Addendum to “The Feynman-Dyson propagators for neutral particles (locality or non-locality?)” *

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Abstract

We answer several questions of the referees and readers arised after publication of the commented article [1]. Moreover, we see that is impossible to consider correct relativistic quantum mechanics without negative energies, tachyons, and without appropriate forms of discrete symmetries.

Why did we consider four field functions in the propagator for spin-1 in Ref. [1, 2]?[1] Let us make some observations for spin-1/2 and spin 1.

We have 4 solutions in the original Dirac equation for $u-$ and 4 solutions for $v = \gamma^5 u$ (remember we have $p_0 = \pm E_p = \pm\sqrt{\mathbf{p}^2 + m^2}$). See, for example, Ref. [3]. In the $S = 1$ Weinberg equation [4] we have 12 solutions.[2] Apart $p_0 = \pm E_p$ we have tachyonic solutions $p_0 = \pm E'_p = \pm\sqrt{\mathbf{p}^2 - m^2}$, i.e. $m \to im$. This is easily to be checked on using the algebraic equations and solving them with respect to p_0:

$$Det[\gamma^\mu p_\mu \pm m] = 0\,, \tag{1}$$

and

$$Det[\gamma^{\mu\nu} p_\mu p_\nu \pm m^2] = 0\,. \tag{2}$$

In construction of field operator [3] we generally need $u(-p) = u(-p_0, -\mathbf{p}, m)$ which should be transformed to $v(p) = \gamma^5 u(p) = \gamma^5 u(+p_0, +\mathbf{p}, m)$. On the other hand, when we calculate the parity properties we need $\mathbf{p} \to -\mathbf{p}$. The $u(p_0, -\mathbf{p}, m)$ satisfies

$$[\tilde{\gamma}^{\mu\nu} p_\mu p_\nu + m^2] u(p_0, -\mathbf{p}, m) = 0\,. \tag{3}$$

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[1] See also references therein.

[2] In Ref. [5] we have causal solutions only for the S=1 Tucker-Hammer equation.

The $u(-p_0, \mathbf{p}, m)$ "spinor" satisfies:

$$[\tilde{\gamma}^{\mu\nu} p_\mu p_\nu + m^2] u(-p_0, +\mathbf{p}, m) = 0\,, \tag{4}$$

that is the same as above. The tilde signifies $\tilde{\gamma}^{\mu\nu} = \gamma_{00}\gamma^{\mu\nu}\gamma_{00}$ that is analogoues to the $S = 1/2$ case $\tilde{\gamma}^\mu = \gamma_0\gamma^\mu\gamma_0$. The $u(-p_0, -\mathbf{p}, m)$ satisfies:

$$[\gamma^{\mu\nu} p_\mu p_\nu + m^2] u(-p_0, -\mathbf{p}, m) = 0\,. \tag{5}$$

This case is opposite to the spin-1/2 case where the spinor $u(-p_0, \mathbf{p}, m)$ satisfies

$$[\tilde{\gamma}^\mu p_\mu + m] u(-p_0, +\mathbf{p}, m) = 0\,, \tag{6}$$

and $u(p_0, -\mathbf{p}, m)$,

$$[\tilde{\gamma}^\mu p_\mu - m] u(p_0, -\mathbf{p}, m) = 0\,, \tag{7}$$

and

$$[\gamma^\mu p_\mu + m] u(-p_0, -\mathbf{p}, m) = 0\,. \tag{8}$$

In general we can use $u(-p_0, +\mathbf{p}, m)$ or $u(p_0, -\mathbf{p}, m)$ to construct the causal propagator in the spin-1/2 case. However, we need not to use both because a) $u(-p_0, +\mathbf{p}, m)$ satisfies similar equation as $u(+p_0, -\mathbf{p}, m)$ and b) we have the integration over $\mathbf{p}$. This integration is invariant with respect $\mathbf{p} \to -\mathbf{p}$:

$$S_F(x_2, x_1) = \sum_\sigma \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{m}{E_p} \tag{9}$$

$$[\theta(t_2 - t_1)\, a\; u^\sigma(p) \overline{u}^\sigma(p) e^{-i\mathbf{p}\cdot(\mathbf{x_2}-\mathbf{x_1})} + \theta(t_1 - t_2)\, b\; v^\sigma(p) \overline{v}^\sigma(p) e^{+i\mathbf{p}\cdot(\mathbf{x_2}-\mathbf{x_1})}]\,.$$

So the result for the causal propagator

$$S_F(x) = \int \frac{d^4p}{(2\pi)^4} e^{-ip\cdot x} \frac{\hat{p} + m}{p^2 - m^2 + i\epsilon} \tag{10}$$

seems to not change.

The situation is different for spin 1. The tachyonic solutions of the original Weinberg equation

$$[\gamma^{\mu\nu} p_\mu p_\nu + m^2] u(p_0, +\mathbf{p}, m) = 0 \tag{11}$$

are just the solutions of the equation with the opposite square of mass ($m \to im$):

$$[\gamma^{\mu\nu} p_\mu p_\nu - m^2] u(p_0, \mathbf{p}, im) = 0\,. \tag{12}$$

We cannot transform the propagator of the original equation (11) to that of (12) just by the change of the variables **p** as in the spin-1/2 case. The mass square changed the sign, just as in the case of $v-$ "spinors". When we construct the propagator we have to take into account this solution and, possibly, the superposition $u(p,m)$ and $u(p,im)$, and corresponding equations.

The conclusion is paradoxical: in order to construct the causal propagator for the spin 1 we have to take acausal (tachyonic) solutions of homogeneous equations into account. It is not surprising that the propagator is not causal for the Tucker-Hammer equation because the latter does not contain the tachyonic solutions. Probably, this statement is valid for all higher spins.

References

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