[An Attempted] Proof of Collatz Conjecture

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Abstract

[This paper gives an attempted proof of [the] Collatz conjecture[.]

$7 \rightarrow 22 \rightarrow 11 \rightarrow 34 \rightarrow 17 \rightarrow 52 \rightarrow 13 \rightarrow 40 \rightarrow 5 \rightarrow 16 \rightarrow 1$

The problem that a number turns into 1 when multiplied by 3 and added by 1 before being divided by 2 for odd numbers and simply being divided by 2 for even numbers, whatever the number, is the same problem as the following:

$$
\begin{aligned}
&7\\
&\rightarrow 7 \cdot 2^0 \times 3 + 2^0 = 22 = 11 \cdot 2^1\\
&\rightarrow 11 \cdot 2^1 \times 3 + 2^1 = 68 = 17 \cdot 2^2\\
&\rightarrow 17 \cdot 2^2 \times 3 + 2^2 = 208 = 13 \cdot 2^4\\
&\rightarrow 13 \cdot 2^4 \times 3 + 2^4 = 640 = 5 \cdot 2^7\\
&\rightarrow 5 \cdot 2^7 \times 3 + 2^7 = 2048 = 2^{11}
\end{aligned}
$$

Multiplying a number by 3 and then repeating the process of adding that number's maximum power factor of 2 will lead to the number being a power of 2.

Now, let's think about the value of $\frac{1}{3}+\frac{1}{3^2}+\frac{1}{3^3}+\frac{1}{3^4}+\cdots-\varepsilon=\frac{2}{3}-\varepsilon$ **added to *N*.**

Let's use the example with 7 again.

$$
\begin{aligned}
&\left(7+\frac{1}{3}+\frac{1}{3^2}+\frac{1}{3^3}+\frac{1}{3^4}+\cdots-\varepsilon\right)\times 3\\
&\rightarrow\left(22+\left(\frac{1}{3}+\frac{1}{3^2}+\frac{1}{3^3}+\frac{1}{3^4}+\cdots-\varepsilon\right)\right)\times 3\\
&\rightarrow\left(67+\left(\frac{1}{3}+\frac{1}{3^2}+\frac{1}{3^3}+\frac{1}{3^4}+\cdots-\varepsilon\right)\right)\times 3\\
&\rightarrow\left(202+\left(\frac{1}{3}+\frac{1}{3^2}+\frac{1}{3^3}+\frac{1}{3^4}+\cdots-\varepsilon\right)\right)\times 3\\
&\rightarrow\left(607+\left(\frac{1}{3}+\frac{1}{3^2}+\frac{1}{3^3}+\frac{1}{3^4}+\cdots-\varepsilon\right)\right)\times 3\\
&\rightarrow 1822+\left(\frac{1}{3}+\frac{1}{3^2}+\frac{1}{3^3}+\frac{1}{3^4}+\cdots-\varepsilon\right)
\end{aligned}
$$

The progression deduced from the above,

7→22→67→202→607→1822 **is identical to adding the minimum value that allows the next term to have a bigger power of 2 as its factor than that of the previous term.**

7→22: **22 already has a power factor of 2, which is bigger than that of 7.**

22→67: **Among natural numbers of 67 and above, the minimum value that has a power factor of 2 bigger than 2 is 68, which has 4 as a factor.**

68→202: **Among natural numbers of 202 and above, the minimum value that has a power factor of 2 bigger than 4 is 202, which has 16 as a factor.**

68→607: **Among natural numbers of 607 and above, the minimum value that has a power factor of 2 bigger than 16 is 640, which has 128 as a factor.**

607→1822: **Among natural numbers 1822 and above, the minimum value that has a power factor of 2 bigger than 129 is 2048, which is a power of 2.**

Let's suppose the first natural number is *N*.

When the process of multiplying the number by 3 and finding the minimum value that has a power factor bigger than the previous value's power factor of 2 is done "A" times, the value would be larger than

$\left\lfloor \left(N+\frac{2}{3}-\varepsilon\right)\cdot 3^{A} \right\rfloor$ **and would have 2^A as its factor. This number shall be called N_A.**

Let's start again at 7. To help with understanding, the same formula has been expressed in two ways, left and right.

$\left\lfloor \left(7+\frac{2}{3}-\varepsilon\right)\cdot\left(\frac{3}{2}\right) \right\rfloor = 11 \quad \left\lfloor \left(7+\frac{2}{3}-\varepsilon\right)\cdot\left(\frac{3}{2}\right) \right\rfloor \cdot 2 = 22$

$\left\lfloor \left(7+\frac{2}{3}-\varepsilon\right)\cdot\left(\frac{3}{2}\right)^2 \right\rfloor = 17 \quad \left\lfloor \left(7+\frac{2}{3}-\varepsilon\right)\cdot\left(\frac{3}{2}\right)^2 \right\rfloor \cdot 2^2 = 68$

$$\left\lceil \frac{\left(7+\frac{2}{3}-\varepsilon\right)\cdot\left(\frac{3}{2}\right)^3}{2} \right\rceil = 13 \quad \left\lceil \frac{\left(7+\frac{2}{3}-\varepsilon\right)\cdot\left(\frac{3}{2}\right)^3}{2} \right\rceil \cdot 2^4 = 208$$

According to the power of 2 in the denominator, the result value moves in the unit of the subsequent power of 2. Due to the addition of the minimum value that gives each term a power of 2 as a factor bigger than that of its previous term, the ceiling function shall be applied instead of the floor function.

$$\left\lceil \frac{\left(7+\frac{2}{3}-\varepsilon\right)\cdot\left(\frac{3}{2}\right)^4}{2^3} \right\rceil = 5 \quad \left\lceil \frac{\left(7+\frac{2}{3}-\varepsilon\right)\cdot\left(\frac{3}{2}\right)^4}{2^3} \right\rceil \cdot 2^7 = 640$$

$$\left\lceil \frac{\left(7+\frac{2}{3}-\varepsilon\right)\cdot\left(\frac{3}{2}\right)^5}{2^3} \right\rceil = 8 \quad \left\lceil \frac{\left(7+\frac{2}{3}-\varepsilon\right)\cdot\left(\frac{3}{2}\right)^5}{2^3} \right\rceil \cdot 2^8 = 2048$$

Thus,

in $7 < \left\lceil \frac{\left(7+\frac{2}{3}-\varepsilon\right)\cdot\left(\frac{3}{2}\right)^A}{2^B} \right\rceil$,

when there is a set of natural numbers *A*, *B*, and *C*, which satisfies $\left\lceil \frac{\left(7+\frac{2}{3}-\varepsilon\right)\cdot\left(\frac{3}{2}\right)^A}{2^B} \right\rceil = 2^C$**, when there is a set of natural numbers *A*, *B*, and *C*, multiplying 7 by 3 and repeating the process of deducing the minimum value that gives a power of 2 as a factor bigger than that of the previous value will lead to the value being a power of 2.**

Generally, regarding a certain natural number called *N*,

when $N < \left\lceil \frac{\left(N+\frac{2}{3}-\varepsilon\right)\cdot\left(\frac{3}{2}\right)^A}{2^B} \right\rceil$,

and there is a set of natural numbers *A*, *B*, and *C*, which satisfies $\left\lceil \frac{\left(N+\frac{2}{3}-\varepsilon\right)\cdot\left(\frac{3}{2}\right)^{A}}{2^{B}} \right\rceil = 2^{C}$,

repeating the process of adding the minimum value that gives each term a power of 2 as a factor bigger than that of its previous term leads to the term being a power of 2. It is evident that such a set of natural numbers, *A*, *B*, and *C*, would always exist for arbitrary *N*.

Therefore, the Collatz Conjecture is true.

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