the context of Weyl's geometry; when $k = 0$ (spatially flat Universe), $a(t) = e^{H_0 t}$ (de Sitter inflationary phase) ; H_o = Hubble constant today; $\phi(t) = \phi_o = constant$, such $16\pi G_N\ \phi_o^2 = 1$, one finds that

$$V(\phi = \phi_o) = 12\ H_0^2\ \phi_o^2\ +\ V_o =\ 2\rho_{vacuum}\ -\ \rho_{vacuum}\ =\ \rho_{vacuum} =$$

$$6H_0^2\phi_o^2 = \frac{3H_0^2}{8\pi\ G_N} \sim 10^{-123}\ M^4_{Planck}. \qquad (29)$$

is precisely the observed vacuum energy density (28) . Therefore, the observed vacuum energy density is intrinsically and inexorably linked to the potential density $V(\phi = \phi_o)$ corresponding to the Jordan-Brans-Dicke scalar ϕ required to build Weyl invariant actions and evaluated at the special point $\phi_o^2 = (1/16\pi G_N)$.

The case of an *ever expanding accelerating universe* (consistent with observations) is so promising because it incorporates the presence of the Hubble Scale and Planck scale into the expression for the observed vacuum energy density via the Jordan-Brans-Dicke scalar field ϕ needed to implement Weyl invariance of the action. Weyl's scaling symmetry principle permits us to explain *why* the observed value of the vacuum energy density ρ_{vacuum} is *precisely* given by the expression (28-29).

In order to introduce true dynamics to the Weyl gauge field, one must add the kinetic term for the Weyl gauge field $F_{\mu\nu}F^{\mu\nu}$. In this case, the integrability condition $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu = 0$ is *no* longer obeyed in general and the rate at which clocks tick may depend on their worldline history. This could induce a variation of the physical constants (even dimensionless constants like the fine structure constant $\alpha = 1/137$). For instance, as the size of the universe grows, ($a(t) = e^{H_0 t}$ increases with time) the variable speed of light, Newtonian coupling and cosmological constant , may vary according to the law $[G(t)/c^4(t)\ \Lambda(t)]\ \sim\ (1/\rho_{vacuum})$ if the vacuum energy density ρ_{vacuum} would remain constant. Many authors have speculated about this last behaviour among c, G, Λ

The most general Lagrangian involving dynamics for A_μ is

$$\mathcal{L} = -\phi^2 \mathcal{R}_{Weyl}(g_{\mu\nu}, A_\mu) + \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}g^{\mu\nu}(D_\mu\phi)(D_\nu\phi) - V(\phi) + L_{matter} + ... \qquad (30)$$

The L_{matter} must involve the full fledged Weyl gauge covariant derivatives acting on scalar and spinor fields contrary to the Cheng-Weyl models of [9] where there is no Weyl gauge field in the derivatives. $L_{radiation}$ terms may be included involving the Maxwell field $\mathcal{A}_\mu$ which must not be confused with the Weyl gauge field A_μ. Once could also add Yang-Mills fields $\mathcal{A}^a_\mu$ and kinetic and potential terms for the Higgs scalars as well. The simplest scenario, of course, was the one given in this section.

There are many differences among our approach to explain the origins of dark energy and that of [6], [2], [3], [1], [9], to cite a few. The Cheng-Weyl approach [9] to account for dark energy and matter (including phantom) does *not* use the Weyl scalar curvature with a variable Newtonian coupling $16\pi\ G = \phi^{-2}$ for the gravitational part of the action, but the ordinary Riemannian scalar curvature with the standard Newtonian gravitational constant . Conformal transformations in accelerated cosmologies have been studied by [10] but their approach is different than the Weyl geometric one presented here. Weyl invariance has been used in [7] to construct Weyl-Conformally Invariant Light-Like p-Brane Theories with numerous applications in Astrophysics, Cosmology, Particle Physics Model Building, String theory,.....

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References

[1] T. Padmanabhan, " Dark Energy : Mystery of the Millenium" astro-ph/0603114.

[2] E. I. Guendelman and A. B. Kaganovich, "k-Essence, Avoidance of the Weinberg's Cosmological Constant No-Go Theorem and Other Dark Energy Effects of Two Measures Field Theory", gr-qc/0606017.

[3] N. Mavromatos, " The issue of Dark Energy in Strng Theory" hep-th/0607006.

[4] C. Castro, Mod. Phys. Lett **A17** (2002) 2095-2103

[5] C. Castro, "On Novel Static Spherically Symmetric Solutions of Einstein equations and the Cosmological Constant Problem " CTSPS preprint, May 2006, submitted to the IJMPD.

[6] L. Nottale, *Fractal Spacetime and Micrphysics : Towards Scale Relativity* (World Scientific, Singapore, 1992)

[7] E. Guendelman, A. Kaganovich, Phys. Rev **D 53** (1996) 7020; Phys. Rev **D 60** , 065004 (1999) ; Int. J. Mod. Phys **A 17** (2002) 417; E. Guendelman, A. Kaganovich, E. Nissimov and S. Pacheva, " Weyl-Conformally Invariant Light-Like p-Brane Theories " hep-th/0409078.

[8] E. Scholz, " On the Geometry of Cosmological Model Building" gr-qc/0511113

[9] H. Wei and R-G Cai, " Cheng-Weyl Vector Field and Cosmological Application" astro-phy/0607064.

[10] J. Crooks and P. Frampton, " Conformal Transformations and Accelerated Cosmologies " astro-ph/0601051.