Theoretical Derivation to Newton's Second Law And the Coulomb Law

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[Abstract] To discuss the possibility of deriving the law of Coulomb and Newton's second law theoretically, according to the law of conservation of energy, the variable dimension fractal method is developed, and used to improved Newton's second law and the law of Coulomb in an example (a small electrification ball moves down along a long incline within the electric field due to an electrification globe). The results suitable for this example with the constant dimension fractal form as follows: the improved law of Coulomb (inverse non-square law of Coulomb); the improved Newton's second law $F=ma^{1.01458}$.
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[Key words] the law of Coulomb, Newton's second law, fractal method, theoretical derivation

The law of Coulomb was summarized with experimental results, it reads

$$f = \frac{kq_1q_2}{r^2} \qquad (1)$$

All appearance, the law of Coulomb presented by Coulomb and the law of gravity presented by Newton are inverse square laws. While, the inverse non-square gravitational law was presented in reference [1] according to the improved Newton's formula of universal gravitation presented in reference [2]. The main results are as follows: the inverse non-square gravitational law with the form of variable dimension fractal reads, $F=-GMm/r^D$, where $D=f(r)$, instead of $D=2$. The values of D are different for different problems. For the problem of gravitational deflection of photon orbit around the sun, $1.954997 \leqslant D \leqslant 2$. For the problem of advance of Mercury's perihelion, supposing $D=2-\varepsilon$, then $2.018165\times10^{-9} \leqslant \varepsilon \leqslant 4.935239\times10^{-9}$.

Similar to inverse non-square law of gravitation, as two electrification bodies run the relative movement(in this case the Newton's second law must be considered), the force between the two bodies will agree with the inverse non-square law of Coulomb.

Newton's second law also was summarized with experimental results, it reads

$$F = ma \qquad (2)$$

Whether or not these two laws can be derived theoretically? It is possible in the case that there is a more extensive law. At present, the law of conservation of energy can be taken on this important task. The reason for this is that the law of Coulomb and Newton's second law can be used for handling the macrocosmic physical phenomenon only, while the law of conservation of energy can be used for handling the macrocosmic and the microcosmic physical phenomenon.

To discuss the possibility of deriving the law of Coulomb and Newton's second law theoretically, according to the law of conservation of energy, the variable dimension fractal method is developed, and used to improved Newton's second law and the law of Coulomb in an example (a small electrification ball moves down along a long incline within the electric field due to an electrification globe). For the reason that the solving process is complicated, the results suitable for this example with the constant dimension fractal form will be given.

1 Variational Principle for Deriving the law of Coulomb and Newton's second law at One time

The law of conservation of energy is a basic one in natural science. Its main content can be stated briefly as follows, in a closed system, the total systemic energy is equal to a constant. Now the variational principle established by the law of conservation of energy can be given with least squares method (LSM).

Supposing that the initial total energy of a closed system equals $W(0)$, for time t the total energy equals $W(t)$, then according to the law of conservation of energy, it gives

$$W(0) = W(t) \qquad (3)$$

it can be written as

$$R_W = \frac{W(t)}{W(0)} - 1 = 0 \qquad (4)$$

according to LSM, for the interval $[t_1, t_2]$, we can get the following variational principle

(5)

where, $\min_0$ denotes the minimum value of functional Π and it should be equal to zero[3].

Besides the time coordinate, another one also can be used. For example, for interval $[x_1, x_2]$, the following variational principle can be given according to the law of conservation of energy

(6)

The above-mentioned principle is established by using the law of conservation of energy directly. Sometimes, a certain principle should be established by using the law of conservation of energy indirectly. For example, a special physical quantity Q is interested, not only it can be calculated by using the law of conservation of energy, but also can be calculated by using other laws (for this paper they are the law of Coulomb and Newton's second law). For distinguishing the values, denotes the value given by other laws as Q, while denotes the value given by the law of conservation of energy as Q', then the value of R_W can be redefined as follows

$$R_W = \frac{Q}{Q'} - 1 = 0 \qquad (7)$$

Substituting Eq. (7) into Eqs. (5) and (6), as Q' is the result calculated with the law of conservation of energy, it gives the variational principle established by using the law of conservation of energy indirectly. Otherwise, it is clear that the extent of the value of Q accord with Q'.

2 Improved Law of Coulomb and Newton's Second Law with the Form of Variable Dimension Fractal and the Like

In the area of Newton's mechanics, the law of gravity has been improved. For example, in reference [2], the following improved formula was given

$$F = -\frac{GMm}{r^2} - \frac{3G^2M^2mp}{c^2r^4} \qquad (8)$$

where: G — gravitational constant; M and m — masses of the two bodies; r — the distance between the two bodies; c — velocity of light; p — half normal chord for body m moving around the body M with a curve, and the value of p reads

$p = a(1-e^2)$ (for ellipse)

$p = a(e^2-1)$ (for hyperbola)

$p = y^2/2x$ (for parabola)

For the problem of gravitational deflection of photon orbit around the sun and the problem of advance of Mercury's perihelion, by using the improved formula of universal gravitation, the same results as given by general relativity can be reached.

Referring to Eq. (8), the general form of improved law of gravity can be written as follows

$$F = -\frac{GMm}{r^2}\left(1 + \frac{a_1}{r^2} + \frac{a_2}{r^4} + \cdots\right)$$

Similarly, besides the static case, the general form of improved law of Coulomb can be written as follows

$$f=\frac{kq_1q_2}{r^2}(1+\frac{a_1}{r^2}+\frac{a_2}{r^4}+\cdots) \qquad (9)$$

In addition, the fractal method has been made good winning in many fields recently. The fractal distribution reads [4]

$$N=\frac{C}{r^D}$$

where: r— characteristic scale, such as length, time and the like; N— a quantity related to r, such as temperature, force and the like; C—a constant to be determined; D—fractal dimension.

For the case of D is a constant, this kind of fractal can be named as constant dimension fractal. For the case of D is not a constant, this kind of fractal can be named as variable dimension fractal[5~7].

The general form of improved law of Coulomb with the form of variable dimension fractal can be written as follows

$$f=\frac{kq_1q_2}{r^D} \qquad (10)$$

where, $D=f(r)$, for example, it may be taken as

(11)

While, in this paper, only the form of constant dimension fractal will be taken, i.e., $D=\text{const}$.

In the area of Newton's mechanics, the Newton's second law also can be improved, the general form may be written as

(12)

Similarly, the general form of the Newton's second law with the form of variable dimension fractal may be written as

$$F=ma^{D'} \qquad (13)$$

where, $D'=f(r)$, for example, it may be taken as

(14)

While, in this paper, only the form of constant dimension fractal will be taken, i.e., $D'=\text{const}$. For the sake of convenience, it may be written as

$$F=ma^{1+\varepsilon} \qquad (15)$$

where, $\varepsilon=\text{const}$.

3 Method for Deriving Improved Law of Coulomb and Newton's Second Law at One Time

Substituting Eq. (9) or Eq. (10), and the related quantities calculated by Eq. (12) or Eq. (13) into Eq. (5) or Eq. (6), the equations derived by the condition of extremum can be written as follows

$$\frac{\partial \Pi}{\partial a_i}=\frac{\partial \Pi}{\partial k_i}=0 \qquad (16)$$

After solving these equations, the improved law of Coulomb and Newton's second law can be reached at one time. According to the value of Π, the effect of the solution can be judged. The more close to zero of the value of Π, the better effect of the solution.

Obviously, the laws derived in this paper are not depending on any experimental result. While, it is a further topic that whether or not the improved law of Coulomb and Newton's second law derived in this way will be suitable for other cases.

It should be noted that besides of solving equations, optimum-seeking methods also could be used for finding the minimum and the constant to be determined. In fact, the optimum seeking method will be used in this paper.

4 Example for Deriving Improved Law of Coulomb and Newton's Second Law at One Time

As shown in Fig.1, let circle O' denotes an electrification globe, m denotes the mass of a small electrification ball (taken as mass point P), the electric charges of the globe and ball equal q_1 and q_2, and there are positive and negative separately, we will assume that the small ball rolls along a long incline from A to B. Its initial velocity equals zero and the gravity and friction are neglected. Supposing that O'A is a plumb line, coordinate x uprights to O'A, coordinate y uprights to coordinate x (parallel to O'A), BC uprights to O'A. The lengths of OA, OB, BC, and AC are all equal to H, O'C equals the radius R of the globe.

For this example, the value of v_{P}^2 which is the square of the velocity for the ball located on point P is interested, for the sake of distinguish, denotes the value given by the improved law of Coulomb and Newton's second law as v_{P}^2, while denotes the value given by the law of conservation of energy as v'^2_{P}, then Eq.（6）can be written as

$$\Pi = \int_{-H}^{0} (\frac{v_{\mathrm{P}}^2}{v'^2_{\mathrm{P}}} - 1)^2 \mathrm{d}x = \min{}_0 \qquad (17)$$

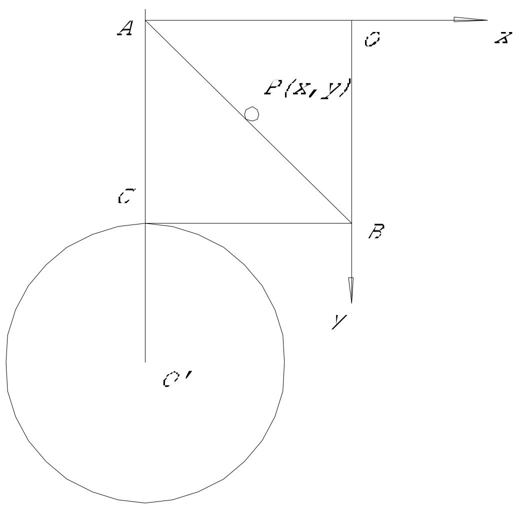

Fig.1 A small electrification ball rolls from A to B

Similar to the gravitational potential energy, from Eq.(10), the potential energy of the electrification ball caused by electricity for the ball located on point P reads

$$V = -\frac{kq_1q_2}{(D-1)r_{\mathrm{O'P}}^{D-1}} \qquad (18)$$

according to the law of conservation of energy, we can get

$$-\frac{kq_1q_2}{(D-1)r_{\mathrm{O'A}}^{D-1}} = \frac{1}{2}mv'^2_{\mathrm{P}} - \frac{kq_1q_2}{(D-1)r_{\mathrm{O'P}}^{D-1}} \qquad (19)$$

therefore

$$v'^2_{\mathrm{P}} = \frac{2kq_1q_2}{m(D-1)}[\frac{1}{r_{\mathrm{O'P}}^{D-1}} - \frac{1}{(R+H)^{D-1}}] \qquad (20)$$

Considering the general case, the rolling curve reads

$$y = y(x) \qquad (21)$$

for the ball located on point P, it gives

$$\mathrm{d}v/\mathrm{d}t = a \qquad (22)$$

because

$$\mathrm{d}t=\frac{\mathrm{d}s}{v}=\frac{\sqrt{1+y'^2}\,\mathrm{d}x}{v}$$

thereupon

$$\mathrm{d}v=a\mathrm{d}t=a\frac{\sqrt{1+y'^2}\,\mathrm{d}x}{v}$$

it gives

(23)

According to the improved law of Coulomb, for point P, the attracted force acted on the ball reads

$$F_{\mathrm{P}}=\frac{kq_1q_2}{r_{\mathrm{O'P}}^{D}}$$

The force along to the tangent reads

$$F_{\mathrm{a}}=\frac{kq_1q_2}{r_{\mathrm{O'P}}^{D}}\frac{y'}{\sqrt{1+y'^2}} \qquad (24)$$

According to improved Newton's second law, for point P, the acceleration along to the tangent reads

$$a=(\frac{F_{\mathrm{a}}}{m})^{1/1+\varepsilon}=(\frac{kq_1q_2y'}{mr_{\mathrm{O'P}}^{D}\sqrt{1+y'^2}})^{1/1+\varepsilon} \qquad (25)$$

from Eq.(23), it gives

$$v\mathrm{d}v=\{\frac{kq_1q_2y'}{m[(H+x)^2+(R+H-y)^2]^{D/2}\sqrt{1+y'^2}}\}^{1/1+\varepsilon}\sqrt{1+y'^2}\,\mathrm{d}x \qquad (26)$$

For the two sides, we run integral operation from A to P, it gives the following result

$$v_{\mathrm{P}}^2=2\int_{-H}^{x_{\mathrm{P}}}\{\frac{kq_1q_2y'}{m[(H+x)^2+(R+H-y)^2]^{D/2}\sqrt{1+y'^2}}\}^{1/1+\varepsilon}\sqrt{1+y'^2}\,\mathrm{d}x \qquad (27)$$

Considering the simplest case, the straight line between A and B reads

$$y=H+x \qquad (28)$$

Substituting Eq.(28) into Eq.(27), and let $x=-z$, it gives the following result

$$v_{\mathrm{P}}^2=2\int_{-x_{\mathrm{P}}}^{H}\{\frac{kq_1q_2}{m[(H-z)^2+(R+z)^2]^{D/2}}\}^{1/1+\varepsilon}(\sqrt{2})^{\varepsilon/1+\varepsilon}\mathrm{d}z \qquad (29)$$

then the value can be calculated by numerical integral method.

Example 1, the given data are assumed as follows: for the electrification globe and ball, $\frac{kq_1q_2}{m}$=3.99×10^{14}m^3/s^2; the radius of the globe R=6.37×10^{6}m, $H=R$/10, try to solve the problem shown in Fig. 1, finding the solution for the value of v_{B}^2, and derive the improved law of Coulomb and the improved Newton's second law at one time.

Firstly, according to the original law of Coulomb and the original Newton's second law (i.e., let D=2 in Eq.(10) and ε=0 in Eq.(15)) and the law of conservation of energy, all the related quantities can be calculated, then substitute them into Eq.(17), it gives

$$\Pi_0=571.4215$$

Here, according to the law of conservation of energy, it gives v_{B}^2=1.0767×10^7, while according to the original law of Coulomb and the original Newton's second law, it gives v_{B}^2=1.1351×10^7, the difference is about 5.4 %.

For the reason that the value of Π_0 is not equal to zero, then the values of D and ε can be decided by the optimum seeking method.

At present the optimum seeking methods can be divided into two kinds, one kind may not depend on the initial values which program is complicated, another kind requires the better initial values which program is simple. One of the second kinds, i.e., the searching method [3] will be used.

Firstly, the value of D is fixed and let D=2，then search the value of ε， as ε=0.0146, the value of Π reaches the minimum 139.3429； then the value of ε is fixed，search the value of D， as D=1.99989, the value of Π reaches the minimum 137.3238；then the value of D is fixed，search the value of ε， as ε=0.01458, the value of Π reaches minimum 137.3231；because the last two results are highly close, the searching can be stopped, the final results as follows

$$D=1.99989,\quad \varepsilon=0.01458,\quad \Pi=137.3231$$

Here the value of Π is only 24% of Π_0. While according to the law of conservation of energy, it gives $v_B^2=1.0785\times10^7$，according to the improved law of Coulomb and the improved Newton's second law, it gives $v_B^2=1.1073\times10^7$, the difference is about 2.7 % only.

The results suitable for this example with the constant dimension fractal form as follows:

the improved law of Coulomb, $f=\dfrac{kq_1q_2}{r^{1.99989}}$;

the improved Newton's second law, $F=ma^{1.01458}$.

Finally we discuss the dimension(unit) of the improved law of Coulomb and the improved Newton's second law. Two precepts can be given.

First one: to prescript the dimensions of $a^{1+\varepsilon}$ and $r^{2-\varepsilon}$ get along with the same of a^1 and r^2 separately.

Second one: to handle the dimension, for each formula, the right side multiplies by a factor, for example, the improved Newton's second law can be written as $F=K'ma^{1+\varepsilon}$， where the value of K' is equal to 1, while the dimension of K' should be chosen to make the dimensions of the left side and right side with the same one.

The first precept is used in this paper for the advantage that the formula form may not be changed, while for the second one the formula form will be changed. Of course, other precept also may be discussed further.

5 Conclusion

The law of Coulomb and Newton's second law were summarized with experimental results. The example given in this shows that these original two laws should be improved. In order to derive these two laws theoretically, the law of conservation of energy can be used, for an example (a small electrification ball moves down along a long incline within the electric field due to an electrification globe), the variable dimension fractal method is developed, and used to derive the improved Newton's second law and the law of Coulomb at one time.

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