Unified Field Theory
by Canonical Gauge Principle

Unified Field Theory by Canonical Gauge Principle. (2024.04)

【preface】

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First of all, I would like to express my gratitude for the good fortune of having the opportunity to study physics. Although I did not come from a particularly rich cultural environment, I was fortunate enough to receive higher education.
After graduating from graduate school, I learned about the laws that govern this world through physics, and had some opportunities to develop my own thoughts and ideas, and these have enriched my life.

This book is the most important work for me, written with the purpose of asking the world about the "unified field theory" that I have been thinking about until now.
The Unified Field Theory presented here takes a fundamentally different approach than Superstring Theory.
The approaches and orientation of thought to understand the nature, like Superstring Theory seems not be appropriate in my view. Also I am concerned that academic societies focusing solely on Superstring Theory and not exploring other possibilities might be becoming authoritarian.

The theory of unified field can be, and should be, constructed more simply by changing the concept of spacetime and by extending the gauge group to the canonical transformations, based on fundamental principles of Quantum Theory. This book is a trajectory of such efforts.

I wonder why it does not be attempted to put quantum structure to the spacetime itself according to the principles suggested by Quantum Theory.

Moreover, the quantum spacetime considered here is an example that could be called the most elementary and simple quantum manifold associated with the translation group as the groups describing the motion of particles, and I expect that this idea may lead to a new geometry.

This time I tried to take a style in which the detailed arguments in the inference process and matters that require attention will be concentrated on footnotes and the appendices. At the same time, it is taken account that the description should not just be a story for the reader, but should provide information and traceability that can withstand verification and recalculation for checking.
Also I aimed to make the content structure and flow of arguments easy to understand, and allow readers to pick out and read only the parts they are interested in.

The Theory of Unified Field in this book consists of the 3+1 parts shown below:

Ⅰ. Principle of Unified Field Theory and Reasons of the Existence of Elementary Particles.
 (Spacetime, Field, Symmetry)
Ⅱ. State-Constructive Formalism of Field Theory.
Ⅲ. Canonical Gauge Gravitational Theory.
Ⅳ. Conceptual Trajectory & Afterword(Thanks).

In Part I, we extract the principles suggested by quantum theory, and revise the traditional concepts of spacetime with the quantization of spacetime in mind. As a result, we obtain a gauge theory that extends

group of coordinate transformations in A. Einstein's general relativity to the spacetime-preserving canonical transformation group.
The concepts of quantum spacetime lead to findings/insights into the following matters:

-1.Reasons for the existence of fields, understanding of the derivation principles of field equations and of the unified formulation

-2.Unification of Bosons and Fermions based on the reasons for the existence of elementary particles

-3.Existence of spinor connection fields and mass mechanisms.

-4.Construction of Unified Field equations and Dirac equations, relationship between mass and chirality.

-5.Origin of symmetry of interactions.

-6.Reality of preon models and the generation problem

From the above, field of Fermions and Bosons including Gravity are unified and the field equations governing them are derived in a unified manner.
Notably, the existence of spinor connection fields as the 5th interaction should be remarked. This seems related to the Higgs field.
It also shows that Majorana particles do not exist and quarks/leptons exist only up to the 3rd generation.

Part I　Keywords:
Quantum spacetime, canonical gauge ring, canonical connection form, canonical curvature, spinor connection field
Preon, equations for Unified Field Theory, chirality and mass mechanism
(See Supplementary note 1)

In Part II, we analyze the basic concepts of quantum theory：redefining the concept of quantum theoretical state, to confirm the tensor algebra on the state space, denying the adiabatic hypothesis/free field concept, constructing a state-explicit theoretical formalism corresponding to this and discuss the approximate solution method based on the variational principle.

In Part I, we introduced the "unified field equation", but even if we can correctly express the interaction, without a general method to solve them, the subsequent development and expansion of the theory would be difficult. Therefore, the development of a general approximate solution method with a mechanism to improve the accuracy, seems essential.

In traditional equation solution methods, the "divergence difficulty" is an obstacle. As a method to remove divergence, there is "renormalization", but this is not a method introduced on principle. It is not a logical consequence as a solution method either. Therefore, I do not think it provides a fundamental solution.

In relation to avoiding the "divergence difficulty", I first think that the adiabatic hypothesis/free field concept should be denied.
This is related to self-interaction. According to the traditional idea, stably existing particles are in a steady interaction process with interaction particles that they themselves absorb/emit, and are assumed

to be in a so-called dressed state. However, according to this, since the interaction always exists, the adiabatic hypothesis and free field concepts are already logically broken here.
Even so, some results can be expected by applying a theory that regards a composite system as a single particle.
In renormalization techniques discovered in QED, the divergence terms consist of several fundamental divergences and this fact is quite suggestive in this regard.

Considering this and the observed fact that quark does not exist alone, It is imagined that Fermion has a 3-color interaction, that has no steady solution unless it is a color- neutral composite system such as an RGB mixture. Except for color-neutral state, the steady solution cannot exist, in the sense that it diverges.
And it seems unreasonable to construct a composite system from the free field using perturbation theory.

Now, abandoning the free field concept/adiabatic hypothesis requires a different calculation method than the traditional diagram technique for calculating transition matrix elements.
Therefore, by introducing an approximate form that explicitly expresses the state on the state space, it is conceivable to construct an approximate solution using the variational method in a finite element manner.
This is based on the idea of the "finite element method" used in engineering fields, but with this method, the state of the system can be expressed as the existing approximation + correction term to construct a successive approximation method.
The equation to be solved is generally an infinite-variable integro-differential equation , but the above-mentioned finite-mode excitation approximation makes it an integro-differential equation with a finite number of oscillation modes as variables.

Ingenuity can also be taken regarding the form of the solution for state. Let us remember the separation of variables method as a solution method for partial differential equations, and by assuming a direct product structure as a tensor for the state of the system, a "state separation method" can be introduced.
By imposing some limitations on the state separation method, quantum mechanical equations appear as approximations, and it should be noted that they do not have the divergence difficulty.
From this fact, since there are no divergent elements in individual approximations, it seems that divergence difficulties can be avoided as a whole.
Even so, advancing the successive approximation, the possibility of divergence cannot be ruled out, but in such cases, it can be assumed that finite terminating conditions like quantization conditions are hidden.

Keywords for Part II:
State space, field operators, divergence difficulty, adiabatic hypothesis/free field, self-interaction, renormalization, scattering problem, steady state problem, state-constructive field theory, state separation method, finite mode excitation
(See Supplementary note 2)

In Part III, we consider the theory of gravity in the theory of unified field.
In unified field theory based on the canonical gauge principle, the "gravitational field" manifests differently from Bosons that mediate interactions between ordinary matter.
As already understood from the introduction of the unified field in Part I, the field variables of unified field gravity originate from the relationship between the Lorentz frame and canonical momentum operators in quantum theoretical spacetime.
They are obtained in a form corresponding to the so-called tetrad(4-frame field), relating to the spacetime metric similar to A. Einstein's theory of gravity, but also containing degrees of freedom related to spin angular momentum density.

While the same results as A. Einstein's theory are derived for light paths and classical particle motions when assuming the presence of a gravitational field, the equation is essentially different from A.Einstein's one in the generation of gravitational field. And in general, the field variables cannot be reduced to the spacetime metric $g_{\mu\nu}$.
The Lagrangian is given by a scalar that is a quadratic form in the 1-st derivatives of the field variables, with multiple theoretically possible gravitational constants (interaction constant) appearing.
However, remarkably, under the assumption of spherical symmetry, if the gravitational constant is chosen to have a certain value, the unified field gravity theory is shown to agree with the A. Einstein theory.
Since A. Einstein's theory has already obtained experimental verification in terms of classical effects, this fact serves as verification of unified field gravity in the classical regime, at least for spherically symmetric fields.

From the above, as long as we assume spacetime states where only spherically symmetric states are excited, A. Einstein's theory can be interpreted as the mean field theory of canonical gauge gravity.
Classical Boson fields are considered to correspond to quantum theoretical expectations of field operators, and in general, the results of classical solutions are only correct when quantum theoretical variance effects can be ignored.

If phenomena are discovered where variance contributes in addition to the mean, there is a possibility that they can be explained as quantum theoretical effects.
Taking this post as an opportunity, the following efforts were taken additionary.

- Consideration about quantum solutions of Schwarzschild spacetime. (using single mode excitation approximation technique ; state-constructive formalism. unified field theory part II).
- Consideration about quantum solutions of the big bang using the wave function of the universe scale factor. (baseed on Hamiltonian deriving the modified Friedmann equation, unified field theory part III)

As for the other cosmological questions, such as dark energy, dark matter, acceleration of cosmic expansion, etc., currently we are not at a stage where definitive answers can be provided, so it seems we have no choice but to let our imaginations fly .
Therefore, let us think about the possible solutions by freely letting our imagination soar based on the unified field theory.
(The existence of spinor gauge connection fields, preon mesons, residual terms in matter field Lagrangian, etc. are imaginatively stimulating as possible explanatory reasons for the above issues.)

Part III Keywords:
Quantum gravity theory, Big Bang, Singularity resolution, Cosmic phase transition,
Dark energy, Dark matter, Preon meson, Black hole, Quantum variance effects
Schwarzschild barrier, Acceleration of cosmic expansion
(See Supplementary note 3)

2024.04 T. Sato Kawasaki, Japan. .

Supplementary note.1:

The equations derived from the unified field theory by canonical gauge principle, are second-order partial differential equations.

On the other hand, the Dirac equation governing the motion of Fermions in existing theories is first-order.

If we write the second-order equations as a system of first-order coupled equations, there should be a spinor degree of freedom of 2×4 in four-dimensional spacetime, but the Dirac equation has a degree of freedom of 1×4.

Initially, just as the Dirac equation was derived as a factorized form of the Klein-Gordon equation, the solution to this problem lies in an approximate factorized decomposition of the solution space.

Each factor is thought to correspond to a particle-preon.

The Dirac equation was initially conceived as a wave function of states, and field operators were constructed based on this. In this sense, it should be noted that the derivation does not originate from a field theory.

The canonical gauge principle implies the possible existence of a spinor connection field not previously known. This connection field is of the form $h \cdot \gamma_\mu dx^\mu$; $h = h_0 + ih_5\gamma^5$ with the gauge group Lie algebra o(1,1) and has a chirality degree of freedom. Furthermore, the expectation value component of its state is related to the (preon level) fermion mass, establishing a relationship between mass and chirality here.

I imagine this could explain why neutrinos are left-handed - the mechanism for it. (Since neutrinos are composite particles of preons, elucidating the above mechanism requires elucidating the composite mechanism.)

The su(3) color symmetry, su(2) flavor symmetry, etc. of the currently observed elementary particles naturally and directly appear in the unified derivation of Bosons and Fermions based on the canonical gauge principle. From the perspective of element reductionism, the primordial field equation describes a structure one step below quarks/leptons, a field representing preons, consistent specifically with the rather simple rishon model.

Supplementary note 2:

Part II "State-Constructive Formalism of Field Theory" mainly considers on solving techniques for equations of fields.

The principle of the unified field and the field equations as its result, can indeed be derived simply and clearly from the concept of "canonical gauge symmetry" as described above. On the other hand, for the theory to be useful, the it should be capable for quantitative calculations.to the actual phenomena and realistic models.

To that end, the concept of state vectors is introduced, and moving from the Heisenberg representation to the Schrödinger representation, while providing a general approximate solution method that avoids divergence difficulties.

"The 'solution technique for the equation of stationary states' considered in Part II is ultimately only the method of explicitly representing the state of the system and applying a simple variational method to it."

Regarding the problem of divergence, an important fact to consider why “renormalization worked well in quantum electrodynamics (QED)” is that all divergences can be written in terms of "elementary divergences" and "elementary divergences" appear to be related to "self-interactions”. This fact seems highly suggestive.

In light of this situation, it seems to be concluded that from theoretical consideration of logical structure of quantum field theory that "the adiabatic hypothesis and the concept of free fields do not hold."

For example, in QED, electrons cannot exist alone and must be regarded as a composite with virtual photons. This is often referred to as "dressed."
Even in scattering problems, the interaction exists as self-interaction before it exists between particles. Therefore, perturbative calculations based on adiabatic hypothesis that take "no interaction" as fundamental lose meaning.

To perform theoretical calculations, methods diverging from the conventional diagram techniques must be introduced. The interaction between two charged particles should occur through the exchange of photons of "dress".
As an alternative to diagram techniques, I propose constructing approximate solutions explicitly giving the state of the system under certain assumptions, using sequential approximate methods for scattering problems and finite element-like methods for steady-state problems.
"I think it is appropriate to call this idea 'state-constructive field theory' or 'state-constructive approach to field theory', which provides approximations by explicitly giving the state of the system."
This method can be expected to be useful in that it has a logical structure of sequential approximations.

PS:

As a future goal of state-constructive field theory, there is a calculation of electrons e (and μ, τ) based on a preon model. An epistemic evolution similar to elucidating the structure of hydrogen atom using the Schrödinger equation will be expected.

Supplementary note 3:

In the theory of the unified field, all fields are generated fundamentally in a principle, so apart from theoretical "indefiniteness", interaction constants (coupling constants) cannot be arbitrarily determined. They are also governed by unified relationships.
The interaction constants, for example in electromagnetic interactions, that is "elementary charge × 1/3", is formally interpreted as the coefficient of the lowest-order term of the interaction when the field operator is normalized and expressed.
Of course field normalization must be done to be consistent with canonical commutation relations. Field normalization is provisionally defined as the scale adjustment to make field operators and canonically conjugate operators have equal coefficients in the differential term in lowest-order approximate representation.

There are some facts to be noted that as follows:

Lagrangian has an arbitrariness (degree of freedom) of a constant multiple as variational functionals.

Field operators and its canonically conjugate operators have a degree of freedom within the range of canonical transformation.

Gravitational field is always involved in the Lagrangian as a scale factor of density.

These seem important when considering the origin of interaction constants (coupling constants).

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partⅣ : Conceptual Trajectory & Afterword

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Unified Field Theory by Canonical Gauge Principle (2024.04)

part I:

Principle of Unified Field and reason of existence of Elementary Particles (Spacetime, Field, Symmetry)

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Overview/Introduction

The basic idea of canonical gauge theory is to change the concept of spacetime from the traditional manifold to the space of operators, following the suggestions of quantum theory - the introduction of "quantum spacetime".
The coordinate functions in manifold theory are reinterpreted as linear operators acting on a linear space, and canonically conjugate linear operators - canonical momentum operators - are also prepared. This extends the general coordinate transformations that were important in general relativity to canonical transformations.
The model considered here is probably the simplest quantum spacetime model, since the canonical commutation relations are represented by symplectic matrices. I imagine that the above idea will probably provide the most fundamental space for a new geometry to replace manifold theory, but that will be the work of mathematicians.

Based on this concept of "quantum spacetime", canonical curvature form can be defined by the commutation relations of operators, and the unified field theory by canonical gauge principle is obtained by taking the quadratic norm of this curvature as the Lagrangean. This is the theme covered in Part I of the unified field theory.
Just by considering canonical parallel translation (canonical gauge connection), many physically important results are obtained, such as the existence of preons, the unification of Bosons/Fermions, the symmetry of elementary particles, and the existence of a 5-th interaction (spinor connection field).

Gravity is also derived in a unified and systematic way, along with other fields. The equation of unified field is obtained in a principled form, and several noteworthy results are obtained.
To briefly introduce some of those results:

-1.The reason of existence of fields, the derivation principles of field equations, and the understanding toward a unified form is obtained.
According to this, preons exist as fundamental matter in lower level of currently known as quarks and leptons, and the interaction fields we know are generated by interactions between preons.

-2.Bosons/Fermions are unified by the reason of existence of elementary particles.
The fields of elementary particle are obtained as connection field on spacetime with canonical transformation group being the gauge group.
From the matrix representations of the connection fields, we notice that there are 2 types of Bosons, corresponding to the FF-type and BB-type components of the matrices.
For Fermions, we also see that 2 types of FB and BF, there are in relation with particle/antiparticle in each other.
As a result, symmetric partners due to supersymmetry and the like do not exist. It is also found that Majorana particles do not exist.

-3.The symmetry of FF-type Bosons (interactions) is derived.
The FF-type Bosons consist of u(3)＝su(3)⊕u(1) symmetric component and o(1,1) symmetric component. su(3) is related to the strong interaction, i.e. color interaction and gluons, but since it is an interaction at the preon level, to avoid confusion, I would like to attach the prefix "pre-" and call it "pre-color interaction".
Theoretically, there is no reason to exclude the above-mentioned u(1), and it cannot be distinguished from the u(1) symmetry of the BB-type Bosons mentioned later.
The o(1,1) symmetric component is a connection field related to spinor degrees of freedom, which has not been recognized before, but from the correspondence with currently known elementary particles, it can be considered to be related to the Higgs field.
That is, the Higgs field is understood to originate from the spinor connection field.

-4.The relationship between chirality and mass of preons is obtained through interaction with the spinor connection field. This is the key to understanding why neutrinos are left-handed.
The parity non-preservation in weak interactions is not because parity violation is built into the equations as a physical law, but because neutrinos are involved.
That is, even effectively replacing $\nu \rightarrow p_L \cdot \nu$ in the equations, where p_L is the left-handed projection operator, should be equivalent. This is an issue of the state of the field, not an operator.

-5.The existence of preons is strongly suggested.
Quarks/leptons are preon composite systems, and generations are understood as excited states of composite systems.
As a preon configuration, the rishon model corresponds to a preon model.
The reason why there are only up to 3 generations of quarks/leptons is naturally explained by the constraint that the emission of 3 pre-color Bosons is limited to white-like combinations.

That is, if there were a 4th generation, it would immediately decay into the 1st generation.
On the other hand, since the emission of 3 white pre-color Bosons is equivalent in color value to the emission of 2 pre-color and the absorption of remaining anti pre-color, the exchange of generations by ν, e- collisions is possible. (ref.section 2.3)
As is well known, admitting the existence of preons also answers the question of the asymmetric existence of particles and antiparticles in the universe, "Where have the antiparticles gone?".
They have not gone anywhere. They are hiding inside the quarks by d=(TVV)*.

-6. Dirac equation is derived and the existence of 2 types of preon model particles is suggested.
It should be noted that the current Dirac equation as the field equation is obtained by simply applying the equation for wave function.
While the equation for the wave function should be linear, the field equation need not be so.
The actual equations must be derived from the unified field theory.
The unified field theory gives a 2nd-order partial differential equation, and by factorizing this, a 1st-order Dirac field equation can be obtained. The two solutions obtained by this factorization may correspond to the pre-flavors T and V of preons. One solution is found to have a chirality-dependent mass by coupling with the spinor connection field.

-7.Under the assumption of spherical symmetry, unified field gravitational theory (the canonical gauge

unified gravitational theory of fields ）is confirmed to coincide with Einstein's theory of gravitation. The gravitational equation given by the unified field theory is essentially different to that from Einstein's gravity.
Since Einstein's theory is a theory concerning the spacetime metric $g_{\mu\nu}$, it cannot provide a representation of Dirac equation in a gravitational field. No scalar Lagrangian consisting of 1st-order differentials of field variables exist other than constants.
On the other hand, unified gravitational theory is not so.
However, when spherical symmetry is assumed for the field, the unified field gravitational theory and Einstein gravitational theory are shown to coincide by adjusting the interaction constants.
From this, it can be considered that the unified field gravitational theory is verified to about the same extent as Einstein's gravitational theory in the classical range.
Quantum gravity theory based on unified field theory will be considered in Part III. In the field theory, it is well known that there are obstacles such as divergence difficulties and no proceduralized solution methods for equations.
Then, in Part II, we will consider about the solution method to field equations.

2024.04／2021.06 T.sato .

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: Principle of Unified Field and reason of existence of Elementary Particles

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1. The Concept of Quantum Spacetime

Quantum theory suggests the following basic principles regarding the nature of the state space of a physical system, or the concept of state:

(1).The state of a system is represented as a vector in a metric linear space.

(2).For each observer, there exist position operators x^j and their canonically conjugate momentum operators p_j.

(3).Physical quantities are represented as self-adjoint linear operators on the aforementioned metric space.

(4) A Lorentz metric can be introduced on the space of formally 1st-order self-adjoint operators with respect to the canonical momenta.

Item (1) refers to the applicability of the state vector concept to physical systems. Two non-zero vectors differing only in length represent the same state.
In general, an observer considers the "state of the system at a certain time t." This state should be interpreted as the projection component onto the space $\ker(x^0\text{-}t)$ $t\in\mathbf{R}$, originating from Item (3).

Item (2) assumes the existence of position x and momentum p as fundamental physical quantities, and the canonical commutation relations between them:

$$[x^j, x^k]=0\ ,\quad [p_j, p_k]=0\ ,\quad [x^j, p_k]=i\cdot\delta^j_k$$ ***1**

According to Item (3), spacetime coordinate values should be understood as continuous eigenvalues of the position operators. ***2**
By considering the canonical commutation relations are fundamental elements, we notice that the classical coordinate transformations on the manifold model of spacetime should be embedded in the canonical transformations as transformations of the position and momentum operators.

The transformations between state representations of multiple observers should be Canonical transformations. By assuming the preservation of spacetime, the generators of these transformations can be assumed to be 1st-order form with respect to the canonical momenta.
The "preservation of spacetime" means the common eigenspace of the position operators. Then, it follows that the general coordinate transformations are subsumed into spacetime-preserving canonical transformations in a quantum theoretical sense. ***3**

The eigenspace ker(x-a) of the position operators corresponds to the space of internal degrees of freedom of the state. Whether Its dimensions are finite or infinite, it cannot be determined a priori.
It is probably finite-dimensional, and it seems reasonable to assume that the dimension is constant, independent of position, from the continuity of spacetime.
However, it could also be imagined to be infinite-dimensional, with only lower-dimensional subspaces being observed in the low-energy region. However, unsubstantiated speculations require caution.

There is also a view that considers these internal degrees of freedom to be like the redundant dimensions of a manifold, discretized by boundary conditions, etc. However, this idea does not reflect the principles suggested by quantum theory.
Currently, it seems that a model similar to the linear spaces assumed in the theory of vector bundles over manifolds, can be applied to the space of positional eigenstate in representing internal degrees of freedom.

Item (4) may seem somewhat different in nature compared to the requirements of the other three items. However, if spacetime coordinate values are to be understood as eigenvalues of the position operators, it is a natural extrapolation from the existence of the Lorentz metric in the manifold model of spacetime in classical theory to introduce an assumption like this.

The metric structure concerning tangent vector basis (∂_x) in manifold theory should firstly be reinterpreted as a metric structure concerning the canonical momentum operators p through the relation $p = -i\partial_x$.
Additionry, from the transformation of metric structure under spacetime-preserving canonical transformations of the momentum operators, it is concluded that the Lorentz metric should be defined on the ring formed by self-adjoint and formally 1st-order operators with respect to the canonical momenta. (say Canonical gauge ring) ***4**

By the way, since a metric structure is already assumed on the state space, this Lorentz metric structure can be expected to be induced from the metric structure of the state space. Therefore, the existence of a metric itself is not an additional requirement, but a requirement based on the metric of the state space, it seems. ***5**
While issues remain regarding the physical interpretation of the metric, introducing a metric already amounts to introducing a gravitational field, as will be discussed later.

***1. Notes:**

- The canonical commutation relation $[x^j, p_k] = i\cdot\delta^j{}_k$ implies that the state space is infinite-dimensional. As is well known, in finite dimensions, for eigenvectors |a>, <a|a>=0 is derived.

- The commutation relations $[x^j, x^k]=0$, $[p_j, p_k]=0$ indicate that the assumed spacetime model is simple and related to the translation group. For example, on a manifold of the rotation group SO(3), the coordinate functions do not commute, the canonical momenta are angular momenta, and they satisfy the well-known commutation relation $[L_\mu, L_\nu] = iL_\lambda$.

- The momentum operator representation in coordinates can also be derived from the canonical commutation relations.
  Representing the position operators x with eigenstates and writing $p_j = -i\partial_j + F_j$. Then, from $[F, x]=0$; $F=F(x)$ and from $[p, p]=0$, the restriction: $-i\partial_j F_k + i\partial_k F_j + [F_j, F_k] = 0$ is obtained. This corresponds to the equation of curvature$=0$, and is the integrability condition for the equation $F_j = i\partial_j S\cdot S^{-1}$. From this, it can be seen that $p_j = S(-i\partial_j)S^{-1}$ ($\exists S$) can be written.

***2. Notes:**

Particles do not occupy a specific position, but rather positions are defined by the localization of particles.

***3. Notes:**

Let W be a generator of an infinitesimal canonical transformation, and consider the transformation to $A=A^*$.

$$A \to A' = A + \delta A = A + [iW, A]$$

By requiring self-adjointness $A'^* = A'$ for the transformation, then $W = W^*$ is required.

Since requirement of preservation of spacetime in the transformation, $\delta x \equiv [iW, x]$ does not contain the canonical momentum operators p.

Therefore, we can write $W = W_0(x) + (p \cdot W_1(x) + W_1(x) \cdot p)/2$, and the resulting infinitesimal canonical transformation is$\delta x = W_1$, $\delta p = -\partial W_0 - (p \cdot \partial W_1 + \partial W_1 \cdot p)/2$

In coordinate representation, p is expressed as $p_j = -i\partial_j + F_j$, and in general there is a 0th-order term F.

In the manifold model of spacetime in the classical sense, the standard representation of canonical momenta in curvilinear coordinates is as follows.

$$p_j = g^{-1/4}(-i\partial_j)g^{1/4}$$

Representing the inner product of states a,b as an integral, taking the volume element to be $g^{1/2}dx$,

$$\langle a|b\rangle = \int a^*(x)(g^{1/2} \cdot dx)b(x) = \int (g^{1/4}a(x))^* \cdot dx \cdot (g^{1/4}b(x))$$

From this,

$$\langle a|p|b\rangle = \int (g^{1/4}a(x))^* \cdot dx \cdot (g^{1/4}\, p(b(x))) = \int (g^{1/4}\, p(a(x)))^* \cdot dx \cdot (g^{1/4}b(x)).$$

Here, $p_j|b\rangle \equiv (g^{-1/4}(-i\partial_j)g^{1/4})b(x)$; self-adjointness in action integrals

***4. Notes:**

"Formally 1st-order self-adjoint operators with respect to the canonical momentum operators" are generators of spacetime-preserving canonical transformations, forming a ring. I would like to call this the canonical gauge ring.

Classical coordinate transformations and gauge transformations become unified.

With $W \equiv \{p_j, \varepsilon^j(x)\}/2 - A$, $(W = W^*)$,

$\delta x = [iW, x] = \varepsilon$, coordinate transformation

$\delta p = [iW, p] = -1/2 \cdot \{p_j, \partial\varepsilon^j\} - \partial A$; coordinate transformation + gauge transformation

While W is the generator of spacetime-preserving canonical transformation, calling the transformation generated by W a "canonical gauge transformation" is valid in the sense that it is a quantum theoretical extension of the definition of a classical gauge transformation.

***5. Notes:**

In finite dimensions, for linear operators A, B, their inner product can be defined as $\langle A|B\rangle = \mathrm{Tr}(A^*B) \times \mathrm{const}$. This can also be interpreted as a statistical average of $\langle A(x)|B(x)\rangle$.

e.g. $\langle A|B\rangle \equiv Z^{-1} \cdot \Sigma_x \exp(-\beta E_x) \cdot \langle A(x)|B(x)\rangle$; $Z \equiv \mathrm{Tr}\exp(-\beta H)$

where, $|x\rangle$ is an eigenbasis of the Hamiltonian H, with $H|x\rangle = |x\rangle E_x$.

Since $\exp(-\beta E_x)\langle x| = \langle x|\exp(-\beta H)$, it can also be written:

$$\langle A|B\rangle \equiv Z^{-1} \cdot \Sigma_x \langle x|\exp(-\beta H)A^*B|x\rangle = Z^{-1} \cdot \mathrm{Tr}(\exp(-\beta H)A^*B)$$

Or $\langle A|B\rangle \equiv Z^{-1} \cdot \mathrm{Tr}(B \cdot \exp(-\beta H) \cdot A^*)$

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2. Reason of existence of elementary particles and unified field

In order to construct a quantum theoretical unified field theory, the concept of spacetime as a manifold model used so far needs to be changed.
What takes the place of manifold is a ring formed by certain kinds of linear operators, and manifold appears as eigenstates of operators.

there, the operator interpretation of concepts In classical differential geometry, such as vector connection form, covariant differentiation, and translation, as well as "canonical curvature" that is a canonically invariant extension of a concept of curvature , etc. can be defined.

The unified field equation is obtained by applying the principle of least action to the quadratic norm of this "canonical curvature". The Lagrangian of a unified field theory is given by a constant times the quadratic norm of the "canonical curvature".
The term "canonical gauge" used in this book comes from the fact that the gauge group is a group of spacetime-preserving canonical transformations .

2.1.Concept of canonical gauge ring

●Construction of canonical gauge ring (algebra)

The linear space formed by formal 1st-order self-adjoint operators related to canonical momentum p constitutes a Lie algebra of spacetime-preserving canonical transformations if the coefficients related to p are commutative.

$D \equiv \{a(x),p\}/2+b(x)$ where $D=D^*$

$\delta x=[iD ,x]=a(x)$; coordinate transformation ($[a(x), x]=0$)
$\delta p=[iD ,p]=-\{\partial a(x),p\}/2-\partial b(x)$; coordinate transformation＋gauge transformation

We define the space formed by formal 1st-order self-adjoint operators related to canonical momentum p which satisfies the above conditions as "canonical gauge ring".
Denoting this by $A_1(x,p)$,
$A_1(x,p)\equiv\{D \mid \exists\, a(x),b(x)\ (D=\{a(x),p\}/2+b(x)),\ D=D^*\}$;
where x, p are position and canonical momentum operators
For $D_1, D_2 \in A_1(x,p)$, if $[a_1, a_2]=0$, then $iD\equiv[iD_1, iD_2]\in i\cdot A_1(x,p)$

In defining the canonical gauge ring $A_1(x,p)$ above, specific self-adjoint operators (x, p) were introduced, but the ring $A_1(x',p')$ can be formed by similar operators. Also in that case, it is obvious that $A_1(x,p)=A_1(x',p')$ if (x',p') and (x,p) are connected with a spacetime-preserving canonical transformation ($\xi \to \xi' \equiv e^{iD}\xi e^{-iD}$ $D\in A_1$).

●Construction of canonical gauge field

The differential operators in classical spacetime models, are replaced by canonical momentum operators p. From the viewpoint of invariance under canonical transformations, the way of defining spacetime metrics like $\langle p_\mu|p_\nu\rangle=g_{\mu\nu}(x)$ as in classical theory loses its meaning.

To Construct the theory, the connection form of tangent vectors is not needed.

When considering how the Dirac equation in a gravitational field should be expressed in a classical spacetime model (getting a little ahead of the subsequent result), assuming the self-adjointness of the Dirac operator D, it should be as follows:

$(\mathcal{D}\text{-m})\psi=0$; $\mathcal{D}=\gamma^M\cdot P_M$ $P_M\equiv\{S^\mu{}_M(x), p_\mu\}/2-U_M(x)$; canonical gauge connection
$\langle P_M|P_N\rangle=\eta_{MN}$; Lorentz metric

By the assumption (Ch.1-(4)) in introducing the quantum theoretical spacetime, there exist self-adjoint 1st-order form to p , that gives Lorentz metric, as follows.

$(P_M)_{M=1..n}$ $P_M\equiv\{S^\mu{}_M(x), p_\mu\}/2-U_M(x)$;

P_M contains $S^\mu{}_M(x)$ that gives the space-time metric $g_{\mu\nu}(x)$ in manifold theory.
That is, $g^{\mu\nu}(x)\equiv S^\mu{}_M(x)\,\eta^{MN}S^\nu{}_N(x)$ (the space-time metric $g^{\mu\nu}(x)$ is defined from $S^\lambda{}_M(x)$)
Expressing S=exp(-V), and V can be thought of as a generalized Newton potential.

If the gauge field is assumed to satisfy the field equation, the above canonical gauge connection can be called a "parallel translation generator" or "infinitesimal parallel translation".
The above-mentioned $S=(S^\mu{}_M(x))$ or $V=(V^\mu{}_M)$ contains the degree of freedom of local Lorentz transformation with respect to index M, related to spin freedom.
This degree of freedom may become important in the interaction between Fermions and gravity.
Especifically, it might appear as a phenomenon that the connection does not match even though the spacetime metric (g) is the same.

The term $U_M(x)$, 0-th order term of the canonical momentum p in the parallel translation generator P, represents gauge fields other than gravity, including not only Boson fields but also Fermion fields.
This will be described later.
In this sense, the parallel translation generators, Lorentz frame $(P_M)^{M=0.3}$, are what becomes the basis of the unified field theory by canonical gauge principle.

●0-th order term of p - canonical connection form -

In classical theory, a physical gauge field is understood as a "connection form" on a vector bundle.
The connection can be expressed as $-id\mathbf{e}=\mathbf{e}\cdot A$ for basis vector **e** ; where $A\equiv A_\lambda(x)dx^\lambda$ (Sup.*1)
This means nothing more than having specified $-id\equiv p+A$ as an element of the canonical gauge ring. In the quantum spacetime, it suffices to give a set of elements of the canonical gauge ring such as $P_M\equiv\{p_k,S^k{}_M\}/2+U_M(x)$ (M=1..n, n=dimension) and declare it as a "connection form".
If this is assumed to satisfy the "field equation", it is a canonical gauge field.
It can be considered to define the "most distortionless connection" based on the principle of least action.

Note*1: Extensibility of connection concept

In classical spacetime manifold models, a connection form is an infinitesimal linear mapping from the fiber linear space at position (x) to the fiber linear space at position (x+dx), defined as a function of position x and displacement dx.
By analogy from the concept of connections on manifolds, it may seem natural to conceive a "connection" in quantum spacetime as a linear mapping between the eigenspaces of the position operator (x), from ker(x-a) to ker(x-(a+da)) , but this idea contains some points to note.

First, note that the coefficient about the momentum operator p in P_μ is restricted to 1.
When considering the canonical transformation generator $P \equiv dx^\mu \cdot P_\mu$; $P_\mu \equiv (p_\mu + A_\mu(x))$ with indeterminates dx , in order to make the above P be invariant, dx should be a differential of x.
(And please note that $[P, x^\lambda] = dx^\lambda$)
On top of that, limiting the coefficient of the momentum operator p to 1 means ensuring that the positional displacement does not depend on the transformation P including the indeterminates dx because of the commutativity of the transformation. That is,

$$[P_\nu,[P_\mu, x]] - [P_\mu,[P_\nu, x]] = [[P_\mu,P_\nu],x] = 0$$

In other words, the coefficient of p is limited to 1 so that the positional displacement by Pμ ,Pv may coincide regardless of the order of operation.

However, from the idea of quantum theoretical spacetime treating position as an eigenvalue of state vector, this limitation is incompatible with the original concept of canonical gauge ring.
This limitation on the coefficient of momentum operator p should be abolished. (The coefficient is not limited to 1, but required to be commutative each other so that they may form a ring.)

Note*2: Concept of parallel translation and field equation

"Parallel translation" is a specific type of connection form. To define "parallel translation" reasonably on a general vector bundle space, at least the fiber space should be a metrical linear space, and the linear mapping by the connection must preserve the metric of vectors. However, even with this requirement, there are still redundant degrees of freedom remaining in the form of length-preserving translations (unitary transformations).

On the other hand, the field equation of a gauge field can be considered to eliminate this kind of redundant degrees of freedom (useless twist for example) by applying the principle of least action defined by the Lagrangian.
Therefore, parallel translation can be understood as a connection form that satisfies the field equation.
Among all metric-preserving connections, the specific connection that corresponds to parallel translation could be selected by the minimality of the Lagrangian.

By the way, it is well known that the Lagrangian of the electromagnetic field is the quadratic norm of the curvature of u(1) connection. Similarly, it is natural to select the quadratic norm of the curvature form as Lagrangian, and it is considered that the minimality of the action forms the condition for the connection to be parallel translation. (Sup*2)

Supplementary*1:

The electromagnetic field $A = A_\lambda dx^\lambda$ can be considered as a model of connection form. If the gauge group is determined as U(1) from the beginning, appearance of the imaginary unit "i" in the connection can be explained to some extent. The origin of group U(1) can be found in the phase indeterminacy of wave function.
On the other hand, there is no room for i to enter in the tangent vector connection in classical gravity theory.

In the canonically gauged unified field theory, the structure group of the connection is canonical transformation group, and the generator is always iW ($W = W^*$,$Y \to [iW, Y]$ for any self-adjoint connection Y preserves self-adjointness.)

Supplementary*2:

For the idea of taking Lagrangian as quadratic norm of curvature as principle, someone may object, citing A. Einstein's theory of gravity, which gives Lagrangian as scalar curvature. However, the curvature referred to there is related to the connection of tangent vectors.
In the case of electromagnetic field, connection coefficients give vector potential, but in the case of gravitational field, potential is spacetime metric, not connection coefficients. This point can also be regarded as an aesthetic defect in Einstein's theory.

2.2.The Reason of Existence of Elementary Particles and Their Unification

As already described, the canonical gauge field contains the gravitational field as a 1-st order coefficient with respect to the canonical momentum p, and the field of the canonical connection form as a 0-th order coefficient.
These two types of fields appear essentially different from the perspective of the order of p they are associated with.
The field of the canonical connection form as a 0-th order coefficient can be interpreted as the connection form on vector bundle in classical theory. Hereafter, let us freely use this interpretation/representation. We will consider the involvement form of the gravitational field later.
The reason of existence of the fields lies in the canonical gauge connection of quantum spacetime.
For a "unified field" theory, it is necessary to simultaneously derive Bosons as interaction fields and Fermions as matter fields.
As seen below, this is possible in canonical gauge unified field theory.

●Bose Frame/Fermi Frame and Canonical Gauge Field

Consider the vector frames in the internal space ker(x-c) (i.e. the spacetime point x=c in quantum spacetime), called the Fermi frame (a) and Bose frame (b), according to the symmetry of the tensor algebra.
Write the connection as follows:

$$-id\mathbf{a}=\mathbf{a}\cdot A+\mathbf{b}\cdot\Omega^*$$
$$-id\mathbf{b}=\mathbf{a}\cdot\Omega+\mathbf{b}\cdot B$$

note: $-idF=[W, F]$ for $\forall F$; $W\equiv\mathbf{a}A\mathbf{a}^*+\mathbf{b}B\mathbf{b}^*+\mathbf{b}\Omega^*\mathbf{a}^*+\mathbf{a}\Omega\mathbf{b}^*$, $W=W^*$

Since canonical transformations preserve commutation relations, transformations like
a→b or b→±a are impossible because of the symmetry of the tensor algebra.
This indicates that the connection coefficients contain anticommuting numbers.

We can understand Fermion field as the canonical gauge connection that transforms the Fermi frame to the Bose frame, or vice versa, the Bose frame to the Fermi frame components.
That is, the connection coefficients Ω ,Ω* are considered to represent Fermion fields in particle/antiparticle relationship with each other.
Similarly, the connection components A, from Fermi frame to Fermi frame, or B, from Bose frame to Bose frame in the canonical gauge connection, should be understood to represent Boson fields.
Thus, there are two types of Bosons, FF type and BB type. These two types of Boson fields should be considered to have different properties based on the difference in their reason of existence.

The Fermi frame (**a**) should have the degree of freedom of a spinor basis. Furthermore, it will have internal degrees of freedom corresponding to color charge. Explicitly by showing the spinor index as j, it becomes as follows:

$$-i\cdot d\mathbf{a}_j=\mathbf{a}_k\cdot A^k{}_j+\mathbf{b}\cdot\Omega^*{}_j$$
$$-i\cdot d\mathbf{b}=\mathbf{a}_j\cdot\Omega^j+\mathbf{b}\cdot B \qquad (j: \text{spinor index})$$

For the connection of spinor degrees of freedom, we assume that the γ matrices are coupled to the spacetime coordinate indices.
This allows the introduction of the spinor field χ by $\Omega^j=(\gamma dx)^j{}_k\cdot\chi^k$.

The connection is an infinitesimal canonical transformation as follows:

$$-idF=[W, F] \quad ; \quad W\equiv\mathbf{a}A\mathbf{a}^*+\mathbf{b}B\mathbf{b}^*+\mathbf{b}\Omega^*\mathbf{a}^*+\mathbf{a}\Omega\mathbf{b}^*$$

By assumption, **a**, **a*** follow the antisymmetric algebra, and **b**, **b*** follow the symmetric algebra. The element "**a**" should commute with A, B, and anti-commute with Ω, Ω*. The element "**b**" commutes with all elements.

- $[\mathbf{a}A\mathbf{a}^*, \mathbf{a}] = \mathbf{a}A$ ・ $[\mathbf{b}B\mathbf{b}^*, \mathbf{a}] = 0$
- $[B, \mathbf{a}] = 0$ ・ $[\mathbf{b}\Omega^*\mathbf{a}^*, \mathbf{a}] = \mathbf{b}\Omega^*$ ・ $[\mathbf{a}\Omega\mathbf{b}^*, \mathbf{a}] = 0$
- $[\mathbf{a}A\mathbf{a}^*, \mathbf{b}] = 0$ ・ $[\mathbf{b}B\mathbf{b}^*, \mathbf{b}] = \mathbf{b}B$ ・ $[\mathbf{b}\Omega^*\mathbf{a}^*, \mathbf{b}] = 0$ ・ $[\mathbf{a}\Omega\mathbf{b}^*, \mathbf{b}] = \mathbf{a}\Omega$ ***1**

Since $\Omega^j = (\gamma_\lambda \cdot dx^\lambda)^j{}_k \cdot \chi^k$, the anticommutativity of **a**, **a*** with Ω means that χ is anticommutative with the Fermi frame and its dual **a**, **a***. Therefore, from this stage,of introduction, we can see that χ(x) is not just a coefficient but already has the nature of an anticommuting operator.

***Note*1**: The Leibniz rule for commutators is as follows:
$[xy,z] = x[y,z] + [x,z]y = x\{y,z\} - \{x,z\}y$; $\{xy,z\} = x\{y,z\} - [x,z]y = x[y,z] + \{x,z\}y$

●Reducibility of Canonical Gauge Field and Spinor Connection Field

Let the basis of Fermi frame be explicitly denoted as $(\mathbf{a}_{mj})$, where j is the spinor index.
Similarly, let the basis of Bose frame be written as $(\mathbf{b}_r)$
The representation of multi-index is reminiscent of tensor product structure.
In actual, the multi-index comes from the subdivision of eigenspaces depending on multiple commutative operators that define internal attributes.

By assuming that the tensor product structure is preserved by the canonical transformation of the connection, we can apply the Leibniz rule to the connection.
In such case that the basis $\mathbf{e}_{xy}$ is obtained with the eigen decomposition of entire space (E) by mutually commutative linear mappings X and Y, isomorphicity to the space as follows in tensor product structure could be assumed:

Assume that $E = E_X \otimes E_Y$ is given. With the linear mappings $X' \in End(E_X)$, $Y' \in End(E_Y)$, and defining the linear mappings X, Y on $E = E_X \otimes E_Y$ as $X = X' \otimes 1$, $Y = 1 \otimes Y'$ respectively.
Then we can construct the tensor space basis by choosing $\mathbf{x} \in \ker(X'-x)$, $\mathbf{y} \in \ker(Y'-y)$, and set $\mathbf{e}_{xy} = \mathbf{x} \otimes \mathbf{y}$.

If $d(E_j) \subset E_j$ $(j = X,Y)$ for the connection "d", the connection form will be decomposed into the following form:

$$d(\mathbf{x} \otimes \mathbf{y}) = d\mathbf{x} \otimes \mathbf{y} + \mathbf{x} \otimes d\mathbf{y} = (\mathbf{x} \otimes \mathbf{y}) \cdot (\Gamma_x \otimes 1 + 1 \otimes \Gamma_y) \quad ;$$

(Here we wrote the linear mapping of the connection as $d\mathbf{x} = \mathbf{x} \cdot \Gamma_x$, $d\mathbf{y} = \mathbf{y} \cdot \Gamma_y$.)

Assuming such reducibility mentioned above for FF connection (Fermi frame to Fermi frame connection) component A, it can be decomposed as follows.

$$-id\mathbf{a}_{mj} = \mathbf{a}_{nk} \cdot A^{nk}{}_{mj} + \mathbf{b}_s \cdot \Omega^{*s}{}_{mj} \quad ; \quad A^{nk}{}_{mj} = \delta^n{}_m \cdot A'^k{}_j + A''^n{}_m \cdot \delta^j{}_k$$

The connection A' represents the connection (spinor gauge field) regarding the spinor degree of freedom. We require that the connection of the spinor degree of freedom couples with the γ matrices. From this we obtain $A' = h \cdot \gamma_\lambda dx^\lambda$. Furthermore, the constraint of self-adjointness $A' = A'^*$ limits h to the form $h = (h_0 + h_5 \cdot i\gamma^5)$.
From the above, we get $A' = (h_0 + h_5 \cdot i\gamma^5)\, \gamma_\lambda dx^\lambda$. (**Note*2**)

The field h is a field that has not been known so far. (The use of h is aware of "hidden".)
From the correspondence with interactions/elementary particles found so far, a relationship with the mass mechanism and Higgs particles can be imagined, but the origin of the field h is totally different from the Higgs mechanism.

Originally, Higgs mechanism was devised to give mass to gauge Bosons, by having Higgs Bosons couple to gauge fields to acquire mass.
On the other hand, the field h is an FF-type Boson as a spinor connection, whose existence is derived in principles by the canonical gauge principle.
As we will see later, the field h couples with Fermions but with no other Boson except for h itself.
Coupling gives a relationship between mass and chirality.
For elemental Bosons derived in principles, apart from effective masses generated by noncommutative higher order self-interactions or interactions with the surroundings, there seems to be no mass that is produced by Higgs mesons coupling as mass terms.
Probably mass is originally an effective concept, and it is necessary to give actual Lagrangian to examine the details.

Note*2: Selection of possibilities for spinor connection

For $A'^j{}_k$ in the connection $-i\cdot d\mathbf{a}_{mj}=\mathbf{a}_{nk}\cdot(\delta^n{}_m\cdot A'^k{}_j+A''^n{}_m\cdot\delta^j{}_k)+\mathbf{b}_s\cdot\Omega^{*s}{}_{mj}$, expand $A'^j{}_k$ as

$A'^j{}_k=A'_M\cdot((\gamma^M)(\gamma_\lambda dx^\lambda))^j{}_k$; (M indicates the multi-index showing the product of γ.)

Requiring the self-adjointness of $A'^j{}_k$, then, according to the commutativity and anticommutativity of γ_λ and γ^M, A'_M is determined to be real or purely imaginary.
In 4-dimensional spacetime, there is no index that gives a uniform commutation relation for all λ except in the case of M=(empty) or (0123).
Therefore, $A'=(h_0\cdot 1+h_5\cdot i\gamma^5)\cdot\gamma_\lambda dx^\lambda$ is obtained. Here, $\gamma^5\equiv\gamma^{0123}$ as abbreviation.
(note that γ is self-adjoint $\gamma=\gamma^*$. $\gamma=\mathbf{a}_j\gamma^j{}_k\mathbf{a}^{*k}$; $\mathbf{a}\equiv$spinor basis. Dirac operator is expressed as $\mathcal{D}\equiv\gamma^\lambda p_\lambda$. The spinor metric is defined so that the Dirac operator D may be self-adjoint and the normalized Dirac spinor of positive energy be +1.
The assumed spacetime metric is diag.(1,-1,-1,-1). Considerations on the representation of the spinor basis selection, matrix representation of γ, and spinor metrics will be shown in the appendix.)

The connection field A'' is the interaction at preon level and is the source of the color interaction at the quark level.
Preons are particles considered to exist in one level lower than quarks, based on the idea of elemental reductionism, and the color degrees of freedom of preons are not the same as the color degrees of freedom of quarks. For this reason, to prevent confusion, the prefix "pre-" is attached to the color degrees of freedom of preons and the associated gauge fields.
Note that the connection field A'' will hereafter be denoted by C with pre-color in mind.

2.3.The Reality of Gauge Fields and Practical Correspondences

The su(3) symmetry of the strong interaction and the u(2) symmetry of the electroweak interaction that are currently observed can be easily understood by correspondence with the degrees of freedom of state vector frame, as follows:

pre-color degrees of freedom of Fermi frame is 3.

pre-flavor degrees of freedom of Bose frame is 2 .

Based on the type of frame connection, we can conclude as follows.

the cause of the field of strong interaction is FF-type connection,

the cause of the field of electroweak interaction is BB-type connection.

h is a new field unknown so far, has chirality, and combines with Fermion.

The fields derived as gauge fields should be considered as elemental entities derived from principle. And the mutual conversion of quarks and leptons observed in weak interactions (β decay) should be understood as a rearrangement of elements from the viewpoints of elemental reductionism.
This suggests that we are in a similar position to 19th century chemists who explored atoms. As substructures of quarks and leptons who mutually convert each other in weak interactions, there exist preons.
Since the reason of existence of gauge fields lies in the connection of state vector in quantum spacetime and they are principled and elemental, the Fermion components of gauge fields can should understood to correspond to preons.
Therefore, the strong and electroweak interactions at the quark/lepton level should be understood as multipolar components of elemental forces by gauge connections.

Among preon models, the rishon model is simple and consistent with the theory of gauge fields.
Expressing the gauge fields explicitly with their components, they become as follows : *A

$$-i\cdot d\mathbf{a}_{mj} = \mathbf{a}_{mk}\cdot(h(\gamma_\lambda dx^\lambda))^k{}_j + \mathbf{a}_{nj}\cdot C^n{}_{m,\lambda}dx^\lambda + \mathbf{b}_s\cdot\chi^{*s}{}_{mk}\,(\gamma_\lambda dx^\lambda)^k{}_j \quad ; \quad h = (h^0 + h_5\cdot i\gamma^5)$$

$$-i\cdot d\mathbf{b}_r = \mathbf{a}_{mk}\cdot(\gamma_\lambda dx^\lambda)^k{}_j\,\chi_r{}^{mj} + \mathbf{b}_s\cdot B^s{}_{r,\lambda}dx^\lambda$$

where, the correspondence between letters for indices and degree of freedom of the gauge fields is:

m, n : pre-color , j: spinor, r, s: pre-flavor

From the pre-color degrees of freedom being 3 in the Fermi frame, the pre-color field $C^m{}_{n,\lambda}dx^\lambda$ forms a u(3)-gauge vector field, and from the Bose frame having 2 degrees of freedom, the pre-flavor field $B^s{}_{r,\lambda}dx^\lambda$ forms a u(2)-gauge vector field.
Although u(3)＝su(3)⊕1 , there is no reason to limit the pre-color interaction gauge group to su(3) from the beginning. Similarly, u(2)＝su(2)⊕1, but the unit basis (1) is a necessary component for the existence of the electromagnetic field. It is natural to consider the unit bases (1) of both to be identical as generators of phase transformations.

The field h is something whose existence has not been known so far and should be carefully examined for its effects in attempts to derive the Dirac equation from the unified field equation.
Fermions appear as $\chi^{*r}{}_{mj}$, $\chi_r{}^{mj}$ in FB-BF type connections and they are spinor field having pre-color 3 × pre-flavor 2 degrees of freedom.
In the rishon model, m runs through R,G,B; and r runs through T, V.
In the rishon model, $e^+ = (T_R T_G T_B)$ $u_R = (V_R T_G T_B)$ $d_R^* = (T_R V_G V_B)$ $\nu = (V_R V_G V_B)$
β decay : n → $p + e^- + \nu^*$ can be understood as a reconfiguration of constituents as follows.

$d(-1/3) = u(2/3) + e^-(-1) + \nu^*(0)$ (n=udd , p=uud)

Here, (#) indicates the charge. Sorting with attention to the charge, it can be written as

$\nu(0)+e^{+}(1)=d^{*}(1/3)+u(2/3)$

(Such idea that this transformation suggests rearrangements of particles with charge 1/3 gives rise to the above model.)

●topics: antiparticle problem, quark/lepton generation problem, dark energy/dark matter

- In rishon model, the antiparticles in the universe have not disappeared but are hidden in combinations. If the probability of pair annihilation occurring cannot be suppressed sufficiently, the world would not exist.
- Think about the 3 types of leptons (e, μ. τ) + 3 types of neutrinos (ν_e ,ν_μ, ν_τ), and the 6 types of quarks (u, d ; c, s ; t, b). These can be interpreted as the excited states of preon composite.
- Since pre-color Bosons cannot exist individually, excited state quarks/leptons in a single environment cannot easily transition to the ground state by emitting 1 Boson from the excited state.
- The reason why the 4th and higher generations of quarks and leptons have not been discovered, is that if there is a difference of more than 3 generations, it would become possible them to decay by emitting the pre-color Bosons in white color combinations.
  With a difference of less than 3 generations there is no transition by decay, but the generation exchange due to collision is possible.
  (It becomes an image like transition of resonantly doubly bonded compounds between composite particles) ***B**
- Since exchanged spin is limited to 1 in the interaction that binds preons to construct quarks and leptons, quarks and leptons composed of T and V are limited to spin 1/2.
- Pre-mesons TcTc* and VcVc* may exist in the universe as dark matter.
  Here c is the pre-color index, which is imagined to be in RGB resonant states. The interaction is by exchange of pre-color and pre-flavor. VcTc*±TcVc* may also exist.
- Considering the relationship between the mass of preons and chirality given by the spinor connection field h, it is natural to think that the field operator of h contains constant component due to Bose-Einstein condensation, spreading throughout the space of the universe.
  It may be a cause of dark energy.
  Also, the composite of white combination pre-color Bosons mentioned in generation problem is also a natural candidate for dark energy.

Note*A: The meaning and decomposition of the connection form can be expressed as follows:

	a	**b**		
(a, b)・	FF (C⊕h)	FB (χ*・γdx)	=	**a**・(FF)+**b**・(BF)
	BF (γdx・χ)	BB (B)		**a**・(FB)+**b**・(BB)

Boson			Lie algebra
	FF	C : pre-color interaction h : spinor connection Field	u(3)=1⊕su(3) o(1,1)
	BB	B : pre-flavor interaction	u(2)=1⊕su(2)
Fermion			
	BF	χ : preon (T,V)	
	FB	antiparticles of BF	

Note*B:

Becoming somewhat explanatory, when considering the elementally resonant bonding transition process, examples like the following can be considered.

Here, (...)' represents an intermediate state. r≡GB*,g≡BR*, b≡RG*

$\mu \rightarrow \mu' + rg$: Emit two Bosons
$\nu e + rg \rightarrow \nu'$: Absorb two Bosons
$\nu' \rightarrow \nu_{\mu} + b^*$: Emit 1 Boson
$\mu' + b^* \rightarrow e$: Absorb 1 Boson

In a form where two Bosons are emitted and one is absorbed, essentially moving one while keeping the pre-color neutral:

..

3. Derivation and Consideration of Canonical Gauge Unified Field Equations

3.1.Construction of the Lagrangian

Here, from the canonical gauge connection, we define the canonical curvature, and from the quadratic norm of this, we give the Lagrangian of the unified field including gravity.
From the standpoint of invariant theory, some arbitrariness remains in the definition of the quadratic norm on the canonical gauge ring.
Also, in the coefficient being a noncommutative linear space (like field operators), careful attention is necessary in the definition of norms.

3.1.1. Concept of Canonical Curvature

Consider a set of elements of the canonical gauge ring that gives a Lorentz metric (P_M ; M=1..dim, $\langle P_M|P_N\rangle=\eta_{MN}$). We will call this the canonical Lorentz frame.
For the canonical Lorentz frame, we can define "canonical curvature" in the following form:
Of these, we consider that the one whose canonical curvature norm satisfies the principle of least action gives the physical field. It is appropriate to call the canonical Lorentz frame that satisfies the principle of least action the "parallel translation generator frame" or "infinitesimal parallel translation basis".
The canonical Lorentz frame (P_M ; M=1..dim) together with the unit 1 become the basis of the canonical gauge ring. Including the unit 1, the commutation relations can be written as follows:

$[i\cdot 1, i\cdot 1]=0$ $[i\cdot 1, iP_N]=0$ $[iP_M, i\cdot 1]=0$ $[iP_M, iP_N]=i\cdot R_{MN}$
$\langle 1|1\rangle=1$ $\langle 1| P_M\rangle=0$ $\langle P_M|P_N\rangle=\eta_{MN}$ (Lorentz metric)

We would like to call the above R_{MN} "canonical curvature of the Lorentz frame (P_M) ".
This naming is inspired by the commutator of covariant differentiation giving the curvature form in classical manifold models of spacetime. However, the imaginary unit is set so that R_{MN} may be self-adjoint. Or should it be better said to consider the 2nd order action of $\mathcal{D}\equiv\varepsilon^M P_M$ ($\mathcal{D}^2=\varepsilon^M\varepsilon^N[P_M, P_N]/2=-i\cdot\varepsilon^M\varepsilon^N R_{MN}/2$) with the elements ε^M that commute with P_M and are anticommutative with each other.
(In manifold theory, for the connection of the vector $\mathbf{e}$, $d^2\mathbf{e}=\mathbf{e}\cdot R$; where $d\equiv dx^\lambda\partial_\lambda$.)

The canonical Lorentz frame (P_M) is expressed by the canonical gauge field as follows:

$P_M=\{p_\mu, S^\mu{}_M(x)\}/2+U_M(x)$ ($P_M=P_M^*$) $\langle P_M|P_N\rangle=\eta_{MN}$ (Loren tzmetric)

Let the 0th order coefficient of canonical connection be expressed by U hereafter.

The relationship with the metric g in manifold as the classical spacetime model is as follows.

$S^\mu{}_M\cdot\eta^{MN}\cdot S^\nu{}_N=g^{\mu\nu}$ (μ,ν, M,N)

From the definition, the canonical curvature R is expressed by its components as follows:

$[iP_M, iP_N]=i\cdot R_{MN}$; $R_{MN}=\{P_L, F^L{}_{MN}\}/2+1\cdot E_{MN}$ { , }=anticommutator
note : $\{P_L, F^L{}_{MN}\}/2=\mathrm{Re}(P_L\cdot F^L{}_{MN})$; $\mathrm{Re}(X)\equiv(X+X^*)/2$

The canonical curvature R of the canonical Lorentz frame is represented in components by the above variables F and E.
The variable E coincides with the curvature form in manifold theory. Note that in quantum spacetime, there is no concept like "curvature of spacetime".
If you look for something analogous to the vector connection, it would only be Levi-Civita connection obtained through the metric g induced from the coefficient S. *1

The concept in classical theory of "parallel translation a tangent vector along a closed curve" can be understood as a special example of canonical gauge transformation of canonical momentum p_μ.
Replacing the tangent vector with the canonical momentum operator pμ and taking account of the torsion of this connection, the following formula will be obtained.
However, as is clear from the equation, the transformation target ξ need not be the canonical momentum operator p_μ.

$$[iP_M, [iP_N, \xi]] - [iP_N, [iP_M, \xi]] = [[iP_M.iP_N], \xi] = [iR_{MN}, \xi] \quad ; \ \xi \quad \text{is arbitrary.}$$

It is trivial from the homomorphism related to canonical transformations that the canonical curvature form R has tensor properties (covariance with respect to canonical transformations).

Note*1:

The canonical curvature should be thought of as the "curvature" of the canonical gauge ring, not of "spacetime manifold". Even in the quantum spacetime model, while "curvature of spacetime" can be represented by Riemann's curvature tensor, this is only given through the Levi-Civita connection defined based on the spacetime metric.

3.1.2. Norm Definition of Canonical Curvature Form

Following Maxwell's electromagnetic theory as the model, the Lagrangian of a gauge field in the spacetime model of a manifold is considered to be given by the quadratic norm of the curvature form.
Also in canonical gauge theory, we can think of applying a similar principle to the construction of the Lagrangian.
Since the canonical curvature is a measure of the distortion of the canonical gauge connection, there is a rational basis for obtaining the field equations by applying the principle of least action to its quadratic norm.
In canonical gauge theory, we take the Lagrangian of the field to be the expectation value of the quadratic norm of the canonical curvature R. ***A**

$$[iP_M, iP_N] \equiv i \cdot R_{MN} \quad ; \quad R_{MN} \equiv \{P_L, F^L{}_{MN}\}/2 + 1 \cdot E_{MN}$$

$$\text{Action} \equiv \int L\,(-g)^{1/2} dx \quad ; \quad L \equiv \langle R|R \rangle \times \text{const.}$$

(Especially when R is considered as a functional defined by the Lorentz frame P, we would like to call this the "curvature form".)
Although the above seems to define the Lagrangian by $L = \langle R|R \rangle$, to actually carry out concrete calculations, we need to resolve the issue of arbitrariness in the definition of the norm.
This issue arises from one, the multiplicity of tensor indices, and two, the non-commutativity/ commutation relation of tensor coefficients.

***A** : **Variational Principle and Expectation Value of Action**

As an elementary but fundamental point, since the variational principle is based on the idea of minimizing the action, the action needs to be a real number.
Therefore, although not explicitly stated, the variation of the action should actually be considered as the variation of the "expectation value" of the action with respect to the variation of the operator. And to take the expectation value, the initial state including the total peripheral environments, is necessary.

●Problem of Arbitrariness in Norm Definition

Example-1: Arbitrariness due to multiplicity of tensor indices

For example, consider a tensor F with component representation $F^C{}_{AB}$ relative to a vector basis **e**.
For example, $F = \mathbf{e}_C F^C{}_{AB} \cdot \mathbf{e}^{*A} \mathbf{e}^{*B}$ can be taken, but if the vector basis **e** has canonical commutative relations, an indeterminacy arises in the representation method of the basis.
Since $\mathbf{e}_C\, \mathbf{e}^{*A} \mathbf{e}^{*B} = \delta_C{}^A \pm \mathbf{e}^{*A}\, \mathbf{e}_C\, \mathbf{e}^{*B}$, depending on the setting of the tensor basis, the component $F^A{}_{AB}$ appears.
Therefore, it is not surprising if the quadratic expression $F^A{}_{AB} \cdot F^C{}_C{}^B$ appears in the norm.
From the standpoint of invariant theory, we can take a linear combination of the following quantities for the norm definition of F.

$$\mathrm{Tr}(F^*F) \equiv F_C{}^{AB} \cdot F^C{}_{AB}, \quad \mathrm{Tr}(F^*)\mathrm{Tr}(F) \equiv F^A{}_{AB} \cdot F^C{}_C{}^B, \quad \mathrm{Tr}(FF) \equiv F^A{}_C{}^B \cdot F^C{}_{AB}$$

Therefore, up to this point in the discussion, as the Lagrangian *L* of the unified field, we can adopt a linear combination of Tr(F*F), Tr(FF), Tr(F*) ・ Tr(F), Tr(E*E), 1.
Here, each term means:

$$\mathrm{Tr}(F^*F) = F_C{}^{AB} \cdot F^C{}_{AB}, \ \mathrm{Tr}(FF) = F^A{}_C{}^B \cdot F^C{}_{AB} \quad \mathrm{Tr}(F^*)\mathrm{Tr}(F) = F^A{}_{AB} \cdot F^C{}_C{}^B$$

$$\mathrm{Tr}(E^*E) = E^{AB} \cdot E_{AB} \quad ; 1 \text{ (constant)}$$

Example-2: Arbitrariness in definition of inner product

let us think about the inner product on a linear space E that consist of generally non-commutative elements such as operators or anti-commutative tensors.
it should be defined for the inner product $\langle x|y \rangle$ as a canonically invariant linear mapping from tensor x^*y or yx^*, to the coefficient ring of E, or further to the field of complex numbers.

If the coefficients are noncommutative, there is a choice to take x*y or yx*. A selection rule needs to be established.
Invariant theory does not provide insight into this matter.
Any arbitrary linear combination of canonical invariant contractions concerning component indices, becomes possible for adoption.
The inner product on E needs to be extended to the tensor space ⊗(E⊕E*) containing definition of canonical commutation relations.

As a tentative guideline, the following can be considered:

- for right-coefficient vector $x = \mathbf{e}_A x^A$, $\langle x|x \rangle = \mathrm{prj}(x^*x) = x^{A*} \mathrm{prj}(e_A{}^* \mathbf{e}_B) x^B$
- for left-coefficient vector $x = x_A \mathbf{e}^A$, $\langle x|x \rangle = xx^* = x_A \cdot \mathrm{prj}(e^A \mathbf{e}^B{}^*) x_B{}^*$

Here, taking $\mathrm{prj}(e_A{}^* \mathbf{e}_B) = g_{AB}$, $\mathrm{prj}(e^A \mathbf{e}^B{}^*) = g^{AB}$ as elements of the coefficient ring would suffice. The above does not have the power to extend the definition of inner product on E to the dual space or tensor space. For example, for the dual space, $\langle x^*|y^* \rangle = \varepsilon \langle y|x \rangle$; $\varepsilon^2 = 1$ the negative sign can also be adoptable. For example, considering $x^A = e^{*B} x_B{}^A$, to map to the coefficient ring, a different principle such as the Tr- operation would be necessary.

note:

When the norm is defined from the inner product, since the inner product is determined by the norm definition as follows, it is sufficient to treat only the norm form as the problem.

$<x|y>+<y|x> = <x+y|x+y> - <x|x> - <y|y>$; Consider y→iy together.

●Influence of Norm Definition on Gravity Theory

The commutation formula $[iP_M, iP_N] = i\cdot R_{MN}$; $R_{MN} \equiv \{P_L, F^L{}_{MN}\}/2 + 1\cdot E_{MN}$ can be viewed as a matrix representation of the action of parallel translation generators iP_M ,with respect to the basis of the translation generator (iP_N) .

From this perspective, it is natural to consider contributions from Tr(F*F) and Tr(F*)Tr(F) to the Lagrangian.

From the standpoint of invariant theory, we can further add the constant 1 and $F^A{}_C{}^B \cdot F^C{}_{AB}$.

However, the role of permutations in coupling and contracting indices seems somewhat peculiar.

That being said, such contracted terms also appear in A. Einstein's theory of gravitation when the gravitational field Lagrangian is written as a quadratic form of the connection coefficients Γ.

($L_E \propto \mathrm{Tr}(\Gamma_\sigma \Gamma^\sigma) - \mathrm{Tr}(\Gamma_\sigma)\mathrm{Tr}(\Gamma^\sigma)$; $\Gamma_\sigma = \mathrm{matrix}\ (\Gamma^\lambda{}_{\mu\sigma})$, $\Gamma^\sigma = g^{\sigma\tau}\Gamma_\tau$)

As in A. Einstein's theory of gravity, the existence of the constant term leads to a "cosmological term", but from the perspective of viewing the Lagrangian as a quadratic norm of the curvature tensor, no theoretical rationale for the "cosmological term" can be found.

When the cosmological term exist, it is multiplied by $g^{1/2}d^4x$, and it participates in the motion of spacetime. But, note that the residual values (vacuum expectation values) of Lagrangians of the fields other than gravity also make similar contributions.

When considering cosmology, a mass point set model is not appropriate for the energy -momentum tensor.

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3.1.3. Representation by Gauge Field of Curvature Tensor Form

Let us represent the canonical Lorentz frame (P_M) by canonical gauge field as follows.

$P_M = \{S^\lambda{}_M (x), p_\lambda\}/2 + U_M(x)$.

S is related to the gravitational field, and U is related to other fields, Boson and Fermion fields.

Canonical curvature form $iR_{MN} \equiv [iP_M, iP_N]$, $R_{MN} = R_{MN}{}^* = E_{MN} - \{F^L{}_{MN} (x). P_L\}/2$

Here, we will calculate how the coefficients E, F are represented by the canonical gauge field.

To facilitate calculation, introduce the following notation:

$(f(M,N))_{2[M,N]} \equiv f(M,N) - f(N,M)$; extract double of antisymmetric component

self.adj = general self-adjoint term ; im = i×self.adj

$\nabla_{\mu\nu}(X) \equiv \partial_\mu X_\nu - \partial_\nu X_\mu + i[X_\mu, X_\nu]$, (for tangent vectorial generator X)

●Calculation of Curvature Tensor Representation

First, $iP_M = S^\lambda{}_M \cdot ip_\lambda + \partial_\lambda S^\lambda{}_M/2 + iU_M$. (Move p_λ to the right of $S^\lambda{}_M$)

$[iP_M, iP_N] = (S^\mu{}_M \partial_\mu S^\nu{}_N \cdot ip_\nu)_{2[M,N]} + \mathrm{self.adj} + i(S^\mu{}_M \partial_\mu U_N)_{2[M,N]} + [iU_M, iU_N] = iR_{MN}$

$R_{MN} = (S^\mu{}_M \partial_\mu S^\nu{}_N \cdot p_\nu)_{2[M,N]} + \mathrm{im} + (S^\mu{}_M \partial_\mu U_N)_{2[M,N]} + i\cdot[U_M, U_N]$

Next, R_{MN} should be expressed as a 1-st order expression of the canonical Lorentz frame (P_M).

Note also that the 0-th order term U_M is $U_M = S^\lambda{}_M U_\lambda$. (λ is coordinate index)

From $P_M = S^\lambda{}_M \cdot p_\lambda - i\partial_\lambda S_M/2 + U_M$, by introducing the inverse matrix T of S, the following is obtained:

$p_\lambda = T^M{}_\lambda P_M - U_\lambda + \mathrm{im}$

$S^{\mu}{}_{M}\cdot\partial_{\mu}S^{\nu}{}_{N}.p_{\nu}=S^{\mu}{}_{M}\cdot\partial_{\mu}S^{\nu}{}_{N}(T^{L}{}_{\nu}P_{L}-U_{\nu})+im=-S^{\mu}{}_{M}S^{\nu}{}_{N}\cdot\partial_{\mu}T^{L}{}_{\nu}P_{L}-S^{\mu}{}_{M}\cdot\partial_{\mu}S^{\nu}{}_{N}\cdot U_{\nu}+im$

$S^{\mu}{}_{M}\partial_{\mu}U_{N}=S^{\mu}{}_{M}\partial_{\mu}(S^{\lambda}{}_{N}U_{\lambda})=S^{\mu}{}_{M}\partial_{\mu}S^{\nu}{}_{N}\cdot U_{\nu}+S^{\mu}{}_{M}S^{\nu}{}_{N}\cdot\partial_{\mu}U_{\nu}$

$\therefore\quad R_{MN}=im-(S^{\mu}{}_{M}S^{\nu}{}_{N}\partial_{\mu}T^{L}{}_{\nu}P_{L})_{2[M,N]}+(S^{\mu}{}_{M}S^{\nu}{}_{N}\cdot\partial_{\mu}U_{\nu})_{2[M,N]}+i\cdot[U_{M},U_{N}]$

The above shows that it is natural to introduce the gravitational field strength variable as

$F^{L}{}_{\mu\nu}\equiv\partial_{\mu}T^{L}{}_{\nu}-\partial_{\nu}T^{L}{}_{\mu}$

By setting $R_{MN}\equiv S^{\mu}{}_{M}S^{\nu}{}_{N}R_{\mu\nu}$, the following is obtained.
(transfer to coordinate frame representation)

$R_{MN}=E_{MN}-\{F^{L}{}_{MN},P_{L}\}/2$; $E_{MN}\equiv S^{\mu}{}_{M}S^{\nu}{}_{N}E_{\mu\nu}$, $F^{L}{}_{MN}\equiv S^{\mu}{}_{M}S^{\nu}{}_{N}F^{L}{}_{\mu\nu}$

$E_{\mu\nu}\equiv\nabla_{\mu\nu}(U)$, $F^{L}{}_{\mu\nu}\equiv\partial_{\mu}T^{L}{}_{\nu}-\partial_{\nu}T^{L}{}_{\mu}$,or $R_{\mu\nu}=E_{\mu\nu}-Re(F^{L}{}_{\mu\nu}\cdot P_{L})$

The 0-th order term $E_{\mu\nu}$ with respect to P has the same form as the curvature in the classical manifold model. However, operators are assumed here as connections for the Fermi frame/Bose frame.

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3.2 Explicit Formulas for the Canonical Gauge Lagrangian (Calculations)

3.2.1. Preparation, Definition of Symbols

Here we enter into the concrete calculations for the Lagrangian of the canonical gauge field (the 0-th order coefficient regarding momentum p) based on the curvature norm in the canonical gauge unified field theory.

In order to get a good outlook, we ignore the coupling with the differentials of gravitational field. Therefore, we can assume $S^A{}_\mu = \delta^A{}_\mu$ hereafter. In reality, calculations taking into account the gravitational field becomes very complex.

For example, looking at $\gamma_\mu dx^\mu$ which appears for spinor connection, μ is the index of the coordinate frame, and γ_μ here should precisely be written as the product of γ- matrices γ_A and the gravitational field variable $T = S^{-1}$ as $\gamma_\mu(x) = \gamma_A \cdot T^A{}_\mu(x)$. Of course, $\gamma_\mu(x)$ and the differential operators become non-commutative.

The Dirac operator $\mathcal{D}$ should also be written as $\mathcal{D} \equiv \{\gamma^\mu, p_\mu\}/2$, $p_\mu \equiv g^{-1/4}(-i\partial_\mu)g^{1/4}$.

Since the calculations only include 1st-order differentials, assuming calculations in a geodesic coordinate system, $g = \eta$ (Lorentz metric), $\partial g = 0$. However, the 1st-order differential of the gravitational field variable $T^A{}_\mu(x)$ remains. In other words, the differential coupling of gravity is neglected. Terms other than this can be reproduced according to general covariance after the calculations.

Next, in order to progress the calculations concisely, several notations and symbols are introduced.

[Notation, Symbols, Conventions]

As an abbreviation for curvature, concerning the tangent vectorial generator X_μ:

$$\nabla_{\mu\nu}(X) \equiv \partial_\mu X_\nu - \partial_\nu X_\mu + i[X_\mu, X_\nu]$$

As a notation for symmetry of indices,

$X_{\mu\nu} - X_{\nu\mu} \equiv (X_{\mu\nu})_{2[\mu\nu]} = 2X_{[\mu\nu]}$; double of anti symmetric component of $X_{\mu\nu}$. These are extensions of $[a,b] = ab - ba$

Concerning products of γ matrices, sometimes $\gamma_A\gamma_B = \gamma_{AB}$ is abbreviated. The antisymmetric component is $\gamma_{[AB]}$.

For the self-adjoint and anti self-adjoint components of linear operators, the following expression will be introduced:

$(X+X^*)/2 \equiv \mathrm{Re}(X)$, $(X-X^*)/(2i) \equiv \mathrm{Im}(X)$

For the equivalence as variational forms between two Lagrangians L and L', i.e. "ignoring differences in divergence", the equivalence relation ≃ is introduced.

If $L' - L = \partial_\mu(X^\mu)$ for ∃X, then $L' \simeq L$.

Order and representation of index characters corresponding to degrees of freedom.

Indices are written in the following order : pre-flavor > pre-color > spinor > coordinate frame

pre-flavor: r, s, ... pre-color: m,n,... spinor: j,k,... coordinates: λ,μ,ν

The up and down placement of indices is done freely according to the metric.

3.2.2. Squared norm of the 0th order component of the canonical curvature form

If the Fermi frame is denoted by **a** and the Bose frame by **b**, the 0th order Lorentz frame component $E_{\mu\nu}$ of the canonical curvature form is given by $E_{\mu\nu} \equiv \nabla_{\mu\nu}(U)$ where U is the generator of the canonical gauge connection given by:

$U = U_\lambda dx^\lambda = \mathbf{a}A\mathbf{a}^* + \mathbf{a}\Omega\mathbf{b}^* + \mathbf{b}\Omega^*\mathbf{a}^* + \mathbf{b}B\mathbf{b}^*$, ($U = U^*$, $P_M \equiv \{S^\lambda{}_M . p_\lambda\}/2 + U_M$)

In the representation using the connection form, $d\mathbf{a}=[iU, a]$, $d\mathbf{b}=[iU, \mathbf{b}]$

i.e. $-id\mathbf{a} = \mathbf{a}A + \mathbf{b}\Omega^*$, $-id\mathbf{b} = \mathbf{a}\Omega + \mathbf{b}B$; $\Omega \equiv (\gamma dx)\chi$

Since the coefficients contain non-commutative terms (operators), we will not use ordinary matrix calculations.

Defining $\boldsymbol{a}_{r\lambda} \equiv \mathbf{a}_{mj}(\gamma_\lambda)^j{}_k \chi_r{}^{mk}$, we omit the index r if there is no risk of confusion.

From this, $U_\lambda = \mathbf{a}A_\lambda\mathbf{a}^* + \boldsymbol{a}_\lambda\mathbf{b}^* + \mathbf{b}\boldsymbol{a}_\lambda{}^* + \mathbf{b}B_\lambda\mathbf{b}^*$; $\boldsymbol{a}_{r\lambda} \equiv \mathbf{a}_{mj}(\gamma_\lambda)^j{}_k \chi_r{}^{mk}$

Defining $B' \equiv -B^T$,

Since χ anti-commutes with $\mathbf{a}$, $\boldsymbol{a}_\lambda$ satisfies $[\mathbf{a}^*, \boldsymbol{a}_\lambda] = \gamma_\lambda\chi$, $[\boldsymbol{a}_\mu{}^*, \boldsymbol{a}_\nu] = \chi^*\gamma_\mu\gamma_\nu\chi$ ($\mu \neq \nu$). (see *1)

Also, $A_\lambda = C_\lambda \otimes 1 + 1 \otimes (h\gamma_\lambda)$ = (pre-color degree of freedom)+(spinor degree of freedom).

***1**: Preparatory calculations ; $\boldsymbol{a}_{r\lambda} \equiv \mathbf{a}_{mj}(\gamma_\lambda)^j{}_k \chi_r{}^{mk}$

$[\mathbf{a}^*, \boldsymbol{a}_{r\lambda}] = [\mathbf{a}^*, \mathbf{a}_{mj}(\gamma_\lambda)^j{}_k \chi_r{}^{mk}] = (\gamma_\lambda\chi)_r$,

$[\boldsymbol{a}^s{}_\mu{}^*, \boldsymbol{a}_{r\nu}] = [\boldsymbol{a}^s{}_\mu{}^*, \mathbf{a}_{mj}(\gamma_\nu)^j{}_k \chi_r{}^{mk}] = [\boldsymbol{a}^s{}_\mu{}^*, \mathbf{a}_{mj}](\gamma_\nu)^j{}_k \chi_r{}^{mk} + \mathbf{a}_{mj}[\boldsymbol{a}^s{}_\mu{}^*, (\gamma_\nu)^j{}_k \chi_r{}^{mk}]$

$= [\chi^{*s}{}_{mk}(\gamma_\mu)^k{}_j\, \mathbf{a}^{*mj}, \mathbf{a}_{m'j'}](\gamma_\nu)^{j'}{}_{k'}\chi_r{}^{m'k'} = \chi^{*s}{}_{mk}(\gamma_\mu)^k{}_j(\gamma_\nu)^j{}_{k'}\chi_r{}^{mk'} = (\chi^*\gamma_\mu\gamma_\nu\chi)^s{}_r$

Approach for Lagrangian calculations:

Policy 1: Calculation of $E_{\mu\nu} \equiv \nabla_{\mu\nu}(U)$

Decomposing $U = \Sigma U^{(j)}$; ($U^{(j)}$ is the j-th term of U), we adopt the following calculation method:

$E_{\mu\nu} = \Sigma \nabla_{\mu\nu}(U^{(j)}) + \Sigma\, i \cdot [U_\mu{}^{(j)}, U_\nu{}^{(k)}]_{2[\mu\nu]}$; $j < k$. We also $[U_\mu{}^{(j)}, U_\nu{}^{(k)}] = U^{[jk]}$ in abbreviation.

Policy 2: Calculation of norm 1

Separating E into aa*, ab*, ba*, bb* components and denoting these components as Eaa*, Eab*,..., since they are orthogonal to each other, the norm of E is the sum of norms Nmn of Emn:

norm(E) $= \Sigma$ Nmn ; Nmn ≡ norm of Emn, m=a,b , n=a*b*

Policy 3: Calculation of norm 2 (individual calculations)

Since Emn is a sum of multiple terms, to perform the calculation we subdivide Nmn and separate it into squared terms and cross terms to calculate them individually and take the sum.

(1). The following is obtained as the curvature term of the generator $U^{(j)}$:

$\nabla_{\mu\nu}(U^{(1)}) = \mathbf{a}\nabla_{\mu\nu}(A)\mathbf{a}^*$

$\nabla_{\mu\nu}(U^{(2)}) = \mathbf{a}_{mj}((\gamma_\nu)^j{}_k \cdot \partial_\mu\chi_r{}^{mk} - (\gamma_\mu)^j{}_k \cdot \partial_\nu\chi_r{}^{mk})\mathbf{b}^{*r} = \mathbf{a}(\gamma_\nu\partial_\mu\chi - \gamma_\mu\partial_\nu\chi)\mathbf{b}^*$

$\nabla_{\mu\nu}(U^{(3)}) = \nabla_{\mu\nu}(U^{(2)})^*$

$\nabla_{\mu\nu}(U^{(4)}) = \mathbf{b}(\partial_\mu B_\nu - \partial_\nu B_\mu + i \cdot [B_\mu, B_\nu])\mathbf{b}^* = \mathbf{b}\varepsilon\nabla_{\mu\nu}(B_\nu)\mathbf{b}^* = -\mathbf{b}\nabla_{\mu\nu}(B')^T\mathbf{b}^*$

χ(2). The following is obtained as the cross product of the generator $U^{(j)}$. note : $\boldsymbol{a}_{r\lambda} \equiv \mathbf{a}_{mj}(\gamma_\lambda)^j{}_k \chi_r{}^{mk}$

$U_{\mu\nu}{}^{[12]} = [\mathbf{a}A_\mu\mathbf{a}^*, \boldsymbol{a}_\nu\mathbf{b}^*] = \mathbf{a}(A_\mu\gamma_\nu\chi)_r\mathbf{b}^{r*}$

$U_{\mu\nu}{}^{[13]} = [\mathbf{a}A_\mu\mathbf{a}^*, \mathbf{b}\boldsymbol{a}_\nu{}^*] = -(U_{\mu\nu}{}^{[12]})^*$

$U_{\mu\nu}{}^{[23]} = [\boldsymbol{a}_{r\mu}\mathbf{b}^{r*}, \mathbf{b}_s\boldsymbol{a}^s{}_\nu{}^*] = [\boldsymbol{a}_{r\mu}\mathbf{b}^{r*}, \mathbf{b}_s]\boldsymbol{a}^s{}_\nu{}^* + \mathbf{b}_s[\boldsymbol{a}_{r\mu}\mathbf{b}^{r*}, \boldsymbol{a}^s{}_\nu{}^*] = \boldsymbol{a}_{r\mu}\boldsymbol{a}^r{}_\nu{}^* + \mathbf{b}_s[\boldsymbol{a}_{r\mu}, \boldsymbol{a}^s{}_\nu{}^*]\mathbf{b}^{r*}$

$= \boldsymbol{a}_\mu\boldsymbol{a}_\nu{}^* - \mathbf{b}_s(\chi^*\gamma_\nu\gamma_\mu\chi)^s{}_r\mathbf{b}^{r*}$; $\chi = (\chi_r{}^{mk})$, $\chi^* = (\chi^r{}_{mk})$

$U_{\mu\nu}{}^{[24]} = [\boldsymbol{a}_{r\mu}\mathbf{b}^{r*}, \mathbf{b}B_\nu\mathbf{b}^*] = \boldsymbol{a}_{r\mu}(B_\nu)^r{}_s\mathbf{b}^{s*} = \mathbf{a}(\gamma_\mu\chi B_\nu)_s\mathbf{b}^{s*}$

$U_{\mu\nu}{}^{[34]} = [\mathbf{b}_s\boldsymbol{a}^s{}_\mu{}^*, \mathbf{b}B_\nu\mathbf{b}^*] = -(U_{\mu\nu}{}^{[24]})^*$

(3). The coefficients of the canonical curvature tensor are summarized as follows.

$E_{\mu\nu} = \Sigma\nabla_{\mu\nu}(U^{(j)}) + \Sigma i\cdot(U_{\mu\nu}{}^{[jk]})_{2[\mu\nu]}$; $j < k$

(The coefficients are denoted as $E_{(\mathbf{aa}*)}$... . The expression of indices (μν) are omitted.)

aa* : $E_{(\mathbf{aa}*)} \equiv \nabla_{\mu\nu}(A) + \varepsilon i\cdot(\gamma_\mu\chi\chi^*\gamma_\nu)_{2[\mu\nu]}$

ab* : $E_{(\mathbf{ab}*)} \equiv \gamma_\nu\partial_\mu\chi - \gamma_\mu\partial_\nu\chi + i\cdot(A_\mu\gamma_\nu\chi + \gamma_\mu\chi B_\nu)_{2[\mu\nu]}$

ba* : $E_{(\mathbf{ba}*)} \equiv E_{(\mathbf{ab}*)}{}^*$

bb* : $E_{(\mathbf{bb}*)} \equiv \nabla_{\mu\nu}(B) - i\cdot(\chi^*\gamma_\nu\gamma_\mu\chi)_{2[\mu\nu]} = \nabla_{\mu\nu}(B) + 2(\chi^* i\gamma_{[\mu\nu]}\chi)$

【**Notes**】:

・The term $\nabla_{\mu\nu}(A)$ in $E_{(\mathbf{aa}*)}$ can be decomposed as follows:

$\nabla_{\mu\nu}(A) = \nabla_{\mu\nu}(C)\otimes 1 + 1\otimes\nabla_{\mu\nu}(h\gamma)$

$\therefore\ E_{(\mathbf{aa}*)} = \nabla_{\mu\nu}(C)\otimes 1 + 1\otimes\nabla_{\mu\nu}(h\gamma) + \varepsilon i\cdot(\gamma_\mu\chi\chi^*\gamma_\nu)_{2[\mu\nu]}$

$\nabla_{\mu\nu}(h\gamma)$ takes the following form:

$\nabla_{\mu\nu}(h\gamma) = \partial_\mu h\gamma_\nu - \partial_\nu h\gamma_\mu + i[h\gamma_\mu, h\gamma_\nu] = \partial_\mu h\gamma_\nu - \partial_\nu h\gamma_\mu + 2hh^*\cdot i\gamma_{[\mu\nu]}$

・$E_{(\mathbf{ab}*)}$ suggests the introduction of the Dirac operator.

$E_{(\mathbf{ab}*)} = (\gamma_\nu\partial_\mu\chi + i\cdot A_\mu\gamma_\nu\chi - i\cdot\gamma_\nu\chi B_\mu)_{2[\mu\nu]}$

Defining $i\mathcal{D}'_\mu \equiv \partial_\mu + i\cdot(A_\mu + B'_\mu)$ where $B' \equiv -B^T$

($(\#)^T$ denotes transpose. $\gamma_\nu\chi B_\mu = \gamma_\nu B_\mu{}^T\chi$). The above gives:

$E_{(\mathbf{ab}*)} = i\cdot(\mathcal{D}'_\mu\gamma_\nu - \mathcal{D}'_\nu\gamma_\mu)\chi$

$\mathcal{D}'_\mu$ is self-adjoint. This seems to invite the $\gamma^\mu\mathcal{D}'_\mu$, but it breaks self-adjointness by multiplying γ^μ to $A_\mu = C_\mu\otimes 1 + 1\otimes(h\gamma_\mu)$ due to spinor related component in A.

Then, $\mathcal{D}_\mu$ is newly defined as follows:

$E_{(\mathbf{ab}*)} = i\cdot(\mathcal{D}_\mu\gamma_\nu - \mathcal{D}_\nu\gamma_\mu)\chi + 2h\cdot i\gamma_{[\mu\nu]}\chi$

・For $E_{(\mathbf{bb}*)}$, note that with $E_{(\mathbf{bb}*)}$, $\nabla_{\mu\nu}(B)^T = -\nabla_{\mu\nu}(B')$.

3.2.3. Calculation of individual elements of the canonical gauge field quadratic norm

Under the preparation in the previous section, let us calculate the Lagrangian $L = (+1/4)\langle E_{\mu\nu}|E^{\mu\nu}\rangle$. The spacetime dimension is 4, so the spinor degree of freedom is also 4. (The coefficient 1/4 is for convenience.)

To calculate the Lagrangian L, it is necessary to determine the quadratic norm for each orthogonal term constituting $E_{\mu\nu}$. As described above, there is arbitrariness in the definition of the inner product on the tensor space, and the choice of its application affects the result of the quadratic norm calculation. E is a right-coefficient type ,with respect to frame **(a)**.

In this case, the relation due to the anti-commutativity of the Fermion field expressed by

$E_{(\mathbf{ab}*)}{}^* E_{(\mathbf{ab}*)} + E_{(\mathbf{ab}*)} E_{(\mathbf{ab}*)}{}^* = 0$ becomes a problem.

As it turns out by actually developing the calculation, it could be considered that the following can be taken as physical requirements for the theory:

One is the existence of the Dirac equation, that is, the factorizability of the quadratic form of the Dirac operator in $R[i\gamma^5]$, and the other is the disappearance of Pauli term in the pre-flavor u(2) interaction.

(1). Calculation of $E_{(aa^*)}$ norm elements;

$N_{(aa^*)} \equiv (1/4)\mathrm{Tr}(E_{\mu\nu(aa^*)} \cdot E^{\mu\nu}{}_{(aa^*)})$

$E_{(aa^*)} = \nabla_{\mu\nu}(C) \otimes 1 + 1 \otimes \nabla_{\mu\nu}(h\gamma) + i \cdot (\gamma_\mu \chi\chi^* \gamma_\nu)_{2[\mu\nu]}$

Let's denote the contribution to $N_{(aa^*)}$ of the product of the j-th term and the k-th term of $E_{(aa^*)}$ by $N_{aa^*}(j\times k)$.

$N_{(aa^*)} = \Sigma N_{aa^*}(i\times i) + 2\Sigma N_{aa^*}(j\times k)$; $j < k$ Calculated according to this expansion.

●$N_{aa^*}(i\times i)$: First, consider the self-square terms $N_{aa^*}(i\times i)$; i=1..3.

$N_{aa^*}(1\times 1) = (4/4)\mathrm{Tr}(\nabla_{\mu\nu}(C)\nabla^{\mu\nu}(C))$

; The factor 4 comes from the trace regarding the spinor degree of freedom.
Take the trace regarding the pre-color degree of freedom.

$N_{aa^*}(2\times 2) = (3/4)\mathrm{Tr}(\nabla_{\mu\nu}(h\gamma)\nabla^{\mu\nu}(h\gamma))$

; The factor 3 comes from the trace regarding the pre-color degree of freedom.
Specific calculations will be described later.

$N_{aa^*}(3\times 3) = (2/4)\mathrm{Tr}(i \cdot (\gamma_\mu \chi\chi^* \gamma_\nu - \gamma_\nu \chi\chi^* \gamma_\mu) \cdot (i \cdot \gamma^\mu \chi\chi^* \gamma^\nu))$

Specific calculations will be described later.

$N_{(aa^*)} \equiv (1/4)\mathrm{Tr}(E_{\mu\nu(aa^*)} \cdot E^{\mu\nu}{}_{(aa^*)})$

$E_{(aa^*)} = \nabla_{\mu\nu}(C) \otimes 1 + 1 \otimes \nabla_{\mu\nu}(h\gamma) + i \cdot (\gamma_\mu \chi\chi^* \gamma_\nu)_{2[\mu\nu]}$

Specific calculations will be described later.

・Calculation of Naa*(2×2):

Consider the expansion terms of $\nabla_{\mu\nu}(h\gamma)$. $\nabla_{\mu\nu}(h\gamma) = (\partial_\mu h\gamma_\nu - \partial_\nu h\gamma_\mu) + 2hh^* \cdot i\gamma_{[\mu\nu]}$

Taking into account that the γ matrix is traceless in $\mathrm{Tr}(\nabla_{\mu\nu}(h\gamma)\nabla^{\mu\nu}(h\gamma))$, the contribution of the cross term disappears. Therefore

$\mathrm{Tr}(\nabla_{\mu\nu}(h\gamma)\nabla^{\mu\nu}(h\gamma)) = 2\mathrm{Tr}((\partial_\mu h\gamma_\nu(\partial^\mu h\gamma^\nu - \partial^\nu h\gamma^\mu)) + 4 \cdot \mathrm{Tr}((hh^*)^2 \times 12)$

Note that $hh^* = h^*h$ is a 4×4 scalar matrix.

The factor 12 comes from $i\gamma_{[\mu\nu]} \cdot i\gamma^{[\mu\nu]} = \gamma_{\mu\nu} \cdot \gamma^{\nu\mu}$; $\mu \neq \nu$.

By writing the first term as the operator form $2\mathrm{Tr}(-h \cdot \gamma_\nu \partial_\mu (\partial^\mu \gamma^\nu - \partial^\nu \gamma^\mu) \cdot h^*)$ to obtain the following.

(First term) $\simeq 2\mathrm{Tr}(-h \cdot \partial_\mu \gamma_\nu \gamma^\nu \partial^\mu h^* + h\partial_\mu \gamma_\nu \gamma^\mu \partial^\nu h^*) = 2\mathrm{Tr}(-3h\partial_\mu \partial^\mu h^*) = -6\mathrm{Tr}(h\partial_\mu \partial^\mu h^*)$

Here, the symbol ≃ means canonical equivalence.; (described above)

$\mathrm{Tr}(\nabla_{\mu\nu}(h\gamma)\nabla^{\mu\nu}(h\gamma)) = 2\mathrm{Tr}((\partial_\mu h\gamma_\nu(\partial^\mu h\gamma^\nu - \partial^\nu h\gamma^\mu)) + 4 \cdot \mathrm{Tr}((hh^*)^2 \times 12)$

Taking into account the commutativity of h and h*, we obtain the following.

$N_{aa^*}(2\times 2) = -(9/2)\mathrm{Tr}(h^* \partial_\mu \partial^\mu h) + 3\times 12 \cdot \mathrm{Tr}((h^*h)^2)$

; Taking the trace regarding the spinor degree of freedom.

・ Calculation of Naa*(3×3):

First, move the terms of χ* to the front. At this time, the sign changes due to the anti-commutativity of χ*. Also, when χ* exceeds χ, infinity may occur, but we assume that it is canceled as a whole and we proceed with the calculation. (χ* intersects χ twice and χ* once)

$N_{aa*}(3\times3) = +(2/4)Tr(i\cdot(\gamma_\mu\chi\chi^*\gamma_\nu - \gamma_\nu\chi\chi^*\gamma_\mu)\cdot(i\cdot\gamma^\mu\chi\chi^*\gamma^\nu))$

$= -(2/4)Tr(\chi^*\gamma^\nu i\cdot(\gamma_\mu\chi\chi^*\gamma_\nu - \gamma_\nu\chi\chi^*\gamma_\mu)\cdot i\gamma^\mu\chi))$

$= -(2/4)Tr(\chi^* i\gamma_{[\nu\mu]}\chi\cdot\chi^* i\gamma^{[\nu\mu]}\chi + 12\chi^*\chi\cdot\chi^*\chi)$

Take the trace regarding the pre-flavor degree of freedom.

We will consider the meaning and interpretation of the 4th order term later.

●$N_{aa*}(i\times j)$; $i<j$ (Successively, let us consider the cross term)

$E_{(aa*)} = \nabla_{\mu\nu}(C)\otimes1 + 1\otimes\nabla_{\mu\nu}(h\gamma) + i\cdot(\gamma_\mu\chi\chi^*\gamma_\nu)_{2[\mu\nu]}$; $\nabla_{\mu\nu}(h\gamma) = (\partial_\mu h\gamma_\nu - \partial_\nu h\gamma_\mu) + 2hh^*\cdot i\gamma_{[\mu\nu]}$

・ $N_{aa*}(1\times2) = (1/4)Tr(\nabla_{\mu\nu}(C)\otimes\nabla^{\mu\nu}(h\gamma)) = 0$;

Due to the γ matrix being traceless.

・ $N_{aa*}(1\times3) = (2/4)Tr(\nabla_{\mu\nu}(C)\cdot i\cdot(\gamma^\mu\chi\chi^*\gamma^\nu)) = -(2/4)Tr(\chi^*\gamma^\nu\nabla_{\mu\nu}(C)\cdot i\cdot\gamma^\mu\chi)$

$= (1/2)Tr(\chi^*\nabla_{\mu\nu}(C)\cdot i\gamma^{[\mu\nu]}\chi)$

$N_{aa*}(2\times3) = (2/4)Tr(\nabla_{\mu\nu}(h\gamma)\cdot i\cdot(\gamma^\mu\chi\chi^*\gamma^\nu)) = -(2/4)Tr(\chi^*(i\gamma^\nu\nabla_{\mu\nu}(h\gamma)\gamma^\mu)\chi)$

$\chi\chi \equiv -(1/2)\cdot Tr(\chi^* K''\chi)$. Here, let $K'' \equiv i\gamma^\nu\nabla_{\mu\nu}(h\gamma)\gamma^\mu$.

Applying the expansion formula of $\nabla_{\mu\nu}(h\gamma)$ and paying attention to $\mu\neq\nu$, we obtain the following.

$K'' = i\gamma^\nu(\partial_\mu h\gamma_\nu - \partial_\nu h\gamma_\mu)\gamma^\mu + i\gamma^\nu(2hh^*\cdot i\gamma_{[\mu\nu]})\gamma^\mu$

$= i(\gamma^\nu\partial_\mu h\gamma_\nu\gamma^\mu - \gamma^\nu\partial_\nu h\gamma_\mu\gamma^\mu) + i\gamma^\nu 2hh^*\cdot i\gamma_{[\mu\nu]}\gamma^\mu = +24h^*h$

∴ $N_{aa*}(2\times3) = -12\cdot Tr(\chi^*h^*h\chi)$

●Summary of $N_{(aa*)}$: $N_{(aa*)} = \Sigma N_{aa*}(i\times i) + 2\Sigma N_{aa*}(j\times k)$; $j<k$

$N_{aa*}(1\times1) = Tr(\nabla_{\mu\nu}(C)\nabla^{\mu\nu}(C))$

$N_{aa*}(2\times2) = -(9/2)Tr(h^*\partial_\mu\partial^\mu h) + 3\times12\cdot Tr((h^*h)^2)$

$N_{aa*}(3\times3) = -(1/2)Tr(\chi^* i\gamma_{[\nu\mu]}\chi\cdot\chi^* i\gamma^{[\nu\mu]}\chi + 12\chi^*\chi\cdot\chi^*\chi)$

$N_{aa*}(1\times2) = 0$

$N_{aa*}(1\times3) = (1/2)Tr(\chi^*\nabla_{\mu\nu}(C)\cdot i\gamma^{[\mu\nu]}\chi)$

$N_{aa*}(2\times3) = -12\cdot Tr(\chi^*hh^*\cdot\chi)$

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(2). Calculation of $E_{(ab*)}$ norm elements ;

$N_{ab*} \equiv +(1/4)Tr(E_{\mu\nu(ab*)}{}^*\cdot E^{\mu\nu}{}_{(ab*)})$

$E_{(ab*)} = i\cdot(\mathcal{D}_\mu\gamma_\nu - \mathcal{D}_\nu\gamma_\mu)\cdot\chi + 2h\cdot i\gamma_{[\mu\nu]}\cdot\chi$; $i\mathcal{D}_\mu \equiv \partial_\mu + i\cdot(C_\mu + B'_\mu)$

The norm formula can be written in operator form as follows.

$N_{ab*} = (2/4)Tr(\chi^* K'\chi)$; $K' \equiv (-i\cdot(\gamma_\nu\mathcal{D}_\mu - \gamma_\mu\mathcal{D}_\nu) + 2i\gamma_{[\mu\nu]}h^*)\cdot(i\cdot\mathcal{D}^\mu\gamma^\nu + h\cdot i\gamma^{[\mu\nu]})$

Expanding K' and considering the sum for $\mu\neq\nu$,

$K' = (\gamma_\nu\mathcal{D}_\mu - \gamma_\mu\mathcal{D}_\nu)\cdot\mathcal{D}^\mu\gamma^\nu - 2\gamma_{[\mu\nu]}h^*\cdot\mathcal{D}^\mu\gamma^\nu - 2\gamma_\mu\mathcal{D}_\nu h\cdot\gamma^{[\mu\nu]} + 2i\gamma_{[\mu\nu]}h^*h\cdot i\gamma^{[\mu\nu]}$

$= 3\mathcal{D}_\mu\mathcal{D}^\mu + \gamma^{[\mu\nu]}\mathcal{D}_\mu\mathcal{D}_\nu - 6\gamma_\mu h\mathcal{D}^\mu - 6\mathcal{D}_\nu h^*\gamma^\nu + 24h^*h$

Here, we introduce the Dirac operator : $\mathcal{D} \equiv \gamma^\nu\mathcal{D}_\nu = \mathcal{D}_\nu\gamma^\nu$.

$\mathcal{D}$ is self-adjoint ($\mathcal{D} = \mathcal{D}^*$) and $\mathcal{D}^2$ is related to $\mathcal{D}_\mu\mathcal{D}^\mu$ as follows.

$\mathcal{D}^2 = \mathcal{D}_\mu \gamma^\mu \gamma^\nu \mathcal{D}_\nu = \mathcal{D}_\mu \mathcal{D}^\mu + \gamma^{[\mu\nu]} \mathcal{D}_\mu \mathcal{D}_\nu$; $\gamma^{[\mu\nu]} \mathcal{D}_\mu \mathcal{D}_\nu = \gamma^{[\mu\nu]} [\mathcal{D}_\mu, \mathcal{D}_\nu]/2 = -i\gamma^{[\mu\nu]} \nabla_{\mu\nu}(C+B')/2$

From the above

$K' = 3\mathcal{D}_\mu \mathcal{D}^\mu + \gamma^{[\mu\nu]} \mathcal{D}_\mu \mathcal{D}_\nu - 6h^*\mathcal{D} - 6\mathcal{D}h + 24h^*h$; $\mathcal{D}_\mu \mathcal{D}^\mu = \mathcal{D}^2 - \gamma^{[\mu\nu]} \mathcal{D}_\mu \mathcal{D}_\nu$

∴ $K' = 3\mathcal{D}^2 + i\gamma^{[\mu\nu]} \nabla_{\mu\nu}(C+B') - 6h^*\mathcal{D} - 6\mathcal{D}h + 24h^*h$

●Summary of $N_{(\mathbf{ab^*})}$:

$N_{\mathbf{ab^*}} = N_{\mathbf{ba^*}} = (1/2)\mathrm{Tr}(\chi^* K' \chi)$;

$K' = 3\mathcal{D}^2 - 6h^*\mathcal{D} - 6\mathcal{D}h + 24h^*h + i\gamma^{[\mu\nu]} \nabla_{\mu\nu}(C+B')$,

where $\mathcal{D}_\mu \equiv -i\partial_\mu + (C_\mu + B'_\mu)$, $\mathcal{D} \equiv \gamma^\nu \mathcal{D}_\nu$

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(3). Calculation of $E_{(\mathbf{bb^*})}$ norm elements;

$N_{\mathbf{bb^*}} \equiv (1/4)\mathrm{Tr}(E_{\mu\nu(\mathbf{bb^*})} E^{\mu\nu}{}_{(\mathbf{bb^*})})$

$E_{(\mathbf{bb^*})} \equiv \nabla_{\mu\nu}(B) + 2(\chi^* i\gamma_{[\mu\nu]}\chi)$

- $N_{bb^*}(1\times1) = (1/4)\mathrm{Tr}(\nabla_{\mu\nu}(B) \cdot \nabla^{\mu\nu}(B))$

  From $\nabla_{\mu\nu}(B) = -\nabla_{\mu\nu}(B')^T$, by using B', $N_{bb^*}(1\times1) = (1/4)\mathrm{Tr}(\nabla_{\mu\nu}(B') \cdot \nabla^{\mu\nu}(B'))$
- $N_{bb^*}(2\times2) = (4/4)\mathrm{Tr}(\chi^* i\gamma_{[\mu\nu]}\chi \cdot \chi^* i\gamma^{[\mu\nu]}\chi)$
- $N_{bb^*}(1\times2) = (2/4)\mathrm{Tr}(\nabla_{\mu\nu}(B) \cdot \chi^* i\gamma^{[\mu\nu]}\chi)$

  In the formula using B', $N_{bb^*}(1\times2) = -(1/2)\mathrm{Tr}(\chi^* \nabla_{\mu\nu}(1B') \cdot i\gamma^{[\mu\nu]}\chi)$

  $\nabla_{\mu\nu}(B)^r{}_s \cdot (\chi^* i\gamma^{[\mu\nu]}\chi)^s{}_r = -\nabla_{\mu\nu}(B')_s{}^r \cdot (\chi^* i\gamma^{[\mu\nu]}\chi)^s{}_r$; $\chi B = -B'\chi$, $B\chi^* = -\chi^* B'$ $(B^* = B)$

● Summary of $N_{(\mathbf{bb^*})}$: $N_{(\mathbf{bb^*})} = \Sigma N_{bb^*}(i\times i) + 2\Sigma N_{bb^*}(j\times k)$; $j < k$

$N_{bb^*}(1\times1) = (1/4)\mathrm{Tr}(\nabla_{\mu\nu}(B') \cdot \nabla^{\mu\nu}(B'))$

$N_{bb^*}(2\times2) = \mathrm{Tr}(\chi^* i\gamma_{[\mu\nu]}\chi \cdot \chi^* i\gamma^{[\mu\nu]}\chi)$

$N_{bb^*}(1\times2) = -(1/2)\mathrm{Tr}(\chi^* \nabla_{\mu\nu}(B') \cdot i\gamma^{[\mu\nu]}\chi)$

.

3.3.Construction of Lagrangian of Canonical Gauge Field

Based on the individual element calculations of the quadratic norm in the previous sections, compose the elements according to the norm interpretation to obtain the Lagrangian. There seems to be two notable facts, as follows:

・The possibility of factorizing the quadratic form of the Dirac operator
・The absence of the Pauli term regarding the electromagnetic field

The terms of the Lagrangian should be classified and organized in consideration of comparison with conventional theories.
However, since the fields dealt with in the canonical gauge unified field theory are, as already described, consistent with preon model (rishon model) and it is elemental, there might be in limitations to complete comparison with conventional concepts.
They are classified as follows:

Classification :
(1). Fermion quadratic term $L(\chi\chi^*1)$ / (2). Fermion quartic term $L(\chi\chi^*2)$ /
(3). Spinor connection Boson L(h) / (4). Pre-color Boson L(C) / (5). Pre-flavor Boson L(B)

Table 3.3.1 Correspondence table of actions in canonical gauge field
(decomposition of actions by classified terms)

No	Cls.	$E_{(aa^*)}E_{(aa^*)}$ ref_aa	$E_{(ab^*)}{}^* \cdot E_{(ab^*)}$ ref_ab	$E_{(bb^*)}E_{(bb^*)}$ ref_bb
(1)	$L(\chi\chi^*1)$	$2 \cdot N_{aa^*}(1\times3)$, $2 \cdot N_{aa^*}(2\times3)$	$2 \cdot N_{ab^*}$	$2 \cdot N_{bb^*}(1\times2)$
(2)	$L(\chi\chi^*2)$	$N_{aa^*}(3\times3)$		$N_{bb^*}(2\times2)$
(3)	L(h)	$N_{aa^*}(2\times2)$		
(4)	L(C)	$N_{aa^*}(1\times1)$		
(5)	L(B)			$N_{bb^*}(1\times1)$

(1).Fermion quadratic term $L(\chi\chi^*1)$

The Fermion quadratic term part of the Lagrangian of the canonical gauge field is calculated as follows:

$+)\ 2N_{aa^*}(1\times3) = \mathrm{Tr}(\chi^*\nabla_{\mu\nu}(C)\cdot i\gamma^{[\mu\nu]}\chi)$
$+)\ 2N_{aa^*}(2\times3) = -24\cdot\mathrm{Tr}(\chi^* hh^*\cdot\chi)$
$+)\ 2N_{ab^*} \equiv \mathrm{Tr}(\chi^* K'\chi)$; $K' = 3\mathcal{D}^2 - 6h^*\mathcal{D} - 6\mathcal{D}h + 24h^*h + i\gamma^{[\mu\nu]}\nabla_{\mu\nu}(C+B')$,
$+)\ 2N_{bb^*}(1\times2) = -\mathrm{Tr}(\chi^*\nabla_{\mu\nu}(B')\cdot i\gamma^{[\mu\nu]}\chi)$

$\therefore\ L(\chi\chi^*1) = \mathrm{Tr}(\chi^* K\chi)$; ref= Tab 3.3.1
$K \equiv 3\mathcal{D}^2 - 6h^*\mathcal{D} - 6\mathcal{D}h + 2i\gamma^{[\mu\nu]}\nabla_{\mu\nu}(C)$; $\mathcal{D}_\mu \equiv -i\partial_\mu + (C_\mu + B'_\mu)$

Regarding the pre-flavor interaction ($u(2) = su(2) \oplus u(1)$), we can confirm by the above that the Pauli term has disappeared.
For leptons, since the existence of the Pauli term in interaction with the electromagnetic field is denied, it is a desirable result that the Pauli term disappears in interaction with an unconfined field.

On the other hand, regarding the pre-color interaction, the Pauli term $2\chi^*(i\gamma^{[\mu\nu]}\nabla_{\mu\nu}(C))\chi$ appears.

Since this pre-color interaction is confined within quarks/leptons which are composite particles of preons, the existence itself cannot be said to be unreasonable.
If " e,μ,τ ; u,c,t ; d*,s*,b* " are viewed as excitation series of preon binding, mass spectrum is expected.

Regarding the mass related matters, the quadratic form of the Dirac operator about Fermions (preons) is:

$$3\mathcal{D}^2 - 6h^*\mathcal{D} - 6\mathcal{D}h = 3(\mathcal{D}-4h^*)\mathcal{D} + 6i(\gamma^\mu\partial_\mu h) = 3\mathcal{D}(\mathcal{D}-4h) - 6i(\gamma^\mu\partial_\mu h)$$

Ignoring the differential coupling of χ with h (terms of $\partial_\mu h$), 4h and 0 become masses (in the sense of unperturbed and elementary state) by factorization.
In the case of deriving Dirac equation from the Klein-Gordon equation , one factor is discarded in the course of factorization of quadratic form.
This is possible and reasonable choice because It is only the process of obtaining the equation for electrons.
On the other hand, for equations derived from the principle of unified field theory, there is no reason why only one should be adopted and the other excluded. Even if the probability of being excited as a spacetime vibration differs due to symmetry breaking, there is no reason why such states do not exist. ***A**

For example, with rishon model, it is quite conceivable from the connection to the chirality that the correspondence of V-particles with $(\mathcal{D}-4h)$ factor and T-particles with $\mathcal{D}$ factor.
Then it would be possible to explain why the pre-flavor degree of freedom is 2 and why neutrinos have a chiral bias..

For the Lagrangian $L \equiv \chi^*\mathcal{D}(\mathcal{D}-4h)\chi$, and if $(4h)(4h)^* = m^2 > 0$, then it is transformed to $L \equiv \chi'^*\mathcal{D}(\mathcal{D}-m)\chi'$ by a canonical unitary chiral transformation $\chi \to \chi' = m \cdot \exp(i\gamma^5\theta)\chi$.
Taking the initial solution of $(\mathcal{D}-m)\chi' = 0$ and separating χ' into left-handed and right-handed component as $\chi' = \chi'_L + \chi'_R$, we can get $\chi = \exp(-i\gamma^5\theta)\cdot\chi' = e^{+\theta}\cdot\chi'_L + e^{-\theta}\chi'_R$, from the formula $\exp(-i\gamma^5\theta) = e^{+\theta}\cdot L + e^{-\theta}\cdot R$.
Left-handed predominance is derived for $\theta > 0$, if χ'_L and χ'_R are in the same order of magnitude.
Where, θ is determined by $4h\cdot\exp(2i\gamma^5\theta) = m$.

The above facts compel a change in conventional thinking regarding the existence of Fermions.
What it means can be written as follows:

- The equation for Fermions is originally 2nd-order with respect to the Dirac operator $\mathcal{D}$.
  This is a consequence of the Lagrangian being a quadratic form with respect to the 1st-order derivative.
  $\ker(\mathcal{D}(\mathcal{D}-4h)) = \ker(\mathcal{D}) \oplus \ker(\mathcal{D}-4h)$ for $h \neq 0$ and the solution space is decomposed into two parts.

- The Fermion equation corresponds to the "infinitesimal oscillation (unperturbed state)" of spacetime, and on the ring $\mathbb{R}[i\gamma^5]$, it is decomposed into a 1-st order equation of $\mathcal{D}$.
  It becomes that the oscillation is expressed by approximation expansion around the zero points of the decomposition factors of the quadratic form related to the Dirac operator $\mathcal{D}$, with the other factors approximated by the 1-st order term of the gauge field h of spinor connection.

The factorization of quadratic form concerning $\mathcal{D}$ is considered to be related to the existence of two types of preons (T, V). However, it might not be so simply easy to associate them 1:1.

For example, in case $(\mathcal{D}\text{-}4h)\chi=0$, χ is coupled with $\gamma^{\mu}C_{\mu}$ and $\gamma^{\mu}B'_{\mu}$, because $\mathcal{D}=\gamma^{\mu}(p_{\mu}+C_{\mu}+B'_{\mu})$. If identifying χ such that $(\mathcal{D}\text{-}4h)\chi=0$ as V, some mechanism that eliminates electromagnetic interaction, there should be, that is, the coupling constant with the electromagnetic field in u(2) must be 0. (since V has no charge).

On the other hand, pre-color interaction C or pre-flavor interaction B' for preon alone do not have much meaning, and their meaning seems to be determined only when the preons have interaction partners.

This can be seen as an example that, in the interaction of T-T and V-V , with $B'=B^0+B^1\sigma_1+B^2\sigma_2+B^3\sigma_3$,because of no pre-flavor exchange, the effective expectation value of B^1, B^2 becomes 0 .

It should also be noted that charge is carried in T-V exchange interaction by the B field.

・The particle concept corresponds to the infinitesimal oscillation of spacetime.

Taking into account higher order effects, the state is a multiparticle hybrid state of various oscillations. (In general, the state vector is a tensor corresponding to the basis vibrations)

This fact is related to the representation of field operators and methods of constructing approximate solutions. Consideration from the viewpoint of solution construction will be investigated in Part II.

・The chiral characteristics of preon is determined by the selection of factors in the Dirac operator quadratic form, that is, the selection of the initial approximation of the infinitesimal oscillation. If the gauge field h of spinor connection contains a constant component as an expected value, Dirac equation can be expressed in the form of definitive mass by taking the spinor representation that diagonalizes the constant component of h. ***B**

If this is cosmic situation, it can be imagined that gauge field h is in a state like Bose-Einstein condensation all over the universe.

h can be a course of dark energy.

(In addition to this, it also seems possible to consider the white combination composite of pre-color Bosons as a cause of dark energy.)

***A :**

Based on the theory of fields, it seems to be interpretable that (D-m) and (D+m) specify the same state. Because they are transformed to each other by $i\gamma^5$.

By $\chi\rightarrow i\gamma^5\chi$; $\chi^*(\mathcal{D}+m)\chi\rightarrow\chi^*(\mathcal{D}\text{-}m)\chi$, $(1+i\gamma^5)\chi\rightarrow(i\gamma^5+1)\chi$

Even if two types are assumed, they could not be distinguished from each other as solitary existence.

***B:**

The action induced by K , $\chi^*\mathcal{D}(\mathcal{D}\text{-}4h)\chi$, is mapped as below by the transform $\chi\rightarrow U\chi$, $U\equiv\exp(i\gamma^5\theta)$.

$\chi^*\mathcal{D}(\mathcal{D}\text{-}4h)\chi \rightarrow \chi^*\mathcal{D}(\mathcal{D}\text{-}4UhU)\chi$. (note : $\gamma^5=\gamma^0\gamma^1\gamma^2\gamma^3$: self-adjoint)

If $hh^*>0$, $\theta\in R$ can be chosen such that $UhU=m>0$.

$\theta\in R$ can be taken as $h=h_0+i\gamma^5\cdot h_5=m\exp(-2i\gamma^5\theta)=m\cdot(ch(2\theta)+i\gamma^5\cdot sh(2\theta))$.

If $hh^*<0$ then $UhU=i\gamma^5\cdot m$ is obtained .

It is possible to change the representation of γ^μ to $\gamma^\mu\gamma^5$.
(note: $\gamma^\mu\gamma^5\cdot\gamma^\nu\gamma^5 = -\gamma^\mu\gamma^\nu\cdot\gamma^5\gamma^5 = \gamma^\mu\gamma^\nu$. $\gamma^\mu \to \gamma^\mu\gamma^5 = w\gamma^\mu w^{-1}$; $w \equiv (1-\gamma^5)/\sqrt{2}$)

(2).Fermion quartic term L(χχ*2)

$L(\chi\chi^*2) = (1/2)Tr(\chi^* i\gamma_{[\nu\mu]}\chi\cdot\chi^* i\gamma^{[\nu\mu]}\chi) - (1/2)Tr(12\chi^*\chi\cdot\chi^*\chi)$; ref=Table 3.3.1

Tr operation shall be taken concerning pre-flavor index.

Historically, the Dirac equation was assumed as the wave function of electrons and was not derived from principles as a field equation. Therefore, it is natural that there are no higher order terms. Probably, the effect of the quartic term appears in the collection of two or more Fermions (preons). The above does not take into account the normalization with respect to the field variables. So considering the normalization, the effect of the quartic term may be very small. Also, if $\chi^* i\gamma_{[\nu\mu]}\chi = 1/2$ $(\mu < \nu)$ then $L(\chi\chi^*2) = -(1/2)Tr(12\chi^*\chi\cdot\chi^*\chi)$

In the canonical gauge unified field theory, the interaction constants are expected to be derived from the unified and appropriate normalization regarding the fields derived in the theory.

(3).spinor connection Boson L(h)

$L(h) = -(9/2)Tr(h^*\partial_\mu\partial^\mu h) + 3\times12\cdot Tr((h^*h)^2)$; ref=Table 3.3.1

h is a 4×4 matrix belonging to $\mathbf{R}[i\gamma^5]$, and in component representation $h = h_0\cdot 1_4 + h_5\cdot i\gamma^5$.

Therefore $h^*h = hh^* = (h_0^2 - h_5^2)\cdot(1)_4$

L(h) is a self-interaction term of h, but in addition to this, for the spinor connection gauge field (h), there is the interaction term $-6\cdot Tr(\chi^*(h^*\mathcal{D}+\mathcal{D}h)\chi)$. This term, contained in L(χχ*1) , shows the interaction with Fermion field χ.

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If the initial approximate solution for χ is $\mathcal{D}\chi \fallingdotseq 0$, the initial approximate value of the above interaction is 0. That is, h considered as a wave function is hardly excited by this action.
On the other hand, if the initial approximate solution is $(\mathcal{D}-4h)\chi \fallingdotseq 0$, then, from $\mathcal{D}\to 4h$, an effective (h*h) quadratic term appears in the Lagrangian of h.
Considering the existence of the quartic term in L(h), the Higgs mechanism might be naturally recalled.
The existence of preon χ such as $(\mathcal{D}-4h)\chi \fallingdotseq 0$ generates the mass of h.
The systematic calculations for them must be carried out through the variational method.

(4).pre-color Boson L(C)

$L(C) = Tr(\nabla_{\mu\nu}(C)\nabla^{\mu\nu}(C))$; $\nabla_{\mu\nu}(C) \equiv \partial_\mu C_\nu - \partial_\nu C_\mu + i[C_\mu, C_\nu]$, ref=Table 3.3.1

This is a typical Lagrangian for Bosons with the quadratic norm of curvature.
The pre-color field C can be represented as a 3×3 Hermitian matrix which is an element of u(3).
(The U(3) infinitesimal transformation is $i\cdot C$)
By introducing the basis Λ of the Lie ring, the structure constant κ is introduced and it becomes as follows:

$C_\mu = \Lambda_b\cdot C^b{}_\mu$, $i[C_\mu, C_\nu] = \Lambda_c\cdot\kappa^c{}_{ab}C^a{}_\mu C^b{}_\mu$, $\nabla_{\mu\nu}(C) = \Lambda_c\cdot(\partial_\mu C^c{}_\nu - \partial_\nu C^c{}_\mu + \kappa^c{}_{ab}C^a{}_\mu C^b{}_\mu)$

Although the component $\Lambda_0 C^0{}_\mu$ as a component of the field of FF-type interaction feels unlikely, theoretically there is no reason to eliminate the phase transformation basis$\Lambda_0 = 1$.
Also, distinction from the phase transformation of BB-type interaction cannot be made.
From the gauge invariance regarding C, for example, $\partial_\mu C^\mu = 0$ should be introduced as a gauge fixing. (A Lagrangian that breaks gauge invariance may be additionally added.)

The fact to be noted as a Boson field Lagrangian is the absence of a mass term like that appearing in the Klein-Gordon equation.
This suggests that to require a re-examination of the concept of mass.
Relevance with confinement mechanisms of interactions is also imagined.

It is more natural that mass terms do not exist in equations of field operators obtained as elementary canonical gauge unified fields. There is no reason to introduce inherent constants to the theory arbitrarily.
Mass is an effective concept that varies depending on the ambient situation, and should be regarded as an effective constant introduced for well approximation when giving linearized equations to infinitesimal oscillations.
As an interaction with preons, the Dirac operator D contains C. Also, it is remarkable that the pre-color interaction has a Pauli term (interaction) $\mathrm{Tr}(\chi^* \cdot 2i\gamma^{[\mu\nu]}\nabla_{\mu\nu}(C)\cdot\chi)$.

In setting the function system for the mode expansion of the space-time representation C(x) of the field operator, we can refer to the linearization of the equation.
In setting the function system for the mode expansion of the space-time representation C(x) of the field operator, we can refer to the linearization of the equation.
If the Boson can be expressed approximately by the Klein-Gordon equation, by expressing the Lagrangian in the form of Klein-Gordon type + correction term, and the mass will be calculated by the variational method making the expectation value of the correction term be minimal. This is a matter related to the solution of field equations, and will be investigated as necessary in Part II.

(5).pre-flavor Boson L(B')

$L(B') = (1/4)\mathrm{Tr}(\nabla_{\mu\nu}(B')\nabla^{\mu\nu}(B'))$; $\nabla_{\mu\nu}(B') \equiv \partial_\mu B'_\nu - \partial_\nu B'_\mu + i[B'_\mu, B'_\nu]$ ref=Table. 3.3.1
L(B') is a typical Lagrangian for Bosons, as the same as the pre-color field C case.
B' can be represented as a 2×2 Hermitian matrix which is an element of u(2). (The infinitesimal transformation of U(2) is i・B')

$B'_\mu = \sigma_b \cdot B'^b{}_\mu$, $i[B'_\mu, B'_\nu] = \sigma_c \cdot \kappa^c{}_{ab} B'^a{}_\mu B'^b{}_\mu$ $\nabla_{\mu\nu}(B') = \sigma_c \cdot (\partial_\mu B'^c{}_\nu - \partial_\nu B'^c{}_\mu + \kappa^c{}_{ab} B'^a{}_\mu B'^b{}_\mu)$

As a generator of BB-type interaction field, the phase transformation basis $\sigma_0 = 1$ should be included, which becomes the component that gives the electromagnetic field. (In addition, as a possibility, there is the scalar component C^0 in u(3) already mentioned.)
Although $B^0{}_\mu$ is free from u(2) commutation relations, the fact that the preon particle V has no charge means that the interaction constant with electromagnetic field is 0.
To explain this, for example, a constraint such as $\langle V | C^0{}_\mu + B'^0{}_\mu + B'^3{}_\mu | V \rangle = 0$ is required at least for infinitesimal oscillations.
For Bosons, the origin of infinitesimal oscillation (center point of expansion) may be a constant rather than $B = 0$.

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3.4.Notices on Lagrangian for canonical gauge fields

●Notes on unified field Lagrangian (ref.3.3)

- The Lagrangian can be decomposed as follows. Interactions between fields are represented as products of field operators.
  (1) Fermion quadratic term L(χχ*1) ／(2) Fermion quartic term L(χχ*2)
  (3) spinor connection Boson L(h) ／(4) pre-color connection Boson L(C)
  (5) pre-flavor connection Boson L(B)

  The meaning investigation of each term constituting the Lagrangian is described in section 3.3.

- Originally, the Fermion equations are 2nd-order with respect to the Dirac operator $\mathcal{D}$. This comes from the fact that the Lagrangian is a quadratic form with respect to the 1st-order derivative. The existence of the Dirac equation depends on the 1st-order factorizability of the quadratic form Lagrangian. This factorizability depends on the definition of the quadratic norm on the noncommutative coefficient metric linear space.

- By applying an appropriate definition of the quadratic norm, it is possible the Lagrangian to factorize , on the ring $\mathrm{R}[i\gamma^5]$, into the 1st-order term related to the infinitesimal oscillations (unperturbed state) of the Fermions.
  This seems to correspond to the existence of two types of Fermions (preons T,V).

- The particle concept corresponds to infinitesimal oscillations of spacetime.
  In general, taking into account higher-order effects, it is a superposed state of multi-particle states of various oscillations. (The state vector is generally a tensor of basis oscillations.)
  This fact is related to the representation of field operators and methods for constructing approximate solutions. It will be investigated in Part II.

- The chiral characteristics of preon depends on the choice of the infinitesimal oscillation as initial solution in approximate.
  (That is, the choice of the 1st-order factor of Dirac operator $\boldsymbol{\mathcal{D}}$ or ($\boldsymbol{\mathcal{D}}$ -4h).)

●Dirac decomposition and symmetry

Possibility of factorization of Dirac operator quadratic form

Consider the case when applying a different definition for quadratic norm and as the result that hh* term has remained, in the quadratic form of Dirac operator .
Omitting constant factors, the quadratic form is:

$$f\equiv\mathcal{D}^2-2h^*\mathcal{D}-2\mathcal{D}h+16h^*h$$

In the mass relation, the above f cannot be decomposed into 1st-order factors on the ring R[iγ5], so it causes the theoretical defect to the requirement of the existence of the Dirac equation. The following can be said:

If $h^*h\neq0$, f cannot be factorized into 1st-order factors on $\mathrm{R}[i\gamma^5]$. ***A**
If $h^*h=0$, h is a constant multiple of the chiral projection operator. ***B**

***A:**

$$f\equiv\mathcal{D}^2-2h^*\mathcal{D}-2\mathcal{D}h+16h^*h$$

In case $h^*h=m^2>0$, then h can be expressed as $h=m\cdot\exp(2i\gamma^5\theta)$.

Ignoring the differential coupling (i.e. $\partial h \fallingdotseq 0$) and mapping χ as $\chi \to \exp(-i\gamma^5\theta)\chi$ by spinor basis transformation, then f is mapped to $f = \mathcal{D}^2 - \mathcal{D}\cdot 4m + 16m^2$ since $\exp(i\gamma^5\theta)\mathcal{D} = \mathcal{D}\cdot\exp(-i\gamma^5\theta)$.

If the quadratic form f could be factor-decomposed, $f = \mathcal{D}^2 + 4m\mathcal{D} + 16m^2 = (\mathcal{D}-\alpha)(\mathcal{D}-\beta)$

$\alpha\beta = 16m^2$, $\alpha^* + \beta = 4m$.

From this $\beta = k\alpha^*$, ($\exists k \in \mathsf{R}$), but also from $\alpha^*(1+k) = 4m$, then $\alpha \in \mathsf{R}$.
However, since f does not have real number zeros, this is a contradiction.

In case $h^*h = -m^2 < 0$, h can be expressed as $h = i\gamma^5 m\cdot\exp(2i\gamma^5\theta)$.
Ignoring the differential coupling ($\partial h \fallingdotseq 0$) and mapping χ as $\chi \to \exp(-i\gamma^5\theta)\chi$ by spinor basis transformation, and set $Z \equiv -i\gamma^5\mathcal{D}$. Then $Z = Z^*$, $Z^2 = -\mathcal{D}^2$. That is, $f = -Z^2 - 4mZ - 16m^2$

Whether or not f can be factorized into 1st-order factors of $\mathcal{D}$ on $\mathsf{R}[i\gamma^5]$ is equivalent to the factorizability concerning Z. It can be seen from the above proof that factorization is impossible on $\mathsf{R}[i\gamma^5]$.

***B:**

For $h = h_0 + h_5\cdot i\gamma^5$, $hh^* = h_0^2 - h_5^2$, so $hh^* = 0 \ \to\ h_0 = \pm h_5$ $h = 2h_0 \times (1 \pm i\gamma^5)/2$
Also in this case, $f = \mathcal{D}^2 - 4\mathcal{D}h = \mathcal{D}(\mathcal{D}-4h)$. note : $\mathcal{D}h = h^*\mathcal{D}$

Taking h as $h = 2h_0 L$, $h_0 \neq 0$ (L: left-handed projection operator) and ignoring differential coupling (i.e. $\partial h \fallingdotseq 0$ approximating initial solution), the following can be said:
Applying the concept of infinitesimal oscillation of spacetime to approximate the Lagrangian $L \equiv \chi^*\mathcal{D}(\mathcal{D}-4h)\chi$, then the following are derived .

In the expansion around the approximate initial solution $\mathcal{D}\chi = 0$, χ is limited to left-handed. (i.e. The right-handed component is not arisen.)

$L = \chi^* f\,\chi \fallingdotseq \chi^*\mathcal{D}(0-4h)\chi = -4\chi^*\mathcal{D}h\chi$; $L \fallingdotseq -4h_0\cdot\chi_L^*\mathcal{D}\chi_L$

In the expansion around the approximate initial solution $(\mathcal{D}-4h)\chi = 0$, since $h^*h = 0$, χ is again limited to left-handed.

$L = \chi^* f\,\chi \fallingdotseq \chi^* 4h^*(\mathcal{D}-4h)\chi = \chi^* 4h^*\mathcal{D}\chi = +4\chi^*\mathcal{D}h\chi$; $L \fallingdotseq +4h_0\cdot\chi_L^*\mathcal{D}\chi_L$

There is no essential difference in Lagrangian due to the approximate initial solution. Also, they map to each other by the transformation $\chi \to i\gamma^5\chi$. Two types of pre-flavors cannot arise.
They seems to show $h^*h = 0$ is unrealistic.

Neutrinos are interpreted as composite particles of preons in canonical gauge unified field theory, but approximately they have zero mass and are limited to left-handed, so the above is an interesting approximate model.

Symmetry in solution space of infinitesimal oscillations

When the Lagrangian is approximately given by $L = \chi^*(\mathcal{D}-\alpha^*)(\mathcal{D}-\beta)\chi$; $L = L^*$, $\alpha, \beta \in \mathsf{R}[i\gamma^5]$ with neglecting higher order terms, similarly to exchange the roots of algebraic equation as in algebra, the idea arises that approximately exchanging α and β .

Although this symmetry will be suppressed by the degree of symmetry breaking due to higher order terms, it might be probable to incorporate the approximate symmetry as a gauge field, or to associate the state basis with the basis of the equation solution space.
Usually, we are only looking at infinitesimal oscillations of spacetime. Fermion higher order terms can be ignored at 0th order approximation. So this symmetry is significant. At large amplitude oscillations, the solutions would become almost indistinguishable in another sense, but large amplitude oscillations should be interpreted as the superposition of multiple excitations of infinitesimal oscillations on the microscopic view. It may be necessary to think actively in this way also for the Big Bang.

●**Issue of origin of mass**

From the fact that electrons do not obey the Klein-Gordon equation, we should doubt the fundamental meaning of the Klein-Gordon equation.
The Klein-Gordon equation originates from the concept of mass point particles in classical theory, based on the relation $v^2=1$ for velocity v and on the momentum $p=mv$ in kinematics.
For charged particle motion in an electromagnetic field, since the canonical momentum is $p=mv+eA$, replacing $p\to p-eA$ and setting $(p-eA)^2-m^2=0$ is also reasonable inference.

However, not only for electrons (Fermions), the equation is not satisfied as a fact, but even for Bosons, the original form does not coincide with the Klein-Gordon equation without adding gauge conditions. Moreover, mass terms must be introduced ad-hoc.

It should be said that the concept itself of "mass as an intrinsic inherent attribute possessed by point particles" has become questionable. The Higgs mechanism is one attempt at a solution, and its concept may survive, but unified field theory does not lead to such a convenient Higgs field.
For leptons and quarks as composite systems of preons, the problem is more complicated as interaction energies are involved.
(It seems reasonable to think that mass is an effective concept, that depends on the interaction with self and with surrounding environments.)

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4. Summary of Part I of Canonical Gauge Unified Field Theory

Briefly summarizing the contents of Part I, the following main results were obtained by introducing quantum spacetime: (See also Overview/Introduction in top page)

-1.Reason of existence of elementary particles/fields.

The reason of existence of fields lies in canonical gauge connection.
Based on this, it can be seen that there are preons as fundamental particles one level below the known quarks/leptons, and the interaction fields become the interactions between preons.

-2.Unification of Bosons and Fermions.

From the reason of existence of elementary particles, Bosons and Fermions are unified.
From the matrix representations of canonical gauge connection fields, it can be seen that there are 2 kinds of interactions for Bosons: FF type and BB type.
Canonical gauge connections of FB type and BF type correspond to Fermions, and they are in particle/antiparticle relationships with each other.
Symmetric partners expected by Supersymmetry theory and the Majorana particles do not exist.

-3.Existence of spinor connection gauge fields.

FF type Bosons consist of components with $u(3)=1\oplus su(3)$ symmetry and components with o(1,1) symmetry.
The su(3) component field is the ultimate cause of strong interactions. The color interaction at the quark level is caused by the interaction at the preon level below it (pre-color interaction).
The scalar component $u(1)=1$ of u(3) is a phase connection component, and there seems to be no theoretical reason to exclude it. It also cannot be distinguished from the phase connection of the BB type mentioned below.

The gauge field of o(1,1) component is a connection field related to the degrees of freedom of spinor, and its form is $h(x)=h_0(x)+i\gamma^5 h_5(x)\in \mathrm{R}[i\gamma^5]$, but this field has not yet been recognized so far.
On the other hand, the Higgs field is known from observations, but in unified field theory, it is not derived the existence of such a field that gives mass to gauge Bosons like the Higgs field.
The Higgs field seems to be related to the gauge field of spinor connection field here, but the recognition about it as a Higgs field is probably not correct.

BB type Bosons consist of components with $u(2)=1\oplus su(2)$ symmetry. The u(2) component field, including the above-mentioned FF-type phase connection, is the ultimate cause of electroweak interactions, the pre-electroweak interaction at the preon level.
Regarding the u(2) symmetry, it seems to include the exchange symmetry of solutions to the field equations. (see next section. -4)
In preon models (rishon models), T and V are considered as pre-flavors, so u(2) components should be recognized as pre-flavor interactions.

-4.Unified field equations, Dirac decomposition, relation between mass and chirality.

The unified field equation for preons is a quadratic equation involving the Dirac operator $\mathcal{D}=\gamma p+C+B'$, and Dirac equations are obtained from its factorization on the ring $\mathrm{R}[i\gamma^5]$.
(Dirac decomposition)

Here, C is the pre-color Boson field with su(3) component, and B is the pre-electroweak Boson field with u(2) component. (B is expressed as B' because it is transpose in matrix representation.)
In unified field theory, the Lagrangian for the preon field χ, is given by the quadratic term χ*𝒟(𝒟-4h)χ, apart from the higher order terms.
Preons acquire chirality and mass through interaction with the spinor connection field h, and the chirality of h causes a bias in the chirality distribution.

Suppose the spinor connection field h forms chiral bias effects over the universe due to Bose-Einstein condensation, the relation between chirality and mass can be easily understood.
This is the key to understanding why neutrinos are almost left-handed.
In unified field theory, it seems to be able to consider a discrete transformation on solution space ker(𝒟(𝒟-4h))＝ker(𝒟)⊕ker(𝒟-4h), similar to the Galois groups in algebra.

-5.Reality of preons, generation problem, neutrino oscillation, particle/antiparticle symmetry

The reality of preons is derived from unified field theory. The rishon model is promising, and generations of quarks/leptons can be understood as excited states of preon composites.

The reason why there are only 3 generations of quarks/leptons can be explained by the fact that pre-color Boson emission becomes a “colorless” combination with 3 pre-color Bosons, as stated at the beginning of Part-I.
If there was a 4th generation, it would immediately decay into the 1st generation.
On the other hand, generation exchange is possible through ν, e^- (μ,τ) collisions.

No proper explanation can be found for neutrino oscillations. It seems unclear that the relationship between the interpretation of generations as energetically excited states of preon composite systems and the concept of generation recognized through phenomenon of generation exchange in particle colliding.

It is impossible for a isolated composite particle system in a steady state to oscillate between different energy eigenstates. That is a logical contradiction.

Taking account that the number of generations can be exchanged between ν, e, μ,τ, then it can be one interpretation that oscillation between 2 energy eigenstates arise through some interacting partners as a catalyst.

Apart from the above, if taking the view that the steady state (energy eigenstate) is a superposition of some generations states and undulation is caused, then a new concept for the generation is required , rather than as "excited states of preon composite system".
However, such a definition is unknown.
Some points of interest that may be related to this matter below.

- Regarding intergenerational undulations, only neutrino ν is talked about.
- In preon composite system, there are two types of interaction: pre-flavor interaction and pre-color interaction. ***A**
- If the concept of generations is that constructed based on the infinitesimal oscillations of spacetime, higher-order terms in the equations for composite systems may contribute to cause

superposition of the various infinitesimal oscillations.

Incidentally, by admitting preons, questions about the asymmetry between particles and antiparticles in the universe, "Where have the antiparticles gone?", can also be answered. They have gone nowhere. They are hiding inside quarks as d＝(TVV)*.

***A** :

in attribute exchange of pre-flavor×pre-color (f,c)←→(f',c') ; c≠c', the exchange of f, f' is indistinguishable from the exchange of c, c'.
f＝f' is a possible combination, and interaction still occurs.
That is, the values of other quantum attributes are exchanged.

In fact, an asymmetry exists between the case f＝f'＝T and the case f＝f'＝V. As a result, for example, even though quark u＝(TTV) is white at preon level, but is colored at quark level due to the pre-color attribute of V.
From this, the quark exists as a steady state, only in a composite system of whiteness, and cannot exist independently.

-6.Unified field equations, Dirac equations, equations for Bosons

The current Dirac equation as the equation of field is given by applying equation for wave functions to field operators.
While the equation for wave function should be linear, this is not the case for the equation of field.
From the unified field equation, a Lagrangian containing higher-order terms for Fermions is obtained, and the Dirac field equation is obtained from the factorization of quadratic form of Dirac operator. (Dirac decomposition).

For Bosons, a conveniently existing field like the Higgs field that directly produces mass is not derived. The fundamental equations for Bosons are derived from the principle of least action concerning the quadratic norm of curvature form of field.

Whether for Boson or Fermion, the mass of a particle is an effective concept, and for composite systems, there arise issues of the mass of the system and confinement and leakage of interactions.
In noncommutative gauges, interaction Bosons are generally thought not to come out unless there is a partner to receive them. This is also related to the nonexistence of isolated quarks.

-7.Unified field equations and quantum gravity

Since unified field theory derives the fields from the 1st principle, it provides a Lagrangian for gravity also. Quantum gravity theory, which will be discussed in Part III, gives equation that is essentially different from Einstein's one. However, it can be concluded that unified field gravity theory and Einstein gravity theory coincide when spherical symmetry is assumed for the field.
(Part III)
From this, it can be understood that some theoretical verifications have been done already in classical level at least on nearly spherically symmetric field.

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part I _Appendix.

A1.Spacetime metric

Lorentz metric

In theories including relativity, the Lorentz metric of spacetime is taken as diag(+,-,-,-). It feels somewhat uncomfortable that the 3-dimensional space we perceive becomes to have a negative metric, but this is rational for expressing the theory mathematically .

Considering the scaling and taking the speed of light as c=1, the Lorentz metric η can be written as $\eta=\mathrm{diag}(+1,-1,-1,-1)$.

For this metric, note that the Hamiltonian H is the canonical energy×(-1). This can be seen from the Lagrange 1-form $\Lambda=p_j dx^j-H dt$; $j=1..3$ $t\equiv x^0$.

If conversely the 3-dimensional metric is made positive as $\eta=\mathrm{diag}(-1,+1,+1,+1)$, the interpretation " H=contravariant energy " makes sense in some extent, because $H=-p_0=+p^0$: (contravariant component), but that understanding is not accurate.

spinor metric

A spinor is an element of the spinor space, which is a linear representation space of Lorentz transformation group. It is wrong to think that the tangent vectors of spacetime are substantial while spinors are not.

Both are linear spaces associated with spacetime points, and transformations on the linear spaces are coupled with coordinate transformations , nothing more.

Initially, the idea of factorizing the Klein-Gordon equation into 1st-order factors,

$$\partial\cdot\partial-m^2=(\mathcal{D}-m)(\mathcal{D}+m) \quad ; \quad \mathcal{D}=\gamma^\mu p_\mu$$

led to the discovery of the Dirac matrices γ and the recognition of the spinor space as a representation space for Lorentz transformations.

(The γ become the representation matrices of the basis of the Clifford algebra described later)

Introducing the basis of the spinor space as (**a**), a Lorentz basis (**e**) can be identified as $e_\mu=\mathbf{a}_j(\gamma_\mu)^j{}_k\mathbf{a}^{*k}$.

The Fermion field $\chi=\mathbf{a}_j\chi^j$, which is a solution of the Dirac equation, is a spinor as an element of the spinor space.

The spinor space must be a metric linear space. Set its metric matrix as $h=\langle\mathbf{a}|\mathbf{a}\rangle$, then the requirement that the Dirac operator be self-adjoint $\mathcal{D}=\mathcal{D}^*$ means that (hγ) must be a Hermite matrix. Furthermore, we require that the norm of a spinor in a positive energy state be positive.

In the representation $\gamma^1=i\sigma_1\otimes 1$, $\gamma^2=i\sigma_2\otimes 1$, $\gamma^3=i\sigma_3\otimes\sigma_1$, $\gamma^0=\sigma_3\otimes\sigma_2$, we can take h as $h=\gamma^0$

$$(h\gamma^\mu)=(h\gamma^\mu)^H=(\gamma^\mu)^H h \quad \therefore\ (h\gamma^\mu)=-\gamma^\mu h \quad \text{for } j=1,2,3\ ,\ (h\gamma^0)=\gamma^0 h$$

spin matrices σ

Pauli's spin matrices are 3 elements ($\sigma_1, \sigma_2, \sigma_3$) of SL2(C) which is a group of 2×2 complex matrices specified by determinant=1 , and together with identity matrix, they constitute an orthogonal basis in 2×2 complex matrix ring.

Initially in early quantum mechanics, spectroscopy revealed that electrons have two spin states, up/dn(down), so a 2-dimensional complex space was provisionally considered, with eigenstates of a linear operator σ_3 represented as the matrix σ_3=diag(1,-1).
If up/dn represents the orientation of the electron spin along the 3rd axis, then σ_1, σ_2 should also exist equivalently, as representation elements for spatial rotation. Such σ1, σ2 would be Hermite matrices with $\sigma_j^2=1$, with orthogonality reflected.

From these , with suitable scaling, it should be $\mathrm{Tr}(\sigma_i\sigma_j)=\delta_{ij}$. (i,j=1..3)
Also, constant component should have extracted due to the orthogonality Tr(σ)=0 relative to the identity matrix.

In addition to the idea on the correspondence$\sigma_j \longleftrightarrow \mathbf{e}_j$ ($\mathbf{e}_j$: 3-dimensional vector basis) from interpreting the electron spin directions, the following correspondence in terms of representation of rotations can be envisaged.

$g\in$SO3 action: $g\cdot\sigma_j \longleftrightarrow g(e_j)$, in an infinitesimal transformation $\mathbf{e}_k\times\mathbf{e}_j \longleftrightarrow \sigma_k\cdot\sigma_j=[i\sigma_k/2, \sigma_j]$

As the commutation relations of the generators, $[i\sigma_a/2, i\sigma_b/2]=i\sigma_c/2$ can be imagined.
With these ideas, 2×2 matrices σ_2, σ_3 can be obtained.
From the commutation relations, it should be $\sigma_1\sigma_2=-i\sigma_3$, and choice of signs as $\sigma_1\sigma_2=+i\sigma_3$ seems not preferable from the viewpoint of correspondence with generators.
Specifically, the following can be taken :

$\sigma_1=$

0	1
1	0

$\sigma_2=$

0	+i
-i	0

$\sigma_3=$

1	0
0	-1

Clifford algebra and Dirac's γ matrices

Although Dirac's γ matrices were born from the idea of factorizing the Klein-Gordon equation, this led to the discovery of spinors. While matrices satisfying $\gamma^\mu\gamma^\nu+\gamma^\nu\gamma^\mu=2\eta^{\mu\nu}$; η=Lorentz metric, can easily be constructed by using Pauli's spin matrices, understanding the essence of the γ matrices requires introducing Clifford algebra.

Consider a n-dimensional real coefficient non-degenerate metric linear space E.
Restricting the coefficient field to real numbers is because the linear space is assumed to be a model of the tangent space of spacetime, the coefficients can be extended to complex but transformations of bases are limited to real numbers. So in that case, basis vectors **e** and **e**・i ($i=\sqrt{-1}$) must be treated as linearly independent.

Introduce the following binary operational relation in the tensor space of E, denoted C(E).

$xy+yx=2\langle x|y\rangle$ for x,y ∈E ; $\langle|\rangle$=a non-degenerate bilinear form as an inner product

From the above, since $\langle x|y\rangle=0$ makes the product antisymmetric, C(E) is the direct sum of antisymmetric tensors. It is easier to see by introducing an orthogonal basis, but it is readily apparent that $C(E)=\oplus_r C^r(E)$, $\dim(C^r(E))={}_nC_r$, $\dim(C(E)))=2^n$.

Since $xx=\langle x|x\rangle$, x∈E can also be interpreted as an element of the dual space.

$x=x^*$ for $x\in E$, $(xy)^*=y^*x^*$ for $x,y\in C(E)$

Using Pauli's spin matrices, an irreducible linear matrix representation of C(E) can easily be constructed as follows. As abbreviation, write $(j1,j2,\ldots)=\sigma_{j1}\otimes\sigma_{j2}\otimes\ldots$, Take $\sigma_0\equiv 1$.
Orthogonal bases $(\mathbf{e}_j)$ can be introduced. $(\mathbf{e}_j)$; $j=1..n$, $\langle e_j|e_k\rangle=\varepsilon_j\cdot\delta_{jk}$ $\varepsilon=\pm 1$
For simplicity, restrict to $\varepsilon_j=+1$ hereafter, but if $\varepsilon_j=-1$ then take $ej\to ej\cdot i$.

Whether the dimension n is even or odd leads to the different structure of Clifford algebra C(E).

- **case of n＝2ν (even number)**

  Set $\mathbf{e}_1=(1,0,\cdots)$, $\mathbf{e}_2=(2,0,0\cdots)$, $\mathbf{e}_3=(3,1,0\cdots)$, $\mathbf{e}_4=(3,2,0\cdots)$, $\mathbf{e}_n=(3,3,\cdots,2)$,
  The size of representation matrix is $2^\nu\times 2^\nu$, the dimension of representation space dim.C(E) is $\dim.C(E)=2^n=2^{2\nu}=2^\nu\times 2^\nu$. So irreducibility of the representation is obvious from dimensional relations. (There are no invariant subspaces under the representation except 0 and the whole space.)
  Each $\mathbf{e}_j$ is anti-commutative. Also, $\mathbf{e}_\#\equiv\mathbf{e}_1\cdots\mathbf{e}_n=(3,3,\cdots,3)$, which anti-commutes with any $\mathbf{e}_j$.

- **case of n=2ν+1 (odd number)**

  Set $\mathbf{e}_1=(1,0,\ldots)$, $\mathbf{e}_2=(2,0,0\ldots)$, $\mathbf{e}_3=(3,1,0\ldots)$, $\mathbf{e}_4=(3,2,0\ldots)$, $\mathbf{e}_{n-1}=(3,3\ldots,2,0)$, and set for final element $\mathbf{e}_n=(3,3\ldots,3,3)$ as exception.

  If applying the same rules as above for $n=2\nu$ to all $\mathbf{e}_k$ $(k=1..n)$, the last element $\mathbf{e}_n$ should end up like $\mathbf{e}_n=(3,3\ldots,3,1)$, but as a representation $\mathbf{e}_n=(3,3...,3,j)$ for any $j=1,2,3$ is possible. The dimension of representation space is $2^{\nu+1}\times 2^{\nu+1}=2^{2\nu+2}=2^{n+1}=2\times\dim C(E)$.
  Since $\mathbf{e}_\#\equiv(\mathbf{e}_1\cdots\mathbf{e}_n)=(3,3\ldots,3,0)\cdot\mathbf{e}_n=(0,0,\ldots,0,3)$, $\mathbf{e}_\#$ commutes with any $\mathbf{e}_j$.
  The representation matrices can be expressed as follows.
  $\mathbf{e}_j=\mathbf{e}'_j\otimes\sigma_0=\mathrm{diag}(\mathbf{e}'_j,\mathbf{e}'_j)$, $\mathbf{e}_n=\mathbf{e}'_j\otimes\sigma_3=\mathrm{diag}(\mathbf{e}'_{n-1},-\mathbf{e}'_{n-1})$. where e' represents the matrices for the $n=2\nu$ case.

spinor group

The linear space S, considered as the space for linear representations of the Clifford algebra, is called spinor space. That is, $C(E)\subset S\otimes S^*$. Especially, as already mentioned, if dim(E)＝even, then $C(E)=S\otimes S^*$ is derived.
Compared to the linear space E considered as a model for the tangent space, the space S might have a somewhat vague feeling. However, even for the tangent space , it is merely a linear space associated with each spacetime point, that obeys the transformation rule related to coordinate transformation, and in this sense, the spinor space S could be understood similarly.
If we take the basis of space S as $(\mathbf{a})$, identifications like $\mathbf{e}=\mathbf{a}\gamma\mathbf{a}^*$ $(\mathbf{a}^*\mathbf{a}=1)$ are possible, where γ is the matrix of the linear representation of $\mathbf{e}$.

Transformations on the spinor space S can be considered arbitrarily, but those related to Lorentz transformations on the tangent space will be useful.

Consider $\alpha\in C(E)$ such that for $x\in E$, the transformation is $x\to y=\alpha x\alpha^{-1}\in E$.
Since $z\in E$ satisfies $z=z^*$, $z^*z=\langle z|z\rangle$, we have $\alpha x\alpha^{-1}=\alpha^{-*}x\alpha^*$, $y^2=\alpha x^2\alpha^{-1}=x^2$
Therefore, $\alpha^*\alpha\in\mathbb{R}$, and the transformation is an orthogonal transformation.
From the expression, the transformation is invariant even if a is scaled by a constant, so $\alpha^*\alpha$ can be limited to 1.

The spinor group is defined as Spin(E)＝{α∈C(E)|α*α=1, E⊂αEα*}.
If the basis of S is taken as (**a**), x can be represented as $x=\mathbf{a}(\gamma_j x^j)\mathbf{a}^*$ in the linear representation space S. Therefore the transformation $x \rightarrow y=\alpha x\alpha^{-1}\in E$ can be interpreted as the transformation on S⊗S* induced by the transformation **a**→α**a** on S.

Considering an infinitesimal transformation, it is easy to see that the Lie algebra of Spin(E) is 2nd-rank antisymmetric tensor space on E. i.e. spin(E)＝C^2(E).
That is, Spin(E) is locally isomorphic to SO(E) special orthogonal transformation group (or rotation group on E in short.)
Considering the homomorphism R from Spin(E) to the rotation group SO(E) (i.e. R: Spin(E) → SO(E)),the the rotation R(α)∈SO(E) (R(α) : x→y＝αxα*∈E for x∈E) ,is invariant under the transformation α→−α.

The kernel of the homomorphism R consists of such α that α*α＝1 and commutative with any element of E. That is, α＝±1. Since SO(E) is arcwise connected, then considering the lift from SO(E) to Spin(E), we can see that the homomorphism R is a surjection. Therefore, Spin(E) is a double covering group of SO(E). Also, in topologically, Spin(E) of dim(E)＞2 is simply-connected. ***A**

1 →±1→Spin(E)→SO(E)→1

Finally, in order to transfer our discussion of rotation groups to Lorentz groups, we replace the orthonormal basis of E , ej∈E to ej'≡i･ej (j=1,2,3). Where, Lorentz metric is η＝(+1,-1,-1,-1) .･
Then, for $x\equiv \mathbf{e}_j' x^j\in E$, $x^2=\langle x|\eta|x\rangle$ is obtained. ***B**

***A** :

Let us consider a closed curve starting from the identity element in Spin(E). The curve is mapped to the space of SO(E). The point on SO(E), rotation, can be visually expressed by the mapped directions of coordinate unit vectors.
Focusing on the closed curve drawn by the k-th coordinate unit vector, and consider the operation contracting the curve continuously to the k-th axis by continuous rotation.
The closed curve on Spin(E) is obtained as the lift of the closed curve in contraction operation on SO(E) . This shows that it is possible to reduce the dimension of the problem.
This shows that it is possible to reduce the dimension of the problem.

By the above method, reduce the dimension to 3. Using Pauli's spin matrices, Spin(3) is expressed as $x\equiv \Sigma\, \sigma_j x^j \rightarrow y=\alpha x\alpha^{-1}$; αα*＝1 $\alpha=i\cdot \Sigma\, \sigma_j\alpha^j$, it can easily be seen that Spin(3) is topologically equivalent (homeomorphic) to 2-sphere S^3. A closed curve on S^3 could be continuously contractible to the identity element.

***B** :

It might be interpreted as partial complexification, but the range of coefficient is still limited to R
In the scope of tangent vector representations, further complexification is unnecessary.
It is freely possible to introduce tensor product with complex fields or operators.
Complexification of Clifford algebra changes some expressions as follows.

$xy+yx=2\langle x^*|y\rangle$ x, y∈E_C≡E⊕iE,

for x∈E, x＝x* ,(xy)*＝y*x* for x,y∈C(E_C)

For x of 0th-rank tensor or coefficient, x* means complex conjugate of x.

Canonical Commutation Relations and Clifford Algebra

Focusing on $x \in E \to x = x^*$, the base space E of Clifford algebra can be constructed from a ν-dimensional real linear space F.

Let f_j be orthonormal basis of F, and take dual basis f*. Set as follows.

$$\sqrt{2} \cdot \mathbf{e}_j \equiv (f_j + f_j^*) , \quad \sqrt{2} \cdot \mathbf{e}_{\nu+j} \equiv (f_j - f_j^*)/i \; ; j=1..\nu$$

Then, anticommutative canonical commutation relations are derive that $\mathbf{e}_j\mathbf{e}_k + \mathbf{e}_k\mathbf{e}_j = \{f_j, f_k^*\} = \delta_{jk}$

$F \oplus F^* = E$ can be constructed.

Conversely from 2ν-dimensional Clifford algebra, ν-dimensional linear space and dual space can be constructed satisfying the canonical anticommutation relations by the following transformation.

$$\sqrt{2} \cdot f_k = \mathbf{e}_k + \mathbf{e}_{\nu+k} \cdot i \, , \; \sqrt{2} \cdot f_k^* = (\mathbf{e}_k - \mathbf{e}_{\nu+k} \cdot i) \quad ; k = 1..\nu \quad .$$

.

A2.Definition and Construction of Metric Structure

Considering examples of field operators in physics, it is evident that we need to expand the coefficients of a linear space from complex numbers to non-commutative elements such as operators. In this case, it is necessary to consider the orientation of action (left and right of the coefficients), the induction of the metric structure onto the tensor space, and how to construct the definitions for them.

-1.Expansion of Linear Space Coefficients and Direction of Action

Let us consider extending the coefficients of linear space from complex number field to the ring R (assuming something like field operators in physics) including the complex number field. As the basis, basis of the complex number field subspace E_C is used in common.

In this case, it becomes necessary to strictly distinguish between right and left coefficients in the representation. The necessity of distinguish the orientation of coefficients can be seen from the following example.

Without distinction of coefficients orientation(left and right) with keeping associative law, like " $\mathbf{x}\alpha=\alpha\mathbf{x}$ for $\mathbf{x}\in E$, $\alpha\in R$ ", the following inconvenience occurs:

$$\mathbf{x}(\alpha_1\alpha_2)=(\mathbf{x}\alpha_1)\alpha_2=\alpha_2(\mathbf{x}\alpha_1)=\alpha_2(\alpha_1\mathbf{x})=(\alpha_2\alpha_1)\mathbf{x}=\mathbf{x}(\alpha_2\alpha_1) \quad ; \quad \mathbf{x}(\alpha_1\alpha_2)=\mathbf{x}(\alpha_2\alpha_1)$$

On the other hand, if orientation of the coefficients is limited to only the left or right, it becomes difficult to introduce a product structure on the tensor space.

In the construction of the tensor product, it is necessary to exchange the ordering of the product including the coefficients.

$$(\mathbf{e}_1\alpha_1)\cdot(\mathbf{e}_2\alpha_2)=(\mathbf{e}_1\mathbf{e}_2)\alpha'_1\alpha_2 \quad ; \quad \text{i.e.} \quad \alpha_1\cdot\mathbf{e}_2=\mathbf{e}_2\cdot\alpha'_1 \quad ,$$

$\mathbf{e}_j\in E_C$, $\alpha_j, \alpha'_j\in R$, E_C : $\mathbb{C}$-linear subspace of E, (j=1,2), $\mathbf{e}_j$: n appropriate basis

We already know, from the example of field operators, the necessities of introducing anti-commutative vector frame (Fermi frame) in the tensor algebra, or coefficients that anti-commute with the Fermi frame. There is a certain exchange relation betweenα_1 and $\mathbf{e}_2$, and it should be considered that $\alpha_1\rightarrow\alpha'_1$ is determined. ($\alpha_1\cdot\mathbf{e}_2=\mathbf{e}_2\cdot\alpha'_1$)

Furthermore, it seems reasonable to assume that the exchange relation can be limited to commutative and anti-commutative by a suitable decomposition.

--

-2.Dual Space

Consider a linear space E with a coefficient of ring R that includes the complex number field and is not necessarily commutative. The dual space E* is defined as a linear mapping from E to t R, independently of the metric concept.

$$\xi\in E^* \; x\in E \; ; (\xi, x) \rightarrow \xi(x)\in R$$

Since E* itself is a linear space, (E*)* can also be considered. The above can also be seen as x acting on $\xi\in E^*$ from the right, so it is reasonable to define (E*)*=E.

When a metric structure $E\times E \rightarrow R$; $x,y \rightarrow \langle x|y\rangle$ is given, a dual mapping E→E* is defined from this action since x corresponds to a linear mapping on E.

i.e. $\langle x|y\rangle=x^*(y)$ for $\forall y\in E$; $x\rightarrow x^*$

If and only if the metric is non-degenerate, $\{x^*|x\in E\}=E^*$.

A degenerate metric is considered a singular case. In that case, a space E0 orthogonal to the entire space should be separated in advance. $E=E_0\oplus E_1$, $E_0=\{x|\langle x|E\rangle=0\}$, E_1=non-degenerate

Hereafter, the metric is assumed to be non-degenerate.

-3.Charge Conjugation Symmetry

In quantum field theory, for the spatial oscillation mode, there exist two states of positive oscillation and negative oscillation, and the annihilation of a particle in the negative oscillation state is interpreted as the generation of an antiparticle in the positive oscillation state.

That is, a more complicated symmetry than a usual linear space is incorporated.

Therefore, the structure like $E = E(+) \oplus E(-)$, $E(-) = E'(+)^*$ may exist in background. In such case, it is not necessarily appropriate to define the initially set linear space E as right-coefficient space uniformly.

The transition of the orientation of coefficients from left to right firstly becomes possible by assuming a commutation relation with the vector.

-4.Symmetry of Tensor Algebra, Fermi Space/Bose Space

When considering the tensor space on the linear space E, we must define the symmetry related to the tensor product. The tensor product is defined as associative and distributive. For the basis of E, the basis of the **C**-coefficient subspace E_C is taken.

The linear space E_C is assumed to be composed of two types of linear spaces, Fermi space and Bose space, from the symmetry related to the product.

$E_C = A_C \oplus B_C$, A_C : Fermi space, B_C : Bose space

With respect to the tensor product, $a \in A_C$ is anti-commutative with elements of A_C.

Any $b \in B_C$ is commutative with all elements.

Let's also assume orthogonality $\langle A_C | B_C \rangle = 0$.

Constructing E* from E, and $E \oplus E^*$ is constructed from this. The symmetry with respect to the product is naturally induced, and $E_C^* = A_C^* \oplus B_C^*$ can be obtained.

Next, we consider inducing a tensor product x, y → xy on $E \oplus E^*$. Since the product of the coefficient and the vector is already defined, it is sufficient to define the product on $E_C \oplus E_C^*$.

The product of x, y is written as xy.

For tensor product on E_C and on E_C^*, it will be sufficient to apply the identification rule derived from the symmetry already described.

For the tensor product with elements of the dual space, canonical commutation relations are set.

For $x \in A_C^*$, $y \in A_C$, $xy + yx = \langle x|y \rangle$

For $x \in B_C^*$, $y \in B_C$, $xy - yx = \langle x|y \rangle$

otherwise , for $x \in A_C^*$, $y \in B_C$, or for $x \in B_C^*$,$y \in A_C$, they commute. : $xy = yx$

-5.Extension of the Metric Structure to the Tensor Space

When defining quantum theoretical concepts of spacetime, it was necessary to endow spacetime with a Lorentz metric and to define the norms for linear mappings derived from commutation relations based on the connection structure in the space.

Given a metric linear space E, one problem to be considered is how to naturally induce a metric on its tensor space.

●Extending the Metric Structure and its diversity/arbitrariness

Let us consider introducing a metric on the tensor space $S \equiv \otimes(E \oplus E^*)$ with the metric linear space E as the base space.

Assume that the canonical commutation relation is defined on $(E \oplus E^*)$.

Thinking on field operators as examples, the coefficients of E should generally be assumed to be noncommutative.

defining an inner product structure as follows:

$x,y \in S \ \rightarrow \langle x|y \rangle \in R$: coefficient ring. complex number field $\mathbb{C} \subset R$ (consider the coefficient ring as the 0th-order tensor space.)

$$\langle x|y \rangle^* = \langle y|x \rangle$$

$$\langle x|y \cdot \lambda \rangle = \langle x|y \rangle \lambda, \quad \langle x|y_1 + y_2 \rangle = \langle x|y_1 \rangle + \langle x|y_2 \rangle$$

The definition extension of dual map/conjugation mapping (*) to the tensor space is given by: $(x \cdot y)^* = y^* \cdot x^*$. For 0th-order tensors $x \in R$, x^* is the conjugate element of x. In particular, if $x \in \mathbb{C}$ then it is the complex conjugate.

Therefore, it must be $(x^*)^* = x$ in order to have a common definition for all orders of tensors,

From the above, expressing x, y as the sums of monomials, it becomes sufficient to assume x, y are monomials to define the inner product $\langle x|y \rangle$.

However, due to the existence of canonical commutation relations, the decomposition into monomials are diverse. Therefore, it is necessary to introduce an ordering concept into the product.

●normal order/reverse order

We say $x \in S$ is in normal order if x is expressed as the sum of tensor monomials represented in normal order.

A tensor monomial is said to be in normal order if it is expressed as the product of elements of $\otimes E$ and elements of $\otimes E^*$.

$$x \in S \ ; \ x = \Sigma\, x(j) \ ; \quad x(j) = x_1 \cdot x_2, \quad x_1 \in \otimes^r E \quad x_2 \in \otimes^s E^* \quad (\exists\, r,s)$$

Dual to the above, reverse order can be introduced.

$$x \in S \ ; \ x = \Sigma\, x(j) \ ; \quad x(j) = x_2 \cdot x_1, \quad x_1 \in \otimes^r E \quad x_2 \in \otimes^s E^* \quad (\exists\, r,s)$$

Fixing the tensor product order determines the rank of the tensor monomials. In the above example, the E-basis rank of x(j) is (r,-s) in normal order representation and (-s,r) in reverse order representation.

●Essence of Inner Product, orientational affinity, canonical Invariance

The inner product $\langle x|y \rangle$, $x,y \in S$ is defined as a bilinear mapping (form) of x^* and y to the coefficient ring. Behind this, there is a relation with the tensor x^*y or yx^*. From this idea, affinity with the left-right orientational attribute of the vector coefficients comes out. ***A**

However, before taking the tensor product, consideration of the combination of tensor orders and their orthogonality is necessary.

Conceivable definition of inner product will be shown below. When there is a metric structure on the base space E, its extension to the tensor space has a diversity by arbitrariness.

･Inner Product as Component index Contraction

Let x, y∈S be decomposed as the sum of monomials expressed in normal order.

$x=\Sigma\,\mathbf{a}X(j)\mathbf{b}^*$, $y=\Sigma\,\mathbf{a}Y(k)\mathbf{b}^*$, $\mathbf{a}\in\otimes^r E_C$ $\mathbf{b}\in\otimes^s E_C^*$ （∃r, s)

(This notation assumes that E has right-coefficient, and dually E* has left-coefficient.)

When the representation order of tensor product is fixed as normal order (or as reverse order), the rank of the monomial tensor becomes to be defined. Here, it seems reasonable to assume that tensor spaces with different rank are orthogonal to each other.

Therefore, in defining the inner product of monomials aX(j)b* and a'Y(k)b'*, we can assume it is 0 unless the lengths of a and a', b and b' are equal respectively.

Then, calculation of the inner product reduces to the problem of contracting the component indices of bX(j)*a* and a'Y(k)b'*.

2 kinds of definition can be made depending on the order of multiplication.

$\langle x|y\rangle=\Sigma\,X(j)^*\langle a|a'\rangle Y(k)\langle b'|b\rangle$; from the idea of forward product x*y

$\langle x|y\rangle=\Sigma\,Y(k)\langle b'|b\rangle X(j)^*\langle a|a'\rangle$; from the idea of transposed product yx*

Where, $\langle a|a'\rangle$, $\langle b'|b\rangle$ are inner products of the same rank tensors on $\otimes E_C$, defined as follows:

$\langle c|c'\rangle\in\mathbb{C}$ $(c,c'\in\otimes E_C)$: coefficient component of a*a in normal order representation '.

In the above ,definition is shown based on normal ordering , but the dual definition based on reverse ordering can be also possible.

Since canonical transformation generally does not preserve the rank of tensors, it is a concern matter whether the assumption of orthogonality is invariant among the tensor spaces of different rank, under canonical transformations. ***B**

･Inner Product as Vacuum Expectation Value

Consider defining the inner product $\langle x|y\rangle$ x, y∈S, by extracting the coefficient component from the tensor (x*y) or (yx*) instead of by contraction.

Symbolically this can be written as $\langle x|y\rangle=\langle z\rangle_0$; $z\equiv x^*y$ or $z\equiv yx^*$

However, this definition has the defect that the norm of a mixed tensor like aXb* cannot be calculated (becomes 0). We will consider a modification for this later.

Let x, y∈S be decomposed to the sum of monomials in normal order.

$x=\Sigma\,aX(j)b^*$, $y=\Sigma\,a'Y(k)b'^*$ $a,a'\in\otimes E_C$ $b,b'\in\otimes^s E_C^*$

Consider an element e*∈E* : the elements of dual space of E ,as an annihilation operator acting 、 as $e^*|0\rangle=0$. When considering vacuum expectation value for component extraction, the only terms that survive in $\langle 0|x^*y|0\rangle$ are those with b, b' empty.

For the forward type:

$\langle x|y\rangle=\langle 0|x^*y|0\rangle=\Sigma\langle 0|X(j)^*a^*a'Y(k)\,|0\rangle=\Sigma\,X(j)^*\langle 0|a^*a'|0\rangle Y(k)$

$\langle 0|a^*a'|0\rangle$ can only survive if the rank of a and a' are equal.

This is the same as in the contraction definition. And from the canonical commutation relations and the relation between inner product, $\langle 0|a^*a'|0\rangle=\langle a|a'\rangle$ holds.

This definition consists of the part of the contraction definition where b, b' are empty.
When the tensor rank of a and a' are the same, $\langle 0|a^*a'|0\rangle = \langle a|a'\rangle$.

$$\langle x|y\rangle = \langle 0|yx^*|0\rangle = \Sigma\langle 0|Y(k)\, b^*b'X(j)^*|0\rangle = \Sigma\, Y(k)\langle 0|b^*b'|0\rangle X(j)^*$$

A similar argument can be made for the case of reverse ordering and for $\langle 0|y\, x^*|0\rangle$, only the terms with a, a' empty can survive.
For the reverse type:

$$\langle x|y\rangle = \langle 0|yx^*|0\rangle = \Sigma\langle 0|Y(k)\, b^*b'X(j)^*|0\rangle = \Sigma\, Y(k)\langle 0|b^*b'|0\rangle X(j)^*$$

・Dual Definition in Reverse Order Representation

Considering the possibility of reinterpreting $\mathbf{e}^*\in E^*$ as a creation operator for antiparticle states by analogy with charge conjugation, $\mathbf{e}^*\in E^*$ is a state creation operator for the space E*, and $\mathbf{e} = (\mathbf{e}^*)^*\in E$ can be reconsidered as the dual space of E*.
In vacuum expectation value calculations, symbolically replace $e^* = \underline{e}$ (generation operator) and follow:

$$\langle 0|\mathbf{e}^* = \langle 0|\underline{\mathbf{e}} = 0\ ,\quad \mathbf{e}|0\rangle = \underline{\mathbf{e}}^*|0\rangle = 0$$

It seems clear that a definition in reverse order can be developed dually with respect to a definition based on normal order, but for the inner product on E*, since the duality mapping is not a canonical transformation, it should be noted that the polarity of the commutation relations of Boson spaces will change. $[\underline{\mathbf{e}}^*, \underline{\mathbf{e}}] = [e, \mathbf{e}^*] = -\langle \mathbf{e}|\mathbf{e}\rangle$
It is possible to interpret this as $\langle \mathbf{e}^*|\mathbf{e}^*\rangle = -\langle \mathbf{e}|\mathbf{e}\rangle$.
The situation regarding the inner product on E* is as follows:

$\langle a^*\|b^*\rangle$	nor.ord.	rev.ord.
forward ab*	0	$-\langle b\|a\rangle$
transposed b*a	$\langle b\|a\rangle$	0

***A:**

The orientationality of the coefficients can also be said to be for convenience. Because the relations as follows can be introduced, in order to represent tensor products under the associative law.

$$\alpha_1\cdot\mathbf{e}_2 = \mathbf{e}_2\cdot\alpha'_1\quad ,\quad \alpha_1, \alpha'_1\in R,\quad \mathbf{e}_j\in E_C\ ,$$

If basically the base space is left-coefficient linear space, it can be said that the dual space is basically right-coefficient one.

$x = \mathbf{e_1}\cdot\alpha\in E\ ,\quad y = \mathbf{e}_2\cdot\beta\in E$ → affinity to forward type definition $x^*y = \alpha^*(e_1{}^*e_2)\beta$

$x = \alpha\cdot\mathbf{e_1}^*\in E^*,\ y = \beta\cdot\mathbf{e_2}^*\in E^*$ → affinity to transposed type definition $yx^* = \beta(e_2{}^*e_1)\alpha^*$

For x, y∈S, which type to associate with $\langle x|y\rangle$ is not uniquely determined.

***B:**

For example, considering the forward case, write an expression including the case where the lengths of a,a' and b,b' do not match, then replace the inner product with the vacuum expectation value as follows:

$$\langle x|y\rangle = \Sigma\, X(j)^*\langle 0|a^*a'|0\rangle Y(k)\langle 0|b'^*b|0\rangle\qquad ;\quad \text{forward product. } x^*y \text{ idea}$$

Considering invariance under the infinitesimal transformation $x \to x + i[w,x]$; $w = w^*$ (self adjoint), since the vacuum maps to 0, invariance can be expected.

Note : Sufficiency of norm definition for inner product definition

When defining an inner product with assuming its sesqui-linearity, it is sufficient to define a quadratic norm for the inner product definition.

$$\langle (a+b)|(a+b)\rangle = \langle a|a\rangle + \langle b|b\rangle + (\langle a|b\rangle + \langle b|a\rangle)$$

According to the above, define the following function n defined by the quadratic norm:

$$2\cdot n(a,b) \equiv \langle (a+b)|(a+b)\rangle - (\langle a|a\rangle + \langle b|b\rangle)$$

$\langle a|b\rangle$ can be obtained from:

$$2\cdot n(a,b) = \langle a|b\rangle + \langle b|a\rangle \quad ; \quad 2\cdot n(a,bi)/i \equiv \langle a|b\rangle - \langle b|a\rangle$$

$$\langle a|b\rangle = n(a,b) + n(a,bi)/i$$

●Interpretations about Inner Product on State Space -Summary

・Consideration Elements

Ordering: normal order/reverse order － defined with duality

orientation of product: forward type base (x^*y) / transposed type base (yx^*) － each has affinity with coefficient orientationality.

・Inner Product Definition by Components Contraction

After fixing the ordering, consider spaces with different ranks as orthogonal, and calculate the inner product by contracting components.

Normal order representation,

decomposition into monomial sum: $x = \Sigma\, aX(j)b^*$, $y = \Sigma\, a'Y(k)b'^*$; $a,b,a',b' \in E_C$

Forward type : $\langle x|y\rangle = \Sigma\, X^*(j)\langle a|a'\rangle Y(k)\langle b'|b\rangle$

Transposed type : $\langle x|y\rangle = \Sigma\, Y(k)\langle b'|b\rangle X^*(j)\langle a|a'\rangle$

・Definition of Inner Product as Vacuum Expectation and its modification

Considering $\mathbf{e}^* \in E^*$ as the annihilation operator of the state space, as acting $\mathbf{e}^*|0\rangle = 0$,and applying the calculation by the vacuum expectation of the tensor product related to the inner product.

Definition due to vacuum expectation contains a defect that the norm of E E*mixed tensors becomes 0.

If considering state expectation not limiting to the vacuum, but to other states, this defect is corrected, and it comes to include the definition by component contraction .

Modification of forward type definition. (example)

$$\langle x|y\rangle = \int \Sigma \langle \xi|b\rangle X^*(j)\langle a|a'\rangle Y(k)\langle b'|\xi\rangle dP(\xi)$$

Definition by the sum of expectation to all states with equal weight (i.e. isotropic distribution) is nothing but a trace operation on tensor space and this coincides to the definition by components contraction.

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Theory of Unified Field by Canonical Gauge Principle (2024.04)

part II :
State-constructive formalism of Field Theory

Link to Table of Contents (Ocal, part Ⅱ) .

return TOC(total) .

Overview / Introduction

In Part I, based on the concept of quantum spacetime, we established a theoretical framework for the unified field and derived the Lagrangian of the unified field. (keywords: spacetime reason of existence of fields/particle, structure, symmetry)

In Part II, as a next step to deal with real-world problems, we consider systematic calculational schemes that can approximately solve equations to derive conclusions, namely practical means.

Aiming to overcome divergence difficulties, we analyze concepts of field theory that appear to be already established, especially conceptual constructions/principles regarding state concepts and constructive definitions of field operators, and we will propose a systematic and general approximate solution method for equations.

With conventional diagram techniques for perturbative calculations, we suffer from "divergence difficulties". As a treatment for this, there is the renormalization prescription, but its effectiveness depends on the type of interaction, and the prescription itself is not principled. Personally, I do not think the renormalization prescription is the final form of the theory.

The principle of the proposed method is based on variational methods, applying approximate methods by state separation, and selectively and explicitly constructing field states, so it is appropriate to call it a "state-constructive field theory".

Approximations can be proceeded on successively by iteration.

probably in mathematics, it would be one of approximate solution methods for partial differential equations of infinite multi-variables,

where the number of variables is equal to that of the modes of fields.

In the approximation by the state separation method, continuity to the particle quantum mechanics appears, and from this, it has probably succeeded in avoiding the divergence difficulties.***A**

In other words, the "state-constructive approximation solution method" for field equations, based on finite element method instead of diagram techniques, limits the solution states to the sum of a few state separation forms, and utilizes variational methods to extract approximate corrections.

The state separation there includes mode separation forms that hypothetically limit states to only a finite number of excited modes, in addition to separations by field type.

Field equations can be seen as a kind of partial differential equations of infinite multivariable function corresponding to the infinite degrees of freedom of a field, which are governed by "field operators" and imagined to form some kind of family.

The proposed method can also be seen as an approximate solution method for such equations.

This "state-constructive approximate solution method" is a direct extension of calculational methods in particle quantum mechanics , and is expected to probably avoid the above divergence difficulties.

Rather than considering the avoidance of these divergence difficulties as an object of proof, we should think about refining the method and making it more effective and systematic.***B**

***A:**

In my personal opinion, thinking about the cause of divergence difficulties, perturbation theory taking the interaction constant as the perturbation parameter is not bad in itself. However, the way to apply perturbation theory is wrong. In the "steady problem" examining the steady state of a composite system, if set the interaction constant = 0, then the coupled system cannot exist as an unperturbed state. In scattering problems, if setting free fields as unperturbed states, then the self interactions are added in perturbative calculations in addition to the interaction among the colliding particles.
From these circumstances, placing free fields at the center of perturbative expansion does not seem physically appropriate.
Moreover, likely Fermi type particles (preons) cannot exist alone. The fact that quarks cannot exist alone in steady state suggests this. Preons bind by Boson exchange, and composite system become "white" as a result of pre-color symmetry, which is a necessary condition for being able to exist alone.

The "self interaction" of electrons seems understandable for the first time as the interaction between preons.
Regarding the gravitational field in general, it is one thing that Einstein's equations do not always agree with the equation of the unified field. But even before that, there are methodological doubts about the applicability of the renormalization prescription itself to nonlinear fields.
The gravitational field is a nonlinear field, and the equations contain self interactions from the beginning. There is no guarantee that self interactions can be canceled out with a finite number of counter terms even in free field calculations, and also that does not seem like the right approach either.
By the way, nonlinearity means that the variance effects also contribute in addition to the product of classical fields as that of the quantum theoretical mean field . Therefore, in states where gravitons are concentrated, such as black holes and the cosmic big bang, the classical theory should be expected in breaking down.

The approximation calculation method In Part II of the theory of the unified field is the method based on variational principles and possible to take account of the variance effects. Successive improvement in approximation accuracy is also possible. The quantum gravity theory will be considered in Part III.

***B:**

Divergence difficulties mean calculations have actually diverged when performed. Whether calculations diverge or converge, and the conditions for that, and concepts to analyze those conditions, are not clear.
From a practical standpoint, proving the divergence avoidance would, even with restrictively defined prerequisites, likely amount to just performing and confirming computational possibility.
In such a case, it is ambiguous what has been proven by that "proof".
While a proof of renormalizability may have logical proof significance, it does not provide a physical solution.

2023.06 T.sato

Unified Field Theory by Canonical Gauge Princiole
part Ⅱ :
State-constructive formalism of Field Theory

Table of Contents(Local ,part Ⅱ).

1.Analysis of the Principles of Quantum Theory

Let us attempt to reconstruct the quantum field theory through the interpretations being faithful to the basic principles of quantum mechanics. Therefore, already known principles in field theory will be re-examined and reconfirmed in light of the basic principles of quantum theory.

1.1.Particularity of Time

The particularity of time originates from our form of state recognition that takes time as a parameter and recognizes motion as a change of state over time.

In classical mechanics, for the motion of 1 particle, we can take the length of the world line instead of time as the parameter. Certainly, there is no need to be constrained by time. However, we would have to accompany it with a constraint condition $dx \cdot dx = ds^2$ in final stage.

Moreover, for multiple particles, we would end up dealing with bundles of world lines, and by the above method, the variables with constraints would only increase, making unified comprehension of the whole difficulty. *1

Rather than that, slicing spacetime (set by an observer) with equal time cross sections and capturing particles as intersections with the equal time cross sections allows obtaining unified geometric perspectives.

The relativity corresponds to the degree of freedom regarding setting equal time cross sections, and relativistic covariance is ensured by this geometric view.

This circumstance is the same in quantum theory as well, with time playing a special role in representing motion.

For example, for 1 particle, the wave function $\psi(t,\mathbf{x})$ corresponds to its motion.

At first glance this appears like a field on 4-dimensional spacetime, but actually t appears as a parameter. $\psi(t,\mathbf{x})$ represents the probability amplitude regarding the state and existence position of the particle at time t. Therefore, the meanings of t and x are completely different.

Furthermore, in multi-particle systems it becomes like $\psi(t,\mathbf{x}_1,\mathbf{x}_2,\cdots,\mathbf{x}_k)$, clearly showing that the state representation is a wave on an abstract space with time t as a parameter. Of course, it could be argued that it is because of Schrödinger representation.

However, directions assigning time to each particle and considering probability amplitudes, like multi-time theories, etc. are not correct directions for capturing reality. There is also the practical problem that representations using wave functions cannot dexterously express particle creation/annihilation. *2

On the other hand, if we take “fields” rather than “particles” as the fundamental objects, the field equations obtained from the variational method become partial differential equations regarding the variational function corresponded to linear operators. The Lagrangian L is given by the spatial integral of density form L^D, and the action I is the time integral of L.

$$L = \int L^D\, d^3x \quad ; \quad \text{Action} \quad I \equiv \int dt\, L = \int d^4x\, L^D$$

The action is given by a 4-dimensional integral with $dt d^3x = d^4x$, resulting in the general covariance of the action form ensures relativistic invariance. Furthermore, as we already know from Part I, this can be expanded to canonical invariance.

By the way, the above action form can be viewed as the action form for 1 point particle in an infinite

dimensional space with the 3-dimensional coordinate variables x as continuous indices.
In this view, the appearance of time is different from space.
Let us note that since the variational principle is based on the principle of least action, the action I should originally be a real number. That is, if the variational variables are linear operators, it should be understood that the notation for the action I abbreviates something that should originally be written like $I' \equiv \langle \Phi | I | \Phi \rangle$, an expectation value with respect to a certain state $|\Phi\rangle$.
Therefore, even when starting the theory from the variational principle, concepts like the space on which the "operators" act, states, state space, etc. are necessary.

When recognizing the obtained partial differential equations as equations of motion for the field operators, it could be said that one rational development of the theory is the Hamilton formalism.
It should be noted that the shifting to Hamilton formalism means having already set equal time cross sections, in other words, having introduced eigenstate representations of the time operator into the state representations, and selecting "special treatment of time" as the formulation for representation.
However, that is a consequence of our state recognition method based on time cross sections.
And as already mentioned, the relativistic invariance and furthermore the canonical invariance of the theory is ensured. ***3**

***1** :
・**World line vs time variable as a motion representation parameter:**
when taking the world line as parameter, the action element dA of 1 mass point motion is given by $dA = (1/2)mv^2 ds$, or $dA = (1/2)m(v^2-1)ds$ (where $v^2 = 1$ is imposed after obtaining the equation.).
Concerning the above formulation, the covariance with respect to the coordinate transformation is clear. On the other hand, the action element dA can also be set as $dA = ds = dt/v^0$ $(dt = v^0 ds)$ with the time variable (t) as parameter, preserving the representation be covariant as well.

While clarity of the transformation's covariance is certainly desirable, ensuring the apparent invariance of the theory in the representation is not so essential, and values like simplicity of the representation or depiction should not be sacrificed just to pursue it.

・**Regarding representation of motion by world line and the constraint conditions:**
In formulations treating world lines like free parameters and quantizing motion, wave functions like $\psi(s;x)$ would be obtained.
However, actually the 4-velocity v must satisfy $v \cdot v = 1$. Therefore there would be a certain 4-velocity operator v and the constraint $(vv-1)\psi = 0$ would be imposed.

・**Regarding action-at-a-distance between particles and breakdown of causality:**
Even within the scope of classical theory, interaction between 2 particles cannot be represented in action-at-a-distance form while preserving causality. Because it becomes interaction between past and future. In that case the Green function that should be selected is (Advanced + Retarded)/2, i.e. it inevitably has to be symmetric with respect to time.
If applying local action theory, introducing the concept of fields is essential.

***2** :
Leaving aside commutativity/anti-commutativity symmetry of elementary particles for now,

without specializing time, representation in spacetime would be $f1(x_1)f2(x_2)$ for 2 particles, $f1(x_1)f2(x_2)f3(x_3)$ for 3 particles ; $x_i \equiv (t_i, \mathbf{x}_i)$, i=1,2,⋯, and the representation becomes multi-time.

On the other hand, we always recognize states in the form “state f(t) at a certain time t”, and in the example of 2 particles, only the case of $t_1 = t_2$ it becomes the object of consideration regarding suitably defined spacetime coordinates. And that is sufficient.
It should be noted also that the above notation based on particles does not have the representational capability for “fields” with infinite degrees of freedom.

***3** :

It does not mean the expressions in Hamiltonian formalism become relativistically covariant.
Selecting the Hamiltonian formalism implies choosing the special treatment of time already.
For example, in past representations of the electromagnetic field, the canonical conjugate momentum of component A_0 being identically 0 was considered problematic, and several remedies were proposed, but the actual issue was simply that “in canonical formalism, A_0 is not a canonical variable”. The time evolution of A_0 is obtained not from canonical equations but from gauge conditions containing $\partial_0 A_0$.
Moreover, gauge fixing is obtained by applying the least action principle to action breaking gauge invariance. Such actions can be considered arbitrarily, for example in the case of Lorentz gauge, it was merely a matter of choosing the action density as $(-g)^{1/2} A_j A^j$ in account of relativistic invariance.

1.2.Adoption of Canonical Formalism and Schrödinger Representation

(1).Variational Method and Heisenberg Representation

Guided by spectroscopic data, Bohr, Heisenberg and others constructed mechanics : matrix mechanics that interpret position and momentum x, p as linear operators. (The "matrix" here refers to linear operators)

It can be said that formally, constructing quantum mechanics starting from the variational principle is done by replacing the canonical variables **x, p** in Hamilton formalism with linear operators.

However, if one says they are "linear operators", then logically it is still necessary, without ending with just calculating rules, to clarify what linear space those operators act on and what kind of meaning it has.

(2) Introduction of Hamilton Formalism

When transitioning from the variational equation to the Hamilton formalism, time is given the character of a state display parameter here. On the other hand, the 3-dimensional position variable x and the canonically conjugate canonical momentum p are linear operators.

In particle mechanics, the canonical variables are the position x and the canonical momentum p, and the canonical commutation relations and canonical equations become as follows:

$[\mathbf{x}, \mathbf{x}]=0$, $[\mathbf{x}, \mathbf{p}]=i$, $[\mathbf{p}, \mathbf{p}]=0$; Expressing Hamiltonian as $H\equiv H(\mathbf{x},\mathbf{p})$,then

$d\mathbf{x}/dt=\partial H/\partial\mathbf{p}=[iH, \mathbf{x}]$, $d\mathbf{p}/dt=-\partial H/\partial\mathbf{x}=[iH, \mathbf{p}]$

; (H becomes the time propagation generator.)

In field theories as well, apart from the point that the canonical commutation relations vary according to the statistical nature of particles, being commutative relations or anti-commutative relations, - this itself is important - the formulation is built up in parallel.

The canonical variables in field theory are the field variable (operator) $\chi\equiv\chi(x)$ and the canonically conjugate momentum density $\pi\equiv\delta L^D/\delta\chi_{,0}$.

The Lagrangian L referred to in masspoint mechanics is given by the 3-dimensional integral (sum) of the Lagrangian density L^D on the equal-time cross-section.

This means that $\chi(x)$: $x\equiv(t,\mathbf{x})$ is like a mass point in an infinite dimensional space with the continuous index of spatial coordinates.

When $\chi(x)$ is expanded as $\chi(x)=\Sigma E_m(\mathbf{x})\cdot z^m(t)$ by a certain base Em(x), z^m can be interpreted as position coordinate variable (operator) in a countable infinite dimensional space.

The canonical commutation relations become the following as equal-time commutation relations:

$[\chi(t,\mathbf{x}), \chi(t,\mathbf{y})]_\pm=0$, $[\chi(t,\mathbf{x}), \pi(t,\mathbf{y})d^3\mathbf{y}]_\pm=\delta_{x,y}$, $[\pi(t,\mathbf{x})d^3x, \pi(t,\mathbf{y})d^3\mathbf{y}]_\pm=0$,

The canonical equations are : $\partial\chi/\partial t=[iH, \chi]$, $\partial\pi/\partial t=[iH, \pi]$: ***1**

It should be noted that even when the canonical commutation relations are given by anti-commutation, the canonical equations are given by commutation relations.

This is related to the fact that for Fermions, the field variables appear as even forms in H.

The corresponding formula for this is $[a, bc]=\{a,b\}c-b\{a,c\}$

Although the Hamilton formalism treats time specially, the relativistic invariance of the theory is guaranteed by the invariance of the Lagrangian density.

Moreover, from the discussion in Part I, it is clear that coordinate transformations should be replaced by canonical transformations.

***1 :** To maintain parallelism with particle systems, the discrete representation is followed, but in the usual continuous system representation, $\delta_{x,y}=\delta^3(\mathbf{x}-\mathbf{y})d^3\mathbf{y}$.

(3) Introduction of Schrödinger Representation

If starting the quantization from the variational principle, the variational variables are interpreted as linear operators, the motion of the system is expressed as the motion of operators, and the so-called Heisenberg representation is obtained.
However, there is no mention of the linear space on which those linear operators act.
To reach the concept of the system state and state vector, the transition to the Schrödinger representation is essential. It is through the Schrödinger representation that the depiction of "the state of the system moving along with the passage of time" first becomes possible.

Taking the wave nature of particles as a clue, Louis de Broglie and E. Schrödinger constructed wave mechanics, but the equations of wave mechanics suggested the operator interpretation of physical quantities like $p^2=-\partial^2=-\nabla^2$. ***1**
Schrödinger's wave equation takes the form $+i\partial f/\partial t=Hf$; $f\equiv$ wave function.

The points to note here are:

- It is a linear equation
- The representation is related to the Hamilton formalism
- It is 1st-order differential with respect to time

In wave mechanics, (x, p) are operators, but they do not move. What moves is the "wave function".
What connects the depiction of "operators that move according to canonical equations obtained from the variational method" (Heisenberg representation) and the motion of the system state vector (wave function) (Schrödinger representation) is "time". This first becomes possible precisely through the "way of looking at the state on the time cross-section" in the Hamilton formalism, the special treatment of time.

Heisenberg's equation of motion become:

$dx/dt=[iH, x]$, $dp/dx=[iH,p]$
$U\equiv\exp(i(t-t_0)H+i\theta)$ gives formal solutions $x=Ux_0U^{-1}$, $p=Up_0U^{-1}$.

For a general operator $A=A(x,p)$ not explicitly containing time t, the motion is $dA/dt=[iH, A]$, and the formal solution is $A=UA_0U^{-1}$; $A_0\equiv A(x, p)|_{(t=t_0)}$.
The transformation $A\rightarrow A_0$ is precisely the conversion of the operator from the Heisenberg representation to the Schrödinger representation.
Therefore, it can be said that the operators in the Schrödinger representation are "frozen in time".

Note here that the above relations are invariant under the transformation $H\rightarrow H+C$ (C: arbitrary commutative constant). This arises from the arbitrariness of the origin of the canonical energy.
Also, the reason the argument of U can be simply expressed in the form $(t-t_0)H$ is that H is the conserved quantity of motion, i.e.$dH(x, p)=0$. This is an important fundamental property in the development of the theory.
Although the above representation transformation formula $A\rightarrow A_0$ is simple, it contains profound meanings. Perhaps the most important thing is that the concepts of the wave function, state vector, and state space, have come to be seriously asked.

For any physical quantity A (Heisenberg representation), the expected value <A> of the observed value obtained when A is observed at time t is expressed as:

$\langle A\rangle = \langle f_0|A|f_0\rangle = \langle U^{-1}f_0|A_0|\, U^{-1}|f_0\rangle = \langle f|A_0|f\rangle$ where $|f\rangle \equiv U^{-1}|f_0\rangle$

$U \equiv \exp(i(t-t_0)H + i\theta)$; $|f\rangle$ = state vector (Schrödinger representation)

$\partial/\partial t\, |f\rangle = -iH|f\rangle$ ($+i\partial/\partial t\, |f\rangle = H|f\rangle$; Schrödinger equation)

While f was understood as the "wave function" when dealing with a single quantum particle, more generally it should be considered as the vector (in linear algebraic sense) representing the state of the system, or, emphasizing the dealing with multi-particle systems, as the tensor.
Especially for the vacuum state $|0\rangle$, it is convenient to assume $|0\rangle$ is an eigenstate of H and use the indeterminacy of the commutative constant ,to set $H|0\rangle = 0$.
In summary, the following can be said:

- In Schrödinger representation, the operators are "time-frozen".
- Utilizing the arbitrariness of the origin of the canonical energy, the action of Hamiltonian H on vacuum can be assumed to be 0.
- In the case of field theories, the state space on which the field operator χ and its canonically conjugate momentum density π act includes multi-particle states and is a tensor space with the single-particle state space as the fundamental space.
  The symmetry and antisymmetry of the tensor algebra corresponds to the statistical nature of particles.

In field theories, the following issues should also be examined as appropriate tasks:

* How the field operator (x) should be constructed.
*The relationship between the front and back (direction of time) of the equal-time cross-section, corresponding to the time-freezing of operators in the Schrödinger representation, and the sign of the energy of the vibration modes. *2
*The relationship between the normalization factors for the field operator $\chi(x)$ and the interaction constants, while taking accounts of the degrees of freedom for constant multiplication to Lagrangian and the canonical commutation relations.

***1: Sign choice in representation** $p=-i\partial$:

In the convention of expressing the plane waves with definite momentum as exp(ipx) ($p_0=\omega$), it follows $p=-i\partial$. As operator commutation relations, the canonical commutation relations are $[x, p]=i$. It is notable that since motion is a canonical transformation, this relation is invariant.

***2 : Relationship between energy and Hamiltonian:**

From the Lagrange 1-form $\Lambda \equiv p dx - H dt$, the correspondance $p_0 = -H$ can be seen.
In the convention where the spacetime metric η has $\eta_{00}=-1$, the interpretation of H as contravariant energy was also possible due to $p^0 = -p_0 = H$, but with $\eta_{00}=+1$ the minus signs would have to be multiplied.
To avoid confusion in mathematical expressions, it will be desirable to maintain the equation representation and change the interpretation.

It can be seen from canonical commutative relation $[p_0, x^j] = -i\cdot\delta^j{}_0$, $[p_0, p_j]=0$, for example, it

should be noted that $H \equiv -p_0$ does never mean an identical relation of operators.
In the Schrödinger representation, $H|f\rangle = -p_0|f\rangle$ is required for the state $|f\rangle$, and from the expression $p_0 = -i\partial/\partial t$, the Schrödinger equation for the state $|f\rangle$: $H|f\rangle = -p_0|f\rangle = -i\partial f/\partial t$ can be obtained . This is the logical structure here.

In Heisenberg representation, the motion of operator A is given by $\partial A/\partial t = [iH, A]$ that is, $\partial/\partial t = \mathrm{ad}(+iH)$, for the operator. On the other hand, in Schrödinger representation, the motion of the state $|f\rangle$ is given by $+i\partial f/\partial t = H|f\rangle$, that is, $\partial/\partial t = -iH$ for the state.
The opposite relationship in terms of sign shall be careful.

1.3. Concept of State Space

Probably the concept of state space would not have emerged without the Schrödinger representation. From the standpoint of pushing through with the Heisenberg representation, the Schrödinger representation is not logically essential. However, that method has limitations in analytic power, and above all, the "state concept" is not established, and without a dynamic perspective of state motion, understanding of the physical content cannot be obtained. *1

Moreover, no matter the representation, the concept of a state vector is essential in order to obtain the expectation values of physical quantities. Below we consider the concept of state space.

*1 : The revolutionary change from classical theory to quantum theory is probably that the state concept has changed from point in phase-space to vector in linear space, and the logical structure moved from distributive lattice to modular lattice formed by linear subspaces.

1.3.1. Construction of State Space and Differential Ring/Tensor Algebra

Introducing the Schrödinger representation means having already selected a specific time surface. The state space is constructed from the base space of single-particle state space E: linear space/**C** as follows, including its dual space E*:

$$\mathbf{C}\to E\to E^*\to (E\oplus E^*)\to\otimes(E\oplus E^*)\equiv S\ ,\qquad S|0\rangle\equiv S_0\quad : \text{State space}$$

Although the dual space E* was originally defined as linear forms on E, by interpreting the action as canonical commutation relations on (E⊕E*), S can be regarded as a differential ring.

Analogous example : $\mathbf{C}\to\mathbf{C}[z]\to\mathbf{C}[z,\partial_z]\ ;\quad [\partial_z, z]=1$

Since the 3-dimensional space as the equal-time cross section has orientation (front/back), each state (vibration mode) a(x) corresponds to the positive/negative energy state according to its orientation.

As negative energy states do not exist physically, particle negative energy state creation/annihilation operators are reinterpreted as antiparticle annihilation/creation for positive energy states.
(Though with this terminology, for Bosons there are cases where particle＝antiparticle)
Through the extension of the tensor algebra, state tensors can become identified with state creation operators. "States" are obtained by having these operators act on vacuum.
As an analogous example of constructing the space of this kind, it is examined next for the differential ring of polynomials.

1.3.2. Relationship between State Space and Operator Space
--- Example of Differential Ring of Polynomials

As a model for understanding the relationship between states and field operators, we give the example of the differential ring of polynomials.

$$\mathbf{C}\to\mathbf{C}[z]\to\mathbf{C}[z,\partial_z]\ ;\quad [\partial_z, z]=1$$

First we consider complex number field **C** and the elemental state z that can be interpreted as a single-particle state. If this corresponds to a Boson state, then the multiparticle state space generated from z can be represented as the polynomial ring $\mathbf{C}[z]$.

We then add (∂_z) the differential operator regarding z to $\mathbf{C}[z]$. This space becomes the differential ring of polynomials $\mathbf{C}[z, \partial_z]$. Since $\mathbf{C}[z]\subset\mathbf{C}[z,\partial_z]$, the polynomial $f\in\mathbf{C}[z]$ can also be interpreted in $\mathbf{C}[z, \partial_z]$ as a multiplication operator. That is, state and operator can be equated.

Applying ∂_z to the state z gives $\partial_z(z)=1$, but as a product of operators, it is $[\partial_z, z]=\partial_z z-z\partial_z=1$,
As immediately seen this is nothing but the canonical commutation relation.

Interpreting z as a state vector, ∂_z can be interpreted as the dual vector of z.

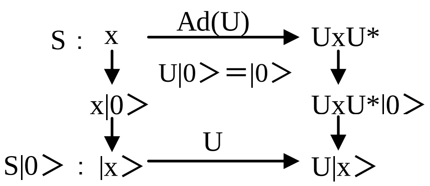
Let a polynomial considered as a state be denoted as $|f(z)\rangle$, the operator f(z) acting on $1\in\mathbf{C}$ can creates the state $|f(z)\rangle$. That is, $f(z): 1\to f(z)(1)=|f(z)\rangle$.
$1\in\mathbf{C}$ corresponds to the vacuum. Extending this, states can be created by having operators $f(z,\partial_z)\in\mathbf{C}[z,\partial_z]$ act on $1\in\mathbf{C}$. $\partial_z(1)=0$ can be interpreted as the action of the annihilation operator on vacuum.

In physics, due to the energy polarity of states, negative energy state creation/annihilation is interpreted as positive energy antiparticle annihilation/creation, so it is slightly more complicated than above. The state space and operator space are connected by the following mappings:

- Standard injection from state space to operator space $|f(z)\rangle\to f(z)$;
  identification of state and operator. / interpreting state as operator.***A**
- Projection from operator space to state space $f(z,\partial_z)\to f(z,\partial_z)\,(1)=f(z,\partial_z)|0\rangle$;
  State creation by an operator acting on vacuum. vacuum "(1)" is frequently expressed by $|0\rangle$

***A:**

If $|f(z)\rangle=|g(z)\rangle$ then $g(z)=f(z)+N$ ($\exists N$ $N|0\rangle=0$), so the injection cannot be set without limiting the domain to $\mathbf{C}[z]$.
If we have $y=U|f(z)\rangle$ by some unitary transformation U, the operator form of y becomes $Uf(z)U^*$. In order to make the operator form explicit, let the symbol ♮ be introduced to express as ♮$(y)=Uf(z)U^*$. (♮:return to original operator.)
In the space of operators, we can consider $\langle f(z)|=f^*(\partial_z)$ as the dual element of $|f(z)\rangle$.



●note : **relativity of particle number - reference state and transformation**

The above state space model can be extended by completion from the polynomial space to ring of formal power series.
Putting aside the issues of convergence, it can also be called an analytic function ring near z=0.
Since z can be considered a local coordinate, another variable $w=w(z)$, equivalent to z ,can also be introduced. Therefore, the concept of a "single-particle state (z)" depends on what is taken as the reference mode, and there is another reference state (w) equivalent to (z), from which for example

$$z=z(w)\equiv c_0+c_1w+c_2w^2+\cdots\ ,\quad c_1\neq 0\quad (\text{i.e}\quad dz/dw\neq 0,\ \infty)$$

then z can be interpreted as a superposition of multiparticle state with respect to w.
For Bosons, it is notable that a constant term c_0 may be mixed in.

1.3.3. Inner Product Structure on Operator Space

(1).Definition of inner product:

In order to calculate the expected value of a physical quantity, it is necessary to define an inner product on the state space.
The state space is embedded into the operator space as we saw in Section 1.3.2.

The operator space can be equated with the tensor space generated from the state space and its dual space.
Considering these situations, it is quite natural to define the metric structure on operator space.
In order to define the inner product on operator space, we introduce "normal ordering" in the expression of tensors, because there can be various expressions about ordering for tensor product in the operator space $S=\otimes(E\oplus E^*)$.
The normal ordering is defined as an expression in which the positive energy term is to the left of the negative energy term in tensor product, and this is possible by using canonical commutation relation.

Any tensors in S can be expressed as the sum of tensor monomials expressed in normal order.
Each monomial tensor m is in the form that $m=ab^*$, a,b : positeve energy monomial (given by tensor product of positive energy tensor monomials in $E\oplus E^*$),and we can define the rank of monomial $m=ab^*$ as (rank(a),-rank(b)) ,in accordance with the introduced ordering.

Suppose the applicability of distributive law, and we define the monomial tensors in normal ordering are orthogonal to each other when their ranks differ. Then, it suffices to define $\langle ab^*|a'b'^*\rangle$ on a, b, a', b' of positive energy monomials. We can set $\langle ab^*|ab^*\rangle\equiv\langle a|a'\rangle\langle b^*|b'^*\rangle$.
The inner product $\langle a|a'\rangle$ is defined in usual way as $\langle a|a'\rangle=\langle 0|a^*a'|0\rangle$.

Therefore, the issue reduces to the define $\langle b^*|b'^*\rangle$. Here, we can set the product as $\langle b^*|b'^*\rangle\equiv\langle b'|b\rangle\cdot\varepsilon$; $\varepsilon=\pm1$.
If and only if $|b\rangle$ is a monomial tensor of Boson, we can take $\varepsilon=-1$ as another definition

$$\langle b^*|b'^*\rangle\equiv\langle 0|[b,b'^*]|0\rangle=-\langle 0|[b'^*, b]|0\rangle=-\langle b'|b\rangle$$

In detail, see part I Apenndix A2.

(2).Norm calculation method for Boson states

Supposing situation where the state tensors of Bosons are represented by analytic functions of a certain reference mode (single-particle state). In such cases, we need to formulate the method of calculating state norms.
Assuming the norm $\langle z|z\rangle=1$ for z, we must calculate $|f(z)|^2=\langle f|f\rangle$ for the analytic function f(z). We consider the following. First, with $\langle z|z\rangle=1$ we have $\langle z^m|z^m\rangle=m!$.
This is essentially the same as $\partial_z^m(z^m)=m!$.

$$\text{For } A(z)=\Sigma A_m\cdot z^m,\quad B(z)=\Sigma B_m\cdot z^m$$

$$\int(d\theta/2\pi)A(z)^*\cdot B(z)=\Sigma A_m{}^*B_m r^{2m}\quad ;\quad z\equiv r\cdot\exp(i\theta)$$

Formally considering a Laplace transform, ignoring convergence we obtain:

$$\langle A|B\rangle=\int d(r^2)\exp(-r^2)(d\theta/2\pi)A(z)^*B(z)=\Sigma\, m!A_m{}^*B_m$$

Note that with $z=r\cdot\exp(i\theta)=x+iy$, $r^2=x^2+y^2$, $2rdrd\theta=2dxdy$.
Also, $dz^*dz=(dx-idy)(dx+idy)=2idxdy=-2idydx$ (note: antisymmetry of differential forms).
From this, the inner product can be written as:

$$\langle A|B\rangle=(1/\pi)\int rdrd\theta\cdot\exp(-r^2)A(z)^*B(z)=(1/\pi)\int\exp(-x^2)dx\int\exp(-y^2)dy(A(z)^*B(z))$$

Expressing to show symmetry explicitly:

$$\langle A|B\rangle=1/(2\pi i)\int A(z)^*\cdot dz^*\exp(-z^*z)dz\cdot B(z)$$

.

2. Concept Analysis Regarding the Construction of Field Operators

In Chapter 2, we will reconfirm the structure, meaning, and properties of field variables (field amplitude operators).

The variational variables in field theory, i.e. the field variables $\chi(t,x)$, can be understood as the motion of a point in an infinite dimensional space with the 3D spatial coordinate x as a continuous index, while maintaining relativistic invariance.

On the other hand, x is also a coordinate variable as an equal-time cross-section of spacetime, and has more meaning than just a continuous index, and is an entity that also follows spacetime transformations.

2.1. Structure and Meaning of Field Amplitude Operators

●Positive Oscillation/Negative Oscillation/Antiparticle

As already mentioned, the field variable $\chi(t,x)$ can be interpreted as a point in an infinite dimensional space with a continuous index (x).

Therefore by applying the prescription of masspoint quantum mechanics there, we can obtain the interpretation on $\chi(t,x)$ as a field operator .

(x) is also a coordinate variable, and $\chi(t,x)$ should be expandable in any complete spatial vibration modes system Em(x). Therefore, it can be written as follows:

$$\chi(t,\mathbf{x}) = \Sigma \ E_k(\mathbf{x}) e^{*k}$$

From this formula, it can be seen that e^{*k} is an amplitude operator for the mode E_k, corresponding to a position operator in an infinite dimensional space in the sense of masspoint quantum mechanics.

Since the square of the absolute value of the amplitude is the physical quantity of vibration intensity, the following formula backs up this fact.

$$|(e^{*k}|f\rangle)|^2 = \langle f| e^k e^{*k} |f\rangle \quad ; f = \text{state vector},$$

Therefore, e^{*k} is an annihilation operator for the state corresponding to mode E_k.

Furthermore, examining the case where the field operator can be explicitly solved (i.e. case of free field) as an example, it turns out that the situation is a little more complicated.

In the case of a free field, $\chi(t,\mathbf{x})$ has a plane wave solution and the solution space can be spectrum decomposed into plane waves. As the mode (E_k), a wave motion $\exp(i\mathbf{kx})\cdot d^3k/(2\omega)$ can be adopted. ***A**

Since plane wave solutions have degrees of freedom (vibration modes) within the range allowed by the dispersion relation $k^2 = m^2$, two degrees of freedom $k_0 = \pm\omega$ arise.

Thinking about it, when slicing spacetime with equal-time cross-section, the cross-section have degrees of freedom of front and back, orientation.

It follows that $E_k(\mathbf{x})$, the vibration mode of spatial momentum $\mathbf{k}$, corresponds to two modes of positive vibration and negative vibration.

This fact is true not only for free fields, but is a general circumstance.

Positive and negative vibrations ($k_0 = \pm\omega$) correspond to the positivity and negativity of vibration energy. On the other hand, a negative energy state is physically unacceptable as a particle state.

This firstly leads one to think to construct a theory with only the positive vibration component, but that is not possible for the following reasons.

In the case where the wave equation is 2nd order in time, like the Klein-Gordon equation, $\chi(0,\mathbf{x})$ and $\partial_0\chi(0,\mathbf{x})$ must arbitrarily be able to specified as initial conditions.

With only the positive vibration component, the degrees of freedom are fixed by specifying $\chi(0,\mathbf{x})$ alone, and $\partial_0\chi(0,\mathbf{x})$ cannot be arbitrarily specified.

That is, the positive and negative degrees of freedom of vibration correspond to the degrees of freedom of the amplitude and also its canonically conjugate momentum.

It is the same for the Dirac equation, which is 1st order in time. With only the positive vibration component, an arbitrary value cannot be given to $\chi(0,x)$. The Dirac equation itself divides the 4-dimensional spinor space in two. ***B**

In the formula $\chi(t,\mathbf{x})=\Sigma E_k(\mathbf{x})e^{*k}(t)$, e^{*k} was the state annihilation operator for mode E_k.

Now that the existence of antiparticles is known, the difficulty of negative energy states has been overcome, by interpreting creation/annihilation of negative energy states in particle base as annihilation/creation of positive energy states in antiparticle base .

Negative energy states are an issue of representation and there are no particles physically in negative energy state.

With the above reinterpretation, as long as state vectors are considered to be mapped to operators, the expression "negative energy state" can be used freely.

$$\chi(t,\mathbf{x})=\Sigma\ E_k(\mathbf{x})e^{*k}(t)=\Sigma E_{(+k)}(\mathbf{x})e^{*(+k)}(t)+\Sigma E_{(-k)}(\mathbf{x})e^{*(-k)}(t)$$

$$E_{(-k)}(\mathbf{x})\equiv E_{(+k)}(\mathbf{x})^* ,\ e^{*(-k)}\equiv e^{C(+k)} \quad e\rightarrow e^C \quad : \text{anti-linear transformation to antiparticle state.}$$

Hereafter, the function system $E(\pm k)(x)$ which is a complete system in each and has the symmetry $E_{(-k)}(\mathbf{x})\equiv E_{(+k)}(\mathbf{x})^*$ will be called "double complete system".

It means a complete system of functions including positive and negative vibration states.

Since the double completeness and the inclusion of antiparticle in field operator, pair creation / annihilation of particles becomes possible in interaction term. Now note that for some particles, particle＝antiparticle.

Also, logically e^{*k} above was not necessarily determined from the beginning as the annihilation operator for state ek, but this will be discussed in section 2.2.

***A**:

The factor $d^3k/(2\omega)$ is not essential to the argument here, but originates from the relativistic invariant form of the density of states $\delta(k^2-m^2)d^4k$.

Here $\omega\equiv(m^2+k_sk_s)^{1/2}$, $s=1..3$; m is the particle mass.

$\Sigma_{(k)} \equiv \int \delta(k^2-m^2)d^4k=\int d^3k/(2\omega)\cdot(\delta_{k0,\omega}+\delta_{k0,-\omega})$: summation over states.

***B** :

With the Dirac operator $\mathcal{D}\equiv\gamma^\lambda k_\lambda$, from $\mathcal{D}\cdot\chi=m\chi$, $\chi\in\ker(D-m)$

The total spinor space S is $S=\ker(\mathcal{D}-m)\oplus\ker(\mathcal{D}+m)$. The subspaces are mapped to each other by γ^5.

●Particle/Antiparticle Conjugation in Canonical Gauge Unified Field Theory

From the considerations in Part I, it is seen that fields other than gravity are obtained as the 0th order components of the canonical connection form, and the particle/antiparticle relation is represented by the diagonally mirror relation in the row representation of the connection. Therefore, for Bosons located on the diagonal line, they themselves become antiparticles.

Since particles and antiparticles are interchanged by diagonal transformations, particle/antiparticle transformation (charge conjugation transformation) is equivalent to the dual transformation on the space of field operators.
The field variables (amplitude/momentum operators) are elements of the operator space since they are linear combinations of creation and annihilation operators for particles and antiparticles.

$$-id \begin{bmatrix} \mathbf{a}_{mj} \\ \mathbf{b}_r \end{bmatrix} = \mathbf{a}_{nk} \cdot \begin{bmatrix} C^n{}_m \cdot \delta^k{}_j + \delta^n{}_m (h\gamma dx)^k{}_j \\ (\gamma dx)^k{}_j \chi_r{}^{nj} \end{bmatrix} + \mathbf{b}_s \cdot \begin{bmatrix} \chi^{*s}{}_{mk} (\gamma dx)^k{}_j \\ B^s{}_r \end{bmatrix}$$

FF-type Boson as pre-color gluon $C = C_\lambda dx^\lambda \in su(3)$ and
spinor connection field $h = h_0 + i\gamma^5 h_5 \in o(1,1)$
BB-type Boson as pre-flavor Boson $B = B_\lambda dx^\lambda \in u(2) = u(1) \oplus su(2)$
FB-type Fermion as $\chi_r{}^{nj}$, its antiparticle as BF-type Fermion $\chi^{*s}{}_{mk}$
Particles corresponding to components on the diagonal line are their own antiparticles.
It is also seen that Majorana-type Fermions cannot exist.

●Mode Expansion Representation of Canonical Commutation Relations

For the field variable $\chi(t,\mathbf{x})$, mode expansion was considered and the introduction of antiparticles was found to be essential, but similar expansion must also be considered for the canonically conjugate variable $\pi(t,x)$. In the canonical formalism, the canonical momentum $\pi(t,\mathbf{y})d^3\mathbf{y}$, canonically conjugate to the field variable $\chi(t,\mathbf{x})$ is introduced.

They must satisfy canonical commutation relations (on equal-time cross-section).
The mode expansion system of canonical variables and their canonical commutation relations can be written in discrete form as follows:
(The (±) in the commutation relations corresponds to the symmetry of the tensor algebra)

$$\chi(t,\mathbf{x}) = \Sigma E_m(\mathbf{x}) e^{*m} \quad ; \quad \pi(t,\mathbf{y}) = i \Sigma e_m E^m(\mathbf{y})^* \quad \text{(including negative energy representation)}$$

$$[\chi(t,\mathbf{x}), \chi(t,\mathbf{y})]_\pm = 0 , \qquad [\chi(t,\mathbf{x}), \pi(t,\mathbf{y})d^3\mathbf{y}]_\pm = i\delta^3{}_{\mathbf{x},\mathbf{y}} \qquad [\pi(t,\mathbf{x}), \pi(t,\mathbf{y})]_\pm = 0$$

$$[e^{*m}, e^{*n}]_\pm = [e_m, e_n]_\pm = 0 , \qquad [e^{*m}, e_n]_\pm = \delta^m{}_n \quad ; \quad \Sigma E_m(\mathbf{x}) E^m(\mathbf{y})^* = \delta^3(\mathbf{x}-\mathbf{y})$$

Focusing on the completeness relation $\Sigma E_m(\mathbf{x}) E^m(\mathbf{y})^* = \delta^3(\mathbf{x}-\mathbf{y})$ derived from the canonical commutation relations, multiplying both sides by Er(x)* and integrating yields:

$$\int d^3\mathbf{x} \cdot E_r(\mathbf{x})^* E_m(\mathbf{x}) \equiv g_{rm} \quad ; \quad \Sigma g_{rm} E^m(\mathbf{y})^* = E_r(\mathbf{y})^*$$

Note here that the notation ensures the duality due to the upper and lower positioning of index m.
The mode sequence $E_m(\mathbf{x})$ is double complete system including positive and negative vibrations, since the index m runs over positive and negative vibration modes.
(For example, assume that there is a vibration identification at the beginning of index m.)

In deriving the canonical momentum $\pi(t,\mathbf{y})d^3\mathbf{y}$ from the Lagrangian, obviously the volume element factor $(-g)^{1/2}\, d^3\mathbf{x}$ due to the gravitational field is involved. Additionally to say, if there is coupling of $\partial_0\chi$ with other types of fields, there will be hybridization of other types of fields to the canonical momentum. It should be notable that the canonical commutation relations hold including such involvements from other types of fields. This itself should be seen as natural considering the relative nature of field types.

Observing the form of the expressing $\chi(t,x)$ and $\pi(t,y)$, it is natural to consider $E_m(x)$ and $E^{*m}(y)$ as the spacetime representations of states $|e_m>$, $<e^{*m}|$. It is almost correct, but that is including the arbitrariness of the canonical transformation representation.
(Details will be discussed in the next section 2.2)

2.2. Construction of Field Operators and its Projectivity

Entering quantum mechanics with the idea from Schrödinger's wave mechanics that particles have accompanying waves, the existence of wavefunctions may feel natural.
However, from the perspective of field theory, the situation is not so simple.
Certainly, as the spacetime coordinate representation of a state, it is a "thing that exists", but particle states are not always single-particle states that can be expanded in spacetime eigenvectors. ***1**

Also, the concept of a "single-particle state" is relative in the view of field theory. For example, it is analogous to the concept of order of function on a manifold. For a function $f=z$, and if another coordinate variable w is introduced, f is expressed also as $f=c_0+c_1w+c_2w^2+\ldots$, and f no longer has a definite order with respect to w. In terms of particle states, changing the state base makes multiparticle superposition states appear, and the state has no longer definite number . ***2**
However, the existence of an analytical inverse transformation is important, and the lowest order excluding constant term is preserved.
In short, the issue is what the state base is for 1-particle to determine the number of particles

With these things in mind, let us now consider the composition of field operators and its important property, projectivity.

***1** : The concept of "multiparticles" has disadvantages as discussed in Section 1.1.

***2** : If the field equation is given, then defining the center of equation expansion, and the infinitesimal oscillation around there can be identified to the single-particle state.
To determine the center of equation expansion, similar idea to Higgs mechanism will be applied

●Background of introducing Field Operator

Let's look back at the considerations we've made so far.
First, starting from the variational method, the field variable (operator) $\chi(x)$ obtained there is expanded into vibrational mode and express $\chi(x)$ as $\chi(t,x)=\Sigma\ E_m(x)e^{*m}(t)$.
This allows us to regard the field as a point in an accountable infinite dimensional space with index m.
According to masspoint quantum mechanics, $e^{*m}(t)$ becomes the position operator corresponding to the coordinate frame of mode $E_m(x)$. Here, it has expressed as $x=(t,x)$.

The energy $k_0(m)$ corresponding to mode $E_m(x)$ becomes positive and also negative due to the degree of freedom of orientation, the front /back with respect to the equal-time cross-section , as $k_0(m)=\pm\omega(m)$.
Due to the requirement of completeness of the solution space, negative energy states cannot be ignored. This difficulty with negative energy was solved by interpreting the operator e^{*m} as a creation/annihilation operator and applying charge conjugation.

●Projectivity of Field Operator

Let the mode $E_m(\mathbf{x})$ be interpreted as a space-time coordinate representation of the space-time vibration mode E_m and write it as $E_m(\mathbf{x})=\langle x|E_m\rangle$.

This gives the following expression. The expression does not depend on the choice of mode basis.

$$\chi(t,\mathbf{x})=\Sigma\ E_m(\mathbf{x})e^{*m}(t)\ =\langle x|\Sigma\,(|E_m\rangle e^{*m}(t))=\langle x|\Sigma\,(|E_m\rangle\langle e^m|)$$

Here, e^{*m} has been equated with annihilation operator, according to the consideration in sec.1.3.2

From this expression, we can see the projectivity of the field amplitude operator, and it is natural to assume a dual relationship between E_m and $\mathbf{e}^{*m}$ at least the reference time $t=t_0$.

Let suppose $e^{*m}(t_0)=\kappa E^{*m}$ with constant κ taking account that the $\chi(t,\mathbf{x})$ must be invariant concerning the set of vibration mode basis (E_m).

$$\chi(t_0,\mathbf{x})=\Sigma\ E_m(\mathbf{x})e^{*m}(t_0)=\Sigma\langle\mathbf{x}\,|E_m\rangle\kappa E^{*m}=\langle\mathbf{x}\,|(\Sigma\,|E_m\rangle\langle E^m|)$$

When projective expression mentioned above is given at $t=t_0$, the expression at time t is given as follows.

$$\chi(t,\mathbf{x})=\kappa\langle\mathbf{x}|\Sigma\,|E_m\rangle U^*E^{*m}\,U\quad ;\quad U\equiv\exp(i(t-t_0)H).$$

To make express the projectivity explicit at any time, it is available to transform the space-time operator x.

$$\chi(t,\mathbf{x})=\kappa\,\Sigma\langle U\mathbf{x}U^*|\,UE_mU^*\rangle(UE^mU^*)^*=\kappa\,\Sigma\langle\mathbf{x}'|\,e_m\rangle e^{*m}$$
$$E_m\ \to e_m=UE^mU^*\,,\quad E^{*m}\ \to\ e^{*m}=UE^m\,U^*\,,\ x\to x'=UxU^*\quad ;\ U\equiv\exp(i(t-t_0)H).$$

From the above formula, there seem to be two things to be noted.

(1).Relativity of particle species

In the Schrödinger representation where time is frozen, expressions with different frozen times are connected by canonical transformation including interaction. This means that, for example, the state recognized as of "electron" at time $t=t_0$ is recognized as a superposed state with other interacting particles at time $t=t_1$.

(2).Relevance between normalization of field operator and coupling constants

Regarding the constant κ, if we transform it as $\chi(x)\to\chi'(x)\equiv\kappa\cdot\chi(x)$, then we can put $\kappa=1$.
This is nothing but a normalization of field operator that causes in Lagrangian the if change the interaction(coupling) constant and the scale of action.
In other words, if the normalizations are set appropriately, $\kappa = 1$ may be sufficient.

In conclusion, the field operator $\chi(t_0,\mathbf{x})$ of the Schrödinger representation frozen in time at time $t=t_0$ is given in the following form. That is space-time coordinate expression of state projection to the base spce of vibration.

Projectivity appears in E_mE^{*m}.

$$\chi(t_0,\mathbf{x})=\Sigma\ E_m(\mathbf{x})E^{*m}=\Sigma\langle\mathbf{x}|E_m\rangle E^{*m}\quad\text{(including negative vibration)}$$

Let's interpret , from the perspective of quantum field theory, the Schrödinger equation that appears in masspoint quantum mechanics and describes the motion of wave function of a single particle.

The Schrödinger equation has the following form.

$$+i\partial_t f(x)=H_x f(x)\,,\quad f(x)\ :\ \text{wave function (probability amplitude)},$$

$H_x = H_x(x,-i\partial_x)$: Hamiltonian (expressed by differential operator)

f is a wave associated with a particle and is called a wave function, but from the perspective of field theory, it is a space-time coordinate expression of a single-particle state, and should be interpreted as $f(x) = \langle x|f.$. $x = (t,x)$

The equation shows that the number of particles remains 1 and is conserved.

$$\langle x|+i\partial_t f\rangle = \langle x|\mathcal{H}|f\rangle \quad ; \quad \mathcal{H} = \int \chi^*(y) H_y \chi(y)\, d^3y$$
$$\langle x|\chi^*(y)|0\rangle = E_m(x)E^{*m}(y) = \delta^3(x-y)$$
$$\langle 0|\chi(y)|f\rangle = E_m(y)\langle E^m|f\rangle = \langle y|E_m\rangle\langle E^m|f\rangle = \langle y|f\rangle = f(t,y) \quad \text{(projectivity)}$$
$$\therefore \quad +i\partial_t|f\rangle = \mathcal{H}|f\rangle \quad \rightarrow \quad +i\partial_t f(x) = H_x f(x)$$

Hamiltonian H_x is given as a self-adjoint linear operator including a differential operator. Corresponding to this, the Hamiltonian in field theory becomes $\mathcal{H}$ below, using the field operator .

$$\mathcal{H} = \int \chi^*(y) H_y \chi(y)\, d^3y$$

Expressing this Schrödinger equation in masspoint quantum mechanics, into quantum field theory, it becomes $+i\partial_t|f\rangle = \mathcal{H}|f\rangle$.

This can be confirmed by taking the spacetime coordinate representations of both sides.

Since it is a single particle, vacuum appears when the annihilation operator is applied, as seen in the above example.

The fact that the Hamiltonian is a quadratic expression of the field operator χ makes it possible to introduce a wave function that follows the Schrödinger equation in masspoint quantum mechanics.

The fact that the Schrödinger equation is linear, is important in the sense that the wave function corresponds to infinitesimal oscillations.

Now let's go back to the variational principle as the starting point and find the Lagrangian L that derives to $\mathcal{H} = \int \chi^*(y) H_y \chi(y)\, d^3y$, quadratic expression of the field operator χ.

Releasing the frozen time, shift to Heisenberg representation and deriving a partial differential equation from the canonical equation.

$$\partial_t\chi(x) = i[\mathcal{H}, \chi(x)] = -i[\chi(x), \int \chi^*(y) H_y \chi(y)\, d^3y]$$
$$= -i\int \delta^3(x-y) H_y \chi(y)\, d^3y \quad : \quad \text{Heisenberg representation}$$
$$[\chi(x), \chi^*(y)]_\pm = \Sigma E_m(x)E^{*m}(y) = \delta^3(x-y) \quad , \quad [\chi(x), H_y\chi(y)]_\pm = 0 \quad ; \ x \equiv (t,x)$$
$$\therefore \ \partial_t\chi(x) = H_x\chi(x) \ , \quad \therefore \ L = \chi^*(+i\partial_t)\chi - \mathcal{H} \quad (\text{note} : +i\partial_t = -p_0)$$

Hamiltonian of quadratic concerning field operator χ is derived from Lagrangian of quadratic concerning field operator χ, and the field equation becomes linear . If the field equation is nonlinear, the state space becomes a Tensor space of the base space consists of solutions as infinitesimal oscillations.

●Infinitesimal oscillations and construction of state base space

In order to express the state of field by a wave function, the field equation must be linear, that is, the Lagrangian must be in quadratic form, but the equation is nonlinear in real world .

if there is interaction including self-interaction, Lagrangian is no longer a quadratic form.

In canonical gauge unified field theory, 4th-order terms of field variables of both Boson and of Fermion, appear in Lagrangian.
In particular, for the fundamental Boson interaction, the Lagrangian has the standard form of the square norm of curvature, and there is no mass term like Klein-Gordon equation.
This suggests that "mass" is an effective concept determined by the interactive field with the surrounding environment. ***A**

When expressing the field operator in mode-expanded form as $\chi(t0,x)=\Sigma$ Em(x)E*m, function system Em(x) is originally arbitrary if that is a complete system including negative oscillations.
So, for example, the vibration mode of the solution space of the Klein Gordon equation should be applicable.
If doing so, the mass m is should be taken as a parameter so that the solution gives a suitable approximation.

Therefore, expressing the Lagrangian as quadratic form + higher-order terms, the solution space of the equation derived from quadratic form, is used as the base space of states, and then the state space is constructed as its Tensor space.
The solution derived from quadratic form can be interpreted as an infinitesimal spacetime vibration.
Therefore, the question is around which state space-time oscillates to an infinitesimal degree, and an idea similar to the Higgs mechanism should be applied for this.

In Field operator, $\chi(t,\mathbf{x}) = \Sigma\ E_m(\mathbf{x}) e^{*m}$; $e^{*m} = U e^{*m} U^*$, the e^{*m} should be interpreted as the annihilation operator of superposition of multiparticle state expressed as a tensor . (Including negative energy expression) ***B**

***A :**

Please refer to part I regarding the existence of nonlinear terms in the unified field equation.
Concerning Boson self-interactions, the presence of a non-commutative component forms a quadratic term in curvature form.
Since the Lagrangian is a quadratic norm of curvature, a 4th-order term arises.

Concerning Fermion (preon), in addition to self-interaction expressed by the norm of the Pauli term, interactions with spinor connection gauge fields are involved. It goes without saying that interactions are involved in composite systems.

***B :**

It is easy to see that the state tensor arises because a time propagation operator appears when the reference freezing time is changed. If there is interaction, the solution space of infinitesimal oscillations will not become a Hamiltonian invariant subspace.
In general, a pure single-particle state should be considered to be a superposition of multiple states including interacting particles when viewed from other standards, and particle specie can be determined only by infinitesimal vibration components.
We can only identify the species and number of particles in an infinitesimal vibration state of space-time, a state as a linear approximate solution of field equation. see 1.3.2 -note.

●Appropriateness of Linear approximation and construction of field operators

From the discussion so far, for general field equations, applying linear approximation, and the field operator can be constructed with the solutions of the linearized equation as the normal modes of infinitesimal vibration, including negative vibrations

There is more than one way to extract a quadratic form from the Lagrangian as an approximation. Moreover, in the case of Boson, the field variables may have a constant term (0-th order tensor component). (See section 1.3.2 note). This will contribute to Fermion's effective mass.
However, in the case of Fermions (preons), the effective mass of composite system should be calculated in actual because they cannot exist alone

Regarding Boson, it seems appropriate to introduce effective mass phenomenologically and set the linearly approximate equation be Klein-Gordon type.
In order to obtain an appropriate value of mass, it must be determined from the variational principle so that the approximation is appropriate (so that the approximation residual may be minimal)..

Overall, it is necessary to establish the concept of an approximation method for effective mass.
(See Part I Section 3.4)

・Supplement: Arbitrariness of setting field operators

The variational variables of field can be transformed arbitrary as long as the inverse transformation exists. For Boson fields, the vacuum expectation value of field operators is not necessarily 0.
In a gravitational field, it may be set as $g = \eta + h$ ($\eta \equiv$ Lorentz metric).
This means that $h = g - \eta$ is defined as a new field operator.
This is an example of a field operator transformation, preserving canonical commutation relation be invariant.
Generalizing the above, we can subtract the expectation about the surrounding environment (background state), instead of the vacuum expectation.

$h \equiv g - \langle \Phi_0 | g | \Phi_0 \rangle$; $|\Phi_0\rangle =$ background state average.

There is also a way of thinking.

In gravitational theory, both g^{-1} and g appear frequently, but removing the constant component of g breaks the symmetry between g and g^{-1}.
From the perspective of a canonical gauge unified field, T is a field variable related as $g = T^* \eta T$, but taking account of compatibility to Newtonian theory , there is also the option of introducing potential V and setting $T = \exp(V)$. Since $(V = 0) \equiv (T = 1)$, the vacuum expected value of V is 0.

Technically, one idea is to use the Möbius transformation and set $T=(1+V/2)/(1-V/2)$ so that V is the field operator. At this time, it becomes $T^{-1} = (1-V/2)/(1+V/2)$.

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3. Difficulty of Divergence and state-constitutive field theory.

3.1. Criticism in scattering problem: denial of free field and adiabatic hypothesis.

It seems that the analytical themes of quantum theory can be broadly classified into two types: scattering problems and stationary problems.

In particular, scattering problems are directly related to particle collision experiments. In conventional quantum theory, for scattering problems, the adiabatic hypothesis is applied and a perturbation calculation of power expansion with respect to the coupling constant is developed.

And QED encounters the difficulty of divergence in second-order perturbation calculations for scattering problems. At present, a renormalization prescription has been developed for this purpose, which gives meaningful physical quantities by canceling out infinities and extracting finite differences, and the results agree with experiments with excellent accuracy.

However, the renormalization prescription is just a prescription operation and does not seem to contribute to the elucidation of actual physics. Therefore, I would like to return to the basics of the scattering problem and consider the validity of the problem setting in the first place.

・**Scattering problem – relationship with classical imagery**

The classical theory of collisions between two particles is that two particles that are far apart approach each other with a specific impact parameter b, and after interacting with each other such as exchanging momentum, they fly away again.

It is a clever choice to think of the problem in the center of mass system and set the center as the origin. This fixes the degree of freedom of translation of the entire system, and is probably a valid idea in quantum theory as well.

Let us start to make some criticisms about the handling of the scattering problem in quantum theory. However, some of them can be refuted.

In calculations in quantum theory, the input before collision, the two-particle state, is usually given by two plane waves, but since plane waves are uniformly spread over the entire space, this is a logical contradiction. It can be criticized that there is no parameter corresponding to the impact parameter that existed in classical theory.

A counterargument to this would be that the plane waves that appear at the input and output are only momentum-spectral decomposed expressions of the states before and after the collision, and do not represent the actual collision phenomenon.

Actual collision phenomena are represented by wave packets of probability amplitude with uncertainties in position and momentum corresponding to classical particles, which spread out over time.

This explanation can be formulated as follows

Two-particle scattering : $|i\rangle \rightarrow |f\rangle = S|i\rangle$, Initial state $|i\rangle$, Final state $|f\rangle$,
S = Scattering matrix
Scattering amplitude : $\langle k'|f\rangle = \Sigma_k \langle k'|S|k\rangle\langle k|i\rangle$

By the way, introduction of plane wave contains more useful meaning than selecting it only as an expression basis for scattering.

The relativity of the interaction can be expressed explicitly by extracting the plane wave component corresponding to the translational motion of the entire system.

This makes it possible to easily express the center of mass fixed at the origin.

A further criticism to the above objection is that when considering, for example, the collision problem of two charged particles (scattering due to electromagnetic interaction), the expression of plane wave decomposition for charged particles as a composite system including virtual photons is not valid. Or at least its applicability is not obvious.

On the contrary, when considering photons as intermediates of electromagnetic interaction, "self-interactions" of charged particles appears and this lose the logical premise of perturbation calculations based on the adiabatic hypothesis.

・Denial of the adiabatic hypothesis

Let us consider the equation of motion of the state vector in Schrödinger representation.

$$+i\partial_t f = H(f) \quad ; \; H \equiv H_F + H_I$$

Here, H_F represents the Hamiltonian without interaction (free field) and H_I represents the Hamiltonian with interaction, but this decomposition $H \equiv H_F + H_I$ is originally artificial.

The adiabatic hypothesis is to set $H_I \to 0$ in the infinite past/infinite future as an asymptotic condition in the scattering problem, but this is not appropriate in two senses.

One is that $H \equiv H_F + H_I$ is a logical contradiction, because the operator is frozen in time in Schrödinger's representation, so $H_I \to 0$ is irrational.

The other one is that $H_I \to 0$ should originally be interpreted as a property of the state vector, for example, as $\langle f|H_I^2|f\rangle \to 0$, and the attenuation of the interaction itself as an operator expression is impossible.

The only interpretation left for justification is to assume that the state vector |f> in the infinite past/infinite future is such that H_I can be ignored. However, considering self-interaction, this is a logical contradiction.

At first glance, the scattering problem appears to be a situation that satisfies the above assumptions. For example, it is possible to draw a picture in which two particles that were sufficiently far apart in the distant past came close to each other, had an interaction represented by H_I, and then eventually moved apart again.

In this assumption, it is important that the infinite past and infinite future states are eigenstates of the H_F, even if asymptotically.

However, if we consider self-interaction, this causes a logical contradiction. This is because particles must be regarded as stationary composite systems dressing interacting Bosons.

This system as a Fermion wearing the dress of interacting Boson, the Boson-Fermion composite, has intrinsic attributes as a particle, as like a conventional elementary particle.

Apart from the propagation and diffusion as the wave packet (due to momentum uncertainty) as a whole , the composite is "stationary" in the sense that it exists, and it should be considered as the steady state of the total Hamiltonian H.

The method of expressing the initial and final states of two colliding particles by plane wave decomposition is a suitable expression for clearly demonstrating the above-mentioned stationarity. (Uncertainty of parallel displacement can easily be factored out)

Electromagnetic interaction between colliding particles should be understood as the exchange of virtual photons between them.

If the adiabatic hypothesis is applied, the self-interaction will be included in the correction calculation

during the interaction period, so it will be necessary to exclude this part to calculate the interaction between two particles. This probably corresponds to the renormalization prescription.
The account of the scattering phenomenon is as above.

However, the validity of considering each of the two colliding particles as Fermion dressed with interacting Bosons, is another matter, and is not completely obvious.
Considering the fact that a single quark does not exist, we can imagine that such a solution does not exist. In other words, there is a strong possibility that conventional Fermions, which appear to be elementary, are actually composite particles, and what appears to be self-interaction is actually leakage of internal interactions between constituent particles to the outside of the system.

3.2.Consideration on the cause of divergence difficulty

●Origin of self-interaction and renormalization

It is well known that the divergence in QED is represented by a small number of fundamental divergences. This makes the prescription possible.
The fundamental divergences form seem to be related to self-interaction from the form of its counter terms.
Related with the denial of the adiabatic hypothesis, followings should be noted:

- The asymptotic disappearance of the interaction Hamiltonian is a logical contradiction.
- Validity of ignoring the interaction Hamiltonian is dependent on the state, but the initial state is not such one due to the self-interaction.
- Because of self-interaction, the initial state should be an eigenstate of total Hamiltonian H, not be a free field nor an eigenstate of the free Hamiltonian H_F.

Even if two particles are far enough apart, as long as the particles themselves have self-interaction, the interaction Hamiltonian cannot be ignored.

●Divergence Difficulty and composite structure of particles

The success of QED renormalization theory shows that, for example, electromagnetic interactions can be dealt with by extracting the leakage effect without going into the internal structure of electrons.
On the other hand, elementary Fermions probably cannot exist on their own. It is difficult to imagine the bare Fermion in self-interacting steady state with virtual Bosons.

For example, an electron remains an electron, in this sense that the "stationarity of attributes" is required.
If we understand electrons as a composite system of "bare electrons" and "virtual photons," we must assume that the relative relationship between the two is stationary.
It seems difficult to understand the electron as a composite stationary solution of the bare electron located near the origin and the resulting electromagnetic field (collection of photons).
On the other hand, if we consider the electron as a composite system of preons, the constancy of the electron can be understood as the constancy of the relative interaction between the preons, which are the constituent Fermions of the electron.
As already mentioned, Fermions that exist stably on their own are actually composite particles, and it is probable what appears to be self-interaction is actually the leakage of internal interactions between the constituent particles, preons, to the outside of the system.

In fact, canonical gauge unified field theory suggests the validity of the Rishon model (one of preon model) for quark-lepton.
The interaction between the constituent particles is an exchange of attributes between the constituent particles by interacting Bosons .

Summarizing the discussion so far, the cause of divergence difficulty seems to come from the application of the free field concept or the adiabatic hypothesis, with ignoring the existence of internal interactions (self-interactions viewed from outside) due to the structure of the particles.
Although the equations correctly represent these interactions, it can be said that the set initial states as an unperturbed solution is not appropriate as the center of the perturbed expansion.
The calculation methods themselves, such as perturbation expansion and introduction of interaction representation, are not found to be inappropriate.

●picture of avoidance of divergence in scattering problems

From the above considerations, interaction in scattering problems has become generally clear.
First, if two colliding particles are far enough apart, the interaction exists as an internal interaction in the collining particle, (self interaction from outside view).
Therefore, the initial state and final state of the collision must be set as steady states.
The interaction that occurs when two particles approach each other can be understood as an exchange of internal Bosons (originally acting between the constituent particles).
Therefore, the key to solving the scattering problem finitely is to express the exchange process of the virtual Boson as an approximate correction of the equation solution, using the tensor product of two steady states as the initial state.

In that case, a steady-state solution is first needed as an initial state, but this becomes a troublesome problem for solving the equation of the scattering problem.
Furthermore, as mentioned above, if no internal structure is assumed for the colliding particles, the existence of such a steady state is doubtful.

The colliding Fermi particles are actually composite particles, and what appears to be self-interaction from the outside is a leakage of the internal interactions of the constituent particles.
However, as a solution, we should look for a method that perturbatively extracts only the effects of Boson exchange, without going into the internal structure of colliding and scattering particles as much as possible.
One can imagine that the success of QED is that it handles this well.
I will discuss the solution in a later chapter. ***A**

***A:**

Depending on the energy of the collision, the constituent particles may recombine, so it is not possible in all cases to avoid accessing the internal structure.
It seems that this should be considered as one of the possible approximate prescriptions.

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4. Approximation method in state-constitutive field theory

State-constitutive field theory is the theory in which the approximation method expressing the states explicitly is applied, as in the case of masspoint quantum mechanics, by introducing the idea of finite element method, while knowing that field theory essentially includes infinite degrees of freedom.

Since canonical commutation relation constructs an differential ring on the state space, depending on how you look at it, it might be said that this theory is the theory on infinitely multivariate differential and integral equations.
On the other hand, the field equations follow the variational principle, and the Lagrangian is invariant to space-time preserving canonical transformations. So it is assumed that a family of high symmetry is probably chosen as its object.

When constructing an approximate solution method , it should be taken account that it is possible to define a successive approximation sequence of calculations in order to improve the approximation accuracy, and that there is a guiding principle in setting the approximate initial solution.

4.1.Perspective of the construction of approximation method

It might be said that there are two perspectives on problems to be solved using quantum field theory: "stationary problems" and "scattering problems."
We believe that these should be solved in center of mass system, but when considering approximate solutions, the extraction/removal of translational degrees of freedom is an issue that should be devised.

Steady-state problems are interested in finding possible quantum states in a modeled composite system.
This typical goal can be seen in the case of E. Schrödinger, as an example, who applied his eponymous equation and the concept of a wave function to the hydrogen atom.
He clarified the configuration of its internal state, and derived the energy spectrum of the system .
it would be a great achievement if similarly deriving the properties of quark/lepton from the preon model.
Mathematically, the method for solving stationary problems corresponds to find the eigenstates of the Hamiltonian. In other words, for a self-adjoint linear operator on a state space, a solution method giving its eigen tensor is required, even if only approximately.
The difference from masspoint quantum mechanics is that the problem of eigenvalues and eigenstates must be solved in a state space (tensor space whose base space is a space of single-particle state).

On the other hand, scattering problems are directly related to scattering experiments interested in verifying the validity of the model, by explanatory reproducing the experimental results.
A rough picture of scattering is that the two particles in a steady state far apart, approach each other, interaction between them occurs, and in the process ends, the final state is reached.
It is supposed that the interaction is caused by the exchange of "latent interacting virtual Bosons," but if the colliding particles are composite systems, the combination of their constituents itself may exchange.

In any case, since the initial state/final state is a steady state, it is a problem of state transition probability of the system consists of two colliding particles, due to the interaction perturbation.

So, it is imagined that the solution is expressed in the form of successive integrals .

In the following, let us consider the solution methods for the stationary problem and scattering problem respectively. It is expected to obtain practical knowledge and deepen thinking by applying the solution methods to actual problems.
Calculations in their methods are basically simple in principle and systematic , but in execution they might be more complex than simple hand-written calculations.

4.2.Approximate solution of stationary problems

4.2.1.Proposal on approximate solution

Let us consider a method to approximately find the eigenstate of Hamiltonian by considering the Hamiltonian expectation as a variational function of states. ***1**
Applying variational technique comes from the reason that the variational method is the basic technique to find an approximate solution under finitely limiting the degree of freedom of the solution.

The above method will be expected to derive the approximation method such that the classical solution method and mass-point quantum mechanics method can be used approximately, that it is a successive approximation method, and that can avoid divergence difficulty.
Suppose Hamiltonian in Schrödinger representation and then operators are time-frozen.
It should be noted that what we need as a solution is the expression of relative motion in the center of mass system.
It must be a kind of canonical gauge fixing to suppose the state in which the momentum of the whole system is zero.

***1 :** For the Schrödinger equation, $A \equiv \int dt \cdot \langle f|(i\partial_t - H|f\rangle$ is a variational action form.
In stationary problems, $J \equiv \langle f|H|f\rangle$ replaces this.

(1).state separation method concerning particle species

Let us consider to set the state tensor in equation restrictedly as the product of the states corresponding to the particle species.
This idea is similar to the separation of variables method for partial differential equations.
Here, the desired state tensor may be a correction term for advancing the approximation.
The Hamiltonian expected value is separated for each particle species by the state separation approximation. A classical field representation can be used as an approximation for the interacting Boson field. ***A**
The field formed by a finite number of Fermions can be applied to a wave function represented due to the projectivity of the field operator. ***B**

Applying variational method on wave function, the determination of the wave function is reduced to the problem of mass-point quantum mechanics. Though, Boson field at this stage is still classical approximation.
By solving the Fermion wave function approximately and then taking variational on Boson field in classical expression (mean field) for the interaction, we can obtain the classical equation for the mean field of Boson. However, in that case, the "variance effect" must be ignored. ***A**

It is also possible to consider a variational on the distribution of excited Bosons rather than a variational on the mean field. However, a finite method is required for actual handling, and this is the "finite mode excitation approximation" described in the next section.
That is, it is a thought of solving quantum theoretical Boson field state using explicitly the "finite mode excitation approximation".

The state separation method is very useful in that mass-point quantum mechanics and classical field theory can be used as an approximation method theoretically.
On the other hand, in order to extract the relative motion, it is necessary to separate the coordinate variable into those of center of mass and the relative one, in the wave function expression, which is rather difficult .
We will see this situation in section. 4.2.2 through an example.

The above can be summarized as follows.

Equation : $\delta\langle f|H|f\rangle = 0$; $\langle f|f\rangle = 1$ ***C**

where, $|f\rangle = |c\rangle + |f_1 \cdot f_2 \cdots f_r\rangle$; state separation is assumed.

$|f\rangle$ may include known bias $|c\rangle$ (e.g. an approximate solution already obtained)

$\chi_j(\mathbf{x})|\, f_j\rangle = |\, 0\rangle f_j(\mathbf{x})$: Wave function representation for Fermion 1 particle state. (projectivity)

$\langle f|B(\mathbf{x})|f\rangle = \langle B(\mathbf{x})\rangle$: Classical field representation (mean field)

Hereafter, "mean field" is not limited to the meaning used in statistical mechanics, but is used in the meaning of "field obtained as a mean".

***A :** The classical expression of Boson field will be given by the expected value (mean field) of field operator. Therefore, the linear term for the field operator is replaced by the classical field when taking the expected value of the Hamiltonian.
Non-linear terms can also approximately be replaced, but with added "variance effect".

This is an essential difference between classical theory and quantum theory. How the singularities appearing in classical gravity theory are interpreted from the viewpoint of this "variance effect" is one of the future research subjects.
According to statistics, the following holds as an approximation.

$\langle F(x)\rangle = F(\underline{x}) + (1/2)\cdot\partial^2 F(\underline{x})\sigma^2 + \cdots$;

$\underline{x} \equiv \langle x\rangle$; expectation of x $\sigma^2 \equiv \langle (x\text{-}x)^2\rangle$; variance

***B :** $\chi(x) = \Sigma\, a_m(x) a^{*m} = \Sigma \langle x\,|\,\mathbf{a}_m\rangle \mathbf{a}^{*m}$; (see sec.2.2)

$a_m(x)$ is the coordinate representation of mode ($\mathbf{a}_m$). m is a mode index and includes negative oscillation, and ($\mathbf{a}_m$) forms a doubly complete system. Duality of state vector needs to be consistent with Canonical conjugation.
The formula $a^{*m}(x) a_n(x) d^3x = \delta_{mn}$ indicates the duality of $a^{*m}(x)$ and $a_n(x)$, but independently to this, in order to execute the summation concerning continuous index (x) with integral, it will be required the concept of density of states.
The series $a_m(x)$ can be set arbitrarily under some restrictions originally, but if it is directly connected to the wave function like $f(x) = a_m(x)$ for $\exists m$ by $\chi(x)|f\rangle = |0\rangle f(x)$, then inner product structure should be taken account of j^0-current. $j^0 = \pi(x)\, \chi(x)\, d^3x$

***C :**

It is not necessary to solve the eigenstate problem strictly. It is sufficient if the approximation can be improved and advanced. Conventionally following Jacobi's idea, we can also consider the eigentensor in the plane spanned by $|f\rangle$ and $H|f\rangle$.

(2)Approximation method of finite mode excitation

Although it is generally considered that the interacting Boson field is a mixture of multiple excited states for a plural oscillation modes, let us consider expressing the field approximately by finite number of oscillation modes.
Under this assumption, the state of Boson field can be expressed as an analytic function of a finite number of variables that are holomorphic at the origin.
Let it be called "mode excitation representation function".
he field operator will actually seem to be represented by finite sum of creation / annihilation operators related to the assumed excitation mode at least in approximation.
The modes orthogonal with assumed excited one have no contribution to the state expectation value of Hamiltonian.
It is also possible to apply the "state separation method" to the excited mode state. ***A**

Since the field operator includes a differential term for the excited mode, a variational on excited mode will derive a partial differential equation.
Considering that the number of excitation modes is as small as possible and the number of excitation modes is increased from the dominant one as a sequential correction, the assumption of approximating the field as superposition of multiple states of only 1-mode excited (single mode excitation approximation) is obtained, that derives 1-variable differential equation.
By solving the differential equation, we can get single mode excitation approximation for the Boson field state. ***B**
To solve the equation by assuming symmetry to the field is interpreted as the state that only the modes that satisfy the symmetry are excited.
For example, the assumption that the field is spherically symmetric is interpreted as the assumption that only the spherically symmetric oscillation modes are excited.

***A** : For example, Boson field operator $B(x)=\Sigma(b_m(x)\cdot b^{*m}+b_m\cdot b^{*m}(x))$
$m=m_1,m_2,\cdots m_s$, index for positive oscillation ,
A finite sum of creation and annihilation operators in the assumed excitation mode.
The state is $f_B=f_B^{(0)}+f_B(b_1,b_2,\cdots b_s)$; $(b_1,b_2,\cdots b_s)$ f_B is the analytic function of (b1, b2,⋯ bs).
If the state separation approximation is brought here, it is limited to $f_B(b)=f_{B1}(b_1)\cdots f_{Bs}(b_s)$.
If the analytic function can be constructed sequentially, it will be effective as an approximation.

***B :** In case of single mode excitation, $\langle f_B|B(x)|f_B\rangle=b(x)\langle b|f_B\rangle+\langle f_B|b\rangle b^*(x)$ holds.
The space-time representation of the excitation mode corresponds to the mean field.

4.2.2.Case Study of Approximate Solution Method-Hydrogen Atom Model

Let's put the above ideas into practice, extract the things to be considered, and check the feasibility.

Using the hydrogen atom as an example, let us consider how to handle it in stationary problems.
In masspoint quantum mechanics, equation regarding the wave function of an electron from the classical Hamiltonian with a fixed center of mass. However from the idea of field theory, the whole Hamiltonian must be given as the sum of all the constituent particles and the interactions between them. In addition, it would be appropriate to start from Lagrangian for consideration of system symmetry.

The Lagrangian density $L^D(x)$ is written down in the non-relativistic range as follows.

$L_p{}^D(x)=\chi_p(x)^*(i\partial_t-(-i\partial_x)^2/(2m_p)))\chi_p(x)$: Lagrangian density of proton
$L_e{}^D(x)=\chi_e(x)^*(i\partial_t-(-i\partial_x)^2/(2m_e)))\chi_e(x)$: Lagrangian density of electron
$L_\gamma{}^D(x)=(1/2)\cdot V(x)(\partial_t{}^2-\partial_x{}^2)V(x)$: Lagrangian density of photon
$L_{EMI}{}^D(x)=-\rho(x)V(x)$; : Lagrangian density of EM interaction
$\rho(x)\equiv e(\chi_p(x)^*\chi_p(x)-\chi_e(x)^*\chi_e(x))$: electro-charge density
(where, $-i\partial$: self-adjoint differential operator in center of mass system. V=EM-potential, c≡1)

Not explicitly restricted, but the coordinate system is assumed to be the center of mass system.
Lagrangian L is the integral of the Lagrangian density $L^D(x)$ on 3-dimensional equal-time space.
Since the total Lagrangian of the system does not include the coordinate variables explicitly, there is a state where the total momentum of P_G is fixed as $P_G=0$.
To set the center of mass as the origin is expected, but if the total momentum is determined, the center of mass will spread due to the uncertainty principle, over the entire 3-dimensional space, so it may be necessary to some kind of extreme operation.

Transform to Hamiltonian from Lagrangian and put stationarity assumptions on the state tensor.
The above is the preparation stage. Hamiltonians are as follows.

$H_p{}^D(x)=\chi_p(x)^*(-i\partial_x)^2/(2m_p)))\chi_p(x)$. $H_e{}^D(x)=\chi_e(x)^*(\,(-i\partial_x)^2/(2m_e)))\chi_e(x)$
$H_\gamma{}^D(x)=(1/2)(\pi(x)^2-\partial_x V(x)^2)$, $H_{EMI}{}^D(x)=+\rho(x)V(x)$
$H^D(x)=H_p{}^D(x)+H_e{}^D(x)+H_\gamma{}^D(x)+\rho(x)V(x)$; $\rho(x)\equiv e\cdot(\chi_p(x)^*\chi_p(x)-\chi_e{}^*(x)\chi_e(x))$,
variational action : $I=\int d^3x\langle f|H^D(x)|f\rangle$; f=state tensor

Here, let us apply the approximation method of "state-constructive field theory" to the action in variational form. If the "state separation method concerning species" is applied according to the theory, separation $f=f_p\cdot f_e\cdot f_\gamma$ should be set.
A concern here is that, considering the extraction of translation degrees of freedom, the representation in center of mass system, or the search for a solution under the condition that total momentum $P_G=0$, then 1 degree of freedom is extra.
If we suppose a composite state of proton p and electron e as $f=f_{pe}\cdot f_\gamma$, then, the wave function $f_{12}(x_1,x_2)$ of 2 particles can be introduced by assuming the following decomposition.

$f_{pe}=\Sigma\,|(x_p,x_e)\rangle\langle(x_p,x_e)|f_{pe}\rangle$; $\langle(x_p,x_e)|f_{pe}\rangle=f_{pe}(x_p,x_e)$

(This prescription might be tentative but not fundamental /systematic.)
For the 2-particle Hamiltonian, the following transformations can be used to extract the center of mass degree of freedom.

$p_1{}^2/(2m_1)+p_2{}^2/(2m_2)=p_x{}^2/(2\mu)+p_G{}^2/(2m)$; $p_x=-i\partial_x$, $p_G=-i\partial_{xG}$
$m=m_1+m_2$ (total mass) $\mu^{-1}=m_1{}^{-1}+m_2{}^{-1}$ (reduced mass)

From the above, it can be seen that the center of mass system condition ($P_G=0$) reduces 1 degree of freedom of the system, but in order to actively use it, appropriate prescription expressing positional relativity is necessary.

The exploration of the eigenstate is obtained by the variational method.

$\delta\langle f|H|f\rangle = \langle\delta f|f\rangle \times \text{const}$, where, $H \equiv \int d^3x \cdot H^D(x)$

Since there is a constraint of $|f|=1$, the variational result must be set as f×const using an indeterminate multiplier.

According to the state separation method, approximate solutions can be obtained sequentially from variations regarding individual states, as follows.

If we first assume classical electromagnetic interaction, we will be able to obtain approximate solutions in wave function representation as $f_p(x)$ and $f_e(x)$ by mass-point quantum mechanics.

Next, consider to obtain an approximate solution of the electromagnetic field in a successive approximation using this solution of Fermions .

First, consider the variation on classical representation of the electromagnetic field, that is, the expected state value (mean field).

Poisson equation is obtained, and the solution will be obtained by using Green's function.

$\langle f_\gamma|V(x)|f_\gamma\rangle = \int d^3y D(x-y)\rho(y)$; expectation of EM-potential(mean field)

It should be noted here that, from the above formula appearing the antisymmetry of Fermion, $\rho(x)\rho(y)\to 0$ as $x\to y$.

From this, considering $\rho(x)V(x)$, we understand why the charged particles are not sensitive to the self-generating field.

As an approximation method, the electromagnetic field at this stage is still a classical solution as the mean field.

Then, by substituting this classical electromagnetic field solution into the variational action again, it is possible to obtain an approximate solution of the wave function sequentially from the variational principle by the technique of mass-point quantum mechanics.

It is also possible to obtain a quantum-theoretical approximate solution of the electromagnetic field from the finite mode excitation approximation.

Below, let us consider the case of single-mode excitation approximation.

From the state separation approximation, variational action can be written as follows.

$\langle f|H|f\rangle = \langle f_{pe}|(H_p+H_e)|f_{pe}\rangle + \langle f_\gamma|H_\gamma|f_\gamma\rangle + \langle f_{pe}|\rho(x)|f_{pe}\rangle\langle f_\gamma|V(x)|f_\gamma\rangle$

$H_\gamma = (1/2)\cdot\int d^3x\cdot(\pi(x)^2 - \partial V(x)^2)$

$V(x) = \Sigma\,(a_m(x)\cdot a^{m*} + a_m\cdot a^{m*}(x))$ → transfer to assuming single mode excitation

$a_m(x)$: non relativistic EM-potential $\omega^2 a_m(x) + \partial^2 a_m(x) = 0$

$\int d^3x\cdot a_m(x)^* a_n(x) = \delta_{mn}\omega_m^{-1}$; mode orthogonality, ω = mode energy

In the approximation of single-mode excitation, it can be written as $f_\gamma = f_\gamma(a)$; 1-variable function, $V(x) = a(x)\cdot a^* + a\cdot a^*(x)$, where, a is the excited vibration mode in the positive energy state.

The equation for the eigenstate is as follows.

$\omega a a^*|f_\gamma\rangle + \int d^3x\cdot V(x)|f_\gamma\rangle\langle f_{pe}|\rho(x)|f_{pe}\rangle = f_\gamma(a)\times\text{const}$

Since $a^*|f_\gamma(a)\rangle = |\partial f_\gamma(a)\rangle$, the above is a differential equation of $f_\gamma(a)$, and the coefficient depends on the distribution of charged particles, energy of the photon itself, and the indeterminate multiplier due to the norm constraint.

Since $\partial f_\gamma / f_\gamma$ is a linear fraction of a, $f_\gamma(a)$ will be a Γ-distribution function of a in general but it can become Gaussian distribution also in case that the denominator of linear fraction is degenerate. *A

The space-time coordinate representation a(x) of the state a is given by variational on function a(x). In single-mode excitation approximation, result is consistent with the classical equation, except for multiples of constant.

***A :**

Let's consider$(dy/dx)/y=(c_{11}x+c_{12})/(c_{21}x+c_{22})$. If $c_{21}\neq 0$, generality is not lost as $c_{21}=1$.

$c_{21}\neq 0$: $d(\ln(y)/dx = c_{11} + c_{12}'/(x+c_{22})$ → $\ln(y) = c_{11}x + c_{12}'\cdot\ln(x+c_{22}) + \text{const}$

$c_{21}=0$: $d(\ln(y)/dx = c_{11}'x+c_{12}'$ → $\ln(y) = c_{11}'x^2/2 + c_{12}'x + \text{const}$

In case c21≠0, then c12' would be required to be a positive integer for y to be a mode excitation representative function. For norm calculation, see para.1.3.3 (2).

4.2.3.Approximation method for Stationary Problems－Summary and themes

In state-constructive field theory, an approximate solution for a stationary problem can be constructed with an idea of the finite element method based on the variational principle. The successive approximation is possible by adding the correction state Δf ($f = f^0 + \Delta f$).

＜**things to consider on approximation solving**＞

- When considering a stationary problem, it is necessary to establish a systematic method for extracting and removing the translation degrees of freedom of the entire system, keeping in mind the expression in the center of mass system.

- As an approximation method for stationary state problems in field theory, the state separation method (by particle species), can be applicable.
  In state separation approximation, classical theory and quantum mechanics can be used as a base for approximation.
  Wave function representation can be used for Fermion fields.
  A classical approximation is available for Boson fields.

- Boson's classical field can be interpreted as the state expectation (mean field) of the field operator.

- By finite mode excitation approximation, the equation of Boson field is given as the equation of calculus concerning mode excitation representation function .

- At the stationary state, charged Fermion has no sensitivity to the self-generating electromagnetic field because of antisymmetry of Fermions. (As is seen from the result of Section 4.2.2)

4.3. Scattering problem

Roughly speaking, the scattering problem can be recognized as an approximate solution method for deriving possible state transitions by collision of 2 particles or a composite system. *A

However, in a broad sense, state transitions such as spontaneous collapse and decoupling of composite systems are also considered.

This is because the state transition a+b→c is dually related to a→c＋b*.
This is interpreted as suggesting that the state transition is a recombination of constituents, from the idea of element reductionism.

***A** : Since the problem is the temporal change of the state, the consideration is naturally done in Schrödinger representation. The time evolution operator for states is $U^*\equiv\exp(-iH(t-t_0))$.

The clues to the approximation are the asymptotic stationarity of the initial / final states.
Initial state $f_I\equiv|f_1^0f_2^0\rangle$ → Final state $f_F\equiv U^*|f_1^0f_2^0\rangle$
$f_F=U^*f_1^0f_2^0|0\rangle=(U^*f_1^0U)\cdot(U^*f_2^0U)|0\rangle$; note. $U|0\rangle=|0\rangle$
each state ; $|f_j(t)\rangle=f_j(t)|0\rangle$, $f_j(t)=(U^*f_j^0U)$ can be considered.
$|f_j(t)\rangle=|f_j^0\rangle\exp(i\omega_j(t-t_0))$, $j=1,2$; f_1, and f_2 are asymptotic stationary states.

If written in the differential form, $\partial_t f_j(t)=|f_j(t)\rangle i\omega+\partial_t n_j$ is obtain, where, n is a term in which the positive energy annihilation operator appears at the right end in normal ordered expression.
Generally, such kind of term can be extracted from the operator z as $z \rightarrow \nu(z)\equiv z-(z|0\rangle)$. where, $(z|0\rangle)$ is a operator give by embedding state tensor $z|0\rangle$ to the operator space.
(canonical injection)

Let us consider the method to find nj with perturbation theory.
Without loss of generality, $|f_j^0\rangle=f_j^0$ (i.e. $\nu(f_j^0)=0$) is assumed.

$$\partial_t f_j(t)=[-iH, f_j(t)] \quad ; \quad f_j(t)\equiv f_j^0\cdot\exp(i\omega_j(t-t_0))+n_j$$

$$\therefore \quad i\partial_t n_j(t)=\nu([H, f_j^0])\cdot\exp(i\omega_j(t-t_0))+[H, n_j] \quad ; \quad n_j(t_0)=0$$

Considering the collision of charged particles within the scope that internal structure does not appear, for example, in the collision and scattering of proton and electron, Hamiltonian H will be expressed as $H=H_p+H_e+H_\gamma+e\cdot jA$.
Probably, the initial state f_j^0 ; j=1,2 is a state in which the charged particles are so called dressed with latent photons. Free field Hamiltonian H_p , H_e , and H_γ have no contribution to $\nu([H, f_j^0])$, and only the electromagnetic interaction Hamiltonian $e\cdot jA$ can contribute. (j is electric current density, A is electromagnetic field.)

Let us focus on the symmetry of the state. From direct expansion, following is obtained

$$f_1f_2=f_1^0f_2^0\cdot\exp(i(\omega_1+\omega_2)(t-t_0))+f_1^0n_2\cdot\exp(i\omega_1(t-t_0))+n_1\cdot f_2^0\exp(i\omega_2(t-t_0))+n_1n_2$$

Taking account of symmetry of the state, it should be $f_1f_2\pm f_2f_1=0$.
In order to obtain final state of scattering calculation of $n_1|f_2\rangle$ or $n_2|f_1\rangle$ is sufficient.

$$f_1f_2|0\rangle=|f_1^0f_2^0\rangle\exp(i(\omega_1+\omega_2)(t-t_0))+n_1|f_2^0\rangle\exp(i\omega_2(t-t_0))$$

$$f_1f_2|0\rangle=|f_1\rangle|f_2\rangle+n_1|f_2\rangle \quad ; \quad n_1|f_2\rangle\pm n_2|f_1\rangle=0$$

--

The scattering problem can be interpreted as a particle exchange that occurs between 2 colliding composite systems. This is because interaction Hamiltonian forms annihilation operator n such that ($n|0\rangle=0$).
Although the adiabatic hypothesis does not hold logically, it can be said that the components of

Hamiltonian involving interaction particle exchange are certainly not the free field components, but interaction Hamiltonian.

For collision and scattering, the initial state of the system is usually assumed as an asymptotically stationary state (as a collision of stable particles), but for the decay phenomenon, the state transition due to the perturbation of the composite system must be considered.
Therefore, it deviates slightly from the situation assumed above.
Even in such case, If we think of $a \to b + c$ as $a + b^* \to c$, we will be able to calculate the transition probability.

In order for a particle to exist in stationary state, the state must be Hamiltonian's eigenstate, including its interaction with environmental surroundings.
If interaction exists, a partner who exchanges (absorbs / releases) the interaction Bosons is required.
Particle wearing Boson dress is a picture comes from setting this partner as himself, but perhaps this is not physical and is considered as convenient simplification to avoid entering the internal structure of the particle. ***B**

As a solution to the scattering problem, one of the themes is how close the state transition can be calculated without entering the details of the internal structure of the composite system.
It is non-trivial for the existence of the solution that guarantees the existence of particles dressed in latent interaction Bosons. In view of the non-existence of a single quark, an elemental Fermions (preon) will not be able to exist stably alone.

***B :**

A calculus equation for the state of Boson field will be obtained by finite mode excitation approximation and probably the solutions of it will also exist, but normalization of the solution may not possible.
As a calculation technique, some kind of extreme operation might be necessary, such that for example, by modifying interaction form so that convergence factor appears, and after calculating the state transition, then set the convergence factor $\to 0$.
In case of QED, perhaps this corresponds to a renormalization prescription.

4.4. Consideration of electronic model in state-constructive field theory

Let's consider what should be taken accounts when solving the electron model by preons in state-constructive field theory, using state separation approximation and finite mode excitation approximation.
(In the electron model by preons, μ,τ are considered to be the excited states, so once the solution method is established, it is expected that the structure of lepton will be clarified. ***A**)
The constituent is as follows. $e+ = (T_R, T_G, T_B)$ For details, please refer to part I section.3.3

Fermion:

degrees of freedom : Pre-flavor＝T×3, pre-color＝R,G,B spin＝±1/2

Boson:

pre-color interaction, su(3)

pre-flavor interaction, u(2) (component σ_0 , σ_3)

interaction with the spinor connection field h.

source mass $4(h^*h)^{1/2}$ as a possibility (different from effective mass)

The Lagrangian of Fermion(preon) is decomposed into 1st-order factors in the approximation of infinitesimal oscillations. The preon combines with the spinor connection field (h) , and one of the primary factors acquire the source mass. Fermion's Lagrangian also has a higher-order term, but its effect will contribute to the effective mass as a correction term in the successive approximation.
As a Fermion(preons) state, we have to take a linear combination of terms 3T, (T_R,T_G,T_B) having each spin, so that it may become su(3) neutral, due to pre-color symmetry . ***B**

Boson's Lagrangian has a 4th-order term for field operators, but no 2nd-order term (related to the mass term). The field operator can be expanded by any double complete system function sequence, but in order to obtain the expression of effective mass/effective range as in traditional theory, following will be needed :
Let the mode basis of the field operator be Klein-Gordon type infinitesimal oscillation.
Determining the effective mass by variational method according to the principle of least action

Note that Boson field operators may have constant components. Especially in relation to the mass of Fermion, the spinor connection field h has that possibility.
(A mechanism like Bose-Einstein condensation can also be assumed.)
The interacting Boson carries spin =1. su(3) swaps the spin and pre-color attributes of T, u(2) swaps spin and phase. ***B**

***A :**

Information on the preon model, reason of quark/lepton is up to the 3rd generation, etc., see part-I. sec.2.3, topics.

***B :**

Assuming a linear combination of spin ±1/2 in each state Tc (c: pre-color) in state $f=T_RT_GT_B$ (antisymmetric product), this creates 2×2×2＝8 bases. Among these, terms that are all 1/2 or -1/2 cannot combine the preons, so 6 bases remain. (Boson carries spin=1)
These are divided into 6＝3+3 depending on whether the total spin is 1/2 or -1/2.
Therefore, considering the state of total spin＝+1/2, the number of modes of Fermion is 3 (as a combination of spin×pre-color).
The state is specified by which of $T_RT_GT_B$ has spin＝-1/2 (all others have spin＝+1/2).
It is assumed that the electrons are in a resonance state of these 3 types of bonds.
Exchange of spin can be caused by both su(3) and u(2) diagonal components.
$(f\to f'=S\cdot f,\quad S\in u(2), su(3))$

Applying state separation approximation to the states of Boson, the mode excitation representation function of pre-color Boson is the analytic function of su(3) matrix elements, and assuming su(3)-symmetry, it can be supposed that it is invariant formula based on power series of matrix su(3) ***1 :**

***1 :**

When we create a power series function of the matrix (x) , y≡f(x) with commutative constant coefficients, $y\to SyS^{-1}$ holds under the similarity transformation $x\to SxS^{-1}$, and as a basic function that is invariant under similarity transformation, we can take the coefficients of the characteristic polynomial $P(\lambda)\equiv\det(\lambda-y)$.

(If we consider $S=1+\varepsilon$; $\varepsilon\to 0$, the above is also a transformation as $x\to x+[\varepsilon,x]$)

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Unified Field Theory by Canonical Gauge Principle (2024.04)

part Ⅲ:

Canonical Gauge Gravitational Theory

Link to Ⅲ-Local Table of Contents .
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Overview / Introduction

In part-III, the construction of quantum gravitational theory will be attempted. This gravitational theory is derived from the unified field theory based on canonical gauge principle.
Introducing the contents of the description, first the validity of the theory is investigated from the aspect of classical theory, and then, considering how to solve the field equation in the aspects of quantum theory .
Finally, extending the imagination and making the guesses about cosmological issues.
The following is an introduction to these themes according to their descriptive order.

(Validity, verification, and development of theory)

The gravitational field equation derived from the canonical gauge unified field theory is essentially different from that of Einstein gravity. *A
On the other hand, since A. Einstein's theory has been verified already within the classical theory, a similar verification is required to canonical gauge gravity theory, at least in the level of classical theory. Under these circumstances, it is shown that the canonical gauge gravitational theory can coincide to A. Einstein's theory under the assumption of static spherical symmetry, by selecting the interaction constants be appropriate.
This can be said to be the determination of the interaction constant in canonical gauge gravitational theory from observation.

Also, assuming spherical symmetry, the quadrature of the canonical gauge gravitational field is shown.
By comparing it with A. Einstein's theory of gravity, we can highlight the essence of Einstein's theory.
For even dynamic but uniformly isotropic 3-dimensional space, canonical gauge gravitational theory can be deployed by introducing an isotropic coordinate system.
This makes it possible to derive the modified Friedmann equation and develop the theory of expanding universe within the framework of quantum theory. Although the problem of the singularity of the universe Big Bang is resolved, unlike the classical theory, the problem of initial conditions appears, so in order to proceed further, a principle for setting initial conditions is required.

(Solving of gravitational field equation／Quantum gravity)

Regarding the solution of field equations, which was one of the themes discussed in Part II, we considered a solution unique to gravitational fields using the Schwarzschild quantum solution of spacetime as a subject.
The reason why the gravitational field is more complicated than other fields is that the gravitational field has self-interaction, the equation is nonlinear with respect to the field variables, and also the point that, the theory contains the variables as S(x), its inverse matrix T(x), and determinant det(T), etc.
Black holes are mainly the subject of Chapter 3, but in Chapter 2 also partially considered regarding

mass distribution, which is the cause of its existence, from the viewpoints of structuring the solution. In canonical gauge gravity theory, the introduction of isotropic coordinates is extremely useful in relation to the canonical Lorentz frame, and in this coordinate representation, matter flows toward the Schwarzschild barrier as time at the point at infinity passes.

(Wings of imagination, cosmology)

In final chapter of Part III (this volume) , speculating freely on the following items regarding the universe, while referring to the canonical gauge unified field theory.

- **dark matter and dark energy**

  Based on canonical gauge unified field theory, the elementary particles unrelated to electromagnetic interactions will be imagined.
  The very existence of Lagrangian density produces an effect similar to the cosmological term.

- **universe model and big bang**

  From canonical gauge gravitational theory, a modified version of the Freinman equation and its expression in canonical formalism can be derived. That solves the Big Bang singularity. This is also the theme discussed in Chapter 1.

- **Expansion mechanism of the universe, expansion acceleration**

  Discussion based on the modified Friedmann equation is argued. From the standpoint of constructing the theory of canonical gauge unified field, it is negative to the existence of the cosmological term as a physical law.

- **Definition and reality of black hole, speculation on its actual situation**

  The term "black hole" has become popular, but let us first start to thought about how to understand and define it.
  And then its actual situation is discussed according to one of the interpretations which we concluded.
  This is also the theme discussed in the course of Chapter

***A :**

The field variables and the form of the expressions are different. it is not enough to match just because the gravitational field is weak.
Requiring that the Lagrangian be a function of the at most 1st order derivative of the field variables and also be a scalar, then, it is true in canonical gauge gravity theory, but in Einstein's theory, such Lagrangian does not exist except for constant. (This is the origin of the cosmological term).
If sacrificing the restriction on differential order, scalar curvature form can be adopted as Lagrangian , except for constant multiplication.
Or sacrificing scalarness, it can be defined covariantly as a quadratic form of Levi-Civita connection coefficients.

2024.01 T.sato

Unifie field Theory byCanonical Gauge Principle
partⅢ：
Canonical Gauge Gravitational Theory

Table of Contents (Local, partⅢ)

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1. Canonical Gauge Unified Field – classical expression of gravitational theory

In Chapter 1, the validity of the canonical gauge unified field gravitational theory will be examined within the scope of classical theory, starting with preliminary considerations and focusing on comparisons with A. Einstein's theory.
Next, we discussed the possibility of verifying and developing the canonical gauge unified field theory, and considered its application to expansion cosmology.
The speculation on the universe will be also developed in Chapter 3.

1.1.Preliminary consideration

●Lagrangian of gravitational field in canonical gauge field unification theory

Through the discussion on field unification in part-I, we concluded that the gravitational field should be considered as the coefficient S of Lorentz frame in canonical gauge ring regarding canonical momentum p.

$P_A = \{S^\mu{}_A, p_\mu\}/2 + U_A$; Lorentz frame in canonical gauge ring, $P = P^*$ (self adjoint.)

p = canonical momentum, $S(x)$ = gravitational field (1st order coefficient of p) ,

$U(x)$ = other gauge field (0-th order coefficient of p)

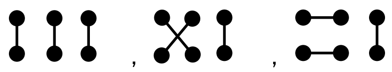
Lagrangian of unified field is derived from commutation relation of Lorentz frame (P_A) .

$[iP_A, iP_B] = i\cdot R_{AB}$; $R_{AB} = \{P_C, F^C{}_{AB}\}/2 + E_{AB}$, $F^C{}_{AB}$ = gravitational field strength

$F^C{}_{AB} = S^\mu{}_A F^C{}_{\mu\nu} S^\nu{}_B$, $F^A{}_{\mu\nu} = \partial_\mu T^A{}_\nu - \partial_\nu T^A{}_\mu$ where, $T = S^{-1}$ (inverse matrix)

Please remember our conclusion in part-I.
The Lagrangian of the 0th-order gauge field is given by the quadratic norm of E, which includes the unification of Boson and Fermion, the symmetry of elementary particles in the preon level, and the unknown field as a spinor connection field.
Lagrangian of unified field should be given in the quadratic norm form of R. but because gravitational field F has 3 indexes, the definition of the quadratic norm is not uniquely defined.
From the viewpoint of invariant theory, the linear combination of the following terms can be adopted.
$F^{ABC} F_{ABC}$ $F^{ABC} F_{BAC}$ $F^A{}_{AC} F^B{}_B{}^C$ 1 （arbitrary for index up/down with Lorentz metric η ）

The above is a problem of index contraction.
Since F_{ABC} is antisymmetric with respect to indexes B and C, then as quadratic expression of F, there can be one term contracting with its own index, and 2 terms contracting with each other.
The diagram showing this situation is as follows. Theoretically, it is possible to add a constant to this.
The constant corresponds to the cosmological constant in A. Einstein's gravitational theory.



In the above, the 2nd term seems a little unnatural, because when considering the following transformation by commutation operation

$iP_A \rightarrow [iP_A, P_B] = \{P_C, F^C{}_{AB}\}/2 + E_{AB}$

and regarding $F^C{}_{AB}$ as a matrix representation of this transformation caused by P_B, that is
$F^C{}_{AB} \equiv F_{(B)}$: matrix. Then, the form of $Tr(F^*F)$, $Tr(F)^*Tr(F)$ are natural and the 2nd term is not.

The 2nd term corresponds to the term of $Tr(F^2)$, and it is also theoretically possible for real field, though it seems something strange as a quadratic norm expression.

If the matrix $F_{(B)}$ is divided into the component of adjoint and anti-adjoint, the following formula may reduce this feeling to some extent.

For real number matrix X, $X = X_S + X_A$ (decomposition to self-adjoint / anti self-adjoint component)

$$Tr(X \cdot X) = Tr(X_S * X_S) - Tr(X_A * X_A) \text{ , } Tr(X^* X) = Tr(X_S * X_S) + Tr(X_A * X_A)$$

$$Tr(X) = Tr(X_S)$$

Considering the fact that gravitational field is a real field, the above may suggest that the symmetric and antisymmetric components of F should be considered independently.
In section 1.3, similar considerations on A. Einstein's theory will be explored.
A. Einstein's theory might be said not aesthetic at the combination of indexes for contraction.

1.2. Analysis on classical representations of gravity in canonical gauge unified field

When interpreting the canonical gauge gravitational field classically, it is not trivial for the field variable $T = S^{-1}$, whether $g_{\mu\nu} = T^A{}_\mu \eta_{AB} T^B{}_\nu$ matches from its meaning to the $g_{\mu\nu}$ of the metric tensor in space-time manifold model.
Concerning the motion of mass points in the gravitational field, it will be necessary to confirm that the motion is given by the geodesic, as in A. Einstein's theory.
Therefore, from this point of view, let us consider the Bianchi identity, the motion of mass points, and the spin-energy relationship in the canonical gauge gravitational theory.

(1).Bianchi identity

Since Lagrangian of the canonical gauge gravitational field is invariant to the canonical gauge, the Bianchi identity can be obtained as in the case of general relativity.
Select the transform generator as $W = 0.5 \cdot (w_\mu p_\mu + p_\mu w_\mu)$. The variation of the gravitational field with respect to the infinitesimal canonical transformation is $\delta T^A{}_\mu = w_\nu \partial_\nu T^A{}_\mu + \partial_\mu w_\nu \cdot T^A{}_\nu$; $(T^A{}_\mu \cdot \eta_{AB} \cdot T^B{}_\nu \equiv g_{\mu\nu})$
Let the variational coefficient of Lagrangian density $L^G = L^G(T,dT)$ be written as $(L^G)^\mu{}_A$.
The following is obtained from the invariance of the action integral.

$$\int dx \cdot (L^G)^\mu{}_A \cdot [w_\nu \partial_\nu T^A{}_\mu + \partial_\mu w_\nu \cdot T^A{}_\nu] = 0 \qquad ; \quad L^G \equiv \det(T) \cdot L \quad (L \text{ is a scalar})$$

By organizing the above using integral by parts, the following identity is obtained.

$$\partial_\mu (L^G)^\mu{}_A \cdot T^A{}_\nu + (L^G)^\mu{}_A \cdot F^A{}_{\mu\nu} \equiv 0 \quad \text{(identity) ;} \quad F^A{}_{\mu\nu} \equiv \partial_\mu T^A{}_\nu - \partial_\nu T^A{}_\mu$$

If the above is written as $\partial_\mu (L^G)^\mu{}_A + (L^G)^\mu{}_B \cdot F^B{}_{\mu A} \equiv 0$; $F^B{}_{\mu A} = F^B{}_{\mu\nu} \cdot S^\nu{}_A$ $(S = T^{-1})$, it can be seen that $F^B{}_{\mu A}$ plays a role like a connection coefficient.

If we interpret the above as the law of conservation of energy and momentum in correspondence with Einstein's theory, we should first investigate the case where the energy and momentum tensors are symmetric.
Set $(L^G)^{\mu\nu} = (L^G)^\mu{}_A \cdot S^{\nu A}$ and assuming that $\cdot (L^G)^{\mu\nu}$ is symmetric with respect to (μ,ν).
It can be transformed as follows.

$$\partial_\mu (L^G)^{\mu\nu} + (L^G)^{\mu\lambda} \cdot T_{B\lambda} (F^B{}_{\mu A} \cdot S^{\nu A} - \partial_\mu S^{\nu B}) \equiv 0$$

The second term "coefficient for $(L^G)^{\mu\lambda}$" gives "connection".
It can be confirmed by direct calculation that the symmetric component of this coefficient is equal to the Levi-Civita connection coefficient. ***1**
Therefore, it can be seen that the above expresses the conservation law of energy momentum with respect to the Levi-Civita connection. (However, it is needed to defined as $g_{\mu\nu} \equiv T^A{}_\mu \eta_{AB} T^B{}_\nu$.)

*** 1:** The coefficient for $(L^G)^{\mu\lambda}$ is transformed as follows.

$$C^\nu{}_{\mu\lambda} \equiv T_{B\lambda}(F^B{}_{\mu A} \cdot S^{\nu A} - \partial_\mu S^{\nu B}) = (F_{B\mu}{}^\nu - \partial_\mu S^\nu{}_B) \cdot T^B{}_\lambda$$

$$C^\nu{}_{\mu\lambda} = \partial_\mu T^B{}_\lambda \cdot S^\nu{}_B + T^B{}_\lambda \cdot F_{B\mu}{}^\nu = (\partial_\mu T^B{}_\lambda \cdot T_{B\sigma} + T_{B\lambda} \cdot F^B{}_{\mu\sigma}) \cdot g^{\sigma\nu}$$

$$= g^{\sigma\nu} \cdot (T_{B\sigma} \cdot \partial_\mu T^B{}_\lambda + T^B{}_\lambda \cdot \partial_\mu T_{B\sigma} - T^B{}_\lambda \cdot \partial_\sigma T^B{}_\mu) = g^{\sigma\nu} \cdot (\partial_\mu g_{\sigma\lambda} - T^B{}_\lambda \cdot \partial_\sigma T^B{}_\mu)$$

$$C^\nu{}_{\mu\lambda} + C^\nu{}_{\lambda\mu} = g^{\nu\sigma} \cdot (\partial_\mu g_{\sigma\lambda} - T^B{}_\lambda \cdot \partial_\sigma T^B{}_\mu + \partial_\lambda g_{\sigma\mu} - T^B{}_\mu \cdot \partial_\sigma T^B{}_\lambda) = g^{\nu\sigma}(\partial_\mu g_{\sigma\lambda} + \partial_\lambda g_{\sigma\mu} - \partial_\sigma g_{\mu\lambda}) = 2\Gamma^\nu{}_{\mu\lambda}$$

Therefore, the symmetric part of the coefficient is equal to the Levi-Civita connection coefficient.
Furthermore, since $C^\nu{}_{\mu\lambda} \equiv \partial_\mu T^B{}_\lambda \cdot S^\nu{}_B + T^B{}_\lambda \cdot F_{B\mu}{}^\nu$ then $C^\nu{}_{\mu\lambda} - C^\nu{}_{\mu\lambda} = F^A{}_{\mu\lambda} \cdot S^\nu{}_A + F_{\lambda\mu}{}^\nu - F_{\mu\lambda}{}^\nu$.
Therefore, $C^\nu{}_{\mu\lambda} = \Gamma^\nu{}_{\mu\lambda} + (1/2)(F_{\lambda\mu}{}^\nu - F_{\mu\lambda}{}^\nu + F^A{}_{\mu\lambda} \cdot S^\nu{}_A)$
Alternatively, this can be written as $C_{\nu\mu\lambda} = \Gamma_{\nu\mu\lambda} + (1/2)[F_{\lambda\mu\nu} - F_{\mu\lambda\nu} + F_{\nu\mu\lambda}]$.

(2).Motion of matter-interaction with gravitational field

In A.Einstein's theory, the mass point motion in the gravitational field is given by the geodesic. This can be obtained also from the viewpoint of the minimal path in space-time, the parallel displacement of tangent vector, and the conservation law of energy momentum, and is intuitively valid.

Even in the gravity of canonical gauge unified field theory, it is concluded that the motion of the mass point in the gravitational field is given by the geodesic in the classical treatment from the viewpoint of the law of conservation of energy momentum.

By analogy with Lagrangian $L_M = (1/2)mv^2$ in mass mechanics, Lagrangian density of the classical material field can be set as $L_M = (1/2)\sigma \cdot (g_{\lambda\mu} v^\lambda v^\mu - 1)$, $(T^A{}_\mu \cdot \eta_{AB} \cdot T^B{}_\nu \equiv g_{\mu\nu})$
This is a continuous fluid model of material field, where σ is the mass density and $\sigma/\det(T)$ is the scalar quantity.
($v^2 \to v^2 - 1$ is set so that the effect of $\delta\sigma/\delta g$ disappears. $\det(T) d^4x = \sqrt{-g}\, d^4x$ is an invariant)

The variational coefficient concerning the gravitational field variable T gives the energy-momentum tensor of the material field.
That is, $(L_M)^\lambda{}_A = \sigma \cdot v^\lambda v_A$ and the mass form dm is given by $dm = \sigma \cdot v^\lambda (d^3x)_\lambda$.

From the law of mass conservation, "equation of continuity " $\partial_\mu(\sigma v^\mu) = 0$ holds. Also, as the field equation, $(L^G)^\mu{}_A + \sigma \cdot v^\mu v_A = 0$ holds from the variational concerning T.
From the field equation and the equation of continuity , the hydrodynamic equation concerning the motion of the material field $\partial_\mu (L^G)^\mu{}_A + \sigma \cdot v^\mu \partial_\mu v_A = 0$ is obtained
The above can be expressed in the covariant derivative form by using Bianchi's identity. That is,

$(L^G)^\mu{}_A + \sigma \cdot v^\mu v_A = 0$ field equation
$\partial_\mu (L^G)^\mu{}_A + \sigma \cdot v^\mu \partial_\mu v_A = 0$ equation of motion
$\partial_\mu (L^G)^\mu{}_A + (L^G)^\mu{}_B \cdot F^B{}_{\mu A} = 0$ identity ; $F^A{}_{\mu\nu} \equiv \partial_\mu T^A{}_\nu - \partial_\nu T^A{}_\mu$

∴ $\sigma \cdot v^\mu \partial_\mu v_A + \sigma v^\mu v_B \cdot F^B{}_{\mu A} = 0$ $(\sigma \cdot v^\mu \partial_\mu v^A + \sigma v^C v^B \cdot F_{BC}{}^A = 0)$

According to $v_A \cdot S^{\lambda A} = v^\lambda$, the above equation can be expressed by the coordinate frame component

using the inverse matrix S of T.
Then, the following equation is obtained.

$$\sigma\cdot v^{\mu}\cdot\partial_{\mu}v^{\lambda}+\sigma v^{\mu}v^{\nu}\cdot T^{A}{}_{\nu}(-\partial_{\mu}S^{\lambda}{}_{A}+F_{A\mu}{}^{\lambda})=0 \quad ; \quad T^{A}{}_{\nu}(-\partial_{\mu}S^{\lambda}{}_{A}+F_{A\mu}{}^{\lambda})\equiv C^{\lambda}{}_{\mu\nu}$$

Concerning $C^{\lambda}{}_{\mu\nu}$,the coefficient of $v^{\mu}v^{\nu}$, only the symmetric part with respect to (μ, ν) will be necessary.
As we have already seen, the symmetric component of $C^{\lambda}{}_{\mu\nu}$ with respect to (μ, ν) reproduces the connection coefficient Levi-Civita connection form in A. Einstein's theory.
Comparing the above results with the symmetric component of the coefficient for vμvν shows that the free masspoint motion in the canonical gauge gravitational field is represented by the geodesic.
As far as classical theory is concerned, the motion of particles in canonical gauge gravity theory is the same as that in the theory of A. Einstein. When it comes to Dirac equation, it becomes necessary of canonical gauge gravitational field.

(3).spin-energy relation

The Lagrangian of the canonical gauge gravitational field is invariant to the global Lorentz transformations to the frame defining the canonical gauge connection. From this, an identity is obtained.

$$\delta(L^{G})=(L^{G})^{\mu}{}_{A}\cdot\delta T^{A}{}_{\mu}+\partial_{\nu}((L^{G})^{\nu,\mu}{}_{A}\cdot\delta T^{A}{}_{\mu})=0 \qquad ; \quad (L^{G})^{\nu,\mu}{}_{A}\equiv\partial L^{G}/\partial(\partial_{\nu}T^{A}{}_{\mu})$$

Let ω_{AB} be an infinitesimal constant parameter specifying the transformation as $\delta T^{A}{}_{\nu}=-\omega^{A}{}_{B}\cdot T^{B}{}_{\nu}$.
From the above the following is obtained.

$$(L^{G})^{\mu A}\cdot\omega_{AB}\cdot T^{B}{}_{\mu}+\partial_{\nu}((L^{G})^{\nu,\mu A}\cdot\omega_{AB}\cdot T^{B}{}_{\mu})=0$$

Here, $-(L^{G})^{\nu,\mu A}T^{B}{}_{\mu}=(L^{G})^{\mu,AB}$ is the current defined by the standard method for the above transformation, and in this case, it should correspond to the spin current of the gravitational field.
$(L^{G})^{\mu}{}_{A}$ is the energy-momentum tensor density obtained from the variational concerning $T^{A}{}_{\mu}$.

If the Lagrangian density of the gravitational field source is written as L^{M} , the equation of the gravitational field is $(L^{G})^{\mu}{}_{A}+(L^{M})^{\mu}{}_{A}=0$, so the relation $(L^{M})^{[AB]}=\partial_{\mu}(L^{G})^{\mu AB}$ is obtained.
That is, the divergence of the gravitational field spin angular momentum density is equal to the antisymmetric component of the energy momentum density of the gravitational source.

1.3.The essence of Einstein's theory of gravity

Here, let us study the essence of Einstein's theory of gravity, especially in its logical construction. That is because Lagrangian in canonical gauge unified field theory is essentially different from that of Einstein's theory. If Lagrangian of field is constructed as a function of quantity up to 1st order derivative of the field variable, Lagrangian of Einstein's theory cannot be a scalar except for a constant. Therefore, even if the weak field approximation is introduced, there still is a situation that the two cannot match.

note:

When writing action integral as $\int L\cdot(-g)^{1/2}d^4x$, we sometimes call L field Lagrangian and $L\cdot(-g)^{1/2}$ Lagrangian density.

Here, g means $g\equiv\det(g_{\mu\nu})$, but from the standpoint of canonical gauge theory, it should be written as $(-g)^{1/2}=\det(T)$, and $(-g)^{1/2}d^4x=\det(T)d^4x$ is an invariant for coordinate transformation. However, for the transformation that changes the direction of space-time, the sign of the square root must be changed from $(-g)^{1/2}\to-(-g)^{1/2}$ in the former. There is no such anomaly in $\det(T)d^4x$ expression.

●Origin of Lagrangian

A. Einstein's theory of gravity is the most rational one as long as the field variable is limited the space-time metric tensor (g). Lagrangian should be constructed by the derivatives of the field variables up to 1st- order. This is related to the rank of the differential equation. Of course, Lagrangian has arbitrariness concerning divergence quantity, so it does not mean that there should be no expression including higher-order derivatives.

For Lagrangian L, L' given from L by a canonical transformation, as follows, is equivalent to L. (canonically equivalence).

$(L\to L'\equiv L+(-g)^{-1/2}\partial_\mu((-g)^{1/2}D^\mu)$ for $\exists D$)

The variational coefficient of the Lagrangian density $L\cdot(-g)^{1/2}$ for the field variable g must be the energy-momentum tensor density of the gravitational source. From this, it is expected that $L\cdot(-g)^{1/2}$ is a scalar density, but this is not true in Einstein's theory.

Of course, logically Lagrangian does not have to be a scalar and so this is not esthetic, but this can be acceptable if such Lagrangian is given by covariant form and is canonically equivalent to a scalar. This can be said to suggest that there may be variables of gravitational field more suitable than $g_{\mu\nu}$.

The following is a list of conditions that can be adopted as Lagrangian.

1. $L=L(g,\partial g)$ consists of at most 1st order derivative of the field variable.
2. $L=L(g,\partial g)$ is a covariant form defined in the same form in any coordinate system.
3. There is a scalar form canonically equivalent to $L=L(g,\partial g)L$.

If Lagrangian of gravitational field $L=L(g, \partial g)$ is scalar, then it cannot be but a constant. Because, by equivalence principle, geodetic coordinate system can be set, and then, $g(P)=\eta$ (Lorentz metric), $\partial g(P)=0$ at any one point P, are derived. i.e. $L=L(\eta, 0)$.

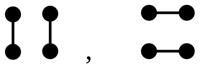
This is the origin of cosmological term in A. Einstein's gravitational theory, and from the above, it can be seen that $L=L(g, \partial g)$ cannot be a scalar other than a constant in Einstein's theory.

Considering canonical equivalency on Lagrangian, and applying integration by parts for ∂g in $L(g,\partial g)$,

then $L \to L' = L'(g, \partial g, \partial^2 g)$ is obtained.
The term $\partial^2 g$ remains in the geodetic system. and L' is linear for $\partial^2 g$. The expression of L' in the geodetic system is as follows.

$$L' = a \cdot \partial^{\lambda}\partial^{\mu} g_{\lambda\mu} + b \cdot \partial^{\lambda}\partial_{\lambda} g_{\mu\nu} \cdot \eta^{\mu\nu} + \text{const} \quad , \quad a, b : \text{constants.}$$

Considering the possible contraction of indexes regarding the $\partial^2 g$ term ($\partial_\lambda \partial_\mu g_{\nu\sigma}$), and taking account of their symmetry, the result is limited to the above. The diagram of this contraction is expressed as follows.



Now, if assuming L' to be a scalar, at least for any coordinate transformations from geodesic system to geodesic system such as x → $x \to x' = x + Cx^3 + O(x^4)$, L' must be transformed as a scalar.
Obviously, a + b = 0 is required for L' to be a scalar for this transformation.
The sufficiency of a + b = 0 is almost self-evident, but in fact it can be confirmed that this matches the scalar curvature apart from constant factor.

●Lagrangian of field in Einstein' gravitational theory

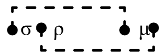
In the following, let us calculate the scalar curvature R, confirm that $L' = a \cdot R + \text{const}$, and at the same time, find $L(g, \partial g)$ that is canonically equivalent to R.
First, let us start to calculate the Riemann curvature tensor. Following are derived from connection formula:

$$d(\partial_\rho) = (\partial_\lambda) \cdot \Gamma^{\lambda}{}_{\rho\nu} dx^{\nu} \quad , \quad d^2(\partial_\rho) = (\partial_\sigma) \cdot \Gamma^{\sigma}{}_{\lambda\mu} dx^{\mu} \cdot \Gamma^{\lambda}{}_{\rho\nu} dx^{\nu} + (\partial_\lambda) \cdot d\Gamma^{\lambda}{}_{\rho\nu} dx^{\nu} = (\partial_\sigma) \cdot R^{\sigma}{}_{\rho,\mu\nu} \cdot dx^{\mu\nu}/2!$$

$$\text{i.e.} \quad R^{\sigma}{}_{\rho,\mu\nu} = \Gamma^{\sigma}{}_{\lambda\mu}\Gamma^{\lambda}{}_{\rho\nu} - \Gamma^{\sigma}{}_{\lambda\nu}\Gamma^{\lambda}{}_{\rho\mu} + \partial_\mu \Gamma^{\sigma}{}_{\rho\nu} - \partial_\nu \Gamma^{\sigma}{}_{\rho\mu} \quad ; \quad dx^{\mu\nu} \equiv dx^{\mu} dx^{\nu} \quad \text{(abbreviation)}$$

From the above, scalar curvature R is obtained by indexes contraction on $R^{\sigma}{}_{\rho,\mu\nu}$, but a little care is needed for the combination of the contracted indexes. (It changes the sign)
As is well known, $R_{\sigma\rho,\mu\nu}$ have remarkable symmetry. That is symmetric concerning double indexes (σ,ρ) ,(μ,ν) , and is antisymmetric concerning σ,ρ and μ,ν. Therefore, the following combination is appropriate as a contraction on double index.

$R_{\sigma\rho,\mu\nu}$:



Contraction of index (σ, μ) gives Ricci tensor $R_{\rho,\mu}$, and contraction of index (ρ, ν) gives the scalar curvature R.

$$R_{\rho,\nu} = \Gamma^{\sigma}{}_{\lambda\sigma}\Gamma^{\lambda}{}_{\rho\nu} - \Gamma^{\sigma}{}_{\lambda\nu}\Gamma^{\lambda}{}_{\rho\sigma} + \partial_\sigma \Gamma^{\sigma}{}_{\rho\nu} - \partial_\nu \Gamma^{\sigma}{}_{\rho\sigma} \quad ; \quad R = g^{\rho\nu} \cdot R_{\rho,\nu}$$

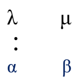
First, let us confirm the expression of R in the geodetic system in which, $\partial g = \Gamma = 0$. the following is obtained

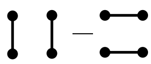
$$R = \eta^{\rho\nu} \cdot (\partial_\sigma \Gamma^{\sigma}{}_{\rho\nu} - \partial_\nu \Gamma^{\sigma}{}_{\rho\sigma}), \qquad ; \ \partial_\nu \Gamma^{\lambda}{}_{\rho\sigma} = (1/2)\eta^{\lambda\tau}(-\partial_\nu\partial_\tau g_{\rho\sigma} + \partial_\nu\partial_\rho g_{\sigma\tau} + \partial_\nu\partial_\sigma g_{\tau\rho})$$

$$+) \quad \eta^{\rho\nu} \cdot \partial_\sigma \Gamma^{\sigma}{}_{\rho\nu} = (1/2)\, \eta^{\rho\nu}\eta^{\sigma\tau}(-\partial_\sigma\partial_\tau g_{\rho\nu} + 2\partial_\sigma\partial_\rho g_{\nu\tau})$$

$$-) \quad \eta^{\rho\nu} \cdot \partial_\nu \Gamma^{\sigma}{}_{\rho\sigma} = (1/2)\, \eta^{\rho\nu}\eta^{\sigma\tau}\partial_\nu\partial_\rho g_{\sigma\tau}$$

$$R = \eta^{\sigma\tau}\eta^{\rho\nu}\partial_\sigma\partial_\rho g_{\nu\tau} - \eta^{\nu\rho}\eta^{\sigma\tau}\partial_\nu\partial_\rho g_{\sigma\tau} \text{ , index contract.} \quad \partial_\lambda\partial_\mu g_{\alpha\beta}$$



R = 

Next, let us find the quadratic form of Γ with canonical equivalence to R.
Concerning the term of $\partial\Gamma$, by integration by parts, following replacement is applicable.

$+g^{\rho\nu}\cdot\partial_\sigma\Gamma^\sigma{}_{\rho\nu} \to -(-g)^{-1/2}\partial_\sigma((-g)^{1/2}g^{\rho\nu})\cdot\Gamma^\sigma{}_{\rho\nu}$, $-g^{\rho\nu}\cdot\partial_\nu\Gamma^\sigma{}_{\rho\sigma} \to +(-g)^{-1/2}\partial_\nu((-g)^{1/2}g^{\rho\nu})\cdot\Gamma^\sigma{}_{\rho\sigma}$

The above is calculated as follows and the following expression is derive for R' that is quadratic form of Γ with canonical equivalence to R.

$-(-g)^{-1/2}d((-g)^{1/2}g^{\rho\mu})=-0.5d\ln(-g)\, g^{\rho\mu}+g^{\rho\alpha}dg_{\alpha\beta}.g^{\beta\mu}$; $0.5\partial_\rho\ln(-g)=\Gamma^\sigma{}_{\sigma\rho}$

$R'=g^{\rho\nu}\Gamma^\sigma{}_{\lambda\sigma}\Gamma^\lambda{}_{\rho\nu}-g^{\rho\nu}\Gamma^\sigma{}_{\lambda\nu}\Gamma^\lambda{}_{\rho\sigma}-(-g)^{-1/2}\partial_\sigma((-g)^{1/2}g^{\rho\nu})\cdot\Gamma^\sigma{}_{\rho\nu}+(-g)^{-1/2}\partial_\nu((-g)^{1/2}g^{\rho\nu})\cdot\Gamma^\sigma{}_{\rho\sigma}$

$=g^{\rho\nu}\Gamma^\sigma{}_{\lambda\sigma}\Gamma^\lambda{}_{\rho\nu}-g^{\rho\nu}\Gamma^\sigma{}_{\lambda\nu}\Gamma^\lambda{}_{\rho\sigma}-g^{\rho\nu}\Gamma^\tau{}_{\tau\sigma}\Gamma^\sigma{}_{\rho\nu}-\partial_\sigma g^{\rho\nu}\cdot\Gamma^\sigma{}_{\rho\nu}+g^{\nu\rho}\Gamma^\tau{}_{\tau\nu}\Gamma^\sigma{}_{\sigma\rho}+\partial_\nu g^{\rho\nu}\cdot\Gamma^\sigma{}_{\rho\sigma}$

$=-g^{\rho\nu}\Gamma^\sigma{}_{\lambda\nu}\Gamma^\lambda{}_{\rho\sigma}+g^{\nu\rho}\Gamma^\tau{}_{\tau\nu}\Gamma^\sigma{}_{\sigma\rho}-\partial_\sigma g^{\rho\nu}\cdot\Gamma^\sigma{}_{\rho\nu}+\partial_\nu g^{\rho\nu}\cdot\Gamma^\sigma{}_{\rho\sigma}$

Let the derivative of g be represented by Γ and the Lagrangian representation be L = L (g, Γ).
Note that $dg^{-1}=-g^{-1}dg\, g^{-1}$. and we get the following.

$\partial_\sigma g^{\rho\nu}=-g^{\rho\alpha}\partial_\sigma g_{\alpha\beta}g^{\beta\nu}$, $\partial_\sigma g_{\alpha\beta}=g_{\tau\beta}\Gamma^\tau{}_{\alpha\sigma}+g_{\tau\sigma}\Gamma^\tau{}_{\beta\sigma}$ $\therefore$ $\partial_\sigma g^{\rho\nu}=-g^{\rho\alpha}\Gamma^\nu{}_{\alpha\sigma}-\Gamma^\rho{}_{\beta\sigma}g^{\beta\nu}$

$R'=-g^{\rho\nu}\Gamma^\sigma{}_{\lambda\nu}\Gamma^\lambda{}_{\rho\sigma}+g^{\nu\rho}\Gamma^\tau{}_{\tau\nu}\Gamma^\sigma{}_{\sigma\rho}+(g^{\rho\alpha}\Gamma^\nu{}_{\alpha\sigma}+\Gamma^\rho{}_{\beta\sigma}g^{\beta\nu})\Gamma^\sigma{}_{\rho\nu}-(g^{\rho\alpha}\Gamma^\nu{}_{\alpha\nu}+\Gamma^\rho{}_{\beta\nu}g^{\beta\nu})\Gamma^\sigma{}_{\rho\sigma}$

$R'=g^{\rho\alpha}\Gamma^\nu{}_{\alpha\sigma}\Gamma^\sigma{}_{\rho\nu}-g^{\beta\nu}\cdot\Gamma^\rho{}_{\beta\nu}\Gamma^\sigma{}_{\sigma\rho}$

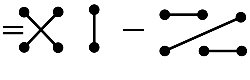
This is one of the well-known final results, but the combination of indexes in contraction is quite strange.
From the equation of mass point motion in gravitational field, it is natural to consider that Levi-Civita connection coefficient expresses apparent field strength. So it is not so strange to give Lagrangian by quadratic form of Γ.
On the other hand, Γ expresses parallel displacement of tangent vector as $d(\partial)=(\partial)\Gamma$, and $\Gamma=\Gamma_\nu dx^\nu$ is the matrix of this transformation.
Therefore, considering $\Gamma(\alpha)=(\Gamma^\lambda{}_{\mu\alpha})$:matrix, the 1st term of R' can be interpreted as $g^{\alpha\rho}\cdot \mathrm{Tr}(\Gamma(\alpha)\cdot\Gamma(\rho))$

This may also be strange since this is not $g^{\alpha\rho}\cdot\mathrm{Tr}(\Gamma(\alpha)^*\cdot\Gamma(\rho))$, but if decomposing the matrix into symmetric / antisymmetric part, this feeling of strangeness may relief a little because the quadratic norm is composed of the norm of each part. Looking the 2nd term , $\Gamma^\sigma{}_{\sigma\rho}=\mathrm{Tr}(\Gamma(\rho))$ seems to be OK, but $\Gamma^\rho{}_{\beta\alpha}g^{\beta\alpha}$ is not, and no rational interpretation found.
Dare speaking, since the motion of the mass point is dominated by $dv^\lambda/ds+\Gamma^\lambda{}_{\mu\nu}v^\mu v^\nu=0$, it may be possible to interpret it as the directional mean of the apparent force $\Gamma^\lambda{}_{\mu\nu}v^\mu v^\nu$

At any rate, Lagrangian $L=L(g,\Gamma)$ of gravitational field in A. Einstein theory is given by R' apart from constant factor, and its expression is as follows.

$R'=g^{\alpha\beta}\cdot\mathrm{Tr}(\Gamma(\alpha)\cdot\Gamma(\beta))-\mathrm{Tr}(\Gamma(\alpha))\cdot(\Gamma^\alpha{}_{\mu\nu}g^{\mu\nu})$, $\Gamma(\alpha)=(\Gamma^\mu{}_{\nu\alpha})$: matrix

The structure of index contraction is as follows.

$\Gamma^\lambda{}_{\mu\nu}$: $\begin{matrix}\lambda & \mu \\ \nu & \end{matrix}$ then R'= 

Next, let us find the normalization constant of A.Einstein's gravity Lagrangian.
Set as the following situation.

Action of mass point $A_m\equiv\int(1/2)mg_{\mu\nu}v^\mu v^\nu ds$,
Action of gravitational field $A_G\equiv\lambda\cdot\int R'\cdot(-g)^{1/2}d^4x$

If we assume a distribution in mass, we can set $mds = \sigma \cdot (-g)^{1/2} d^4x$, σ is a scalar quantity that represents the mass density seen in a stationary system, but due to the law of conservation of mass, $\delta m \equiv 0$ must be set in the variational with respect to g.

Here, when applying the variational principle to $A \equiv A_m + A_G$, it is required that the weak gravitational field approximation agrees with Newtonian theory of gravity.
From the definition of R', of course $\delta A_G \equiv \lambda \cdot \delta \int R'(-g)^{1/2} d^4x \equiv \lambda \cdot \delta \int R \cdot (-g)^{1/2} d^4x$; $R = R(g, \Gamma)$.
If we consider g and Γ as independent in the expression $R = g^{\rho\nu} \cdot R_{\rho,\nu}$ $R_{\rho,\nu} = R_{\rho,\nu}(\Gamma)$ and take the variational, we can use Palatini's variational method as $\delta_\Gamma \int R \cdot (-g)^{1/2} d^4x = 0$ is known to give a Levi-Civita connection between Γ and g.

Therefore, $\delta A_G \equiv \lambda \cdot \delta_g \int R(-g)^{1/2} d^4x = \lambda \cdot \int (\delta g^{\rho\nu} \cdot R_{\rho,\nu} + (1/2) g^{\rho\nu} \delta g^{\rho\nu})(-g)^{1/2} d^4x$, $\delta g^{\rho\nu} = -g^{\rho\alpha} \delta g_{\alpha\beta} g^{\beta\nu}$.
We obtain $\delta A_G = -\lambda \cdot \int \delta g_{\alpha\beta} \cdot G^{\alpha\beta} (-g)^{1/2} d^4x$; $G^{\alpha\beta} \equiv R^{\alpha\beta} - (1/2) g^{\alpha\beta} R$.
The variational equation becomes $\int (1/2) m \cdot \delta g_{\alpha\beta} v^\alpha v^\beta ds - \lambda \cdot \int \delta g_{\alpha\beta} \cdot G^{\alpha\beta} (-g)^{1/2} d^4x = 0$. note : $\delta m \equiv 0$

From here on, it is necessary to assume that the mass point is stationary at the origin and the gravitational field is static and spherically symmetric.
The coordinate system is an orthogonal one, and then $g_{\alpha\beta} = 0$ except g_{00}, $g_{11} = g_{22} = g_{33}$.

$(1/2)\sigma \cdot v^\alpha v^\beta - \lambda(R^{\alpha\beta} - (1/2) g^{\alpha\beta} R = 0$: variatinal equation (Einstein equation)

Taking 00 component, $(1/2)\sigma - \lambda(R^{00} - (1/2)R = 0$,
Taking the trace, $\sigma/2 - \lambda(R - 2R) = 0$. i.e. $\sigma/2 + \lambda R = 0$
Therefore, following are derived. $R^{00} - (1/2)R = -R$. i.e. $R^{00} = -(1/2)R$.

In geodetic representation, R is expressed as follows.

$$R = \eta^{\sigma\tau}\eta^{\rho\nu}\partial_\sigma\partial_\rho g_{\nu\tau} - \eta^{\nu\rho}\eta^{\sigma\tau}\partial_\nu\partial_\rho g_{\sigma\tau} = \partial_\sigma\partial_\sigma g_{\sigma\sigma} - \eta^{\nu\nu}\eta^{\sigma\sigma}\partial_\nu\partial_\nu g_{\sigma\sigma} \quad ; \quad \eta = \mathrm{diag}(1,-1,-1,-1)$$

$$\therefore \quad R = \partial_s\partial_s g_{11} + \partial_s\partial_s(g_{00} - 3g_{11}) = \partial_s\partial_s g_{00} - 2\partial_s\partial_s g_{11}$$

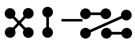
Taking the case for a weak stationary gravitational field, in geodetic representation, we obtain
from $R^{00} = -(1/2)R$, then, $(1/2)\partial_s\partial_s g_{00} = -(1/2)\partial_s\partial_s g_{00} + \partial_s\partial_s g_{11}$ $\therefore$ $\partial_s\partial_s g_{00} = \partial_s\partial_s g_{11}$
from $\sigma/2 + \lambda R = 0$, $R = \partial_s\partial_s g_{00} - 2\partial_s\partial_s g_{11}$, $\partial_s\partial_s g_{00} = \partial_s\partial_s g_{11}$, then $\sigma/2 - \lambda\partial_s\partial_s g_{00} = 0$

Here, from the equation of mass point motion (geodesic) in the gravitational field, we know that $g_{00} = 1 + 2U$ gives U=Newton potential, so $\sigma/2 - 2\lambda \cdot \partial_s\partial_s U = 0$
Corresponding to the expression in 3-dimensional Euclidean metric, we obtain

$$\nabla^2 U = 4\pi G\sigma = +\sigma/(4\lambda) \qquad \text{i.e. } 1/\lambda = 16\pi G$$

Finally, we obtain the normalization constant of R' concerning gravity is $+1/(16\pi G)$.
The conclusions are expressed as follows.

Lagrangian of gravitational field $L_G = R'/(16\pi G)$;

$R' \equiv g^{\alpha\rho}\Gamma^\nu{}_{\sigma\alpha}\Gamma^\sigma{}_{\nu\rho} - \Gamma^\sigma{}_{\sigma\rho} \cdot \Gamma^\rho{}_{\beta\nu} g^{\beta\nu}$ = 

Lagrangian of masspoint motion $L_m = (1/2) g_{\mu\nu} v^\mu v^\nu$; $\sigma(-g)^{1/2} d^4x = m \cdot ds$

$$(-g)^{-1/2}\delta((-g)^{1/2} \cdot (L_m + L_G)) = (\sigma v^\mu v^\nu - G^{\mu\nu}/8\pi G) \cdot \delta(g_{\mu\nu}/2) \quad ; \quad G^{\mu\nu} \equiv R^{\mu\nu} - (1/2) g^{\mu\nu} \cdot R$$

-

1.4.Problems of verification in the level of classical theory

1.4.1.Verification materials and methods

Gravitational theory is classically verified by the motion of mass points in the gravitational field.
As long as light is considered like a particle and treated geometrically, verification by bent of light rays in gravitational field can be considered as a kind of verification by mass point motion.
Though most of the verification methods are based on motion of mass points or light, but the theory of gravity itself is a theory on the generation of a gravitational field, and the variables of the gravitational field differ according to theory.
Therefore, in order to make a theoretical comparison with each, it is necessary to clarify the mutual relationship.

Theory	Lagrangian of mass point	variables of Gravitational field	Remarks
I. Newton theory	$L=(1/2)v^2$; $v=dx/dt$ $t\equiv x^0$	U : Newton potential $U=\ln(T)$ 00 component Lagrangian$=(\partial U)^2 \times$const	
A. Einstein theory	$L=(1/2)v^2$; $v=dx/ds$ $v^2=1$: ***A**	g : metric tensor $g\equiv T^*\eta T$ Lagrangian= scalar curvature R×const	***A** : in case of light $(dx)^2\equiv g_{\mu\nu}v^\mu v^\nu \to 0$
Canonical. Gauge Unified Field theory	the same as Einstein's	$T=S^{-1}$: S=1st degree coefficient of canonical gauge connection ***1** Lagrangian=quadratic form of canonical curvature form	***1** : Lorentz Frame (P_c) $P_c\equiv\{S,p\}/2+U_c(x)$

In A.Einstein's theory, and canonical gauge unified field theory, the equation of motion of mass point is represented by geodesic line equation, but in Newtonian theory, this must be approximated.

Geodesic line equation :

$$dv^\lambda/ds+\Gamma^\lambda{}_{\mu\nu}v^\mu v^\nu=0\,,\quad v^\lambda=dx^\lambda/ds \quad ; \quad \Gamma^\lambda{}_{\mu\nu}=0.5g^{\lambda\sigma}(-\partial_\sigma g_{\mu\nu}+\partial_\mu g_{\nu\sigma}+\partial_\nu g_{\sigma\mu})$$

Assuming that the velocity of mass point is extremely small compared with light velocity, the metric $g_{\mu\nu}$ is close enough to the Lorentz metric, and the system is stationary, then

$$\Gamma^\lambda{}_{00}=-0.5\eta^{\lambda\sigma}\partial_\sigma g_{00} \quad ; \quad dv^\lambda/ds \fallingdotseq -0.5\eta^{\lambda\sigma}\partial_\sigma g_{00} \ 、$$
$$dv^\lambda/dt-0.5\cdot\eta^{\lambda\sigma}\partial_\sigma g_{00}=0\,,\quad v^\lambda=dx^\lambda/dt \quad (\lambda=1..3),\quad t\equiv x^0$$

The above shows that $0.5\cdot(g_{00}-1)$ corresponds to Newton potential U. That is, as relation of the field variables in each theory, $g_{00}\fallingdotseq 1+2U$ or $T^0{}_0\fallingdotseq 1+U$ ***1**

The above is also an approximation derived from the fact that the moving velocity of the mass point is extremely slow compared with the speed of light. In Newtonian theory, gravitational field variable is scalar potential U only, and there is no corresponding object other than g_{00}.
Therefore, U will be applied to light rays as well.
As is well known, the hyperbolic solution is correspond to the spherically symmetric central force field.

***1:** This approximation suggests that $V : T = \exp(V)$, can be introduced as a matrix variable of gravitational field, corresponding to Newton potential. (where, $U = V^0{}_0$)

The above is a discussion of the interpretation of mass point motion for verification purposes, but as stated above, the essence of gravitational theory lies in the generation equation of the gravitational field. The conventional gravitational theory has been verified by observing the motion of mass points (planets) and light within the gravitational field generated by the sun.
Regarding the Black Hole, its existence is believed to be certain (although its exact definition is somewhat ambiguous), but we are not aware of any quantitative theoretical verification.

The 2 items described below (bending of light rays due to solar gravity and Mercury's perihelion shift) are well-known verification materials.
Both are based on the solar gravity that is thought to be spherically symmetric with high accuracy.
In the next section1.5, the gravitational field generated by canonical gauge gravitational theory in the case of spherical symmetry is explained.
A notable result is that the two theories of A.Einsteins Gravity and Canonical Gauge Gravity agree, if the interaction constant Λ in the theory of canonical gauge gravitational theory satisfies $\Lambda + 1 = 0$.

From the discussion in Section 1.2, it has been obtained that the two theories agree regarding the mass motion in the gravitational field, assuming that $g_{\mu\nu} \equiv T^A{}_\mu \eta_{AB} T^B{}_\nu$ coincides with the metric tensor $g_{\mu\nu}$ in the space-time manifold model.
Therefore, the verification of A.Einstein's theory is also the verification of canonical gauge gravity theory, (at least for static and spherically symmetric case).

- **Relationship between solar ray bending and distance (verification material 1)**
  The bending of light rays passing through the gravitational field is measured based on the amount of change in the apparent position of the stars visible around the sun during a solar eclipse, and the problem to be argued is the relationship between the bending angle and the distance to the sun.
  Since the bending due to gravity is calculated as the residue excluding the contribution of refraction by the solar atmosphere, the calculation of an accurate value seems complicated.
  An important point in verifying the canonical gauge gravitational theory is that this observation determines Λ, the interaction constant in the theory, and requires $\Lambda + 1 = 0$.

  Regarding the bending angle of light, Einstein's theory gives twice the value of the result according to Newton's theory, and the actual measurement supports this. The reason is that for the distortion of space-time metric, Newton's theory affects only the time-directional component g_{00}, but Einstein's theory affects the spatial component as well.
  Quantitative discussions will be developed in apdx-1.

- **Mercury's perihelion shift (verification material 2)**
  The perihelion shift of Mercury can be explained by general relativity. Schwarzschild solution is used as a spherically symmetric gravitational field, but the verification is limited to the range of the weak gravitational field.
  The actual verification seems to be quite complicated because it calculates the shift as a residue excluding the contribution of the perturbation effect of other planets.

The mass motion in a spherically symmetric central force field is limited to planar motion, and from the conservation law of angular momentum and energy, variables separation equation can be obtained for the 1st derivative of true anomaly and of the orbit radius.
The essence of the perihelion shift lies in the deviation from 2π of　true anomaly given by integral concerning (the reciprocal of) the orbit radius variable
(Integral can be written in the form of complex contour integral).
Quantitative discussion will be developed in apdx-1b.

1.4.2.Prospects for next verifications

In the previous section, the verification of canonical gauge gravity theory is done by confirming the consistency with A. Einstein's theory in terms of items that have already been verified within the scope of classical theory and under the special condition of spherical symmetry.

Ideally, the next step would be to focus on the differences between these 2 theories and to decide which is correct through observation and experimentation.
Candidates for verification materials include cases where the gravitational field deviates significantly from spherical symmetry, is dynamic, and involves spin angular momentum.
We are looking forward to observing the gravitational waves emitted by the binary star. ***A**
The rotational motion of galaxies is related to the discovery of dark matter, and much of the analysis is explained using Newtonian theory.
It might be necessary to analyze and evaluate the dragging effect of the gravitational field generated by high-speed massive matters and the resulting interaction between the stars.

***A :**
When setting $T = 1 + \Delta T$,　then　$g = T^{*}\eta T \fallingdotseq \eta + \Delta T^{*}\eta + \eta\Delta T$, so the antisymmetric component of T does not appear in A. Einstein's theory even within the range of linear approximation.
If there is a phenomenon in which the antisymmetric component is excited, the effects not found in A.Einstein's theory might appear.

1.4.3.Comparison of Canonical Gauge Gravitational theory with Einstein theory

Canonical gauge gravity (Canonical gauge unified field)	**Einstein theory (general relativity)**
・a part of unified field theory ・classical field is interpreted as state-expected value of field operator. ・The gauge group : space-time preserving canonical transformation group ***A** ・Space-time appears as the expected value of quantum theoretical operators ・Gravitational variable : coefficient T of canonical momentum operator p concerning Lorentz frame P of canonical gauge ring ; p＝{T,P}/2-Uc ・Mass motion in classical theory: geodesic --Same as Einstein theory ・Introduction of Dirac equation : possible in principle ・Lagrangian $L=L(T, \partial T)$: quadratic of canonical curvature. scalar form. -- essentially different from Einstein theory ・Remarks: For spherically symmetric field , theory with $\Lambda=-1$ agrees with Einstein theory. ***B** ・Verification is investigated in section 1.5.	・Gravity theory ・Classical theory ・The gauge group: general coordinate conversion group ***A** ・**B**ased on space-time manifold model ・Gravitational variable: metric tensor g ,related with canonical gauge theory by $g=T^*\eta T$. ・Mass motion is geodesic ・Introduction of Dirac equation : ad hoc (Vierbein / tetrad) ・Lagrangian $L=L(T, \partial T)$: Levi-Civita Quadratic form with canonical equivalence to scalar curvature R ・Remarks: there are some observational verifications in weak gravity and no negative material has been found.

***A :** Here, "gauge group" is used in a broader sense than in the theory of connection form.
This is based on the idea that there must be a universal transformation rule between expressions of nature because the expression of natural phenomena is based on the gauge that human beings have introduce to nature arbitrarily.

***B :** Lagrangian $L \propto (1/2!)\cdot F_{ABC}F^{ABC} + \Lambda\cdot F_A{}^A{}_C \cdot F_B{}^{BC}$
See detail in section.1.5.3.

1.5. Verification of canonical gauge gravitational theory in classical level

1.5.1. Lagrangian for static spherically symmetric field

There shall be a static spherically symmetric solution in the gravitational field equations.
If so, a solution can be obtained by applying the variational principle to Lagrangian constructed assuming static spherical symmetry.
Therefore, first of all, the expression of spherical symmetry becomes a problem. The following forms can be assigned as space-time metric when spherical symmetry is assumed.

$$ds^2 = a(r)^2 dt^2 - b(r)^2 \cdot (dx^\mu \cdot dx^\mu) \quad ; \quad \mu=1..3$$

This is derived from the following considerations.
Let the time coordinate variable be t and the 3-dimensional space coordinate variable be the spherical coordinate variable (r, θ, φ).
Due to the invariance with respect to time inversion $(t \to -t)$, the time/space displacements are orthogonal.
For the displacement in time direction, $ds^2 = a(r)^2 dt^2$ can be set.
The line element ds^2 for angular displacement is proportional to the form in Euclidean space.

$$ds^2 = -c(r)^2 \cdot (d\theta^2 + \sin^2(\theta) d\varphi^2)$$

It can be assumed that line element ds^2 for the radial displacement is $ds^2 = -b(r)^2 dr^2$.
The displacements in angular direction and radial direction are orthogonal.
From the above, the following is obtained.

$$ds^2 = a(r)^2 dt^2 - b(r)^2 dr^2 - c(r)^2 (d\theta^2 + \sin^2(\theta) d\varphi^2)$$

It can be adjusted so that $c = b \cdot r$ by transformation of the radial coordinate variable r.
Therefore, following holds

$$ds^2 = a(r)^2 dt^2 - b(r)^2 (dr^2 + r^2 (d\theta^2 + \sin^2(\theta) d\varphi^2))$$

Further, by applying standard variable transformation from spherical coordinates to Cartesian coordinates, following is obtained.

$$ds^2 = a(r)^2 dt^2 - b(r)^2 \cdot (dx^\mu \cdot dx^\mu) \quad ; \quad \mu=1..3 \qquad r^2 \equiv x^\mu \cdot x^\mu$$

From the above, the following form can be introduced as canonical gauge gravitational field variable. In order to avoid confusion, the variable $x_\mu = -x^\mu$ is never used.
First, let us find expression of $F^A{}_{BC}$.
From the assumption of "static", the operator is $\partial_0 \to 0$. (i.e. The time derivative term does not appear.)
Let T be expressed as $T^A{}_\mu = a_A(r) \cdot \delta^A{}_\mu$; $S^\lambda{}_A = a_A(r)^{-1} \cdot \delta^\lambda{}_A$ ($a_0 \equiv a$, $a_A \equiv b$ for A=1..3)
Then, $\partial_\lambda T^A{}_\mu = a_A' \cdot x^\lambda r^{-1} \cdot \delta^A{}_\mu \cdot (1-\delta^0{}_\lambda)$. Hereafter, introduce n as $n^\lambda \equiv x^\lambda r^{-1} (1-\delta^0{}_\lambda)$.
Since $F^A{}_{\lambda\mu} = a_A' \cdot (n^\lambda \delta^A{}_\mu - n^\mu \delta^A{}_\lambda)$, then $F^A{}_{BC} = a_A' \cdot (n^\lambda \delta^A{}_\mu - n^\mu \delta^A{}_\lambda) \cdot a_B^{-1} \delta^\lambda{}_B \cdot a_C^{-1} \delta^\mu{}_C$ is given

$$\therefore \quad F^A{}_{BC} = a_A' a_B^{-1} a_C^{-1} \cdot (n^B \delta^A{}_C - n^C \delta^A{}_B)$$

Let us write $\alpha \equiv \ln(a)$ hereafter. That is, $a = \exp(\alpha)$. where, α is considered to correspond to Newton potential. By this substitution, $F^A{}_{AC} = -(\alpha_0 + 2\alpha_1) \cdot a_1^{-1} n^C$ is obtained for example.
Using these notations to find the quadratic norm of F gives:

$$2L^C(1) \equiv (1/2!) \cdot F^A{}_{BC} F_A{}^{BC} = -(\alpha_0'^2 + 2\alpha_1'^2) a_1^{-2}$$

$$2L^C(2) \equiv F^A{}_{BC} F^B{}_A{}^C = -(\alpha_0'^2 + 2\alpha_1'^2) a_1^{-2} \quad \cdots \text{note: not independent to } L^C(1)\ !$$

$$2L^C(3) \equiv F^A{}_{AC} \cdot F^B{}_B{}^C = -(\alpha_0' + 2\alpha_1')^2 \cdot a_1^{-2}$$

$2L^C \equiv 2L^C(1) + \Lambda \cdot 2L^C(3) = -((1+\Lambda)\alpha_0'^2 + 4\Lambda\alpha_0'\alpha_1' + 2(1+2\Lambda)\alpha_1'^2) \cdot a_1^{-2}$

In the above, the factor (1/2!) comes from anti-symmetry with respect to the index (B,C), and the factor 2 for L^C is introduced since L^C is a quadratic form of F.

Here, in order to compare the canonical gauge gravity theory with A. Einstein's theory , let us calculate Lagrangian in Einstein's theory. First, the Levi-Civita connection coefficient shall be considered. From $g_{\mu\nu} \equiv T^A{}_\mu \eta_{AB} T^B{}_\nu$,,and according to $g_{\mu\nu} = a_\mu a_\nu \eta_{\mu\nu}$, $(T^A{}_\mu = a_A(r) \cdot \delta^A{}_\mu)$, the following is obtained from the direct calculation.

$\Gamma^\lambda{}_{\mu\nu} = -\eta_{\mu\nu}(a_\nu/a_\lambda)^2 \partial^\lambda \ln(a_\nu) + \delta^\lambda{}_\mu \partial_\nu \ln(a_\lambda) + \delta^\lambda{}_\nu \partial_\mu \ln(a_\lambda)$; note. $\partial_\mu \ln(a_\lambda) = a_\lambda'/a_\lambda \cdot x^\mu r^{-1}$

If you lower the index λ, replace the index, and write as follows, the symmetry is easy to see.

$\Gamma_{\lambda\mu\nu} = -\eta_{\mu\nu} a_\nu^2 \partial_\lambda \alpha_\nu + \eta_{\nu\lambda} a_\lambda^2 \partial_\mu \alpha_\lambda + \eta_{\lambda\mu} a_\mu^2 \partial_\nu \alpha_\mu = -\eta_{\mu\nu} a_\nu^2 n^\lambda \alpha_\nu' + \eta_{\nu\lambda} a_\lambda^2 n^\mu \alpha_\lambda' + \eta_{\lambda\mu} a_\mu^2 n^\nu \alpha_\mu'$

The normalization factor will be considered later, and Einstein's Lagrangian R', which is equivalence to the scalar curvature R, is calculated as follows.

$R' = R(1) + R'(2)$;

$R'(1) \equiv \eta^{\lambda\lambda}\eta^{\mu\mu}\eta^{\nu\nu} a_\lambda^{-2} a_\mu^{-2} a_\nu^{-2} \cdot \Gamma_{\lambda\mu\nu}\Gamma_{\mu\lambda\nu}$, $R'(2) \equiv -\eta^{\lambda\lambda} a_\lambda^{-2} \Gamma^\sigma{}_{\sigma\lambda} g^{\mu\nu} \Gamma_{\lambda\mu\nu}$ である。

$\Gamma^\sigma{}_{\sigma\lambda} = n^\lambda \Sigma_\sigma \alpha'_\sigma = n^\lambda(\alpha'_0 + 3\alpha'_1)$

$g^{\mu\nu}\Gamma_{\lambda\mu\nu} = -n^\lambda \Sigma_\nu \alpha'_\nu + 2n^\lambda \alpha_\lambda' = -n^\lambda(\alpha'_0 + \alpha'_1)$, note : $\Sigma_\lambda \alpha_\lambda = \ln(a_0 a_1 a_2 a_3) = \ln(\det(T))$

$R'(1) = (a_0'^2 + a_1'^2) a_1^{-2}$

$R'(2) = -(\alpha'_0 + 3\alpha'_1)(\alpha'_0 + \alpha'_1) a_1^{-2}$

$\therefore$ $R' = -(4\alpha'_0\alpha'_1 + 2\alpha_1'^2) \cdot a_1^{-2}$

As mentioned above, α_0 is a term corresponding to Newton potential, but it is surprisingly noted that R' does not contain the quadratic term of α_0.

Let us summarize the results of Lagrangian calculations for a spherically symmetric gravitational field. It becomes as follows.

Theory	**Lagrangian of spherically symmetric gravitational field**	**Remarks**
Unified Field Theory by canonical gauge	$2L^C = -((1+\Lambda)\alpha_0'^2 + 4\Lambda\alpha_0'\alpha_1' + 2(1+2\Lambda)\alpha_1'^2) \cdot a_1^{-2}$	$2L^C = -R'$ for $\Lambda = -1$
A. Einstein Theory	$R' = -(4\alpha'_0\alpha'_1 + 2\alpha_1'^2) \cdot a_1^{-2}$	none of α'_0 quadratic term

In the above, L^C is a scalar and R' is not. It is quite noteworthy that the two, which have completely different transformation property, match each other in terms of specific coordinates called isotropic coordinate system under the condition of static spherical symmetry.

Under the assumption of static spherical symmetry, the gravitational theory derived from unified field theory (by canonical gauge principle) is consistent with A. Einstein's theory when $\Lambda = -1$.

The way in which the observational facts require $\Lambda = -1$ can be seen, for example, in the analysis of the relationship between the bending of a ray and the distance. (ref = apdx-1a)

Verification material: For an analytical discussion of the relationship between ray bending and distance, please refer to apdx-1a.

Verification material: For an analytical discussion of Mercury's perihelion shift, please refer to apdx-1b.

1.5.2. Structural Analysis of static spherically symmetric gravitational field

Suppose that a gravitational field is only formed when there is a gravitational source, then $\nabla^2\Phi = 0$ and $\nabla^2\Phi = \delta(x)$ are completely different equations, and it is reasonable to interpret that the former physically leads to $\Phi =$ const. At this time, when dealing with a spherically symmetric gravitational field, we should assume that the mass is spherically symmetrically distributed at the origin. Otherwise, we are forced to think of spacetime itself as a holey object with a topological defect. This is also related to the interpretation of black hole. (This matter will be revisited in Section 2.3)

On the other hand, the source of gravity should originally be obtained from the Lagrangian of fields other than gravity using the variational principle, and the "mass distribution" as a source of gravity is a classical and convenient concept.
Therefore, to confirm the basics, I would like to consider the expression of mass distribution, the solution of spherically symmetric gravitational fields, and the quadrature possibility.
From the viewpoint of solving the equation, it is not necessary to obtain a final solution. This is because we already know the Schwarzschild solution for the spherically symmetric gravitational field in classical theory, and we have obtained the result that canonical gauge gravitational theory agrees with A.Einstein's theory with an interaction constant $\Lambda=-1$.

Preliminary Consideration

- **Expression of Mass distribution of gravity source**

In classical theory, $dA \equiv (1/2)m g_{\mu\nu} v^\mu v^\nu \cdot ds$ is considered as the action element of the mass point for the source of gravity. The expression of this in distribution form, that is, the relationship between mass form m and scalar mass density σ_0, is as follows.

$$mds = \sigma_0\sqrt{-g}\cdot d^4x \qquad \therefore \quad m = \sigma_0\sqrt{-g}\cdot v^0 d^3x$$

σ_0 : mass density at static coordinate system (scalar quantity)

When taking the variational regarding the gravitational field, from the law of conservation of mass, the restriction $\delta m = \delta(\sigma_0\sqrt{-g}\cdot v^0 d^3x) = 0$ is imposed.
This is a noteworthy point. That is, it is not possible to set $\delta\sigma_0 = 0$.
In the variational with $\delta\sigma_0 = 0$, a hydrostatic pressure term $\sigma_0 g^{\mu\nu}$ is added to the variational coefficient as an energy-momentum tensor. To eliminate this, it is possible to modify the action term as following dA'.

$$dA' \equiv (1/2)\sigma_0\sqrt{-g}\cdot v^0(g_{\mu\nu}v^\mu v^\nu - 1)d^3x$$

According to this modification, the action form of the mass point will be transformed to $(1/2)m(g_{\mu\nu}v^\mu v^\nu - 1)\cdot ds$. Even so, regarding mass point motion, there is no problem since there is no change in variationals.
If we consider the mass density $\sigma \equiv \sigma_0\sqrt{-g}$ at the adopted coordinates and ignore its variation of σ with respect to g, the above modification becomes unnecessary.
The trick comes from $\delta m = 0$, but it seems to be a little unnatural.

Considering an action element dA_m for the mass m distributed on a spherical shell, we get the following. *1

$$dA_m = (1/2)m(g_{\mu\nu}v^\mu v^\nu)\cdot(dt/v^0)\delta(r-r_0)drd^2\Omega/(4\pi)$$

The variational coefficient of the above by (g/2) should give the energy-momentum tensor density. Considering spherical mirror image transformation and introducing the coordinates of $u=r^{-1}$, the action elements of the mass distribution are as follows.

$$dA_m=(1/2)dm(g_{00}v^0v^0)(dt/v^0) \quad ; \quad dm\equiv m\delta(u-u_0)du d^2\Omega/(4\pi)$$

After obtaining the equation by variational, the stationary condition $g_{00}v^0v^0=1$ is imposed.

***1 :** The model of mass distribution on spherical shell is only to study the relationship between mass and field. There is no purpose to study the so-called interior solution here.

・Spherical mirror image transformation

The action for Lagrangian L^C is expressed in the form $\int L^C\cdot\sqrt{(-g)}d^4x$, and for spherical coordinates, the volume element includes an r^2dr term. Introducing the variable $r^{-1}=u$, we get

$$r^2dr=r^3dr/r=-u^{-3}du/u=-u^{-4}du$$

On the other hand, due to the relationship $\partial/\partial r=-u^2\partial/\partial r$, a situation exists where the density coefficient cancels in the quadratic form of the 1st derivative with respect to u, as follows.

$$h_{ij}(f)(\partial_r f_i\cdot\partial_r f_j)\cdot r^2dr=-h_{ij}(f)(\partial_u f_i\cdot\partial_u f_j)\cdot du$$

The transformation $r \to u=r^{-1}$ while preserving the angular variable is well known as the mirror image transformation with respect to the unit sphere.

・Equation of spherically symmetric gravitational field and its quadrature possibility

From the result in Section 1.5.1, the Lagrangian L^G of the spherically symmetric gravitational field in canonical gauge gravitational theory is given as follows.

$$L^G=-L^C/(8\pi G) \quad : \ *2$$

$$L^C=-((1+\Lambda)\alpha_0'^2+4\Lambda\alpha_0'\alpha_1'+2(1+2\Lambda)\alpha_1'^2)\cdot a_1^{-2} \quad ; \quad '\equiv\partial/\partial r$$

***2 :**

The calculation of interaction constant in section 1.3 for the Lagrangian of gravitational field in A.Einstein Theory gives , $L^G=R'/(16\pi G)$.

The results in section 1.5.1.shows : $\Lambda=-1 \to 2L^C=-R'$. $\therefore\ L^G=-L^C/(8\pi G)$

From the above Lagrangian, action form becomes as follows

$$L^G\cdot\sqrt{-g}\cdot d^4x=L^G\cdot a_0a_1^3dt\cdot r^2drd^2\Omega$$
$$=-(8\pi G)^{-1}\cdot((1+\Lambda)\alpha_0^{\bullet 2}+4\Lambda\alpha_0^{\bullet}\alpha_1^{\bullet}+2(1+2\Lambda)\alpha_1^{\bullet 2})\cdot\exp(\alpha_0+\alpha_1)\cdot dtdud^2\Omega \quad ; \quad {}^{\bullet}\equiv\partial/\partial u$$

On the other hand, the action form due to the mass of gravity source becomes as follows.

$$-(1/2)m\delta(u-u_0)(g_{00}v^0v^0)(dt/v^0)dud^2\Omega/(4\pi)$$

The negative sign comes from orientation of u due to $u\equiv r^{-1}$.

The total Lagrangian density $\mathcal{L}_u$ for the adopted coordinate system (t, u, θ, φ) is as follows :

$$\mathcal{L}_u=\mathcal{L}_u{}^C+\mathcal{L}_u{}^m$$

$\mathcal{L}_u{}^C \equiv -(8\pi G)^{-1}\,((1+\Lambda)\alpha_0{}^{\bullet 2}+4\Lambda\alpha_0{}^{\bullet}\alpha_1{}^{\bullet}+2(1+2\Lambda)\alpha_1{}^{\bullet 2})\cdot\exp(\alpha_0+\alpha_1)$

$\mathcal{L}_u{}^m \equiv -(1/2)mv^0\delta(u-u_0)\cdot a_0{}^2/(4\pi)$ …Note that u is included explicitly.

Since $\mathcal{L}u$ is formally isomorphic to the Lagrangian of mass point motion when the variable u is viewed as time, we will shift to Hamiltonian formalism. Since there is a term exp(α0+α1)=a0a1, we will convert the variables and set it as $\beta_0=\alpha_0$, $\beta_1\equiv\alpha_0+\alpha_1$ This results :

$\mathcal{L}_u{}^C \equiv -(8\pi G)^{-1}\,((3+\Lambda)\beta_0{}^{\bullet 2}-4(1+\Lambda)\beta_0{}^{\bullet}\beta_1{}^{\bullet}+2(1+2\Lambda)\beta_1{}^{\bullet 2})\cdot\exp(\beta_1)$

$\mathcal{L}_u{}^m \equiv -(1/2)mv^0\delta(u-u_0)\cdot\exp(2\beta_0)/(4\pi)$ ⋯ u is included explicitly.

Canonical momentum p that is conjugate to β is obtained as follows. ; rel. apdx1

$-(4\pi G)p_0 \equiv ((3+\Lambda)\beta_0{}^{\bullet}-2(1+\Lambda)\beta_1{}^{\bullet})\exp(\beta_1)$

$-(4\pi G)p_1 \equiv (-2(1+\Lambda)\beta_0{}^{\bullet}+2(1+2\Lambda)\beta_1{}^{\bullet})\exp(\beta_1)$

If expressing the above in matrix form as $-(4\pi G)p=h\cdot\exp(\beta_1)\cdot\beta^{\bullet}$, then since $\det(h)=2(1+3\Lambda)$, it can be seen that the theory is singular when $\Lambda=-1/3$.

In this case, $p_0+2p_1=0$ for $\Lambda=-1/3$.

h=

(3+Λ)	−2(1+Λ)
−2(1+Λ)	2(1+2Λ)

$h^{-1}=(2(1+3\Lambda))^{-1}$

2(1+2Λ)	2(1+Λ)
2(1+Λ)	(3+Λ)

Since Hamiltonian $H=\mathcal{L}_u{}^C-\mathcal{L}_u{}^m$ contains variable (u) explicitly, it is not constant over the entire range of u, but in each section of $0\leqq u<u_0$, $u>u_0$, H is a constant, that is, H is piecewise constant. (piecewise conserved.).

The canonical equation is as follows.

$d\beta/du=-(4\pi G)\cdot\exp(-\beta_1)\cdot h^{-1}p$; $\beta^{\bullet}\equiv d\beta/du$

$\delta\beta_0$: $dp_0/du=-mv^0\delta(u-u_0)\cdot\exp(2\beta_0)/(4\pi)$; $v^0a_0=1$, $a_0=\exp(\beta_0)$

$\delta\beta_1$: $dp_1/du=\mathcal{L}_u{}^C$; piecewise constant.

Inside the spherical shell concerning r, the range $u>u_0$, no gravity exist , so $\beta\equiv$const

∴ $p=0$, $L_u{}^C=0$. For $u\to 0$, i.e. at infinity concerning $r\to\infty$ ($u\to 0$) then $\beta\to 0$.

It is also found that $\Lambda=-1/3$ (i.e. $p_0+2p_1=0$) is impossible.

Obviously, the canonical equations are quadrature-possible.

First, by integrating p with $u=\infty$ as starting point, p can be obtained as a piecewise constant.

$p_0=\theta(u_0-u)\cdot m/(4\pi)\cdot\exp(\beta_0(u_0))$, $p_1=\theta(u_0-u)\cdot(u-u_0)L_u{}^C$

Subsequently, β_1 is obtained first from $d\beta/du=-(4\pi G)h^{-1}\cdot\exp(-\beta_1)p$ and then β_0 is derived.

It is already known that canonical gauge gravity theory agrees with A.Einstein's theory at Λ=-1 assuming spherical symmetry.

Therefore, when Λ=-1 the solution of the equation is Schwarzschild spacetime.

Other Λs are of theoretical interest, but they seem to be unrealistic. Schwarzschild spacetime will be argued from a quantum theoretical perspective in Section 2.3.

・Themes to be considered regarding spherically symmetric gravitational field.

To find the quantum solution for Schwarzschild spacetime will be expected.

Gravitational field is nonlinear and self-interacting. The classical field is the expected value of the field operator, and since the expected value of the product does not match the product of the expected values in general, the solution should be corrected by quantum mechanical variance effects.

Steady state problems in the sense of quantum theory must be considered as Hamiltonian eigenvalue problems, rather than applying variationals to the Lagrangian assuming a stationary solution as in classical theory.
However, it may beyond eigenstate problem of system energy and becomes necessary to the interpretation with the sense of statistical mechanics .

Regarding gravitational collapse and the formation of black holes, it may be necessary to discuss through stability of the static solutions even from a classical perspective.
Classical theory predicts that it will shrink up to the Schwarzschild radius. However, complete contraction is expected to take an infinite amount of time (as observed from the outside). These considerations will be discussed in Section 2.3.

1.5.3. Conclusion from observational facts (summary)

Concerning canonical gauge gravitational theory (formally it should probably be called as gravitational theory based on unified field theory by canonical principle), following can be concluded from the observational facts .

(1). $4\pi G=1$ is appropriate choice for nondimensionalization of Plank unit system.
(2). Concerning coupling constant ratio Λ that appears in gravitational Lagrangian L^C , $\Lambda=-1$ is consistent with observations.
The normalization constant is $1/16\pi G$ for R' , -1/2 for L^C .

According to (2) above, gravitational Lagrangian L^G, in canonical gauge gravitational theory, is as follows in dimensionless notation ($4\pi G=1$).

$$L^G=-L^C/2=(1/8)(F_{ABC}F^{ABC}/2!-F^A{}_{AC}F^B{}_B{}^C) \quad ; \quad \text{where,} \quad F^A{}_{BC}\equiv(\partial_\mu T^A{}_\nu-\partial_\nu T^A{}_\mu)S^\mu{}_B S^\nu{}_C$$

S is the inverse matrix of T. Index A, B, C… indicates that component is Lorentz Frame representation. The indices are arbitrarily up/down with Lorentz metric η.

In the canonical gauge gravitational theory, that should formally be called "gravitational theory of unified field theory by canonical gauge principle", the theoretically possible form of Lagrangian is as follows.

$$L^G=-L^C/2=(1/8)\cdot((1-\Lambda_0)F_{ABC}F^{ABC}/2!+\Lambda_0\cdot F^A{}_{BC}F^B{}_A{}^C+\Lambda\cdot F^A{}_{AC}F^B{}_B{}^C+\Lambda_C)$$

・expressed in dimensionless notation ($4\pi G=1$)
・$F^A{}_{BC}$ is field strength. $F^A{}_{BC}\equiv(\partial_\mu T^A{}_\nu-\partial_\nu T^A{}_\mu)S^\mu{}_B S^\nu{}_C$
・3 interaction constants (Λ, Λ_0,Λ_C) can be allowed to include by invariant theory

Λ_C corresponds to so-called cosmological term and is required to be a sufficiently small value, but even taking account of the fact of the accelerated expansion of universe, or considering its existence reason, it seems that Λc as the interaction constant, should be zero ($\Lambda c=0$).
(That is, as a physical law, it seems that Λc should not exist).
The effect like Λ_C can be interpreted as being formed by the Lagrangian expectation value in a field other than gravity.

Λ_0 is the weight for the invariance $F^A{}_{BC}F^B{}_A{}^C$, given by theoretically possible contraction of index. In case of spherically symmetric gravitational field, it cannot be determined by observation since $F_{ABC}F^{ABC}/2! = F^A{}_{BC}F^B{}_A{}^C$.
Apart from invariant theory, it seems that $\Lambda_0 = 0$ when taking account of the character of Lagrangian L^C, as a quadratic norm of canonical curvature form.

The denominator 2! of the term $F_{ABC}F^{ABC}/2!$ comes from antisymmetry of F_{ABC} with respect to indexes B and C. If $F^A{}_{BC}$ is viewed as $F^A{}_{BC} = F(_C)$: matrix, it can be written as follows.

$$\Sigma\ F_{ABC}F^{ABC}/2! = \Sigma\,\eta^{CD}\cdot \mathrm{Tr}(F(_C)^*F(_D))/2! \quad ; \quad \Sigma\,F^A{}_{BC}F^B{}_A{}^C = \Sigma\,\eta^{CD}\cdot \mathrm{Tr}(F(_C)F(_D))$$

Each term is represented by a linear combination of trace of the square of symmetric component and the antisymmetric component of matrix $F(_C)$ respectively, but there no reason is found to consider a linear combination of the above two terms ($F_{ABC}F^{ABC}$ and $F^A{}_{BC}F^B{}_A{}^C$).

It seems that the normalization of field variables related to interactions should be decided in relation to canonical commutation relations.

[Supplement] : Λ+1＝0. － from a theoretical viewpoint

The ambiguity of the definition concerning quadratic norm of tensor is mentioned in part-I, but the above result $\Lambda+1=0$ can be understood as a result comes from quadratic norm definition.
Consider the following tensor f expressed in reverse ordering.

$$f \equiv \Sigma_{(B<C)}\,F^A{}_{BC}\ e^{*C}e^{*B}e_A = (1/2!)\,\Sigma\,F^A{}_{BC}\ e^{*C}e^{*B}e_A$$

Because of the anti-commutativity between tensor basis or commutation relation with dual basis, the normal ordered representation of f becomes as follows.

$$2f = \Sigma\,F^A{}_{AC}\ e^{*C} - \Sigma\,F^A{}_{BC}\ e^{*C}e_A e^{*B} = \Sigma\,F^A{}_{AC}e^{*C} - \Sigma\,F^A{}_{BA}e^{*B} + \Sigma\,F^A{}_{BC}\cdot e_A e^{*C}e^{*B}$$

$$2f = 2\,\Sigma\,F^A{}_{AC}e^{*C} - \Sigma\,F^A{}_{BC}\cdot e_A e^{*B}e^{*C} \quad \therefore\ f = \Sigma\,F^A{}_{AC}e^{*C} - \Sigma_{(B<C)}F^A{}_{BC}\cdot e_A e^{*B}e^{*C}$$

If adopting the norm definition based on normal ordered representation, and assuming the metric of dual basis is inversely sign of metric of base space such as $\langle e^{*C}|e^{*C'}\rangle = -\eta^{CC'}$, the following is obtained. ($\Lambda=-1$)

$$\langle f|f\rangle = -\,\Sigma\,F^A{}_{AC}\cdot F^B{}_{BC'}\eta^{CC'} + \Sigma_{(B<C)}F^A{}_{BC}\cdot F^{A'}{}_{B'C'}\cdot \eta_{AA'}\eta^{BB'}\eta^{CC'}$$

Observations of ray bending only conclude that $\Lambda \fallingdotseq -1$, but theoretical validity suggests $\Lambda = -1$. (under the above assumption)

[Additional note] :

Problems of scaling and interaction constants from the perspective of unified field theory.

If the normalization factor of the Lagrangian (or, action of field) is a number with dimensions, it can be made dimensionless or unitized by setting the unit. Regarding Einstein's theory of gravity, discussed in Section 1.3, $L_G \equiv R'/(16\pi G)$ has been derived as the normalized Lagrangian of gravitational field. By combining the Lagrangian of the mass point $L_m \equiv (1/2)\sigma g_{\mu\nu}v^\mu v^\nu$, we confirmed that the Lagrangian of the system can be given by a constant multiple of $(L_G + L_m)$.
(However, the mass point is interpreted as a concentrated distribution of mass, $m ds = \sigma\cdot(g)^{1/2}d^4x$, and

$\delta m = 0$ for variational because of mass conservation law.)

In unified field theory, all fields are obtained at once, so scaling adjustments of individual component fields lose meaning, and there is a certain relationship between interaction constants.
In this sense, it is better to write the gravity equation as $-G^{\mu\nu}/\kappa_E + T^{\mu\nu} = 0$ rather than as $G^{\mu\nu} = \kappa_E \cdot T^{\mu\nu}$.
Regarding the elementary charge e, within the natural unit system, $e^2/4\pi = 1/137$ (dimensionless).
In addition, if we make the gravitational interaction constant dimensionless ($4\pi G=1$), all units become dimensionless. All that remains is the scaling adjustment by a constant multiple of the total.

From the viewpoint of a unified field theory, it is reasonable to adjust the coefficients of the quadratic term in the Lagrangian linear approximation concerning the field. Additionally to saye, it is preferable to scale that gives the symmetry between the canonical conjugate momentum operator and the field (amplitude) operator.

At present, the coupling constant of elementary charge of electromagnetic interaction is not 1, but this coupling constant can be unitized by dimensionless scaling. Since the Lagrangian has an indeterminacy that is multiplied by a constant, if we convert it to $\varphi \to e^{-1}\varphi$, $L \to e^2 L$, the coefficient for the 2nd-order term of the infinitesimal oscillation remains unchanged. (where, φ = field operator)
While the interaction constant g of 3rd order coupling coefficient will be converted as g→e-1g, for example.

In the case of the Dirac equation, the conventional Lagrangian does not have a quadratic differential term, so the situation is a little complicated. According to the unified field theory, Fermion's Lagrangian is obtained from the 2nd-order differential by approximate factorization. In $\chi^*(\mathcal{D}-\alpha)(\mathcal{D}-\beta)\chi$, a scale factor of $(\mathcal{D}-\alpha) \fallingdotseq (\beta-\alpha)$ appears near the vibration of $(\mathcal{D}-\beta)\chi = 0$.
If α and β are interaction fields, their state expectation becomes the scale factor. Therefore, the normalization factor for the Lagrangian (or should we say for the action) is dependent on the surrounding state.

1.6. Expanding universe and quantum Friedmann universe in canonical gauge unified field

Now let us consider the expanding universe ,using Lagrangian of the canonical gauge in unified field theory, while keeping in mind the comparison with A. Einstein's theory.
As already seen, under the assumption of static spherical symmetry, the Lagrangian of the gravitational field coincide, between the canonical gauge unified field gravitational theory and A.Einstein's gravitational theory.
(To be exact, we assume that there is no spin field and that the coupling constant ratio in canonical gauge gravity is $\Lambda = -1$).

Regarding the accelerated expanding universe, there might be not so much enough observational materials to verify it , but it is interesting to consider this matter using the canonical gauge unified field theory of gravity.
However, even in a dynamic universe, if we assume uniform space isotropy, the canonical gauge unified field gravity theory can be derived as an equation corresponding to the Friedman equation.

In the canonical gauge unified field, the scale factor is a canonical variable, so it is naturally interpreted as a field operator. Therefore the "Quantum Friedmann universe" will naturally be considered from the perspective of quantum gravity.
Regarding the expanding universe, we will discuss the resolution of the singularity at the Big Bang.
Expanding acceleration of the universe will be mentioned in Chapter 3 as a speculation for cosmology.

1.6.1.Assumption on cosmological time and symmetry

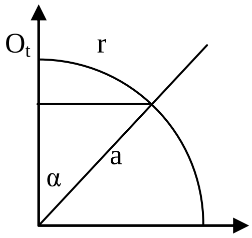
When considering dynamic space-time from a cosmological viewpoint, the following assumptions are made.

ⅰ.Existence of global cosmological time (t)
ⅱ.Isotropic uniformity of 3-dimensional equal-time cross section

From viewpoint of field theory, the above are requirements for symmetry of excited gravitational field. The concept of time is set by observer and is more fundamental than the classical concept of proper time.
Let line element in 3-dimensional equal-time space(cross-section) be denoted by $d\ell$
The following holds for the space-time line element ds.

$$ds^2 = -d\ell^2 \quad \text{for} \quad dt = 0$$

$d\ell$ can be set as follows with the help of the figure.



$d\ell^2 = dr^2 + \mathbf{a}^2 \cdot \sin^2(\alpha) d\Omega^2$　;

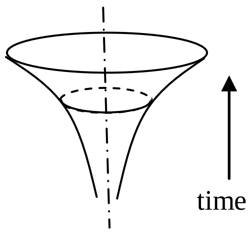
$d\Omega \equiv$ line element by angular coordinate component

$r = \mathbf{a} \cdot \alpha$　(set by assuming 3-dimensional sphere S^3)

a can be interpreted as "the radius of the universe".

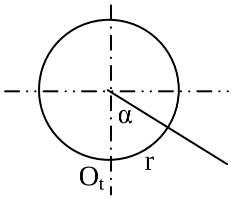
The above keeps sense also with the conversion $a \to ia$,$\alpha \to -i\alpha$ that makes possible to transit interpretation from 3-dimensional sphere to 3-dimensional hyperboloid.

Consider constructing 4-dimensional space-time by stacking 3-dimensional spaces in the direction of the time axis.



When expressing 3-dimensional space as a circumference (ring) in the figure, it seem possible for the rings to rotate and stack while ensuring isotropic uniformity in equal-time cross section, but not for 3-dimensional space.

The rotation of the ring would correspond to SO (3) transformation in 4-dimensional space-time. This breaks the uniform isotropic relation concerning inclination between the time axis and space.



It is natural to adopt the angle α with respect to the center of cone as a the coordinates of 3-dimensional space.

$d\ell^2 = a^2(d\alpha^2 + \sin^2(\alpha) d\Omega^2)$　;　$ds^2 = -d\ell^2 \ / dt^2 = 0$

$ds^2 = dt^2 - d\ell^2$　; time and space are orthogonal

The coefficient of dt becomes 1 by the conversion of the time variable. (cone generatrix length)

The 3-dimensional equal-time space is expressed by a cross section of an axial-symmetric cone cut vertically to the center axis.

Static point in space is represented as a generatrix of cone.　Space is orthogonal to the time axis.

It can also be seen that it is natural to adopt the central angle α with respect to the cone center axis as a coordinate of the equal-time 3-dimensional space.

In the canonical gauge unified field theory, the Lorentz frame is constructed as a 1st-degree polynomial of canonical momentum operator.

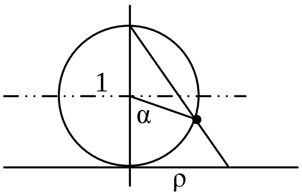
Lorentz frame corresponds to tetrad in classical theory. Therefore, we introduce gravitational variable by setting the isotropic coordinate system from the above. Without spin fields, we can assume that the Lorentz rotational degrees of freedom are not excited.

1.6.2.Introduction of isotropic coordinates in 3D space

From the formula of $d\ell^2$, put it as follows.

$d\alpha = \kappa d\rho$,　$\sin(\alpha) d\Omega = \kappa\rho d\Omega$　;　$(d\Omega^2 \equiv d\theta^2 + \sin^2(\theta) d\varphi^2)$

From the above, $2\tan(\alpha/2) = \rho$ can be obtained by $d\alpha/\sin(\alpha) = d\rho/\rho$.　Here, integration constant was chosen so that the transformation would be a spherical projection.



$ds^2 = dt^2 - a^2(d\alpha^2 + \sin^2(\alpha)d\Omega^2) = dt^2 - (a\kappa)^2(d\rho^2 + \rho^2 d\Omega^2)$
By considering tetrad (frame: E_A) ,following can be obtained,
$x^3 \equiv \rho \cdot \cos(\theta)$, $x^2 \equiv \rho \cdot \sin(\theta)\sin(\varphi)$, $x^1 \equiv \rho \cdot \sin(\theta)\cos(\varphi)$
$\partial_\mu = \mathbf{E}_A \cdot T^A{}_\mu$; $\partial_0 = \mathbf{E}_0$, $\partial_j = \mathbf{E}_A \cdot a\kappa\delta^A{}_j$ (j=1..3)

As the variables $T^A{}_\mu$ of gravitational field related to Lorentz frame are given as follows.

$T^A{}_\nu dx^\nu = \delta^A{}_0 dt + (1-\delta^A{}_0)\delta^A{}_\nu \cdot (a\kappa)dx^\nu$;

$\therefore$ $T = \mathrm{diag}(1, a\kappa, a\kappa, a\kappa)$, $S = T^{-1} = \mathrm{diag}(1, (a\kappa)^{-1}, (a\kappa)^{-1}, (a\kappa)^{-1})$

$d(T^A{}_\nu dx^\nu) = (1/2!) \cdot F^A{}_{\mu\nu} dx^{\mu\nu}$, $F^A{}_{\mu\nu} = (1-\delta^A{}_0)(\delta^A{}_\nu \cdot \partial_\mu(a\kappa) - \delta^A{}_\mu \cdot \partial_\nu(a\kappa))$, esp. $F^0{}_{\mu\nu} = 0$

i.e. $F^A{}_{0j} = (1-\delta^A{}_0)\delta^A{}_j \cdot \partial_0 a \cdot \kappa$,

$F^A{}_{jk} = (1-\delta^A{}_0)a\kappa'(\delta^A{}_k \cdot n_j - \delta^A{}_j \cdot n_k)$; $\kappa' \equiv \partial\kappa/\partial\rho$, $n_j = x^j/\rho$, j,k=1..3

1.6.3.Calculation of Lagrangian

The Lagrangian of the gravitational field in canonical gauge theory is calculated as follows with coupling constant $\Lambda = -1$.

Lagrangian $L^C = -(1/4) \cdot (L(1) - L(2))$;

$L(1) \equiv (1/2!) \cdot \Sigma F({}_{ABC})F({}^{ABC})$, $L(2) \equiv \Sigma F({}^A{}_{AC})F({}^B{}_B{}^C)$; note $F({}^A{}_{BC}) = F^A{}_{\mu\nu} S^\mu{}_B S^\nu{}_C$

Please note that in the above expression, indexes A, B and C relate to the Lorentz frame.
The expression above shows the indexes relate to which frame, that is, Lorentz frame or coordinate frame. Up and down of the index is arbitrarily done by using Lorentz metric.
The following results can be obtained by calculations using the representation of the coordinate frame index.

$L(1) = (1/2!)\Sigma \eta_{AA} \cdot g^{\mu\mu} g^{\nu\nu} F^A{}_{\mu\nu} F^A{}_{\mu\nu} = \Sigma (F^A{}_{0j})^2 (a\kappa)^{-2} - \Sigma (F^A{}_{jk})^2 (a\kappa)^{-4}$; A, j, k=1..3, $j < k$

$\therefore$ $L(1) = 3(\partial_0 a \cdot \kappa)^2 (a\kappa)^{-2} - 2(a\kappa')^2 (a\kappa)^{-4}$

$L(2) = (\Sigma F^A{}_{\mu\nu} S^\mu{}_A)^2 g^{\nu\nu} = (\Sigma F^A{}_{\mu 0}\delta^\mu{}_A (a\kappa)^{-1})^2 + (\Sigma F^A{}_{jk}\delta^j{}_A (a\kappa)^{-1})^2 (a\kappa)^{-2}$

$\therefore$ $L(2) = (-3\partial_0 a \cdot \kappa(a\kappa)^{-1})^2 - (\Sigma a\kappa'(\delta^j{}_k - 1)n_k (a\kappa)^{-1})^2 (a\kappa)^{-2} = (-3\partial_0 a \cdot \kappa(a\kappa)^{-1})^2 - 4(a\kappa')^2 (a\kappa)^{-4}$

scalar Lagrangian $4L^C = L(2) - L(1) = 6(\partial_0 a \cdot \kappa)^2 (a\kappa)^{-2} - 2(a\kappa')^2 (a\kappa)^{-4}$, $\det(T) = (a\kappa)^3$
Lagrangian density $\mathcal{L} \equiv L^C \cdot \det(T) = (3/2)a(\partial_0 a)^2 \cdot \kappa^3 - (1/2)a\kappa'^2\kappa^{-1}$
The sphere / hyperboloid transformation is given by $a \to ia$, $\alpha \to -i\alpha$, $\rho \to -i\rho$, $x \to -ix$.

Lagrangian is obtained from the spatial integral of the Lagrangian density by measure d^3x.
From the definition, $\kappa = d\alpha/d\rho$, $2\tan(\alpha/2) = \rho$, so $\kappa^{-1} = d\rho/d\alpha = 1/\cos^2(\alpha/2) = 1 + (\rho/2)^2$ holds.
For the spatial integration of Lagrangian density, the integration can be executed by setting $d^3x \to 4\pi\rho^2 d\rho$, and as the integrand, the following is available. However, if the universe is a hyperbolic space, it will be converted as $\alpha \to i\alpha$, and the meaning of spatial integration becomes unclear.

$\kappa^3\rho^2 d\rho = \kappa^2\rho^2 d\alpha = 4\cos^2(\alpha/2)\sin^2(\alpha/2)d\alpha = \sin^2(\alpha)d\alpha$

$\kappa'^2\kappa^{-1}\rho^2 d\rho = (d\kappa/d\alpha \cdot d\alpha/d\rho)^2\kappa^{-2}\rho^2 d\alpha = (d\kappa/d\alpha)^2\rho^2 d\alpha = 4\sin^4(\alpha/2)d\alpha$

By the way, assuming that the 3-dimensional space is S^3, the spatial integral is given by the integral for $\alpha\in[0,\pi]$, and the following is obtained.

$L_a = \int \mathcal{L}\, d^3x = 3\pi^2(a(\partial_0 a)^2 - a)$ canonical conjugate variable of a : $b \equiv 6\pi^2 a\cdot(\partial_0 a)$

$H_a = b(\partial_0 a) - L_a = 3\pi^2(a(\partial_0 a)^2 + a) = 3\pi^2(b'a^{-1}b' + a)$; $[a, b'] = i\cdot(6\pi^2)^{-1}$

By introducing the scale transformation $a = \lambda^{-1}a'$, $b = \lambda b'$, preserving the canonical commutation relation, and let us consider the simplification of the Hamiltonian.

$H_a = (12\pi^2)^{-1}(\lambda^3 b'a'^{-1}b' + 36\pi^4\lambda^{-1}a') = (12\pi^2)^{-1}\lambda^3(b'a'^{-1}b' + 36\pi^4\lambda^{-4}a')$

If we choose $36\pi^4\lambda^{-4} = 1$ i.e. $\lambda = (6\pi^2)^{1/2}$, we get the following.

$H_a = (3/2)^{1/2}\cdot\pi\cdot(b'a'^{-1}b' + a')$;

$a' = a\cdot(6\pi^2)^{1/2}$, $b' = b\cdot(6\pi^2)^{-1/2} = a\cdot(\partial_0 a)\cdot(6\pi^2)^{1/2} = a'\cdot(\partial_0 a')\cdot(6\pi^2)^{-1/2}$

1.6.4.Expanding universe and singularity problem

From the viewpoint of canonical gauge unified field theory, it should be confirmed that the classical interpretation of the theory leads to the Friedmann expansion universe in A. Einstein's theory.
(The analysis of the classical solution itself is not the main subject here.)

Since Hamiltonian is a constant of motion, we obtain $a(\partial_0 a)^2 + a = \text{const}$, and Friedmann equation without the cosmological term in general relativity textbooks is $(\partial_0 a)^2 = ma^{-1} - k$.
Therefore, it can be seen that they agree at least for $k = +1$ (finite universe). ***A**

For classical solution, $a = 0$ at $t = 0$ is interpreted as a big bang.
From the viewpoint of quantum theory $a = 0$ is unreasonable. This is because Hamiltonian has a term of $ba^{-1}b$, and due to the uncertainty principle, a and b cannot be 0 at the same time. ***B**

***A** : Note that $a(\partial_0 a)^2 - a = \text{const}'$ (i.e. $k = -1$) is obtained by the spherical/hyperboloid transformation : $a \to ia$.
By the transformation $a \to ia$, $L_a \to -i\cdot L_a$, the following transformation is induced.
$L_a = 3\pi^2(a(\partial_0 a)^2 - a) \to -iL_a = -i\cdot 3\pi^2(a(\partial_0 a)^2 + a)$,
To transfer to the Hamiltonian formalism shall be applied to the transformed Lagrangian above.
$b = 6\pi^2 a\cdot(\partial_0 a)$, $[a, b] = i$, $H_a = 3\pi^2(b_1 a^{-1} b_1 - a)$ i.e. $k = -1$

***B** : For classical solution, $a \to +0$ should be interpreted as convergence to 0 including variance.
By the way, if (a) also takes a negative eigenvalue, it is possible that the energy is finite and that $a \to +0$ in expectation value of (a).
On the other hand, if (a) can be defined as a positive-definite operator and $a^{1/2}$ can be defined as a self-adjoint operator, $a \to +0$ is impossible.

In quantum theory, scale factor (a) is a field operator, and when considering the state tensor f in Schrödinger representation with (a)-diagonal representation, the following interpretation can be made from a quantum mechanical viewpoint.

Note that f(a) becomes so-called probability amplitude.

In classical theory, $a \to +0$ means $\int da \cdot a|f(a)|^2 \to 0$, $\int da \cdot a^2|f(a)|^2 \to 0$

Energy $\langle f|H_a|f\rangle$ is finite only if $\langle f|ba^{-1}b|f\rangle = \int da \cdot a^{-1}|\partial f(a)|^2$ is finite

Such a function f (a) can exist if $a < 0$ is allowed in domain. For example, something like a square root of a Gaussian distribution can obviously realizes $\langle f|a|f\rangle = 0$ from its parity. However, even in classical theory, (a) is considered as $a \geqq 0$ according to the definition of scale factor, so it is possible to consider (a) as field operator that $a = a^{1/2} \cdot a^{1/2}$; $a^{1/2} = a^{1/2*}$

Therfore,considering $ba^{-1}b = (ba^{-1/2})(a^{-1/2}b) \equiv vv^*$; $v^* \equiv a^{-1/2}b$, we can obtain following from commutation relation.

$v^* = (-i/2)a^{-3/2} + v$, $v = (+i/2)a^{-3/2} + v^*$

$v^*v = (1/4)a^{-3} + ba^{-1}b - i/2(a^{-3/2}v^* - va^{-3/2}) = (1/4)a^{-3} + ba^{-1}b - i/2[a^{-2},b]$

$v^*v = (1/4)a^{-3} + ba^{-1}b - a^{-3} = ba^{-1}b - (3/4)a^{-3}$

$\langle f|v^*v|f\rangle = \langle f|ba^{-1}b|f\rangle - (3/4)\langle f|a^{-3}|f\rangle > 0$; $\langle f|a^{-3}|f\rangle \to +\infty$ as $\langle f|a|f\rangle \to +0$

Furthermore, for $\langle f|a^{-3}|f\rangle$, $\langle f|a^{-3}|f\rangle \geqq \langle f|a|f\rangle^{-3}$ can be concluded. ***1**

If the kinetic energy term <f | ba-1b | f> is finite, then <f | a | f> cannot be zero.

***1** : From the Cauchy-Schwarz inequality $\langle f|a^{\mu+\nu}|f\rangle^2 \leqq \langle f|a^{2\mu}|f\rangle\langle f|a^{2\nu}|f\rangle$ holds

For $\mu = -3/2$, $\nu = 1/2$ then $\langle f|a^{-1}|f\rangle^2 \leqq \langle f|a^{-3}|f\rangle\langle f|a|f\rangle$

For $\mu = -1/2$, $\nu = 1/2$ then $\langle f|f\rangle^2 = 1 \leqq \langle f|a^{-1}|f\rangle\langle f|a|f\rangle$

$\therefore$ $1 \leqq \langle f|a^{-3}|f\rangle\langle f|a|f\rangle^3$

Ref : The speculations on Big Bang and Expanding Universe are available in section 3.2-3.3.

3.2.Big Bang 3.3.Expanding Universe

1.6.5.Quantum Friedmann Universe

(1).Summary of noteworthy points

How expanding universe should be interpreted in terms of quantum theory is a problem that should be considered including observation facts.

It might goes beyond the theme of this book, which is to compare the canonical gauge gravitational theory with the existing theory and verify its validity. Still, some remarks would like to be mentioned.

-The scale factor (a) is a variable that appears when a uniform and isotropic symmetry is imposed for the excited mode of state space of the universe, and its basic character is a field operator.

-When the space-time metric is adopted as a gravitational field variable, the field operator contains a constant component. In general, Boson field operators contain constant components.

It is natural to interpret that the constant component is vacuum expectation value, while it may come from the definition of variables.

In the case of gravitational field, it can be thought that the original variables is $V \equiv \ln(T)$.

Where, $g = T\eta T^*$ and when $T = 1$ then $V = 0$. (η = Lorentz metric)

-The Schrödinger equation for expanding universe imposed uniform and isotropic symmetry will be $+i\partial_t f = H_a f$ by using Hamiltonian H_a (described in sec.1.6.4).

When we write the quantum state of the universe as $|\Phi_U\rangle$, this is a superposition of tensor multiple states formed by many particles. The wave function $f \equiv f(a,t)$ means the expansion coefficient when $|\Phi_U\rangle$ is expanded in the eigenstate of (a):

$$|\Phi_U\rangle = \Sigma_a |a\rangle f(a,t) \quad \text{or} \quad \langle a|\Phi_U\rangle = f(a,t)$$

(In general, the state $|a\rangle$ is not a state vector of the basic space but a linear subspace, so it should be considered that the inner product of the linear subspace is defined.)

Since H_a does not include time variable explicitly, the solution is expanded by energy eigenstates.
It seems that an oscillating universe is possible for energy eigenstates. Moreover, multiple modes superposed make it possible to generate a swell in the expectation value of (a).

・While the above Schrödinger holds true, the expected value of a : $\underline{a} \equiv \langle \Phi|a|\Phi\rangle$ satisfies $d\underline{a}/dt \equiv \langle f|\partial H_a/\partial b|f\rangle$ because of the general relationship between classical mechanics and quantum mechanics. ***A** ((b) is the canonical conjugate momentum of (a))

・For the universe to be in an unsteady state, it is necessary that its energy is uncertain.
Even if it is uncertain, in case it is a superposition of a finite number of energy eigenstates, the expansion will be oscillatory, and there may swell . That is, cyclic stationarity with respect to scale factor (a) is maintained. Only when it is an infinite sum, non-periodic solution can be obtained.
The actual state of our universe depends on the initial conditions.

From the observational facts, our universe is not stationary.
In the 1st-order ordinary differential equation with translation invariance, the degree of freedom of the initial condition is absorbed by the time translation, so the motion is 1 type substantially.
On the other hand, in the case of quantum theory, the motion is diverse depending on initial conditions.
Should we consider the contingency that all modes are in phase at the big bang?
When taking into account that energy is finite, isn't it reasonable to consider Boltzmann distribution ?

***A :**
In quantum theory, the following canonical equation (Heisenberg representation) holds for operators.

$$dx/dt = [iH,x] = \partial H/\partial p \quad , dp/dt = [iH, p] = -\partial H/\partial x ,$$

Or in general $d\xi/dt = [iH,\xi] = \partial H/\partial p \cdot \partial\xi/\partial x - \partial H/\partial x \text{-} \partial\xi/\partial p$

If we take the expected values of both sides $\langle f0|\cdots|f0\rangle$, we obtain the equation of motion of the expected values.
Since Heisenberg representation and Schrödinger representation are corresponded by canonical transformation, the representation also holds true in the Schrödinger representation.
The major difference from the classical theory is that the classical theory replaces the expected value

of the operator's function with the function of the operator's expectation value.
Then, as a result, it follows that the lowest-order correction for this is the quantum mechanical covariance regarding the surrounding environment.

(2).Consideration of quantum Friedmann equation

Whether the expansion motion of the our universe is oscillatory or transient, that is a quite interesting question. In classical Friedmann equation, the result depends on the cosmic curvature sign $k=0, \pm 1$.
Let's adjust the scaling and write $H_a = -\partial_a a^{-1} \partial_a + ka + N(a)$ as an expanded expression of the eigenstate of (a). The equation of quantum Friedmann universe becomes $H_a f_1 = +i\partial_t f_1$. Here, the wave function (probability amplitude) is expressed as f_1.
Introducing $a^{-1}\partial_a f_1 \equiv f_2$ and the following simultaneous equation is obtained.

$$+i\partial_t f_1 = -\partial_a f_2 + (ka + N(a)) f_1 \; ; \quad \partial_a f_1 = a \cdot f_2$$

set $+i\partial_t f_1 = \omega f_1$ by using variable separation method

$$\begin{pmatrix} \partial_a f_1 \\ \partial_a f_2 \end{pmatrix} = \begin{pmatrix} 0 & a \\ ka + N(a) - \omega & 0 \end{pmatrix} \begin{pmatrix} f_1 \\ f_2 \end{pmatrix}$$

First, note that f is normal at $a=0$.
Next, considering the asymptotic behavior at $a \to \infty$, we see that $\lambda^2 \sim a \cdot (ka + N(a))$ for the eigenvalue λ of the transition matrix.
Therefore, the wavefunction boundary condition restricts the solution only to the case $\lambda < 0$.
Therefore, at least $\lambda^2 \sim a \cdot (ka + N(a)) > 0$, and in a negative curvature universe, it is required $N > a$ for the expectation value N of the Lagrangian other than the gravitational field.
From now on, let's limit our discussion to $k=+1, N=0$.

Now, by analogy with the Schrödinger equation for the hydrogen atom, if the eigenvalues of the quantum Friedmann equation are discrete, the question arises as to why our universe is not in a steady state, or in the lowest energy state. Therefore, it seems possible to think that the Hamiltonian eigenvalues form a continuous spectrum.
Assuming $k=+1$, $N=0$, and taking account of the boundary condition, let us take transform as

$$f_1 = \exp(-a^2/2) \cdot (1 + \delta g_1), \quad f_2 = -\exp(-a^2/2) \cdot (1 + \delta g_2) .$$

Also assuming that δg_1 and δg_2 are infinitesimal quantities of O(1/a), and we obtain the following by transforming it into an iterative form.

$$\partial \delta g_1 = a(\delta g_1 - \delta g_2) \qquad \partial \delta g_2 = -a(\delta g_1 - \delta g_2) + \omega \delta g_1 + \omega \quad ; \quad \partial \delta g = aE \cdot \delta g + \omega(1 + \delta g_2) e_2$$

$$\begin{pmatrix} \partial_a \delta g_1 \\ \partial_a \delta g_2 \end{pmatrix} = a \cdot \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} \delta g_1 \\ \delta g_2 \end{pmatrix} + \omega \cdot \begin{pmatrix} 0 \\ 1 + \delta g_1 \end{pmatrix} \qquad E \equiv \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} = 1 - \sigma_1$$

When calculating the propagation matrix U, from $U \partial_a U^{-1} = \partial_a - aE$, we get $\partial_a U = aEU$.
Since $(E/2)^2 = E/2$, the following is obtained.

$$U = \exp(a^2 E/2) = 1 + (\exp(a^2) - 1) \cdot E/2 \qquad U^{-1} = \exp(-a^2 E/2) = 1 + (\exp(-a^2) - 1) \cdot E/2$$

The iterative expression using this becomes as follows. Approximate initial value: $\delta g_2 = 0$.

$$\delta g = -\omega \cdot U(a) \int_C U^{-1}(a') e_2 \cdot (1 + \delta g_2) da' \quad ; \quad a' \in C : [a, \infty]$$

note : $f_1 = \exp(-a^2/2) \cdot (1 + \delta g_1)$, $f_2 = -\exp(-a^2/2) \cdot (1 + \delta g_2)$

(3).Quantum theoretical view of universe

Summarizing the results obtained so far

- The Hamiltonian governing the expansion of the universe is as follows, which leads modified Friedmann equation.

  $H=-ba^{-1}b+ka+N$; a＝scale factor, b＝canonical conjugate variable of (a).

  k＝cosmic curvature sign

  N＝Effect of fields other than gravitational field (state expectation of Lagrangian)

- The wave function (probability amplitude) f_1 regarding scale factor (a) is given by the following equation according to quantum mechanics.

  Equation: $+i\partial_t f_1=Hf_1$

  (in the text,the coefficients appeared in H has been simplified by scaling)

  f_1＝probability amplitude of scale factor ;

  i.e. expansion coefficient in scale factor eigenstate of the state of the universe

  Assume uniform isotropy. (Means the assumption that mode excitation in other states can be ignored.) Scale factor (a) is originally a field operator, but we considered its diagonal representation and made it an independent variable in the Schrödinger representation.

- In a negative curvature universe, $N>a$ is required to satisfy the norm boundedness condition of probability amplitude.

- If the universe is a superposition of a finite number of scale factor eigenstates in the above equation, a periodic stationary universe can be obtained. The entire spectrum of superposition depends on the initial conditions.
- The classical solution is obtained by replacing the expected value of the function of the state variable with the function of the expectation value of the state variable in the quantum theoretical equation. This is a kind of approximation operation, but it must be considered that this approximation is violated due to quantum mechanical variance effects in extreme quantum concentration states such as the big bang and black holes.

  It is understood that this effect contributes to the resolution of the big bang singularity that classical theory leads to. How this will affect the revision of estimates of the age of the universe will be an important observation-related issue.

- The sequence of successive approximation solutions for scale factor (a) is as follows.

  Basic equation $+i\partial_t f_1=Hf_1$; H＝Hamiltonian : already described.

  variable transformation $f_2\equiv a^{-1}f_1$ $f=\exp(-a^2/2)(\mathbf{e}_1\cdot g_1-\mathbf{e}_2\cdot g_2)$; $g=1+\delta g_j=O(1/a)$

  Iterative expression $\delta g(a,\omega)=-\omega\cdot\int_C U(a,a')\mathbf{e}_2\cdot g_2 da'$, C : $a'\in[a,\infty]$

  where, $U(a,a')=\exp((a^2-a'^2)\cdot E/2)=1+(\exp(a^2-a'^2)-1)\cdot E/2$,

  $E\equiv 1-\sigma_1$ (σ_1 : Pauli's spin matrix)

  Expressing excitation amplitude concerning $g_1(a,\omega)$ by $C(\omega)$, following is obtained.

  $g_1(a,t)=\Sigma_\omega C(\omega)g_1(a,\omega)\exp(-i\omega t)$, note : $f_1=\exp(-a^2/2)g_1$,$f_2\equiv a^{-1}f_1$

  To determine the coefficient $C(\omega)$, initial conditions are required.

・According to the transformation from f to g already mentioned, Schrödinger equation can be also transformed as follows.

$+i\partial_t f_1 = H f_1 = \omega f_1 \ \rightarrow \ +i\partial_t g_1 = \exp(-a^2/2) H \exp(+a^2/2) \cdot g_1$

Therefore, the following holds true regarding the coefficient C(ω) of the above-mentioned initial condition. See para.(2)

big bang wave function (initial probability amplitude) : $g_1(a;t=0)$ See para.(2)

scale factor probability amplitude (in time representation) fj(a,t):

$g_1(a,t) = \Sigma_\omega C(\omega) g_1(a,\omega) \exp(-i\omega t)$, note : $f_1 = \exp(-a^2/2) g_1$

Quadratic norm of normalized denominator: $\langle g_1(a,\omega)|g_1(a,\omega)\rangle_a \equiv N^2(\omega)$

$\langle g_1(a,\omega)|g_1(a,t)\rangle_a = C(\omega) N^2(\omega) \exp(-i\omega t)$ $\therefore$ $\langle g_1(a,\omega)|g_1(a,t=0)\rangle_a = C(\omega) \cdot N^2(\omega)$

note. $\Sigma_\omega |g_1(a,\omega)\rangle N^{-2}(\omega) \langle g_1(a',\omega)| = \delta(a-a')$

・It can be said that there was no big bang like the picture by classical theory at the beginning of the universe. In reality, it is governed by the equations mentioned above. In fact, the existence of a singularity violates the uncertainty principle of quantum theory.
On the other hand, we still do not know the constructive principle of the initial conditions.

・It is an interesting question what happened in quantum theoretical view when a＝0 in classical theory. The time behavior of the expectation and variance of a is of particular interest, but nothing can be concluded unless the initial state is known.
It seems necessary to look for plausible construction principle for the initial state distribution, such as maximum entropy. ***B**

***B :**

Setting the initial conditions is considered to be the most important problem, but an approximation method for solution concerning time evolution of the wave function can also be constructed as follows, by using the variational method.

Action of variational object $A \equiv \int d^4x \cdot (f_1^* \partial_t f_1 - f_1^* H f_1)$;

$f_1 = \exp(-a^2/2 + ka) \cdot pol$; $pol(a,c) \equiv (c_0 + c_1 \cdot a + c_2 a^2 + \cdots)$:Polynomial of a. c

k, c is a function of time t

If we limit the shape of the wave function as described above and apply variational method, differential equations regarding k(t) and cj(t) will be obtained. They will show the approximate behavior of quantum mechanical state.
It should be noted that the domain is $a \geqq 0$ and that $\exp(ka) \cdot pol = g_1 = O(1/a)$ as $a \rightarrow \infty$.

Initial variance of (a) is related to the energy of the universe. (see 1.6.4)
The above is an approximation based on the assumption that the initial state of the wave function can be approximated by the linear sum of the modified Γ-distribution.
As long as the initial conditions of the wave function are unknown, or the principle of its construction is unknown, the meaning of solving the equation is questionable.

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2. Canonical gauge gravitational theory / unified field theory as quantum theory

The canonical gauge gravitational theory is a part of the unified field theory based on the canonical gauge principle. There, the gravitational field is understood as the coefficient of first-order component with respect to the canonical momentum of the Lorentz frame in canonical gauge ring.
The theory is based on the concept of space-time quantum theory, the derivation of the field is principle, the reason for the existence of the field is clear, and the symmetry of the observed elementary particles is also derived. Therefore, the reality of quantum gravity theory depends on the reality of this unified field theory.
To say about gravity, the 1st touchstone is the classical application of the gravitational field equations.
From the discussion in Chapter 1, we have known that :

- The Lagrangian of canonical gauge gravitational theory is essentially different from that of A. Einstein's theory. In canonical gauge gravitational theory, Lagrangian of the field is a scalar function composed of derivatives up to the 1st order of field variables.

In A. Einstein's theory, if expressing Lagrangian as a function composed by up to the 1st order derivative of the field variable, the Lagrangian cannot be taken as a scalar. (though It is in canonical equivalence to scalar)

- Despite the essential difference in Lagrangian between canonical gauge gravitational theory and A. Einstein's theory, when imposing static spherical symmetry on the gravitational field, Lagrangian of canonical gauge gravitational theory with coupling constant ratio $\Lambda = -1$, matches that of A. Einstein's theory.

Some observational verifications concerning A. Einstein's theory of gravity are already established, and since they generally relate to almost spherically symmetric fields, it could be considered by the above facts that canonical gauge gravitational theory is also observationally supported.

On the other hand, regarding the validity of the canonical gauge gravitational theory as quantum theory, the internal consistency of the theory and the elimination of the divergence difficulty will be the problems. This will be considered in the next section (Section 2.1).

2.1. Validity as a Quantum Field Theory

The reason generally considered why constructing a quantum theory of gravity is difficult, seems to be due to the occurrence of divergence difficulties and the lack of effective renormalization prescriptions against divergence difficulties under the conventional ideas.

From the perspective of the canonical gauge unified field theory, the causes of divergence difficulties lie in two things as possibility : the field equations are physically incorrect, and the application of adiabatic hypothesis and free field concepts are inappropriate.
The renormalization prescription itself can hardly be recognized as a legitimate calculation method to begin with. We have already discussed this to some extent in Part II (and Part I as well).

Based on canonical gauge principle, starting from the perspective of the unified field, we have been able to provide equations that seem physically correct.

As a result, we have come to know the symmetries of elementary particles, the existence of preon substructures, the unification of Bosons/Fermions, the existence of spinor connection fields, and have also been able to deepen our understanding towards the Dirac equation.

Furthermore, aiming to solve the equations, we have decide to leave the diagram technique and have conceived a state-constructive field theory, that consists of finite element method-like approximation method, finite mode excitation approximation method, and state separation method.
It seems extremely important that this approximate solution method appears to avoid divergence difficulties.
It will be a difficult thing to prove the success of avoiding divergence difficulties, including defining it precisely. However, with the approximate method based on the conception of a state-constructive field theory, from the continuous relationship with classical theory and mass-point quantum mechanics, there seems to be no reason to be concerned about divergence difficulties.
This is the same for canonical gauge fields other than gravitational fields as well.

Supplemental Explanation:

The word "state-constructive" shows the method of solving the field equations, that is different from the conventional diagram technique, and that is a method to obtain the state tensor explicitly, mainly by an approximate solution construction method based on the variational method.
For the actual techniques, state separation method and finite mode excitation approximation are mainly applied.***A**

The applicability of state-constructive field theory to the canonical gauge unified field theory without divergence difficulties is considered to suggest the existence of approximate solutions in theory. Regarding the initial solutions, these approximation methods induce the equations of classical theory and masspoint quantum mechanics.
For Bosons, the classical field is the mean field due to the state, and quantum mechanically, that is a superposition of excited states.
It is possible to find approximate solutions under the assumption that only a few main modes have multiple excitations.
Regarding the existence of solutions, the problem of boundedness of the norm of the state tensor remains, but conversely, this may give new quantum constraints on the interaction.***B**
That is, similarly to the mechanism by which the boundedness condition of norm leads to the principal quantum number in solving the Schrödinger equation concerning the H-atom, there is a similar possibility that the states of interaction , not only of gravity, will be quantized.

***A** : For state-constructive field theory, see Part II
The validity of the finite mode excitation assumption depends on whether the state tensor space generated by finite basis vectors, becomes an invariant subspace due to the symmetry of the Hamiltonian.

***B** : For multiple states in the single mode excitation approximation, analytic (holomorphic) functions at the origin correspond.
The mode is given as an infinitesimal oscillation mode around the equilibrium state.
Therefore, depending on how the equations are linearized, the basis modes differ for each higher-order term.

The inner product of analytic functions f, g takes the following form : Canonical Gauge Unified Field Theory part-II 1.3.3.(2)

$\langle f|g \rangle = 1/(2\pi i) \int f(z)^* \cdot dz^* \exp(-z^* z) dz \cdot g(z)$, note : $dz^* dz = (dx - idy)(dx + idy) = 2i \cdot dxdy$

Assume: The basis mode vector $|z\rangle$ is normalized, $\langle z|z \rangle = 1$.

2.2 Considerations on solutions to the gravitational field

Let us overview the situations assumed when applying the state-constructive theory of fields discussed in Part II to the gravitational field, by theme. Also the relationship between classical solutions and quantum theoretical solutions are briefly reviewed.

▪ **Construction of field operators**

Concerning the construction of field operators, the following have been learned from the considerations in Part II:

The field operator $\chi_A(x)$ is obtained by mode decomposition as $\chi_A(x) = \Sigma f_{mA}(x) e^{*m}$, where, $f_{mA}(x)$ is a 3D spatial representation of the vibration mode, and corresponding to the existence of spatial orientation, there are two kinds of vibration as positive and negative one, which correspond to particle/antiparticle relation.

Also, the creation/annihilation of negative vibration particle states is interpreted as the annihilation/creation of positive vibration antiparticle states.

$f_m(x)$ forms a complete system in the single particle state space.

$e^{*m} = e^{*m}(t)$ is the mode amplitude operator, but from the canonical commutation relations imposed on the algebra in the dual space, it can be identified with the state tensor in the state space.

Also, it is generally transformed over time by the Hamiltonian. Or, the basis of the field moves over time, and the state at time $t = t_0 + dt$ is a superposition of the state at $t = t_0$ and the interaction field.

One way to obtain such $\chi(x)$ is to account for infinitesimal oscillations, linearize the equation, and use a solution basis of the linear equation to construct χ. Of course, if the dominantly approximate oscillation modes are already known, there is no need to follow this.

$\chi(x)$ can be expanded in any complete function system, as long as it satisfies the symmetry of positive/negative oscillations and particles/antiparticles.

▪ **Construction of canonical conjugate operators**

From the Lagrangian density $\mathcal{L}$, the canonical conjugate momentum $\pi(x)$ of the (amplitude) field operator χ can be obtained by the standard method $\pi(x) \equiv \partial\mathcal{L}/\partial\chi_{,0}$. This is derived by finding the canonical conjugate variable of the mode amplitude e^{*m}.

With f*m(x) as the dual basis function for fm(x) with respect to the d3x measure, the following is obtained:

Dual basis function $f^{*m}(y)$: $\Sigma f_m(x) f^{*m}(y) = \delta^3(x-y)$ $(\because \int d^3x \cdot f_n(x)^* f_m(x) \cdot f^{*m}(y) = f_n(y)^*)$

Field amplitude operator : $\chi(x) = \Sigma f_m(x) e^{*m}$; $f_m(x) = \langle x|e_m \rangle$

Field amplitude momentum operator : $\pi(x) = i \cdot \Sigma e_m f^{*m}(x)$

The expression $\pi(x) = i\cdot \Sigma e_m f^{*m}(x)$ originates from the definition: $i\cdot e_m \equiv \int d^3x\cdot f_m(x)\pi(x)$,based on

$\delta\mathcal{L} = \int d^3x\cdot\pi(x)\cdot f_m(x)\delta(e^{*m}{}_{,0})$ with $\pi(x) \equiv \partial\mathcal{L}/\partial\chi_{,0}$.

That is, $i\cdot e_m$ is the canonical conjugate of e^{*m}. At the same time, in the state linear space, e_m is regarded as the dual vector of e^{*m}.

$$i\cdot \Sigma \mathbf{e}_m f^{*m}(y) = \int d^3x\cdot \Sigma f_m(x)\pi(x)f^{*m}(y) = \pi(y) \quad \text{i.e.} \quad \pi(x) = i\cdot \Sigma \mathbf{e}_m f^{*m}(x)$$

$$\Sigma f_m(x)f^{*m}(y) = \delta^3(x-y) \quad \therefore \quad \Sigma h_{nm}\cdot f^{*m}(y) = f_n(y)^* \quad ; \quad h_{mn} \equiv \int d^3x\cdot f_n^*(x)f_m(x)$$

The canonical commutation relations of $\chi(x)$, $\pi(y)$ can be derived from the commutation relations of the mode amplitudes.

$$[\chi(x),\pi(y)]_\pm = i\cdot \Sigma f_m(x)[\mathbf{e}^{*m}, \mathbf{e}_n]_\pm\cdot f^{*n}(y) = i\cdot \Sigma f_m(x)f^{*m}(y) = \delta^3(x-y)$$

In the case of applying a finite mode excitation approximation, the mode index m runs within a finite range. However, positive and negative vibrations should be included.
(The indices are paired as m, -m)

Note here that $\mathcal{L}$ contains the gravitational field variable $(-g)^{1/2}$. Due to the involvement of the gravitational field, it is found that for the oscillation modes constituting $\chi(x)$, the canonical conjugate modes to the oscillation modes consisting $\chi(x)$, are generally not in the range of the dual space of the oscillation modes, and $\mathbf{e}_m$ contains $(-g)^{1/2}$ relative to $\mathbf{e}^{*m}$.

From the properties of the dual basis functions, the following is obtained by multiplying $f^*{}_n(y)$ on both sides and integrating :

$$\Sigma f_m(x)f^{*m}(y) = \delta^3(x-y),\ \Sigma h_{nm}\cdot f^{*m}(y) = f_n(y)^* ; \quad h_{mn} \equiv \int d^3x \cdot f_n^*(x)f_m(x)$$

Example in the case of Electromagnetic Field:

When viewing the field as a multi-excitation state of infinitesimal oscillations, the mode basis functions of the electromagnetic field can be written as, for example,

$\exp(ikx) \times N(\mathbf{k},k_0)$ $(\exists N)$

The amplitude N, wavenumber k, and the sign of the oscillation (sgn(k0)) or mode index. Then, the field operator A can be expressed as follows:

$$A_\mu(x) = \Sigma f_m(x)\cdot e^{*m} \quad ; \quad f_m = f_{\mu,k}(x) \equiv a^r{}_\mu\cdot(k_0)^{1/2}\exp(ikx) \quad ; \quad \text{index } m \equiv (r, k, k_0)$$

$\Sigma \equiv \int (2\pi^3)^{-1}(d^3k/k_0)\cdot \Sigma_r$ (The microscopic state density is involved to the sum for the continuous index)

In the case where there is no involvement of the gravitational field in the metric, the metric (inner product) of f becomes as follows:

$h_{mn} = \delta rs\cdot\delta(k_0,k_0')\cdot\delta^3(k-k')d^3k/k_0$; $m \equiv (a_\mu, k_0, k)$ positive/negative vibrations are orthogonal.

With the canonical momentum density $\pi_\mu = \partial_0 A_\mu - \partial_\mu A_0$, if we ignore $-\partial_\mu A_0$ as the gauge term, we can set:

$$\pi_\mu(x) = \partial_0 A_\mu = \Sigma \mathbf{e}_m\cdot ik_0 f^{*m}(x) , \quad [A(x),\pi(y)] = \delta^3(x-y)\cdot N_{\mu\nu} , \quad N = \text{projection matrix}$$

In the case of the electromagnetic field, due to gauge invariance, $\pi_0 \equiv 0$ means that A_0 cannot be regarded as an independent canonical variable, so the time evolution of A_0 is determined from the Gauge condition.

The setting $A_0 \equiv 0$ can also be used as gauge fixing. Even when the Lorentz condition is imposed, one gauge degree of freedom remains, so the internal degree of freedom of the electromagnetic field becomes 2. The above-mentioned projection N means projection to the constrained subspace by gauge fixing.

・Relationship between classical solutions and quantum solutions

First, let's look at the case of mass-point quantum mechanics. The motion of physical quantities in the Heisenberg representation is:

$\partial_t x = i[H, x] = \partial H/\partial p \quad \partial_t p = i[H, p] = -\partial H/\partial x \quad (\partial_t \xi = i[H, \xi]$ in general.)

Here, considering the quantum theoretical average of both sides, if the quantum theoretical variance is small enough to be negligible, the equations of classical theory will hold.

$\partial_t \underline{x} = \underline{\partial_p H(x,p)} \doteqdot \partial_p H(\underline{x},\underline{p}) \ ; \ \partial_t \underline{p} = -\underline{\partial_x H(x,p)} \doteqdot -\partial_x H(\underline{x},\underline{p})$

where, $\underline{x} \equiv \langle f_0|x|f_0\rangle$; $|f_0\rangle =$ initial state

Conversely from the above, it can be seen that corrections due to quantum theoretical variance effects are necessary for the classical theoretical expectation value solutions.
This classical approximation can also be seen to reduce to similar conclusions in the Schrödinger representation as well, from the fact that the Hamiltonian does not depend on time.
($\underline{x} \equiv \langle f_0|x|f_0\rangle = \langle f|x_0|f\rangle$)
The expected value and variance are independent variables, and their values depend on the initial quantum mechanical states. The quantum theoretical variance value cannot be estimated from the classical solution.

On the other hand, assuming a minimum uncertainty state for x and p, it can be assumed that the space-time representation of the state, i.e. the wave function, is Gaussian. (see below)

The uncertainty principle is nothing more than the Cauchy-Schwarz inequality for vectors, but its derivation utilizes the positivity of the norm. Let's introduce the deviations $\Delta x \equiv x - \underline{x} \quad \Delta p \equiv p - \underline{p}$.

$Y \equiv |(\Delta x - i\lambda\Delta p)|f\rangle|^2 \quad , Y \geqq 0 \quad$ for $\forall \lambda \in \mathbf{R}$

$Y = \langle f|\Delta x^2|f\rangle + \langle f|i\lambda(-\Delta x\Delta p + \Delta p\Delta x)|f\rangle + \lambda^2\langle f|\Delta p^2|f\rangle \geqq 0$

note: canonical commutation relations for deviations $[\Delta x, \Delta p] = i$, $\langle f|i\lambda(-\Delta x\Delta p+\Delta p\Delta x)|f\rangle = \lambda$

$\therefore \ D \equiv 1 - \langle f|\Delta x^2|f\rangle\langle f|\Delta p^2|f\rangle \leqq 0$ Equality only when $\exists \lambda \in \mathbf{R}$ st.$(Y=0)$.

Assuming the minimum uncertainty of (x, p), from $\exists \lambda \in \mathbf{R}.(Y=0)$, $(\Delta x - i\lambda\Delta p)|f\rangle = 0$

Considering the space-time representation (wave function) of f, we obtain $\Delta x \cdot f - \lambda(\partial_x - i\underline{p})f = 0$.

From this, assuming that the domain of x is the entire real number space, we obtain

$f \propto \exp(\Delta x^2/\lambda + i\underline{p}x)$.

The above assumptions are valid when the wave function f(x) is the space-time representation of the state $|f\rangle$ with the independent variable x , but as seen in sec.1.6.5 - quantum Friedman universe, the variable corresponding to x might be a variable like scale factor (a),of which domain is restricted as $a > 0$. such cases will be noted also possible.
The above assumptions are valid when the wave function f(x) is assumed for x as an independent variable of the space-time representation of the state $|f\rangle$, but as seen in 1.6.5 - quantum Friedman universe, the variable corresponding to x, may be the scale factor (a) with the domain $a > 0$.
It should be noted since such cases will arise.

When the assumption holds, considering the normalization, the following is obtained:

$f=\det(2\pi S)^{-1/4}\cdot\exp(-0.5\langle x-\underline{x}|S^{-1}|x-\underline{x}\rangle+i\langle\underline{p}|x\rangle)$,

where S is the quantum theoretical variance (covariance matrix) of x

The behavior of the above x(t), p(t), S=S(t) can be approximated using variational methods under the assumption of minimum uncertainty of (x, p).

i.e. $A\equiv\int d^3x\cdot(g)^{-1/2}\langle f|i\partial_t-H|f\rangle$; $\delta A=0$

x(t), p(t) are given by classical solutions if S is ignored, and the contribution of S becomes the quantum variance effect.

In particular, for static equilibrium states, since $\partial_p H(\underline{x},\underline{p})\fallingdotseq-\partial_x H(\underline{x},\underline{p})\fallingdotseq 0$, it is clear that the classical solution of the equilibrium point is the minimal of the potential, but this is generally corrected by quantum variance effects.

Equilibrium point equation: $\underline{\partial H(\xi)}=0$

Expanding up to 2nd order around the expected value gives

$(\partial H)_0+0+(1/2)(\partial^3 H)_0\cdot\underline{\Delta\xi^2}\fallingdotseq 0$; $\Delta\xi\equiv\xi-\underline{\xi}$

Approximating the first term to 2nd order around the classical solution ξc, we obtain

$(\partial H)_0=0+(\partial^2 H)c\cdot\Delta'\xi$; $\Delta'\xi\equiv\underline{\xi}-\xi c$

So it can be seen that $\Delta'\xi\equiv\underline{\xi}-\xi c$ can be treated as a 2nd order correction amount.

Also, $\underline{\Delta\xi^2}$ is nothing but the covariance matrix S. However, since the covariance is given from the initial state, the correction amount cannot be calculated unless the initial state is given.

For the equilibrium state vector, from the variational principle

$\delta\langle f|H|f\rangle=0$ ／ $\langle f|f\rangle=1$ i.e. $H|f\rangle=-|f\rangle\omega$; $\exists\omega$ i.e. $H|f\rangle=-|f\rangle\omega$; $\exists\omega$

so the equilibrium state is an eigenstate of the Hamiltonian, and $-|f\rangle\omega=|i\partial_t f\rangle$ i.e. $f=f_0\cdot\exp(i\omega t)$ is derived.

Next, considering the case of quantum field theory, $\mathcal{H}$(x,p) will be replaced by $\int d^3x(-g)^{1/2}\cdot H(\chi(x),\pi(x))$, and similar discussion can be developed in parallel with the above.

However, the wave function is difficult to handle as it becomes a function of infinite commutative and non-commutative variables. Therefore, it is necessary to approximate the state as a function of a finite number of mode variables.

At this time, there is some arbitrariness in the selection of the mode basis representing the state, and a variational method based on the classical solution method is considered to be effective in constructing an effective approximation.

For static equilibrium states, the solution of the classical field equation corresponds to the expected value of the equilibrium state when quantum variance effects are ignored. The corrections to the expected value might relax the singularities in classical solutions.

・Difficulties with the gravitational field

For Boson fields assuming high degrees of symmetry, it is conceivable to approximate the field by the excitation of a few modes satisfying that symmetry. The single mode excitation approximation, or more generally the finite mode excitation approximation, is a method of restricting the domain of field operators to a finitely generated subspace by a few modes.

While it is possible to introduce wavefunctions for a few fermions, to introduce finite mode excitation approximation is rational for Bosons since multiple excitation occurs.

・・**Definition of orthogonality of modes**

For oscillation modes other than the gravitational field, inner products of modes can be defined by ignoring or assuming as known the gravitational field. Then, norm calculations are possible for mode excitations.

For the gravitational field, norm calculations cannot be performed unless it is assumed as known, so an iterative method must be taken for calculations of inner products of states.

The inner product of the modes of the field amplitude operator is as follows:

Field amplitude operator: $\chi(x) = \Sigma f_m(x)e^{*m}$; $f_m(x) = \langle x|e_m\rangle$

$\langle e_m|e_n\rangle = \Sigma \langle e_m|x\rangle\langle x|e_m\rangle = \Sigma_x f_m(x)^* f_n(x)$

$\Sigma_x = \int d^3x(-g)^{1/2}$ (Involvement of state density for continuous eigenvalue) There is gravitational field involvement due to g.

When expressing the sum with respect to a continuous index (x) by the integral of the coordinate representation, the concept of "microscopic density of states" regarding the coordinate variables is required.

fm(x) is interpreted as the space-time coordinate representation of state as $f_m(x) = \langle x|e_m\rangle$, ($e_m$ is state vector of vibration mode.).

However, as the function basis for expanding the field operator $\chi(x)$, any function system on an equal-time cross section can be taken.

When $f_m(x)$ is set with consideration to infinitesimal oscillations, the inner product structure should be given according to j^0-current.

In the case of Boson, $\partial_t f_m$ appears in j^0-current form, but since f_m is a function on the equal-time cross section $(t-t_0=0)$, it is held $-i\partial_t f_m = k_0 \cdot f_m$. Following expression will also be available

$\langle e_m|e_n\rangle = \Sigma_x f_m(x)^* f_n(x) = \Sigma_x f_m{}^W(x)^*(h^*h)\cdot f_n{}^W(x)$ $f_n \equiv h_n \cdot f_n{}^W$; $f_n{}^W$ = wave function

$(h_n{}^* h_n) = \omega_n$: frequency(energy) of vibration for Boson, $(h_n{}^* h_n) = \gamma^0$ for Dirac particle, etc.

・・**Emergence of inverse operator**

Regarding the gravitational field, the field operator includes a constant component in some natural form, and also it is necessary to introduce an inverse element or an inverse matrix of the field operator.

When adopting the coefficient T related to the Lorentz frame of the canonical momentum p as the gravitational field variable (operator), the inverse matrix S of this is required.

If we consider the fundamental mode of the gravitational field is an infinitesimal oscillation around $T=1$, then T will contain a constant component of 1.

As already mentioned in Part II, these are general circumstances for Boson fields.

Also, note that adding a constant to the Boson field operator does not change the canonical commutation relation.

Regarding the existence of a constant component, if the creation of a constant component can contribute to minimizing the effect, there is a strong possibility that a constant component will be formed in a form similar to Bose-Einstein condensation in statistical mechanics.

In the case of a gravitational field, the field variable corresponding to Newton potential is $V \equiv \ln(T)$. (The actual Newton potential is its time component $V^0{}_0$.)

In this case, since $T=\exp(V) \rightarrow T^{-1}=\exp(-V)$, there is no problem for reciprocal representation of the field operator T. However, due to issues of non commutativity, it leads that ∂T etc. cannot be expressed easily.
Similarly, expression using linear fractional transformation, $T=(1-V/2)^{-1}(1+V/2)$, $T^{-1}=(1+V/2)^{-1}(1-V/2)$ is also adoptable, but this has similar difficulties.

In a prescription like linear approximation, there is no prospect of successive improvement in accuracy. The idea of eigenstates expansion for the field operator $\chi(x)$ may be useful for the solution.

in case that particle itself is an antiparticle, like as a gravitational field or an electromagnetic field, the field operator has the following form.

$$\chi(x)=\text{const}+\Sigma\,(u^m(x)e_m+e_m(x)u^{*m})$$

Taking account of applying single mode excitation approximation and focusing on one mode, $\chi=1+u^*z+z^*u \quad \chi=\chi^*$ will be obtained.

(Here, * shows not canonical conjugation but duality on the state space.
z is the state vector/tensor of mode. z* is a dual element of z. As a canonical commutation relation, $z^*=\partial_z$. $u=(u_j(x))$ includes the component index.)

If we require that the product of the field components is commutative, we find that the argument arg(u) does not depend on the components.
By performing phase transformation, we can set $\chi=1+u(x)(z+z^*)$ from the beginning.
The following canonical transformation can be considered.

(Note : The commutation relation is preserved by the transformation as $\chi\rightarrow R\chi R^{-1}$, but the norm is not invariant because $RR^*\neq 1$.)

Also note that the second formula depends on u(x).

・・Notes on equation solving techniques

-1. Action of Hamiltonian to Vacuum

Constants commute with all operators, and the time evolution of any operator is expressed by the canonical commutation relation with the Hamiltonian as shown by the equations of motion. Therefore, it can be considered that Hamiltonian contains a constant with arbitrariness. That is, the setting of the origin of canonical energy is arbitrary. From this fact, when assuming that the vacuum is mapped to the vacuum by the Hamiltonian, the action of the Hamiltonian on the vacuum can be assumed to be 0. Combining this fact with the operatorization of states, the following calculation technique is obtained:

$$H|f\rangle=[H,f]|0\rangle \quad (\quad Hf|0\rangle=H|f\rangle, \quad fH|0\rangle=0 \quad)$$

-2. Successive approximation search for eigenstates.

The method of approximating diagonalization of a quadratic form by successive SO(2) transformations is known as the Jacobi method and is applied to such things as diagonalization of symmetric matrices. Adopting this idea allows construction of the following approximation method:

Initial solution $|f0\rangle$, Correction direction $|f1\rangle=H|f0\rangle-|f0\rangle\mu \quad ; \quad |f1\rangle\perp|f0\rangle$

Then, unless |f0＞ is an eigenstate, |f1＞≠0 and eigenvectors with smaller eigenvalues can be searched for within the 2D plane spanned by |f0＞ and |f1＞.
By the iteration of this method, if the energy is lower bounded, the eigenstate with minimum energy can be searched for.

This method assumes that *H*|f＞ is calculable, but in the context of state-constructive field theory where field operators have infinite degrees of freedom, it is necessary to devise ways to effectively narrow down freedoms by appropriate approximations so as to make the search as effective as possible. Finite mode excitation approximation is an effective method, but in order for successive calculations, it must be possible to express approximations errors explicitly.

-3.Finite degrees of freedom and variational methods
In state-constructive field theory, generally for Bosons, it takes the form of handling the analytic functions of infinitely many variables, so efficiently approximating them with analytic functions of a finite number of variables becomes an issue, and variational methods become the guiding principle.

・・**Problem of Gauge fixing**
The essence of gauge fixing is to exclude the arbitrariness of the solution of the equation of motion that corresponded to a transformation(gauge transformation) by which the original Lagrangian is invariant.
The gauge fixing can be done by artificially adding the action that breaks the symmetry concerning the gauge transformation and applying least action principle concerning the gauge transformation to the total action.
As with the electromagnetic field, the degrees of freedom of field variables will be reduced by gauge constraints, and the time evolution of variables that have lost qualifications as canonical variables will be given by gauge conditions.***1**

***1**：For the electromagnetic field, if an additional Lagrangian for gauge fixing is taken as $A_\mu(\sqrt{-g})A^\mu$, the Lorenz condition is obtained.
From the least action principle for $\delta A=\partial W$, $\partial_\mu(\sqrt{-g}\cdot A^\mu)=0$ is obtained.
From the existence of $\partial_0 A^0$, the time propagation of A^0 is given not by the canonical equation but by the Gauge condition. Since $\pi_0\equiv 0$, A_0 loses its qualification as a canonical variable.
Relativistically, the time direction is treated specially, but there is no problem in the Hamiltonian formalism.

2.3. Quantum theoretical consideration on static gravitational field

As a quantum theoretical consideration on dynamic gravitational field, we treated the Quantum Friedmann universe in Section 1.6.5 by using the probability amplitude of the scale factor. Here, we consider Schwarzschild spacetime as an example of static gravitational field with particularly high symmetry.

●Schwarzschild Black Hole―its representation and the mass distribution as the source of gravity

Usually, Schwarzschild spacetime is specified by the way of finding a spherically symmetric solution of the gravitational equation, and determining the constant of integral by its asymptotic form in

infinity far that gives the mass observed at infinity far.
In other words, such an approach does not delve into the source of gravitational field itself.
However, this is incompatible with the quantum theoretical approach, which determines field states based on variational principles on total Lagrangian.

The gravitational field is formed by the mass of spacetime, and it must be given as the expected value of the energy-momentum tensor of fields other than the gravitational field. Before transfer to on to quantum theory considerations, let's first consider the possible interpretation of Black Hole.

Considering Schwarzschild's solution as a reference, there are at least three possible interpretations of Black Hole.
The 1st is the position that assumes that the mass distribution that is the source of gravity is outside the Schwarzschild radius and denies the existence of the Schwarzschild barrier as physical reality.
In this case, a black hole is a kind of limit concept, and can be interpreted as a state where mass is concentrated near the Schwarzschild radius and a strong gravitational force is generated.
This view is reasonable because even if mass flows toward the center from a distance, it takes an infinite amount of time to reach the Schwarzschild radius. When we say that the huge mass in a galaxy forms a black hole, this first position is valid.

The 2nd position is that the Schwarzschild barrier of existence is real , and that the inside of it is also a real, separate world.
This position is, "We cannot see inside. However, space-time there exists."
When discussing black hole evaporation, event horizons, etc., it is of course assumed that the existence of space-time that accommodates them.

The 3rd position is that spacetime itself has holes or topological defects, and that when viewed from the outside, a gravitational field that matches the Schwarzschild external solution is observed, but there is no such thing called as the inside of the Schwarzschild barrier. If according to the idea that many mini blackholes were formed at the time of the Big Bang, it should be interpreted as bubbles/topological defects in the birth of space-time.

However, all 3 of these positions are based on classical theory.
In quantum theory, the mass of a gravitational source should be interpreted as the expectation value of an energy-momentum tensor formed by a field other than the gravitational field especially in a state-separation sense. (The variational coefficient with respect to the gravitational variable, of the Lagrangian expectation value of the fields other than the gravitational field).

From the standpoint of canonical gauge quantum gravity, the existence in real of the Schwarzschild barrier is highly doubtful.
In the case of a black hole caused by a huge mass, it is reasonable to take the 1st standpoint.
If large spots were created during the materialization process of energy at the beginning of the universe, it would become difficult to deny the hypothesis that black holes existed before the formation of galaxies.
In the 3rd standpoint, if saying that the source of gravity is a topological defect in space-time, and that the gravitational field is the result of the topological defect in space-time, then the question arises how to understand fields other than gravitational one.

At the same time, this position seems to abandon the perspectives of unified field theory.

According to the 2nd standpoint, which acknowledges the existence of space-time itself, Schwarzschild's spherically symmetric solution is given by solving the equation of not "$G=0$ for $r>0$", but "$-\sqrt{-g}\cdot G/2\kappa+m\delta^3(x)vv/v^0=0$ ".
(Where, G is Einstein tensor. note : $\delta\int(1/2)mv^\mu v^\nu g_{\mu\nu}ds=\int m\delta^3(x)\, v^\mu v^\nu/v^0\cdot\delta(g_{\mu\nu}/2)d^4x$)

According to this idea, the gravitational source mass m should be obtained by integral the coefficient of variational $\sqrt{-g}\cdot G$ for the 00 component over an arbitrary spherical volume centered on the origin, and converting the divergence term to a surface integral.
Let's actually do some calculations within the scope of classical theory.

First, we will observe the calculation flow using an approximation using a normal static weak gravitational field.
With the Newtonian potential as Φ, the Lagrangian of the gravitational field is expressed as

$\mathcal{L}^G=(4\pi G)^{-1}\cdot(1/2)(\nabla\Phi)^2$, then

$\delta\int\mathcal{L}^G\cdot d^4x=(4\pi G)^{-1}\int\nabla\Phi\cdot\nabla\delta\Phi\, dtd^3x=-(4\pi G)^{-1}\int\nabla^2\Phi\cdot\delta\Phi\, dtd^3x$

$\delta\int(1/2)mv^\mu v^\nu g_{\mu\nu}ds\fallingdotseq\delta\int m(1+\Phi)dt=\int m\delta\Phi dt=\int m\delta^3(x)\cdot\delta\Phi dtd^3x$

$-(4\pi G)^{-1}\nabla^2\Phi+m\delta^3(x)=0 \quad \therefore\ -(4\pi G)^{-1}\int\nabla\Phi dS+m=0$

The solution is $\Phi=-Gm'/r$ ($\exists m'$) for $r>0$. From this, $m=m'$; i.e. Iit is possible to identify the source mass of the gravitational fields.

Next, we perform calculations in parallel to the above for the Schwarzschild classical solution.
Use the symbols defined in Section 1.5.1. We only deal with the case of $\Lambda=-1$.
Including the normalization factor, the scalar Lagrangian is $-0.5L^C$ shown below, the Lagrangian density for the isotropic coordinate system $(x^j)_{j=0..3}$ is $\mathcal{L}^G$, and its expression is as follows.

$-0.5L^C=-(\alpha_0'\alpha_1'+0.5\alpha_1'^2)\cdot a_1^{-2}$, $\sqrt{-g}=\det(T)=ab^3=a_0a_1^3=\exp(\alpha_0+3\alpha_1)$

$\therefore\ \mathcal{L}^G=-(\alpha_0'\alpha_1'+0.5\alpha_1'^2)\cdot\exp(\alpha_0+\alpha_1)$,

$\delta(\int\mathcal{L}^G d^4x+\int(1/2)m(v^0v^0a_0^2)ds)=0$

$v^\mu=dx^\mu/ds$. From static condition $v^\mu=0$ for $\mu\neq0$ $v^0ds=dx^0=dt$

The Schwarzschild classical solution corresponding to the Lagrangian above has been obtained and is expressed as follows in isotropic coordinate representation:

$ds^2=a_0^2dt^2-a_1^2(dr^2+r^2d\Omega^2)$

$a_0=(1-u)/(1+u)$, $a_1=(1+u)^2$; $u\equiv r_0/r$, $r_0\equiv0.5Gm'$

(note : the value of r_0 is 1/4× so called Schwarzschild radius. m'=mass observed in infinit far).

Here, let (n) be the radial direction, then $(\#)\cdot\delta\alpha_A'=(\#)\cdot\delta(n^j\partial_j\alpha_A)\ \rightarrow\ -\partial_j(n^j\cdot(\#))\cdot\delta\alpha_A$.
So 3D integrals can be transformed into surface integrals.
Furthermore, since the spherically symmetric 3D integral can be transferred as $d^4x\rightarrow4\pi r^2dtdr$, the part of $\partial_j(n^j\cdot(\#))$ can be replaced to $r^{-2}\partial_r(r^2(\#))$.

$\delta\mathcal{L}^G/\delta\alpha_0=0$: $+\partial_j(n^j(\alpha_1'\cdot\exp(\alpha_0+\alpha_1)))+\mathcal{L}^G+m(v^0a_0^2)\delta^3(x)=0$

$\delta\mathcal{L}^G/\delta\alpha_1=0$: $+\partial_j(n^j((\alpha_0'+\alpha_1')\cdot\exp(\alpha_0+\alpha_1)))+\mathcal{L}^G=0$

$-\partial_j(n^j\cdot\alpha_0'\exp(\alpha_0+\alpha_1))+m(v^0a_0^2)\delta^3(x)=0$; $v^0v^0a_0^2=1$ (line element condition, applied after variational) i.e. $-r^{-2}\partial_r(r^2\alpha_0'\cdot\exp(\alpha_0+\alpha_1)))+m(v^0a_0^2)\delta^3(x)=0$

Here, using the relationship $r^2\partial_r=-r_0\partial_u$, we get $r^2\alpha_0'\cdot\exp(\alpha_0+\alpha_1)=-r_0\partial_u\alpha_0\cdot\exp(\alpha_0+\alpha_1)=2r_0$
As a result, we obtain $-4\pi\cdot 2r_0+m(v^0a_0^2)=0$ at $r=0$.
From $a_0=(1-u)/(1+u)$, $a_0(r=0)=-1$, and from the line element condition, $v^0v^0a_0^2=1$ (at $r=0$),
so we get the following final result.

$m'=m/v^0(0).$, $v^0v^0a_0^2=1$ (at $r=0$) $a_0(r=0)=-1$
$(8\pi r_0=m/v^0(0)$, $r_0\equiv 0.5Gm'=m'/8\pi\cdot(4\pi G)$ $m'=m/v^0(0)$; $(4\pi G)=1)$

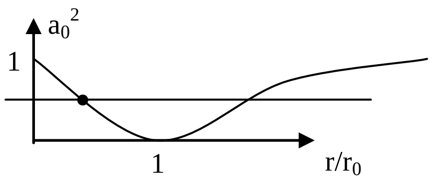
From the above, $v^0(0)=\pm1$, but physically $v^0(0)=+1$ is natural to take.
In that case, the proper time of the stationary mass point is $ds=dt/v^0$, and time flows at the origin in synchronization with the infinite distance. If connected analytically as a line element, we obtain that $ds=-a_0dt$ inside of the barrier. ($r/r_0<1$).

Let us consider the motion of a mass within a barrier.
Since masspoint energy is conserved a static field (i.e. $dp_0=0$) we get the following.

$a_0^2v^0=\text{const}$, and $(a_0dt)^2-(a_1dr)^2=ds^2$, $ds/dt=1/v^0=a_0^2/\text{const}$.

Here, we consider the falling motion of a mass point that was in a stationary state at $r=r_1$.
The time coordinate (t) is adopted as the motion parameter.
Regardless of the sign selection of $v^0(0)$, we get the following:

$1=(a_1/a_0\cdot dr/dt)^2+(a_0/a_0(r_1))^2$; potential $a_0=(1-r^0/r)/(1+r^0/r)$, $a_1=(1+r^0/r)^2$



Schwarzschild BH
potential diag

Fig.2.3-1

From the above, it can be seen that inside the barrier $r<r_0$, the mass point falls toward the barrier.
Of course, starting from the origin takes an infinite amount of time. (the below described.)

$-a_0(1-a_0^2)^{1/2}/a_1=dr/dt$; $dt=-a_1dr\ a_0^{-1}(1-a_0^2)^{-1/2}=O(r^{-2.5})dr$
for $r\to0$, $a_1=O(r^{-2})$, $(1-a_0^2)^{-1/2}=O(r^{-1/2})$

By the way, please note that falling means movement over time, so if there is a time reversal inside the barrier, the masspoint will fall toward the center and come to rest depending on its initial kinetic energy. This suggests the possibility of a spherically symmetric mass distribution inside the barrier, but this is difficult to imagine unless there is a barrier as a premise at the first stage.
In other words, there is no such possibility from the viewpoint that black hole is formed by the gravity generated by the concentration of masses.

First, it should be noted that the concentration of mass at the origin is only a limit extrapolation model.
However, at any rate, if masspoints inside the barrier fall toward the barrier, a shell-like mass distribution is assumed to the Black Hole, and from the viewpoint of gravity generating due to mass distribution, it seems to be unlikely that the barrier exists as physical reality.
Based on the above results, and considering on the classical interpretation of the black hole, it will lean toward that black hole is a topological defect in space-time, or that, black hole is a phenomenon of strong gravitational field due to extreme mass concentration near the barrier, that exists only as

limit concept.

It is also important to pay attention to the ambiguity of the relationship between the origin and coordinates, while the mass of the gravity source is supposed to concentrate at origin.
The expression $ds^2=(1-a/r')dt^2-(1-a/r')^{-1}dr'^2-r'^2d\Omega^2$ is also often used as the expression for the Schwartzschild solution. The relationship with the isotropic coordinate is as follows.

$r'-a/2=r+a^2/(16r)$ i.e. $r'=r\cdot(1+a/(4r))^2$。

First, the origin does not correspond to the origin. The minimum value on the right side of the above equation is a/2, and $r'-a/2\geqq a/2$.
The Schwarzschild barrier $r'=a$ corresponds to $r=a/4$ in isotropic coordinates, and the domain inside the barrier, $r<a/4$ in r-space is mapped to wrap around the outside of the barrier in r'-space. (Doubly covering strucure.)
Conversely, the domain $r'<a$ of inside the barrier in r'-space corresponds to the imaginary number domain in r-space, and its reality is extremely doubtful.
From the concept of canonical gauge theory, it is appropriate to introduce isotropic coordinates in view of the existence of the Lorentz frame, and therefore we should consider that spacetime with $r'<a$ does not exist.
From this perspective, arguments such as the evaporation of black holes will lose their basis.

The classical solution in isotropic coordinates has no points that diverge as field variables.
Physical singularity is at $r=r_0=a/4$, where it is singular that the proper time ds is static: $ds=a_0dt=0$ $(a_0(r_0)=0)$.

●Quantum Schwarzschild spacetime

Let's consider quantum theoretical understanding and its expression of Schwarzschild spacetime, from viewpoints of state-constructive field theory, rather than conventional diagram techniques.
Assuming spherical symmetry, calculating the Hamiltonian, and we will apply the single mode excitation approximation, and finally we will find the integro-differential equations to be solved.
Probably, it would be difficult to actually solve the equation. Furthermore, at this stage, it is unclear how much significance this solution has. The important thing now would be to confirm that a solution exists and that successive approximations expression is possible.

・Hamiltonian of static spherically symmetric field

Use the symbols in Section 1.5.1 for calculations. Unlike the classical theory, it is necessary to calculate the time derivative term for obtaining the Hamiltonian.

$T^A{}_\mu=a_A(t,r)\cdot\delta^A{}_\mu$; $S^\lambda{}_A=a_A(t,r)^{-1}\cdot\delta^\lambda{}_A$ ($a_0\equiv a$, $a_A\equiv b$ for A=1..3) we can express as such.

$\partial_\lambda T^A{}_\mu=a_A'\cdot n^\lambda\cdot\delta^A{}_\mu+a_A^\bullet\cdot\delta^0{}_\lambda\delta^A{}_\mu$ is held. $'\equiv\partial/\partial r$, $\bullet\equiv\partial_0=\partial/\partial t$ $n^\lambda\equiv x^\lambda r^{-1}(1-\delta^0{}_\lambda)$ 。

$F^A{}_{\lambda\mu}=a_A'\cdot(n^\lambda\delta^A{}_\mu-n^\mu\delta^A{}_\lambda)+a_A^\bullet\cdot(\delta^\lambda{}_0\delta^A{}_\mu-\delta^\mu{}_0\delta^A{}_\lambda)$

$F^A{}_{BC}=F^A{}_{\lambda\mu}\cdot a_B{}^{-1}\delta^\lambda{}_B\cdot a_C{}^{-1}\delta^\mu{}_C$

$\therefore$ $F^A{}_{BC}=a_A'a_B{}^{-1}a_C{}^{-1}\cdot(n^B\delta^A{}_C-n^C\delta^A{}_B)+a_A^\bullet a_B{}^{-1}a_C{}^{-1}\cdot(\delta^0{}_B\delta^A{}_C-\delta^0{}_C\delta^A{}_B)$

Expressing $\alpha\equiv\ln(a)$,and $F^A{}_{AC}=-(\alpha_0+2\alpha_1)\cdot a_1{}^{-1}n^C-3a_1^\bullet a_1{}^{-1}a_0{}^{-1}\delta^0{}_C$ is obtained.
The quadratic form of F becomes as follows.

$2L^C(1)\equiv(1/2!)\cdot F^A{}_{BC}F_A{}^{BC}=-(\alpha_0'^2+2\alpha_1'^2)\cdot a_1^{-2}+3\alpha_1^{\bullet 2}\ a_0^{-2}$
$2L^C(2)\equiv F^A{}_{BC}F^B{}_A{}^C=-(\alpha_0'^2+2\alpha_1'^2)\cdot a_1^{-2}+3\alpha_1^{\bullet 2}\ a_0^{-2}\ \ldots\ L^C(1)$ と独立ではない！
$2L^C(3)\equiv F^A{}_{AC}\cdot F^B{}_B{}^C=-(\alpha_0'+2\alpha_1')^2\cdot a_1^{-2}+9\alpha_1^{\bullet 2}\ a_0^{-2}$

Even including the time derivative term, $L^C(1)$ and $L^C(2)$ are the same. (not linearly independent)
Proceeding the Lagrangian calculations gives the following.
$2L^C\equiv 2L^C(1)+\Lambda\cdot 2L^C(3)=-((1+\Lambda)\alpha_0'^2+4\Lambda\alpha_0'\alpha_1'+2(1+2\Lambda)\alpha_1'^2)\cdot a_1^{-2}$
$2L^C\equiv 2L^C(1)-2L^C(3)=(4\alpha_0'\alpha_1'+2\alpha_1'^2)\cdot a_1^{-2}-6\alpha_1^{\bullet 2}\ a_0^{-2}$

It is remarkable that Lagrangian does not contain $a_0^\bullet$. Therefore, the canonical conjugate momentum for a_0 becomes 0 as identity. Or perhaps we should understand that a_0 has lost its qualification as a canonical variable due to the initial assumption of "static gravitational field".
Since $a_1^\bullet$ and a_1 are non-commutative, quantum theory requires adjustment so that L^G becomes self-adjoint.
Since we have already obtained the result that the Lagrangian L^G of gravity is $L^G=-0.5L^C$, so considering normalization, and let us adjust the coefficients to this, hereafter. Let the field variables be a_0, a_1.
In order to use the finite mode excitation approximation, it is reasonable to calculate the Lagrangian corresponding to that in masspoint mechanics, by integral the Lagrangian density on 3D space.
Let's express this as $L^{G(3)}$.

$$-4L^{G(3)}=\int d^3x\cdot\det(T)((4\alpha_0'\alpha_1'+2\alpha_1'^2)\cdot a_1^{-2}-6\alpha_1^{\bullet 2}a_0^{-2})$$
$$=\int d^3x\cdot(4a_0'a_1'+2a_0a_1'^2a_1^{-1}-6a_1^{\bullet 2}a_0^{-1}a_1)\quad;\quad -4\pi_1=-12\,a_1^\bullet a_0^{-1}a_1\quad\therefore\ (1/3)\pi_1a_0a_1^{-1}=a_1^\bullet$$

Let the Hamiltonian corresponding to the above be expressed as $H^{G(3)}$. Self-adjointness is considered.
$$+4H^{G(3)}=\int d^3x\cdot(4a_0'a_1'+2a_0a_1'^2a_1^{-1}+(2/3)\pi_1a_0a_1^{-1}\pi_1)\ (+\text{const})\qquad(d^3x=4\pi r^2dr)$$

Here, a_j is a field operator and can be expressed as follows in mode expansion representation.
$a_j=1+\Sigma u_{jm}(r)\cdot z^{(m)*}$; $(j=0,1)$, $z_{(m)}$: positive/negative oscillation mode, $\pi_1=\Sigma z_{(m)}u_1{}^{m*}(r)$

Furthermore, let us now apply the single mode excitation approximation. If a calculation arises no discrepancy assuming single mode excitation, it is not an approximation but an exact one, that may occur for system with high symmetry. (i.e. The space of excited states generated by single mode can be an invariant subspace of the Hamiltonian.) Calculation can be done as follows.
$a_j=1+u_j(r)\cdot z^*+u_j(r)^*\cdot z$; $(j=0,1)$, $z\equiv e_M(t)$: Excitation mode (1 mode)
$\pi_1=i\omega(-u^1(r)\cdot z^*+u^1(r)^*\cdot z)$
note : for the state $|f\rangle\equiv|f(z)\rangle$, mode $e'\perp z\ \rightarrow\ e'^*|f\rangle=0$, $\langle f|\,e'=0$.

From $[a_0, a_1]=0$, we get $u_1^*/u_1=u_0^*/u_0$, and by adjusting the phase of (u_j) and (z), we can set $a_j=1+u_j(r)(z+z^*)$, $\pi_1=i\,u^1(r)(z-z^*)$. In this case, $[a_0, \pi_1]\neq 0$, but since a_0 is not a canonical variable, we think this is no problem. ($2u_1u^1=1$ as clarified later.)

Assume that mode (z) is normalized. This means that the coefficient function u(r) is imposed a normalization condition, but as mentioned in Section 2.2, the gravitational field is involved in the inner product calculation of u(r). Remarks: distribution of state : $dn=d^3x\cdot a_0a_1^3$

●Quantum Equation of Stationary Gravitational Field

-1.Overview of the equation

・Equation to be solved

The equation for the quantum gravitational field in Schwarzschild spacetime can be expressed as

$\delta\langle f|H^{G(3)}|f\rangle=0 \quad / \langle f|f\rangle=1$

where, the variational is taken with respect to $u_0(r)$, $u_1(r)$ and the gravitational field state f.

$H^{G(3)}$ is the Hamiltonian for the stationary spherically symmetric gravitational field.

$H^{G(3)}\equiv\int d^3x\cdot(a_0'a_1'+(1/2)a_0a_1'^2a_1^{-1}+(1/6)\pi_1a_0a_1^{-1}\pi_1) \quad (d^3x=4\pi r^2dr\ , \quad '\equiv\partial/\partial r\ )$

$a_j=1+u_j(r)(z+z^*) \quad \pi_1=iu^1(r)(z-z^*) \quad u_1u^1=1/2$

: a, π are field operators and canonical conjugate operators

・Existence of kinetic energy term

If there were no canonical momentum term $(1/6)\pi_1a_0a_1^{-1}\pi_1$ in $H^{G(3)}$, by interpreting a_0, a_1 as expectation values of the field operators and taking the variational with respect to a_0, a_1 instead of $u_0(r)$,$u_1(r)$, the classical solution of the field would be obtained.

Therefore, the quadratic term of canonical momentum $(1/6)\pi_1a_0a_1^{-1}\pi_1$ gives a quantum correction to the classical solution.

This term seems to be interpretable as a kind of zero-point motion/oscillation energy of the field.

If the classical solution provides a highly accurate approximation, then from the classical viewpoint, this zero-point motion term should be considered small. However, in extreme situations such as black holes, it is expected to act to alleviate the singularity.

・Quantum variance effect and non commutativity

Assuming that the commutativity among the components of the Boson field amplitude operators, the terms consist of amplitude operators in Hamiltonian will have common eigenstate and each of them will be able to have a definite value for the eigenstate.

In this case, there is surely no quantum variance effect . However, in case zero-point motion term exists even in the static field as described above, the eigenstate of Hamiltonian will not match to that of the field amplitude operator.

The field exhibited in classical theory is the quantum expectation value of the field operator, and since the classical solution is obtained by replacing the expectation value of the product of operators with the product of expectation values, quantum corrections become inevitable. *A

***A: Quantum variance effect**

In classical theory, the expectation value of the product of operators is replaced by the product of expectation values. For a state $|f\rangle$, the difference is expressed as follows.

$\langle f|AB|f\rangle=\langle f|A|f\rangle\langle f|B|f\rangle+\langle f|ANB|f\rangle \quad ; \quad N=1-|f\rangle\langle f| \ , \quad$ (note. $N^2=N=N^*$)

where $\langle f|ANB|f\rangle=\langle f|AN^*NB|f\rangle$ is the covariance of A and B.

If A=B, it becomes the variance. In the case of a stationary equilibrium state, since the expectation value of the momentum π_1 itself is 0, if the effect of the zero-point fluctuation term is negligible, the approximation means $\langle f|AN^*\cdot NB|f\rangle\fallingdotseq0$.

To evaluate the correction term, the state $|f\rangle$ is needed.

Supplementary:

Effectiveness of the finite mode excitation approximation and necessity of correction

The finite mode excitation approximation for Boson fields is a method for obtaining the quantum solution using the variational principle, under the assumption that it is possible to approximate the state space of the field by the tensor space generated by finite number of states excited oscillation modes.
In particular, the single mode excitation approximation can be understood as the initial stage of the finite mode excitation approximation.
The calculation is done in Schrödinger representation with the operators in time freezing.

Due to the restriction of state space , the field amplitude operators and their canonically conjugate momentum operators are partially truncated, and are constructed only by the creation and annihilation operators of assumed excited modes, while satisfying charge conjugation symmetry and canonical commutation relations. Completeness holds by definition within the assumed mode space.
If the Hamiltonian without mode truncation keeps the assumed excited state space as invariant subspace, the mode excitation approximation holds strictly.

-2.Consideration on solving method for the equations of quantum gravitational field

As can be seen from the derived equations, this becomes a problem of solving integro-differential equations.
By using the Schwarzschild spacetime as an example, let us extract the points to be taken accounts for obtaining the quantum solution of the gravitational field.
First, by applying variational method, the equations to be solved are as follows. Where, δu denotes the variation with respect to u.

$H^{G(3)}|f\rangle = |f\rangle \cdot \lambda \quad (\exists\lambda)$, $\quad \delta_{u0}\langle f|\, H^{G(3)}\, |f\rangle = 0$, $\quad \delta_{u1}\langle f|\, H^{G(3)}\, |f\rangle = 0$

$|f\rangle$ is an analytic function f(z) of z that is regular at the origin.

$H^{G(3)} \equiv \int 4\pi r^2 dr \ \cdot (a_0' a_1' + (1/2) a_0 a_1'^2 a_1^{-1} + (1/6)\pi_1 a_0 a_1^{-1} \pi_1) \quad (\ ' \equiv \partial/\partial r\ )$

$u_j(r)$; $a_j \equiv 1 + u_j(r)(z+\partial_z) \quad (j=0,1) \quad \pi_1 = iu^1(r)(z-\partial_z)$

- If f(z) is given, since inner product calculations with respect to the mode (z) can be performed, the differential equations of $u_0(r)$, $u_1(r)$ will be obtained.
  If the classical solutions of a_0, a_1 (the expectation value with respect to f) have become good approximations, the contribution of the expectation value of the field zero-point motion must be small.

  If $u_0(r)$, $u_1(r)$ are given, an integro-differential equation with respect to f(z) is obtained.
  $H^{G(3)}\, f(z) = f(z) \cdot \lambda$
  However, there is a difficulty caused by the inverse operator a_1^{-1}. The calculation can be somewhat simplified by the follow transformation :
  According to the transformation formula $a_j \equiv 1 + u_j(r)(z+\partial_z) = \exp(-z^2/2)(1+ u_j(r)\partial_z) \cdot \exp(+z^2/2)$, let us define $f_R(z)$ as $f(z) \equiv \exp(-z^2/2) \cdot f_R(z)$.
  $f_R(z)$ is a function of (z) only and not containing (r).

  Let this transformation (similarity transformation) be denoted by $(\#)_R$.
  $a_{jR} \equiv \exp(z^2/2)\, a_j\, \exp(-z^2/2) = 1 + u_j(r)\partial_z$
  $\pi_{1R} \equiv iu^1(2z-\partial_z)$

$H^{G(3)}|f\rangle = \exp(-z^2/2)\cdot(H^{G(3)})_R(f_R(z))$ $(H^{G(3)})_R$ (All operators in $(H^{G(3)})_R$ are transformed.)

For the inverse operator a_{1R}^{-1}, an explicit expression is available:

$$a_{1R}^{-1}(\xi) = \exp(-z/u_1)(c0 + \int dz\cdot\exp(z/u_1)\,\xi)$$

From the above, it can be seen that the equation $H^{G(3)}|f\rangle = f\cdot\lambda$ becomes a integro-differential equation with respect to f(z). The solution method seems difficult except for successive approximation. f(z) must satisfy the bounded norm condition. This may lead to a quantization condition for the energy eigenvalue λ.

・**Bounded norm condition**: $f(z)^*f(z) < o(1)\cdot\exp(z^*z)$

f(z) is expressed as $f = \exp(-z^2/2)\, f_R(z)$, and must be normalizable such that $\langle f|f\rangle = 1$.
For the inner product of Boson excited state is given by the 2D integral on the complex plane, as shown in Section 2.1/Note B, and the convergence factor is exp(-z*z).
Therefore, it is required the increase of f(z)*f(z) be suppressed by exp(-z*z).

$$f^*\exp(-z^*z)f = \exp(-z^{*2}/2 - z^2/2 - z^*z))f_R^*f_R = \exp(-2\rho^2\cos^2(\theta))\cdot(f_R^*f_R) \quad ; \quad z = \rho\exp(i\theta)$$

It is necessary that $(f_R^*f_R)$ is slowly increasing to make the above integral convergent.
(f_R is an analytic function other than constant, so it cannot be bounded.)

-3.Conclusion on quantum solution of the static gravitational field

As an example of finding the quantum solution of the gravitational field equation, we considered the Schwarzschild space-time that is a spherically symmetric stationary field with the mass concentrated at the origin.
Let us summarize the insights obtained from this example.

【Conclusion】

- Even in a stationary field, there may be a zero-point motion term. This seems to be a general situation. The existence of this term forms a correction term to the classical solution. Furthermore, by causing the eigenstate of Hamiltonian to deviate from the definite state of the amplitude operator, a quantum variance effect is induced.

- Regarding the derivation of the equation, it can be derived using the variational method based on the state-constructive field theory.
  This method gives integro-differential equations, and the development of a systematic and general solution method is expected.
  The classical solutions are useful as the initial value for a successive approximation solution method.

- By applying the state-constructive field theory, the quantum solution of gravitational field can be obtained while avoiding divergence difficulties.
  There is a possibility that the energy eigenvalue of the field is quantized due to the condition of norm existence for the excited states.

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3. Cosmological speculation

3.1.speculation on dark energy／dark matter

Observations have revealed that the universe is filled with dark energy, dark matter / missing mass. However, the true nature of dark energy and dark matter has not yet been established.
The word "dark" suggests that it has nothing to do with electro-magnetic interaction. The difference in energy-matter suggests a difference in the character of Boson-Fermion in the constituent particles of the field. And all of them spread throughout the universe, suggests that they have been very common objects since the creation of the universe.

When associating Boson with dark energy, it is taken account that Boson consist of FF type (spinor connection field (h) and pre-color interaction) and BB type (pre-flavor interaction).
Concerning the fields not confined into the particles, the following are considered.
the electromagnetic interaction component in the pre-flavor interaction,
the spinor connection field (h),
the whiteness composite of pre-color Bosons.
The latter 2 of them seem to be promising candidates of dark energy.
(This is the Boson composite particle mentioned when imagined that if there is the 4th generation of quark/lepton, it will emit whiteness composite pre-color Bosons and decay into the 1st generation. The possibility of such a thing existing should be confirmed by equation.)

It is important nature that the h-field operator will have constant component , i.e. forming a kind of condensation.
Considering the interaction of the h-field with matter (Fermion), the non-uniformity of condensation may be induced due to the correlation with matter. Because the constant component is decided by the minimal action principle. (The Lagrangian of unified field is shown in vol. I.)
The variational coefficient of interaction Lagrangian with respect to h-field, can be interpreted as the interaction term generating h-field due to coupling as current.

If the universe is filled with particles of composite pre-color Boson with whiteness, then preon will interact with this particle to exchange pre-color Boson. To the outside observer, this should appear as if preon is interacting with the vacuum. However, the frequency of Boson exchange depends on the particle density and the effective coupling constant.

If the dark matters associated with Fermions, the existence of preon mesons is conceivable, since preon is nothing but Fermion. TT*, TV*±VT*, V*V, etc. will be considered **<u>*A</u>**
The pre-color states of particles should be in resonance.
As the pre-color interaction su(3) and pre-flavor interaction u(2) will be possible. Since T and V are not symmetrical in original mass, (interaction with the h field) and in EM charge(interaction with electromagnetic field) , the exchange ratio of the pre-flavor degree of freedom might be relatively low.

Since preon T has an EM-charge of 1/3, preon meson containing T should not be completely free from electromagnetic interaction and should have a dipole character. (That is, the possibility of being visible is not 0)
According to the report saying that V-3 triplet ν＝(VVV) is too light in mass for dark matter, the contribution of V-mesons may be too small.

What are some clues to quantitatively consider these speculations?

Originally, the existence of dark matter was concluded by fact that the observed rotational velocity distribution of galaxy does not match the galaxy's mass distribution assumed by optical observations, and that, in order to explain this observational results, it is necessary to assume the dark mass distributed far from the center of the galaxy.
Therefore, it is another notable feature that the mass distribution is not so condensed in the center. This means that the energy dissipation of dark matter is not progressing. As can be seen in the example of the two-body problem, in order for objects that attract each other to condense, the energy of the system must decrease.

For a large number of particles, condensation becomes possible by energy dissipation. for example in fluid model by considering energy dissipation mechanisms such as inelastic collisions between constituent particles or friction model between fluids.
The wide distribution of dark matter within galaxies implies that energy dissipation due to interactions is slow.

A : attribute exchange between X and X

preon＝(fc) ; f＝T,V , c＝r,g,b (preon has attribute on pre-flavor and pre-color)

pre-color exchange : c → c'＋su3.(cc'*) , c* ＋su3.(cc'*) → c'*

pre-flavor exchange : f → f'＋su2.(ff'*) , f* ＋su2.(ff'*) → f'*

If the interaction Boson cannot exist alone, pair annihilation will be less likely to occur if not in dense state.

3.2.Creation of universe - Big Bang

In Section 1.6, from viewpoint of validating the canonical gauge gravity theory, Lagrangian of gravitational field was calculated and modified Friedmann equation was derived .
While considering cosmology based on canonical gauge gravity, especially on Big Bang situation, calculations that include the Lagrangian of fields other than gravity will be required.
The energy-momentum tensor derived by the Lagrangian is neither that from fluid model nor from dominated by electromagnetic radiation model . It also includes a term caused by Lagrangian state expectation value. (This term is often called the cosmological term, but it is incorrect to think that it exists as a law of gravity.)
Furthermore, when we consider the so-called cosmological term (Lagrangian hydrostatic pressure term), the positive or negative curvature of the universe and expansion or contraction, become completely different issues.

Regarding the initial state of the universe, it is not possible to assume that it is uniformly isotropic in a priori, but observations have confirmed that this holds true approximately with a high degree of accuracy. To apply the variational principle on this premise can also be interpreted as analysis using mode decomposition approximation.
The scale factor (a) is an object that should be interpreted as a field operator, and by freezing time, and transferring to Schrödinger representation, and the related wave function can also be introduced. (Section 1.6)

As it is natural in quantum theory interpretation, a state with definite energy is a stationary state, and the states of the universe in all times are superimposed, resulting in a state of completely uncertain time. Of course, the real universe is not like that.
Since the general solution is a superposition of energy eigenstates, the expansion/contraction behavior in the classical mean-field sense depends on the selection of the excitation mode that is, on the settings for initial state.
Let's take a look at these circumstances below.

・Modified Friedmann equation

The symbols used in Section 1.6.3 will be used here.
Let Lagrangian of the field other than gravity be denoted as L^0 and add this to the Lagrangian of gravitational field. At the time of big bang, the fields other than gravity were probably in the phase of preon / interaction Boson pair creation and annihilation.

The state expectation value of the Lagrangian L^0 produces an effect similar to the so-called cosmological term, and its value is expected to vary greatly depending on the Fermion and Boson component ratio of the state. (Regarding the phase transition of expanding universe, will be extended the wings of imagination in the next section "Expanding universe".)

In any case, considering L^0, the Lagrangian density is modified from the formula of section.1.6.1 as follows.

$$\mathcal{L}\equiv L^C\cdot\det(T)\rightarrow\mathcal{L}\equiv(L^C+L^0)\cdot\det(T)\quad(\text{ref.sec.1.6.3.})$$

$$\det(T)=(a\kappa)^3\quad;\quad\kappa^3\rho^2d\rho=\kappa^2\rho^2d\alpha=4\cos^2(\alpha/2)\sin^2(\alpha/2)d\alpha=\sin^2(\alpha)d\alpha$$

Lagrangian of the field other than gravitational field becomes as follows :

$$\int L^0(a\kappa)^3\rho^2d\rho d\Omega=a^3\int L^0\sin^2(\alpha)d\alpha d\Omega\equiv a^3N(a)$$

Assuming that the three-dimensional space is S^3, the spatial integral runs $\alpha \in [0,\pi]$.

Lagrangian L_a is $L_a = \int \mathcal{L} d^3x = 3\pi^2(a(\partial_0 a)^2 - a) + a^3 N$, but N is not the cosmological constant as commonly said but the state expectation value of Lagrangian (the spatial integral of scalar Lagrangian L^0), which is generally contains the gravitational field variables in the form of following:

- For the Fermion field operator χ, form of $g^{\mu\nu}\partial_\mu \chi^* \partial_\nu \chi$.
- About the Boson field operator B, form of $g^{\mu\alpha} g^{\nu\beta} E_{\mu\nu} E_{\alpha\beta}$; $E_{\alpha\beta} = \partial_\alpha B_\beta - \partial_\beta B_\alpha + i[B_\alpha, B_\beta]$
- Strictly speaking, in the form of differential coupling such as $\partial\gamma_\mu = \gamma_A \partial T^A{}_\mu$ $\gamma_\mu \equiv \gamma_A T^A{}_\mu$ via γ matrices.

Here, the metric g has the following relationship with scale factor a, under the assumption of uniform and isotropic symmetry.

$T = \mathrm{diag}(1, a\kappa, a\kappa, a\kappa)$, $\tan(\alpha/2) = \rho/2$, $\sin(\alpha) = \kappa\rho$

$L_a = \int \mathcal{L} d^3x = 3\pi^2(a(\partial_0 a)^2 - a) + a^3 N$, $a^3 N = \mu a^3 N'$ (μ will be decided conveniently later)

Assuming that differential coupling is negligible, the canonical conjugate variable of a is :

$b \equiv 6\pi^2 a \cdot (\partial_0 a)$, $[a, b] = i$

Hamiltonian $H_a = b(\partial_0 a) - L_a = 3\pi^2(a(\partial_0 a)^2 + a) = (12\pi^2)^{-1}(b a^{-1} b + 36\pi^4 a) - \mu a^3 N'$

Consider the scale transformation $a = \lambda^{-1} a'$, $b = \lambda b'$ that maintain the canonical commutation relation, and simplify the Hamiltonian.

$H_a = (12\pi^2)^{-1}(\lambda^3 b' a'^{-1} b' + 36\pi^4 \lambda^{-1} a') - \mu a^3 N' = (12\pi^2)^{-1}\lambda^3(b' a'^{-1} b' + 36\pi^4 \lambda^{-4} a') - \mu\lambda^{-3} a'^3 N'$

$36\pi^4 \lambda^{-4} = 1$, $\mu\lambda^{-3} = (3/2)^{1/2} \cdot \pi$, i.e. $\lambda = (6\pi^2)^{1/2}$.

if set $\mu = 18\pi^4$, then following are obtained.

$H_a = (3/2)^{1/2} \cdot \pi \cdot H_a'$; $H_a' \equiv b' a'^{-1} b' + a' - a'^3 N'$

$a' = a \cdot (6\pi^2)^{1/2}$, $b' = b \cdot (6\pi^2)^{-1/2} = a \cdot (\partial_0 a) \cdot (6\pi^2)^{1/2} = a' \cdot (\partial_0 a') \cdot (6\pi^2)^{-1/2}$

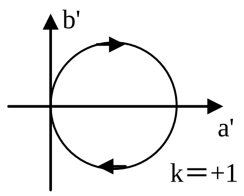
By setting Hamiltonian = constant, the modified Friedmann equation including fields other than gravitational field is obtained as follows.

$H_a' \equiv b' a'^{-1} b' + a' - a'^3 N' = E$ (curvature sign $k = +1$)

By spherical / hyperboloid transformation to the above

$H_a' \equiv b' a'^{-1} b' - a' - a'^3 N' = E$ (curvature sign $k = -1$)

When drawing a trajectory on phase space by classical theory, it will be as follows.



Trajectory: $b'^2 + (a' - E/2)^2 - a'^4 N' = (E/2)^2$, (in classic theory)
Range of motion: $b'^2 = aE - a'^2 + a'^4 N' \geqq 0$.
When $N = 0$, it is a circle : center (E/2, 0) , radius (E/2) (left fig).
The origin is the Big Bang. The canonical conjugate variables a', b' cannot become 0 simultaneously. Also, $a' \geqq 0$. ***A**

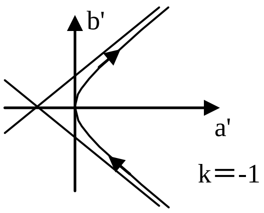
Fig.3.2-1. phase diagram of universe expansion

***A** : See Section 1.6.3.

Although the argument is in classical theory, the range of motion is $b'^2 \geqq 0$, and then, even if the spatial curvature is positive $k = +1$, the expansion will continue on the range of $b'^2 = aE - a'^2 + a'^4 N'(a) \geqq 0$.

Canonical equation:

$da'/dt' = 2b'a'^{-1}$; $db'/dt' = (b'a'^{-1})^2 - 1 + 3a'^2N' + a'^3\partial_a N'$, $t' \equiv t \times (3/2)^{1/2} \cdot \pi$



Trajectory: $b'^2 - (a'+E/2)^2 - a'^4N = (E/2)^2$, (in classic theory)
Range of motion: $b'^2 = a'E + a'^2 + a'^4N' \geqq 0$.
When N'=0, it is a hyperbola : (left fig).
The origin is the Big Bang. The canonical conjugate variables a', b' cannot become 0 simultaneously. Also, $a' \geqq 0$. :***A**

Fig.3.2-2. phase diagram of universe expansion

***A** : See Section 1.6.3. .

Although the argument is in classical theory, the range of motion is $b'^2 \geqq 0$, and then, even if the spatial curvature is negative k=-1, the motion will be confined in the range of $b'^2 = a'E - a'^2 + a'^4N'(a) \geqq 0$.

Canonical equation:

$da'/dt' = 2b'a'^{-1}$; $db'/dt' = (b'a'^{-1})^2 + 1 + 3a'^2N' + a'^3\partial_a N'$, $t' \equiv t \times (3/2)^{1/2} \cdot \pi$

The existence of so-called cosmological term (not constant and not appropriate naming in strict sense) N=N (a) makes the problems of curvature and expansion/contraction completely independent.
Actually, N (a) is not the cosmological constant, but the state expected value of the Lagrangian of the field other than gravity, and that value should change significantly due to the phase change in the universe. $(N' \equiv (18\pi^4)^{-1} N)$

Introducing quantum theoretical viewpoint, Schrödinger equation concerning the scale factor (a) can be given by time-freezing of the field operators. The wave function is the expansion coefficient with respect to eigenstate, and the eigenstate itself is considered to be a multiple superposed state of gravitons.

$+i\partial f/\partial t' = H_a' f$; $H_a' \equiv b'a'^{-1}b' + k \cdot a' - a'^3N'$, $b' = -i\partial/\partial a'$,

N' replaced with quantum mechanical expectation value.

In equation solving, the variable separation method about time is of course effective, and the energy eigenstate gives a stationary universe.
Generally,the state of superposition with several energy eigenstates will cause undulation.
Therefore, it is possible for (a') to cause undulation depending on the initial conditions.
For the quantum Freiman universe, we have already considered it in Section 1.6.5.

・Solving the singularity problem

Already discussed in Section.1.6.4.
Apart from constant multiple of Hamiltonian for expanding motion is

$H_a' \equiv b'a'^{-1}b' + ka' - a'^3N'$ (k=curvature sign)

Since $a' \geqq 0$ is the domain, the eigenstate of a'=+0 cannot be realized with finite energy.

・materialization of energy

We can consider that materialization is the change from interaction Boson-dominated situation to

preon-dominated state in the chaotic state of pair generation and annihilation at the preon level. ***B**
This transition should be seen as one of the phase transition from the creation of the universe, and could be considered that it corresponds to the hierarchy of matter :

hierarchy of matters: preon> quark, lepton> baryon, lepton> atom

In the large-scale structure of the universe, the presence of voids seems to suggest that materialization suppresses expansion, and that the phase transition of materialization significantly changes the expectation value of the Lagrangian of the field.
Let us reconsider this in the section on the discussion of the expansion mechanism of space in Section 3.3.

--

***B** : preon level ー elementary level in canonical gauge unified field theory.
＋BF : preon (T,V)
＋FF : pre-color Boson＋spinor connection (h)
＋BB : pre-flavor Boson

3.3.Expanding Universe

3.3.1.Mechanism of space expansion

Regarding the expansion of the universe, it seems that the following should be taken into account.

- Lagrangian other than the gravitational field should be included in deriving the equation of the expanding universe.
- The presence of voids in the large-scale structure of the universe.
- Several phase transitions should be possible, in the term from materialization of energy at big bang to the generation of atoms.

The following can be said.

-1.As an energy-momentum tensor term of the cosmic expansion equation, there is a limit to the application of the fluid model. (Concerning fluid model, some consideration will be taken in appendix-2)

-2.The identity of so-called cosmological term (or constant, in sometime) is state-expected value of Lagrangian, and there exists by no means cosmological term/constant as the law of gravity.
It is the standpoint of the canonical gauge unified field theory that field Lagrangian is defined by quadratic norm of canonical gauge curvature form. Also, it is negative to introduce an unfounded scalar field in the theory of cosmic inflation, since it destroys the unity of the theory. The behavior of the expectation value of Lagrangian other than gravity will replace this.

-3.Existence of voids as large-scale structure of the universe - super galaxy clusters are densely packed between voids like bubbles. This fact suggests that Lagrangian's potential energy term dominance causes the rapid expansion of space, and that materialization or Lagrangian's kinetic energy term dominance of matter's one causes the slowing down of the expansion rate.
The above is an explanation when Lagrangian is understood in the form of L＝T-U, but it might be able to explain as the energy partition between Fermion/Boson.

-4.Concerning generation of matters, as following phases and phase transitions might be considered.

Phase-1:
A world in which pair creation / annihilation of preons are repeated, and FF-type Bosons of su(3) pre-color interaction and BB-type Bosons of u(2) pre-flavor interaction are flying freely.
In addition, there FF-type Bosons of spinor connection field (h-field) should be there without condensed.

phase-2:
Preon combining has progressed and materialization at quark/lepton level has been completed. Quarks/leptons are flying freely. Color gluons and weak Bosons as interaction Bosons exist as multi-pole interaction of preon level interaction.

The reason why matter did not disappear by pair annihilation is not because of the symmetry breaking of the interaction, as was conventionally explained, but because the anti-preon was hidden in the quark.
It can be seen from the fact d*＝(TVV).

The total charge of the universe is 0, which suggests that the numbers of T and T* are equal, and that the universe was born from vacuum. Similarly, the number of V and V* in the universe should be equal.
It is imagined that a large amount of preon mesons were generated through materialization of energy.

Quarks/leptons over 3rd generation will emit the whiteness- composite of pre-color Bosons and transition to lower generation state, so they are virtually not generated. The whiteness-composite of pre-color Bosons become free from confinement and can spread in the universe all over.
Regarding the spinor connection field (h-field), Bose-Einstein condensation has occurred as the universe has cooled down and the field was left throughout the universe.

	constituents	1st	2nd	3rd
quark	(TTV)	u	c	t
	(TVV)*	d	s	b
lepton	(TTT)*	e	μ	τ
	(VVV)	ν_e	ν_μ	ν_τ

Phase 3:
Quark combining has progressed and materialization in Lepton+Baryon level has been completed. The strong interaction Boson (color gluon) has been confined in baryon as interacting particle between quarks.
On the other hand, the spinor connection field (h-field) remains there.

Phase 4:
A state in which atoms are formed from leptons＋nucleons and materialization mainly up to H and He, is completed.
The spinor connection field (h-field) has probably been condensed and remains in space. (Bose-Einstein condensation)
That is a candidate , along with the whiteness-composite of pre-color Boson, for the dark energy that is believed to cause accelerated expansion of the current universe.

Now let us consider what the equation of cosmic expansion should be like.
In section 1.6, Lagrangian of the field other than gravitational field (the field as canonical gauge 0-order term), was omitted.
For the correction of this, let the scalar Lagrangian of the field be denoted by $L^{(0)}$, and then, the action $A^{(0)}$ generated by $L^{(0)}$, becomes $A^{(0)} \equiv \int L^{(0)} \cdot (-g)^{1/2} d^4x$:

$$\delta A^{(0)} = \int (\delta L^{(0)}/\delta g_{\mu\nu} + L^{(0)} g^{\mu\nu}) \cdot \delta(g_{\mu\nu}/2)(-g)^{1/2} d^4x$$

The existence of the term $(-g)^{1/2}$ is the origin of the so-called "cosmological term", which is actually $L^{(0)} g^{\mu\nu}$. There exists by no means cosmological term/constant as a constant of the law of gravity.
As mentioned above, L(0) can be either positive or negative depending on the distribution of the expected values of the kinetic term and interaction energy term. The value should change significantly due to the phase transition. That is, the control of the cosmic expansion by so-called "cosmological term" is actually the control by state expectation value of the Lagrangian other than gravitational field.

Since $L = T - U$, we should also consider the approximate validity of the Virial theorem.

On the other hand, term $\delta L^{(0)}/\delta g_{\mu\nu}$ has traditionally been expressed in terms of energy density and pressure by applying a fluid model, but this is not always valid depending on the phase of the universe.
The essential problem is how the space-time metric (correctly, the canonical gauge gravitational variable $T^A{}_\mu$) appears in Lagrangian $L^{(0)}$.

For example, in kinematic term of field operator, it is contained like $\partial_\mu\chi^* g^{\mu\nu}\partial_\mu\chi$ as a metric for quadratic form of coordinate differentiation.
Concerning su(3) or u(2) gauge fields, since the terms of them contain coordinate frame indexes, it appears as the metric to define quadratic norm with respect to coordinate frame index.
Strictly speaking, as mentioned in Section 3.2, it is not without derivative coupling of $T^A{}_\mu$..
(Due to gravitational field, the change : $\gamma_\mu \cdot dx^\mu \rightarrow \gamma_A T^A{}_\mu \cdot dx^\mu$ is caused in spinor gauge connection)

Moreover, when a fluid model is applied, the above-mentioned term of $L^{(0)} g^{\mu\nu}$ disappears.
In the light of these circumstances, let us consider the conceptual significance of the fluid model in appendix-2.

Note : The terms Lagrangian , Hamiltonian are somewhat confusing:
It is a common to multiply "field Lagrangian (L)" by volume element factor $(-g)^{1/2}$ to call it "Lagrangian density".
On the other hand, in canonical theory, time is treated specially as a parameter describing state evolution, so it is correct to call $L' \equiv \int L \cdot (-g)^{1/2} d^3x$ as "Lagrangian", that is the integral of L on the 3-dimensional equal time space.
The quantity called "energy momentum tensor" is given as variational coefficient of the action with respect to the half of space-time metric ($g_{\mu\nu}/2$).
For example, for the gravitational field, $\delta(R'(-g)^{1/2})) = -G^{\mu\nu}\delta g_{\mu\nu}.(-g)^{1/2} = +G_{\mu\nu}\delta g^{\mu\nu}.(-g)^{1/2}$
Note the sign of the formula. (note: $\delta x^{-1} = -x^{-1} \cdot \delta x \cdot x^{-1}$. In the above, G is Einstein tensor.)

3.3.2.Acceleration of expansion

In 1998, observations of Type Ia supernovae raised suspicions that the expansion of the universe might be accelerating. This is an example of how the actual measurement brings us a shocking awakening to the belief. From this, it becomes certain to seem that the universe is filled with unknown energy so-called "dark energy".
Let us consider this problem below from the equation of motion related to the scale factor.
Using the symbols used in section 3.2.the discussion of Big Bang, and then the formulation becomes as follows.

Hamiltonian $H_a = (3\pi^2/2)^{1/2} \cdot H_a'$; $H_a' \equiv b'a'^{-1}b' + ka' - a'^3 N'$, (k = sign of curvature)

$a' = a \cdot (6\pi^2)^{1/2}$, $b' = b \cdot (6\pi^2)^{-1/2} = a \cdot (\partial_0 a) \cdot (6\pi^2)^{1/2} = a' \cdot (\partial_0 a') \cdot (6\pi^2)^{-1/2}$

$[a, b] = [a,b] = i$ (canonical commutation relation)

$N' = N'(a)$; state expectation value of Lagrangian $N \times (18\pi^4)^{-1}$, (sec.3.2)

$t' \equiv (3\pi^2/2)^{1/2}\, t$ (note: $H_a dt = H_a' dt'$)

By using a canonical equation, the expression of expansion acceleration $d^2a/dt^2 = [iH_a , da/dt]$ is easily obtained. For the simplicity, let us use scaled variables and then following are obtained:

$da'/dt' = [iH_a', a'] = \partial H_a'/\partial b' = 2a'^{-1}b'$,　$d^2a'/dt'^2 = -2a'^{-2}(2a'^{-1}b')\, b' + 2a'^{-1}\, db'/dt'$,

$db'/dt' = [iH_a', b'] = -\partial H_a'/\partial a' = a'^{-2}b'^2 - k + \partial_{a'}(a'^3N')$

$d^2a'/dt'^2 = -2a'^{-3}b'^2 - 2ka'^{-1} + 2a'^{-1}\partial_{a'}(a'^3N') = -(1/2)a'^{-1}[(da'/dt')^2 + 4k] + 2a'^{-1}\partial_{a'}(a'^3N')$

Taking account of the relationship with observed values for universe expanding rate, let us introduce the expanding velocity da'/dt' with $da/dt \equiv a^\bullet$. Then, $da'/dt' = da/dt \cdot (6\pi^2)^{1/2}(3\pi^2/2)^{-1/2} \equiv 2a^\bullet$ holds.
The following expression will be easy to understand.

$d^2a'/dt'^2 = -2a'^{-1}(a^{\bullet 2} + k) + 2a'^{-1}\partial_{a'}(a'^3N')$　;

note : $a^\bullet = a'^{-1}b' = da/dt$,is gradient of the line passing through origin and (a', b'), in phase space

The 1st term $-2a'^{-1}(a^{\bullet 2}+k)$, in the expression of acceleration d^2a'/dt'^2, is negative in Friedmann solution of $N' \equiv 0$.　It is natural in the positive curvature space $k = +1$, but also even in the case of the negative curvature space $k = -1$, since $a^\bullet$ can be seen as the slope of the line passing through the origin and hyperbolic solution, it follows that $a^\bullet$ gradually decreases from ∞ to 1+0, and the 1st term changes like $-\infty \rightarrow -0$.
(ref : phase space figure in section. 3.2)

After that, the cause of acceleration will be sought in the 2nd term $+2a'^{-1}\partial_{a'}(a'^3N')$.
(where, $N' \equiv N/(18\pi^4)$; N is the state-expectation of Lagrangian of 0th gauge field, that is, the field other than gravity.)
Due to the effect of the factor a^{-1}, if the 1st term decreases the absolute value with expansion, the 2nd term becomes relatively effective.

Roughly considering on deceleration/acceleration of the expansion of the universe, it is possible to say that the matter field is related to the contraction (deceleration of expansion) and the energy field is related to the expansion.
From the standpoint of the canonical gauge unified field theory, the contribution of the spinor connected gauge field (h) , or whiteness-composite of pre-color su(3) Bosons could be candidates of energy field.
On the other hand, the contribution of the "undulations" mentioned in Sections 1.6.5 or 3.2 cannot be immediately excluded.
The sign of the Lagrangian state expectation value is surely important, but perhaps we should recall the Virial theorem regarding the fate of the universe.. (See Section 3.3.1)

●**Supplementary information** :

Meaning and limitations of the fluid model
There seems to be a fundamental defect in using a fluid model for distribution of cosmic energy.
Especially, it should be remarked that the hydrostatic pressure term is inevitably omitted from the law of mass conservation.
Apart from cosmology, let us investigate the concept of fluid model in Apdx-2.

3.4.Black Hole

3.4.1. Consideration on Definition

We have already discussed the interpretation of Black Hole in Section 2.3 using Schwarzschild space-time as an example.
Black Hole is a term that has become very common also in journalism.
Black Hole seems to be a concept that has emerged since the Schwarzschild solution was obtained as a static spherically symmetric solution of Einstein's gravitational equation, and the impossibility of a light escape from it was recognized.
After that, several exact solutions have been known for Einstein equations, but never heard the story of what so-called Black Hole is strictly defined in relation to them.

●Schwarzschild barrier

As is a well-known fact, when analyzing the fall of a free particle in Schwarzschild space-time, the particle reaches Schwarzschild radius within a finite proper time of fallen particle, but not within a finite time of outside. ***A**
Therefore, when thinking on mass condensation, it is doubtful how meaningful to consider the Schwarzschild barrier from the perspective of us living in the outside world.
It can also be considered as follows.
When light is emitted from the bottom of a well of Gravitational field, the light loses energy and become red shifted. The frequency shift of light expresses the time correspondence between the light emitting point and the observing point. If the bottom of the well is on the Schwarzschild barrier, the frequency of light will be zero and the time seen from the outside will stop. That is, when viewed from the outside, time remains still.
From the above, it seems to be possible for black hole to consider as not a real existence but a kind of limit concept in classical theory.

The Schwarzschild solution is often given through the following process.
Obtaining a stationary spherically symmetric solution of Einstein's gravity equation with radius r>0, then treating it as an equation for the entire space, and interpreting it as having a gravitational source mass at the center. From an asymptotic form at a distance, its mass is defined .
Although this is understood as a solution technique, it is inconsistent with the idea of applying the principle of variational to the entire space as the scope of existence of the field.
We have already confirm in Section 2.3 that such a solution is consistent with the solution for the limit situation supposing a mass concentration as $dm=\delta^3(x)d^3x$ at the origin.

***A : Schwarzschild vertical free fall in space-time**

Let's express Schwarzschild space-time as follows.

$$ds^2=g_0dt^2-g_0^{-1}dr^2+r^2d\Omega^2 \quad ; \quad g_0\equiv 1-a/r \quad d\Omega^2=d\theta^2+\sin^2\theta d\varphi^2 、\quad a=\text{Schwarzschild radius}$$

Let us consider the vertical fall of the mass point in this space-time. Therefore, $d\theta=d\varphi=0$
Lagrangian L of mass-point with unit mass is $L=(1/2)g_{\mu\nu}v^\mu v^\nu$; $v^\mu=dx^\mu/ds$. Energy is conserved from the time translation invariance of the system, and $g_0\, dt/ds=E$: const.
∴ $dt=Eds/g_0$, From this, $ds^2(1-E^2/g_0)=-dr^2/g_0$

Considering the still state $dr=0$, then $(1-E^2/g_0)=0$ is obtained. i.e. $E^2=g_0$. So considering the fall from the radius $r=r_0 > a$ position, $E=(1-a/r_0)^{1/2}$ is turns out.

The falling motion is quadrature possible and the variables are separated as follows.

$ds = -dr \cdot (E^2 - g_0)^{-1/2} = -rdr \cdot (r \cdot (rE^2 - rg_0))^{-1/2}$; $(rE^2 - rg_0) = a(1 - r/r_0)$

Integral from $r = r_0$ to $r = a$, and s is finite. On the other hand, for t, from $dt = dsE/g_0$, it is easy to see that $r = a$ results in a logarithmic divergence integral. $dt = -rdr \cdot E(a/r_0 \cdot r(r_0 - r))^{-1/2}(r-a)^{-1}$

When imagining the matter falling into a black hole, in order to fall, the matter must dissipate the energy through interactions such as many-body effects, as can be seen from solutions of 2-body problem other than vertical fall.
In terms of external time, the closer the matter to the Schwarzschild barrier, the slower the energy decay rate, and not reach the Schwarzschild barrier in finite time.
From the energy expression $E = (1 - a/r_0)^{1/2}$, it is impossible for the mass point to exceed $E = +0$ and enter the imaginary region. (internal region $r_0 < a$ with respect to this coordinate representation).

3.4.2.Existence of black holes

Followings are the speculation on the existence and its meaning of black hole.

●Black Hole produced by Gravitational collapse

For massive stars, it is thought that a black hole is formed by gravitational collapse at the end of their life. Even if we cannot directly follow the gravitational collapse using equations of motion, such consideration as follows will be possible:

By considering the classical test particle, and by observing the quasi-static behavior of the test particle corresponding to the change of the pressure tensor of matter field, the collective motion of mass points as a whole will be predicted.

The pressure tensor of the matter field is imagined to act as a deterrent against free fall, and the decay of the pressure will cause condensation of the mass distribution, but this will not lead immediate formation of Schwarzschild spacetime.

●Giant Black Hole in the center of the galaxy

Considering how the giant Black Hole was created, which is said to be in the center of the galaxy, it would have to assume a different mechanism from gravitational collapse.
First, was it already in the universe's creation period, or was the group of stars condensed after the universe was created?
If thinking already it was in the early universe, it is just saying that it was, and it is nothing but a stop thinking. Unless its existence is derived naturally from the mechanism of creation of universe, there is no worth to be talked about.

Concerning the giant black holes, following questions are arisen.

- When considering that the spots of the mass distribution are divided into lumps and eventually become the raw material of the stars, it is unlikely that a lump is be formed with an extremely large mass compared to the average star.
- Energy dissipation mechanism sufficient to the contraction of mass distribution must be identified. As seen from 2-body problem, just because gravitational attractive force works does not mean that

contraction occurs immediately.

That is, as is understood by the fact that semi-major axis is related with energy of mass points system, for orbit contraction, the interaction between the constituent particles must have an energy dissipation mechanism. ***A**

According to the conventional theory, after the creation of universe, matters are distributed in space with the materialization of energy. It is imagined that the distribution will have spots as non-uniformity, and the spots grow due to gravity and become clumps.
The spots should also be formed when materializing of energy, and this will be a matter of large-scale structure.
We can also imagine that there is a kind of spectrum in the distribution of spots, a scale structure that resembles a small wave on top of a large wave.

It is imagined that the mass distribution is divided into clusters based on the relationship between the initial distribution, gravitational interactions due to dark energy and dark matter, and initial density.
However, a method to systematically handle this type of problem seems to have not yet been established.

Conceivable theoretical goals will be as follows.

- On spots/non-uniformity in mass distribution, to find the appropriate expression and to define the magnitude of it.
  The expression may become similar to the spatial power spectrum of mass distribution.
- To establish the equations on the time development of the spots/non-uniformity, taking into account of the type of interaction, energy dissipation mechanism, space expansion, and initial conditions. The fluid model might be a reference.

***A :**

In general, in case of many-body problems, the interaction will cause the orbital angular momentums of constituents to converge around the mean value. The eccentricity will also approach zero. This means the existence of interactions that exchange orbital angular momentums and kinetic energies between the constituent particles. The most explicit interaction is "collision", but in many-body problems, interactions can occur with viscous effects.
For example, the fact that the galaxy is disk-shaped and the orbital angular momentums of the constituent stars are distributed around its mean value means that the interaction of orbital angular momentum exchange between the constituent stars has occurred in the galaxy size.
Therefore, considering something like a gas model, it may be possible to contract at the galaxy level with energy dissipation due to "viscous effect".
about initial distribution density, interaction cross section, and energy dissipation that are sufficient to explain mass contraction, there is no quantitative outlook now.

3.4.3. Speculations on Real Black Hole

Speculations on some themes related to actual black holes.

・On selection of coordinate system

It is noteworthy that the meaning of the divisions such as inside/outside of the Schwarzschild barrier

differs depending on the adopted coordinate system.
We have already seen such a case in Section 2.3 (2).
From the viewpoint of unified field theory, isotropic coordinate systems are directly related to the Lorentz frame of quantum space-time, so they are a particular choice of coordinate systems.

From a quantum theoretical view, it is not permissible to violate the requirement of monovalent continuity regarding the space-time coordinates of the space-time metric that expresses the gravitational potential.

The position that Schwarzschild barrier is real existence as a topological defect in space-time, is unacceptable, since it destroys the conventional basic understanding of physical fields.
If it is true, the basic structure of space-time operators and canonical commutative relations are all overturned.
A quantum space-time with a complex topological structure is not inconceivable, but the coordinates of the space-time should be related to the motion. ***A**

***A:**

For example, the perspective of viewing spacetime as a manifold is related to the translational groups.　First of all, the such state there is, that the point exists there, and next, the position coordinates of the point is replaced with the translation that maps the initial point to there.
Similarly, in the case of rotation, the object of motion is a rigid body at this time, and the motion is expressed as an element of SO(3), which transfers the initial attitude to the current state.
In other words, the position coordinates are the variable on SO(3), and the canonical conjugate variable is the angular momentum. Therefore, the canonical commutation relation also changes.
Ordinary Poisson brackets cannot be applied. It is not represented by a symplectic matrix.

・**Permanent fall**

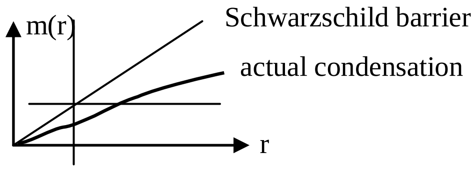
In the Schwarzschild space-time, free fall takes infinite time when viewed from the outside.

A real Black Hole is a state of permanent fall/condensation of the masses when viewed in terms of time outside of mass, and the Black Hole concept is a kind of limit concept.

Based on the idea that black holes are formed by condensing mass, it should be assumed that in the observed mass distribution, the radius of the Schwarzschild barrier corresponding to mass m(r) within radius r is smaller than r. In this sense, the Schwarzschild barrier does not exist with reality.

From an energetic viewpoint, at least as long as the radius of the mass distribution is larger than the Schwarzschild radius due to internal mass, and also if energy dissipation mechanism act effectively, then the mass distribution will condense.
This can be interpreted as equivalent to a permanent fall.

radius r with isotropic coordinate
r(m)＞0.5Gm



If the Schwarzschild barrier is unrealistic, then the evaporation of BH as discussed by S. Hawking would lose its inferential basis.

- **Zero-point motion of fields**

From the discussion in Section 2.3.(3), from a quantum theory point of view, zero-point motion energy exists even in a static gravitational field, which alleviates the blackhole singularity. It also induces quantum dispersion effects.

- **Quantum variance effect**

The classical field is a quantum theoretical mean field, and there is a quantum dispersion contribution to nonlinear terms.

This is a correction for the process replacing the expected value of the product of operators with the product of expected values.

Quantitative evaluation requires solutions to quantum states. ref: Note in section 2.3 (3).

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Apdx-1. Observational verification of canonical gauge gravitational theory

Currently, the verification materials are almost limited to the case of weak gravitational field in classical theory. Though the theory of Black Hole has been talked about, it has not been used as material for quantitative theory verification. While A.Einstein's theory, some exact solutions are found and are theoretically interesting, appropriate approximations would be effective for testing the theory.

Below, we would like consider, as the well known 2 verifications, on the bending angle of light and the Mercury's perihelion shift as verification materials.
In the order of consideration, it is first necessary to clarify the parameters that govern these phenomena.
Next, we must approximately solve the gravitational field equation and extract such parameters.
The theories of I.Newton and A.Einstein should be referred to for comparison.

●Bending angle of light rays

Apply the concept of light rays according to geometrical optics, which will follow the geodesic equation.
In the case of light, since the line element ds of the world line is null, we consider that the rest mass $m\to 0$, momentum $v\equiv m\cdot dx/ds$ is finite, $gvv = m^2 \to +0$.
Introduce the free parameter σ and assume $ds/m = d\sigma$.
The problem of ray bending in a spherically symmetric gravitational field can be easily determined by the successive approximation method. Set the coordinates where the space-time metric is as follows.

$$ds^2 = a_0^2 dt^2 - a_1^2 (dx^\lambda dx^\lambda) \ ; \ \lambda=1..3 \ , \ g_{\mu\nu} = \eta_{\mu\nu} a_\mu^2 \ .$$

First, it can be limited to in-plane motion of $x^3=0$. From the successive approximation method we obtain $v^\lambda = -\int \Gamma^\lambda{}_{\mu\nu} v^\mu v^\mu d\sigma$.

The 0th-order approximate solution is $v^1=1, v^2=0, v^3=0, v^0=1, \quad x^1=\sigma, x^2=b$
(b : impact parameter. For simplicity, adjust the scale so that $v^1=1$.)

The 1st-order approximation solution is given by substituting the right-hand side to the no-bent solution as a 0th-order approximation.
To calculate the bending angle, it is sufficient to consider only v^2 component.
The 1st-order approximate solution : $v^2 = -\eta^{22}\int \Gamma_{2\mu\nu} v^\mu v^\mu d\sigma$. (effective only $\mu,\nu=0,1$).

$$\Gamma_{\lambda\mu\nu} = -\eta_{\mu\nu} a_\nu^2 n^\lambda \alpha_\nu' + \eta_{\nu\lambda} a_\lambda^2 n^\mu \alpha_\lambda' + \eta_{\lambda\mu} a_\mu^2 n^\nu \alpha_\mu' \quad ; \quad ' \equiv d/dr \ , \quad n^\lambda \equiv (1-\delta^\lambda{}_0)\cdot x^\lambda/r \ , \ \alpha \equiv \ln(a)$$

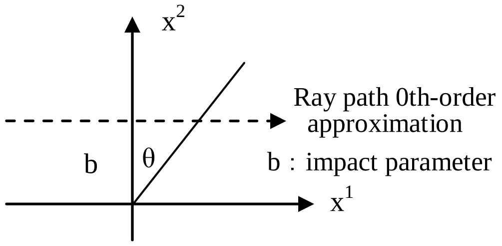
$\Gamma_{\lambda\mu\nu}$ is expressed as above, but only Γ_{200}, Γ_{211} left for $\lambda=2$, $\mu,\nu=0,1$.
The lowest-order approximation calculated with $a \fallingdotseq 1$ in mind are as follows:

$$\Gamma_{200} = -n^2\alpha_0' \ , \quad \Gamma_{211} = +n^2\alpha_1' \quad , \quad v^2 = -\int n^2(\alpha_0' - \alpha_1') d\sigma \qquad dx^1 = d\sigma$$

$(\alpha_0 - \alpha_1)$ contributes to the bending angle. There is no contribution of α_1 in Newton's theory.
In Einstein's theory, $\alpha_1 = -\alpha_0$, which gives 2 times bending angle of Newton's theory, which is consistent with the actual observation.
Therefore, it is necessary that $\beta_1 \equiv \alpha_0 + \alpha_1 = 0$, at least in the approximation of weak field.

$$x^1 = b\cdot\tan(\theta) \quad n^2 = \cos(\theta) \quad r = b/\cos(\theta) \quad v^2 = -b\int(\alpha_0' - \alpha_1') d\theta/\cos(\theta)$$

Let us perform the above integral.
First, from the correspondence with Newton theory, $\alpha_0 = -Gm/r$ must be satisfied.
From $\alpha_0' = Gm\cdot r^{-2}$, $\alpha_1 = k\cdot\alpha_0$, $v^2 = -(1-k)Gmb^{-1}\int d\theta\cdot\cos(\theta) = -2(1-k)Gmb^{-1}$ is obtained.
As mentioned above, the observation requires $\beta_1 \equiv \alpha_0 + \alpha_1 = 0$ (within the observation accuracy).



In the light-path bending problem, lowest-order approximation by the successive approximation method can be applied, assuming the optical path is a null geodesic.
At the start, the 0th-order approximate solution, is a straight line.

According to the calculation formula (ref) in section.1.5.2, the solution of weak gravitational field neglecting higher order terms is:

$-(4\pi G)p_0=((3+\Lambda)\beta_0{}^{\bullet}-2(1+\Lambda)\beta_1{}^{\bullet})$,

$-(4\pi G)p_1=(-2(1+\Lambda)\beta_0{}^{\bullet}+2(1+2\Lambda)\beta_1{}^{\bullet})$; $\bullet\equiv\partial/\partial u$, $u\equiv r^{-1}$

$p_0=\theta(u_0-u)\cdot m/(4\pi)$, $p_1=0$ (2nd-order infinitesimal)

To obtain $\beta_1\equiv\alpha_0+\alpha_1=0$, $1+\Lambda=0$ is required for the interaction constant ratio Λ.
Already known, when selecting interaction constant ratio Λ in canonical gauge unified field theory as $1+\Lambda=0$, then, at least in the scope of spherically symmetric static gravitational field, the coincidence (in classical sense) of the theory with A. Einstein's is derived. (Section.1.5.1)

Since A.Einstein's theory seems to have been verified for weak spherically symmetric fields, so it can be said that this is also a verification of canonical gauge gravitational theory.
In the following, for the sake of simplicity, let us limit $\Lambda=-1$.

Regarding κ, the normalization constant for dimensional adjustment, following are derived from the above with the range $u<u_0$.

$d\beta_0/du=\kappa p_0/2=\kappa m/(8\pi)$; by Newto's theoy $\beta_0\equiv\alpha_0=-Gmu$ $\therefore$ $\kappa=-8\pi G$

When considering nondimensionalization , $4\pi G=1$ should be taken because it must be disappeared for the coefficient which appears in Poisson equation in Newtonian gravitational theory ($\triangle\alpha_0=\sigma$).. The Plank unit system should be formed by adding $4\pi G=1$ to the natural unit system.

The canonical gauge connection field (canonical gauge 0th-order term) can be redefined as $\phi\rightarrow k\phi$, but for gravitational potential $\alpha=\ln(T)$ it is not. It shall be noted that the scale transformation for α is induced not by a constant multiple but by a constant difference.

- To making dimensionless of Planck unit system should be $4\pi G=1$.
- Lagrangian $L^G=-L^C/2$ of canonical gauge gravitational theory, including normalization factor (see. Sec.1.5.3) becomes as follows in dimensionless notation. ($\Lambda=-1$)

  $L^C=-(1/4)(F_{ABC}F^{ABC}/2!-F^A{}_{AC}F^B{}_B{}^C)$; $F^A{}_{BC}\equiv(\partial_\mu T^A{}_\nu-\partial_\nu T^A{}_\mu)S^\mu{}_B S^\nu{}_C$, $T^A{}_\nu S^\nu{}_C=\delta^A{}_C$

  This is Lorentz Frame representation. Index conversion between top and bottom are based on Lorentz metric.

●Mercury's perihelion shift

As an observational verification material for A. Einstein's theory of gravity, there is the Mercury's perihelion shift. The canonical gauge gravitational theory is consistent with A. Einstein's theory in

case to spherically symmetric fields when setting the interaction constant ratio $\Lambda=-1$.
Therefore, the verification of A.Einstein's theory by Mercury's perihelion shift is also the verification of canonical gauge gravitational theory.

In order to test the gravitational theory by perihelion shift, it is needed to derive an expression for the planet's perihelion shift in a spherically symmetric gravitational field and compare it with actual measurements. The actual verification seems to be quite complicated, because it must calculate the shift as a residual amount after excluding the perturbation effects of other planets.

Though the Mercury's perihelion shift is already well known, we will derive here an expression for perihelion shift and reconfirm the structure of its expression formula.
Let's start by solving the equation of mass point motion in a spherically symmetric central force field.
Considering a mass point of unit mass, and set Lagrangian of its motion be $L=(1/2)g_{\mu\nu}v^{\mu}v^{\nu}$,
$v\equiv dx/ds$, where, ds is a free parameter and put to be the line element after applying variational.
The space-time metric is set as follows.

$$ds^2=a_0^2dt^2-a_1^2(dr^2+r^2d\theta^2+r^2\sin^2\theta d\varphi^2)$$

Because of the invariance concerning time translation and rotation the conservation of energy and angular momentum is derived. The motion can be limited to the planar motion in $\theta=\pi/2$.
From the above、following are obtained.

$$a_0^2(dt/ds)^2-a_1^2(dr/ds)^2-a_1^2r^2(d\varphi/ds)^2=1$$
$$p_0\equiv a_0^2(dt/ds)\ :\ \text{const}\ ,\qquad p_\varphi\equiv a_1^2r^2(d\varphi/ds)\ :\ \text{const}$$

From the above, $d\varphi/ds$ and dr/ds are expressed as functions of r, and the ratio of both is taken to obtain the expression of $d\varphi/dr$.

$$d\varphi/ds=a_1^{-2}r^{-2}\cdot p_\varphi \quad ; \quad a_1(dr/ds)=\pm(a_0^{-2}p_0^2-a_1^{-2}r^{-2}p_\varphi^2-1)^{1/2}$$
$$\therefore\ d\varphi/(a_1dr)=\pm a_1^{-2}r^{-2}\cdot p_\varphi(a_0^{-2}p_0^2-a_1^{-2}r^{-2}p_\varphi^2-1)^{-1/2}$$
$$\text{i.e.}\ \ d\varphi=\pm a_1^{-1}r^{-2}dr\cdot p_\varphi(a_0^{-2}p_0^2-a_1^{-2}r^{-2}p_\varphi^2-1)^{-1/2}$$

Putting the above $p_\varphi r^{-1}=u$ and rearranging it a little, we can obtain the following formula with selecting the sign of the root so that it becomes ＋ with respect to du.

$$d\varphi=du/\sqrt{f(u)} \quad ; \quad f(u)\equiv a_1^2a_0^{-2}p_0^2-u^2-a_1^2$$

In the above, $u\equiv p_\varphi r^{-1}$ reciprocates in the range of $f(u)\ \geqq 0$. The deviation of φ from 2π that advances in the meantime is the perihelion shift value per 1 revolution. In that sense, the calculation of the perihelion shift is equivalent to execute the following, but it would be necessary to consider an appropriate approximate expression for comparison with the observed amount.

$$\delta\equiv\oint du/\sqrt{f(u)}-2\pi \quad ;\ \text{(contour integral on complex plane)}$$

In Newtonian theory, $a_1=1$, $a_0^{-2}=1+2Gm/r=1+2Gmp_\varphi^{-1}\cdot u$ holds. In this case, f(u) is a quadratic polynomial with a quadratic coefficient of -1, then, exactly $\delta=0$. .
Therefore, in order to obtain the perihelion shift in post Newtonian theory, it is necessary for a_0 and a_1 to expand at least to the 2nd order of u. That is, the perihelion shift is a 2nd-order effect that does not appear in the linear approximation theory. As can be seen from the formula of perihelion shift δ expressed by contour integral, the brunch points of $\sqrt{f(u)}$, that are, the zero points of f(u) are important for the calculation.

In Newtonian theory, it is calculated as follows.

$f(u)\equiv a_1{}^2 a_0{}^{-2} p_0{}^2 - u^2 - a_1{}^2 = -(u-u_1)(u-u_2)$

Set $u_C \equiv (u_1+u_2)/2$, and then completing the square $f(u)=(u_2-u_C)^2-(u-u_C)^2$ is obtained.

Completing the square for quadratic polynomial and then introducing trigonometric functions as usual. Let us introduce e such that $u=u_C(1+e\cdot\cos(\nu))$, $u_1,u_2=u_C(1\pm e)$. This e corresponds to the eccentricity of the orbit.

$f(u)=(e\cdot u_C)^2\sin^2(\nu)$ $\therefore$ $d\varphi=du/\sqrt{f(u)}=d\nu$

The relationship between the introduced parameters and the orbit is as follows.
From $p_\varphi{}^{-1}u=r^{-1}$, e=eccentricity, ν=true anomaly. a=Semi-major axis . a is ,according to the definition, $a\equiv p_\varphi(u_1{}^{-1}+u_2{}^{-1})/2=p_\varphi/((1-e^2)u_C)$; $p_\varphi u_C=Gm$.

Next, let us consider the case that f(u) is different from Newtnian's.
Taking account of correction, let f(u) be expressed as follows.

$f(u)=-(u-u_1)(u-u_2)f_1(u)$

The values of the zero-points u_1, u_2 of f(u) can only be obtained approximately, but the zero-points of f(u) are important as mentioned above. Introduce trigonometric functions as in Newtonian theory.
From this we get:

$f(u)=(e\cdot u_C)^2\sin^2(\nu)\cdot f_1(u)$; $u=u_C(1+e\cdot\cos(\nu))$ $\therefore$ $d\varphi=d\nu/\sqrt{f_1(u)}$

From the above, it can be seen that the dominant element of the perihelion shift δ is the deviation of $f_1(u)$ from 1. If $f_1(u)$ is approximated by its 0th-order term, the value of $f_1(u_C)$ is important.
Then, how should it be calculated ?
The Newton-Raphson method can be used for the approximate values of the zeros u_1, u_2, and then, $u_C\equiv(u_1+u_2)/2$ will be obtained.
$f_1(u_C)=f(u_C)\cdot(eu_C)^{-2}$ for $e>0$ is obtained, and the following is obtained from this.

$\delta\fallingdotseq 2\pi(eu_C/\sqrt{f(u_C)}-1)=2\pi(1/\sqrt{f_1(u_C)}-1)$; $f(u)\equiv a_1{}^2 a_0{}^{-2} p_0{}^2 - u^2 - a_1{}^2$

It is a little worried that the above equation is $f(u_C)\to 0$ as $e\to 0$. (To get $f_1(u_C)$ in case of $e\to 0$, we need the value of $f''(u_C)$)

Summary and Remarks

- The canonical gauge gravitational theory with interaction constant ratio $\Lambda=-1$ agrees with A.Einstein's gravitational theory in a spherically symmetric gravitational field. (ref: 1.5.1)

- From the discussion of comparison with the observation of the bending angle of light rays, it is concluded that "$\Lambda=-1$ " and "$\beta_1\equiv\alpha_0+\alpha_1=0$ " is equivalent, where the former Λ is interaction constant ratio in canonical gauge gravitational theory, and the latter is an observational result given by an approximation of weak field.

- The dominant parameter for the planetary perihelion shift is the difference from -1 of the quadratic coefficient of f(u) with respect to u.

  $f(u)\equiv\exp(2\beta_1)a_0{}^{-2}(a_0{}^{-2}p_0{}^2-1)-u^2$; $a_0=a_0(u,\Lambda)$

$a \equiv \exp(\alpha)$, $a_1 = \exp(\beta_1 - \alpha_0) = \exp(\beta_1) a_0^{-1}$

In Newton's theory, $a_1 = 1$, $a_0^{-2} = 1 + 2Gm/r = 1 + 2Gm p_\varphi^{-1} \cdot u$。

- Observational verification of A. Einstein's gravitational theory by planetary perihelion shift also a verification of the canonical gauge unified field theory of $\Lambda = -1$.

Apdx-2. Validity of applying the fluid model in cosmology

-1. Conceptual structure and variational principle of fluid model

When thinking about what a fluid model is, it can be said that it is a method for statistically processing the collective motion of a large number of mass points by imitating the motion of a fluid. Since the motion of an extremely large number of mass points is described using a small number of statistical parameters, the internal motion and relative motion in the mass point system cannot be of course described. These shall be described as thermodynamic quantities such as temperature and pressure.

Therefore, to obtain the equation of motion for a system of mass points, the starting point is to process the sum of the actions of the mass point system statistically. The action must be written in the form of a local action theory.
From the viewpoint of adopting the variational principle as the foundation of mechanics, the approach that starting with a prototype equation for fluid motion and setting formulas of external force term in the equation, according to the situation, lacks unity.
and also, such approach is not the only possible method neither.
Depending on the definition of words and conventional usage, the approach that using a set of equations with mass density, flow velocity, and thermodynamic quantities as field variables as a "fluid model" is not at all self-evident.
Considering that the Lagrangian for mass point motion is a function of position and velocity, it might seem more natural to consider the distribution function of the position and velocity of mass points, and apply the variational principle to the average action related to this.

From the above, the first problem is how to determine the Lagrangian to use as the basis.
For example, if we take $L \equiv (1/2)g_{\mu\nu}v^\mu v^\nu$, this formula already incorporates the interaction with the gravitational field. However, conversely, nothing other than gravity is taken into account.
Then if we deal with a plasma fluid, for instance, it can be said that the interaction with the electromagnetic field is lacking. The term of +evA should be added. In other words, we need to provide the action form in accordance with the external interaction assumed. ***A**

Then, how it should be incorporate that the internal interactions, (or in the language of fields theory, self-interactions), caused by phenomena such as collisions? regarding interaction caused by collisions, that energy dissipation will be thought as the most important and problematic item. ***B**
To incorporate energy dissipation into the equation, we must consider the probability of collision occurrence and excitation of internal motion modes due to collisions, and the provide the expected value of the energy dissipation rate.

If the state distribution variables include not only position but velocity also, then a collision term like the Boltzmann equation should probably be introduced.
If the state distribution variables include only position, then we will assume local equilibrium and estimate the collision frequency from the model variables of thermodynamics. ***C**
The problems regarding internal interactions, especially on the energy dissipation seem to be subtle and difficult issues in constructing the fluid model.

Now, let us consider the applicability of the variational principle. The introduction of the position, or position and velocity (momentum) distribution is used in the form of an integral approximation of the

sum of Lagrangians for the constituent mass points. Therefore, the distribution initially assumed is related to the initial position or initial position and momentum.
The action term involves the transformed values of these initial conditions, i.e.:

$x = \varphi(t, x0)$, $v = \partial_t \varphi(t, x0)$.

When the state distribution is continuously approximated and written as dn(x), dn0(x0)＝dn(x) holds from the invariance of the state number (conservation of the number of constituent mass points、or conservation of mass). Therefore, $\delta dn(x) = 0$.
For example, if the mass form $mds = \sigma(-g)^{1/2} d^4x$, $\delta(mds) = 0$

$\Sigma L(x,v) \to \int L(x,v) dn0(x0) = \int L(x,v) dn(x)$, $\delta dn(x) = 0$

$\delta v = d\delta x/ds \to v\partial(\delta x)$ (i.e. $d/ds \to v\partial_x$ ：differential operator along the average direction）

If distribution considered on (position, velocity) , that is , considered as the collection of the points on phase space, in case without collision, it becomes merely a collection of motion of numerous points in phase space, and there is no direct interaction. The mutual influence can only occur via the external field. (i.e. Excitation of internal degrees of freedom of fluid particles is out of scope of consideration.) For example, the mass point distribution forms a gravitational field, and the mass points move according to gravity.

The distribution function f follows the Liouville equation without a collision term.
When regarding the external field as given, it is simply a superposition of single mass point motions.

$df/dt = (\partial_t + v\partial_x + (F/m)\cdot\partial_p) f = 0$

As for the usefulness of the variational formulation, we can point out the applicability to approximate solution methods. This is because we can use methods such as approximating the action term of interest, or determining the coefficients of the approximate expansion of the solution by the principle of least action.
In general, it is not so fruitful to attempt to solve difficult equations directly. It is thought to be more useful for understanding the essence to first investigate the forms or characteristics of the solution by auxiliary means, and then consider effective approximations.

***A：Supposed targets to be applied of the fluid model:**

The evolution of matter distribution clumps in stars, the formation process of the solar system, the condensation of matter, matter inflow into black holes, and so on.
In considering these phenomena, energy dissipation cannot be ignored.

***B：Energy dissipation**

Initially, while the modeled object was treated as a mass point, it actually has internal degrees of freedom, and the translational energy is converted into internal motion or heat due to collisions. Similarly, for example, part of the relative orbital angular momentum may be converted into rotational angular momentum. This can be considered as one of the phenomena of exciting internal motion modes and exchanging conserved quantities.

Considering the equivalence of mass and energy, energy dissipation appears to be a phenomenon

strictly that violates the mass conservation law of the mass point model. In order to recover the mass conservation law, the variation of the energy must be treated as that of internal energy.
(An example of this can be seen in the next section 2)Moreover, we should consider the possibility that this energy is easily carried away into the external space by radiation.

***C： Local equilibrium assumption**

Physically, this would mean assuming local equilibrium and determining the collision frequency , for example, by treating the velocity distribution as a Boltzmann distribution.
From an information theory perspective, it can be stated as assuming the distribution that maximizes entropy under the condition of satisfying the known data.

-2. Fluid Dynamical Interpretation of Energy-Momentum Tensor

Let us first interpret a single mass point as a fluid with mass concentrated at a point, and confirm the correspondence between the motion of a mass point and the continuum representation of a fluid.
Representing the worldline by $x=x(s)$ with the line parameter s, the mass distribution can be obtained by superposing the 4-dimensional delta function mδ4 weighted by mass m along the worldline.
The action A for the motion of a mass point can be expressed as follows:
(Here, we only consider the gravitational interaction, which is included in g(x).)

$$A=\int ds\cdot(1/2)mg_{\alpha\beta}v^{\alpha}v^{\beta}=\int d^4x\cdot\int ds\cdot m\delta^4(x-x(s))\,(1/2)g_{\alpha\beta}(x)v^{\alpha}v^{\beta}$$

Introducing the scalar mass density $\sigma(x)(-g)^{1/2}$, we find $\sigma(x)(-g)^{1/2}=\int m\delta^4(x-x(s))ds$.

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Next, by superposing a uniform flow of mass points based on a kinetic theory of gas molecules, let us consider the fluid dynamical interpretation of the energy-momentum tensor.
As mentioned earlier, the fluid model is understood as a method for statistically treating the collective motion of a system of mass points.
Recognizing a fluid as a flow of particles, in the relativistic sense, we end up considering a bundle of worldlines.
However, treating the motion of a system of mass points as a bundle of worldlines is difficult, and the bundle of worldlines cannot be controlled. Assigning a proper time to each mass-point individually loses its meaning.

Therefore, based on the idea of "superposing uniform flows," let us find the energy-momentum tensor instead of the action form.
Additionary to say, applying the mass-point model for constituent particles of fluid is nothing but ignoring the internal degrees of freedom of fluid particles, so separate consideration is needed for the interaction between particles due to collisions.

Since the momentum conservation law holds in interparticle collisions, it is possible to take the collisions into account from kinematic view, but since changes occur in the internal motion, the energy-momentum tensor terms will change. It should be noted that energy dissipation and other phenomena need to be treated as thermodynamic properties of the fluid..

Below, we will denote a volume element as dV_α.

$$dV_\alpha \equiv \Sigma (-g)^{1/2} \varepsilon_{\alpha\lambda\mu\nu} \cdot dx^{\lambda\mu\nu}/3!$$

While the definition of the volume element is as above, dV0 is particularly important due to the special nature of time.

dV_α transforms as a covariant vector with respect to the index α.

The mass flow jα and the energy-momentum tensor ταβ corresponding to the mass m and momentum p within the associated volume element have the following forms. (For a uniform flow element, we will denote the quantity by (').)

$m = \Sigma dV_\alpha \cdot j^\alpha$; $j^\alpha \equiv \Sigma \sigma' v'^\alpha$ (v' is the relativistic 4-velocity)

$p^\beta = \Sigma dV_\alpha \cdot \tau^{\alpha\beta}$; $\tau^{\alpha\beta} \equiv \Sigma \sigma' v'^\alpha v'^\beta$ (the sum is taken over the distribution of flow directions)

When considering a coordinate system comoving with the fluid flow, care is needed in defining the rest frame.

First, we need to prioritize the mass flow. Therefore, it is not a system in which the 3-dimensional momentum is zero, but a system in which the mass flow is at rest. jr = 0 (r≠0)

Statistically, $j^\alpha \equiv \Sigma \sigma' v'^\alpha = (\Sigma \sigma) \cdot \underline{v}^\alpha$ defines the statistical average velocity v'. However, due to the directional dispersion of the flow, the length of $\underline{v}'^\alpha$ generally becomes greater than 1.

In other words, $\underline{v}'^\alpha$ cannot be regarded as a 4-velocity.

By adjusting the Lorentz norm to 1 as $\underline{v} = \lambda u$, $(\lambda \geqq 1)$, we can introduce the fluid 4-velocity u.

$\Sigma \sigma (v' - \underline{v}') = 0$; $\underline{v} = \lambda u$, $\lambda \geqq 1$; i.e. $j = \rho u$, $\rho = \lambda \Sigma \sigma' \geqq \Sigma \sigma'$

From the concepts of mean and variance, expressing the energy-momentum tensor τ in terms of v yields:

$\tau^{\alpha\beta} = (\Sigma \sigma') \cdot \underline{v}'^\alpha \underline{v}'^\beta + \Sigma \sigma' \Delta v'^\alpha \Delta v'^\beta = \lambda \rho u^\alpha u^\beta + \Sigma \sigma' \Delta v'^\alpha \Delta v'^\beta$; $\Delta v'^\alpha \equiv v'^\alpha - \underline{v}'^\alpha$

The 1st term is the product of expectations, and the 2nd term is the variance term. While it is common to separate the energy-momentum tensor of a fluid into the motion and pressure terms, we cannot immediately identify the 1st term as the motion term. This is related to the equation of motion for the fluid.

As is well known, the equation of motion for a masspoint can also be obtained by interpreting the mass point as a fluid with concentrated mass and applying the energy-momentum conservation law to it. ***D**

Therefore, it is acceptable to regard the equation of motion for a fluid as being given by the conservation of energy-momentum.

$\tau^{\alpha\beta} = \lambda \rho u^\alpha u^\beta + \Sigma \sigma' \Delta v'^\alpha \Delta v'^\beta$; $(\Delta v'^\alpha \equiv v'^\alpha - \underline{v}'^\alpha)$, $\tau^{\alpha\beta} = \rho u^\alpha u^\beta + P^{\alpha\beta}$;

$P^{\alpha\beta} \equiv (\lambda - 1) \rho u^\alpha u^\beta + \Sigma \sigma' \Delta v'^\alpha \Delta v'^\beta$

In the expression for the energy-momentum tensor, taking into account the mass conservation law $\nabla_\alpha j^\alpha = \nabla_\alpha (\rho u^\alpha) = 0$, the motion term must be the $\rho u^\alpha u^\beta$ part, and the remaining part must be separated as the so-called pressure tensor.

Based on the equation of motion for the fluid, one might be tempted to interpret P as the pressure tensor, but in reality, P contains elements other than the pressure term in the kinetic theory of gases.

$\tau^{\alpha\beta} = \lambda \rho u^\alpha u^\beta + \Sigma \sigma' \Delta v'^\alpha \Delta v'^\beta$; $(\Delta v'^\alpha \equiv v'^\alpha - \underline{v}'^\alpha)$, $\tau^{\alpha\beta} = \rho u^\alpha u^\beta + P^{\alpha\beta}$;

$P^{\alpha\beta} \equiv (\lambda - 1) \rho u^\alpha u^\beta + \Sigma \sigma' \Delta v'^\alpha \Delta v'^\beta$

From now on, let us consider the rest frame, and examine the physical meaning of P.
(Rest frame: $u^\alpha = \delta^\alpha{}_0$, $\Delta v'^\alpha = v'^\alpha - \lambda\delta^\alpha{}_0$)

P^{00} :

The energy p^0 contained in the volume element dV_0 ,is given by $p^0 = dV_0 \cdot \tau^{00} = dV_0(\rho + P^{00})$.
Therefore, P^{00} is interpreted as the internal energy. The increased mass due to the velocity dispersion of the constituent particles is also included in this term.
(If the mass of the constituent particles is set to 1, $j^0\, dV_0$ is simply the number of worldlines passing through dV_0.)

P^{0b} :

The momentum p^b $(b \neq 0)$ contained in the volume element $p^b = dV_0 \cdot \tau^{0b} = dV_0 P^{0b}$.
Therefore, $P^{0\beta}$ should be interpreted as the "internal momentum", but since the fluid is at rest, this interpretation does not clearly reveal its physical meaning.
From $P^{0b} = P^{b0}$, we can write $p^0 = dV_b \tau^{b0} = dV_b P^{b0} + dV_0 P^{00}$, so P^{b0} can be interpreted as the energy flow.
Since the fluid is at rest with no material flow, P^{b0} is interpreted as the heat flow. The internal momentum of a stationary fluid can be interpreted as the heat flow.

P^{ab} :

$P^{ab} = \Sigma\, \sigma' v'^a v'^b$, $(a,b \neq 0)$ is called the pressure tensor. Writing $P^{ab} = -p \cdot \eta^{ab} + P_C{}^{ab}$, and we extract the scalar part p, which is treated as the pressure in the equation of state.
(The minus sign arises from the Lorentz metric $\eta = \mathrm{diag}(1,-1,-1,-1)$.)
The interpretation of p as pressure comes from the kinetic theory of gases, but the original definition of pressure as "the force per unit area acting on the system through the boundary surface dS^b " is not so clear.
If we replace the definition with an ideal measurement, the definition may become more concrete.
However, in reality, various effects need to be considered, so it does not immediately become clear.

In the fluid model, mass conservation is assumed as a premise of the model.
This leads in variational method to $\delta(\sigma(-g)^{1/2}) = 0$, so no hydrostatic pressure term (like the cosmological term) appears in the coefficients of variational with respect to the gravitational field variables $g_{\mu\nu}$.
To make the fluid model realistic, it is necessary to model the phenomenon of interparticle collisions. This is particularly important for elucidating the dissipation mechanism of kinetic energy and the condensation of the mass distribution by that.
As an approach for the above, it will be possible that the introduction of state variables of thermodynamics, and establishing of thermodynamical modeling.

***D :**

Introducing the covariant derivative ∇, we have:

Energy-momentum conservation : $\nabla_\alpha(\sigma' v'^\alpha v'^\beta) = 0$

Mass conservation: $\nabla_\alpha(\sigma' v'^\alpha) = 0$ (note : $\nabla_\alpha \xi^\alpha = (-g)^{-1/2} \partial_\alpha((-g)^{1/2} \xi^\alpha)$)

∴ Equation of motion: $v'^\alpha \nabla_\alpha v'^\beta = 0$ i.e. $m(dv'^\beta/ds + \Gamma^\beta{}_{\alpha\gamma} v'^\gamma v'^\alpha) = 0$

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Ⅳ. Conceptual Trajectory & Afterword(Thanks)

return TOC(total) .

The unified field theory developed in this book represents the culmination of my personal research, and there are also recognitions and concepts that have changed and developed during the writing.
In the following, let me look back the perspective of " theory of unified field based on the canonical gauge principle", focusing on the ideological trajectory.

Conceptual Trajectory

part-I: Principle of Field Unification and existence logic of elementary particles.

First, let us attempt to establish the concept of space-time based on the quantum mechanical principle, departing from the manifold model. Assuming the existence of a pair (x, p) of coordinate variables operator x and canonical conjugate momentum operator p, and we consider that space-time is given by diagonalized representation of the state space by the operator x.
The internal space corresponding to the state where the position coordinates x = a, is given by ker(x-a).
As a group (gauge group) that requires the theory be covariant, we assumed a space-time preserving canonical transformation. In the theory based on (x, p), introduction of canonical transformation is quite natural. Also, if interpreting the transformation as the conversion of world image between 2 observers, common space-time between them will be required.

By considering the infinitesimal transformations of gauge group, the concept of "canonical gauge ring" can be obtained. The canonical gauge ring is also an extended concept of tangent vector space in manifold theory, and at the same time can be interpreted as an extension of the space formed by all connection forms on the state space.
The metric can be induced from the metric of state space, and canonical gauge ring forms a metric linear space, in which the Lorentz frame can be introduced.
The relationship between the Lorentz frame P and the canonical momentum operator p is in the form $P=\{S(x), p\}/2+U(x)$. The metric of tangent vector space in manifold theory is defined by S, where S is related to the gravitational field. The 0-order term U corresponds to the connection coefficient in the manifold theory.

We can consider the existence logic and unified field of elementary particles Based on the above-mentioned space-time interpretation.
According to symmetry of elementary particles, we assume the existence of Fermi frame / Bose frame in state space. This is a category by anti-commutative / commutative symmetry with respect to the tensor product.
Considering the canonical connection (i.e. action of canonical gauge operator) for this frame, it is possible to unify Fermion and Boson.
In this sense, both Fermion and Boson are gauge fields of infinitesimal canonical transformation.
Fermion is a connection component (FB or BF component) from the Fermi frame to the Bose frame(or vice versa), and FB/BF component is in a charge-conjugated relationship with each other.
Similarly, it can be seen that Boson has FF type and BB type.
Fermi frame has spinor degrees of freedom and 3 degrees of freedom as representation space of su(3)

(pre-color degrees of freedom), and Bose frame has 2 degrees of freedom as the representation space of u(2) (pre-flavor degrees of freedom). Fermion is generated as a spinor field, and preon model (rishon model in it) fits well.

unified field : Lorentz frame $P_A=\{S^{\lambda}{}_A, p_{\lambda}\}/2+U_A$			
	0th-order term U		
		FF-canonical gauge field (Fermi frame to Fermi frame)	
			su(3) pre-color Boson
			$R[i\gamma^5]$ spinor connection field
		BB-canonical gauge field (Bose frame to Bose frame)	
			u(2) pre-flavor Boson
		FB/BF-canonical gauge field	
			preon (T,V) FB＝(BF)* ; charge conjugation
	1st order coefficient		
		gravitational field	

The preon model gives a substructure to quark/lepton and understands quark/lepton as a composite system of preons. This model can explain the reason for the parallelism of quark/lepton generations and the limitation of 3 generations.
Also it solves the problem of cosmological particle/antiparticle asymmetry.

On FF-Boson, in addition to the fields corresponding to su(3) degrees of freedom, the new field h(hidden) corresponding to spinor gauge connection emerges with the gauge algebra of $\mathbf{R}[i\gamma5]=o(1,1)$.

Additionally to say in technical view, concerning canonical gauge operator action, reducibility to degrees of freedom and γ matrices involvement in spinor connection have been assumed.
Finally, in order to obtain the unified field equation, we must give the Lagrangian of the fields, and we have arrived at the concept of canonical gauge curvature form.

The concept of the canonical curvature can be defined by using the commutation relation of Lorentz frames on the canonical gauge ring that gives extension of the curvature concept.
Since unified field is given as the coefficient of Lorentz frame, it should be considered as essential to obtain the Lorentz frame considered as "translation", that minimize quadratic norm of canonical curvature.
Thus, Lagrangian, apart from the constant factor, is given by the quadratic norm of canonical curvature.
However, since the metric in tensor space is not uniquely determined by the metric of basic vector space, there remains a degree of freedom that cannot be eliminated by invariant theory alone.
Since Lagrangian is a quadratic form of 1st derivative of field variable with respect to space-time, the field equation obtained from variational method is 2nd-order differential equation with respect to space-time.
It is one of the essential results in Part I that clarifies the relationship between unified field equation and Dirac equation concerning Fermion.
The unified field equation of Fermion and Dirac equation are related through approximate factorization concerning Dirac operator, and it results in the relationship between the Fermion mass and chirality.

Of course, mass is an effective concept and is generated by interaction with surroundings.

However, the interaction with the spinor connection field (gauge field (h)) corresponding to the mass, appears at the zero point of the factor in the approximate initial solution.
This raises the possibility that the spinor connection gauge field is in the state of Bose-Einstein condensation in space. Naturally, this is a potential cause of dark energy.
Since the factors are different for $h \neq 0$, it can be understood that two types of Fermions are defined at initial solution level and should be considered to correspond to two preons T and V.
Since the correspondence between equation and elementary particle can be defined only with approximation, we come to the idea that the infinitesimal oscillation of field corresponds to identification of elementary particles. This is an important idea when thinking about the method of solving the equation.

The part-I finished with calculating unified field Lagrangian and comparing it with the existing Lagrangian other than the gravitational field.
Whereas the Dirac equation was initially conceived as a wave function, equation of Fermion in unified field theory is derived from principle and therefore the equation has higher-order terms
The result is that the Pauli term exists in pre-color interaction and disappears in pre-flavor interaction.
Looking at a little more detail, it might have been possible to investigate the normalization of field variables and to consider the magnitude of the contribution of higher-order terms.

Considering the mathematical foundations in Part I, it seems that theories based on canonical formalism have been developed beyond the framework of manifold theory. Therefore I hope that the tools and concepts will become more sophisticated, and that they will develop into new geometric theories that go beyond manifolds.
Secondly, I hope to have a systematic theory about the extensibility of metrics in linear algebra.
Given a metric linear space E, define the metric on the tensor space $\otimes(E \oplus E^*)$ containing its dual space E^*. The purpose is to clarify what definitions are possible.
Considering the applicability to the physics, E shall contain Fermions and Bosons, that is, the tensor product is a mixture of elements with anti-commutative and commutative symmetry, and the tensor product with the elements of dual space should satisfy canonical commutation relation. Since the coefficient is in general field operator, it cannot be assumed to be commutative, but a predetermined commutation relation can be assumed.
It would be grateful if a systematic theory of metric expansion in such a situation above could be presented.

part-II: Foundation of state-constructive Field Theory

Part-2 is a search for how to solve the equation.
Even if the correct equation to be solved is obtained by unified field theory, the conventional perturbative diagram technique encounters the divergence difficulty. It seems natural to solve equations according to more substantive state representations.
In perturbative diagram techniques, the continuity and approximate relationships with quantum mechanics and classical theory are unclear, that I have been dissatisfied with.
With regard to time, we have to give special attention to time in order to recognize the state, and I have come to think that time is a part of our recognition form.

In this idea, the action of field can be interpreted as action of mass point on a continuous infinite

dimensional space having a continuous index of space.
i.e. $\int d^4x \cdot L^D = \int dt \cdot (\int d^3x L^D)$; $\int d^3x L^D \equiv$ Lagrangian

In the above idea, the field equation (in Heisenberg representation) is also the equation of motion for high-dimensional mass point. This becomes the basis of constructing "field operator".
Generalized, by normal mode expansion, it comes to the recognition that field is equivalent to a mass point with infinite degrees of freedom with mode excitation amplitude as a variable.
From this, the field operator become naturally associated with the creation and annihilation of mode excitation.

The field operator acts on the state space, which becomes the tensor space of the oscillation state represented by mode, which also includes the dual space. It should be noted that the 3-dimensional oscillation mode corresponds to 2 frequencies, positive and negative, depending on the orientation of 3-dimensional space, and the concept of charge conjugation symmetry can be obtained from this.
The following are notable results/concepts in the construction of field operators.

- **Projectivity of field operator**：
Projectivity means the structure of field amplitude operator $\chi = \Sigma\, e_m(x) e^{*m}$. (where, index m includes negative oscillation components)
Property of projection becomes explicit by writing the above as $\chi = \Sigma \langle x | e_m \rangle \langle e^{*m} |$

- **Double duality of field operator**：
Double duality means that negative oscillations are included in the above index m, and if the mutual conversion correspondence is m→-m, then $\mathbf{e}_{-m}(x) = \mathbf{e}^m(x)^*$, that is, it means the fact that antiparticle is included and that the charge conjugation symmetry is satisfied.
The particle/antiparticle relationship is represented by the charge-conjugated transformation, which is nothing but transposing of the matrix representing the canonical gauge connection of the state frame.

- **Interpretation of state as an operator :**
interpretation of state as operator means the fact that the state space can be embedded in the operator space through a vacuum. The ring (differential algebra) $C[z, \partial_z]$, which is the polynomial ring $C[z]$ with a differential operator added, provides an example of this which is easy to understand .
Considering the polynomial $f(z) \in C[z]$ in $C[z, \partial_z]$, it is also an operator $f(z) : a(z) \to f(z)a(z)$, and $f(z) \in C[z, \partial_z]$ can be interpreted as $f(z) \cdot 1 \in C[z]$ by acting on 1 (vacuum). $C[z] \hookrightarrow C[z, \partial_z]$

- **Introducing center of mass system**：
When solving the field equations, the choice of coordinate system is a problem.
Excluding the translational degrees of freedom due to the translational invariance of action is nothing but a kind of gauge fixing. It is natural to introduce a center of mass system, but fixing the translational degrees of freedom is fairly difficult in terms of formulation.
When solving a two-body problem with mass point quantum mechanics, it can be written only with relative coordinate variables in the wave function representation.
Attempting similar things, first, interpretation of relative wave functions becomes a problem.
In many-body problem, there is a difficulty that the representation of relative coordinates cannot be expressed in symmetric form.
At present, we are satisfied with adding the condition that the state of the system $|f\rangle$ makes the total

momentum P be 0 , i.e. P|f＞＝0.

Transformation to Schrödinger representation

The field operator obtained by the variational principle is in Heisenberg representation, which satisfies the following equation of motion.

$\partial_t\chi=[iH,\chi]$; $\chi\equiv$ field (amplitude) operator, $H\equiv$ Hamiltonian

It should be noted that the above is invariant for conversion $H\to H+\text{const}$, which is also an appearance of an arbitrariness of origin of canonical energy.

From this, it can be assumed that $H|0\rangle=0$ for vacuum $|0\rangle$.

The fact that Hamiltonian does not explicitly include the time variable t is important.

If set $U\equiv\exp(i(t-t_0)H)$, the expected value of any operator A in Heisenberg representation is

$\langle f_0|A|f_0\rangle=\langle f_0|UA_0U^*|f_0\rangle=\langle f|A_0|f\rangle$.

In Schrödinger representation , the operator A is time-frozen at time $t=t_0$ as A_0 , and the state moves as $|f\rangle=U^*|f_0\rangle$. It should be noted that $U^*=\exp(-(t-t_0)iH)$, which is inversely related to the motion of the operator. Also the fact that $H=-p_0$: canonical momentum 0 component × (-1) is also note. ($-H=p_0=-i\partial_0 \;\to\; H=+i\partial_0$) It should be remarked that $H|0\rangle=0$ allows the following calculation method.

$H|f\rangle=Hf|0\rangle=[H,f]|0\rangle$ （ $|f\rangle\to$ f ; operatorization）

Approximation method/general theory

Regarding approximation methods, rather than the thought that they are simply as methods for obtaining approximate solutions individually, but following thought is growing.

Approximation methods should be systematized by organizing solution methods, from viewpoint as the general method that giving a successive method to improve accuracy, or giving series expanded expressions of solutions.

It is also important to distinguish the nature of the problem. this is one insight that in quantum theory, problems can be broadly divided into stationary problems and scattering problems.

Considering approximation method from a perturbation theory perspective, following natures are important: generality/ explicit expression of correction terms /successive accuracy improvement. Furthermore, it is preferable to be able to calculate by some methods whose relations are elucidated, just as seen in optics, for example, that the idea and calculation method based on geometric optics/wave optics are both available, according to the object in problem.

Regarding the problem of quantum field theory, it seems necessary to clarify the meaning, the relationship, and the possibility to use the solutions of classical theory and mass-point quantum mechanics.

Approximation method/Steady problem

The stationary problem is to find the eigenstates of Hamiltonian H. Regarding field equations, the degrees of freedom are infinite, and since it is unmanageable as it is, it seemed necessary to introduce the finite element method.

From the idea of finding an optimal solution within a limited number of finite degrees of freedom, the use of variational methods is applied.

Using an idea similar to the variable separation method in partial differential equations, we considered a solution method that limits the state |f＞ to the form of a tensor product for each type of the field. Let's call this the "(type) state separation method."

Assume that we have used the state separation method.
Regarding Fermion fields, if we know the number of the particles, we can express them as a wave function using the projectivity of field operators.
Determining the number of particles corresponds to determining the number of Fermion excitation modes. We can recognize once again that the wave function is nothing but the expansion coefficient (excitation amplitude) of the state concerning the positional eigenstate.

Regarding the Boson field, we considered limiting the number of excitation modes to a finite number. In particular, let us call the approximation in which the number of excitation modes is limited to 1 "single mode excitation approximation." This can be interpreted as the first step in the multimode excitation approximation. The mode excited state of the Boson field is expressed by a multivariable analytic function at the origin, and the equation for this can be obtained from the variational method. In the Boson field, the excitation may be quantized due to the normalizability condition for mode excitation.
This is the same mechanism by which the principal quantum number is obtained when applying the Schrödinger equation to the hydrogen atom.

Applying the above method to the preon model, it will be expected to solve the lepton model of (TTT)＝e+, μ+, τ+.

Approximation method/scattering problem

In scattering problems, it is important to avoid divergence difficulties. First of all, it seems necessary to clarify the picture of interactions between particles.
Logically, Hamiltonian interactions cannot be adiabatically reduced to 0 at the operator level. Whether or not the interaction term can disappear asymptotically depends on the nature of the state. Since the interaction is not limited to colliding particles but includes self-interactions of individual colliding particles, the adiabatic hypothesis does not hold and the free field concept breaks down.

The interaction in a collision is due to a part of the Boson flying around in self-interaction going to interact with the collision partner, and calculating the interaction due to a collision becomes a problem of extracting the difference of the interaction. This is a view close to the original idea of ” renormalization." It is necessary to devise a calculation method that extracts only the contribution of Boson exchange.

However, calculating the steady state with self-interaction in a straightforward manner, probably it will fail. This is because the energy of particles in a steady state of self-interaction diverges, which means that such a state does not exist.
For example, if we consider a steady state in which a quark exists alone, its energy will diverge. Therefore, quark cannot exist on its own. Similarly, if electrons diverge in energy and charge, due to self-interaction, then electrons cannot exist in the current model form, which suggests to introduce a preon model.

So, isn't it possible to calculate the scattering problem without going back to the preon level?
If so, that is also problematic.
Matter has a hierarchical structure of molecules, atoms, electrons, nuclei, etc., and approximations must be devised to enable closed calculations at each hierarchical level.
Until now, interaction between hierarchical particle sets has been dealt with by adding several attributes to mass points, such as dipoles and multi-poles.

It seems that a hierarchical structure should be introduced to the approximation method.
An approximation method that allows calculation in a closed form at each level is expected. On the other hand, there is also a phenomenon in which new particles appear due to particle recombination in the substructure of colliding particles, and a theory of interactions between layers due to interactions between constituent elements will also be needed. A theory that systematically deals with these issues is eagerly awaited.

Remaining themes:

Calculating stationary state of the electron preon model.
Systematization of approximation method

part-III. Canonical Gauge gravitational theory

The unified field theory based on the canonical gauge principle is a unified theory, and of course includes the theory of gravity.
It will be described as canonical gauge gravitational theory in part Ⅲ.

The first topic that comes to mind is the verification of the theory of gravity.
The structure of the Lagrangian is completely different between canonical gauge gravitational theory and A. Einstein's theory of gravity (theory of general relativity). Nevertheless, in canonical gauge gravity, if we set the interaction constant ratio $\Lambda = -1$, it follows that the two coincide for a static spherically symmetric field.
Since A. Einstein's theory has obtained some quantitative verification in spherically symmetric fields, this also serves as a verification of canonical gauge gravity theory in the field of classical theory.
On the other hand, observational verification of cosmological phenomena, in which the difference between the two is conspicuous, is awaited.

Regarding the expanding universe, quantum theoretical considerations can be developed using the Hamiltonian.
We have investigated the wave function related to the scale factor.
Of course, Big Bang's singularity is resolved. However, unlike the classical solution, the behavior of the universe can be anything from a steady universe to a drastic big bang depending on the initial conditions. Guiding principles regarding setting initial conditions are highly needed.

Regarding Black Hole, we have considered the quantum equations of Schwarzschild spacetime. The equations followed the state-constructive field theory formulation which we proposed in Part II , with single mode excitation approximation. As an expression of the gravitational field, integro-differential equations were obtained that are difficult to solve. At this moment, even if the equation is solved, its significance is unknown.
It is recognized that zero-point motions exist even in static fields, and this will induce quantum variance effects. This effect is expected to alleviate the singularity that emerges in classical theory.
When expressing Schwarzschild space-time in isotropic coordinate system, there is no divergence or discontinuity in the gravitational potential $g_{\mu\nu}$. The singularity in a physical sense lies in the fact that on the barrier, the time is still.

Taking account that the unified field theory takes the existence of the Lorentz Frame on canonical gauge ring, as its starting point, the isotropic coordinate system might be given a privileged status than other one. From a quantum theoretical point of view, the monovalent continuous condition to field variables cannot so easily be violated.

There are various interesting themes related to cosmology. They are described as cosmological free speculation. Some of them are introduced below.

- The spinor connection gauge field is in the state of Bose-Einstein condensation, in whole space of universe.

- According to the unified field theory, considering the process of materialization of energy during the creation of the universe, the possible candidates for dark energy are spinor gauge connection fields, and whiteness composite of pre-color interaction Bosons.

A possible candidate for dark matter are preon mesons.

- The speed of the expansion of the universe changes according to the phase transition of the universe. The expansion speed is governed by the difference of kinetic terms and energy term of fields regarding the Lagrangian other than gravity.
  The void in large scale would have been formed by this effect.
  (The part where the materialization of energy was delayed rapidly expanded.)

- The current acceleration of the expansion of the universe can be interpreted as a result that the expansion makes the kinetic energy term reduced and the potential energy term predominant in the Lagrangian other than gravitational field. Alternatively, it seems possible to interpret the solution of the modified Friedmann equation as a waviness effect of the vibration mode.
  There are modified Friedmann equation and the expression of universe expanding acceleration, in the text.

- In the discussion of the expansion of the universe, it is concluded that the validity of applying a fluid model to the energy-momentum tensors, is extremely doubtful. Originally, the variational principle should take priority. In the fluid model, the so-called cosmological term disappears due to the mass conservation law on the constituent particles.

- A black hole should be understood as the limit concept of permanent fall.
  The fall never reaches the event horizon.

- Probably due to the quantum variance effect, there is no Schwarzschild barrier. there is no evaporation of Black Hole also.

- Formation of the giant black hole at the center of the galaxy requires the introduction of an effective energy dissipation mechanism.
  Applying a fluid model seems to be effective for the phenomena like mass condensation, formation of galaxies, formation of solar system, black hole formation, etc., but the introduction of energy dissipation mechanisms require careful attention to reflect reality.

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Afterword (Thanks)

This paper is a personal study that I wrote while exploring something that I have been interested in as my life's work for many years. I am not a professional in this area.
I have always had an interest in physics, but it was only after I retired that I had free time for research.
And now, I feel blessed and grateful to be able to spend time doing something that I am passionate about.

There are several fascinating theories in physics.
It is a canonical theory called the flower of mechanics, a relativity with a formal unified beauty, and a quantum theory that teaches the depth of existence and recognition.

It seemed very attractive to construct a unified field theory. It was the goal was to unify the field by a single principle, to provide correct equations, and to find a logical solution to the "divergence difficulty" that plagues quantum field theory. Also I secretly expected that field unification would lead to a unified understanding of the origin of fermions/bosons and the relationship between mass and chirality.

It would have been correct to start field unification with the quantization of space-time.
As a result, the general coordinate transformation, which plays an important role in A. Einstein's general theory of relativity, has been extended to the space-time preserving canonical transformation of operators.
Noting that the theory of gravity is derived from covariance with respect to general coordinate transformations, this is certainly an advance.

Considering that general relativity was inspired by Riemannian geometry, the conceptual shift to space-time quantization might advance traditional manifold theory and open a window into a new geometry, which can be called quantum manifold theory. I secretly look forward to the efforts and success of mathematicians.

Looking back, it seems that one of the things that deserves special mention is the deepening of the understanding of the Dirac equation.
In unified field theory, the variational principle is applied to the quadratic form Lagrangian, so the resulting equation is originally a 2nd-order partial differential equation. The Dirac equation is obtained by approximately factorizing with respect to the Dirac operator. Therefore, the Dirac equation, which was conventionally obtained from factorization of the Klein-Gordon equation as a wave function, was for the first time obtained as an equation for field operators.
Akin to Galois theory's root substitution for algebraic equations, it seems natural to posit a gauge field substituting the factored Dirac equation solution.

This time, what seemed important beyond physics was the importance of the concept of approximation and the need for systematic techniques for constructing approximate solutions.
First, I realized that the finite element method will be needed to handle the infinite degrees of freedom.
Next, we found that the variational method is effective as a method for obtaining the optimal solution within the limited degrees of freedom for approximation.

This means that field theory problems lead to conventional methods of quantum mechanics and classical theory as approximation methods.
Conversely, it shows that conventional methods of quantum mechanics and classical theory are included

in field theory problems in a continuous manner as approximation methods.

What is important for an approximation method is that it has a successive correction formula to improve accuracy, and that it has a method for determining coefficients in the approximation expansion. In this respect, I think that the variational method is a truly great guiding principle.

Finally, regarding gravitational theory, the consideration of the expanding universe and the concept of black holes this time. Also the solutions to quantum equations related to these were investigated. For the equations, applied the formulation in state-constructive field theory and the integro-differential equations emerged.
It has confirmed that no phenomena threatening the existence of solutions like a divergence difficulty, but, due to the existence condition of the state norm, there is a possibility that the quantization condition is imposed on the Boson field.
The studies on the integro-differential equations of these types will proceed, I hope.
It was recognized that even static fields, there is a like zero-point motion of the field. As a result, the quantum variance effects work, and the singularity in classical solution seems to be alleviated and eliminated, at least concerning the Schwarzschild barrier.
Regarding theory on gravity, it should be originally seen as a cosmic phenomenon and is inseparable from astrophysical observational facts, I realize. However, I have described the cosmology as speculations in Part Ⅲ.

Looking back on this article, I see that there are still many issues to be solved in developing the theory, but on the other hand, I feel that we have come so far.
I would like to thank the environment and circumstances that made this possible, as well as the people around me.
For further development of the theory, I would like to wait for someone who understands the unified canonical gauge field theory and feels the necessity and will for further evolution.

2024.04 T.sato Kawasaki, Jpn .

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