<u>On the Nonintegrable Regime of Particle Physics</u>

Ervin Goldfain

Ronin Institute, Montclair, New Jersey 07043

Email: ervin.goldfain@ronininstitute.org

Abstract

Recent research points out that the unavoidable approach to Hamiltonian chaos well above the Fermi scale leads to a spacetime having continuous (fractal) dimensions. Here we analyze a toy model of inflationary Universe comprising of a Higgs-like scalar in interaction with a pair of vector bosons. The model is manifestly nonintegrable as it breaks the perturbative unitarity of scattering processes and evolves towards Hamiltonian chaos and fractal spacetime.

Key words: Hamiltonian chaos, fractal spacetime, continuous dimensionality, Higgs boson, vector boson scattering.

1. Background and Motivation

Over the last couple of decades, many (yet almost forgotten) studies have concluded that the dynamics of primordial cosmology is prone to turn into *Hamiltonian chaos.* Even lesser known is that the nearly universal onset of Hamiltonian chaos leads to a spacetime endowed with *fractal dimensions* [5 - 7]. In this sense, both primordial cosmology and the far ultraviolet sector of field theory echo the open-ended evolution of large nonlinear systems outside equilibrium, which is a hallmark of *complex dynamics*.

The aim of this work is to provide further motivation for the emergence of fractal spacetime well above the Fermi scale of electroweak interactions. To this end, we consider here a toy model of inflationary Universe consisting of a Higgs-like scalar in interaction with a pair of vector bosons. The model breaks perturbative unitarity and exhibits the attributes of Hamiltonian chaos and fractal spacetime.

The paper is partitioned as follows: the second section details the main assumptions underlying the model; third section develops the equation of motion for vector bosons and highlights its similarity to the gravitational behavior of primordial cosmology. The Appendix section briefly covers the perturbative theory of vector boson scattering in flat spacetime.

The reader is urged to keep in mind the introductory nature of this paper. Further work is needed to develop, modify, or debunk the ideas detailed below.

2. Working assumptions

A1) the toy model developed below replicates the simplest chaotic inflation scenario, in which the classical inflaton field is replaced by a Higgs-like scalar field with effective potential [1],

$$V(\varphi) = \pm m_H^2 \varphi^2 + \frac{\lambda}{4}\varphi^4 \tag{1}$$

A2) The Higgs-like scalar decays into a condensate of gauge bosons χ due to the interaction term $-(1/2)g^2\varphi^2\chi^2$, where $\chi=(W^+,W^-)$. The rate of this process is taken to be [1]

$$\Gamma(\varphi\to\chi)=\frac{g^4\sigma^2}{8\pi m_H} \tag{2}$$

where the vacuum of the Higgs-like field is proportional to

$$\sigma=\frac{m_H}{\sqrt{\lambda}} \tag{3}$$

A3) The mass of the Higgs-like scalar greatly exceeds the mass of χ, on account of the *gauge hierarchy ansatz.* The rationale of this ansatz is the cumulative contribution of radiative corrections above the Fermi scale. It is thus reasonably to assume that the condition $m_H >> m_\chi$ holds well in this regime.

A4) To simplify the presentation and capture the main point of the argument, the self-interaction coupling λ is considered negligible, $\lambda<<1$.

A5) *Decoherence* of quantum processes and the subsequent transition to classicality is expected to occur at some scale larger than the Fermi scale. G*ravitational decoherence* is presumed to play a significant role in this setting.

3. High energy coupling of the Higgs-like field to vector bosons

The equation determining the behavior of classical fluctuations of χ assumes the form [1]

$$\ddot{\chi}_k + 3H\dot{\chi}_k + \Omega_k^2(t)\,\chi_k = 0 \tag{4}$$

According to this scenario, the production of χ follows from the time-dependent oscillator frequency of (4), whose square is given by,

$$\Omega_k^2(t) = \frac{k^2}{a^2(t)} + g^2\Phi^2\sin^2(m_H t) \tag{5}$$

Here, H stands for the Hubble constant, k is the wavenumber, $a(t)$ the time-dependent expansion parameter and Φ the amplitude of the Higgs-like scalar. The main contribution to the production of vector bosons comes from field excitations of large momentum $k >> a(t)m_H$, which is far greater than the contribution of the Hubble constant. Thus, to a leading order approximation, one can drop the second term in (4) and takes $a(t) = a$ to be a constant. The result is a Mathieu equation modeling a harmonic oscillator with variable mass, that is,

$$\boxed{\ddot{\chi}_k + \mu_k^2 \chi_k = 0} \tag{6}$$

in which

$$\mu_k^2 = \frac{k^2}{m_H^2 a^2} + \frac{g^2 \Phi^2}{2m_H^2}[1 - \cos 2(m_H t)] \tag{7}$$

The oscillator model (6) replicates the equation describing curvature fluctuations in the primordial Universe (eq. (14) in [4]). One can reasonably infer that (6) follows a similar route to Hamiltonian chaos and fractal spacetime as the one discussed in [4], upon letting (6) be driven by the random fluctuations of the early Universe.

APPENDIX

Vector boson scattering in the Standard Model

There are several diagrams in the Standard Model that contribute to the scattering of longitudinal vector bosons $W_L^+ W_L^- \to W_L^+ W_L^-$ [2 - 3]. Fig. 1 below shows the so-called $s-$channel diagram, which represents the predominant contribution to scattering at $m_{WW} = O(m_H)$. Unlike (6), for $m_H = 125$ GeV, perturbative unitarity in flat spacetime is preserved up to arbitrarily high energies.

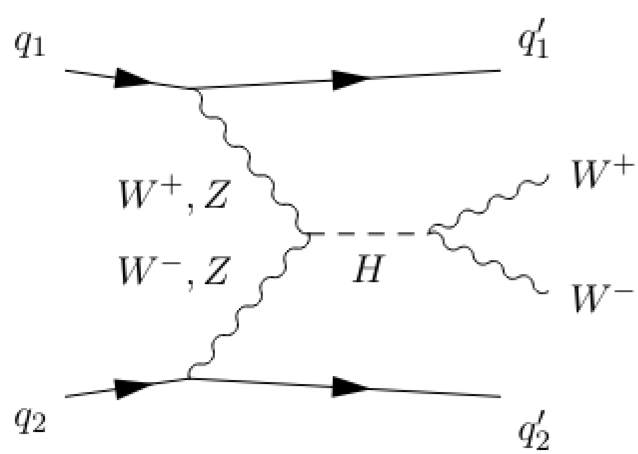

Fig. 1: Vector boson scattering in the Standard Model

References

1. https://arxiv.org/pdf/hep-th/9405187.pdf

2. B. W. Lee, C. Quigg, and H. Thacker, 'Weak Interactions at Very High Energies: The Role of the Higgs Boson Mass.' *PhysRev, D16*: p. 1519, 1977.

3. M. Veltman, 'Second Threshold in Weak Interactions.' *Acta PhysPolon, B8*: p. 475, 1977.

4. https://www.researchgate.net/publication/379534697

5. Zaslavsky, G. M. Hamiltonian Chaos and Fractional Dynamics, Oxford Univ. Press 2006.

6. https://www.researchgate.net/publication/369584882

7. https://www.researchgate.net/publication/369814408