Proofs for Collatz Conjecture and Kaakuma Sequence

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Apr 2024

Abstract

The objective of this study is to present precise proofs of Collatz conjecture and to introduce some interesting conjectures on Kaakuma sequence. We Proposed a novel approach that tackles the Collatz conjecture in different techniques with different point of views. The study also shows some general truths and behaviors of Collatz conjecture. Finally, the proof discovers Qodaa ration test that works for all complex and complicated Kaakuma sequence and new characteristics of the sequence. This proof help to changes some views of researchers on unsolved problems and some point of views on probability in infinite range. In this study we discovered the dynamicity of Collatz sequence and reflection and interpretation of the probabilistic proof Collatz Conjecture.

The Collatz Conjecture, proposed by Lothar Collatz in 1937, remains one of the most intriguing unsolved problems in mathematics. The conjecture posits that, for any positive integer, applying a series of operations will eventually lead to the number 1. Despite decades of rigorous investigation and countless computational verifications, a complete proof has eluded mathematicians. In this research endeavor, we embark on a comprehensive exploration of the Collatz Conjecture, aiming to shed light on its underlying principles and ultimately establish its validity. Our investigation begins by defining the Collatz function and its recursive nature. We analyze the behavior of the function for various integer inputs and identify recurring patterns within the resulting sequences.

Our investigation culminates in the formulation of a set of conjecture encompassing lemmas and postulates, which we rigorously prove using a combination of analytical reasoning, numerical evidence, and exhaustive case analysis. These results provide compelling evidence for the veracity of the Collatz Conjecture and contribute to our understanding of the underlying mathematical structure. Finaly we derived some behaviors of Collatz sequence like translation, reflection, divisibility, constants, successive division, evenly distribution, iteration groups, huge untrivial cycle and we constructed point of views on Consistency of Constants, instantaneously falling values, contradictions on density and Qodaa ratio test.

Keywords: Collatz Conjecture, number theory, mathematical proof, recursive sequences, computational analysis, modular arithmetic, Kaakuma Sequence, Qodaa ratio test, Stopping Time

1.Introduction

The Collatz Conjecture, also known as the 3n+1 Conjecture, Hailstone Problem, Kakutani's Conjecture, Ulam's Conjecture, Hasse's Algorithm, and the Syracuse Problem, it is one of long-standing an unsolved mathematical problem that has fascinated mathematicians for 87 years and it is one of the most dangerous unsolved problems in mathematics. The conjecture is named after German mathematician Lothar Collatz, who first proposed it in 1937. involves the mapping

The Collatz Conjecture originally states iterative sequence of natural numbers. Take a natural number and if it is even, make it half, if it is odd multiply it by 3 and add 1, continue the process repeatedly taking the result as next impute, and continue iterating. The conjecture states that regardless of the starting value, the sequence of numbers will eventually reach the value 1.

Eg 14, 7, 22, 11, 34, 17, 52, 26, 13, 40, 20, 10, 5, 16, 8, 4, 2, 1

The Collatz Conjecture has captured the minds of mathematicians for almost a century Many have attempted to prove or disprove it, employing various techniques and approaches. Despite its apparent simplicity, the conjecture has resisted all attempts at a definitive solution. The search for a solution to the Collatz Conjecture continues, driven by the allure of a seemingly simple problem harboring immense complexity. It serves as a reminder that even in the vast realm of mathematics, profound mysteries still await discovery.

Even if the Collatz Conjecture is simple to express and understand the problem, it tantalized scientist for around a century to be solved. Mathematicians have extensively tested the conjecture using computers for a billions of billion values, and it holds true for all tested cases. The Collatz Conjecture has fascinated mathematicians due to its apparent simplicity combined with its elusiveness. Many attempts have been made to prove or disprove the conjecture, involving various mathematical techniques and concepts. However, the conjecture remains one of the most enduring unsolved problems in mathematics.

Kaakuma sequence is vast general form of Collatz sequence

$$n = \begin{cases} \dfrac{\pm k_1 n \pm c_1}{b_1} & case1 \\ \dfrac{\pm k_2 n \pm c_2}{b_2} & case2 \\ \dfrac{\pm k_3 n \pm c_3}{b_3} & case3 \\ \vdots & \\ \dfrac{\pm k_i n \pm c_i}{b_i} & casei \end{cases}$$

With certain rule of iteration to give meaningful iteration

2. Expressions of Collatz sequence

Collatz conjecture notated in different ways while they have the same meaning

$$n_{i+1} = \begin{cases} 3n_i + 1 & if\ n_i\ is\ odd \\ \frac{n_i}{2} & if\ n_i\ even \end{cases}$$

n_i is any number that begin an orbit and finally reaches 1 by iterating following the rule.

$$c(n) = \begin{cases} 3n + 1 & if\ n\ is\ odd \\ n/2 & if\ n\ even \end{cases}$$

in this the result used as next value for iteration until the value reaches 1.

$$n = \begin{cases} 3n + 1 & if\ n\ is\ odd \\ n/2 & if\ n\ even \end{cases}$$

When we use in simple format as used in coding assignment, the right side of equation is impute and the left side of equation is output and the iteration continues by using the output as next impute until it gets 1.

$$n = \begin{cases} \frac{3n+1}{2} & if\ n\ is\ odd \\ \frac{n}{2} & if\ n\ even \end{cases}$$

This the shorter form

$$n = \begin{cases} \frac{3n+1}{2} & n\ \equiv 1(mod2) \\ \frac{n}{2} & n\ \equiv 0(mod2) \end{cases}$$

When we express the conditions of iteration in modular form, and there are some more complex notations of Collatz conjecture.

When the conjecture reversed, start from 1 and double a number any time and divide a number minus one when it is possible to get positive integer. All natural numbers covered in tree map, or no natural number left out of reverse tree map

$$n = \begin{cases} \dfrac{n-1}{3} & n \equiv 1(mod3) \\ 2n & \forall n \ n \epsilon \mathbb{N} \end{cases}$$

		1	2, 4, 8, 16, 32, 64, 128, 256, 512, 1024
	16	5	10, 20, 40, 80, 160, 320, 640, 1280
	10	3	6, 12, 24, 48, 96, 192, 384, 768, 1536
	40	13	26, 52, 104, 208, 416, 832, 1664
	52	17	34, 68, 136, 272, 544, 1088
	34	11	22, 44, 88, 176, 352, 704, 1408
	22	7	14, 28, 56, 112, 224, 448, 896, 1792
	28	9	18, 24, 48, 96, 192, 384, 768, 1536
	64	21	42, 84, 164, 328, 656, 1312
	88	29	58, 116, 232, 464, 928, 1856
	58	19	38, 76, 152, 304, 608, 1216
	76	25	50, 10, 200, 400, 800, 1600
	112	39	78, 156, 312, 624, 1248

In This tabular form of tree, the nodes make new line

3. Behavior of Collatz sequence

Before preceding to the proof Collatz conjecture it is mandatory to know some basic behaviors of Collatz trajectory

3.1. Transformation

The Collatz trajectory map can be shifted by using transformation rule

3.1.1. Translation:

translation is a transformation that shifts every value in the orbit by a fixed distance forward or backward. Example 7, 22, 11, 34, 17, 52, 26, 13, 40, 20, 10, 5, 16, 8, 4, 2, 1 when it is shifted two steps forward 9, 24, 13, 36, 19, 54, 28, 15, 42, 22, 12, 5, 18, 10, 6, 4, 3 and when it is shifted three points backward 4, 19, 8, 31, 14, 49, 23, 10, 37, 17, 7, 2. 13, 5, 1. -1. -2

$$n = \begin{cases} 3n+1 & n \equiv 1(mod2) \\ \dfrac{n}{2} & n \equiv 0(mod2) \end{cases} + 2 \equiv n = \begin{cases} 3n-3 & n \equiv 1(mod2) \\ \dfrac{n+2}{2} & n \equiv 0(mod2) \end{cases}$$

$$n = \begin{cases} 3n+1 & n \equiv 1(mod2) \\ \dfrac{n}{2} & n \equiv 0(mod2) \end{cases} - 3 \equiv n = \begin{cases} 3n+7 & n \equiv 1(mod2) \\ \dfrac{n+2}{2} & n \equiv 0(mod2) \end{cases}$$

It is done by c=c-l(k-d). If a conditional equation is $\frac{kn+c}{d}$ and it is translated by length l then translated by $\frac{kn+c-\ (k-d)}{d}$ we perform this in all cases and is used with its sign or direction

Lemma1 n next term $\frac{kn+c}{d}$ when we shift it by translating length l, the two terms are n+l, $\frac{kn+c}{d}+l$ $=\frac{kn+c+}{d}$ when we use direct formula $\frac{k(n+l)+c-l(k-d)}{d}=\frac{kn+c+}{d}$ we can use here proof induction

by applying this the short form of Collatz sequence

$$n=\begin{cases}\dfrac{3n+1}{2} & n\equiv 1(mod2)\\ \dfrac{n}{2} & n\equiv 0(mod2)\end{cases}+\ 1\ \equiv\ n=\begin{cases}\dfrac{3n}{2} & n\equiv 0(mod2)\\ \dfrac{n+1}{2} & n\equiv 1(mod2)\end{cases}$$

because of its simplicity we used it and its inverse on this study

$$n=\begin{cases}\dfrac{3n}{2} & n\equiv 0(mod2)\\ \dfrac{n+1}{2} & n\equiv 1(mod2)\end{cases}$$

this orbit converges to 2 or (2, 3) cycle

example 8, 12, 18, 27, 14, 21, 11, 6, 9, 5, 3, 2 it is easy to express cases by power of 2

its inverse is

$$n=\begin{cases}\dfrac{2n}{3} & n\equiv 0(mod3)\\ 2n-1 & \forall n\ \ n\in\mathbb{N}\end{cases}$$

3.1.2. Reflection on y-axis

A reflection of Collatz orbit on y-axis is done by multiplying constant terms by -1and the sequence started by reflected value of original sequence, negative value of n.

$$-1\times\left(\frac{kn+c}{d}\right)\leftrightarrow\frac{kn-c}{d}$$

$$-C(n)=\begin{cases}\dfrac{3n}{2} & n\equiv 0(mod2)\\ \dfrac{n+1}{2} & n\equiv 1(mod2)\end{cases}\leftrightarrow\ n=\begin{cases}\dfrac{3n}{2} & n\equiv 0(mod2)\\ \dfrac{n-1}{2} & n\equiv 1(mod2)\end{cases}$$

$$n = \begin{cases} \dfrac{3n}{2} & n \equiv 0(mod2) \\ \dfrac{n-1}{2} & n \equiv 1(mod2) \end{cases}$$

-8, -12, -18, -27, -14, -21, -11, -6, -9, -5, -3, -2 for all negative integers converges to -2, -3 cycle

3.1.3. Scaling

Scaling is multiplying constant term by scaling number.

$\frac{kn+c}{d} \leftrightarrow \frac{kn+sc}{d}$ when it is scaled up by s or when the Collatz orbit multiplied by s

8, 12, 18, 27, 14, 21, 11, 6, 9, 5, 3, 2 multiplied by 5: 40, 60, 90, 135, 70, 105, 55, 30, 45, 25, 15, 10

$$5\left(n = \begin{cases} \frac{3n}{2} & n \equiv 0(mod2) \\ \frac{n+1}{2} & n \equiv 1(mod2) \end{cases} \right) \equiv n = \begin{cases} \frac{3n}{2} & n \equiv 0(mod2) \\ \frac{n+5}{2} & n \equiv 1(mod2) \end{cases}$$

When we scale Collatz sequence by a number that is different from power of 3 the scaled map of Collatz sequence has two or more cycles.

$$n = \begin{cases} \dfrac{3n}{2} & n \equiv 0(mod2) \\ \dfrac{n+3^i}{2} & n \equiv 1(mod2) \end{cases}$$

the trajectory converges to 2×3^i or $(2 \times 3^i,\ 2 \times 3^i)$ cycle for all positive integers.

$$example\ n = \begin{cases} \dfrac{3n}{2} & n \equiv 0(mod2) \\ \dfrac{n+27}{2} & n \equiv 1(mod2) \end{cases}$$

converges to 54 or (54, 81) cycle for all natural numbers n. 1, 14, 21, 24, 36, 54, 81, 54, 81

3.2. Divisibility of Collatz sequence

In selective mapping only selected part of Collatz sequence or nearby node mapped to new sequence. Example from the Collatz sequence when $n \equiv 0(mod3)$ extracted and divided by 3

$$\left(n = \begin{cases} \frac{3n}{2} & n \equiv 0(mod2) \\ \frac{n+1}{2} & n \equiv 1(mod2) \end{cases} \quad n \equiv 0(mod3)\right)/3 \quad \leftrightarrow \quad n = \begin{cases} \frac{3n}{2} & n \equiv 0(mod2) \\ \frac{3n+1}{4} & n \equiv 1(mod4) \\ \frac{n+1}{4} & n \equiv 3(mod4) \end{cases}$$

It converges to 1 for all integers

28, 42, 63, 32, 48, 72, 108, 162, 243, 122, 183, 92, 138, 207, 104, 156, 234, …mapped to

14, 21, 16, 24, 36, 54, 81, 61, 46, 69, 52, 78, 117 …

$$\left(\frac{n = \begin{cases} \frac{3n}{2} & n \equiv 0(mod2) \\ \frac{n+1}{2} & n \equiv 1(mod2) \end{cases} \quad n \equiv 0(mod5)}{5}\right) \leftrightarrow n = \begin{cases} \frac{3n}{2} & n \equiv 0(mod2) \\ 3n & n \equiv 3(mod4) \\ \frac{3n+1}{4} & n \equiv 1(mod8) \\ \frac{n+3}{16} & n \equiv 13(mod16) \\ \frac{9n+7}{4} & n \equiv 5(mod32) \\ \frac{n-1}{4} & n \equiv 21(mod32) \end{cases}$$

It converges to 1 for all natural numbers

28, 42, 63, 32, 48, 72, 108, 162, 243, 122, 183, 92, 138, 207, 104, 156, 234, … next node inverse

55, 83, 125, 63, 95, 143, 215, 323, 485, 243, 365, 183, 275, 413, 311. 467 … mapped to

11, 33, 25, 19, 57, 43, 129, 97, 73, 55, 165, 373, 93, 6, 9, 7, 21, 5, 13, 1

$$\frac{n = \begin{cases} \frac{3n}{2} & n \equiv 0(mod2) \\ \frac{n+1}{2} & n \equiv 1(mod2) \end{cases} \quad n \equiv 0(mod9)}{9} \leftrightarrow n = \begin{cases} \frac{3n}{2} & n \equiv 0(mod2) \\ 3n & n \equiv 3(mod4) \\ \frac{3n+1}{8} & n \equiv 5(mod8) \\ \frac{3n+1}{4} & n \equiv 1(mod16) \\ \frac{3n+5}{32} & n \equiv 9(mod32) \\ \frac{3n+5}{16} & n \equiv 25(mod64) \\ \frac{n+7}{64} & n \equiv 57(mod64) \end{cases}$$

It converges to 1 for all natural numbers

28, 42, 63, 32, 48, 72, 108, 162, 243, 122, 183, 92, 138, 207, 104, 156, 234, …mapped to

7, 21, 8, 12, 18, 27, 81, 61, 23, 69, 26, 39, 117, 44, 66, 99….

3.3. Successive Division and squeezing stopping time.

Here we can divide each case in two or more sub case with their results.

$$n = \begin{cases} \frac{9n}{4} & n \equiv 0(mod4) \\ \frac{3n+2}{4} & n \equiv 2(mod4) \\ \frac{3n+3}{4} & n \equiv 3(mod4) \\ \frac{n+1}{4} & n \equiv 1(mod4) \end{cases}$$

28, 63, 48, 108, 243, 183, 138, 104, 234, 176, 396, 891, 669, 168

$$n=\begin{cases}\dfrac{27n}{8} & n\equiv 0(mod8)\\ \dfrac{9n+4}{8} & n\equiv 4(mod8)\\ \dfrac{9n+6}{8} & n\equiv 2(mod8)\\ \dfrac{3n+6}{8} & n\equiv 6(mod8)\\ \dfrac{9n+9}{8} & n\equiv 7(mod8)\\ \dfrac{3n+7}{8} & n\equiv 3(mod8)\\ \dfrac{3n+9}{8} & n\equiv 5(mod8)\\ \dfrac{n+7}{8} & n\equiv 1(mod8)\end{cases}$$

We can do this partially

3.4. Evenly Distribution of 3 powers and 2 powers

When we map the inverse tree of Collatz trajectory the gaps between 3 powers are evenly distributed with few exceptions, first time occurrences of higher powers that will substitute lower powers is uncertain. That means $3^{i+1}k\ separated\ by\ two\ of\ 3^{i}j$

27, 53, 105, 209, 417, 833, 1665, 3329, 6657, 13313, 26625, 53249, 106497, 212993, 425985, 851969, 1703937, 3407873, 6815745,

Lemma2 $(2^{k}+1\ \equiv 0(mod(3^{k}))$

This shows expansion rate of inverse tree map is almost fair. It has a great role on density of inverse of Collatz map

3.5. Constants

3.5.1. Nearly Constant Expansion Rate of inverse tree map.

The average growth Collatz inverse tree map is 1/3. We can see this in two ways by using Collatz inverse map and list format

$$n=\begin{cases}\dfrac{2n}{3} & n\equiv 0(mod3)\\ 2n-1 & \forall n\ \ n\in\mathbb{N}\end{cases}$$

Let us start from 5 because 2 and 3 have cycling case and they can distort true image of expansion

[5], [9], [17, 6], [33, 11, 4], [65, 22, 21, 7], [129, 43, 41, 14, 13], [257, 86, 85, 81, 27, 25] this is 1/3 expansion rate in average when the list has more than 30 elements

When we do only one (the first element of list) 1000 new leaf created in 3000 of operations done in average with very small standard deviation

[5], [9]. [17. 6]. [6, 33], [33, 11, 4], [11, 4, 65, 22] in this sequence the first node creates one or two leaf and append at the end next list.

List length	new	round		list	new	round		list	new	round
1(9)	0	3		159	39	19		15844	3966	35
2(6, 17)	1	4		211	52	20		21122	5278	36
3	1	5		282	71	21		28150	7028	37
4	1	6		381	99	22		37535	9385	38
5	1	7		505	124	23		50060	12525	39
6	1	8		665	160	24		66747	16687	40
8	2	9		885	220	25		88948	22201	41
12	4	10		1187	302	26		118396	29448	42
18	6	11		1590	403	27		157670	39274	43
24	6	12		2122	532	28		209600	51930	44
31	7	13		2829	707	29		278863	69263	45
39	8	14		3765	936	30		369741	90878	46
50	11	15		5014	1249	31		487132	117391	47
68	18	16		6682	1668	32		643631	156499	48
91	23	17		8902	2220	33		844576	200945	49
120	29	18		11878	2976	34		1100940	256364	50

[2], [3], [5], [9], [17, 6], [33, 11, 4], [65, 22, 21, 7], [129, 43, 41, 14, 13], [257, 86, 85, 81, 27, 25]

The above table show list count in each step with new branches in previous list finally new branch approaches 1/3 of previous list in new list.

3.5.2. Average Stopping Time

$$n = \begin{cases} 3n+1 & n \equiv 1(mod2) \\ \frac{n}{2} & n \equiv 0(mod2) \end{cases}$$

Average stopping time of this sequence is 4.98898

$$n = \begin{cases} \frac{3n+1}{2} & n \equiv 1(mod2) \\ \frac{n}{2} & n \equiv 0(mod2) \end{cases}$$

Average stopping time of this sequence is 3.49269

$$n = \begin{cases} \frac{9n}{4} & n \equiv 0(mod4) \\ \frac{3n+2}{4} & n \equiv 2(mod4) \\ \frac{3n+3}{4} & n \equiv 3(mod4) \\ \frac{n+1}{4} & n \equiv 1(mod4) \end{cases}$$

Average stopping time of this sequence is 2.155207

3.5.3. The ratio of stopping time to log(n, 2) is bounded.

The ratio of stopping time to log(n, 2) bounded and less than 10, when starting numbers are big like more than 8 digits it is bound and less than 6

For starting number 2^p and stopping time t the ratio is 3.67 to 5.15

p	187	188	189	190	191	192	193	194	195	196	197	198	199
t	693	690	753	753	753	749	994	994	994	994	747	745	745
t/p	3.71	3.67	3.98	3.96	3.94	3.90	5.15	5.12	5.10	5.07	3.79	3.76	3.74

3.6. Stopping Time Iteration Groups

When we group number by iteration, some numbers have the same number of stopping time and grouped by $2^t k + c$. If the iterations of c's stopping time is t and $2^t > c$ then all the numbers formed by $2^t k + c$ have stopping time t. They are also falling points of numbers.

T = (4, 7, 5, 7, 5, 59, 56, 8, 54, 7, 54, 51, 8, 45, 8, 42, 31, 8, 15, 40, 21, 29, 8, 13, 8, 13, 8, 31, 12)

c = [4, 8, 12, 16, 24, 28, 32, 40, 48, 60, 64, 72, 80, 92, 96, 104, 112, 124, 128, 156, 160, 168, 176, 192) with corresponding values t to c, $2^t k + c$ have the same stopping time of t. t of $2^5 k + 12 = 5$, t of $2^7 k + 16 = 7$, t of $2^5 k + 24 = 5$, t of $2^{59} k + 28 = 59$

3.7. Huge iteration if there exist non trivial cycle

if there is untrivial cycle then it is very huge iterations that is nearly equal to starting number and it easy is to get huge starting numbers.

what are the expected points and values to check c(n) has no more loop

- ✓ low density of non c(n) numbers in loop
- ✓ long iteration more than trillions of trillions
- ✓ big height in loop
- ✓ bounded value of c

the loop has ups and downs while it gets back to starting number or small number in loop

$n, \frac{3n}{2}, \frac{9n}{4}, \frac{27n}{8}, \frac{27n+8}{16}, \frac{81n+24}{32}, \frac{81n+}{64}, \frac{243n+168}{128}, \frac{729n+504}{256}, \frac{729n+760}{512}, \frac{2187n+2280}{1024}, ---, \frac{3^u n+c}{2^t}$

$$n = \frac{3^u n + c}{2^t - 3^u}$$

$$n = \frac{c}{2^t - 3^u}$$

c is partially geometric series of ratio r=3/4 and
$g_1 = 2^2 3^{u-2}, 2^3 3^{u-3}, 2^4 3^{u-4}, ---, 2^i 3^{u-i}$

$c < 3^{u+1}$

$n < \frac{3^{u+1}}{b^t - a^u}$ or $n_{max} = \frac{3^{u+1}}{b^t - a^u}$

$here\ to\ get\ n_{max}, t = \lceil u \times \log_2 3 \rceil\ and$

$\lceil u \times \log_2 3 \rceil - u \times \log_2 3\ supposed\ to\ be\ too\ small$

$so\ that\ 2^t - 3^u\ to\ be\ small$

from computer search maximum n is less than 10t, from that if 20 digits number is non c(n) and it is looped at least it must have 19 digits number of iterations 45 digits number height c(h) from the analysis.

Using this it is simple to search up to 2^{300} if there non trivial cycle the starting number is greater than 2^{300} and the number of iterations is greater than 2^{296}

4) Proofs

4.1. Existence of falling Values with Stopping time

If Collatz's conjecture is incorrect, there are two possibilities, the first is to simply diverges to infinite, and the other is a sequence with non-trivial cycle with huge starting number greater than $2^{300}$and huge number of iterations greater than 2^{296}. In both cases, it should grow rapidly so that to balance the density of numbers in existing sequence, that is, the number of odd numbers should be rare and the occurrence of even numbers that are powers of 2 with high 2 powers should be frequent. Otherwise, reverse map of existing sequence going to be diminished that is impossible from the behavior that seen in "3.4" and constant growth of expansion rate that seen in "3.5.1".

It is impossible a number of values to have infinite or huge amount of stopping time because almost all values are in Iteration groups in behavior shown in 3.4. Negligible

These two conditions lead to existence of instantaneous falling values and all stopping values in sequence. As small iteration stopping time expected like $k2^4 + 4$ stopped after 4 iterations and $k2^7 + 8$ stopped after 7 iteration these are very short falling values and $k2^{56} + 32$ stopped after 56 iterations and $k2^{59} + 28$ short falling values and $k2^{497} + 2^{113}$ stopped after 497 iteration and $k2^{699} + 2^{137}$ stopped after 699 iteration this are long falling values and there are some instantaneously falling values after 100,000s of iteration like $k2^{65070} + \frac{2^{16667}-68}{81}$ stopped after 65070 iterations either the stopping time is big or small, all numbers have stopping time. In non c(n) sequence falling of a single term increases the number of values that tends to fall. We can show that 2k+1, 4k+2, 8k+4, 16k+8 --- fall after certain points iteration. If we take the fastest growth of non C(n) in both cases and count non falling values it exceeds it limit. For example when we check 8k+4 less than 40 billion 0.123714965/0,125 = 98.971972% 8k+4 in trivial Collatz sequence has limited falling values. With the same method we can count all.

- ✓ *The sequence must grow as fast as possible*
- ✓ *All values tend to fall after long or short iterations limits fast growth*
- ✓ *The falling of single value decreases growth speed increases more falling values*
- ✓ *All local maximum values create new branches of tree that overflows the density.*

4.2. The Matter of Consistency of Constants

There are some distinct constants if Collatz conjecture as we have seen behavior of Collatz sequence in "e"

4.2.1. The average stopping time of Collatz sequence is constant.

it is similar to constants of pi and e

$$n = \begin{cases} \dfrac{3n+1}{2} & n \equiv 1(mod2) \\ \dfrac{n}{2} & n \equiv 0(mod2) \end{cases}$$

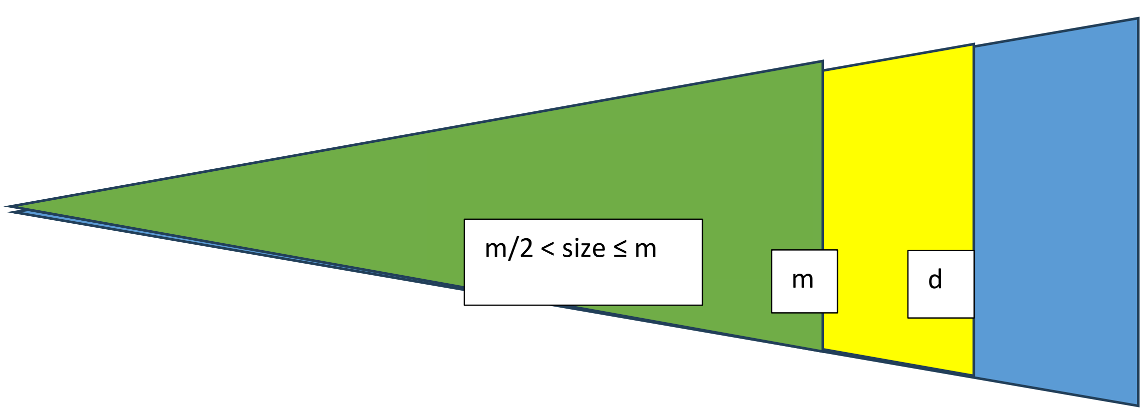
Average stopping time of this sequence is 3.49269 the point is if the average of stopping time is constant and consistent that approaches in both sides with very small variation it is impossible to divert after 10^{20} or 10^{40} if the Collatz conjecture is invalid it implies that , let 2^{120} is non c(n) and

$$\left(\sum_{n=2}^{2^{120}-1} t\right) \Big/ 2^{120} - 1 = 3.49269 \text{ but}$$

$$\left(\sum_{n=2}^{2^{120}} t\right) \Big/ 2^{120} = \infty \text{ that is impossible}$$

4.2.2. The inverse map of Collatz covers all natural numbers starting root 1,

on the process its expansion rate is 30% of as height gets higher. As it is expected from pack of numbers 1/3 is 3k, 1/3 is 3k+1and 1/3 is 3k+2 from this 1/3 that is 3k by percentage 33.33% expected to expansion rate to form next leaves but around 3% is tolerance value the higher 3 powers comes later to go deep. Using projection, we can check validity of Collatz conjecture by long and huge number calculation. We can use scaling and translation to simplify the solution



If the Collatz conjecture is true, we can get minimum value of leaves m and the size of tree below minimum value at any height or steps by using projection of expansion rate. If we can show size is greater than half of minimum value the Collatz conjecture is correct it is hardly possible to show it. If the Collatz conjecture is not correct half of minimum number in leaves is greater than size of tree less than minimum number.

4.2.3. The ratio of stopping time to log(n, 2) is bounded and less than5

The ratio of stopping time to log(n, 2) is bounded and less than 5 and we can adjust odd in small numbers like 28 and 32 by translation:: this can be checked by computer programs by taking high rate stopping time values like 2^k this constant is like the ratio of primes in natural numbers pi(x)

In this case we can also categorize what kind of starting values have long stopping time. Like

2^k	4(2^6k-1)/9	8(2^18k-1)/27	16(2^54k-1)/81	32(2^162k-1)/243	64(2^162k-1)/729	64(2^(486k+394))-7)/729

4.3. Contradiction in Density

All the above studies and some more studies in other documents show that from all natural numbers, more numbers are occupied in Collatz sequence than number in non-Collatz sequence if exists. It mentioned in behavior “3.4” and “3.5.1” but the behavior of Collatz sequence mentioned in “3.2” shows the opposite. Actually, beyond the behavior in “3.2” it is possible to count share of non-trivial part of Collatz sequence and it is more than the share of trivial part of Collatz sequence.

When the behavior of Collatz sequence in “3.2” can be expressed well by invers map for any natural number g and h, to simplify the condition we use h=0

$$\frac{n = \begin{cases} \frac{n-1}{3} & n \equiv 1(mod3) \\ 2n & \forall n \;\; n \in \mathbb{N} \end{cases}}{g} + h \qquad n \equiv h(modg)$$

$$\frac{n = \begin{cases} \frac{n-1}{3} & n \equiv 1(mod3) \\ 2n & \forall n \;\; n \in \mathbb{N} \end{cases}}{g} \qquad n \equiv 0(modg)$$ the objective is to ground non trivial sequence

Even though it is not proved a sequence that diverges to the infinite like 5n+1 is not divisible or not grounded and a sequence with a number of cycles decreases it steps or merged to one when the divisor is part of new cycle part

Eg1:- we cannot get proper equation for $\dfrac{n=\begin{cases}\frac{n-1}{5} & n\equiv 1(mod3)\\ 2n & \forall n\ n\in\mathbb{N}\end{cases}}{5}$ $\quad n\equiv 0(mod5)$

It is known that $n=\begin{cases}\frac{3n}{2} & n\equiv (mod2)\\ \frac{n-1}{2} & n\equiv 1(mod2)\end{cases}$ has three cycles and

{0}, {4, 6, 9}, {16, 24, 36, 54. 81, 40, 60, 90, 135, 67, 33}

Eg2:- when it is divided by 4 or 9

$$\frac{n=\begin{cases}\frac{3n}{2} & n\equiv 0(mod2)\\ \frac{n-1}{2} & n\equiv 1(mod2)\end{cases}\quad n\equiv 0(mod4)}{4}\leftrightarrow n=\begin{cases}\frac{3n}{2} & n\equiv 0(mod2)\\ 3n & n\equiv 1(mod4)\\ \frac{3n-1}{8} & n\equiv 3(mod8)\\ \frac{3n-1}{4} & n\equiv 15(mod16)\\ \frac{3n-5}{32} & n\equiv 23(mod32)\\ \frac{3n-25}{16} & n\equiv 39(mod64)\\ \frac{n-7}{64} & n\equiv 7(mod64)\end{cases}$$

4,6,9,27,10,15,11,4 this is cycle of 16 in original sequence after division by 4 and 1,3,1 map 4,6,9,4 after grounded and it is merged with third cycle. By doing the same grounding rule we can check the density of a non-trivial of Collatz is thousands of times or millions of times of trivial sequence

By principle it is supposed all cycles in Kaakuma sequence to have equivalent number of numbers of natural numbers when the limit is higher, to satisfy this the cycles or diverging sequence starts earlier. In 3n-1 sequence it has three portions and new cycles get earlier this solves complication and conflicts. Hence in 3n+1 problem new cycle will be after 100 digits of number with 100-digits number of iterations or after 20 digits of number if a new sequence diverges to infinite. How the density of this new sequence occupies thousands or millions of parts of the whole natural numbers?

i. If non-Collatz sequence is non-trivial sequence it divided by half first number it starts from the beginning with big circle to hold many numbers at the begging to be expanded with the same function that makes it thousands and millions time massive of trivial part Collatz sequence.
ii. When a new sequence diverges to infinite it can be trillions time trillions massive than trivial part of Collatz sequence Because each local peaks are new branches in inverse

tree that neve share nodes with other local peak that makes infinitely diverging sequence trillions time trillions massive than trivial part. and

4.4. Qodaa Ratio Test

Before starting we need to realize three points as much as Qodaa Ratio Test chronic solution of Collatz conjecture will supporting harder and adjacent functions in Kaakuma Sequence with infinite Examples.

I. Dynamic behavior of Collatz Sequence and probabilistic proof Collatz conjecture.

From thousands of studies on approaches on proof of Collatz conjectures different individuals interested on different approaches based on study are and level of understanding in addition to logical method and used tools of researcher. As I am amateur probabilistic approach is best for me even though the term probabilistic is inappropriate for the sequence and complicated way of getting probabilistic value ¾ that will be impossible for complicated sequences that we are going to see in Kaakuma sequence.

The terms dynamic and probabilistic are interrelated the existence of one arises the other on its background. This concept is related to interplay between determinism and free will in our lives falls within the realm of philosophy. Are all things beyond our determination dynamic? Is Collatz sequence partially or totally Dynamic. What about after determination still it is Dynamic or deterministic?

The dynamism of Collatz sequence based different want: - number of steps to reach 2, number of steps to reach less than starting number (stopping time in this study case), how many time up and how many times down successively, what number will occur after certain number of iterations, these can be determined by applying the behaviors seen in "c" and "f"

Eg. The stopping time of any number can be set and determined by t for a number in the form of $2^t k + c$ in the same way we can determined other by successive division of sequence function in required level even though the existence of limit in time and space. This shows Collatz sequence is not dynamic as written in many articles and papers.

$$n = \begin{cases} \dfrac{3n+1}{2} & n \equiv 1(mod2) \\ \dfrac{n}{2} & n \equiv 0(mod2) \end{cases}$$

This is to mean as we can determine the next value of odd and even like $\frac{3n+1}{2}$ and $\frac{n}{2}$ and the stopping time of even that is in first step or it is 1 we can determine also

$$n = \begin{cases} \frac{27n}{8} & n \equiv 0(mod8) \\ \frac{9n+4}{8} & n \equiv 4(mod8) \\ \frac{9n+6}{8} & n \equiv 2(mod8) \\ \frac{3n+6}{8} & n \equiv 6(mod8) \\ \frac{9n+9}{8} & n \equiv 7(mod8) \\ \frac{3n+7}{8} & n \equiv 3(mod8) \\ \frac{3n+9}{8} & n \equiv 5(mod8) \\ \frac{n+7}{8} & n \equiv 1(mod8) \end{cases}$$

Next value 8k, 8k+1, 8k+2 … 8k+7 as mentioned in equation and the stopping time of 8k+6, 8k+3, 8k+5, 8k+1 is 1 in the same we can break each line successively as much as required and we can apply the first rule 3n/2 and (n+1)/2 now the sequence's wanted points are determined.

By consequence of this the occurrence of even and odd are counted in each line and there is no probabilistic occurrence of even and odd number for any starting number in Collatz sequence. Hence, we cannot say Collatz conjecture is probably true.

On the other hand, what is its reflection and interpretation of probabilistic proof of Collatz conjecture with value ¾=0.75, what if its value changed 0.25 for weaker versions and 0.9999 for harder version. In six successive divisions the value becomes 0.75^{1024} that is less than 1.6^{-128} that is almost zero and what is its interpretation probabilistic approach. In other word the implication the probabilistic proof Collatz conjecture is to express the occurrences of odd number in infinite range is probably negligible. But this is false because of concepts based on proof 4.1 and it is well defined by **Qodaa Ratio Test** in deterministic way.

II. Formations and Views on Equivalent, Weaker and Harder Version of Collatz Conjecture. The most crucial thing to state Qodaa ratio test is form infinite number of equations that are distinct and similar in form of Collatz conjecture that can give us the patterns converging and converging sequences. At the same time, it guides us to compare which sequence is harder. To start this

$$n = \begin{cases} 3n+1 & n \equiv 1(mod2) \\ \frac{-n}{2} & n \equiv 0(mod2) \end{cases}$$ this converges to 1, 4, -2, 1 for all integers

when all positive integers derived from it

$$n = \begin{cases} 3n+1 & n \equiv 1(mod2) \\ \frac{3n-2}{4} & n \equiv 2(mod4) \\ \frac{n}{4} & n \equiv 0(mod4) \end{cases}$$ converges to 1 for all positive integers

Its reflection on y axis is

$$n = \begin{cases} 3n-1 & n \equiv 1(mod2) \\ \frac{3n+2}{4} & n \equiv 2(mod4) \\ \frac{n}{4} & n \equiv 0(mod4) \end{cases}$$ converges to 2 for all positive integers

$$n = \begin{cases} \frac{3n+1}{2} & n \equiv 1(mod2) \\ \frac{-n}{2} & n \equiv 0(mod2) \end{cases}$$ this converges to 0 for all integers

when all positive integers derived from it

$$n = \begin{cases} \frac{3n+1}{2} & n \equiv 1(mod2) \\ \frac{3n-2}{2} & n \equiv 2(mod4) \\ \frac{n}{4} & n \equiv 0(mod4) \end{cases}$$ converges to 2 for all positive integers

Its reflection on y axis is

$$n = \begin{cases} \frac{3n+1}{2} & n \equiv 1(mod2) \\ \frac{3n+2}{2} & n \equiv 2(mod4) \\ \frac{n}{4} & n \equiv 0(mod4) \end{cases}$$ converges to cycle 1,2,4 for all positive integers

C(n)/3 for n $n \equiv 0(mod3)$

$$n = \begin{cases} \frac{3n}{2} & n \equiv 2(mod2) \\ \frac{3n+1}{2} & n \equiv 1(mod4) \\ \frac{n+1}{4} & n \equiv 3(mod4) \end{cases}$$ converges to 1 for all positive integers

They are nearly similar and related and we can drive more similar sequences. When a coefficient increases by its base then it is harder version and when a coefficient decreased by it base it is weaker version.

$$n = \begin{cases} \frac{(k_1 - b_1)n + c_1}{b_1} & case1 \\ \frac{k_2 n + c_2}{b_2} & case2 \\ \frac{k_3 n + c_3}{b_3} & case3 \\ \vdots & \\ \frac{k_i n + c_i}{b_i} & casei \end{cases}$$ Weaker version and

$$n = \begin{cases} \frac{k_1 - n + c_1}{b_1} & case1 \\ \frac{k_2 n + c_2}{b_2} & case2 \\ \frac{k_3 n + c_3}{b_3} & case3 \\ \vdots & \\ \frac{k_i n + c_i}{b_i} & casei \end{cases}$$ Original version and

$$n = \begin{cases} \frac{(k_1 + b_1)n + c_1}{b_1} & case1 \\ \frac{k_2 n + c_2}{b_2} & case2 \\ \frac{k_3 n + c_3}{b_3} & case3 \\ \vdots & \\ \frac{k_i n + c_i}{b_i} & casei \end{cases}$$ Harder version

III. Theory of fairness

Even though there are some complicated equations to drive proportional occurrences of cases, in feasible conditions cases are produced and produce proportionally in long run. To consider this the case must be fully divided by base on a case. This is to mean that $\frac{3n+1}{8} \quad n \equiv 5(mod8)$ but not $\frac{3n+1}{4} \quad n \equiv 5(mod8)$ in this case the equation never neglects any case and produces each case in proportion. Its output of 8k, 8k+1, 8k+2, 8k+3, 8k+4, 8k+5, 8k+6, 8k+7 are proportional because behavior in 3.4, 3.5.1, 3.6 and proof 4.1. This leads to Qodaa Ratio Test.

Qodaa ratio test is the best clear cut to check if any Kaakuma sequence is divergent or convergent. It uses two principles that leads to calculate the ratio of product of numerators to product of denominators, evenly distribution of inverse Kaakuma sequence tree map and non-negligibility nature of decent share of iterations. And it helps to identify weaker and harder version of Collatz conjecture and it gives detail explanation of probabilistic approaches of Collatz conjecture.

$$n = \begin{cases} \dfrac{\pm k_1 n \pm c_1}{b_1} & case1 \\ \dfrac{\pm k_2 n \pm c_2}{b_2} & case2 \\ \dfrac{\pm k_3 n \pm c_3}{b_3} & case3 \\ \vdots & \\ \dfrac{\pm k_i n \pm c_i}{b_i} & casei \end{cases}$$

We can express it in integer values

$$n = \begin{cases} \dfrac{k_1 n + c_1}{b_1} & case1 \\ \dfrac{k_2 n + c_2}{b_2} & case2 \\ \dfrac{k_3 n + c_3}{b_3} & case3 \\ \vdots & \\ \dfrac{k_i n + c_i}{b_i} & casei \end{cases}$$

For all integers k, n, c, and b and k and b are not 0

Qodaa Ratio test states that if $\prod k_i < \prod b_i$ then the sequence never diverges to infinite or it does not have big trivial cycle more than 1000000. But it is not proved for the inverse proposition.

Kaakuma sequence have many categories among them we can check simple and complex

In simple Kaakuma sequence the number of cases is equal to base and the power of all cases are equal and each case produces approximately equal amount of all cases. The simplest form of Kaakuma sequence is Collatz sequence with base two.

Let us start the simple form of Kaakuma sequence from base two to base eight

Example 1, Base two: - $n = \begin{cases} \frac{kn+c}{2} & n \equiv 1(mod2) \\ \frac{n}{2} & n \equiv 0(mod2) \end{cases}$

to get all possible values of k we need to get powers of each case first, that can be done by producing amount and produced amount proportionality table.

produced	producing	
	A	B
a	a	b
b	a	b

We use letters as order of cases and we equate producing equal to produced. A produce a amount of odd in case 1 and a amount of even in case 2 and B produce b amount of odd in case 1 and b amount of even in case 2 when we equate producing equal to produced means horizontal line is equal to vertical line in each case.

$a + a = a + b \text{ that means } a = b \text{ in case} 1 \text{ and } b + b = b + a \text{ in case } 2 \text{ the same is } a = b$
this help us to get power in each

produced	producing					
	A	B	A	B	sum	Simplified
a	a	b	b	b	2b	1
b	a	b	b	b	2b	1

From Qodaa ratio test $\left(\frac{k}{2}\right)^1 \times \left(\frac{1}{2}\right)^1 < 1 \rightarrow \frac{k}{4} < 1 \rightarrow k < 4$ from this the absolute value of k can be 1, 2 or 3 the sequence not to be divergent or not have big cycle. We can check constant term in -k and k and k can be factorized and distributed in cases.

Therefore $n = \begin{cases} \frac{3n+1}{2} & n \equiv 1(mod2) \\ \frac{n}{2} & n \equiv 0(mod2) \end{cases}$ has only one cycle and with, ratio 0.75

And $n = \begin{cases} \frac{3n-1}{2} & n \equiv 1(mod2) \\ \frac{n}{2} & n \equiv 0(mod2) \end{cases}$ has only three cycles

Example 2, Base three: - $n = \begin{cases} \frac{kn+c}{3} & n \equiv 2(mod3) \\ \frac{n+2}{3} & n \equiv 1(mod3) \\ \frac{n}{3} & n \equiv 0(mod3) \end{cases}$

With the same principle above

produced	producing							
	A	B	C	A	B	C	sum	Simplified
a	a	b	c	c	c	3c	3c	1
b	a	b	c	c	c	c	3c	1
c	a	b	c	c	c	c	3c	1

When we use Qodaa ratio test rule $\left(\frac{k}{3}\right)^1 \times \left(\frac{1}{3}\right)^1 \times \left(\frac{1}{3}\right)^1 < 1 \rightarrow \frac{k}{27} < 1 \rightarrow k < 27$ from this k can be integers between -26 and 26 except 0 the sequence not to be divergent or not have big cycle.

$$n = \begin{cases} \frac{26n-25}{3} & n \equiv 2(mod3) \\ \frac{n+2}{3} & n \equiv 1(mod3) \\ \frac{n}{3} & n \equiv 0(mod3) \end{cases} \quad \text{converges to 1 for starting numbers } n \in \mathbb{N}$$

with Qodaa ratio 0.962962…

$$n = \begin{cases} \frac{n+1}{3} & n \equiv 2(mod3) \\ \frac{26n-20}{3} & n \equiv 1(mod3) \\ \frac{n}{3} & n \equiv 0(mod3) \end{cases} \quad \text{converges to 1 for starting numbers } n \in \mathbb{N}$$

$$n = \begin{cases} \frac{n+1}{3} & n \equiv 2(mod3) \\ \frac{26n-1}{3} & n \equiv 1(mod3) \\ \frac{n}{3} & n \equiv 0(mod3) \end{cases} \quad \text{converges to 1 for starting numbers } n \in \mathbb{N}$$

$$n = \begin{cases} \frac{n+1}{3} & n \equiv 2(mod3) \\ \frac{20n+2}{3} & n \equiv 1(mod3) \\ \frac{n}{3} & n \equiv 0(mod3) \end{cases} \quad \text{converges to 1 for starting numbers } n \in \mathbb{N}$$

$$n = \begin{cases} \frac{n+1}{3} & n \equiv 2(mod3) \\ \frac{20n-2}{3} & n \equiv 1(mod3) \\ \frac{n}{3} & n \equiv 0(mod3) \end{cases} \quad \text{converges to 1 for starting numbers } n \in \mathbb{N}$$

Example 3, Base four: - $n = \begin{cases} \frac{255n-261}{4} & n \equiv 3(mod4) \\ \frac{n+2}{4} & n \equiv 2(mod4) \\ \frac{n+3}{4} & n \equiv 1(mod4) \\ \frac{n}{4} & n \equiv 0(mod4) \end{cases}$ converges to 1 with Qodaa ratio 255/256=0.996

We can compare this with original Collatz sequence after first Division

$n = \begin{cases} \frac{9n}{4} & n \equiv 0(mod4) \\ \frac{3n+2}{4} & n \equiv 2(mod4) \\ \frac{3n+3}{4} & n \equiv 3(mod4) \\ \frac{n+3}{4} & n \equiv 1(mod4) \end{cases}$ converges to 2 or 3 with Qodaa ratio 81/256=0.3045

The difference in product of their coefficients 255-81=164 and the base is increased 81 times that is much harder.

Base five: - $n = \begin{cases} \frac{3124n-313}{5} & n \equiv 4(mod5) \\ \frac{n+2}{5} & n \equiv 3(mod5) \\ \frac{n+3}{5} & n \equiv 2(mod5) \\ \frac{n+4}{5} & n \equiv 1(mod5) \\ \frac{n}{5} & n \equiv 0(mod5) \end{cases}$ converges to 1 with Qodaa ratio 0.99968

Base six: - $n = \begin{cases} \frac{46655n-46657}{6} & n \equiv 5(mod6) \\ \frac{n+2}{6} & n \equiv 4(mod6) \\ \frac{n+3}{6} & n \equiv 3(mod6) \\ \frac{n+4}{6} & n \equiv 2(mod6) \\ \frac{n+5}{6} & n \equiv 1(mod6) \\ \frac{n}{6} & n \equiv 0(mod6) \end{cases}$ converges to 1 Qodaa ratio= 0.999978

Base seven: - $n = \begin{cases} \frac{823542n-4200008}{7} & n \equiv 6(mod7) \\ \frac{n+2}{7} & n \equiv 5(mod7) \\ \frac{n+3}{7} & n \equiv 4(mod7) \\ \frac{n+4}{6} & n \equiv 3(mod7) \\ \frac{n+5}{7} & n \equiv 2(mod7) \\ \frac{n+6}{7} & n \equiv 1(mod7) \\ \frac{n}{7} & n \equiv 0(mod7) \end{cases}$ converges to 1, Qodaa ratio=0.999998

Base eight: - $n = \begin{cases} \frac{16777215n-1164404}{8} & n \equiv 7(mod8) \\ \frac{n+2}{8} & n \equiv 6(mod8) \\ \frac{n+3}{8} & n \equiv 5(mod8) \\ \frac{n+4}{8} & n \equiv 4(mod8) \\ \frac{n+5}{8} & n \equiv 3(mod8) \\ \frac{n+6}{8} & n \equiv 2(mod8) \\ \frac{n+7}{8} & n \equiv 1(mod8) \\ \frac{n}{8} & n \equiv 0(mod8) \end{cases}$ converges to 1, Qodaa ratio = $\frac{16777215}{16777216}$

We can also compare this original Collatz sequence after second division.

$n = \begin{cases} \frac{27n}{8} & n \equiv 0(mod8) \\ \frac{9n+4}{8} & n \equiv 4(mod8) \\ \frac{9n+6}{8} & n \equiv 2(mod8) \\ \frac{3n+6}{8} & n \equiv 6(mod8) \\ \frac{9n+9}{8} & n \equiv 7(mod8) \\ \frac{3n+7}{8} & n \equiv 3(mod8) \\ \frac{3n+9}{8} & n \equiv 5(mod8) \\ \frac{n+7}{8} & n \equiv 1(mod8) \end{cases}$ converges to 2,3 cycle Qodaa ratio = 0.0317

Complex Kaakuma sequence

In complex Kaakuma sequence part of bases are powered and producing amount and produced amount are not equal in some cases a case may not produce another case completely and number cases are greater than base.

Base two partially with sub cases: - $n = \begin{cases} \frac{n}{2} & n \equiv 0(mod2) \\ \frac{kn+c}{4} & n \equiv 3(mod4) \\ \frac{n+3}{4} & n \equiv 1(mod4) \end{cases}$

In this case base is two and number of line is 3 and producing power is depend on ratio of each case in natural number that means there exist half of even, quarter of 4k+1, and quarter of 4k+3 numbers in natural number each produces with ratios 1/2:1/4:1/4 when we change it in natural number form 2:1:1 we can take this as cases power directly $\left(\frac{1}{2}\right)^2 \times \left(\frac{k}{4}\right)^1 \times \left(\frac{1}{4}\right)^1 = k/64$

We use qooda ratio test to get values of k, k/64<1 1<k<64 for positive integer values of k

We use tabular for to get powers of each case that will be more useful on complicated cases.

Base two partially with sub cases: - $n = \begin{cases} \frac{n}{2} & n \equiv 0(mod2) \\ \frac{63n-9}{4} & n \equiv 3(mod4) \\ \frac{n+3}{4} & n \equiv 1(mod4) \end{cases}$

produced	producing							
	A	B	C	A	B	C	sum	Simplified
a	2a	2b	2c	4c	2c	2c	8c	2
b	a	b	c	2c	c	c	4c	1
c	a	b	c	2c	c	c	4c	1

When we equate them producing and produced in cases

a=b+c, 3b=a+c, 3c=a+b from this b=c and a=2c

$n = \begin{cases} \frac{n}{2} & n \equiv 0(mod2) \\ \frac{63n-59}{4} & n \equiv 1(mod4) \\ \frac{n+1}{4} & n \equiv 3(mod4) \end{cases}$ converges to 1 for all natural numbers

Base two partially with sub of sub cases : - $n = \begin{cases} \frac{n}{2} & n \equiv 0(mod2) \\ \frac{n+1}{4} & n \equiv 3(mod4) \\ \frac{n+7}{8} & n \equiv 1(mod8) \\ \frac{kn+c}{8} & n \equiv 5(mod8) \end{cases}$

The ratio of cases 1/2 : 1/4 : 1/8 : 1/8 : when we put in positive integer form 4:2:1:1

$$\left(\frac{1}{2}\right)^4 \times \left(\frac{1}{4}\right)^2 \times \left(\frac{1}{8}\right)^1 \times \left(\frac{k}{8}\right)^1 = \frac{k}{16384} \; using\ qooda\ ratio\ test\ 1 < k < 1684$$

$$n = \begin{cases} \frac{n}{2} & n \equiv 0(mod2) \\ \frac{n+1}{4} & n \equiv 3(mod4) \\ \frac{n+7}{8} & n \equiv 1(mod8) \\ \frac{16383n-81907}{8} & n \equiv 5(mod8) \end{cases} \quad \text{converges to 1}$$

If we set k in line 2 the product of coefficients value differs because of difference in power

$$n = \begin{cases} \frac{n}{2} & n \equiv 0(mod2) \\ \frac{kn+c}{4} & n \equiv 3(mod4) \\ \frac{n+7}{8} & n \equiv 1(mod8) \\ \frac{n+3}{8} & n \equiv 5(mod8) \end{cases}$$

$$\left(\frac{1}{2}\right)^4 \times \left(\frac{k}{4}\right)^2 \times \left(\frac{1}{8}\right)^1 \times \left(\frac{1}{8}\right)^1 = \frac{k^2}{16384} \; using\ qooda\ ratio\ test\ 1 < k < 128$$

$$n = \begin{cases} \frac{n}{2} & n \equiv 0(mod2) \\ \frac{127n-369}{4} & n \equiv 3(mod4) \\ \frac{n+7}{8} & n \equiv 1(mod8) \\ \frac{n+3}{8} & n \equiv 5(mod8) \end{cases} \quad \text{converges to 1}$$

On the same way if we set k in line 1

$$n = \begin{cases} \frac{kn+c}{2} & n \equiv 0(mod2) \\ \frac{n+1}{4} & n \equiv 3(mod4) \\ \frac{n+7}{8} & n \equiv 1(mod8) \\ \frac{n+3}{8} & n \equiv 5(mod8) \end{cases}$$

$$\left(\frac{k}{2}\right)^4 \times \left(\frac{1}{4}\right)^2 \times \left(\frac{1}{8}\right)^1 \times \left(\frac{1}{8}\right)^1 = \frac{k^4}{16384}\ using\ qooda\ ratio\ test\ 1 < k < \sqrt{128}$$

$$n = \begin{cases} \frac{11n-2}{2} & n \equiv 0(mod2) \\ \frac{n+1}{4} & n \equiv 3(mod4) \\ \frac{n+7}{8} & n \equiv 1(mod8) \\ \frac{n+3}{8} & n \equiv 5(mod8) \end{cases}$$ converges to 1

Base two partially with sub of sub cases : - $n = \begin{cases} \frac{n}{2} & n \equiv 0(mod2) \\ \frac{n+7}{8} & n \equiv 1(mod8) \\ \frac{n+5}{8} & n \equiv 3(mod8) \\ \frac{n+3}{8} & n \equiv 5(mod8) \\ \frac{kn+c}{8} & n \equiv 7(mod8) \end{cases}$

With ratio 1/2 : 1/8 : 1/8: 1/8 : 1/8 : when we put in positive integer form 4:1:1:1:1

$$\left(\frac{1}{2}\right)^4 \times \left(\frac{1}{8}\right)^2 \times \left(\frac{1}{8}\right)^1 \times \left(\frac{1}{8}\right)^1 \times \left(\frac{k}{8}\right)^1 = \frac{k}{65536}\ using\ qooda\ ratio\ test\ 1 < k < 65536$$

- $n = \begin{cases} \frac{n}{2} & n \equiv 0(mod2) \\ \frac{n+7}{8} & n \equiv 1(mod8) \\ \frac{n+5}{8} & n \equiv 3(mod8) \\ \frac{n+3}{8} & n \equiv 5(mod8) \\ \frac{65535n-42596}{8} & n \equiv 7(mod8) \end{cases} \quad converges\ to\ 1$

When we move the coefficient in first line

$$\left(\frac{k}{2}\right)^4 \times \left(\frac{1}{8}\right)^2 \times \left(\frac{1}{8}\right)^1 \times \left(\frac{1}{8}\right)^1 \times \left(\frac{1}{8}\right)^1 = \frac{k^4}{65536}\ \textit{using qooda ratio test}\ 1 < k < 16$$

$$n = \begin{cases} \frac{n-4}{2} & n \equiv 0 (mod 2) \\ \frac{n+7}{8} & n \equiv 1 (mod 8) \\ \frac{n+5}{8} & n \equiv 3 (mod 8) \\ \frac{n+3}{8} & n \equiv 5 (mod 8) \\ \frac{n+1}{8} & n \equiv 7 (mod 8) \end{cases} \quad \textit{converges to}\ 1$$

$$n = \begin{cases} \frac{n}{2} & n \equiv 0 (mod 2) \\ \frac{n+1}{4} & n \equiv 3 (mod 4) \\ \frac{n+7}{8} & n \equiv 1 (mod 8) \\ \frac{n+11}{16} & n \equiv 5 (mod 16) \\ \frac{n+3}{16} & n \equiv 13 (mod 16) \end{cases}$$

With ratio 1/2: 1/4: 1/8: 1/16: 1/16: when we put in positive integer form 8:4:2: 1:1

$$\left(\frac{1}{2}\right)^8 \times \left(\frac{1}{4}\right)^4 \times \left(\frac{1}{8}\right)^2 \times \left(\frac{1}{16}\right)^1 \times \left(\frac{k}{16}\right)^1 = \frac{k}{2^{30}}\ \textit{using qooda ratio test}\ 1 < k$$

$$\begin{cases} \frac{n}{2} & n \equiv 0 (mod 2) \\ \frac{n+1}{4} & n \equiv 3 (mod 4) \\ \frac{n+7}{8} & n \equiv 1 (mod 8) \\ \frac{n+11}{16} & n \equiv 5 (mod 16) \\ \frac{kn+c}{16} & n \equiv 13 (mod 16) \end{cases}$$

With ratio 1/2:1/4: 1/8: 1/16: k/16 in integer form 8:4:2:1:1

$$\left(\frac{1}{2}\right)^8 \times \left(\frac{1}{4}\right)^4 \times \left(\frac{1}{8}\right)^2 \times \left(\frac{1}{16}\right)^1 \times \left(\frac{k}{16}\right)^1 = \frac{k}{2^{30}} \rightarrow 1 < k < 2^{30}$$

$$n = \begin{cases} \frac{n}{2} & n \equiv 0(mod2) \\ \frac{n+1}{4} & n \equiv 3(mod4) \\ \frac{n+7}{8} & n \equiv 1(mod8) \\ \frac{n+11}{16} & n \equiv 5(mod16) \\ \frac{(2^{30}-1)n-13\times(2^{30}-1)+32}{16} & n \equiv 13(mod16) \end{cases} \quad \text{Converges to 1}$$

When we shift k in line 3

$$\left(\frac{1}{2}\right)^8 \times \left(\frac{1}{4}\right)^4 \times \left(\frac{k}{8}\right)^2 \times \left(\frac{1}{16}\right)^1 \times \left(\frac{k}{16}\right)^1 = \frac{k^2}{2^{30}} \to 1 < k^2 < 2^{30} \to 1 < k < 2^{15}$$

$$n = \begin{cases} \frac{n}{2} & n \equiv 0(mod2) \\ \frac{n+1}{4} & n \equiv 3(mod4) \\ \frac{(2^{15}-1)n-(2^{15}-1)+16}{8} & n \equiv 1(mod8) \quad \textit{converges to } 1 \\ \frac{n+11}{16} & n \equiv 5(mod16) \\ \frac{n+3}{16} & n \equiv 13(mod16) \end{cases}$$

When we shift k in line 2

$$\left(\frac{1}{2}\right)^8 \times \left(\frac{k}{4}\right)^4 \times \left(\frac{1}{8}\right)^2 \times \left(\frac{1}{16}\right)^1 \times \left(\frac{k}{16}\right)^1 = \frac{k^4}{2^{30}} \to 1 < k^4 < 2^{30} \to 1 < k < 2^{15/2}$$

$$n = \begin{cases} \frac{n}{2} & n \equiv 0(mod2) \\ \frac{181n-535}{4} & n \equiv 3(mod4) \\ \frac{n+7}{8} & n \equiv 1(mod8) \\ \frac{n+11}{16} & n \equiv 5(mod16) \\ \frac{n+3}{16} & n \equiv 13(mod16) \end{cases}$$

When we shift k in line 2

$$\left(\frac{1}{2}\right)^8 \times \left(\frac{k}{4}\right)^4 \times \left(\frac{1}{8}\right)^2 \times \left(\frac{1}{16}\right)^1 \times \left(\frac{k}{16}\right)^1 = \frac{k^4}{2^{30}} \to 1 < k^4 < 2^{30} \to 1 < k < 2^{15/2}$$

$$n = \begin{cases} \frac{n}{2} & n \equiv 0(mod2) \\ \frac{181n-535}{4} & n \equiv 3(mod4) \\ \frac{n+7}{8} & n \equiv 1(mod8) \text{ converges to 1} \\ \frac{n+11}{16} & n \equiv 5(mod16) \\ \frac{n+3}{16} & n \equiv 13(mod16) \end{cases}$$

When we shift k in line 1

$$\left(\frac{k}{2}\right)^{8} \times \left(\frac{1}{4}\right)^{4} \times \left(\frac{1}{8}\right)^{2} \times \left(\frac{1}{16}\right)^{1} \times \left(\frac{k}{16}\right)^{1} = \frac{k^4}{2^{30}} \to 1 < k^8 < 2^{30} \to 1 < k < 2^{15/4}$$

$$n = \begin{cases} \frac{13n-4}{2} & n \equiv 0(mod2) \\ \frac{n+1}{4} & n \equiv 3(mod4) \\ \frac{n+7}{8} & n \equiv 1(mod8) \\ \frac{n+11}{16} & n \equiv 5(mod16) \\ \frac{n+3}{16} & n \equiv 13(mod16) \end{cases}$$

Complicated cases: in complicated cases a case never produce at lest one case.

Base two partially with sub cases: - $n = \begin{cases} \frac{n}{2} & n \equiv 0(mod2) \\ \frac{kn+c}{4} & n \equiv 3(mod4) \\ \frac{n+1}{2} & n \equiv 1(mod4) \end{cases}$

In this case it is impossible to take simply coefficients to get ratio like

1/2:1/4:1/2 in integer form 2:1:2

$$\left(\frac{1}{2}\right)^{2} \times \left(\frac{k}{4}\right)^{1} \times \left(\frac{1}{2}\right)^{2} = \frac{k^1}{2^6} \to 1 < k^1 < 2^6 \to 1 < k < 2^6 \ this\ is\ false$$

The right method is using producing and produced amount equality of each case.

produced	producing							
	A	B	C	A	B	C	sum	Simplified
a	2a	2b	0	2a	2a	0	4a	1
b	a	b	c	a	a	2a	4a	1
c	a	b	c	a	a	2a	4a	1

2a=2b, 3b=a+c, c=a+b → a=b, c=2a

$$\left(\frac{1}{2}\right)^1 \times \left(\frac{k}{4}\right)^1 \times \left(\frac{1}{2}\right)^1 = \frac{k^1}{2^4} \rightarrow 1 < k^1 < 2^4 \rightarrow 1 < k < 2^4$$

- $n = \begin{cases} \frac{n}{2} & n \equiv 0(mod2) \\ \frac{15n-37}{4} & n \equiv 3(mod4) \\ \frac{n+1}{2} & n \equiv 1(mod4) \end{cases}$ converges to 1

$$n = \begin{cases} \frac{n}{2} & n \equiv 0(mod2) \\ \frac{n+1}{4} & n \equiv 3(mod4) \\ \frac{n+3}{4} & n \equiv 1(mod8) \\ \frac{kn+c}{8} & n \equiv 5(mod8) \end{cases}$$

produced	producing									
	A	B	C	D	A	B	C	D	sum	Simplified
a	4a	4b	0	4d	12d	8d	0	4d	24d	3
b	2a	2b	2c	2d	6d	4d	4d	2d	16d	2
c	a	b	c	d	3d	2d	2d	d	8d	1
d	a	b	c	d	3d	2d	2d	d	8d	1

a=b+d,
3b=a+c+d
3c=a+b+d
7d=a+b+c, from this c=2d, b=2d, a=3d

$$\left(\frac{1}{2}\right)^3 \times \left(\frac{1}{4}\right)^2 \times \left(\frac{1}{4}\right)^1 \times \left(\frac{k}{8}\right)^1 = \frac{k^1}{2^{12}} \rightarrow 1 < k^1 < 2^{12} \rightarrow 1 < k < 2^{12}$$

$$n = \begin{cases} \frac{n}{2} & n \equiv 0(mod2) \\ \frac{n+1}{4} & n \equiv 3(mod4) \\ \frac{n+3}{4} & n \equiv 1(mod8) \\ \frac{4095n-20459}{8} & n \equiv 5(mod8) \end{cases} \quad converges\ to\ 1$$

When we shift k in line 2

$$\left(\frac{1}{2}\right)^3 \times \left(\frac{k}{4}\right)^2 \times \left(\frac{1}{4}\right)^1 \times \left(\frac{1}{8}\right)^1 = \frac{k^2}{2^{12}} \rightarrow 1 < k^2 < 2^{12} \rightarrow 1 < k < 2^6$$

$$n = \begin{cases} \dfrac{n}{2} & n \equiv 0(mod2) \\ \dfrac{63n-181}{4} & n \equiv 3(mod4) \\ \dfrac{n+3}{4} & n \equiv 1(mod8) \\ \dfrac{n+3}{8} & n \equiv 5(mod8) \end{cases} \quad converges\ to\ 1$$

When we shift k in line 1

$$\left(\frac{k}{2}\right)^{3} \times \left(\frac{1}{4}\right)^{2} \times \left(\frac{1}{4}\right)^{1} \times \left(\frac{1}{8}\right)^{1} = \frac{k^3}{2^{12}} \rightarrow 1 < k^3 < 2^{12} \rightarrow 1 < k < 2^4$$

$$n = \begin{cases} \dfrac{15n-4}{2} & n \equiv 0(mod2) \\ \dfrac{n+1}{4} & n \equiv 3(mod4) \\ \dfrac{n+3}{4} & n \equiv 1(mod8) \\ \dfrac{n+3}{8} & n \equiv 5(mod8) \end{cases} \quad converges\ to\ 1$$

$$n = \begin{cases} \frac{n}{2} & n \equiv 0(mod2) \\ \frac{n-1}{2} & n \equiv 3(mod4) \\ \frac{n+1}{2} & n \equiv 1(mod8) \\ \frac{kn+c}{8} & n \equiv 5(mod8) \end{cases}$$

produced	producing									
	A	B	C	D	A	B	C	D	sum	Simplified
a	4a	0	0	4d	4d	0	0	4d	8d	1
b	2a	2b	0	2d	2d	4d	0	2d	8d	1
c	a	b	c	d	d	2d	4d	d	8d	1
d	a	b	c	d	d	2d	4d	d	8d	1

a=d,
b=a+d, b=2d
c=a+b+d, c=2b
7d=a+b+c, from this c=4d, b=2d, a=d

$$\left(\frac{1}{2}\right)^{1} \times \left(\frac{1}{2}\right)^{1} \times \left(\frac{1}{2}\right)^{1} \times \left(\frac{k}{8}\right)^{1} = \frac{k^1}{2^6} \rightarrow 1 < k^1 < 2^6 \rightarrow 1 < k < 2^6$$

$$n = \begin{cases} \frac{n}{2} & n \equiv 0(mod2) \\ \frac{n-1}{2} & n \equiv 3(mod4) \\ \frac{n+1}{2} & n \equiv 1(mod8) \\ \frac{55n+1}{8} & n \equiv 5(mod8) \end{cases} \quad converges\ to\ 1$$

$$n = \begin{cases} \frac{n}{2} & n \equiv 0(mod2) \\ \frac{n-1}{2} & n \equiv 3(mod8) \\ \frac{n-5}{2} & n \equiv 7(mod8) \\ \frac{n+1}{2} & n \equiv 1(mod8) \\ \frac{kn+c}{8} & n \equiv 5(mod8) \end{cases}$$

produced	producing											
	A	B	C	D	E	A	B	C	D	E	sum	Simplified
a	4a	0	0	0	4e	4e	0	0	0	4e	8e	4
b	a	0	0	0	e	e	0	0	0	e	2e	1
c	a	0	0	0	e	e	0	0	0	e	2e	1
d	a	b	c	d	e	e	e	e	4e	e	8e	4
e	e	b	c	d	e	e	e	e	4e	e	8e	4

a=b+a=e
2b=a+e
2c=a+e
d=a+b+c+e
7e=a+b+c+d, from this a=b=c=e, d=4e

$$\left(\frac{1}{2}\right)^4 \times \left(\frac{1}{2}\right)^1 \times \left(\frac{1}{2}\right)^1 \times \left(\frac{1}{2}\right)^4 \times \left(\frac{k}{8}\right)^4 = \frac{k^4}{2^{22}} \rightarrow 1 < k^4 < 2^{22} \rightarrow 1 < k < 2^{5.5}$$

$$n = \begin{cases} \frac{n}{2} & n \equiv 0(mod2) \\ \frac{n-1}{2} & n \equiv 3(mod8) \\ \frac{n-5}{2} & n \equiv 7(mod8) \\ \frac{n+1}{2} & n \equiv 1(mod8) \\ \frac{45n-3}{8} & n \equiv 5(mod8) \end{cases} \quad \text{converges to1}$$

$$n = \begin{cases} \dfrac{n}{2} & n \equiv 0(mod2) \\ \dfrac{n+1}{4} & n \equiv 3(mod4) \\ \dfrac{\mathrm{kn} + \mathrm{c}}{2} & n \equiv 1(mod4) \end{cases}$$

produced	producing							
	A	B	C	A	B	C	sum	Simplified
a	2a	2b	0	2a	2a	0	4a	2
b	a	b	c	a	a	2a	4a	2
c	a	b	0	a	a	0	2a	1

a=b

3b=a+c

c=a+b

c=2a=2b

$$\left(\frac{1}{2}\right)^2 \times \left(\frac{1}{4}\right)^2 \times \left(\frac{\mathrm{k}}{2}\right)^1 = \frac{k^1}{2^7} \rightarrow 1 < k^1 < 2^7 \rightarrow 1 < k < 2^7$$

$$\mathrm{n} = \begin{cases} \frac{n}{2} & n \equiv 0(mod2) \\ \frac{n+1}{4} & n \equiv 3(mod4) \\ \frac{126n-120}{2} & n \equiv 1(mod4) \end{cases}$$ converges to1

all this different kind examples shows how Qodaa Ratio Test Works even in complicated equations.

Qodaa ratio test is simpler than others by showing figurative values. But we can get for all proofs exact figurative values by using maxima, minimum and other tools even though they more time and space.

References

1. J. Lagarias. The 3x + 1 Problem: An Annotated Bibliography(1963–1999). Department of Mathematics,University of Michigan Ann Arbor,(2011), 48109–1109, lagarias@umich.edu.
[2] J. Lagarias. The 3x + 1 Problem: An Annotated Bibliography, II (2000–2009). Department of Mathematics,University of Michigan Ann Arbor,(2011), 48109–1109, lagarias@umich.edu.
3. Tao, T. Almost all orbits of the Collatz map attain almost bounded values. arXiv **2019**, arXiv:1909.03562v3
4. Andrei, S.; Kudlek, M.; Niculescu, R.S. Chains in Collatz's Tree; Technical Report; Department of Informatics, Universität Hamburg: Hamburg, Germany, 1999.
5 J.C. Lagarias (2010) “The 3x+1 problem: An overview”, The Ultimate Challenge: The 3x+1 problem:
3–29.